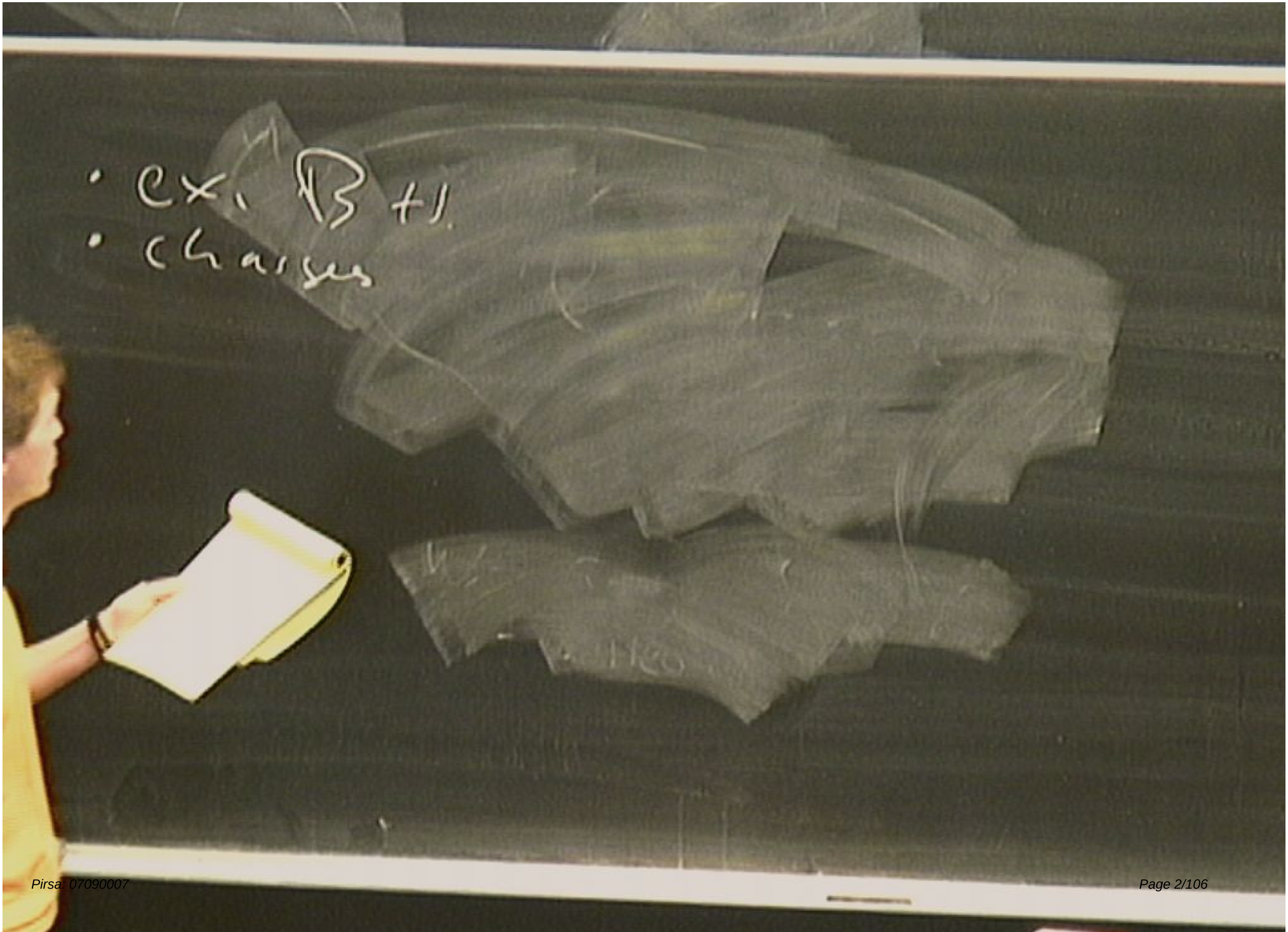
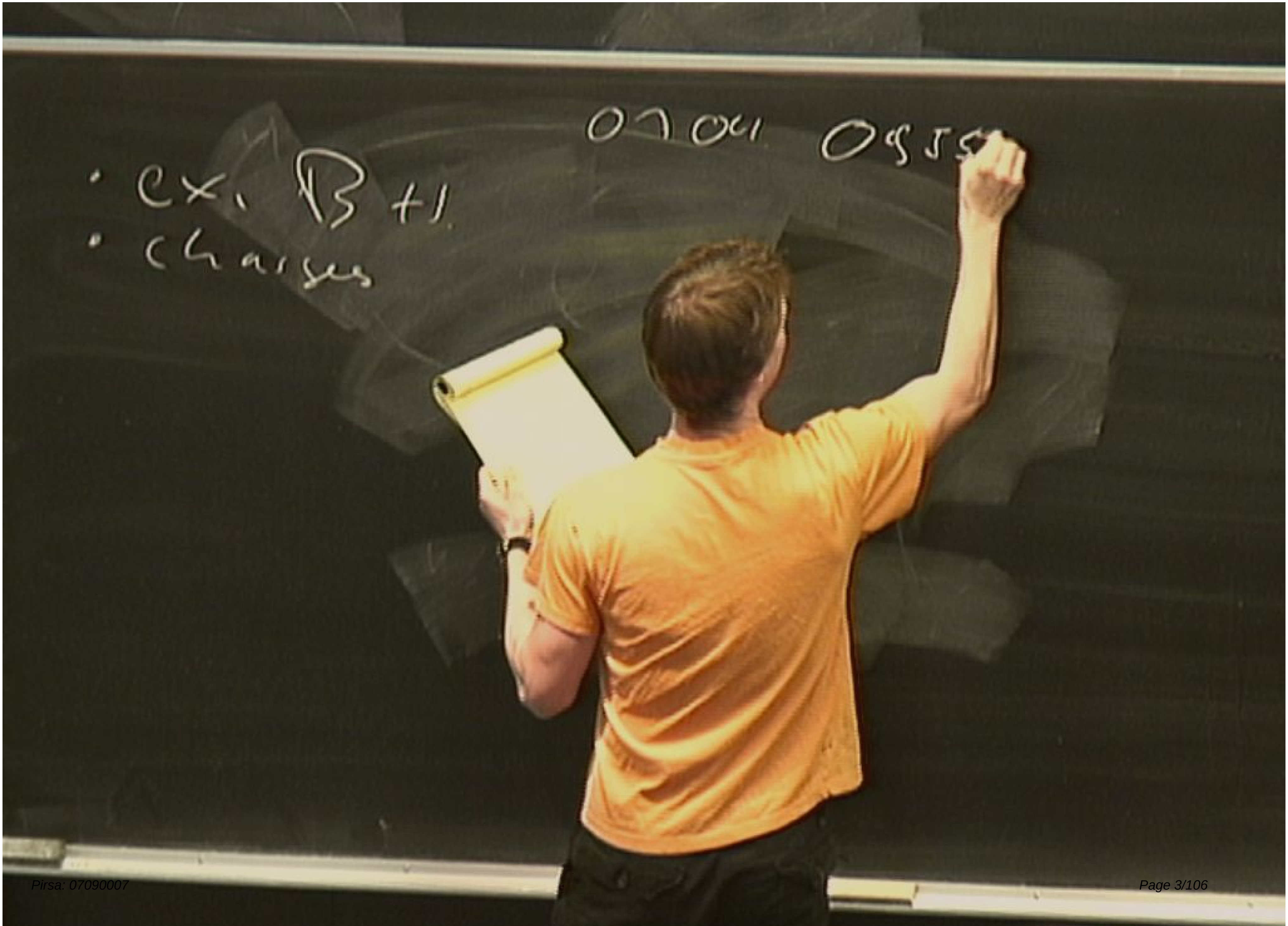
Title: Charges from Attractors

