Title: Entanglement Entropy in Loop Quantum Gravity

Date: Sep 08, 2007 09:30 AM

URL: http://pirsa.org/07090012

Abstract: The entanglement entropy between quantum fields inside and outside a black hole horizon is a promising candidate for the microscopic origin of black hole entropy. I will explain the motivation behind this interpretation of black hole entropy, and why it requires quantum gravity. I will then apply these ideas to loop quantum gravity and show how to compute the entanglement entropy of spin network states. The result of this calculation agrees asymptotically with results from the isolated horizon framework, and I will give the reason for this agreement. Finally, I will show that the entanglement entropy gives extensive corrections to the area law, suggesting corrections to the gravitational action.

1. Black Hole Entropy and Entanglement
   - Black Hole Entropy
   - Entanglement Entropy
2. Loop Quantum Gravity
   - Link States
   - Wilson Loops
   - Spin Networks
3. Isolated Horizons
4. Outlook

Black Hole Entropy

Observers outside a black hole horizon see thermal Hawking radiation.

A black hole of horizon area A has an entropy of

$$S_{\mathrm{BH}} = \frac{A}{4}\frac{c^3}{\hbar G}$$

Entropy occurs for *cosmological horizons* and *acceleration horizons*. This suggests that entropy is *universal* for *all causal horizons*. [1]

What is the statistical origin of this entropy?

[1] Ted Jacobson, Renaud Parentani. Horizon Entropy. gr-qc/0302099.

Entanglement

Let $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$.

If $|\psi\rangle$ cannot be written as $\left|\psi^A\right\rangle \otimes \left|\psi^B\right\rangle$ then $|\psi\rangle$ is *entangled*.

When ignoring system B, we get a mixed state of system A

$$\rho_A = \mathrm{Tr}_{\mathcal{H}_B} |\psi\rangle\langle\psi|$$

Definition

The *entanglement entropy* of system A is the von Neumann entropy of ρ_A

$$S_E \equiv S(\rho_A) = -\mathrm{Tr}\rho_A \log \rho_A$$

System AB has zero entropy. System A has positive entropy.

The Schmidt Decomposition

Every state $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$ has a *Schmidt decomposition*:

$$|\psi\rangle = \sum_{i \in \mathcal{I}} \sqrt{\lambda_i} \left|\psi_i^A\right\rangle \otimes \left|\psi_i^B\right\rangle$$

Where

- $\left\{\left|\psi_i^A\right\rangle\right\}$ is an orthonormal set in $\mathcal{H}_A$.
- $\left\{\left|\psi_i^B\right\rangle\right\}$ is an orthonormal set in $\mathcal{H}_B$.
- $\lambda_i > 0$ and $\sum_{i \in \mathcal{I}} \lambda_i = 1$.

The numbers $\left\{\sqrt{\lambda_i}\right\}$ are called the *Schmidt coefficients*.
The number of elements in $\mathcal{I}$ is the *Schmidt rank*.

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The Schmidt Decomposition

Suppose we know the Schmidt decomposition

$$|\psi\rangle = \sum_{i\in\mathcal{I}} \sqrt{\lambda_i}\,|\psi_i^A\rangle \otimes |\psi_i^B\rangle$$

Then we can compute the reduced density matrices in diagonal form

$$\rho_A = \sum_{i\in\mathcal{I}} \lambda_i\,|\psi_i^A\rangle\langle\psi_i^A| \qquad \rho_B = \sum_{i\in\mathcal{I}} \lambda_i\,|\psi_i^B\rangle\langle\psi_i^B|$$

Note: both reduced density matrices have the same nonzero spectrum.

