Title: PenroseÂ's Space of Quantized Directions

Date: Sep 07, 2007 04:00 PM

URL: http://pirsa.org/07090010

Abstract: In the sixties, Roger Penrose came up with a radical new idea for a quantum geometry which would be entirely background independent, combinatorial, discrete (countable number of degrees of freedom), and involve only integers and fractions, not complex or real numbers. The basic structures are spin-networks. One reason we might believe that space or space-time might be discrete is that current physique tells us that matter is discrete and that matter and geometry are related through gravity. Once a discrete theory is decided on, it seems awkward that the dynamics would retain "continuous elements" in the form of real numbers (used for the probabilities for example). The great achievement of Penrose's theory is that there is a well defined procedure which gives the semi-classical limit geometry (always of the same dimension) without any input on topology (the fundamental theory does not contain a manifold).

OUTLINE

1. Introduction and Motivation
2. Spin Networks
3. Scalar Product
4. Dynamics
5. Dynamics
6. Dynamics
7. The Spin Geometry Theorem, or the (partial) Semi-Classical limit
8. Conclusion and Outlook

Introduction and Motivation

- Introduction: Roger Penrose proposed this model in the 60's then pretty much forgot about it. J. Moussouris did a bit more in the 80's.
- Properties:
  - : Quantum theory of geometry
  - : Background and Topology independent
  - : Discrete degrees of freedom and discrete mathematics
  - : Evolution and measurement might two aspects of the same thing

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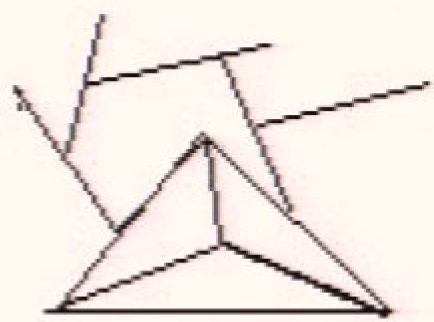

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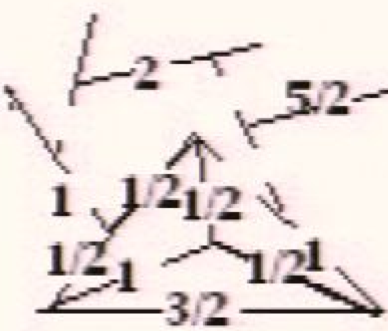

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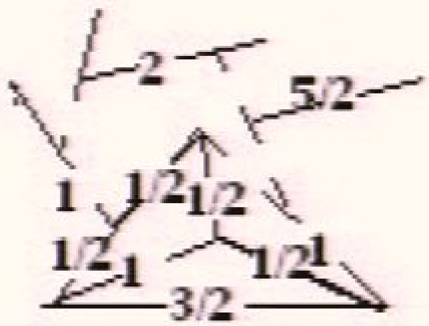

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In what follows we have see the spin network edges as either lengths or areas.

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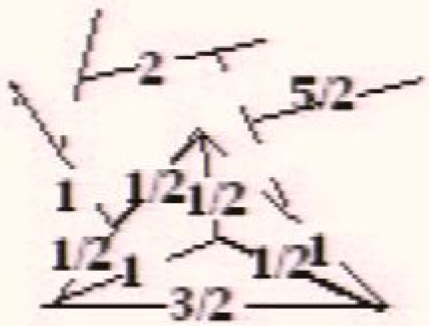

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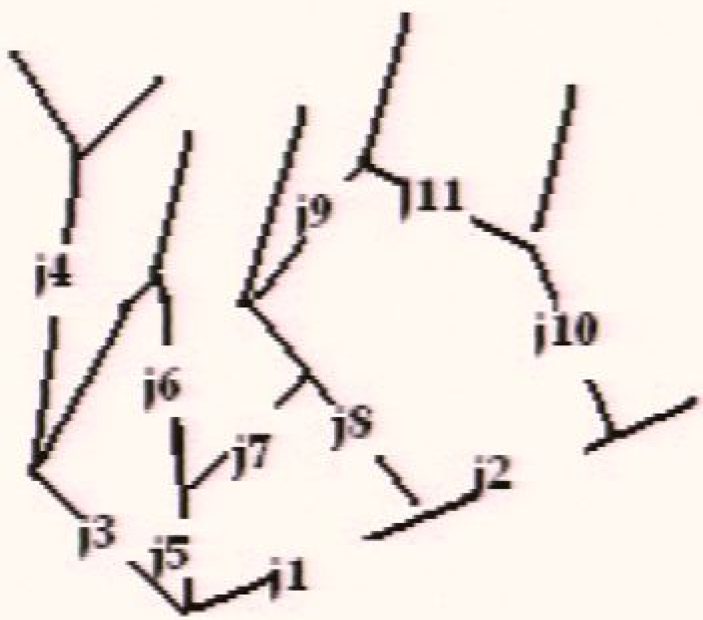

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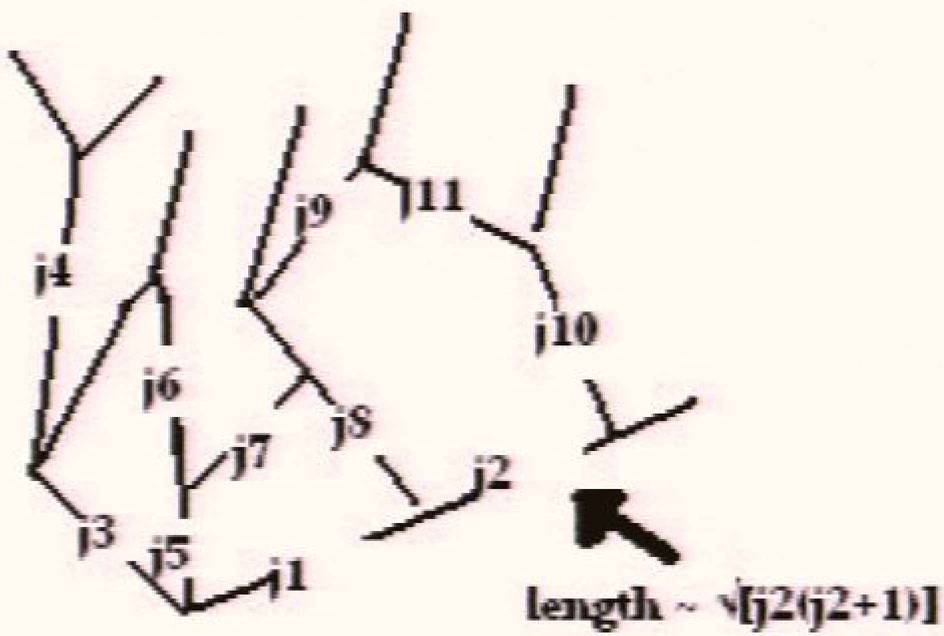

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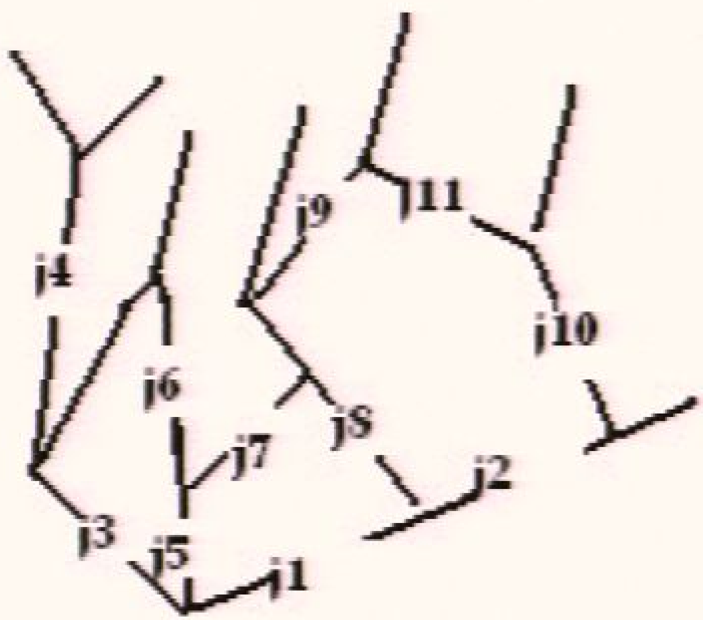

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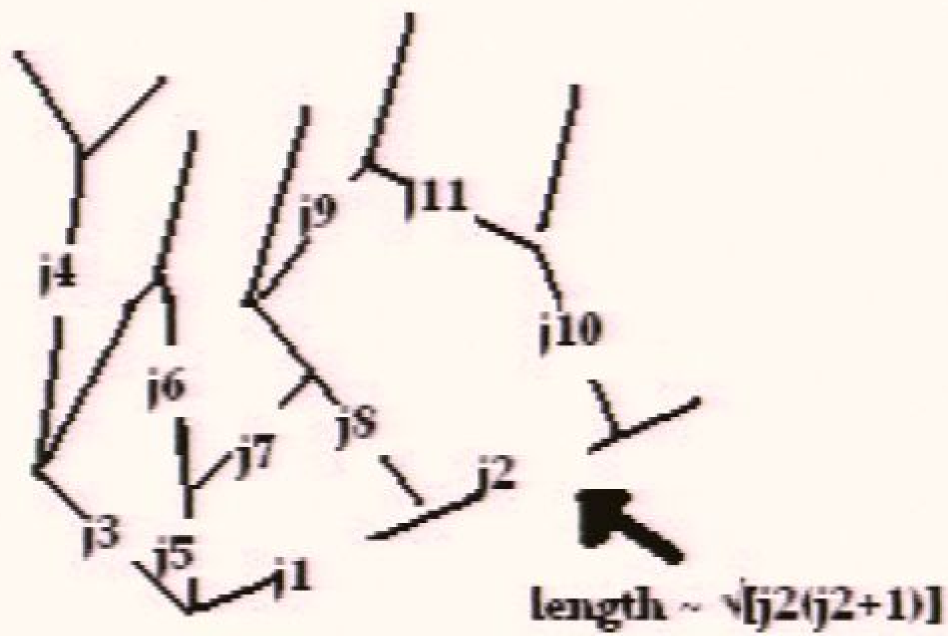

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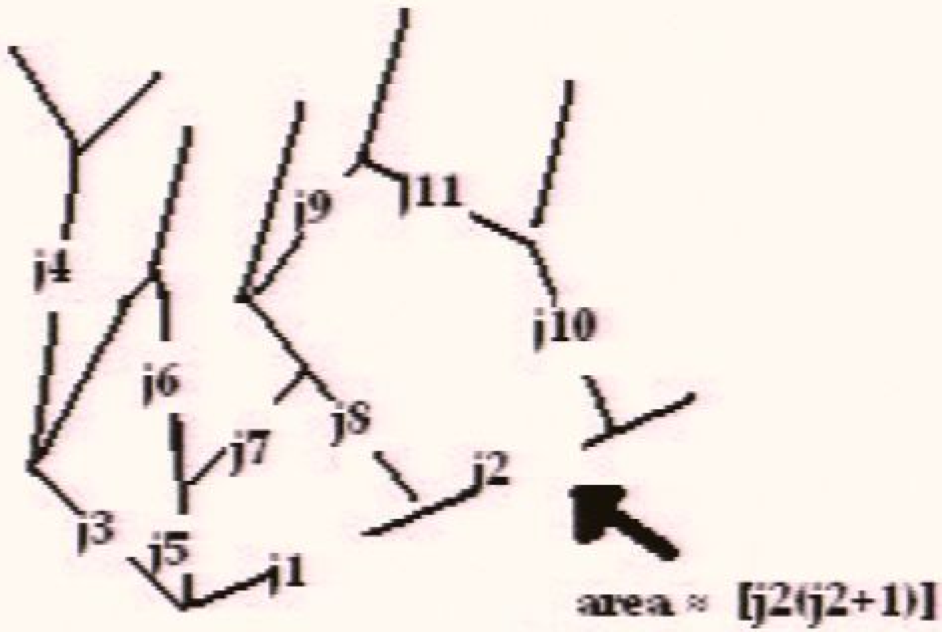

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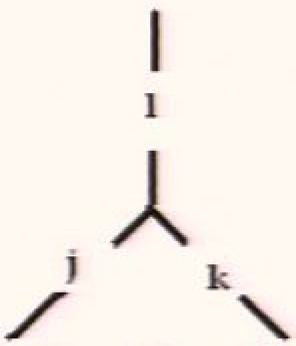
Scalar Product



$L = J + K \Rightarrow J \cdot K = \frac{1}{2}[L^2 - J^2 - K^2]$

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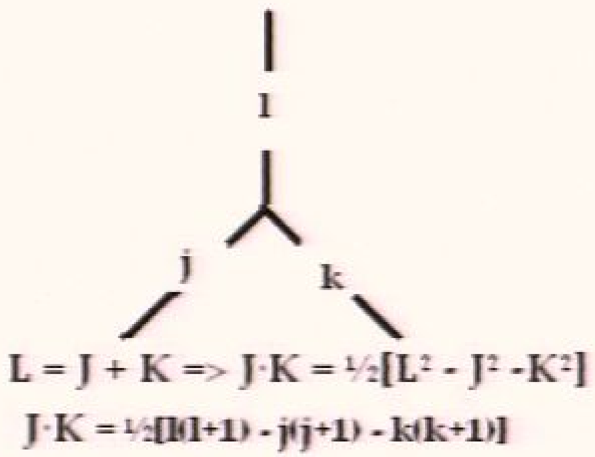

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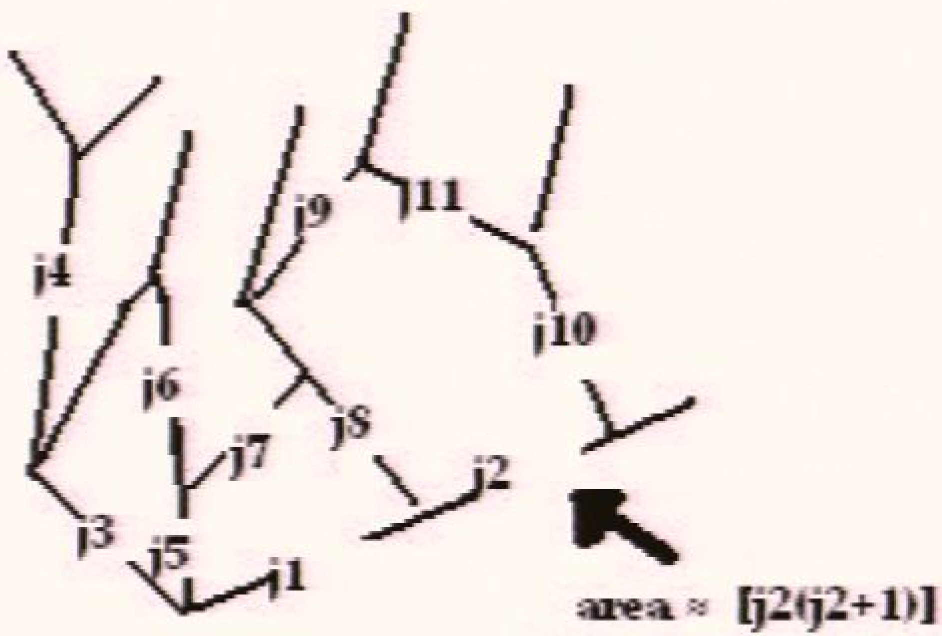

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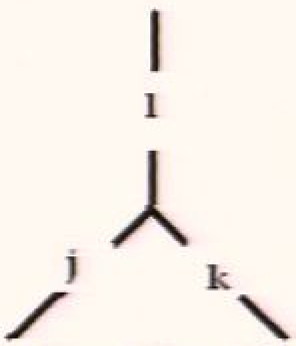

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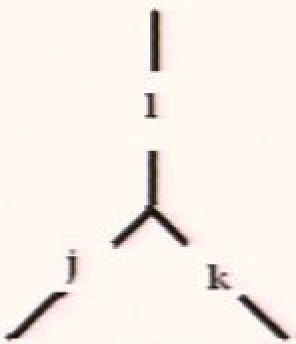

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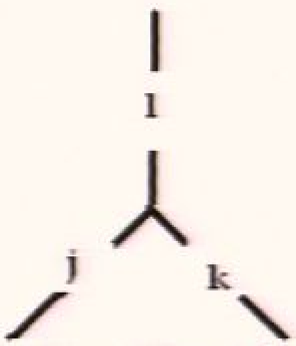

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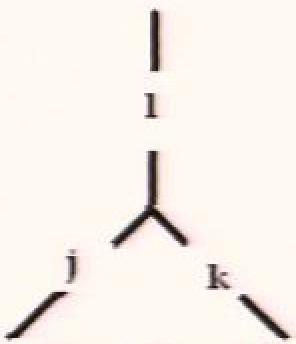
Note that the cosine of the angle between j and k :

$$cos\theta_{j,k} = \frac{l(l+1) - j(j+1) - k(k+1)}{2jk} \quad (2)$$

can take on only a finite number of values (because $l \in [|\ |j-k|, j+k\ |])$ all of which are necessarily rational.