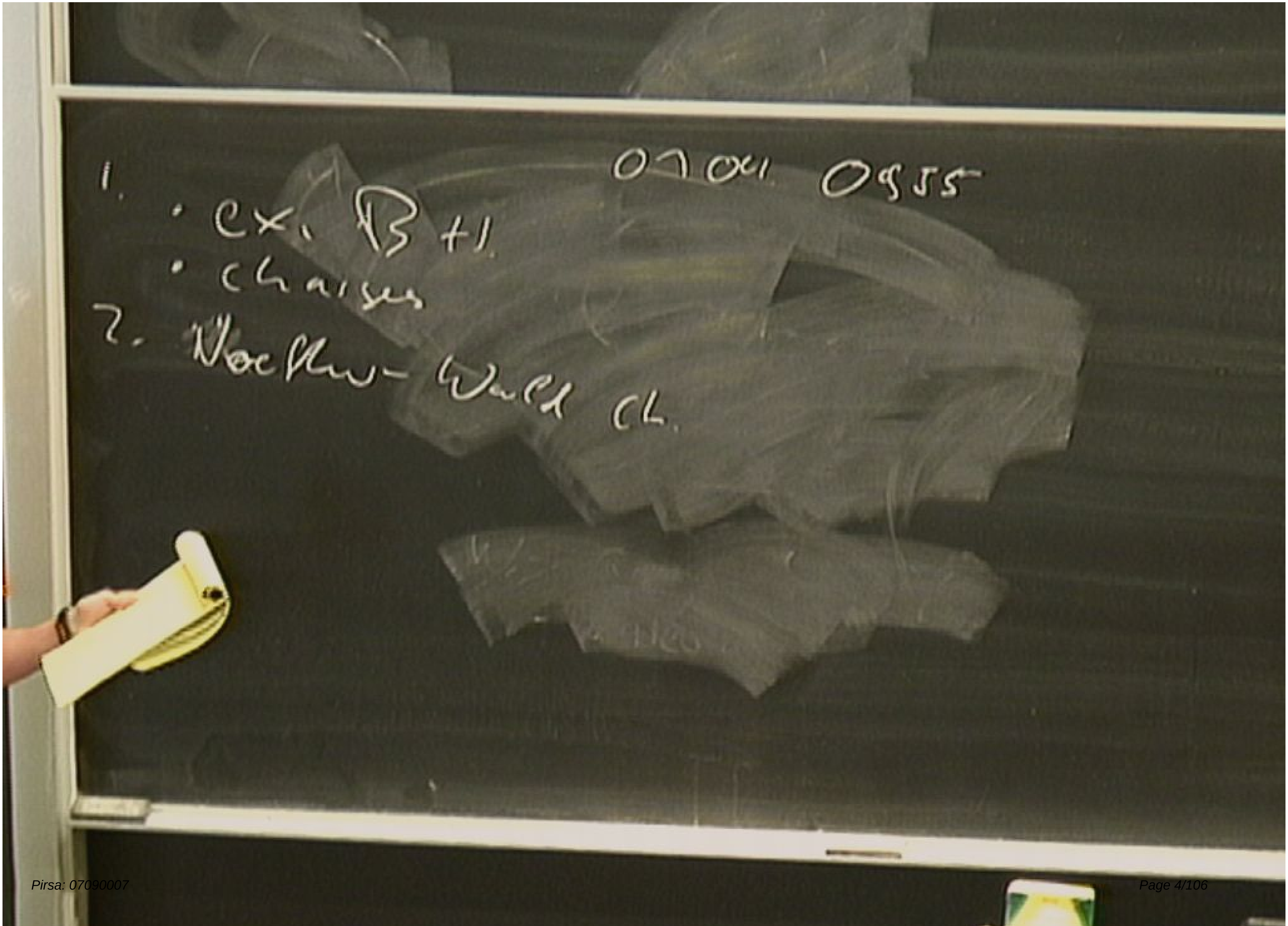
Date: Sep 07, 2007 11:00 AM

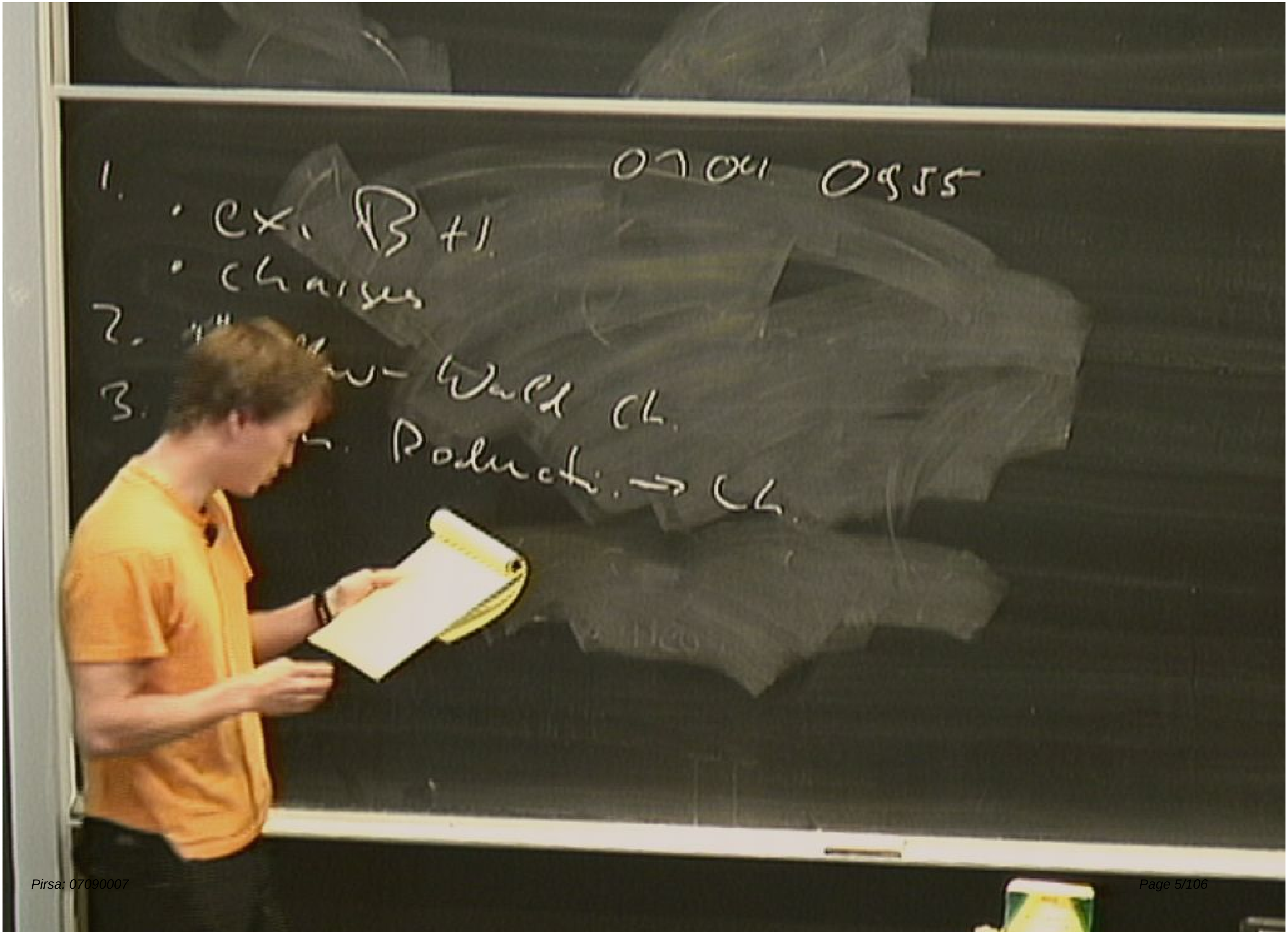
URL: http://pirsa.org/07090007

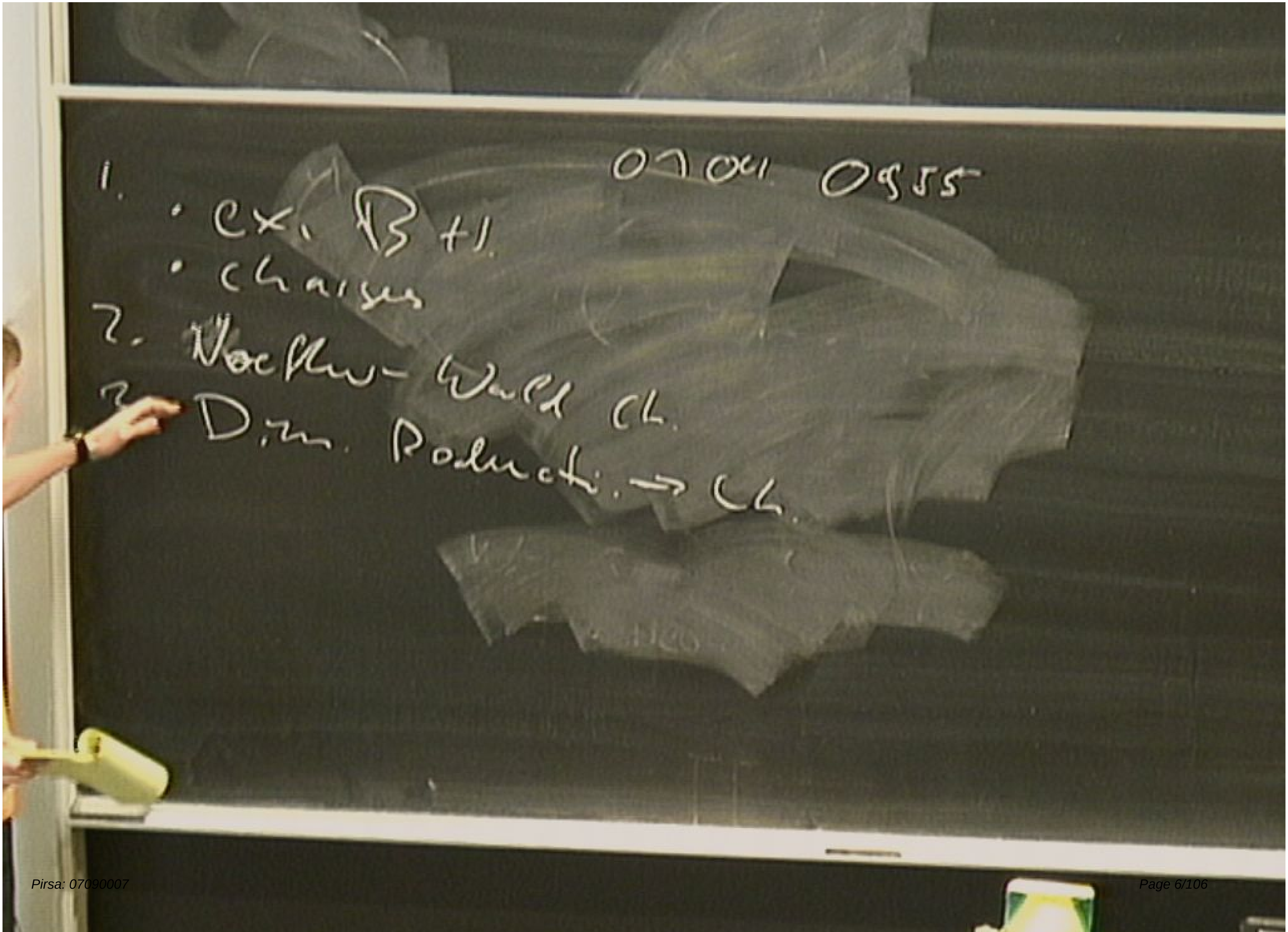
Abstract: We describe how to recover the quantum numbers of extremal black holes from their near horizon geometries. This is achieved by constructing the gravitational Noether-Wald charges which can be used for non-extremal black holes as well. These charges are shown to be equivalent to the U(1) charges of appropriately dimensionally reduced solutions. Explicit derivations are provided for 10 dimensional type IIB supergravity and 5 dimensional minimal gauged supergravity, with illustrative examples for various black hole solutions. We further discuss how to derive the thermodynamic quantities and their relations explicitly in the extremal limit, from the point of view of the near-horizon geometry. We relate our results to the entropy function formalism.


• ex. B +1
• charges


· charges


0955
1.
• charges
2. Noether-Wald ch.


1.
• charges
2.
3.
World ch.


0955
1.
• ex. B +1
• charges
2. Noether-Wald ch.
3. Dim. Reduction → ch.

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1\.
- ex. B +1
- charges

2\. Wess-Wald ch.

3\. Dim. Reduction $\Rightarrow$ ch.

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1. • ex. $\mathcal{B}$ +1.
   • charges
2. Noether-Wald ch.
3. Dim. Reduction $\Rightarrow$ ch.

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1\. • ex. $\mathcal{B}$ +1.
   • charges

2\. Noether-Ward Ch.

3\. Dim. Reduction → Ch.

4\. Entropy.