The entanglement entropy is *symmetric*:

$$S(\rho_A) = S_E(\rho_B) = -\sum_{i\in\mathcal{I}} \lambda_i \log \lambda_i$$

Entanglement Entropy in Quantum Field Theory

In Quantum Field Theory, states are functionals:

$$\psi : \{\text{Field configurations on } \Sigma\} \to \mathbb{C}$$

Each region of space $\Omega \subseteq \Sigma$ is a subsystem

$$\mathcal{H}_\Sigma = \mathcal{H}_\Omega \otimes \mathcal{H}_{\overline{\Omega}}$$

An observer with a horizon only has access to $\mathcal{H}_\Omega$, and sees the mixed state ρ_Ω.

Proposal: Black hole entropy is entanglement entropy [2]

$$S_{\text{BH}} = S_E$$

[2] Luca Bombelli, Rabinder K. Koul, Joohan Lee, and Rafael D. Sorkin. Quantum source of entropy for black holes. Phys. Rev. D 34, 373 - 383 (1986).

Entanglement Entropy in Quantum Field Theory

Why entanglement entropy?

- It satisfies the Generalized Second Law:
  S_E increases if the exterior evolves independently of the interior.[3]
- It scales like the horizon area:
  For a scalar field in the vacuum state on a flat background

$$S \propto A/\mu^2$$

  Where μ is the minimal length.[4]

But,

- It is infinite:

$$S \to \infty \text{ as } \mu \to 0$$

Quantum gravity could provide the required UV cutoff at $\mu \approx \ell_P$.

[3] Rafael D. Sorkin. Toward a Proof of Entropy Increase in the Presence of Quantum Black Holes. Phys. Rev. Lett. 56, 1885 - 1888 (1986)

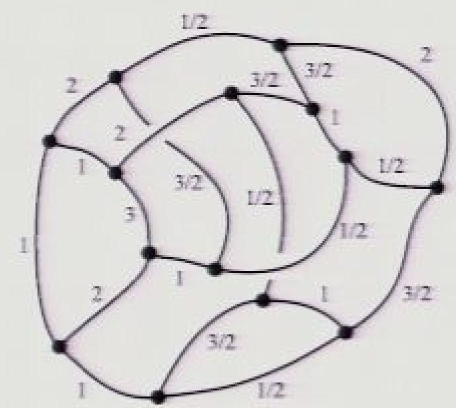
[4] Mark Srednicki. Entropy and Area. hep-th/9303048.

Loop Quantum Gravity

Geometry of space is described by Spin Network States:

- Labelled by embedded Spin Networks.
- Show discrete structure at short distances.
- Have a well-defined area operator

$$A(\partial\Omega) = 8\pi\gamma \sum_{p\in\mathcal{P}} \sqrt{j_p(j_p+1)}$$



Plan: Compute S_E for spin network states, using the Schmidt decomposition.

Link states

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The Hilbert space is spanned by *link states* $|\gamma, j, a, b\rangle$ defined by

$$\langle A|\gamma, j, a, b\rangle = R^j(U(A, \gamma))^a_b$$

Where

$$U(A, \gamma) = \mathcal{P}\exp \int_\gamma A$$

$$R^j(\cdot)^a_b = \text{the (a,b) matrix element in the spin j representation}$$

Link states are *orthogonal* and *normalized*.

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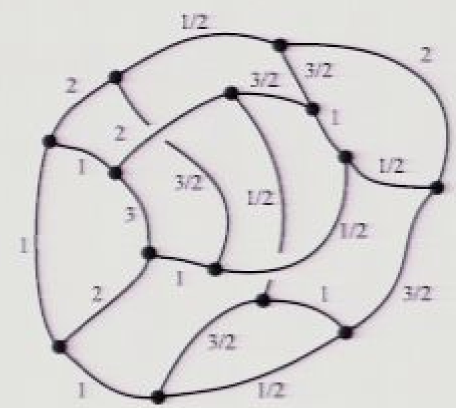

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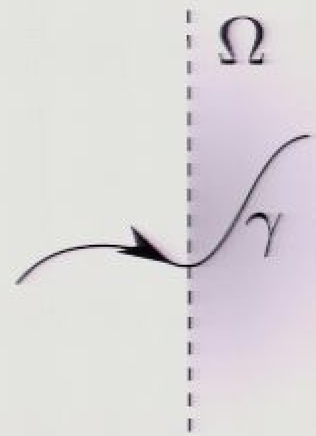