Scalar Product



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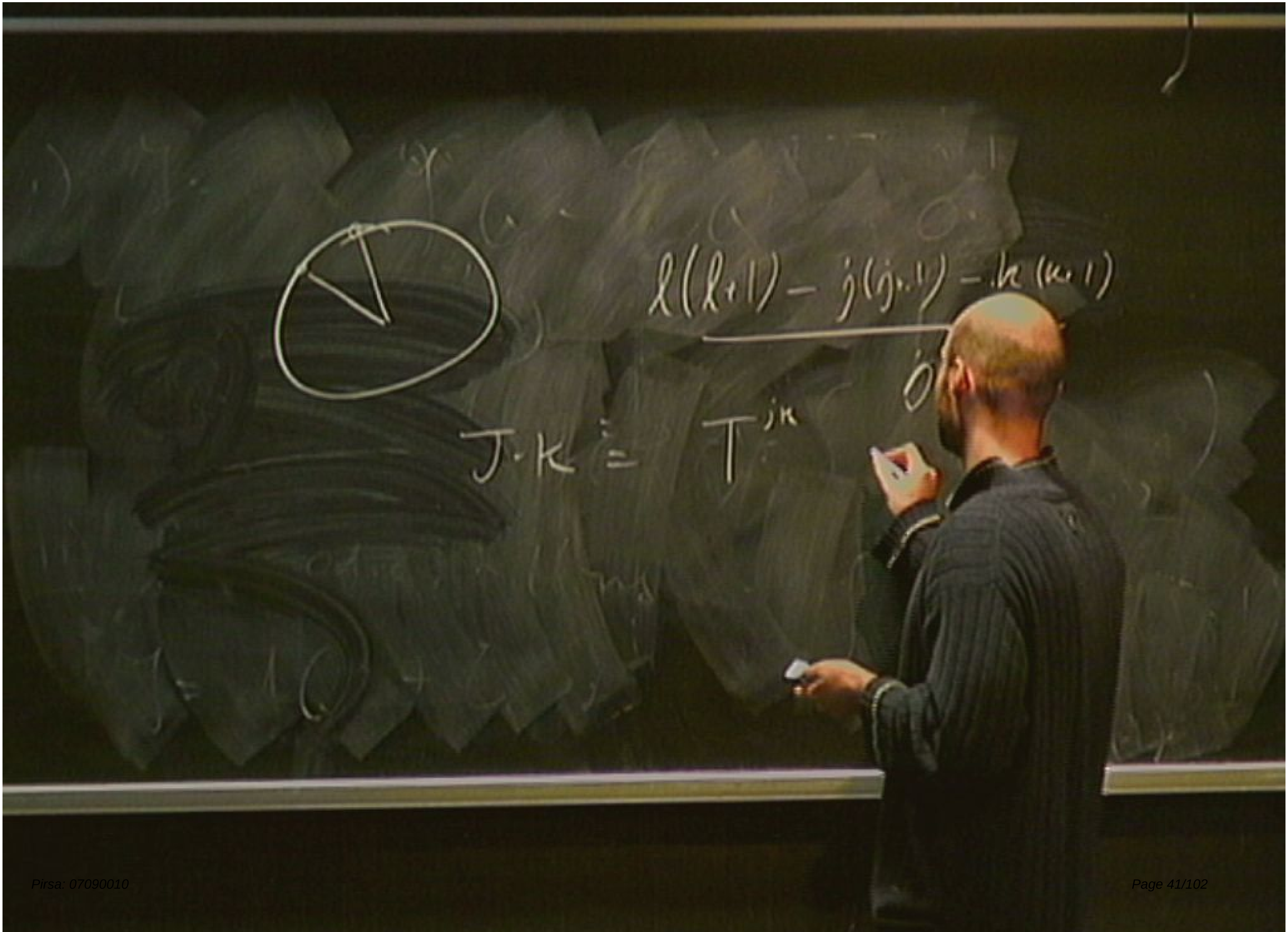
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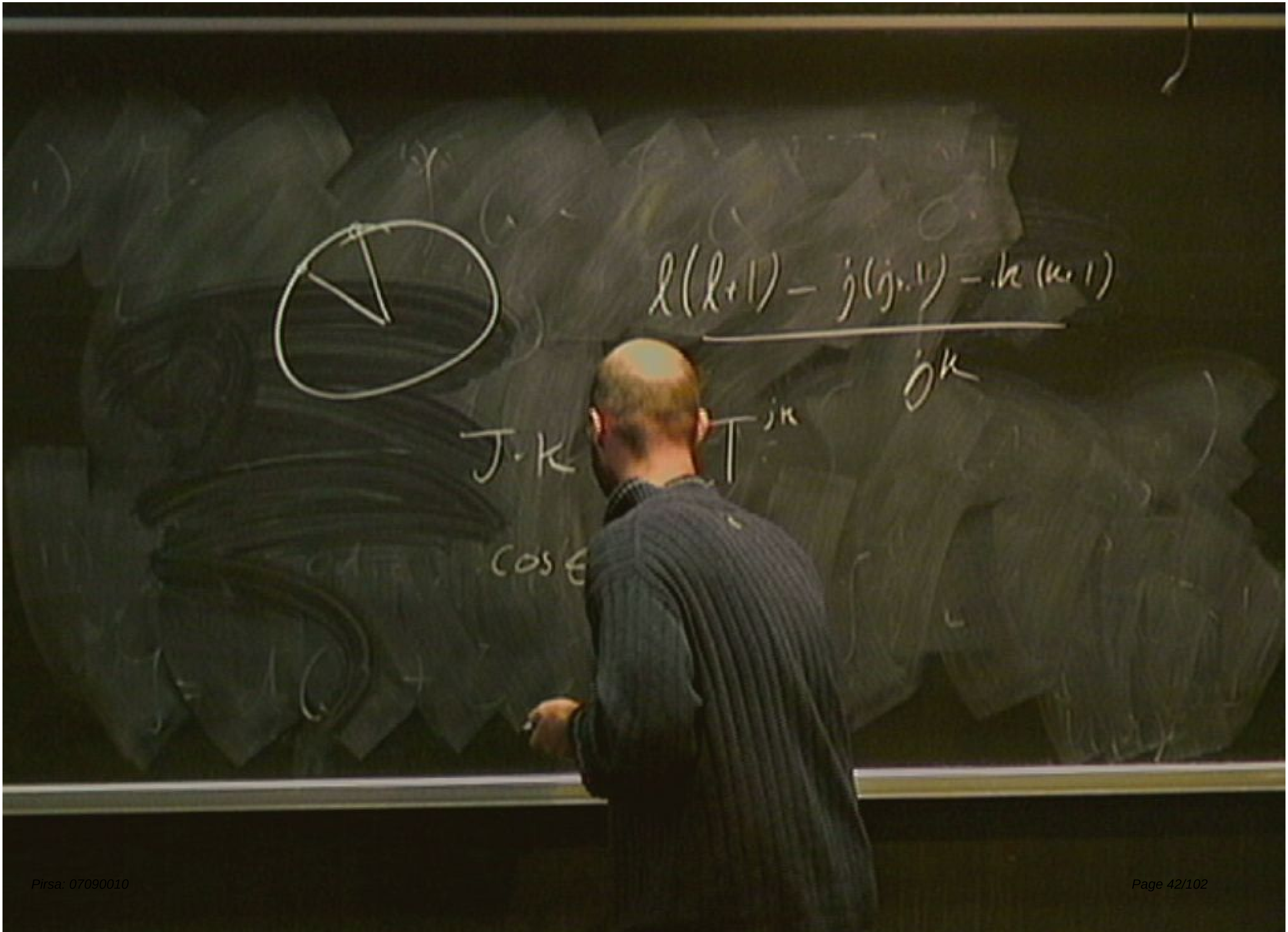
J·K =
T jk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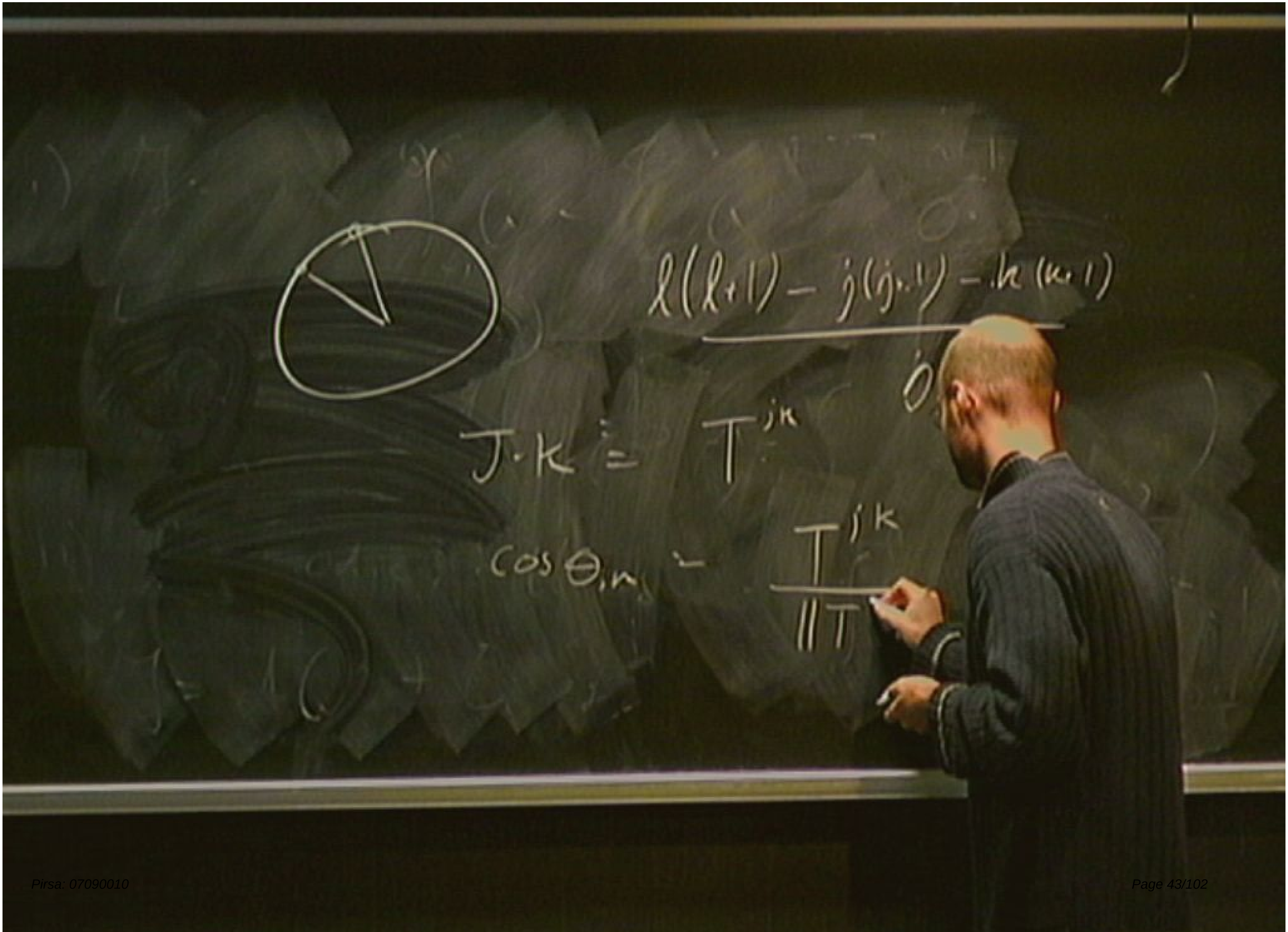
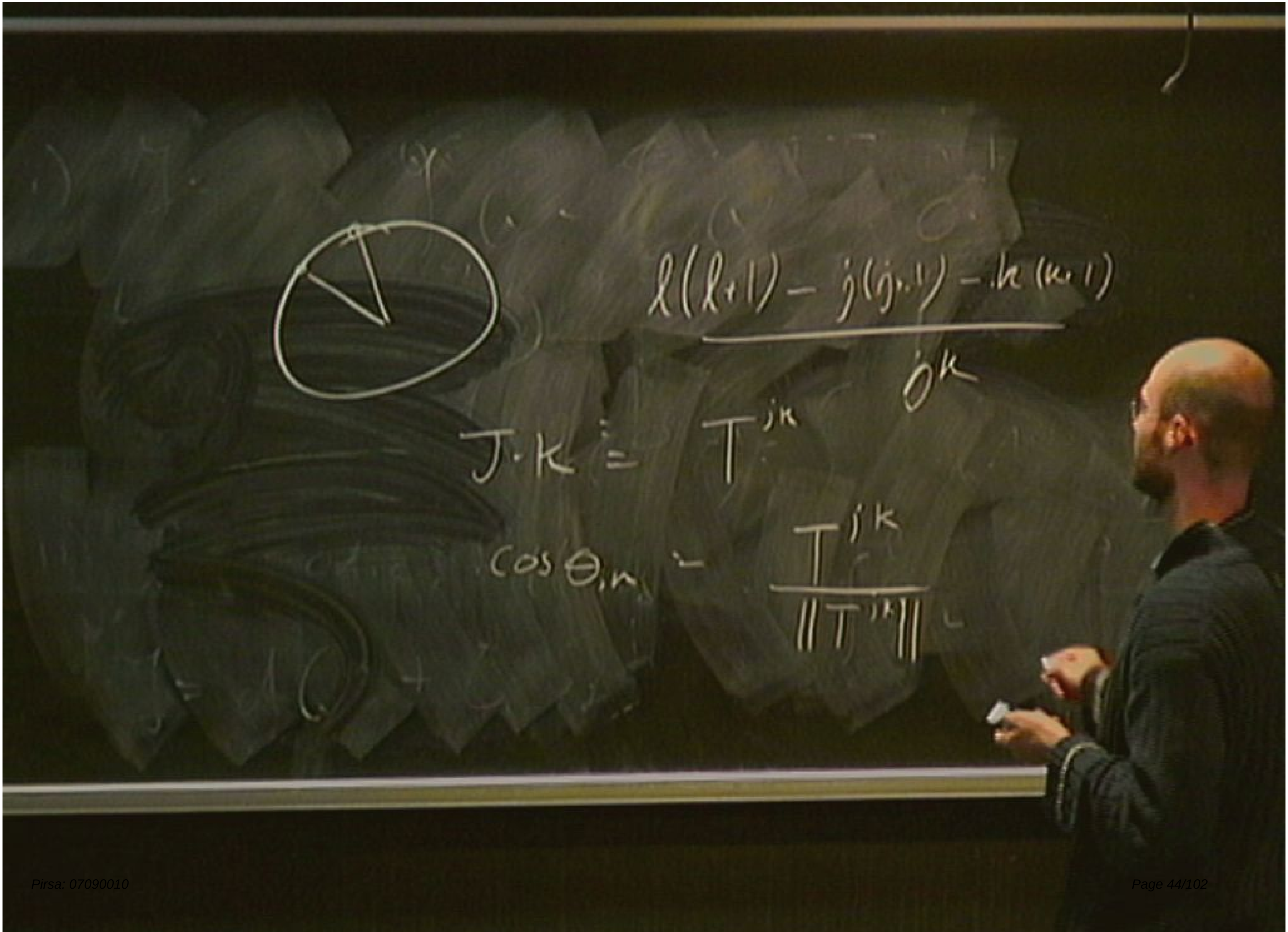
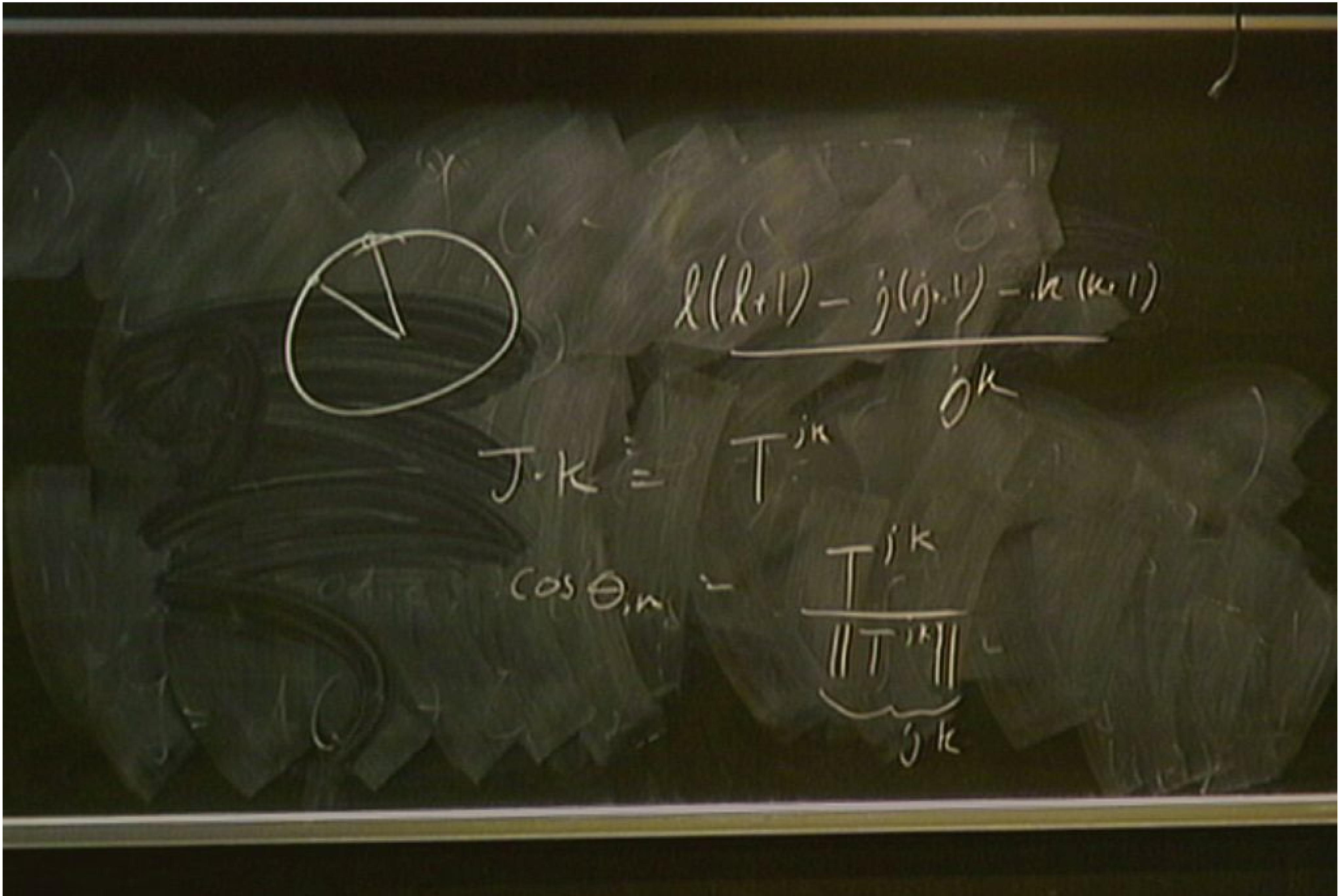

l(l+1) − j(j+1) − k(k+1)
J·K =
T^jk
cos θjk =
T^jk
||T


l(l+1) − j(j+1) − k(k+1)
J·K = T^jk
cos θ_jk = T^jk / ||T^jk||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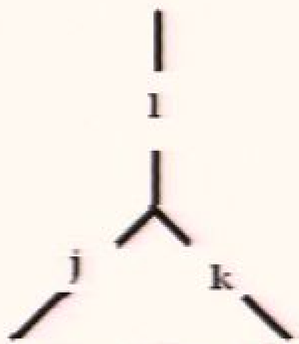

L = J + K => J·K = ½[L² - J² - K²]

J·K = ½[l(l+1) - j(j+1) - k(k+1)]

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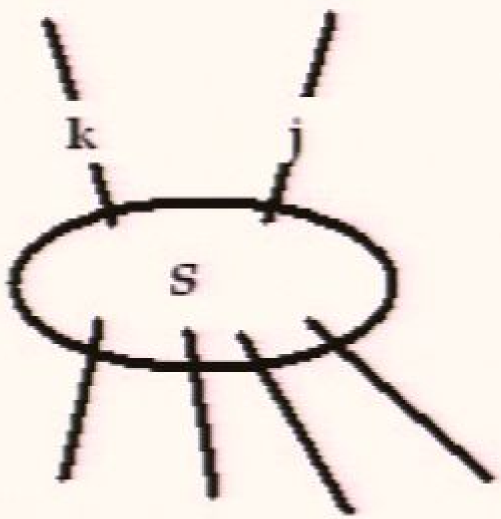
For edges not intersecting at a vertex it is more complicated but it basically the same principle. For example, if we have the following spin network:



Then the expected or “average" scalar product can be calculated as follows.

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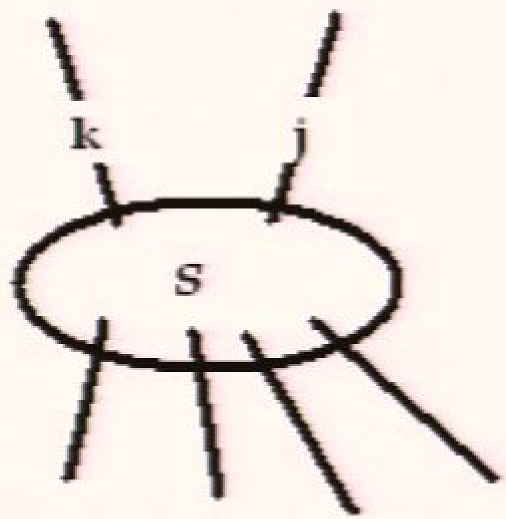

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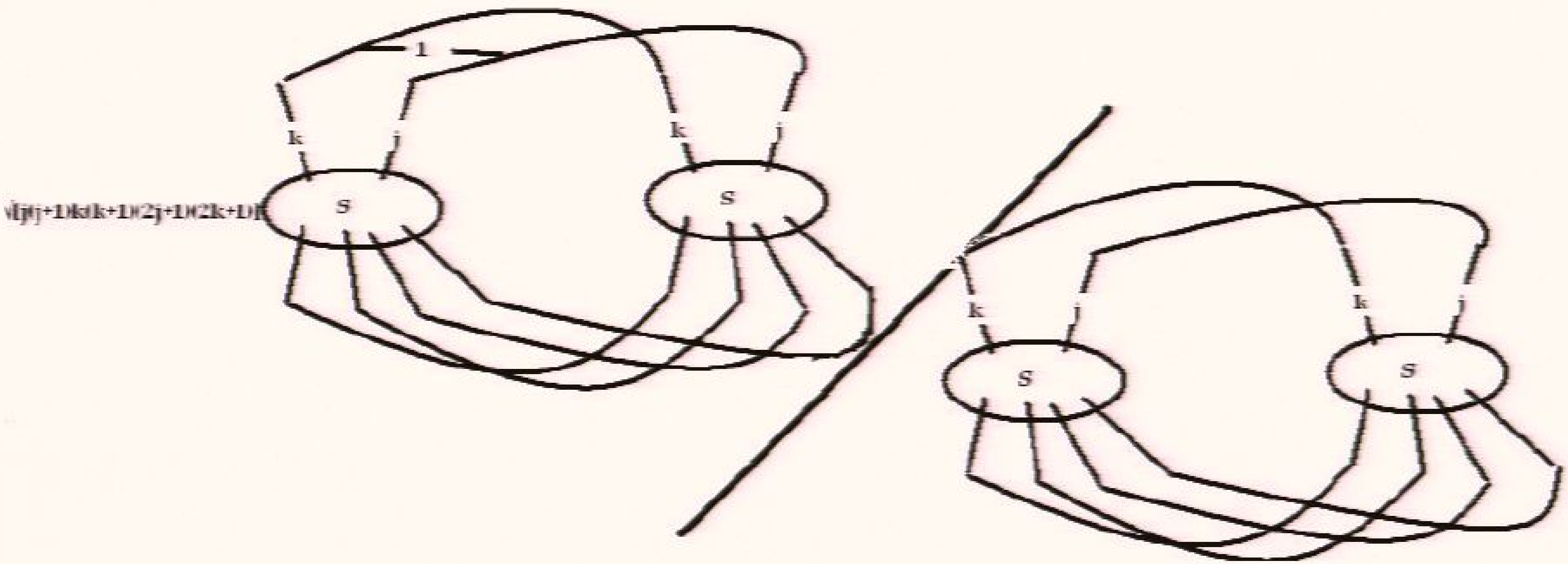

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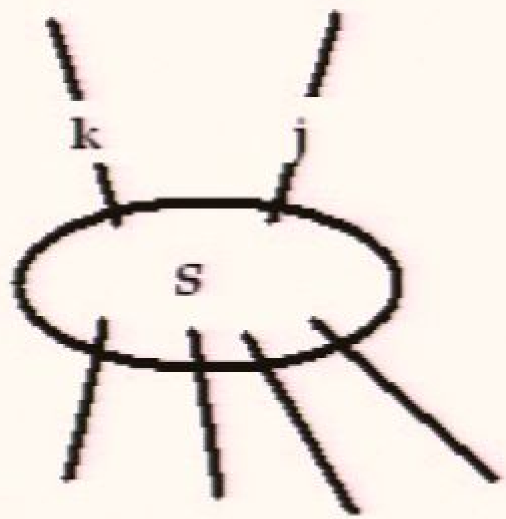

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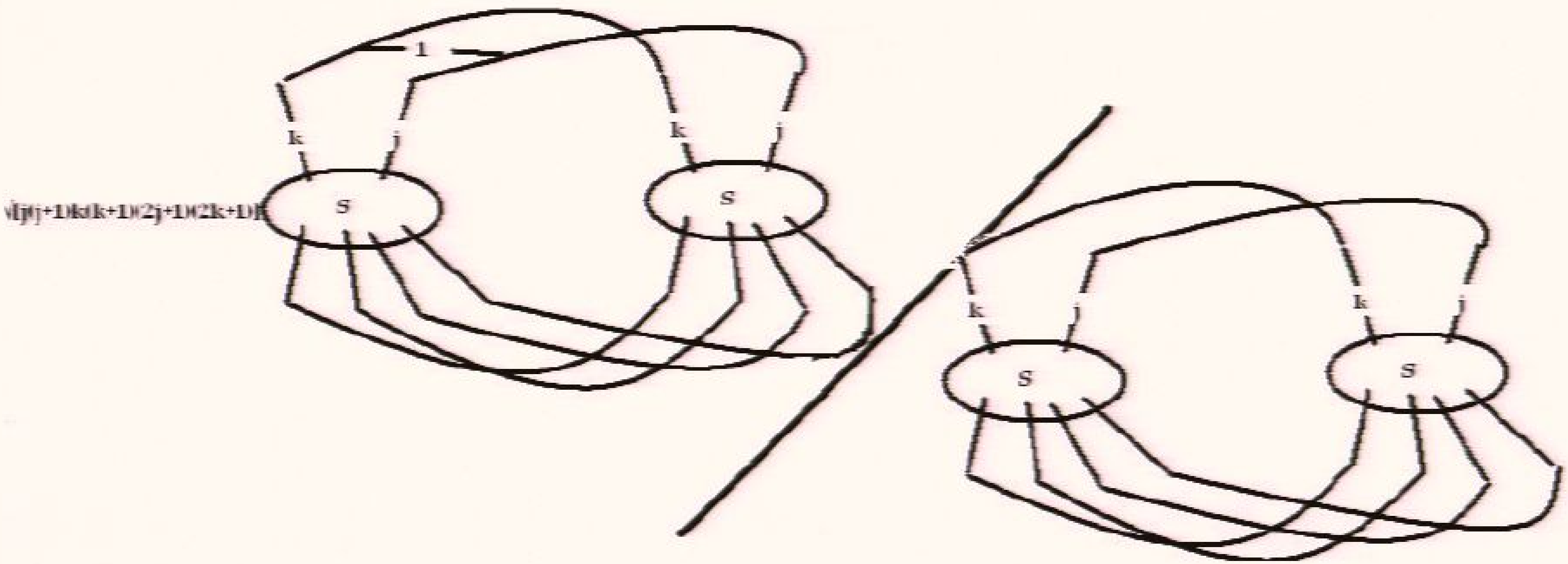