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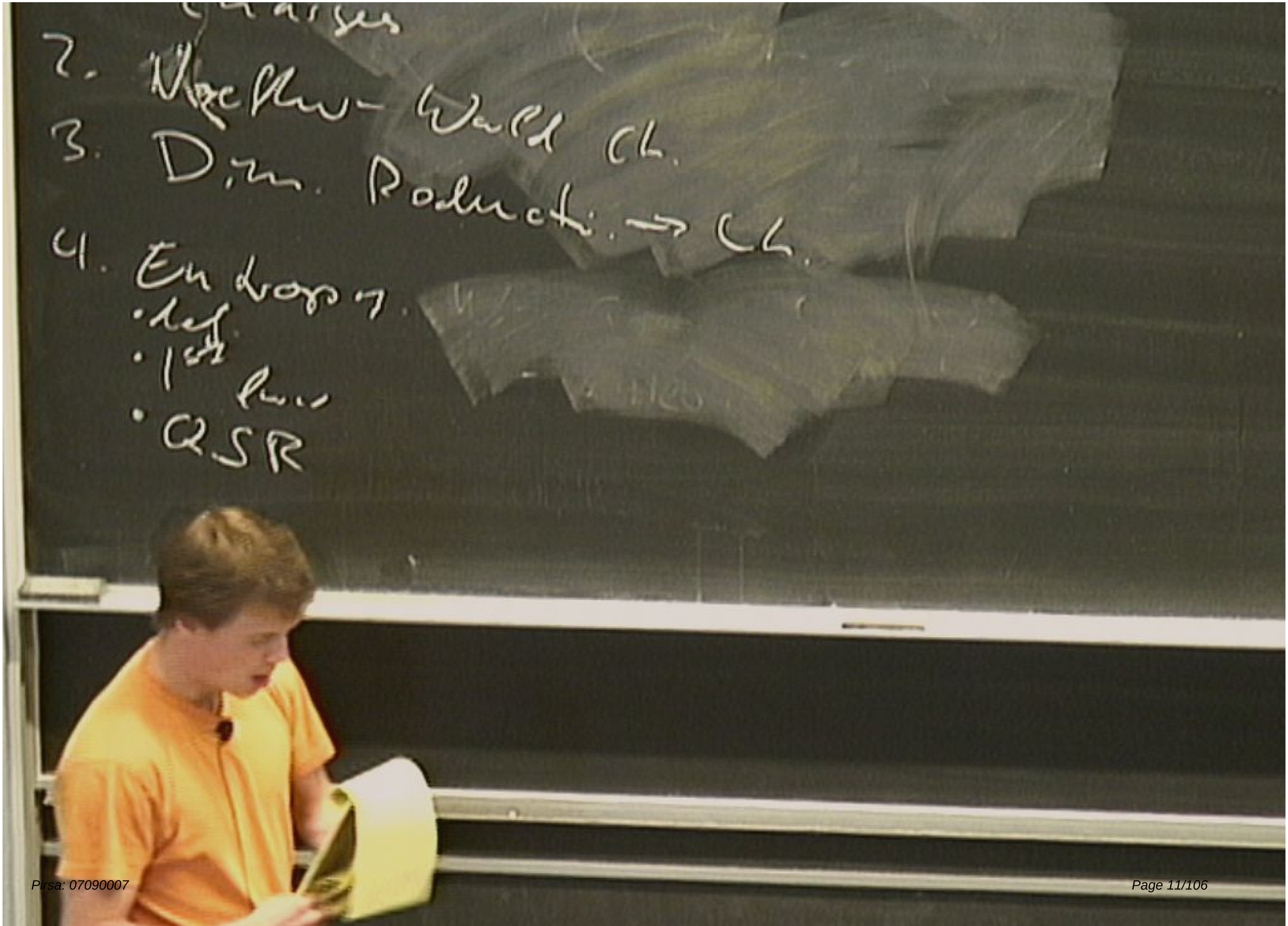
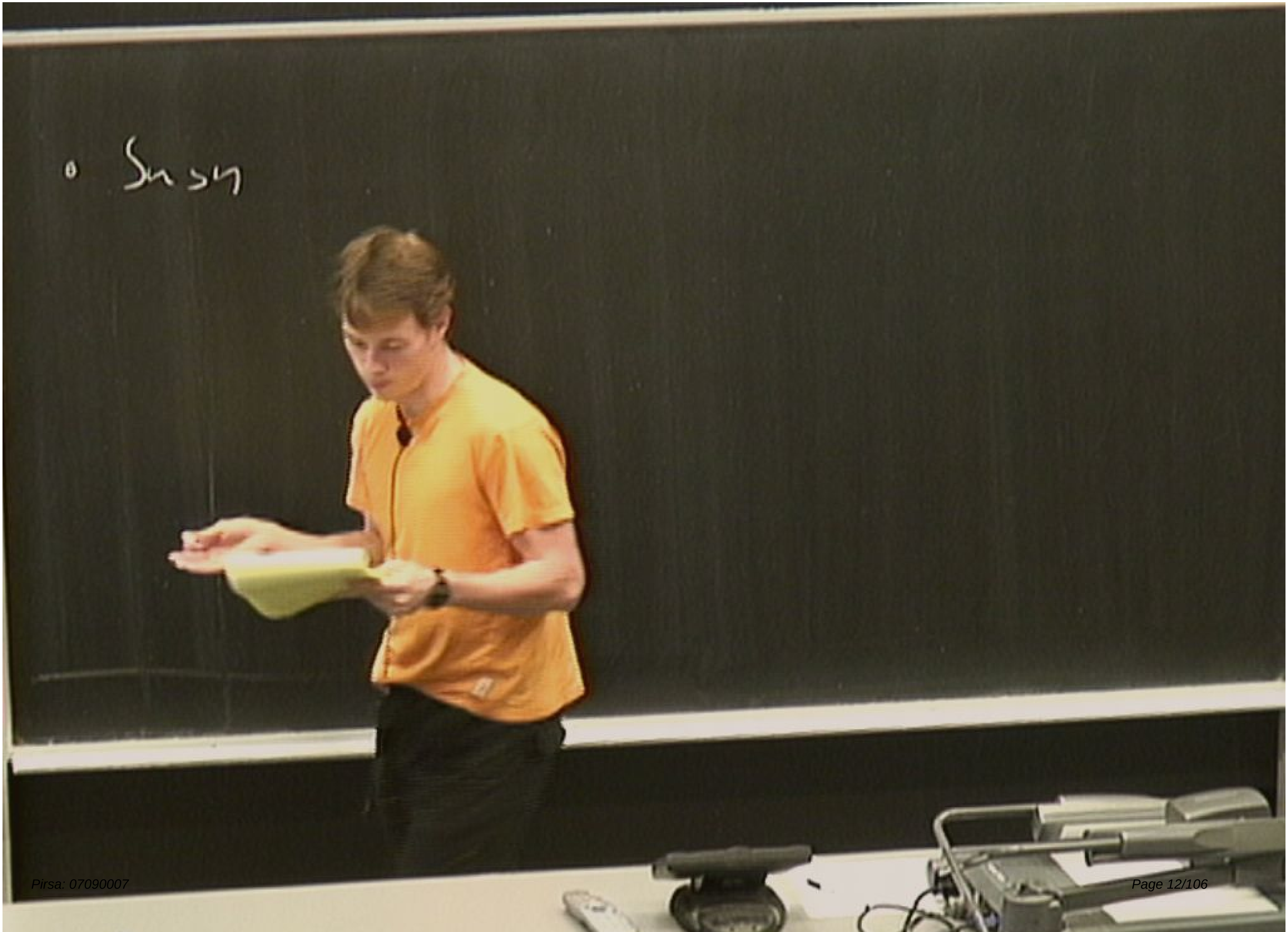
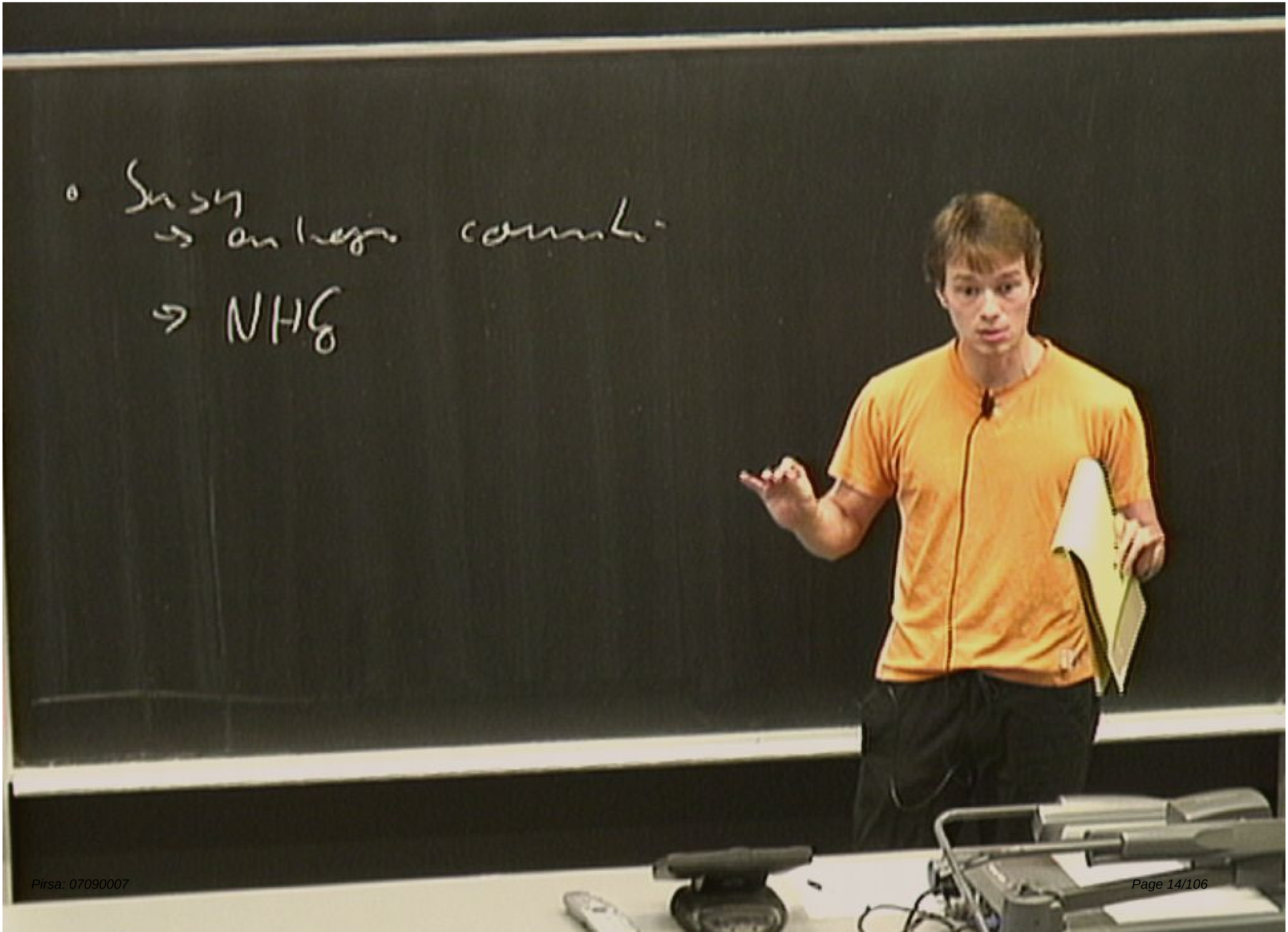
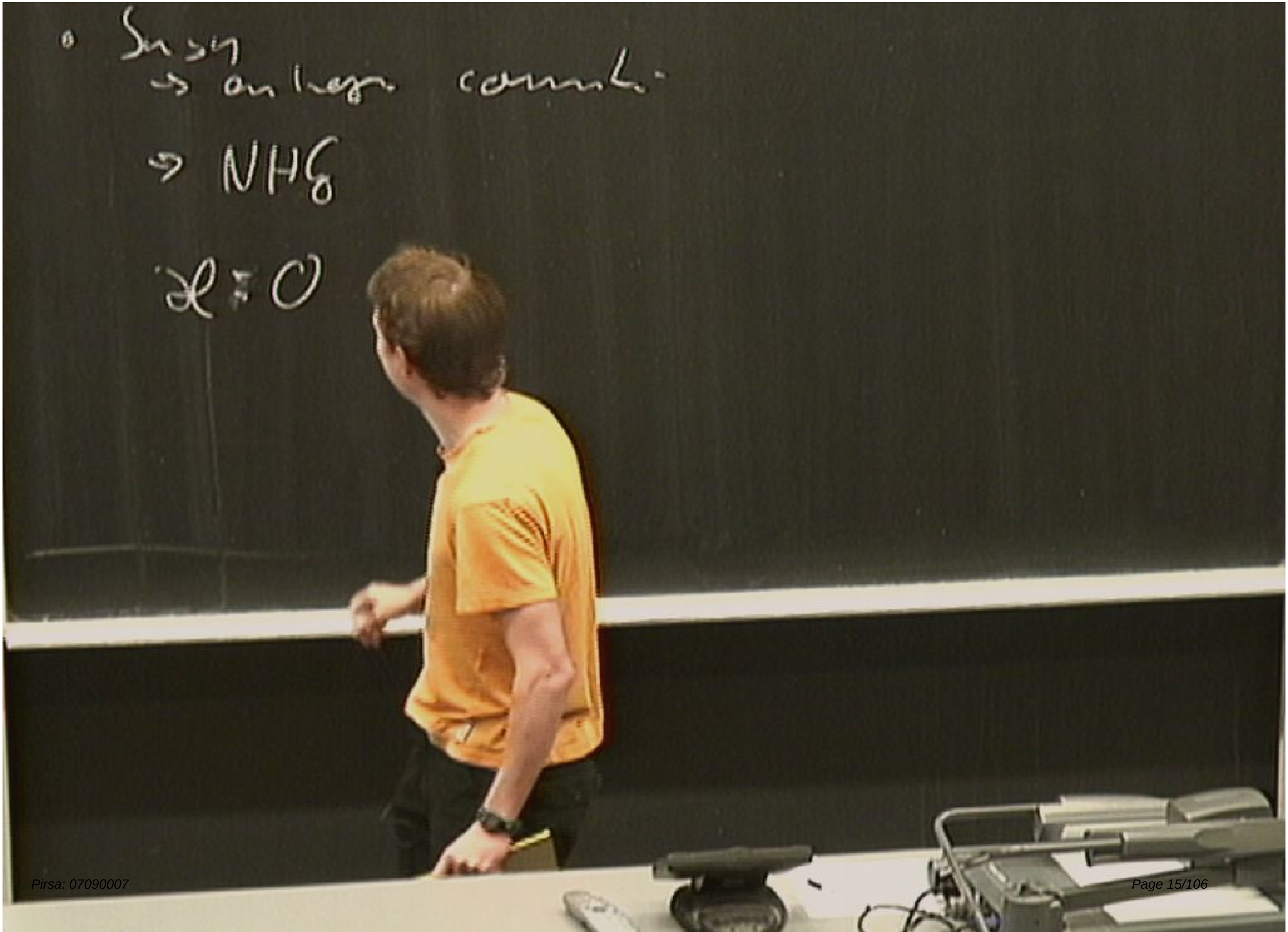
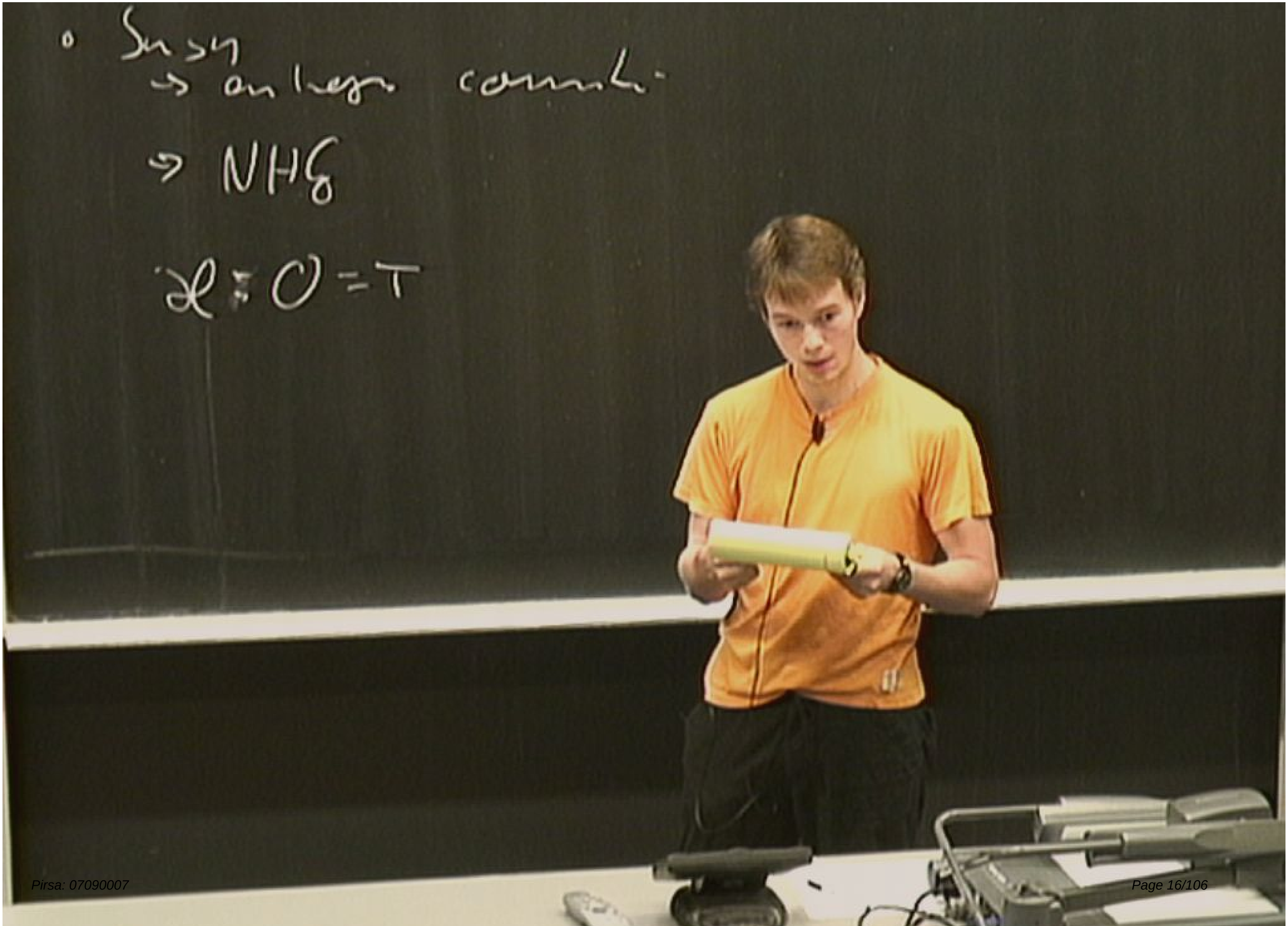
1. • ex. B +1.
   • charges
2. Noether-Ward id.
3. Dim. Reduction → id.
4. Entropy.
   • def.
   • 1st law
   • QSR