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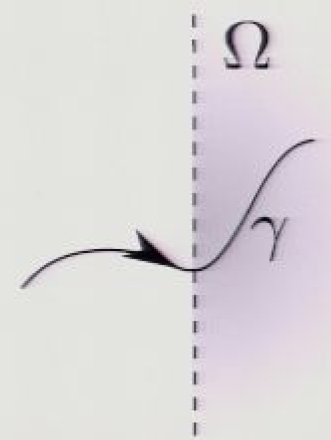

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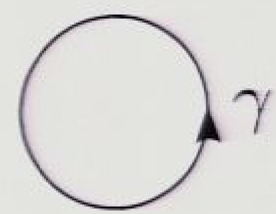
Need *gauge-invariant* linear combinations of link states.

Wilson loop states:



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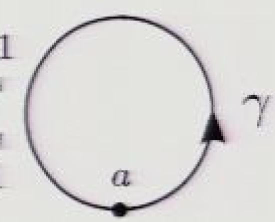

Where γ is a closed curve.

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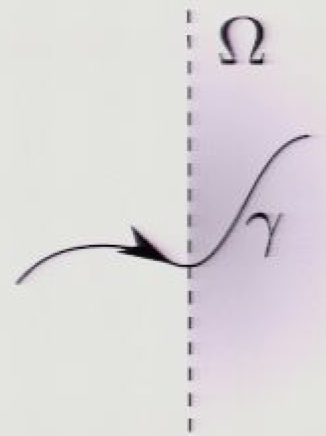

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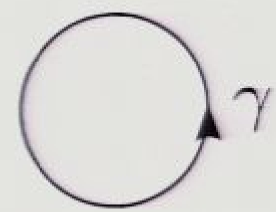

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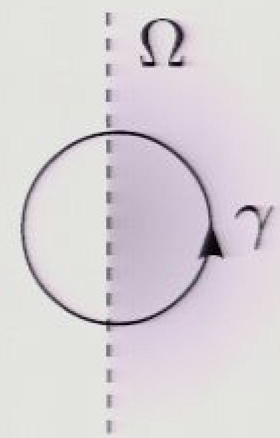

Apply the Schmidt decomposition of the link state twice

$$|\gamma, j\rangle = \frac{1}{2j+1} \sum_{a,b=1}^{2j+1} \underbrace{|\gamma_1, j, a, b\rangle}_{\in \mathcal{H}_\Omega} \otimes \underbrace{|\gamma_2, j, b, a\rangle}_{\in \mathcal{H}_{\overline{\Omega}}}$$

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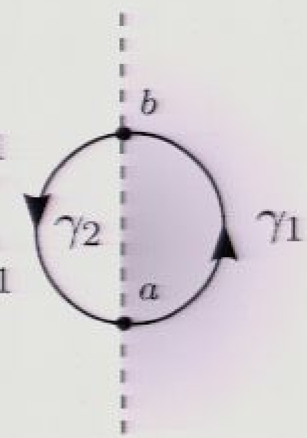

This is the Schmidt decomposition of $|\gamma, j\rangle$

$$S_E(\Omega) = 2 \log(2j+1)$$

Wilson loops

Suppose γ intersects $\partial\Omega$ at n points

$$|\gamma, j\rangle = \frac{1}{\sqrt{2j+1}^n} \sum_{a_1,\dots,a_n} |\gamma_1, j, a_1, a_2\rangle \otimes \cdots \otimes |\gamma_n, j, a_n, a_1\rangle$$

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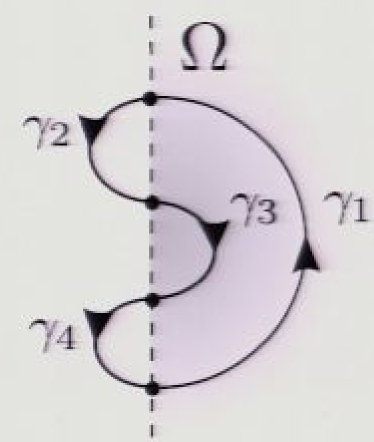

The Schmidt rank is $(2j+1)^n$.