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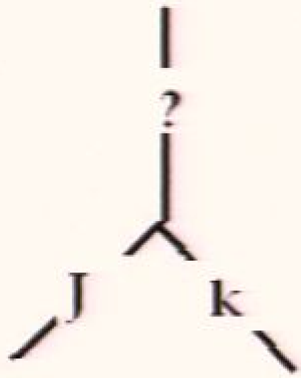

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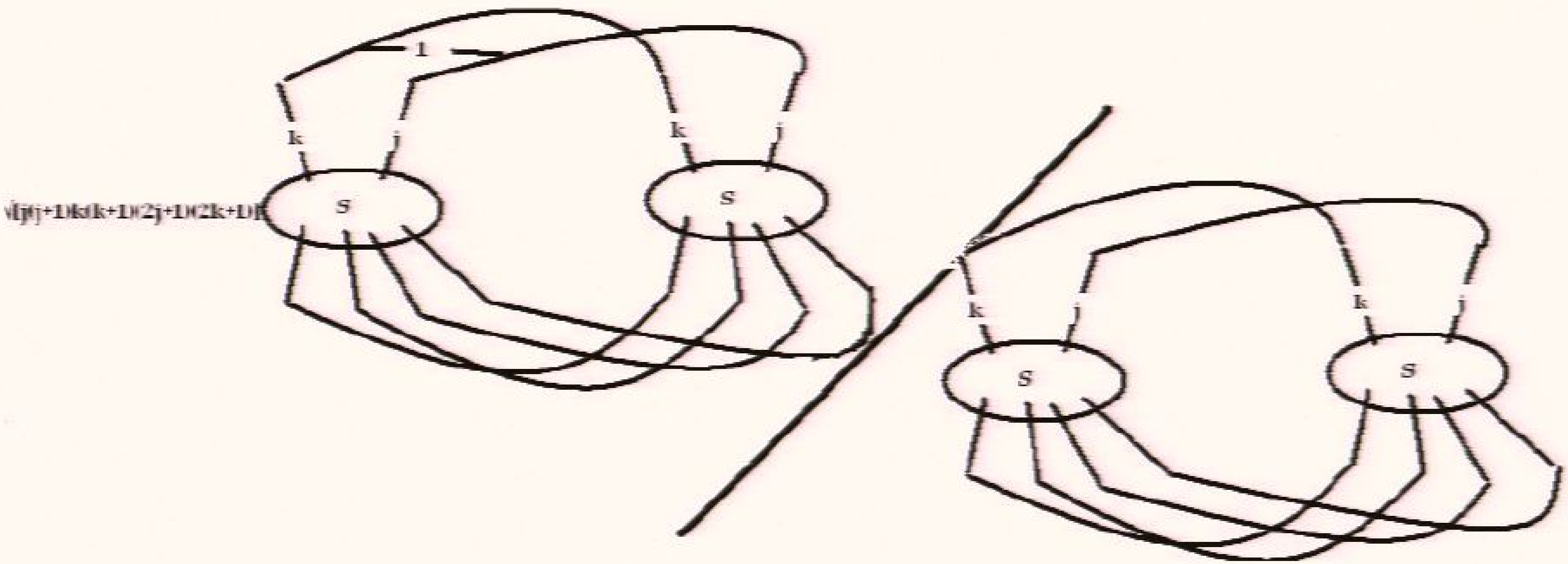
Answer:

Scalar Product



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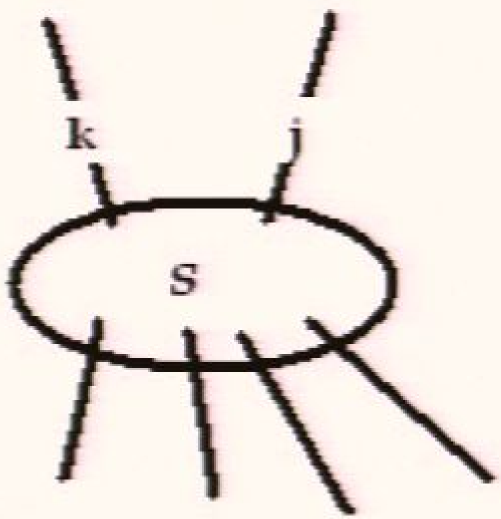

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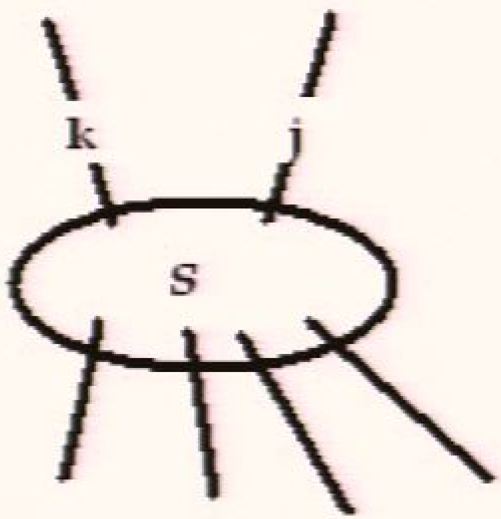

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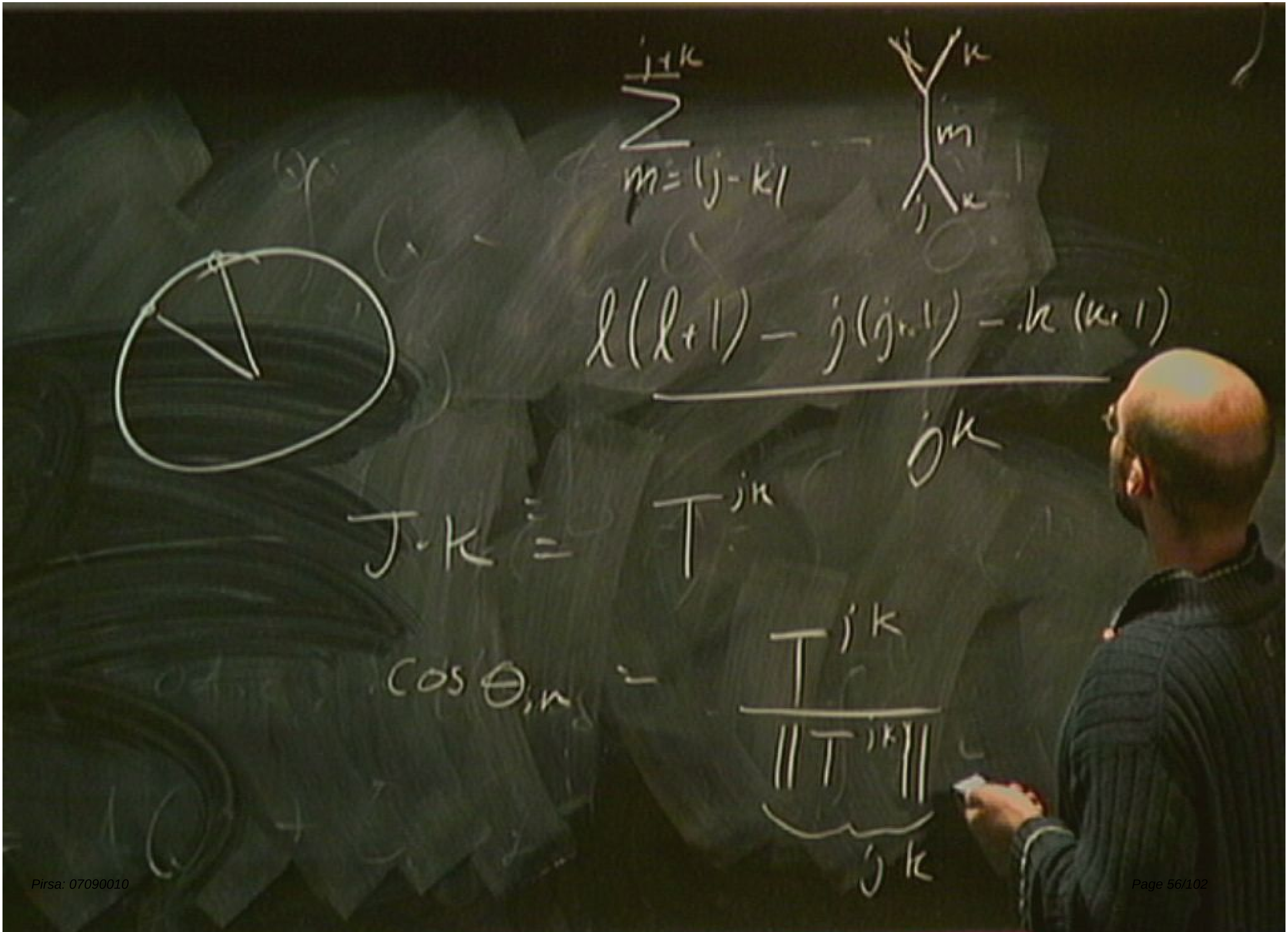
j+k
m=|j-k|
m
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k
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jk
J·K = T^jk
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||T^jk||
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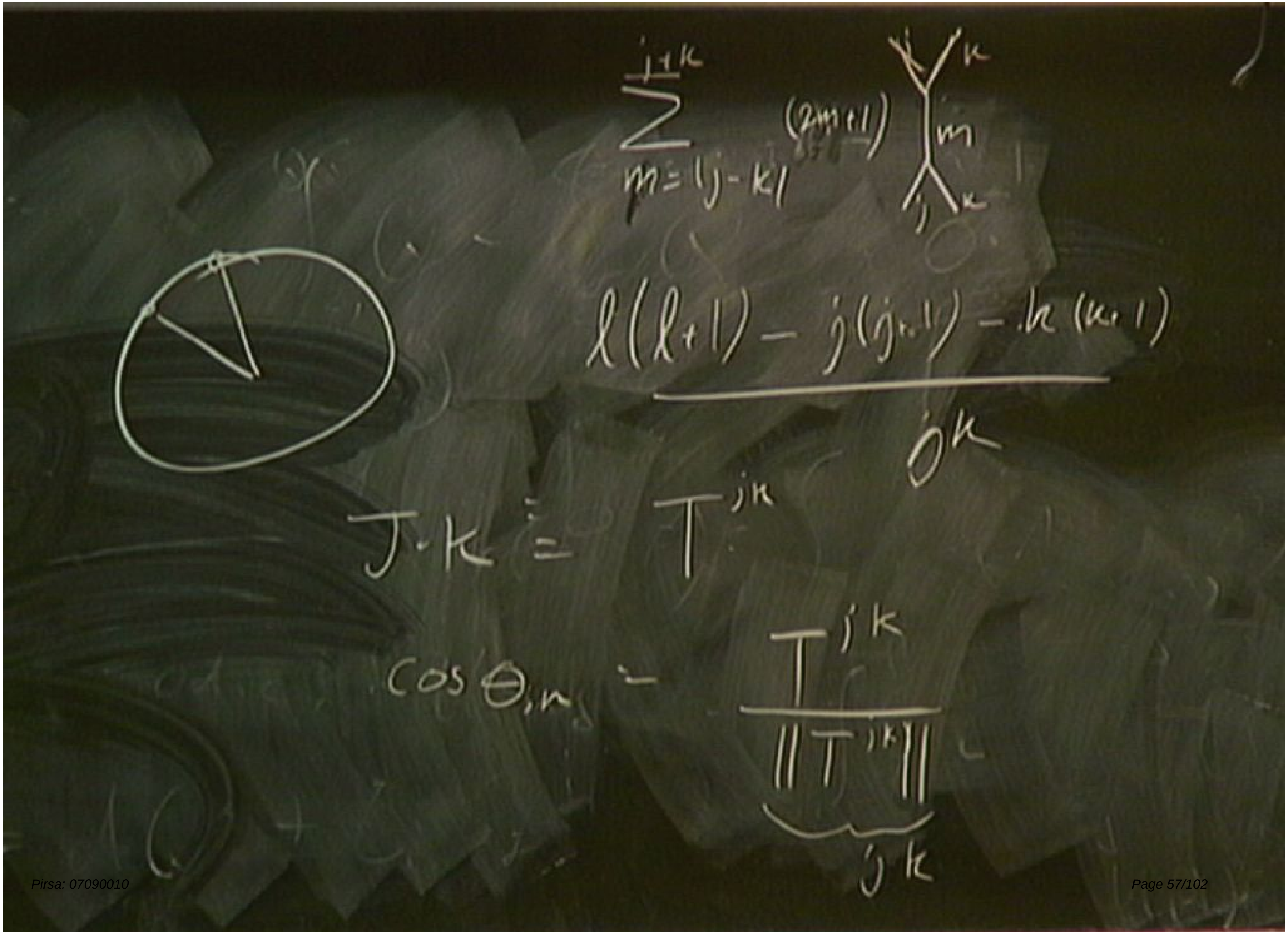
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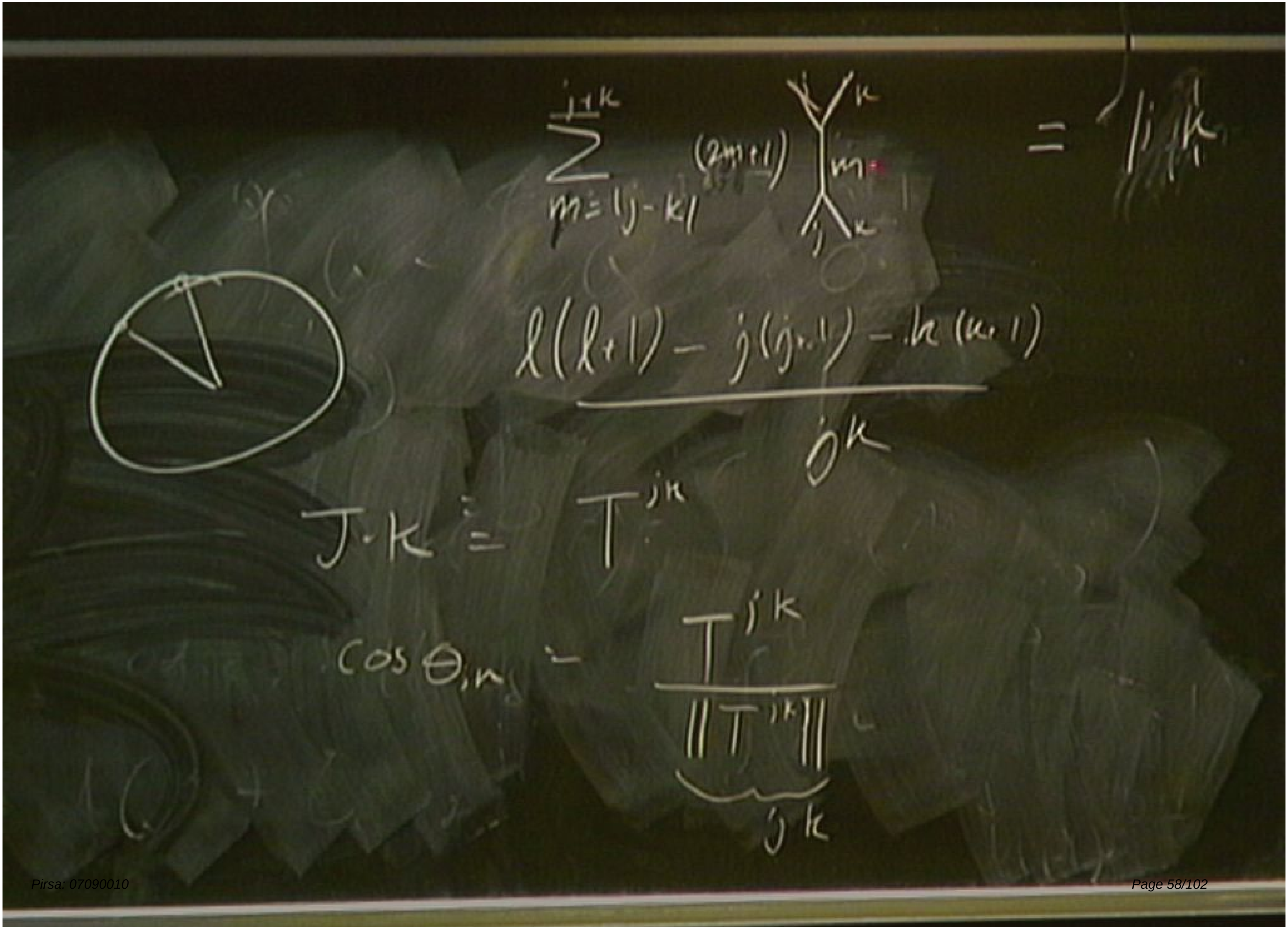
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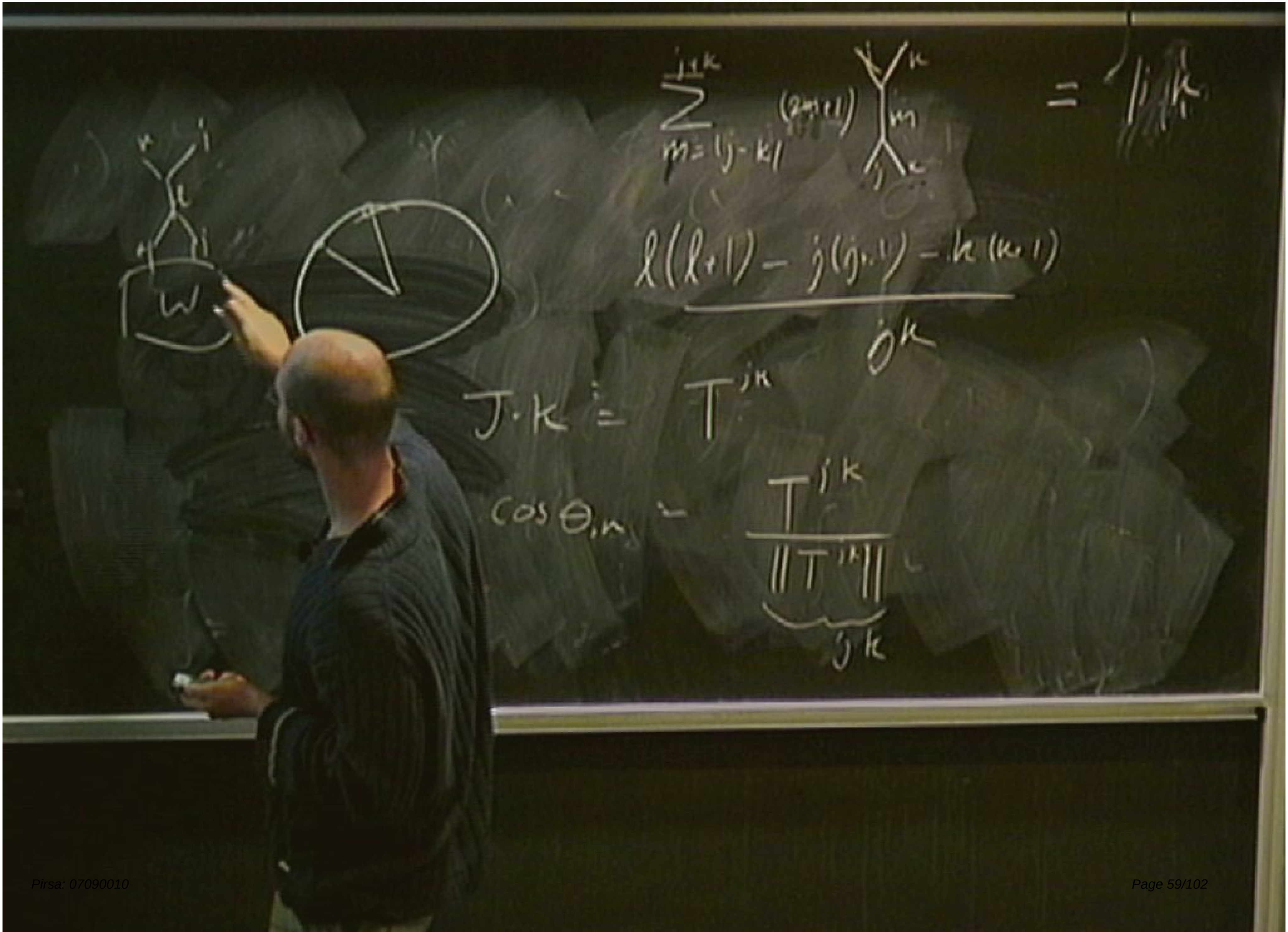

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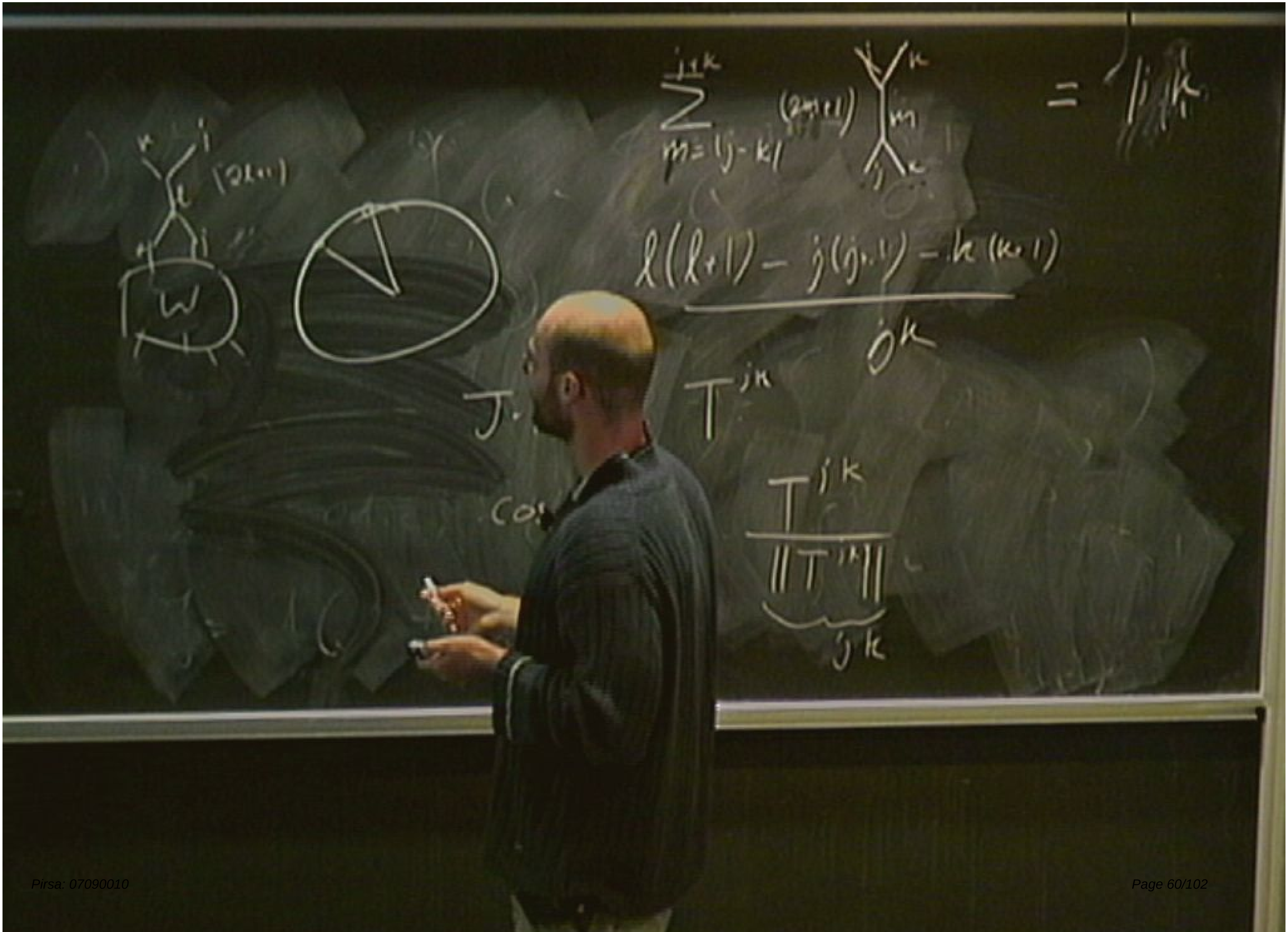