2. Noether-Wald Ch.
3. Dim. Reduction → Ch.
4. Entropy
• def.
• 1st law
• QSR


Susy


NHG


NHG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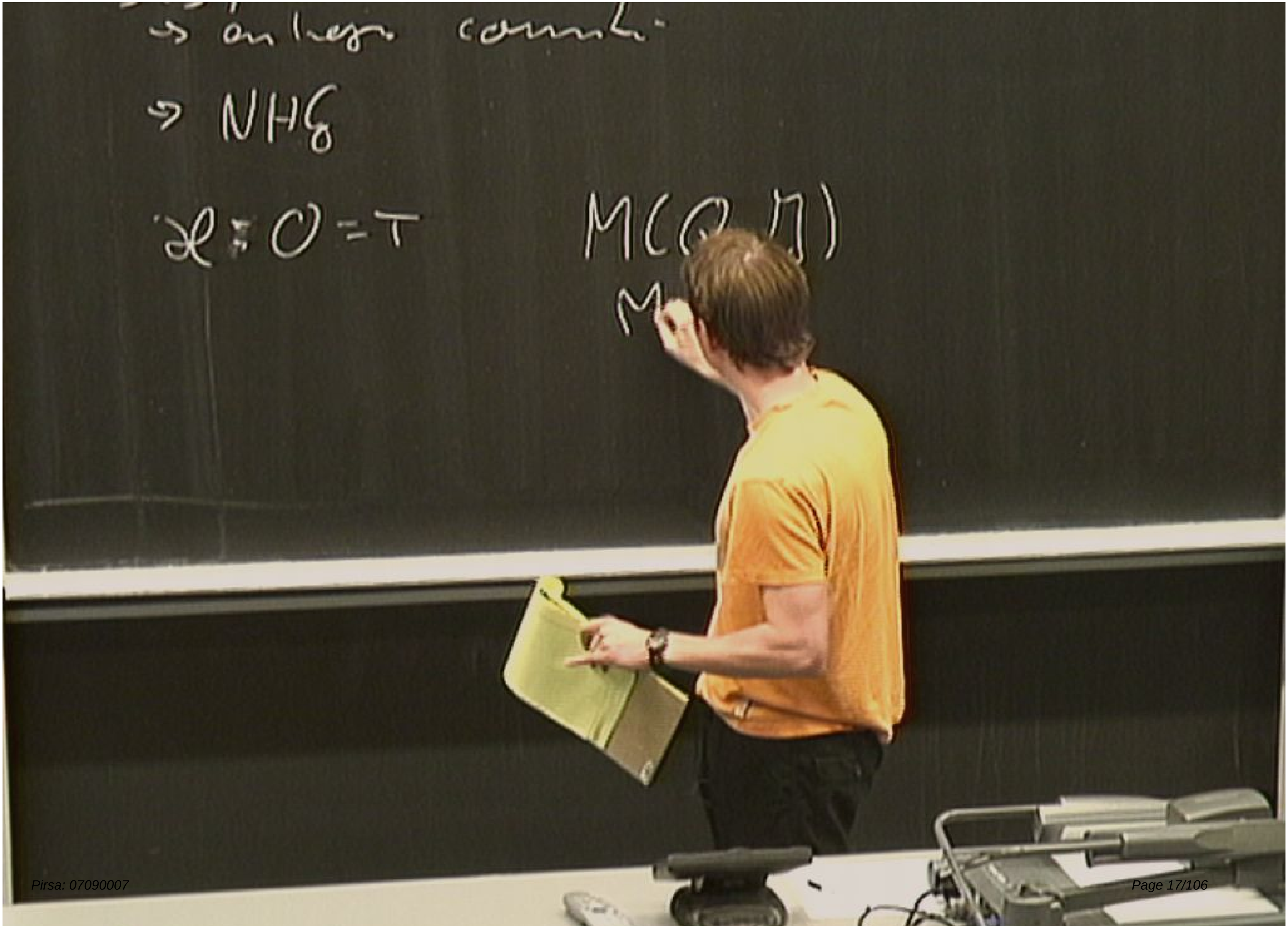