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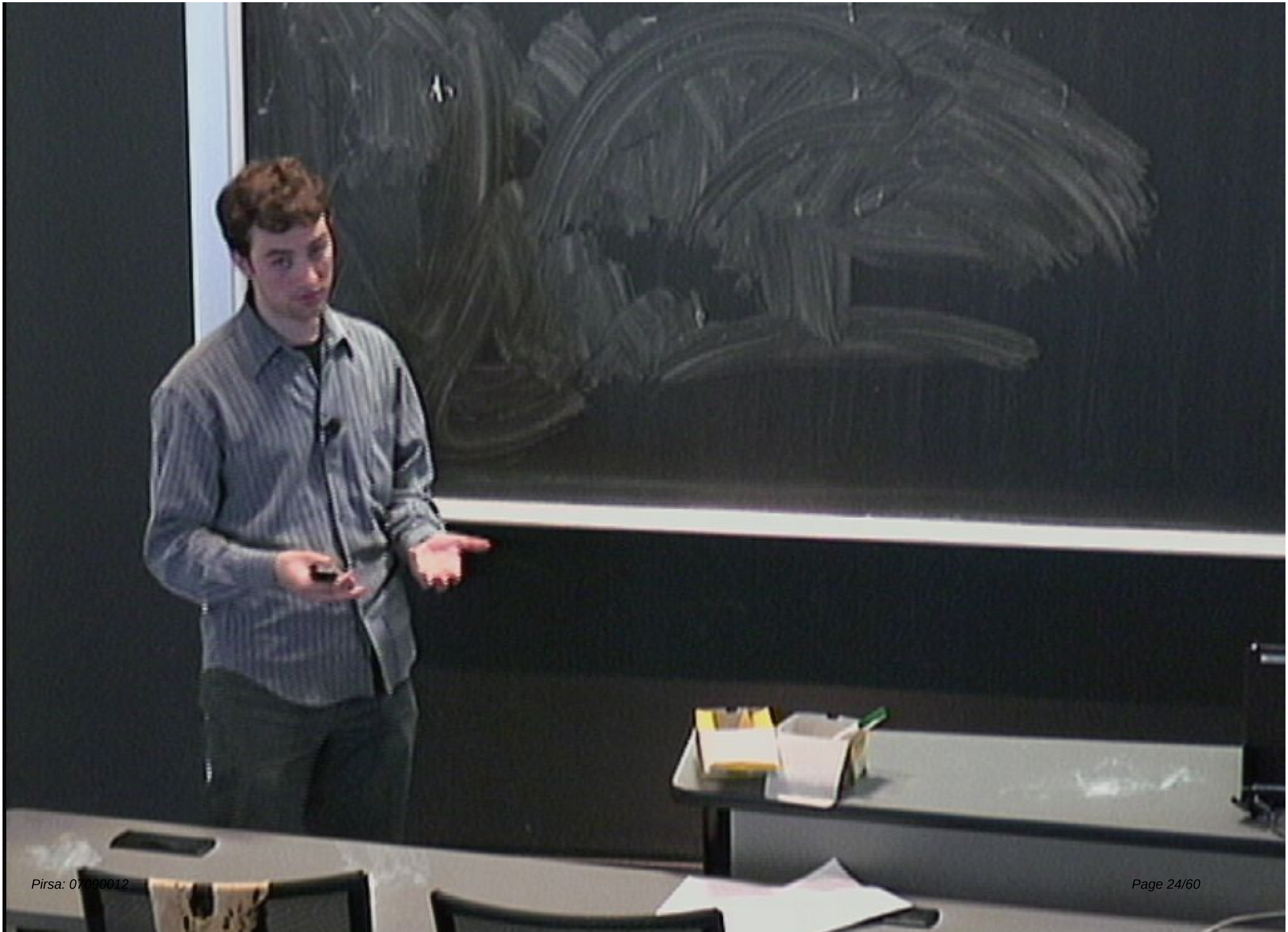
Entropy counts intersections of γ with $\partial\Omega$