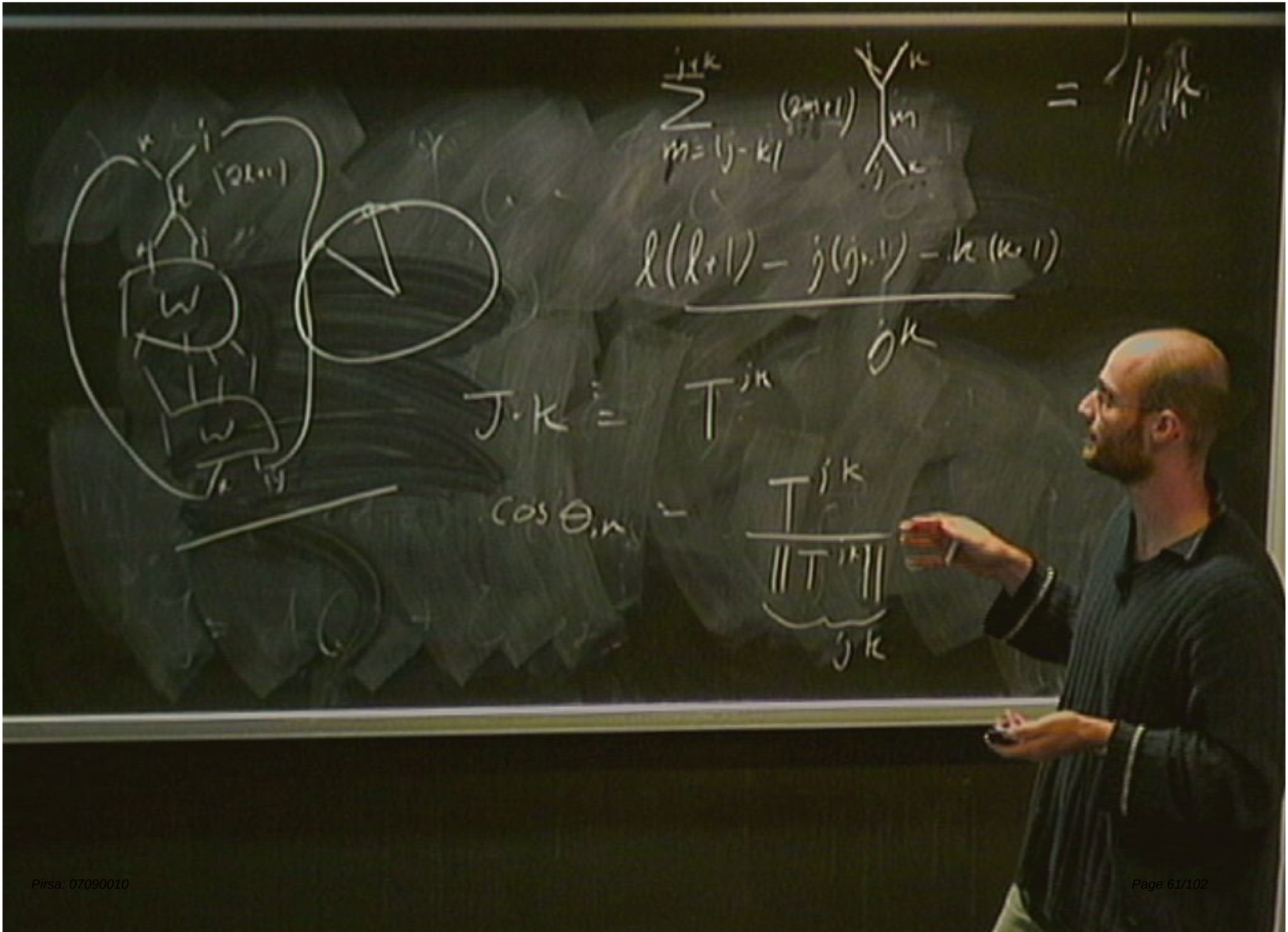

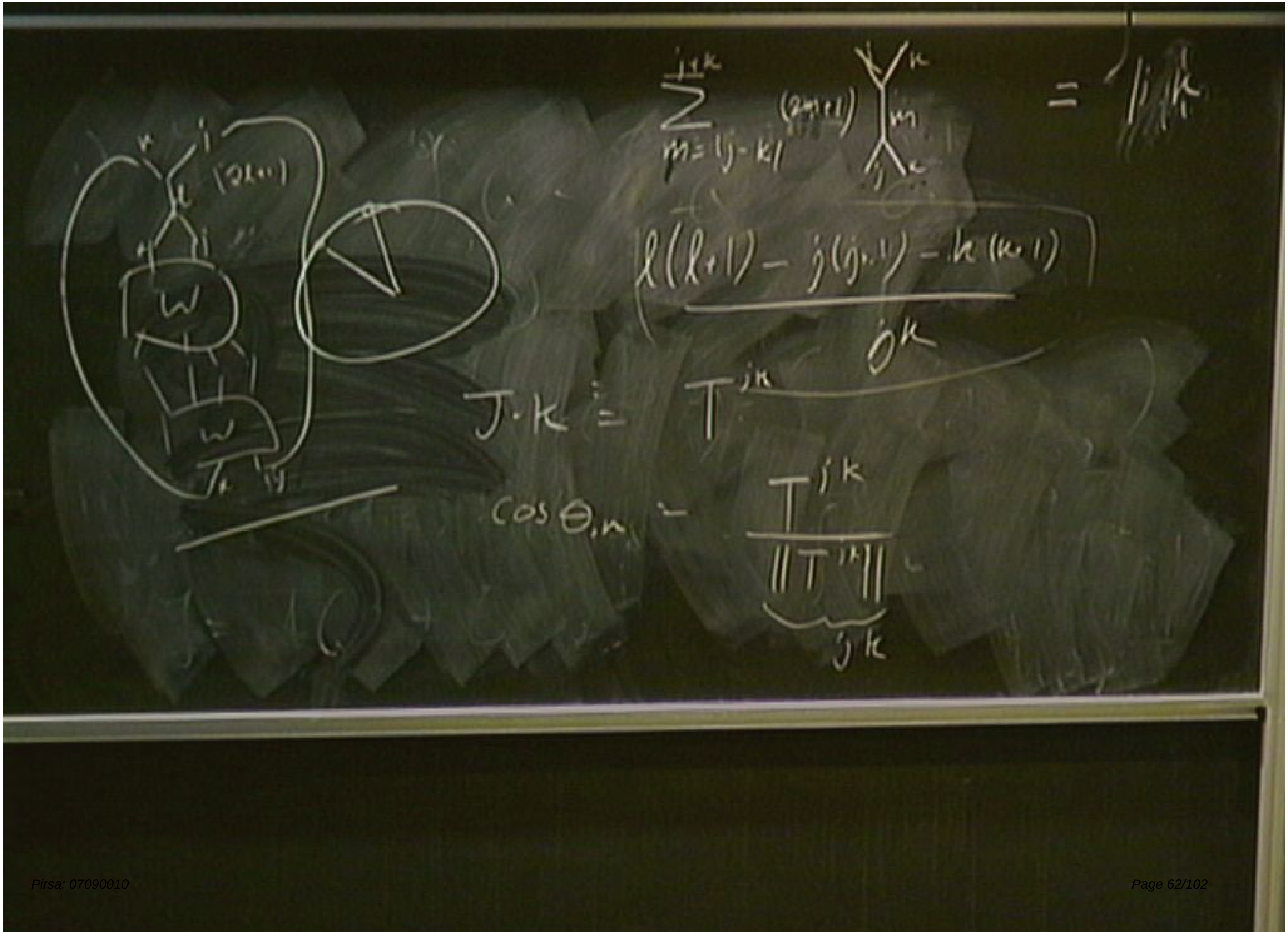
m = |j - k|
j + k
(2m+1)
(2k+1)
W
W
l(l+1) - j(j+1) - k(k+1)
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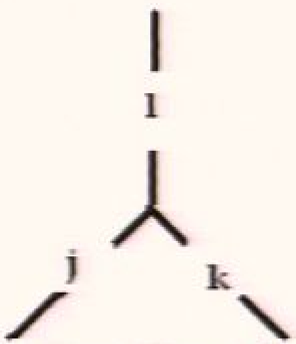
||T^{jk}||

Scalar Product



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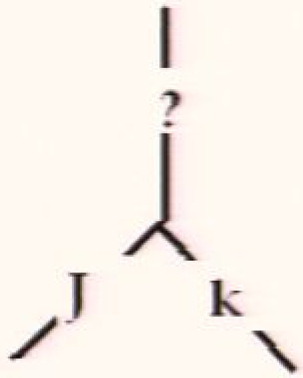

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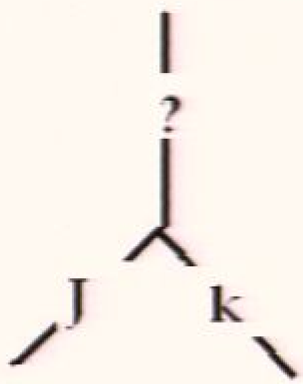

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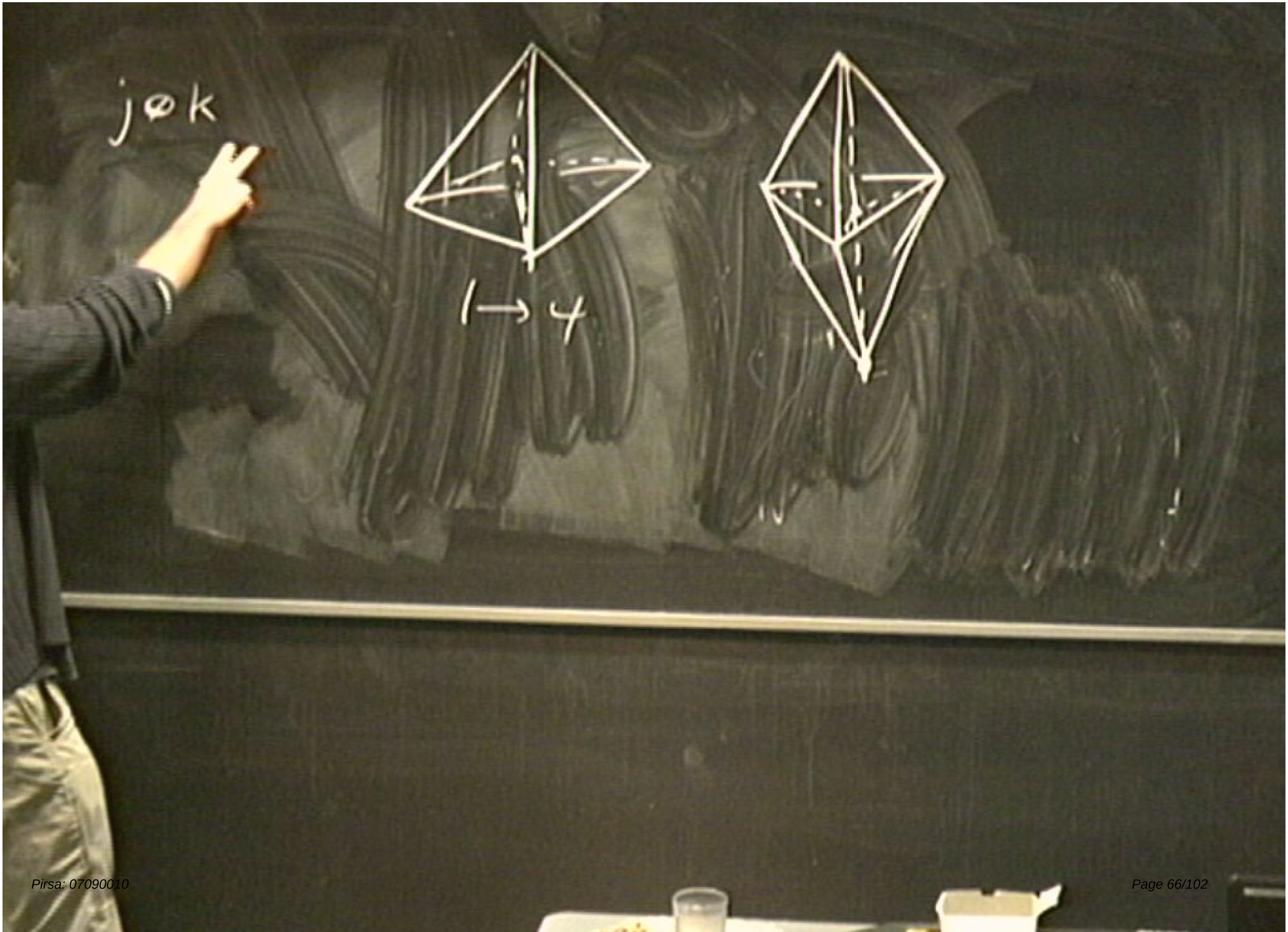
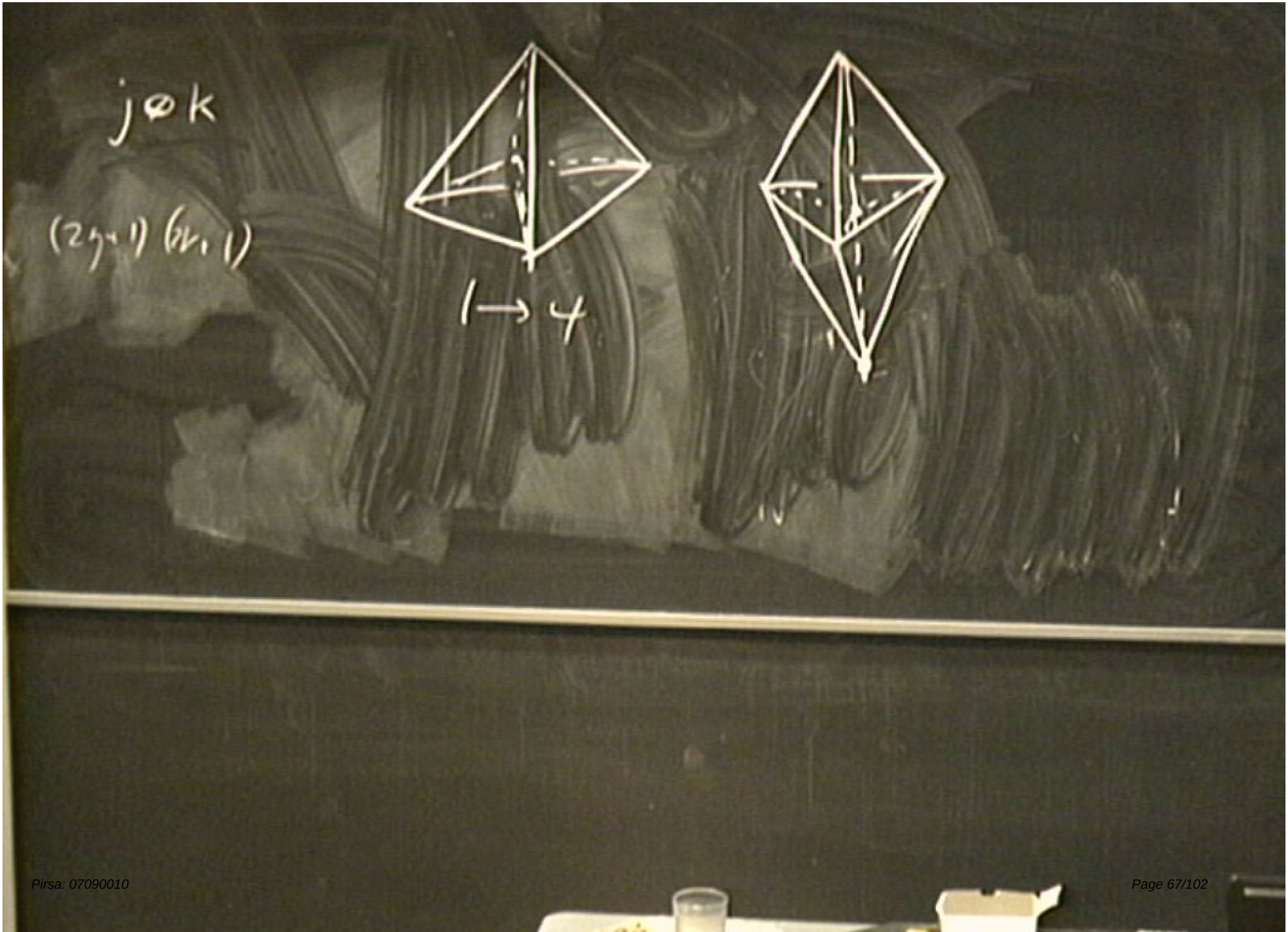
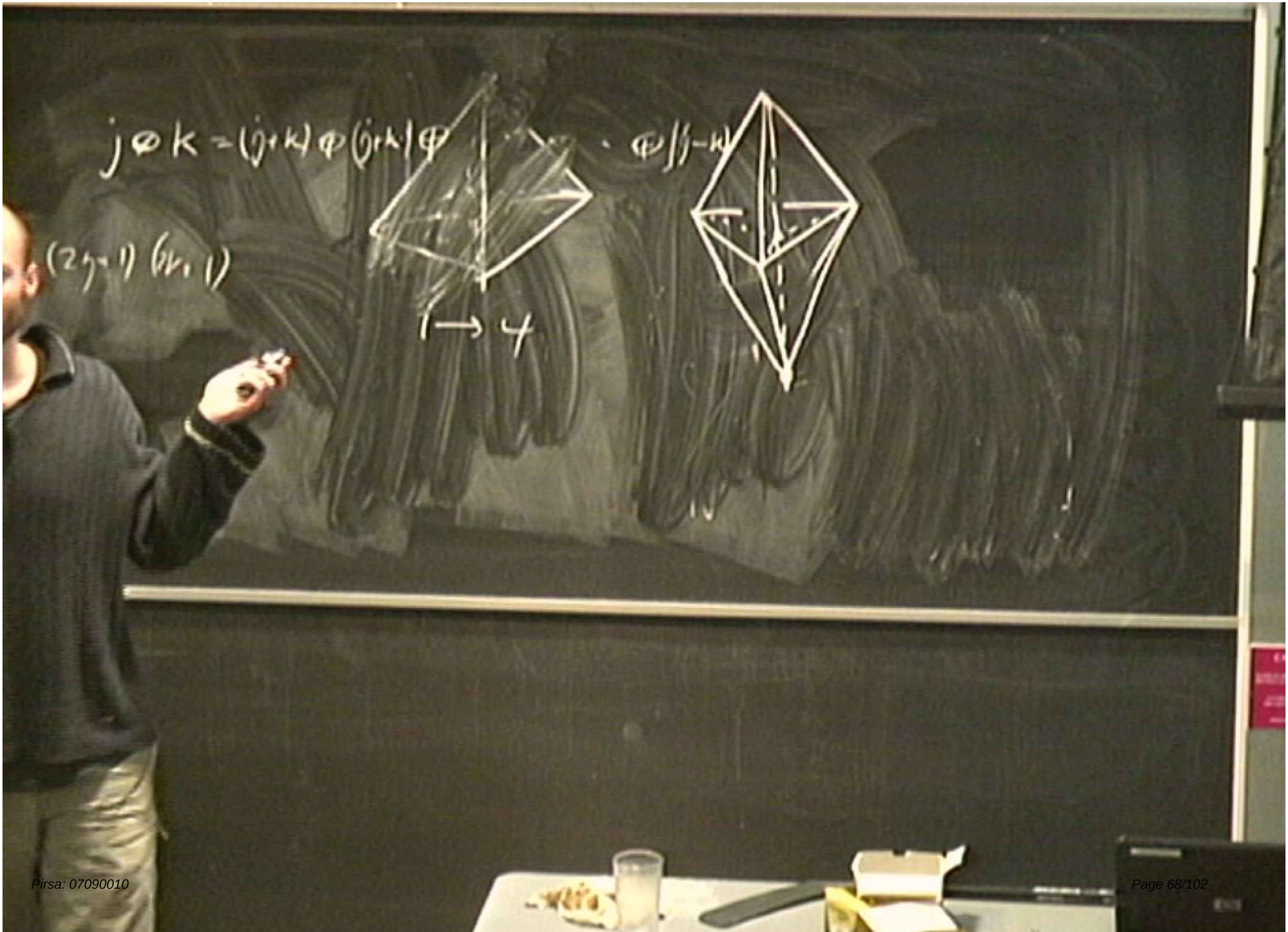
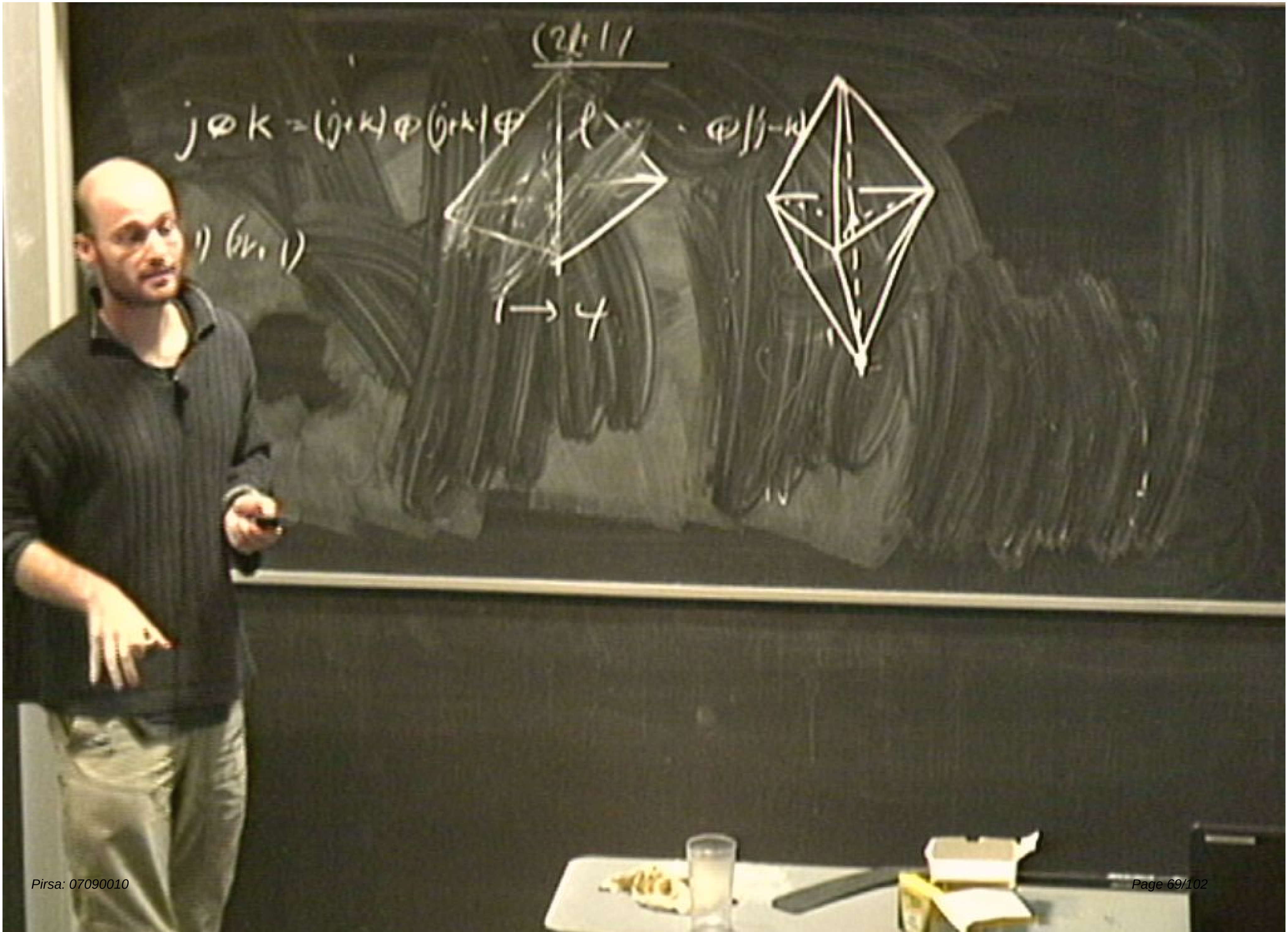
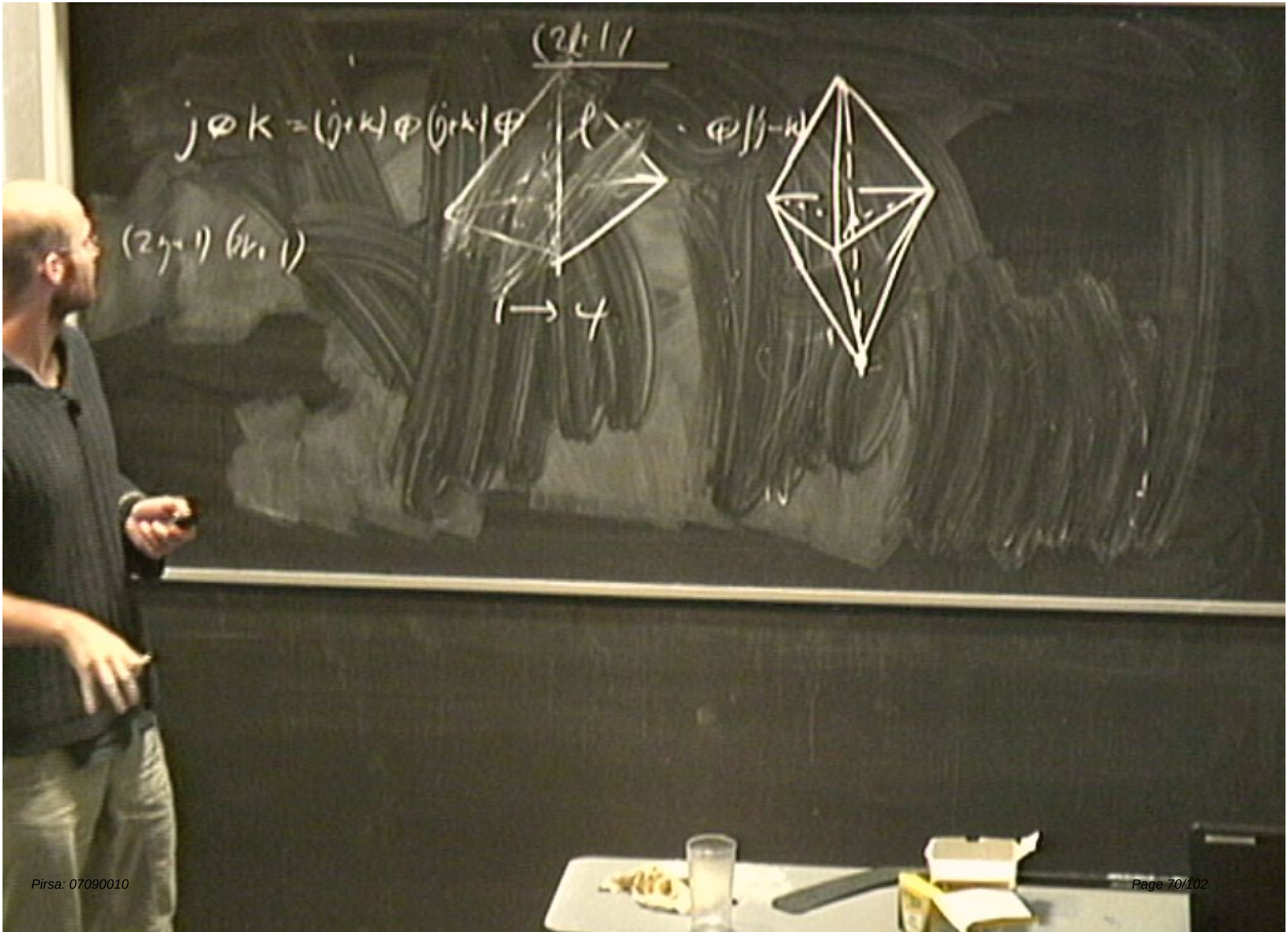
Answer:


jøk
1 → 4


j⊗k
(2j+1)(2k+1)
1 → 4

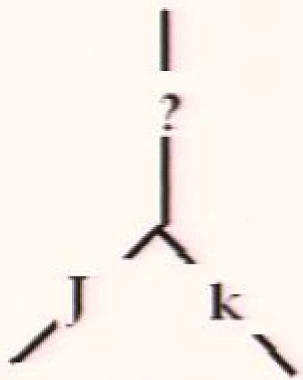






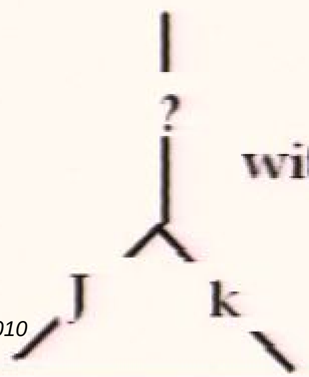
Dynamics

Suppose we have the following spin network, what value should



we attribute to "?" ?

Answer:

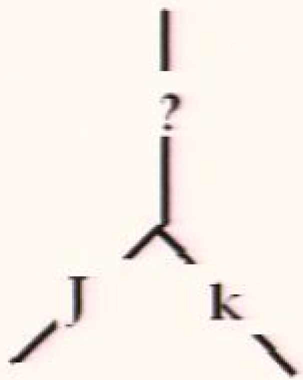


?=m

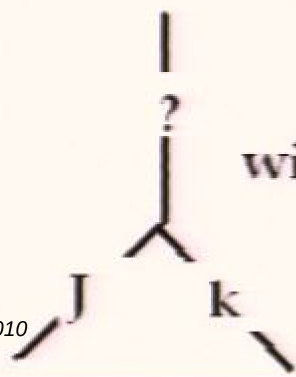
with $P = \dfrac{2m+1}{(2k+1)(2J+1)}$

Dynamics

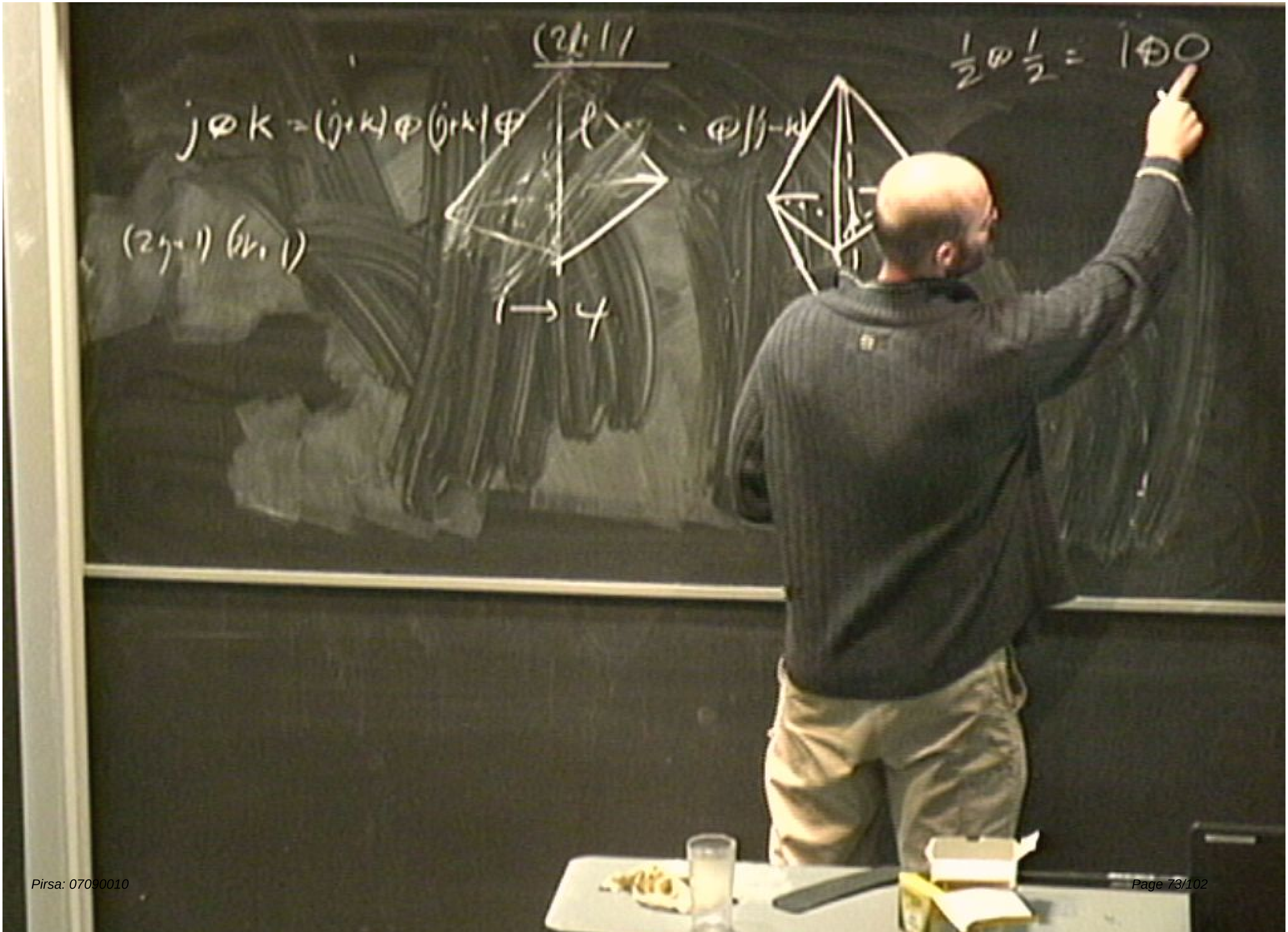
Suppose we have the following spin network, what value should

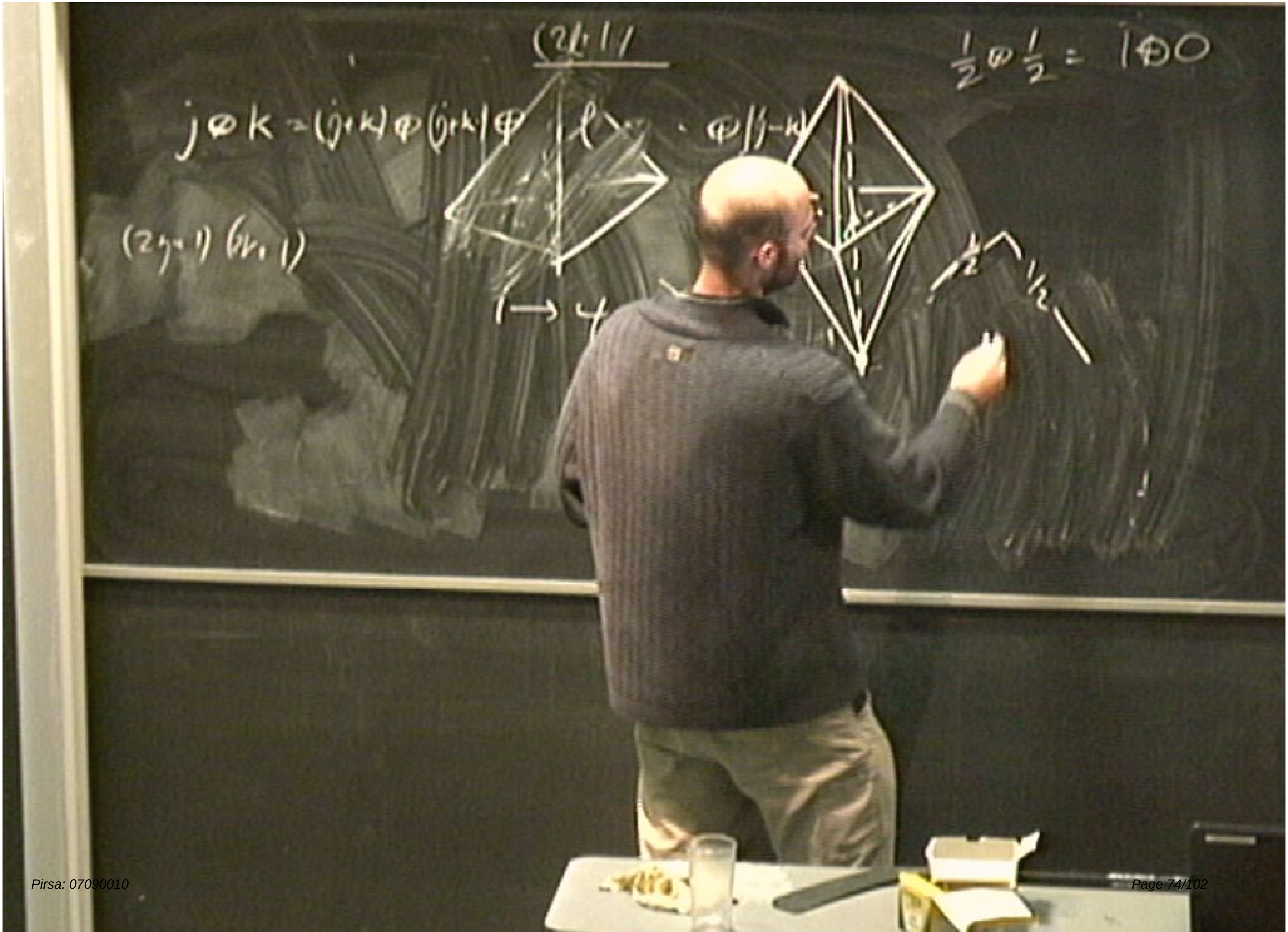


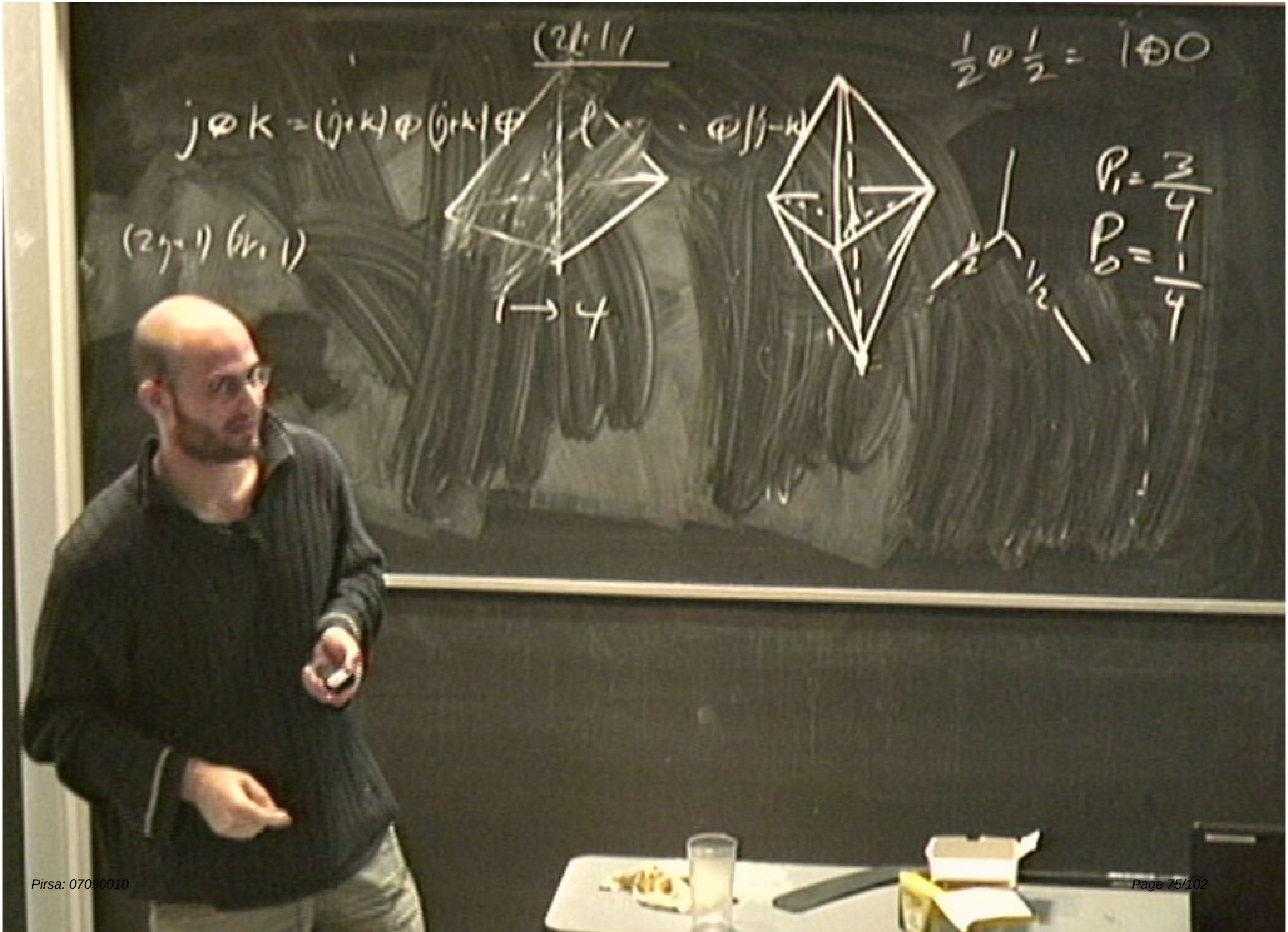
we attribute to “?" ? Answer:



$$? = m \quad \text{with } P = \frac{2m+1}{(2k+1)(2J+1)}$$



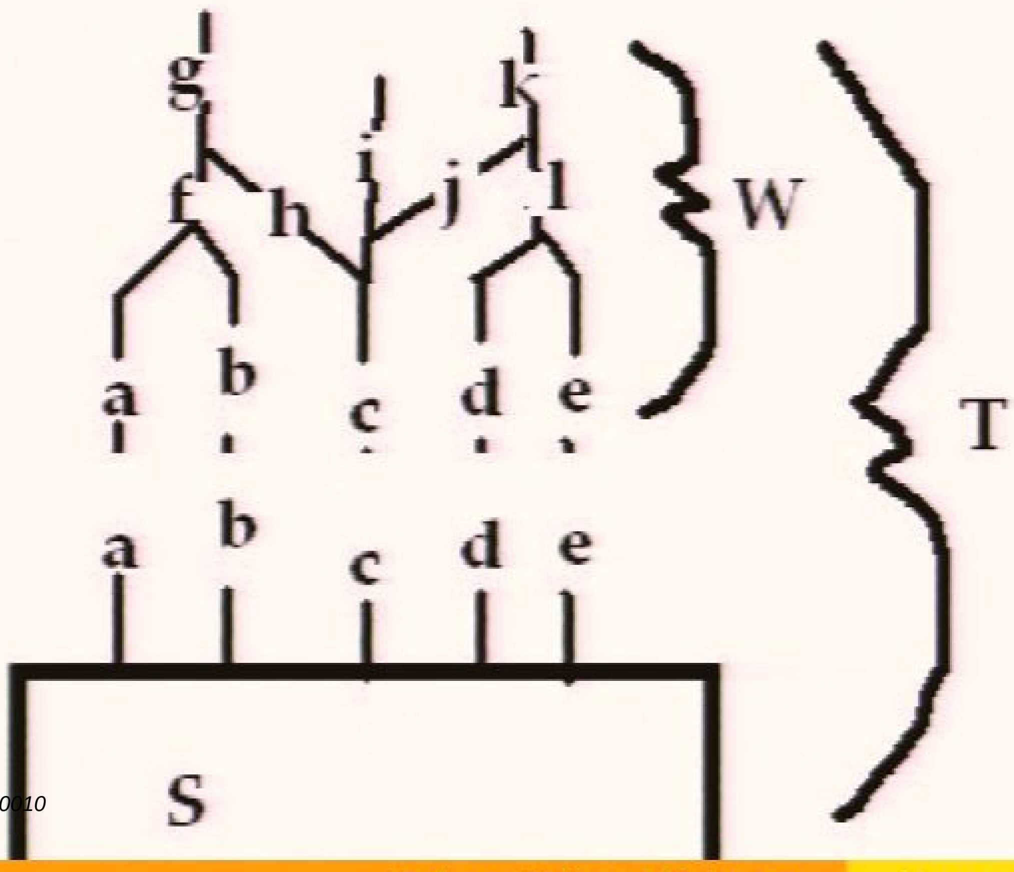




½ ⊗ ½ = 1 ⊕ 0
1 → 4

Dynamics

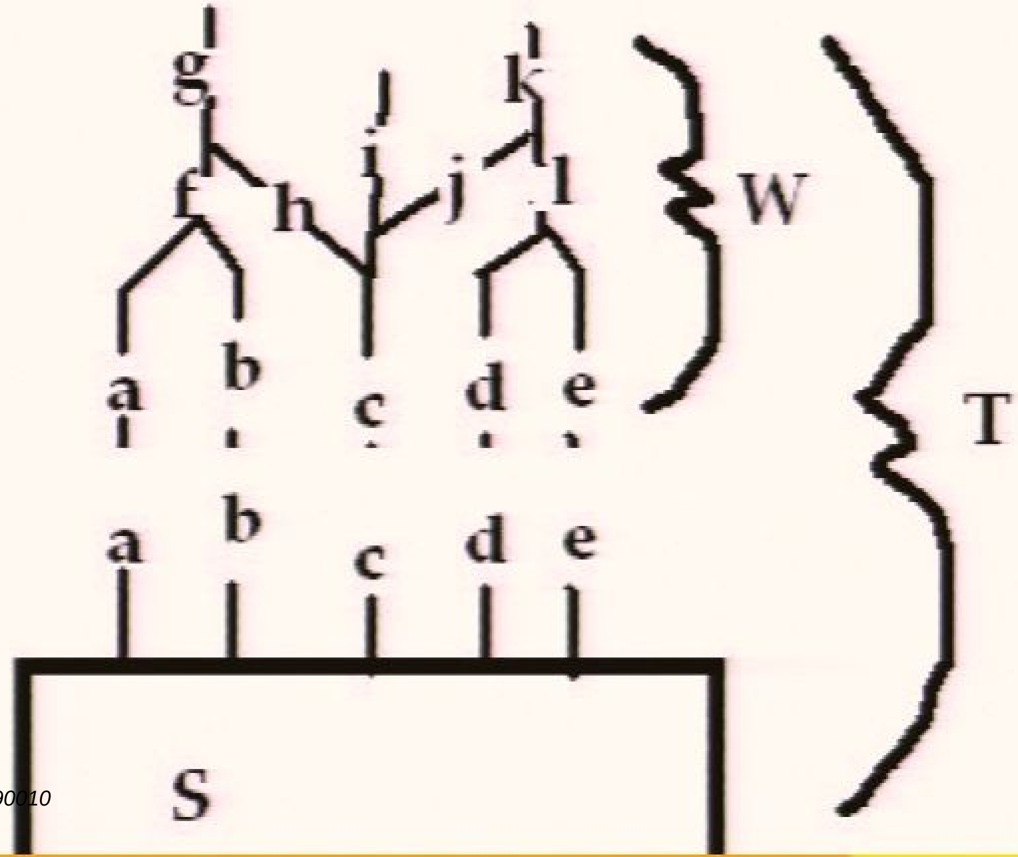
More generally:



$P = ?$

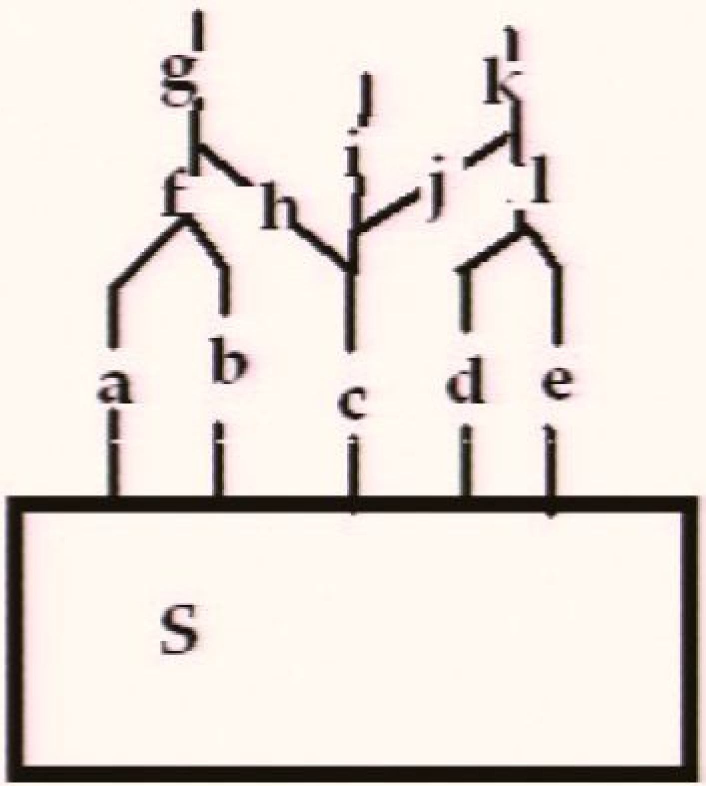
Dynamics

More generally:

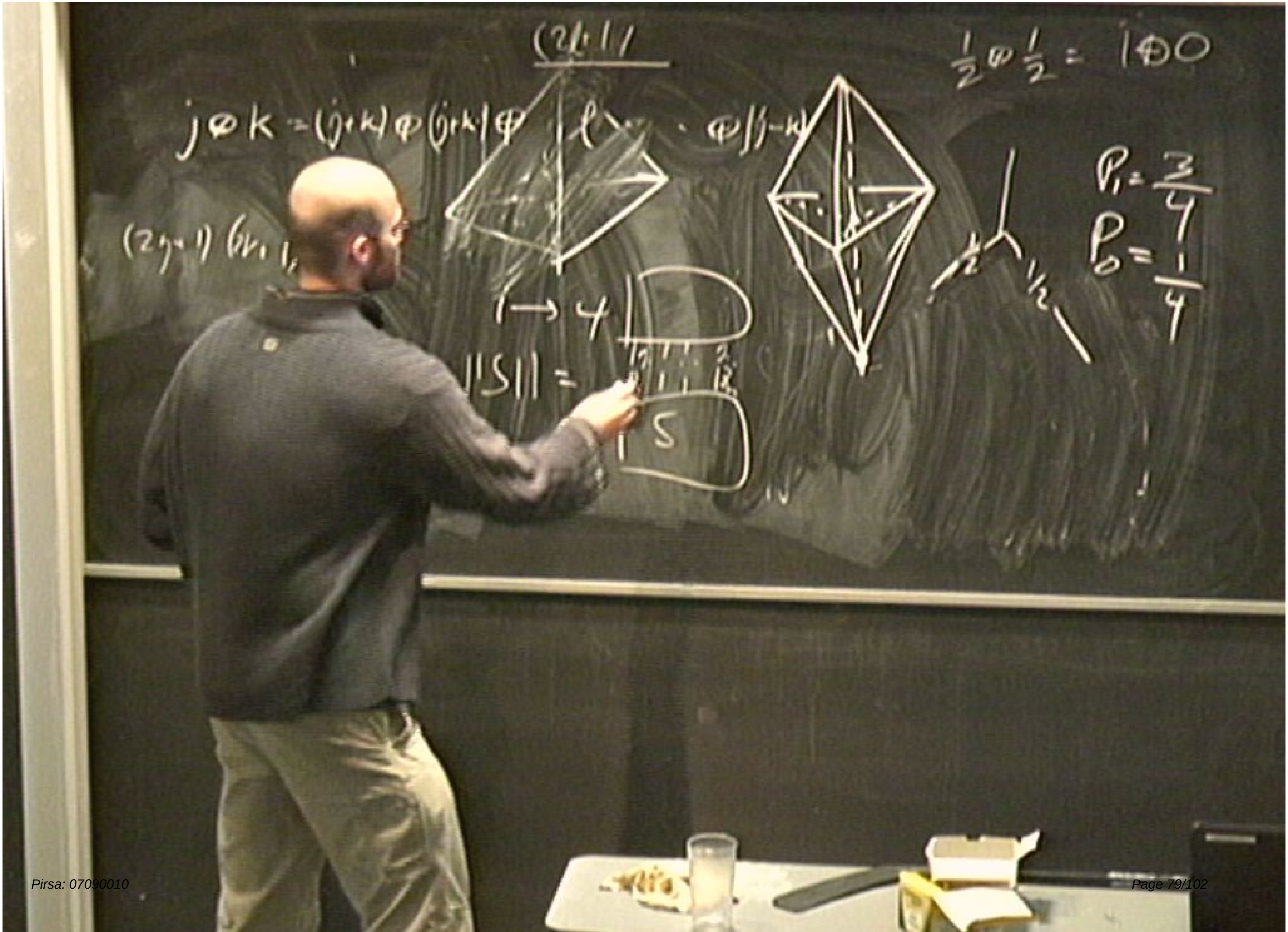


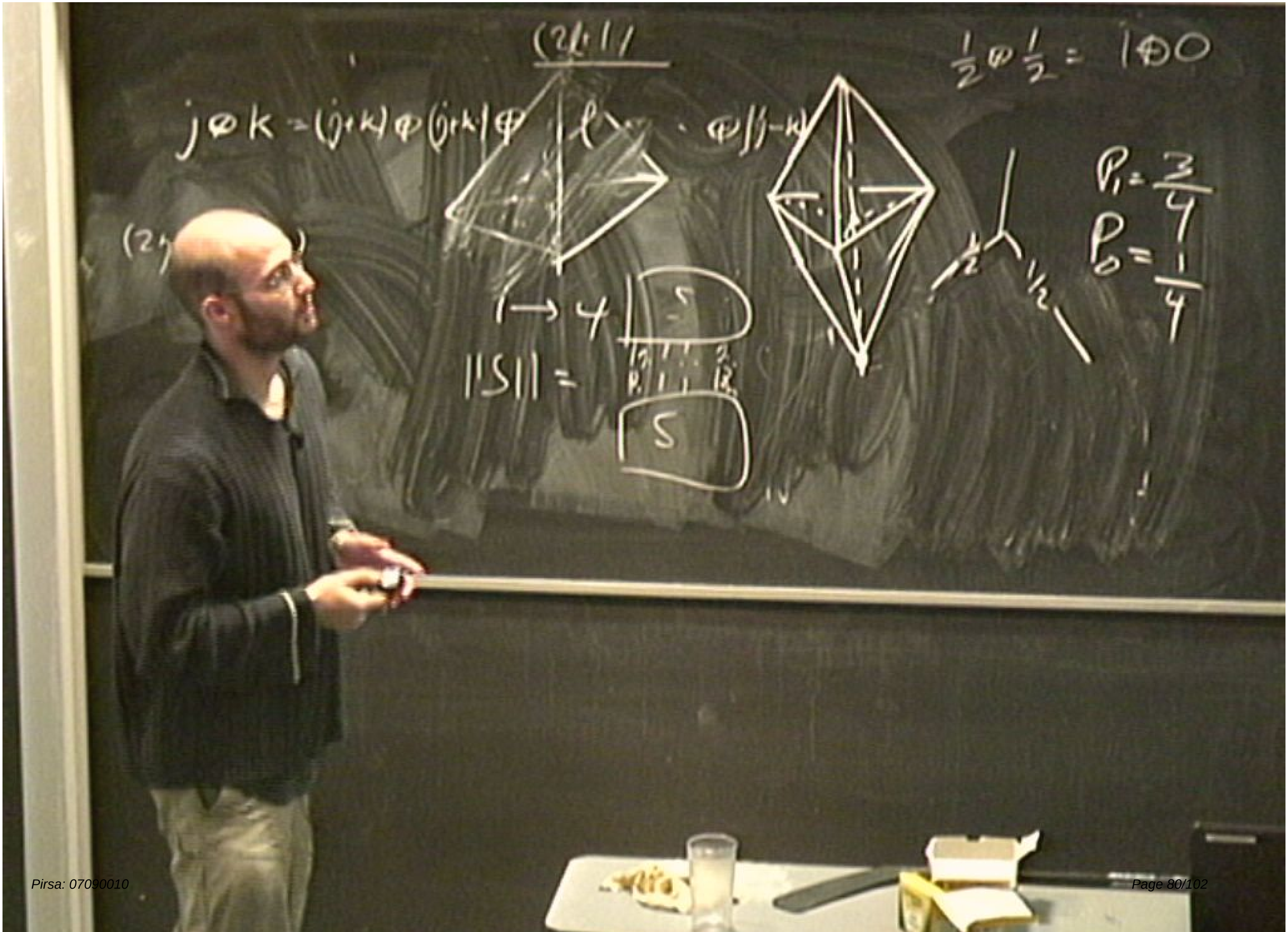
$P = ?$

Dynamics

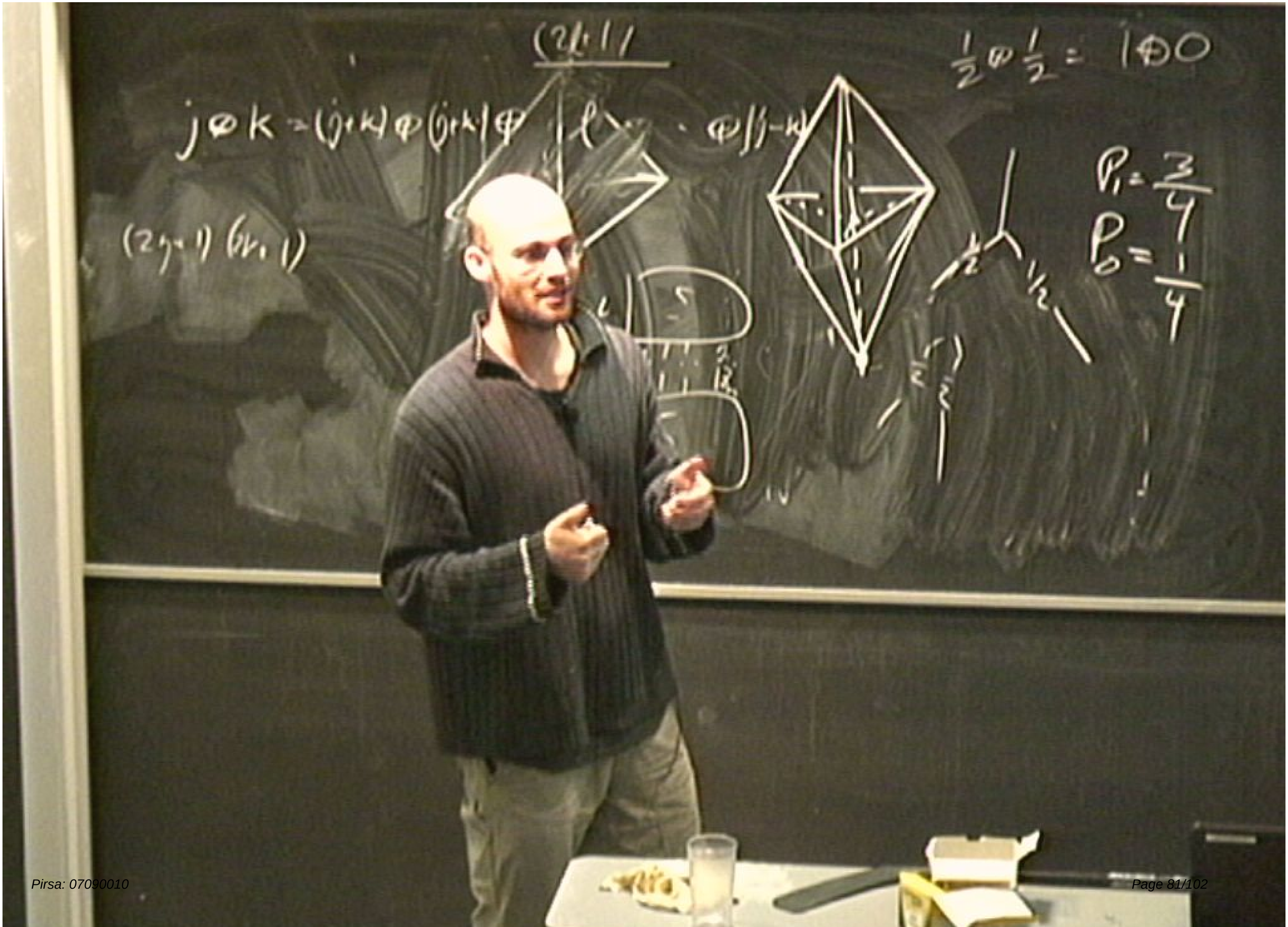


$$P = \frac{\|T\|\,(2g+1)(2i+1)(2k+1)}{\|S\| \quad \|W(f,g,h,i,j,k,l)\|}$$

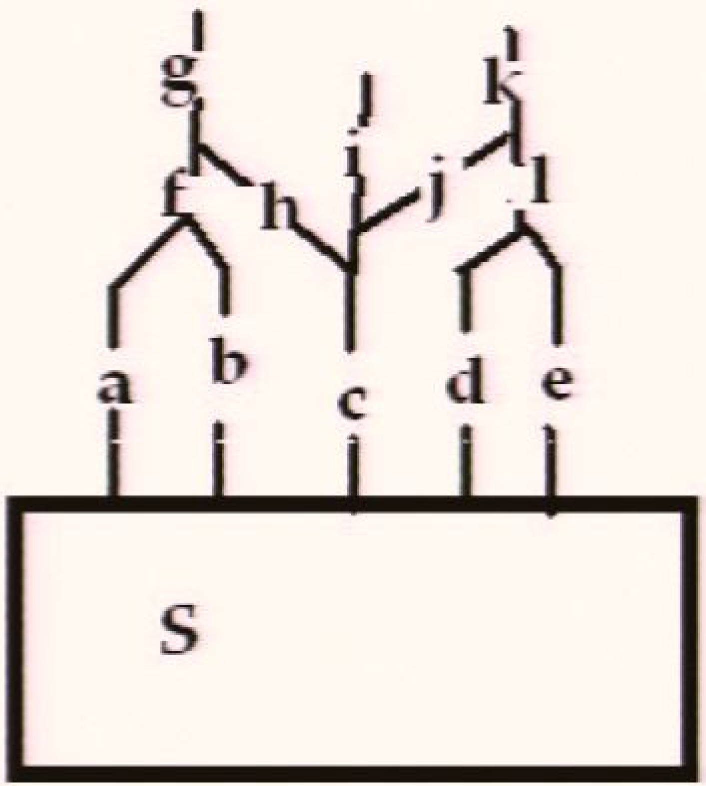

1/2 ⊗ 1/2 = 1 ⊕ 0
ρ1 = 3/4
ρ0 = 1/4
1/2
1/2
S



½ ⊕ ½ = 1 ⊕ 0
ρ1 = 3/4
ρ0 = 1/4
||S|| =

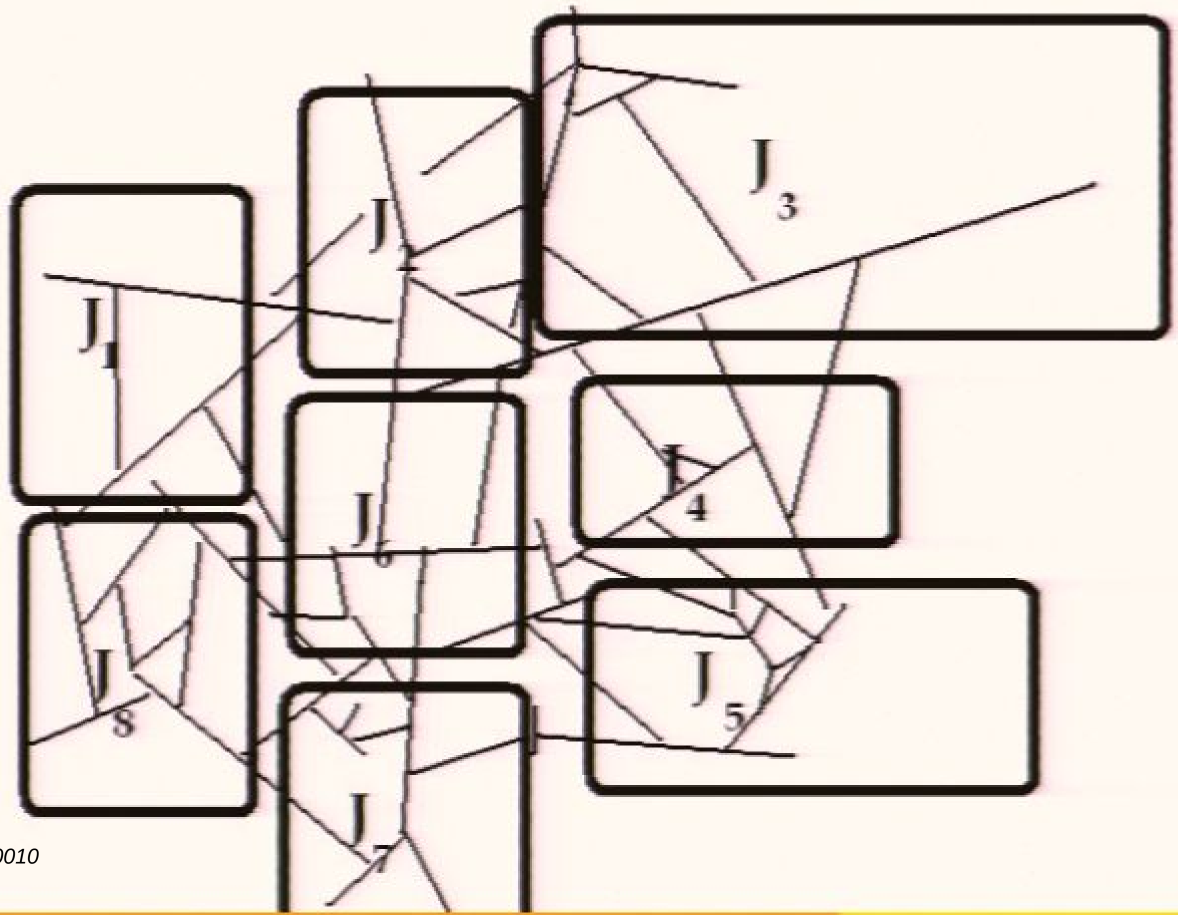

½ ⊗ ½ = 1 ⊕ 0
ρ1 = 3/4
ρ0 = 1/4

Dynamics

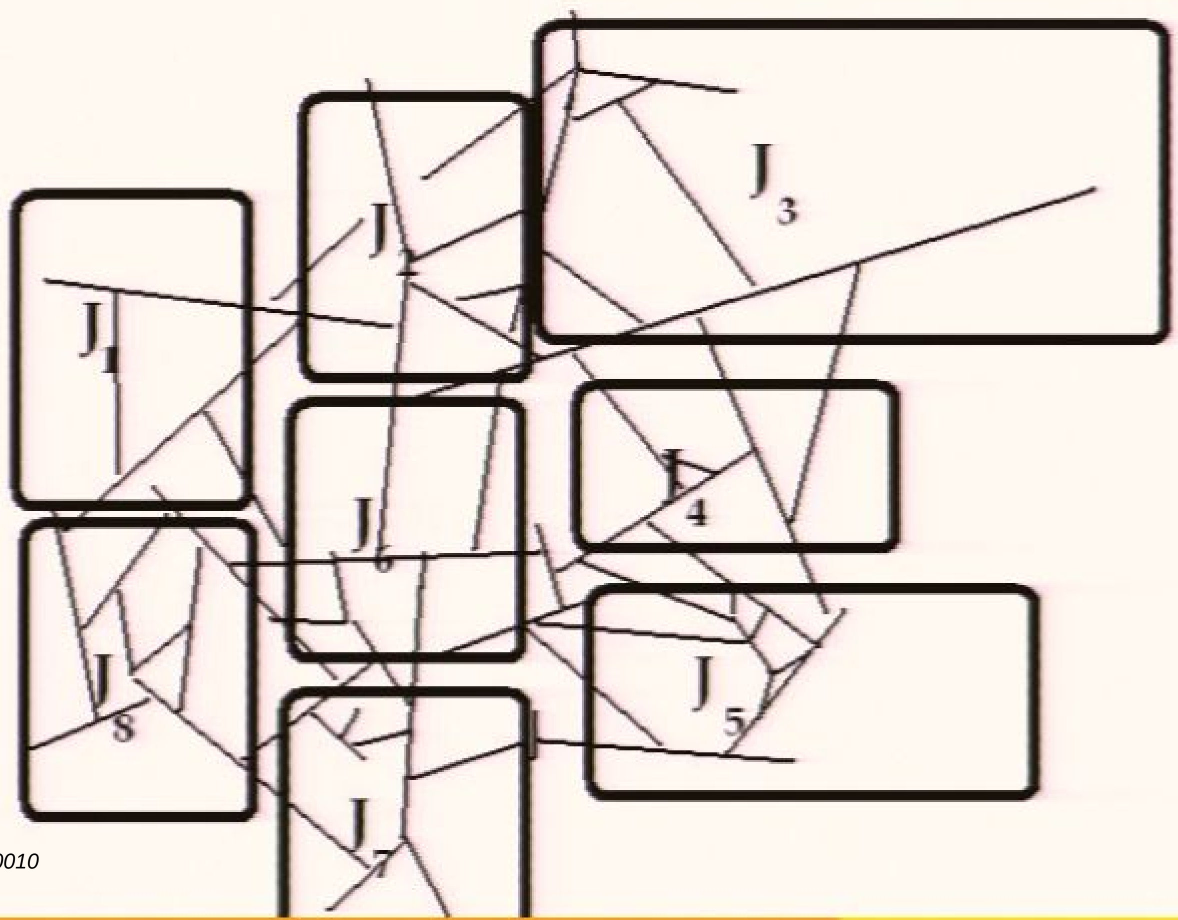


$$P = \frac{\|T\| \, (2g+1)(2i+1)(2k+1)}{\|S\| \quad \|W(f,g,h,i,j,k,l)\|}$$

Spin Geometry Theorem



Spin Geometry Theorem



Spin Geometry Theorem

Define:

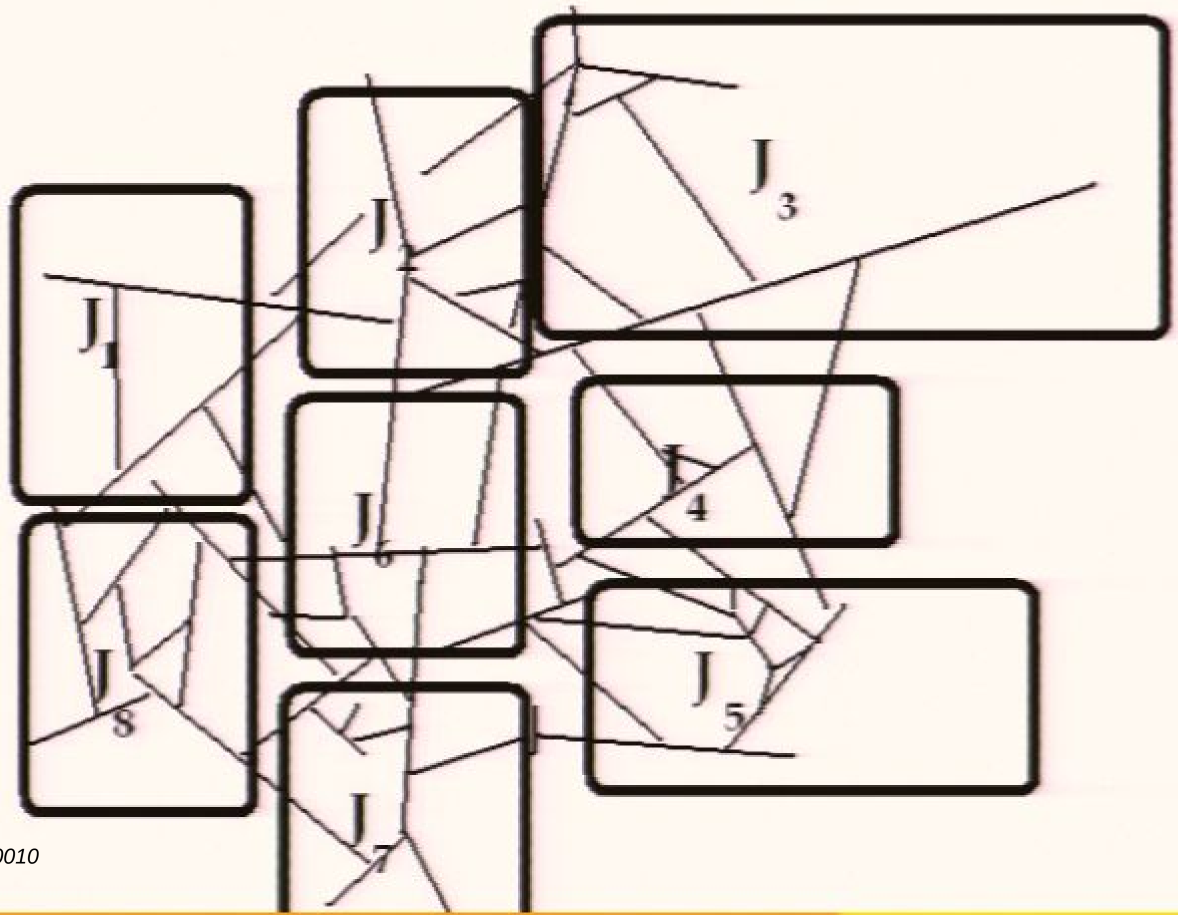
$$T_{a,b} = \mathbf{J_a} \cdot \mathbf{J_b}$$
$$\widehat{T_{a,b}} = \frac{T_{a,b}}{||T_{a,b}||} \qquad (3)$$

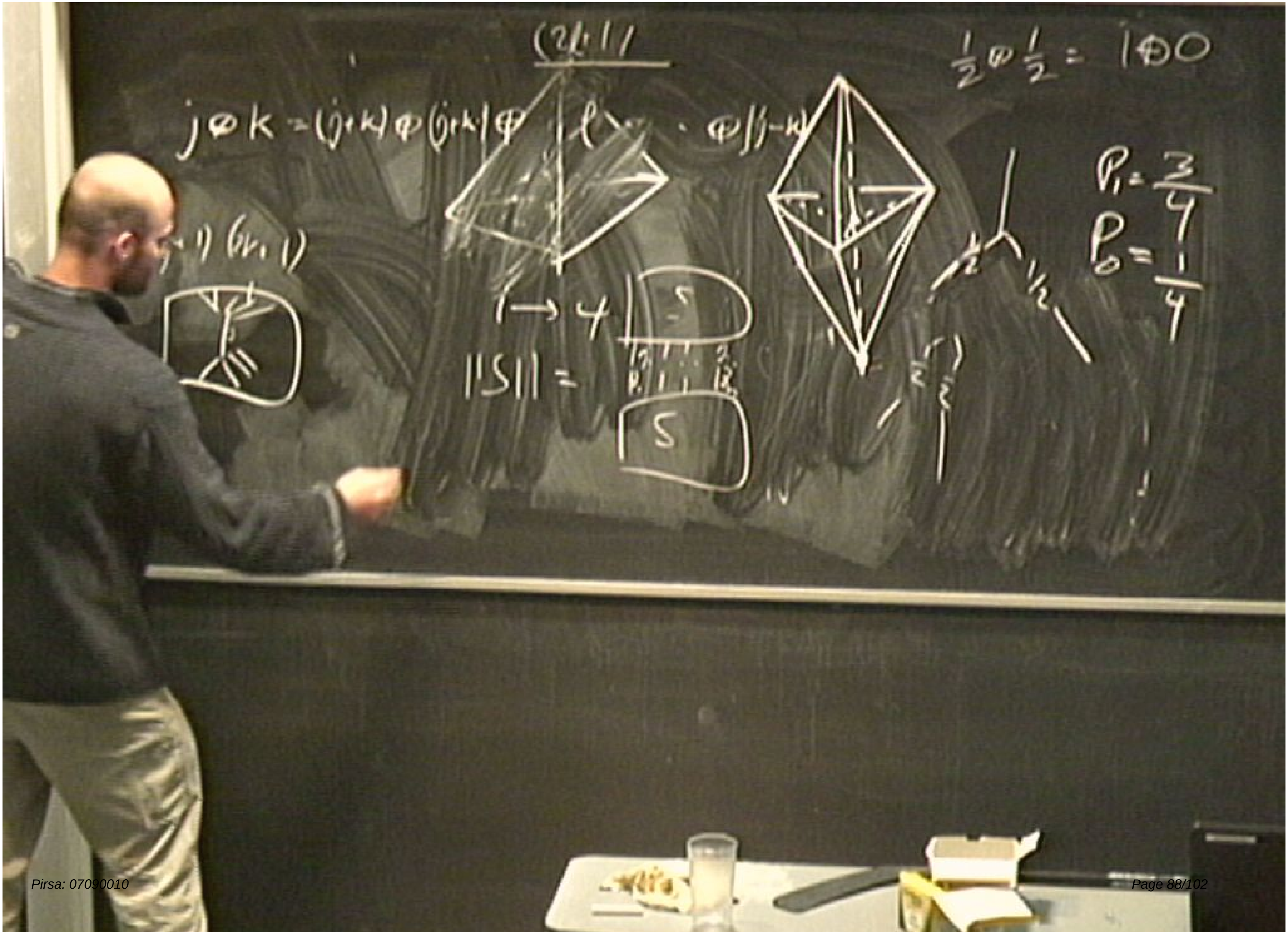
Spin Geometry Theorem

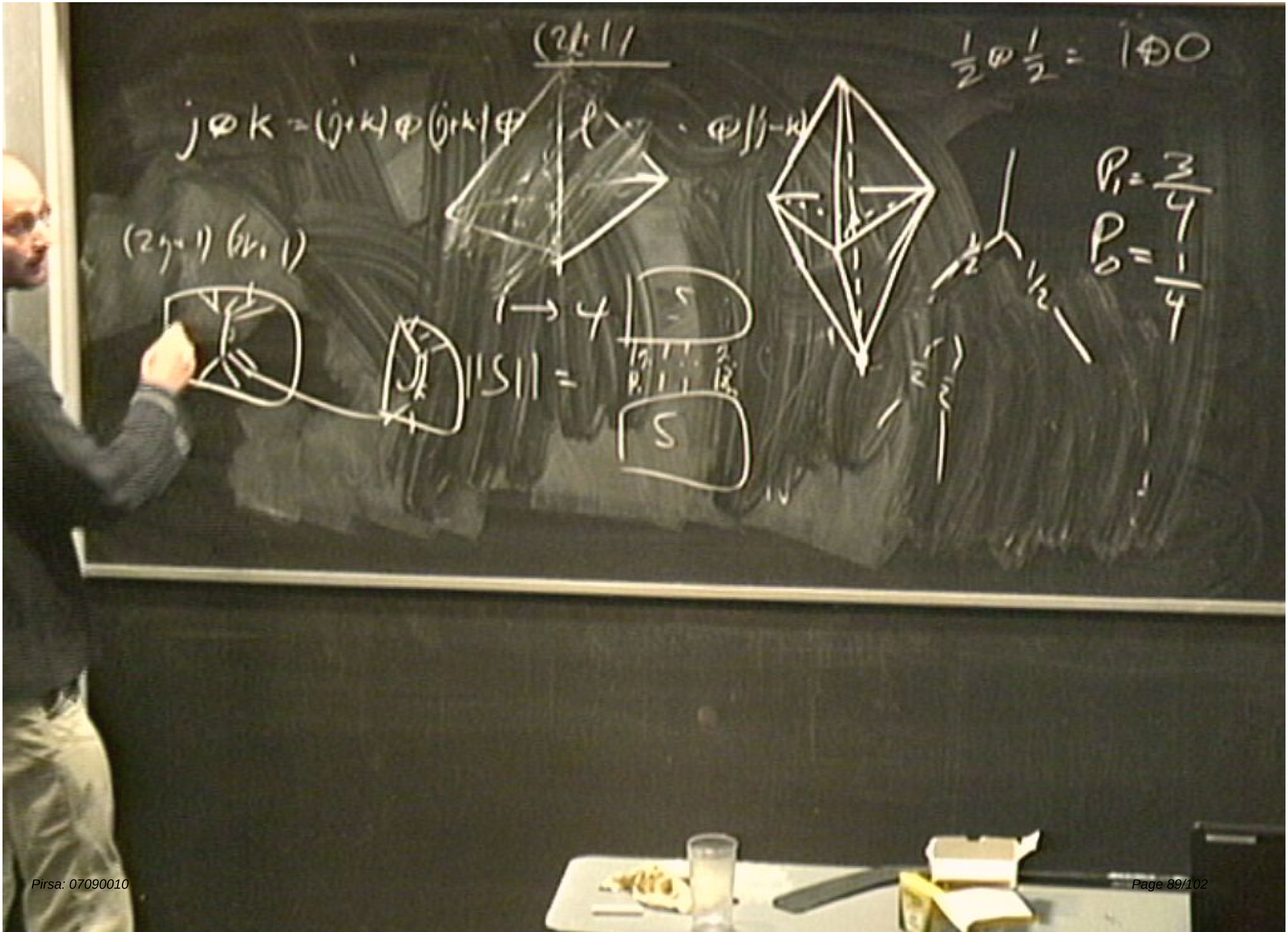
Define: Then the Spin Geometry Theorem states that if I is a set containing 4 elements then

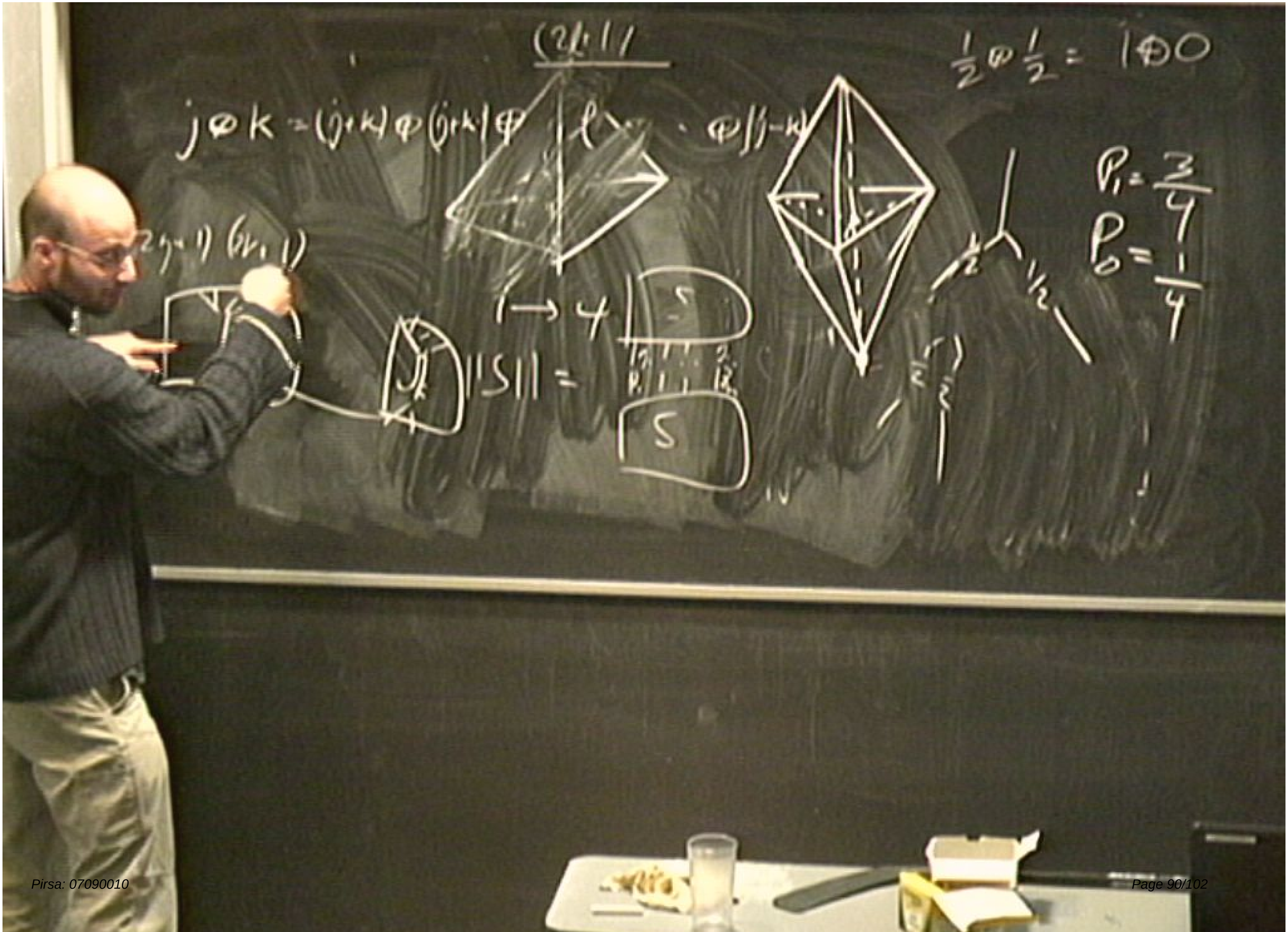
$$\begin{aligned} &\forall \varepsilon > 0,\ \exists \delta > 0\ s.t. \\ \forall\, a,b \in I \quad &< \widehat{T_{a,b}}^{2} - < \widehat{T_{a,b}} >^{2} >< \delta\ then \\ < det \widehat{T_{a,b}}_{a,b\in I} >< \varepsilon\ &and\ det < \widehat{T_{a,b}}_{a,b\in I} >< \varepsilon \end{aligned} \tag{4}$$