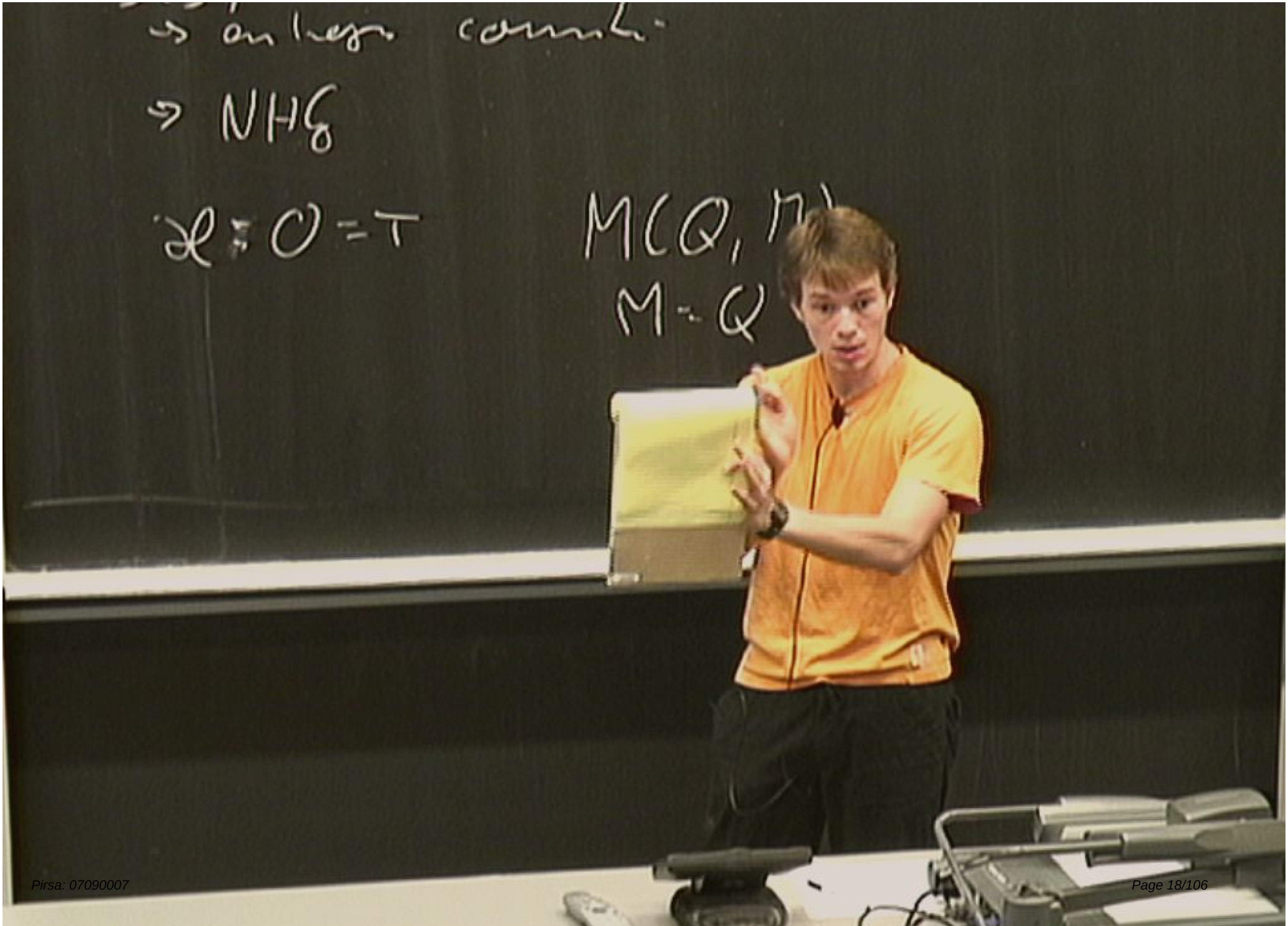
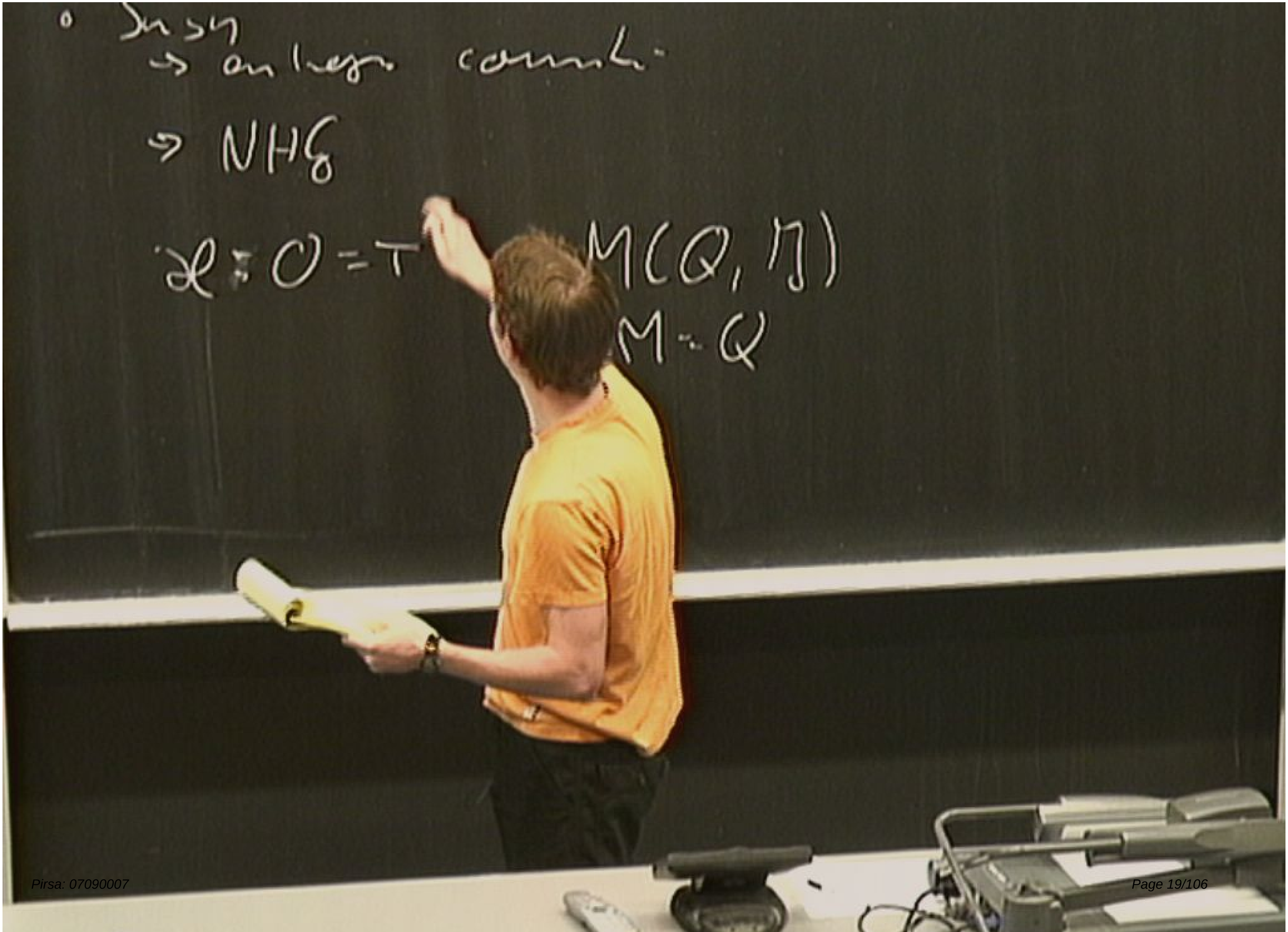
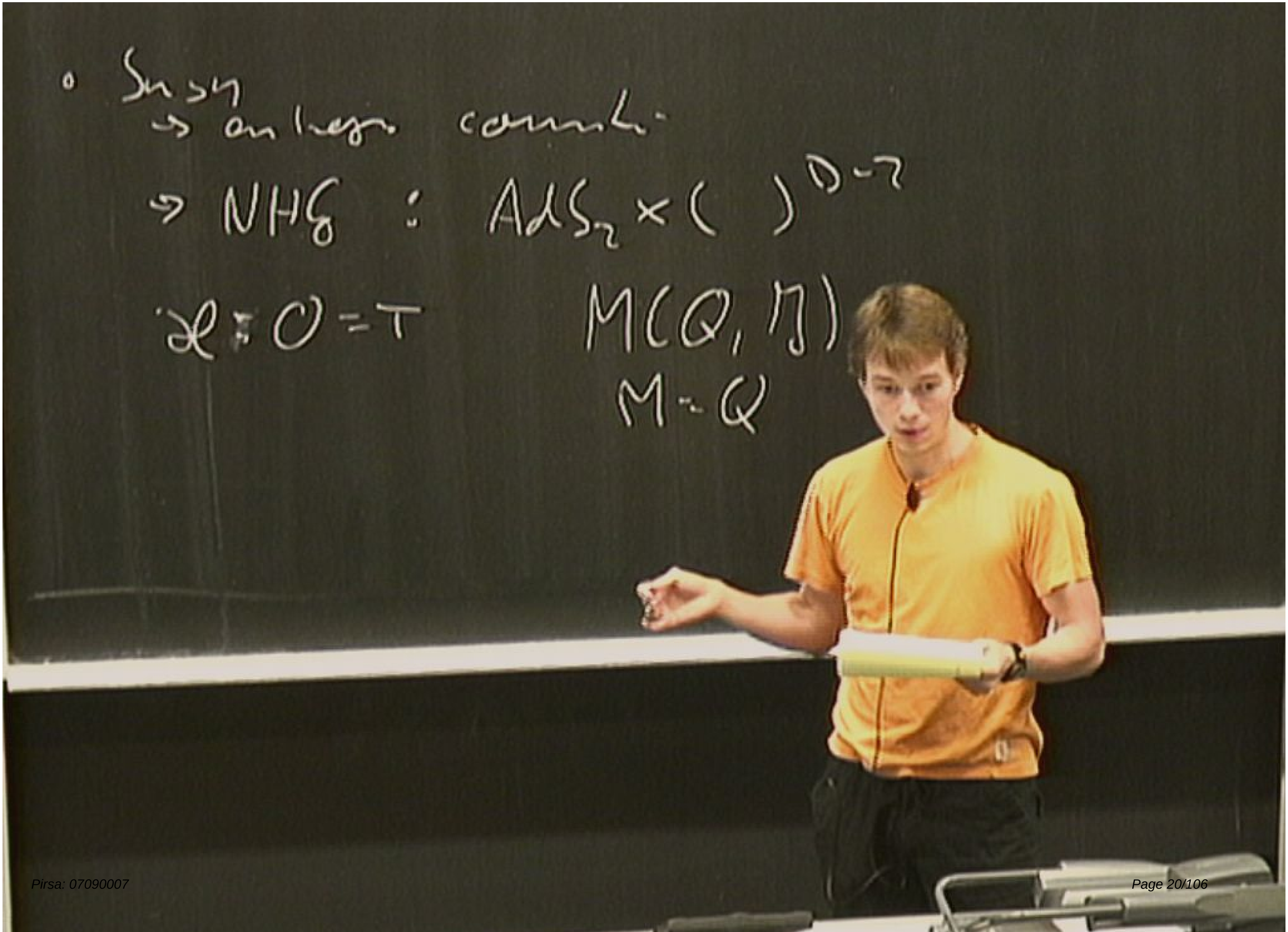
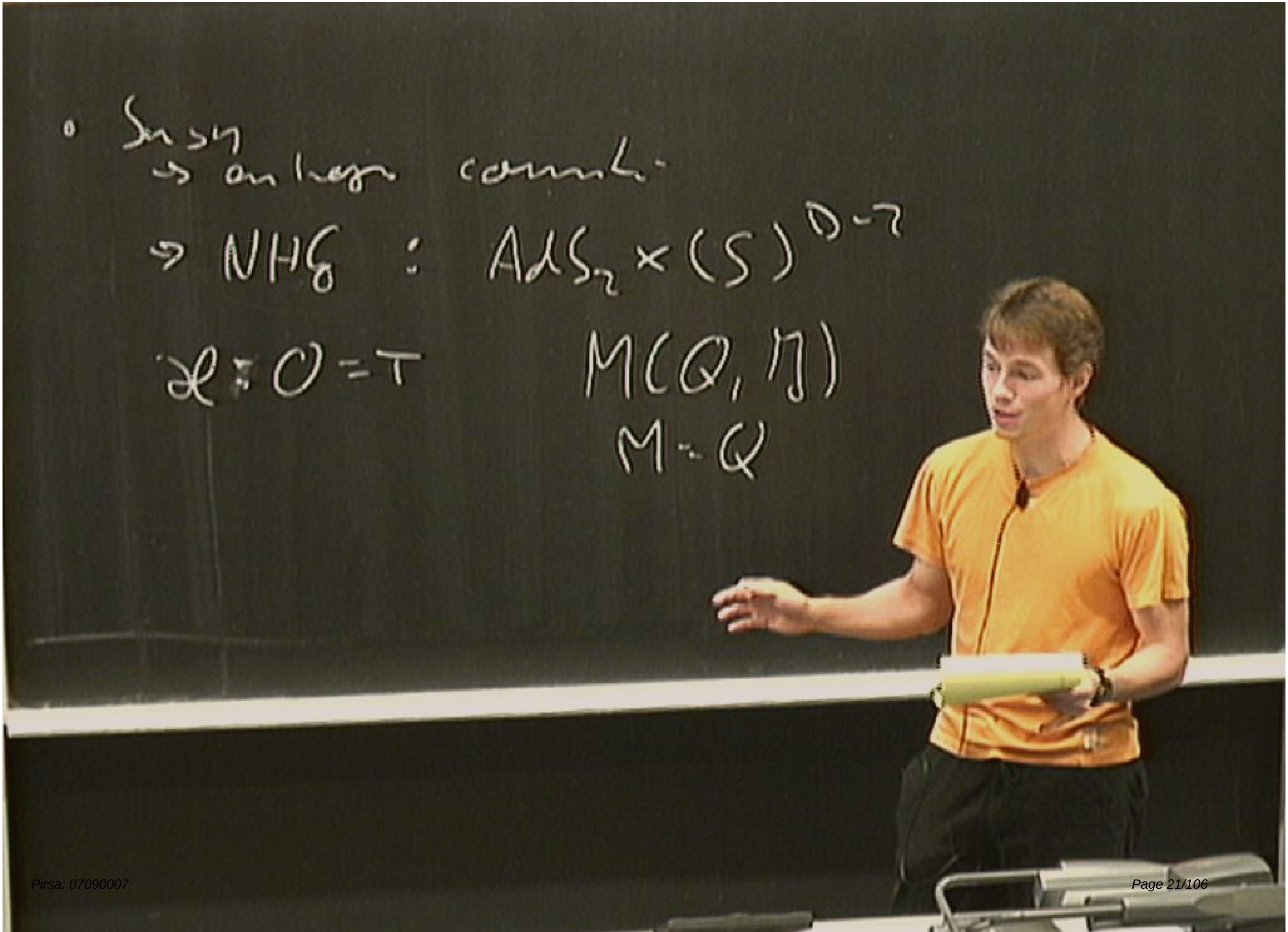
→ NHG