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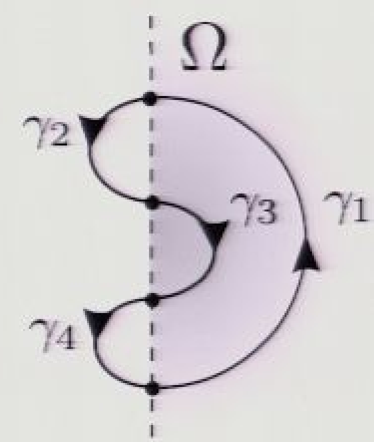


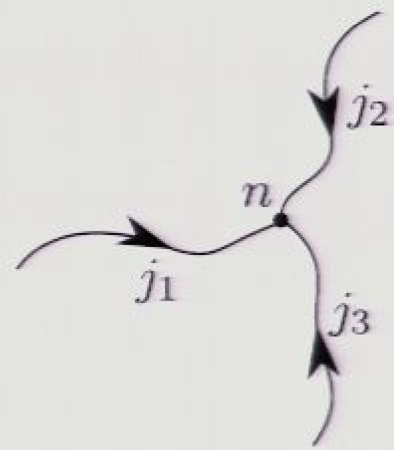
Spin Networks

Wilson loops are not the only gauge-invariant states.

Can have multiple edges meet at a node n

Need a group-invariant map, an *intertwiner*

$$i_n : \mathcal{H}_{j_1} \otimes \cdots \mathcal{H}_{j_m} \to \mathbb{C}$$



This leads to the Spin Network states

$$|S\rangle = \left(\bigotimes_n i_n \right) \circ \left(\bigotimes_l |\gamma_l, j_l, a_l, b_l\rangle \right)$$

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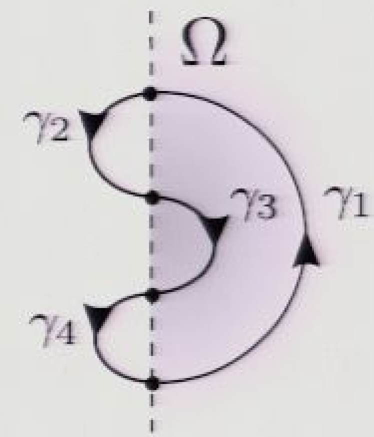

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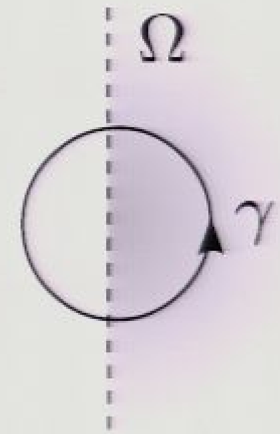

Apply the Schmidt decomposition of the link state twice

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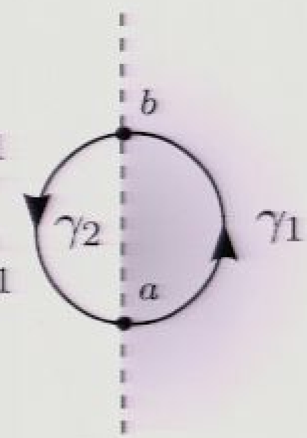

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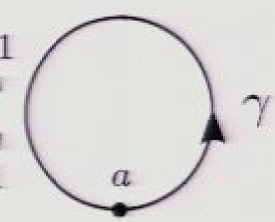