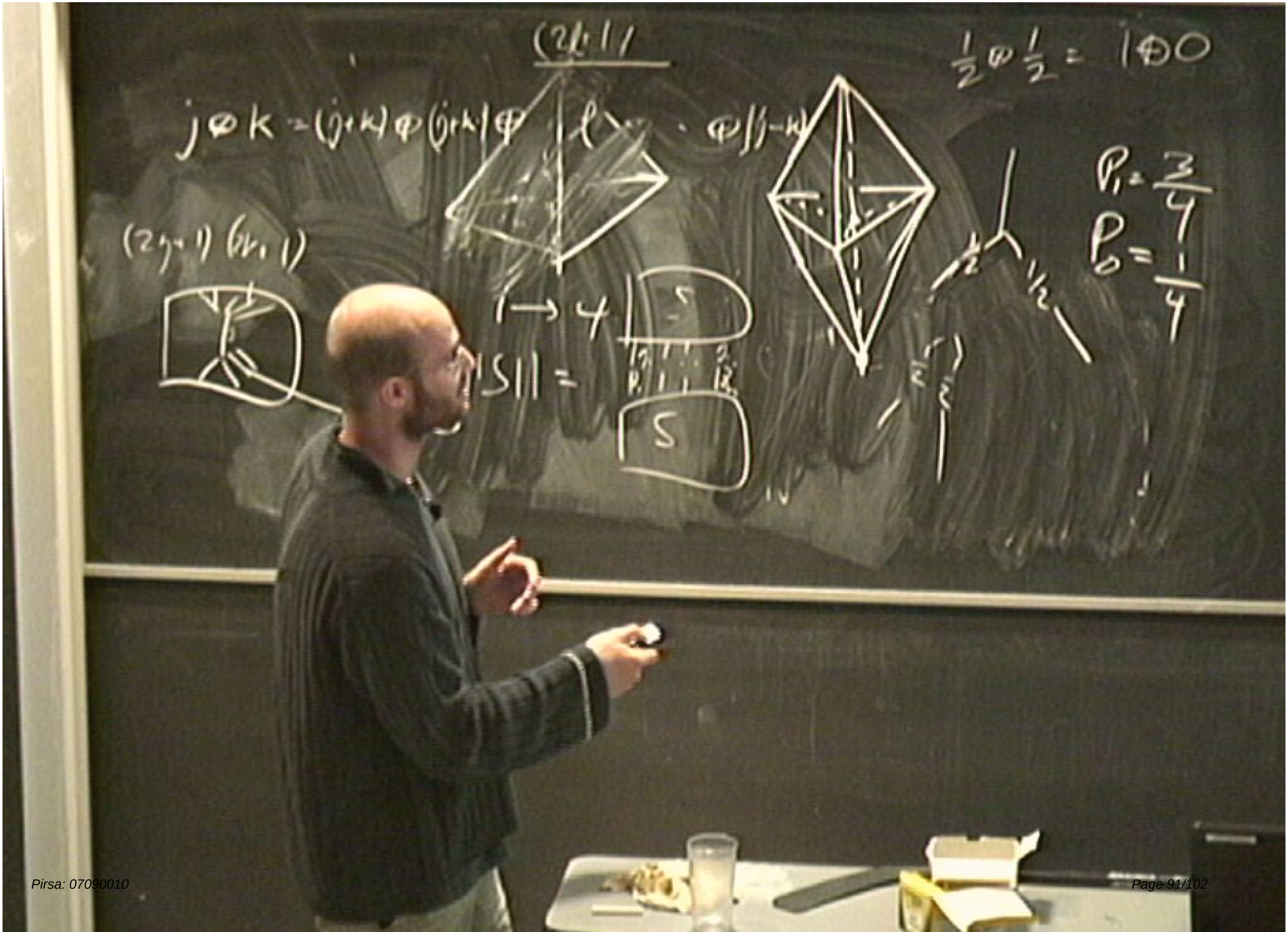
Spin Geometry Theorem




½ ⊕ ½ = 1 ⊕ 0
ρ1 = 3/4
ρ0 = 1/4
‖S‖ =


½ ⊕ ½ = 1 ⊕ 0
P₁ = 3/4
P₀ = 1/4
S
S


1/2 ⊗ 1/2 = 1 ⊕ 0
j ⊗ k = (j+k) ⊕ (j+k-1) ⊕ ... ⊕ |j-k|
P1 = 3/4
P0 = 1/4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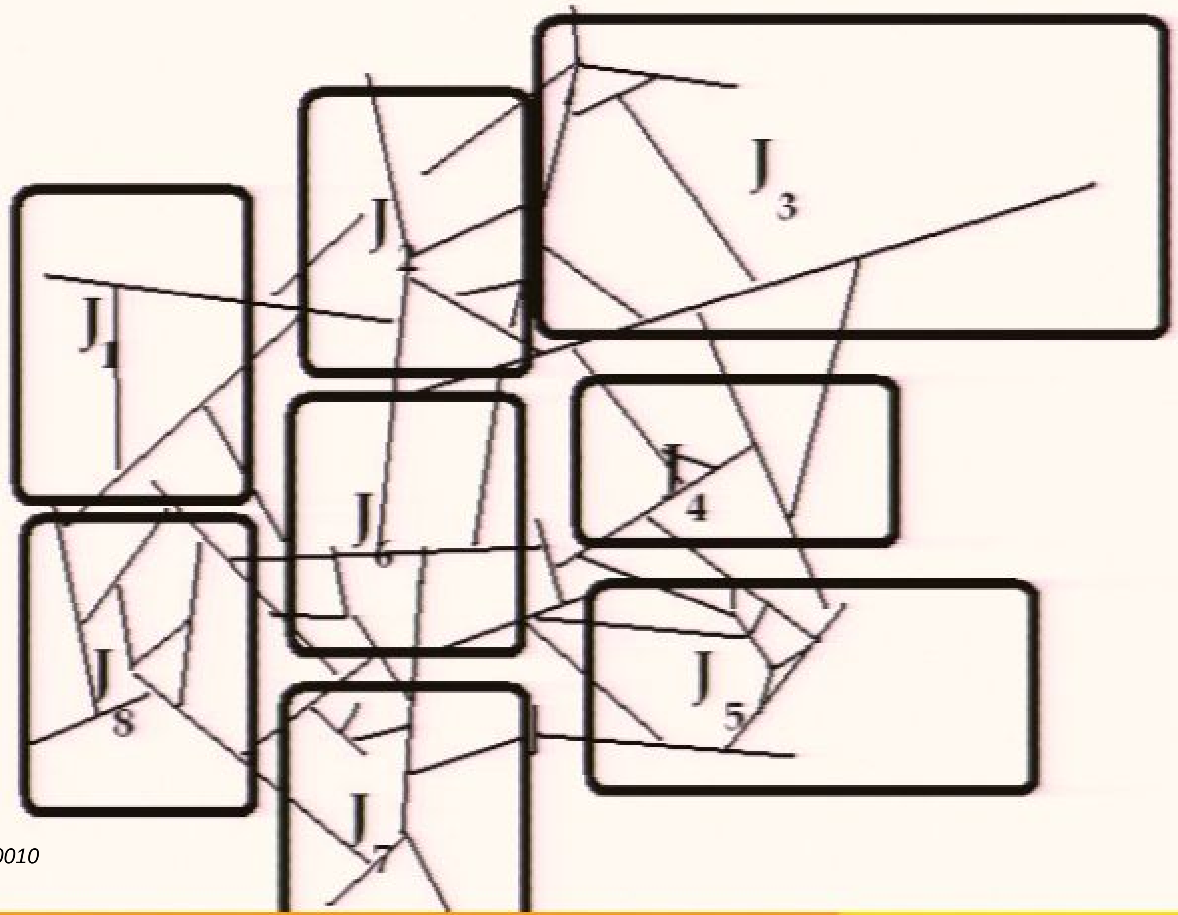
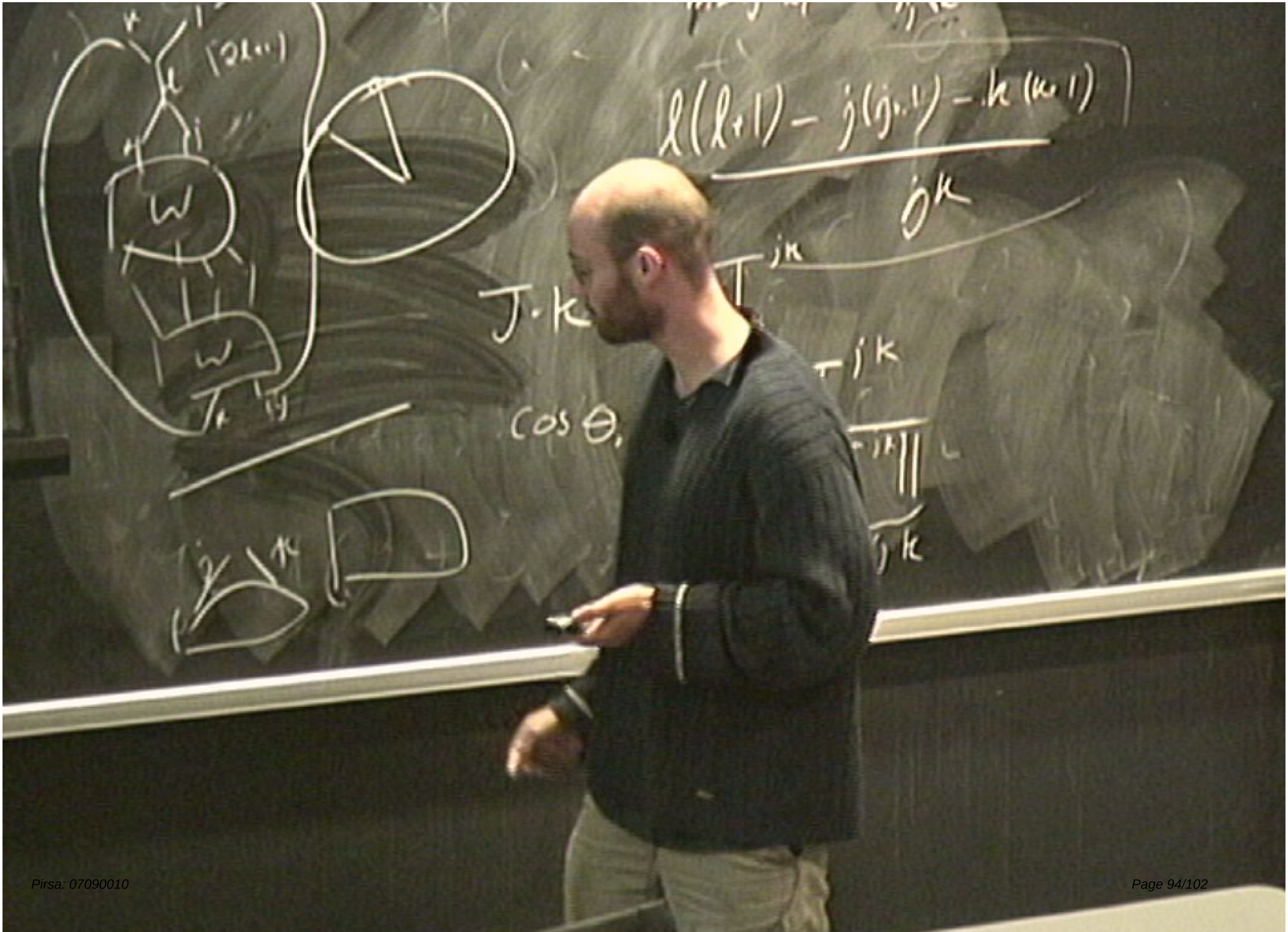
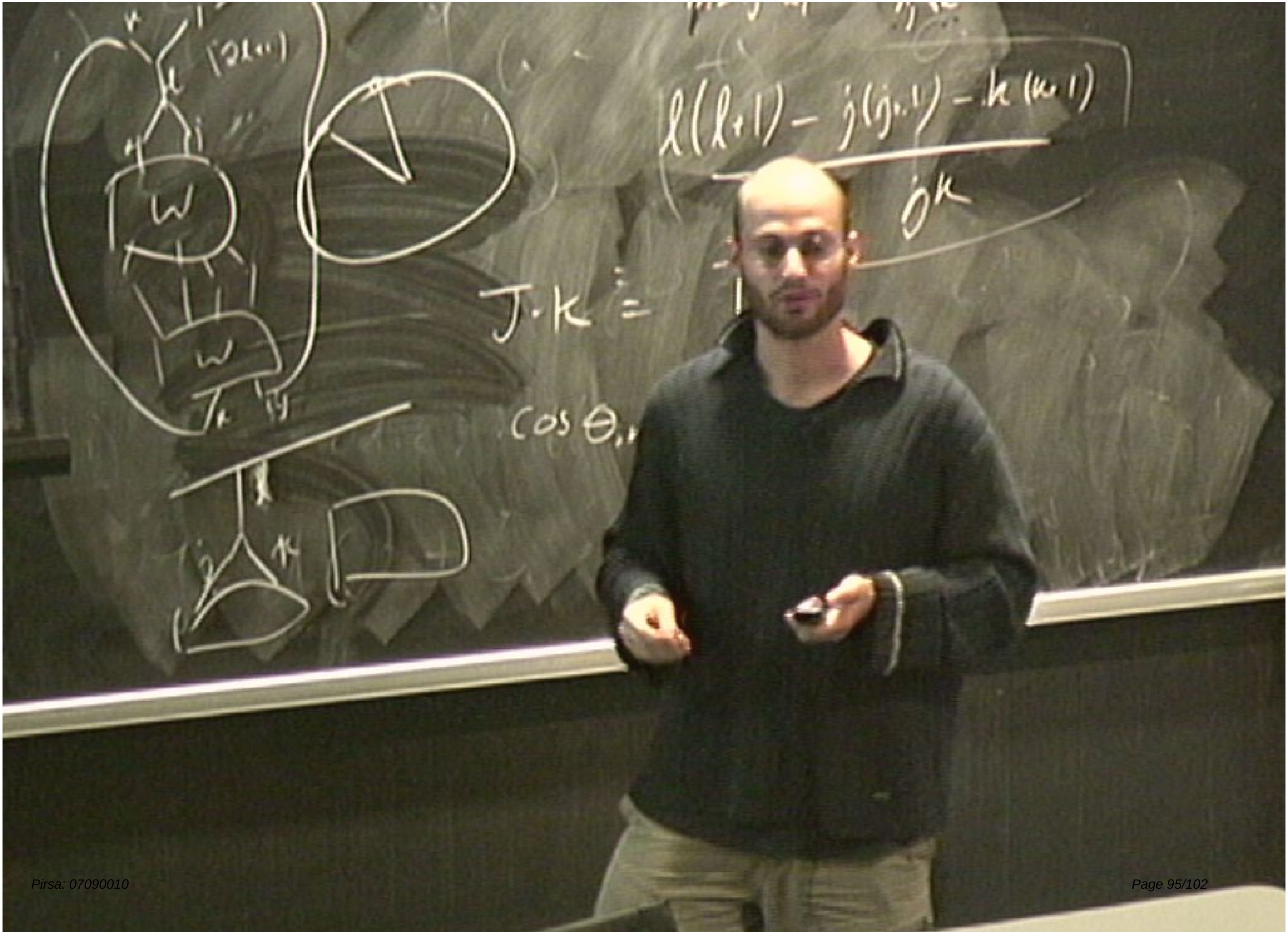
Spin Geometry Theorem



Conclusion and Outlook

- Discrete Kinematics
- Dynamics and measurement unite
  - Dynamics not yet really complete Space changes.
  - Semi-Classical limit
  - Generalization


l(l+1) - j(j+1) - k(k+1)
jk
J·k
cos θ


J·K =
cos θ

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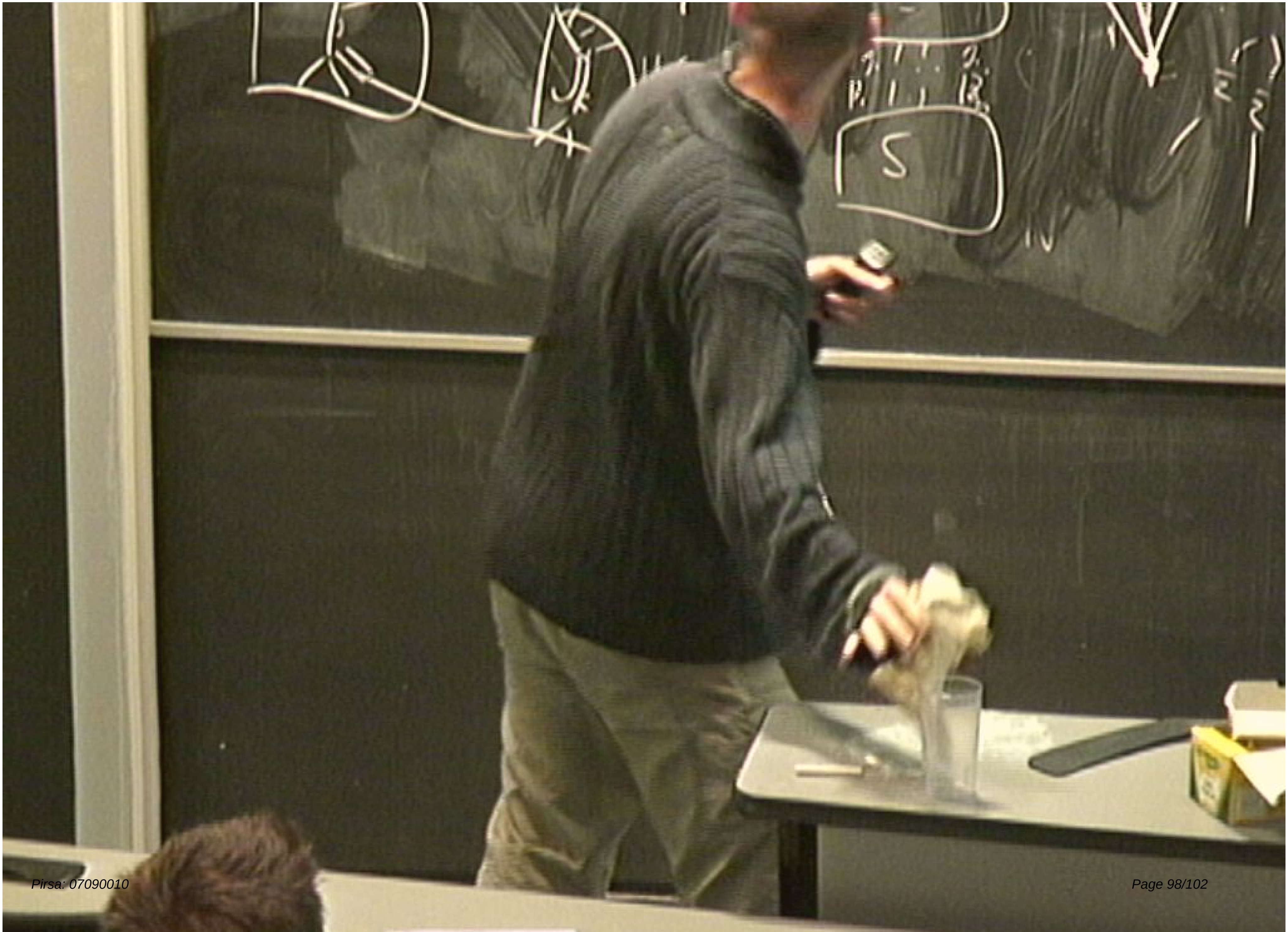

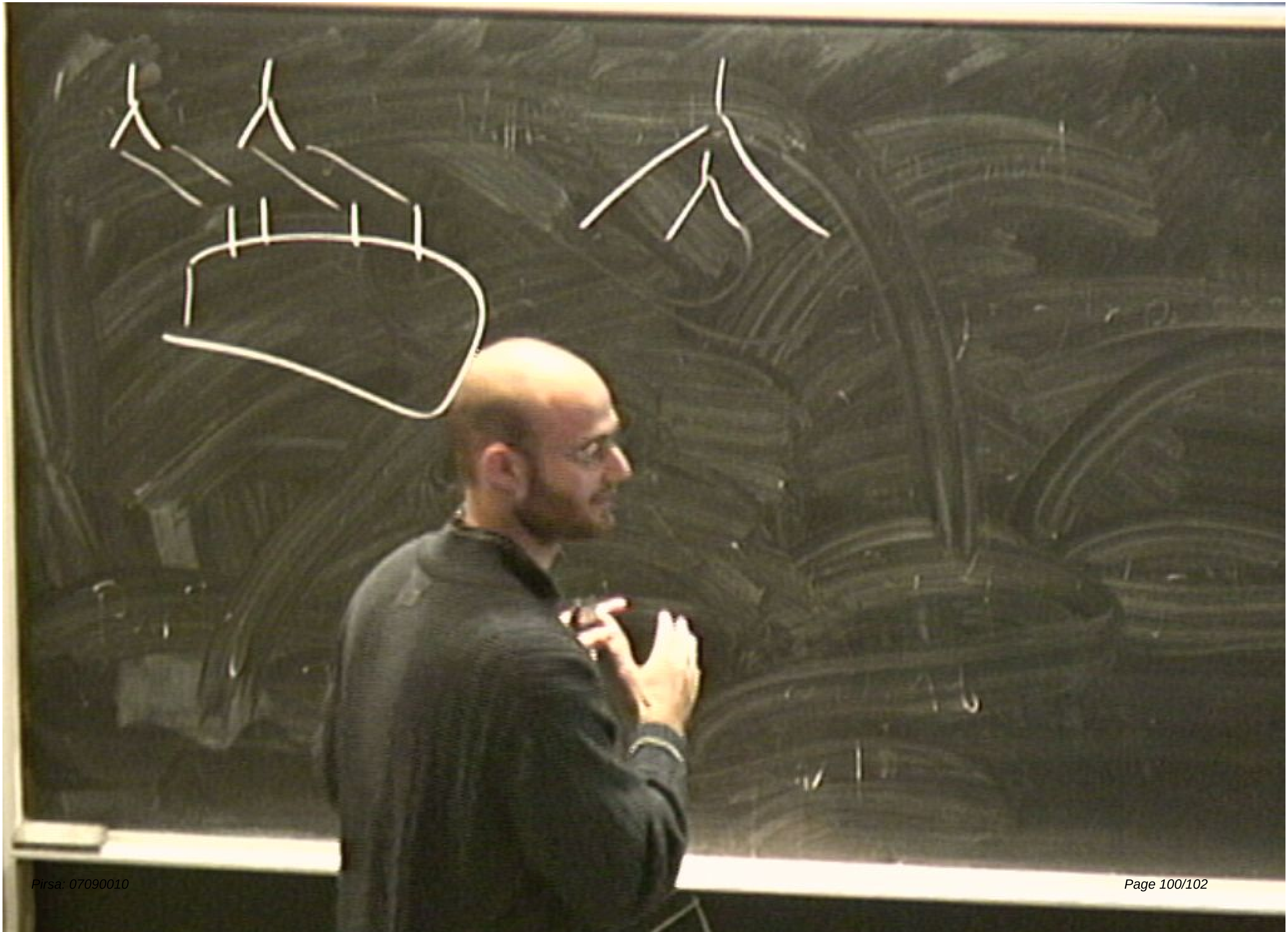

Conclusion and Outlook

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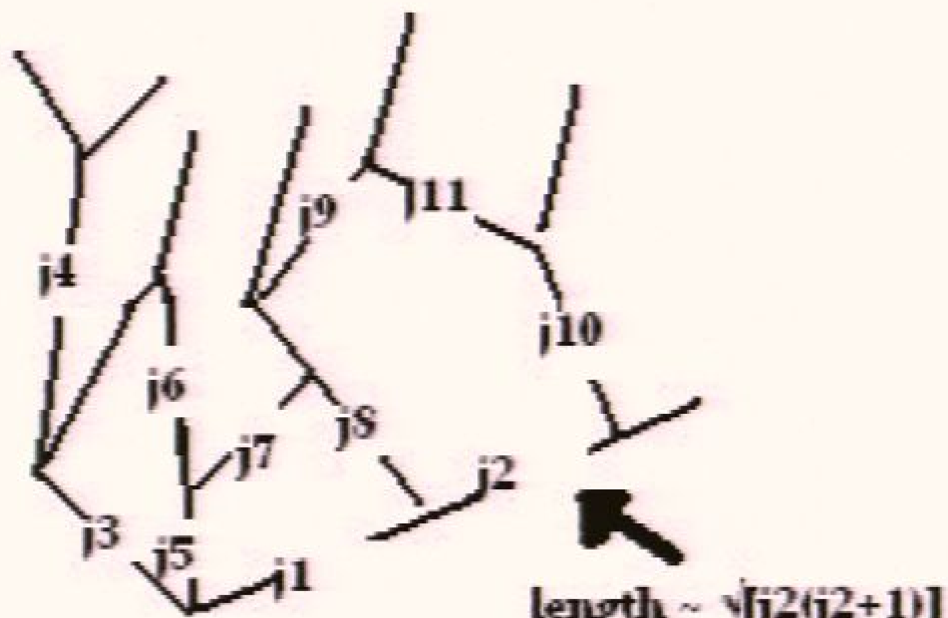
Spin Networks

In what follows we have see the spin network edges as either



lengths or areas.