→ NHG


→ NHG
M = Q


→ NHG
M = Q


Susy
NHG : AdS2 × ( )D-2
M(Q, J)
M ~ Q



Susy
→ NHG : AdS2 × (S)D-2
M(Q, J)
M = Q

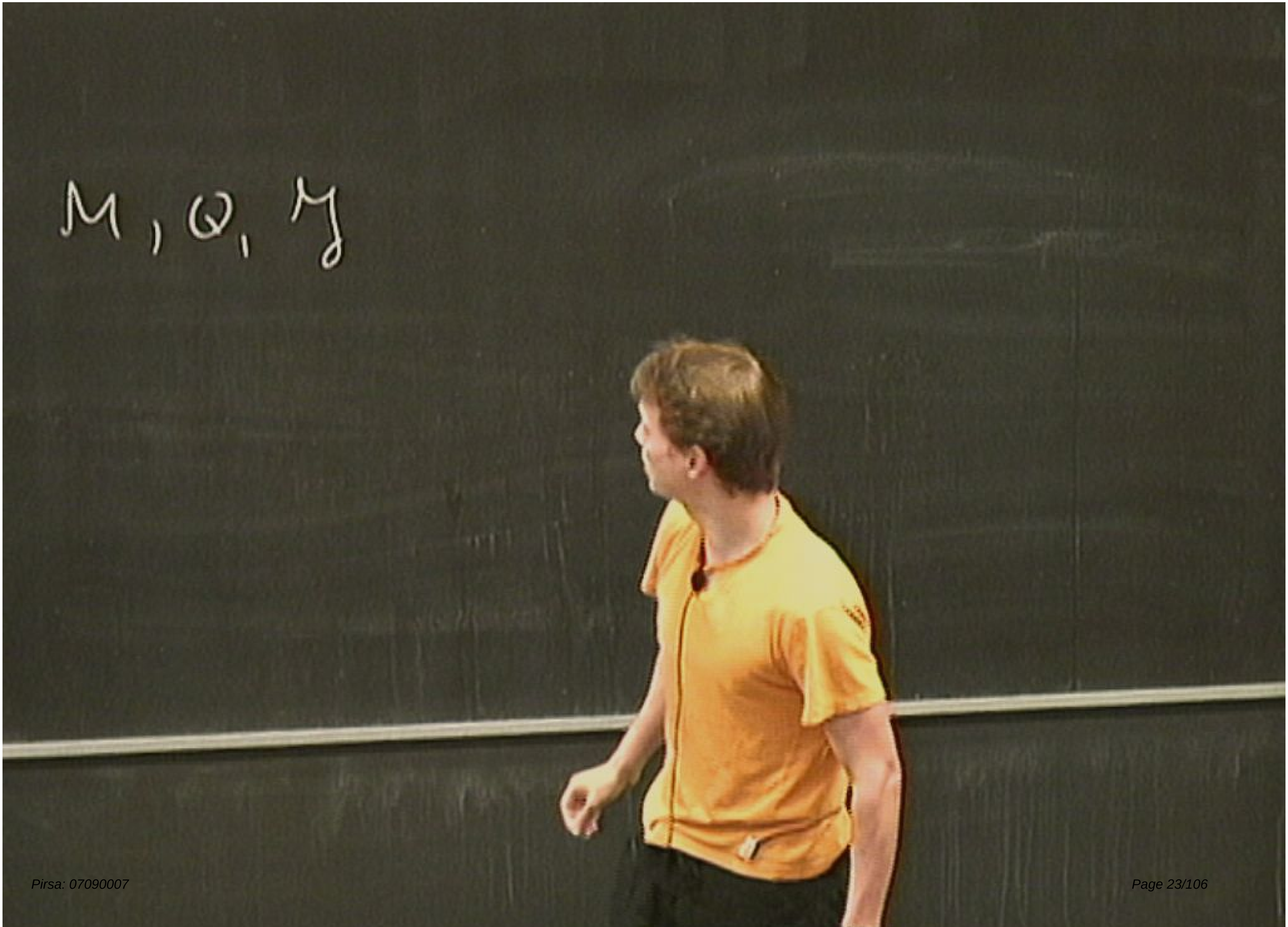
• $S_n > 4$

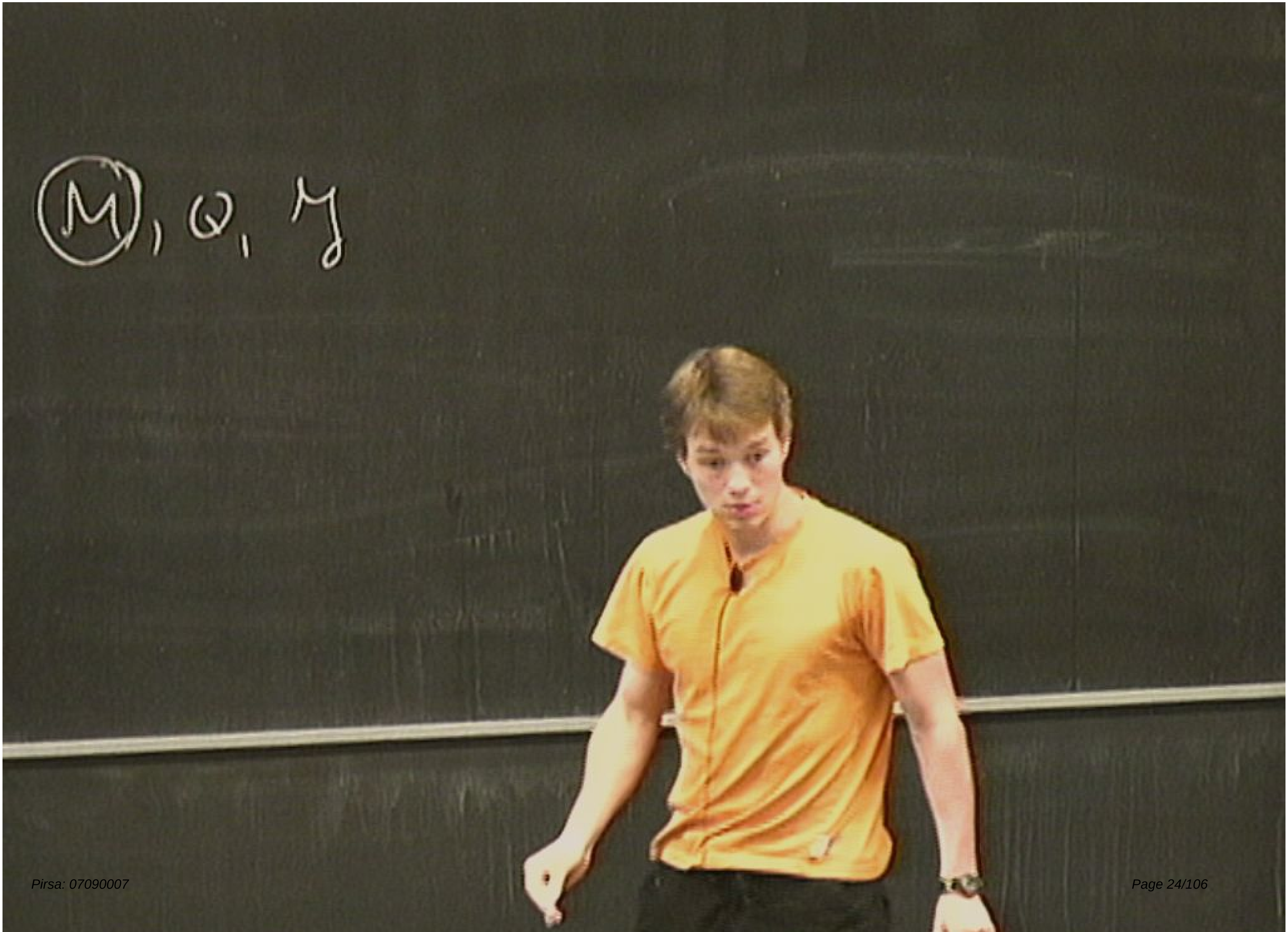
→ on large comm.