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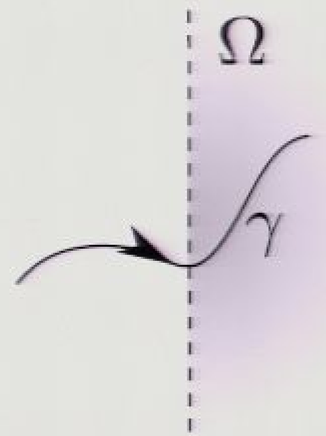

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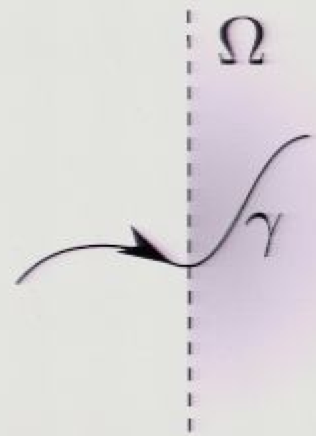

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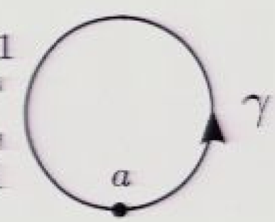

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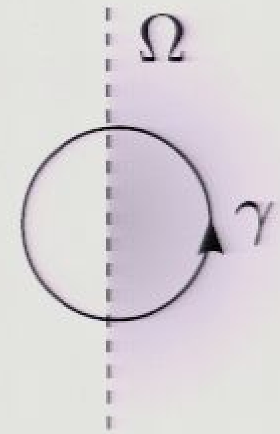

Apply the Schmidt decomposition of the link state twice

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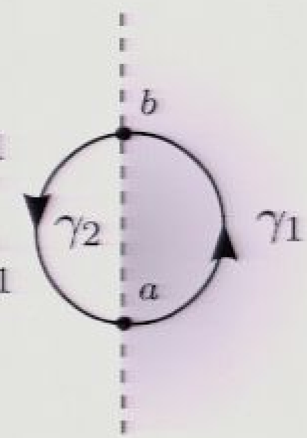

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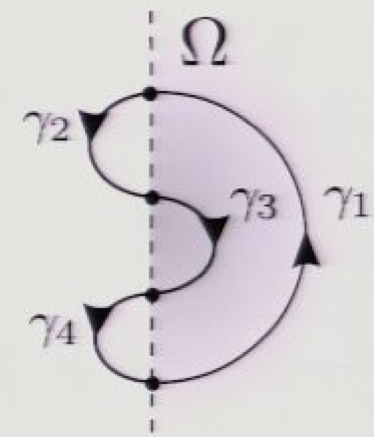
\begin{aligned}
|\gamma, j\rangle &= \frac{1}{\sqrt{2j+1}^{\,n}} \sum_{a_1,\ldots,a_n} |\gamma_1, j, a_1, a_2\rangle \otimes \cdots \otimes |\gamma_n, j, a_n, a_1\rangle \\
&= \frac{1}{\sqrt{2j+1}^{\,n}} \sum_{a_1,\ldots,a_n} \underbrace{\left(|\gamma_1, j, a_1, a_2\rangle \otimes \cdots\right)}_{\in \mathcal{H}_\Omega} \otimes \underbrace{\left(|\gamma_2, j, a_2, a_3\rangle \otimes \cdots\right)}_{\in \mathcal{H}_{\overline{\Omega}}}
\end{aligned}
$$

The Schmidt rank is $(2j+1)^n$.

$$S_E(\Omega) = n \log(2j+1)$$

Entropy counts intersections of γ with $\partial\Omega$



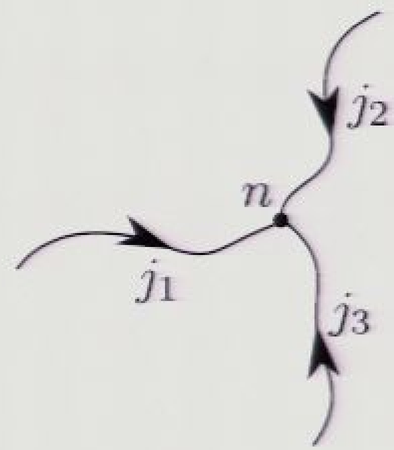
Spin Networks

Wilson loops are not the only gauge-invariant states.

Can have multiple edges meet at a node *n*

Need a group-invariant map, an *intertwiner*

$$i_n : \mathcal{H}_{j_1} \otimes \cdots \mathcal{H}_{j_m} \to \mathbb{C}$$



This leads to the Spin Network states

$$|S\rangle = \left(\bigotimes_n i_n \right) \circ \left(\bigotimes_l |\gamma_l, j_l, a_l, b_l\rangle \right)$$

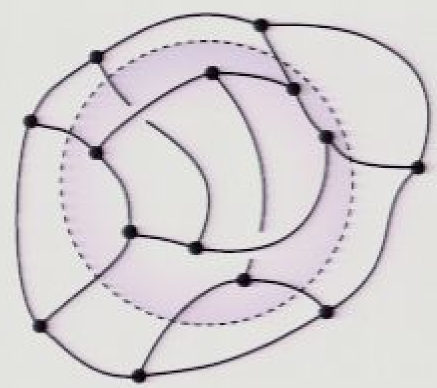
Spin Networks

Apply the link state Schmidt decomposition to each puncture p:

$$|S\rangle = \frac{1}{\sqrt{N}} \sum_{a_p=1}^{2j_p+1} |S_\Omega, a_p\rangle \otimes |S_{\overline{\Omega}}, a_p\rangle$$

Where we have defined

$$|S_\Omega, a_p\rangle \equiv \left(\bigotimes_{n\in\Omega} i_n\right) \circ \left(\bigotimes_{l\in\Omega} |\gamma_l, j_l, a_l, b_l\rangle\right)$$



The Schmidt rank is $N = \prod(2j_p + 1)$

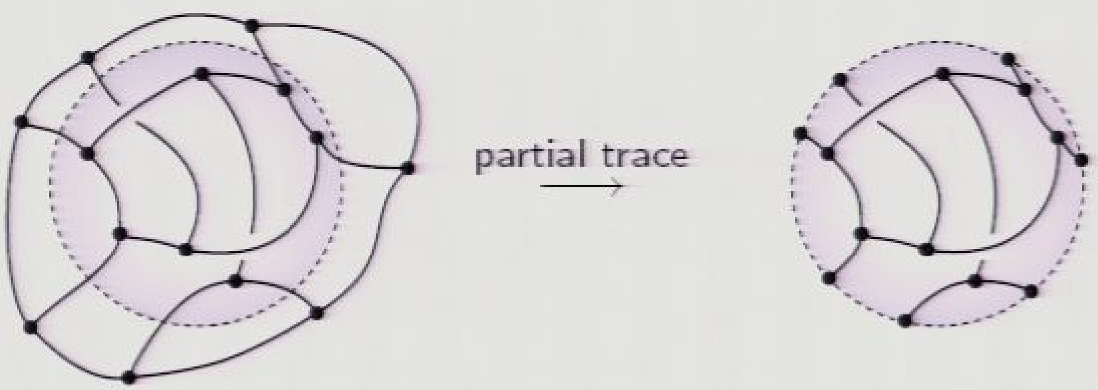
$$S_E(\Omega) = \sum_{p\in\mathcal{P}} \log(2j_p + 1)$$

Entropy is extensive over the boundary.

Spin Networks

The density matrix ρ_Ω is a gauge-invariant "mixed spin network state".

$$\rho_\Omega = \frac{1}{N} \sum_{a_p=1}^{2j_p+1} |S_\Omega, a_p\rangle\langle S_\Omega, a_p|$$



A pure spin network cannot have endpoints; a mixed spin network can.

Relation to Isolated Horizons

S_E depends only on the boundary. Can we describe S_E using a theory living on $\partial\Omega$?

Yes. The Isolated Horizon framework[5] has exactly such a Hilbert space.