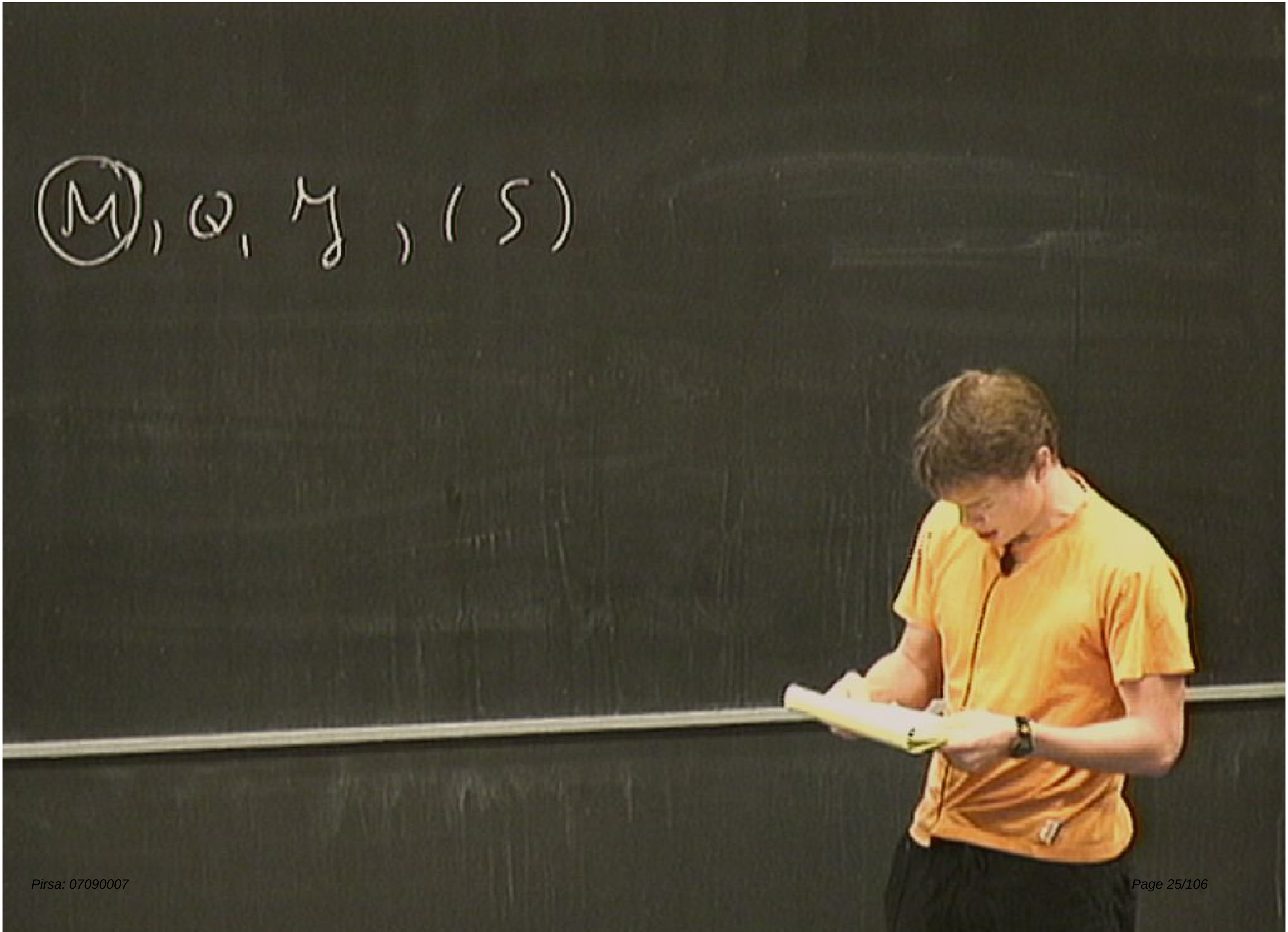
→ NHG : $AdS_2 \times (S)^{D-2}$

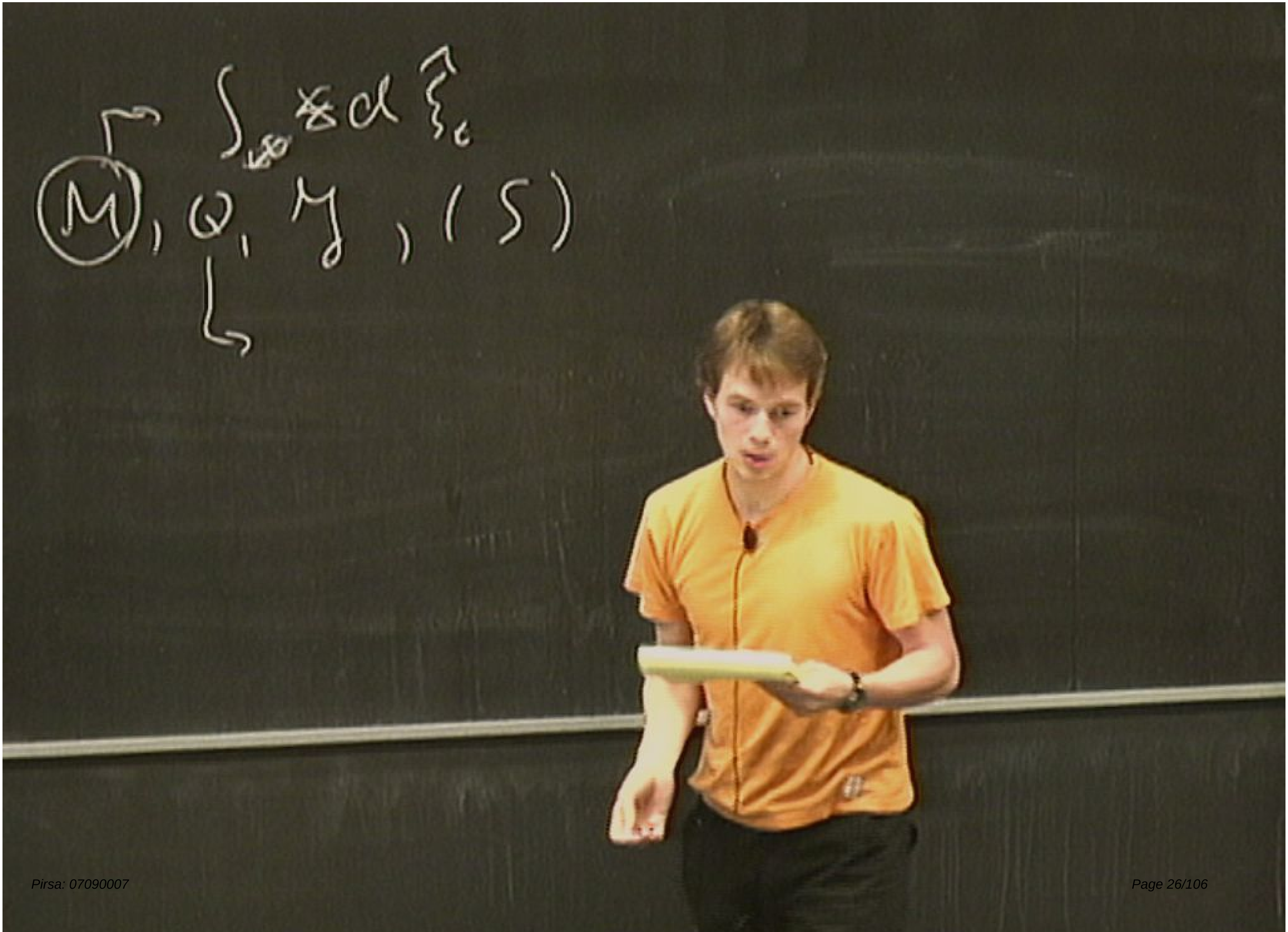
$\mathcal{L} = O = T$ $\qquad M(Q, J)$

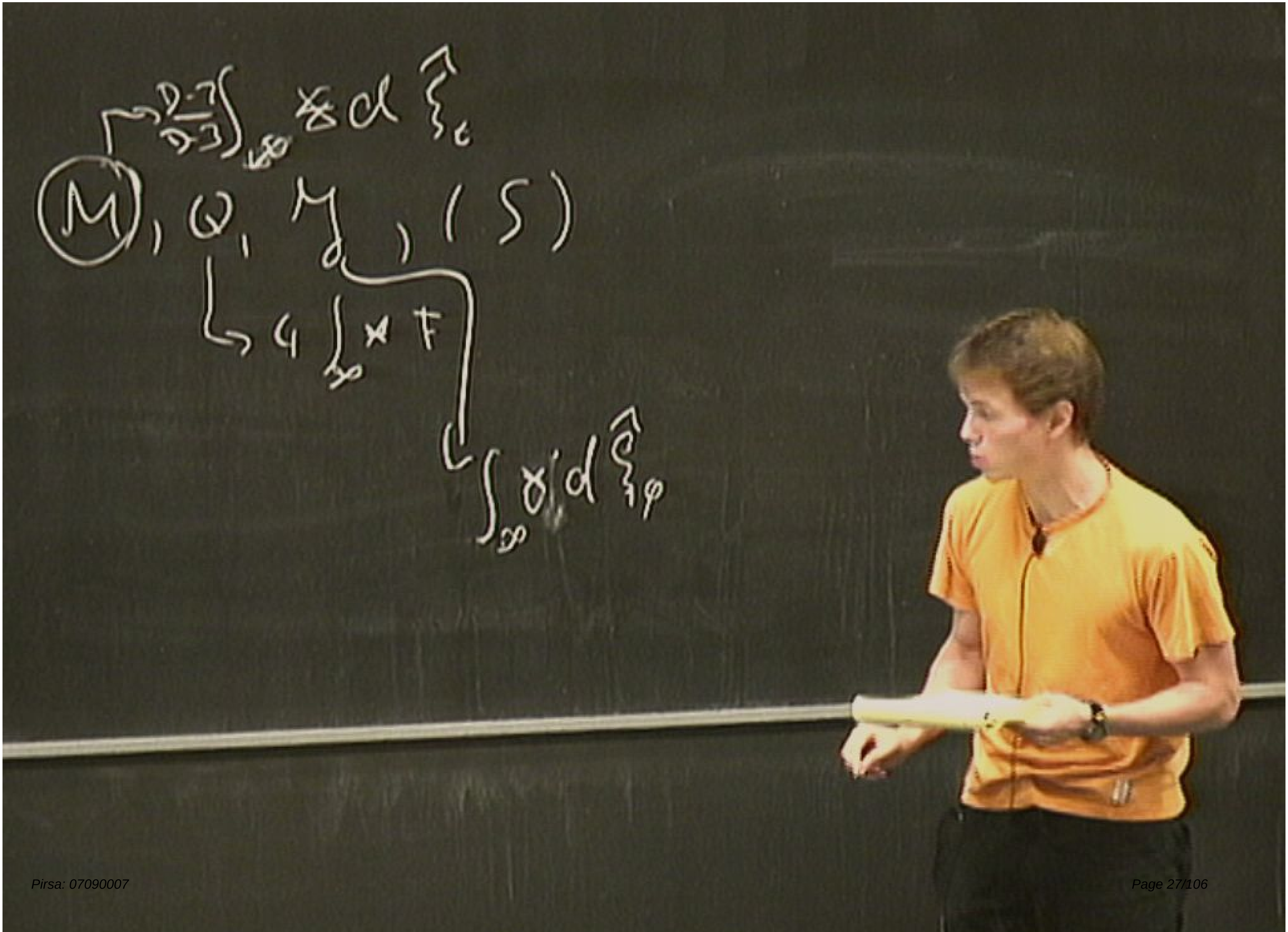
$M = Q$

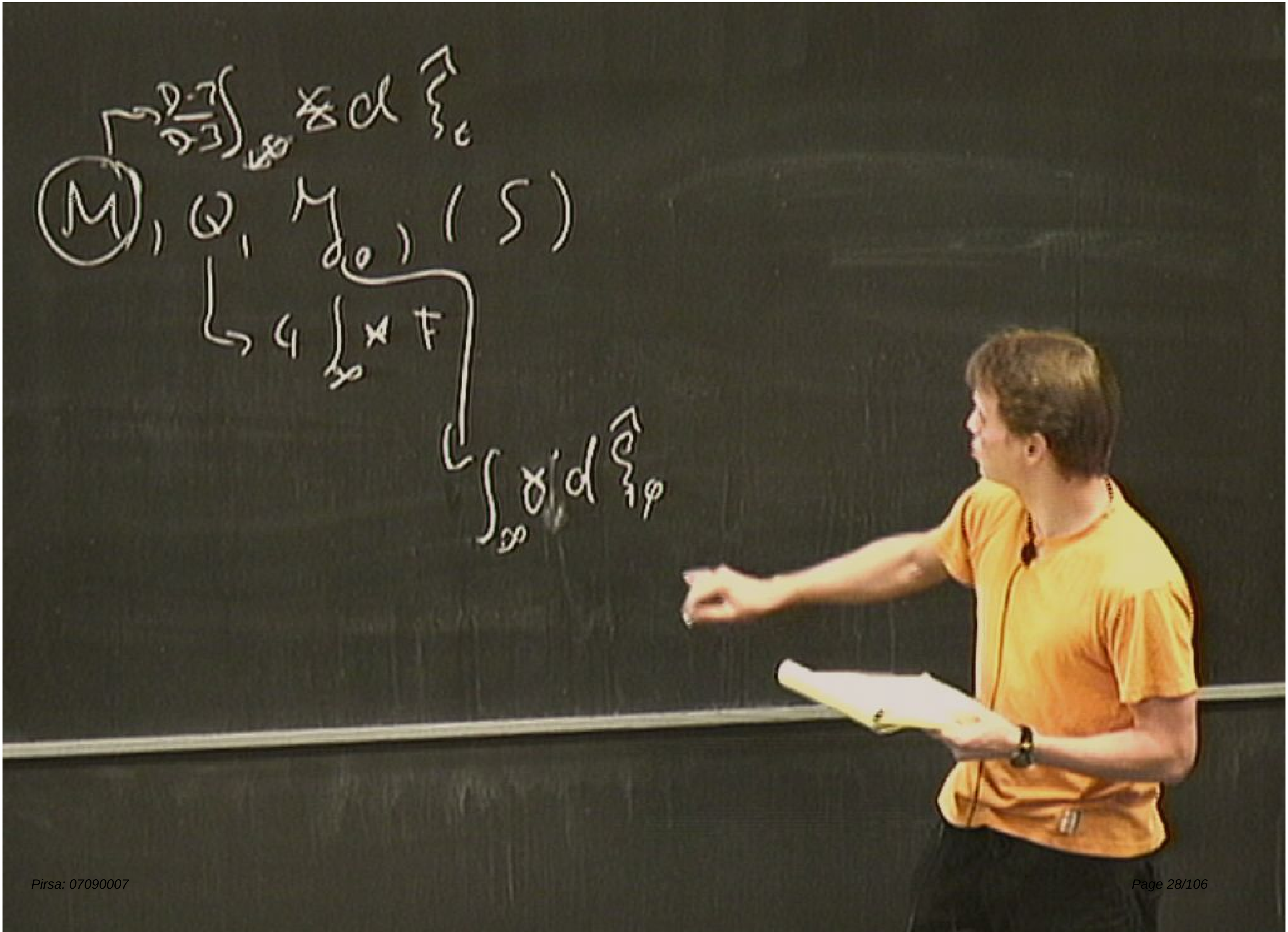