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Trace over $\mathcal{H}_{\Omega}^{\mathcal{P}}$ gives a maximally mixed density matrix $\rho_{\partial\Omega}$

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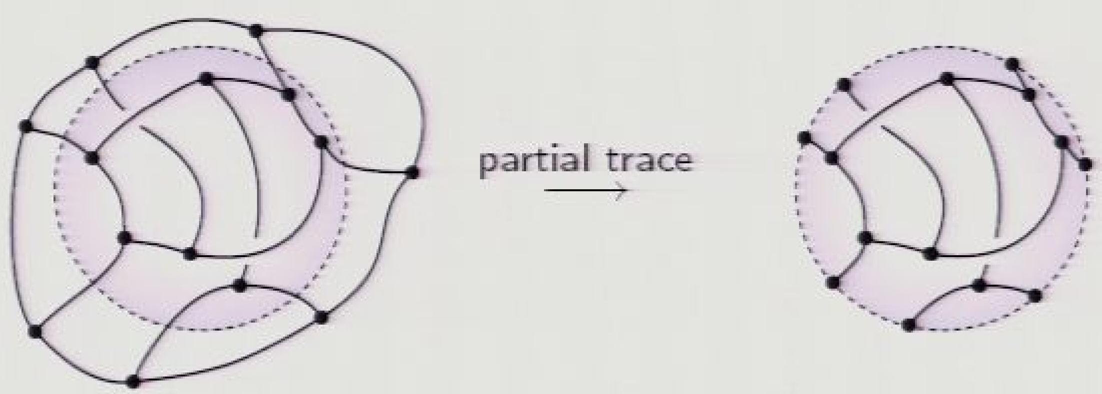

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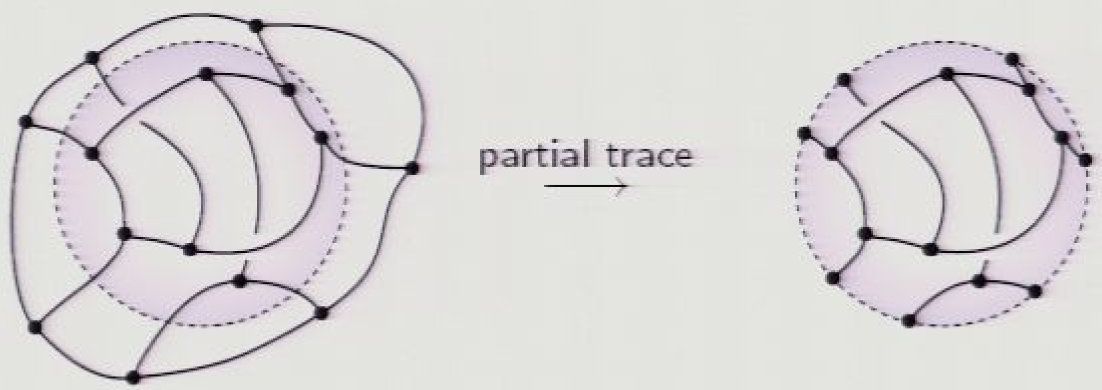

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Outlook

Black hole entropy is naturally explained in terms of entanglement.

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- It's **extensive**; it can be computed as a sum over punctures.
- It's **holographic**; it agrees with Chern-Simons theory in 2+1.

But,

- It isn't exactly proportional to area.

$$S_E(\Omega) = \sum_{p \in \mathcal{P}} \log(2j_p + 1)$$

$$A(\partial\Omega) = \sum_{p \in \mathcal{P}} 8\pi\gamma\sqrt{j_p(j_p + 1)}$$

So now what?

Corrections to S_{BH}

For an arbitrary diffeomorphism-invariant Lagrangian, the classical black hole entropy is [6]

$$S = 2\pi \oint_{\partial\Omega} Q$$

Where Q is a Noether charge depending on the Lagrangian.

Open Question: Is there a quantity Q such that when quantized

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