M, Q, J

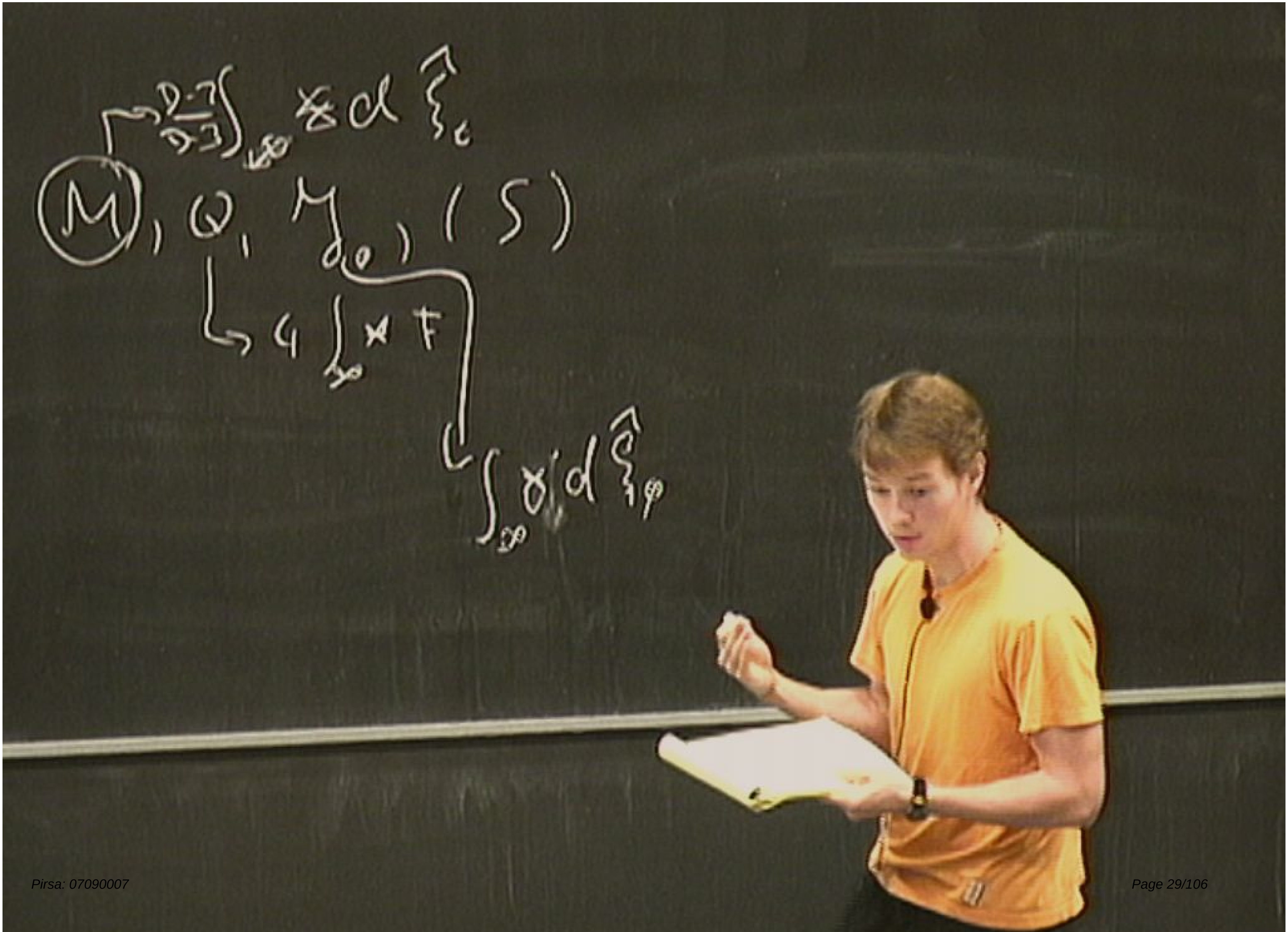

M, Q, J


M, Q, G, (S)


M, Q, J, (S)


M , Q , J , (S)
4 ∫ * F


$\frac{D-2}{D-3}\int_{\infty} * d\hat{\xi}_t$
$M, Q, J_0, (S)$
$4\int_{\infty} * F$
$\int_{\infty} * d\hat{\xi}_{\varphi}$

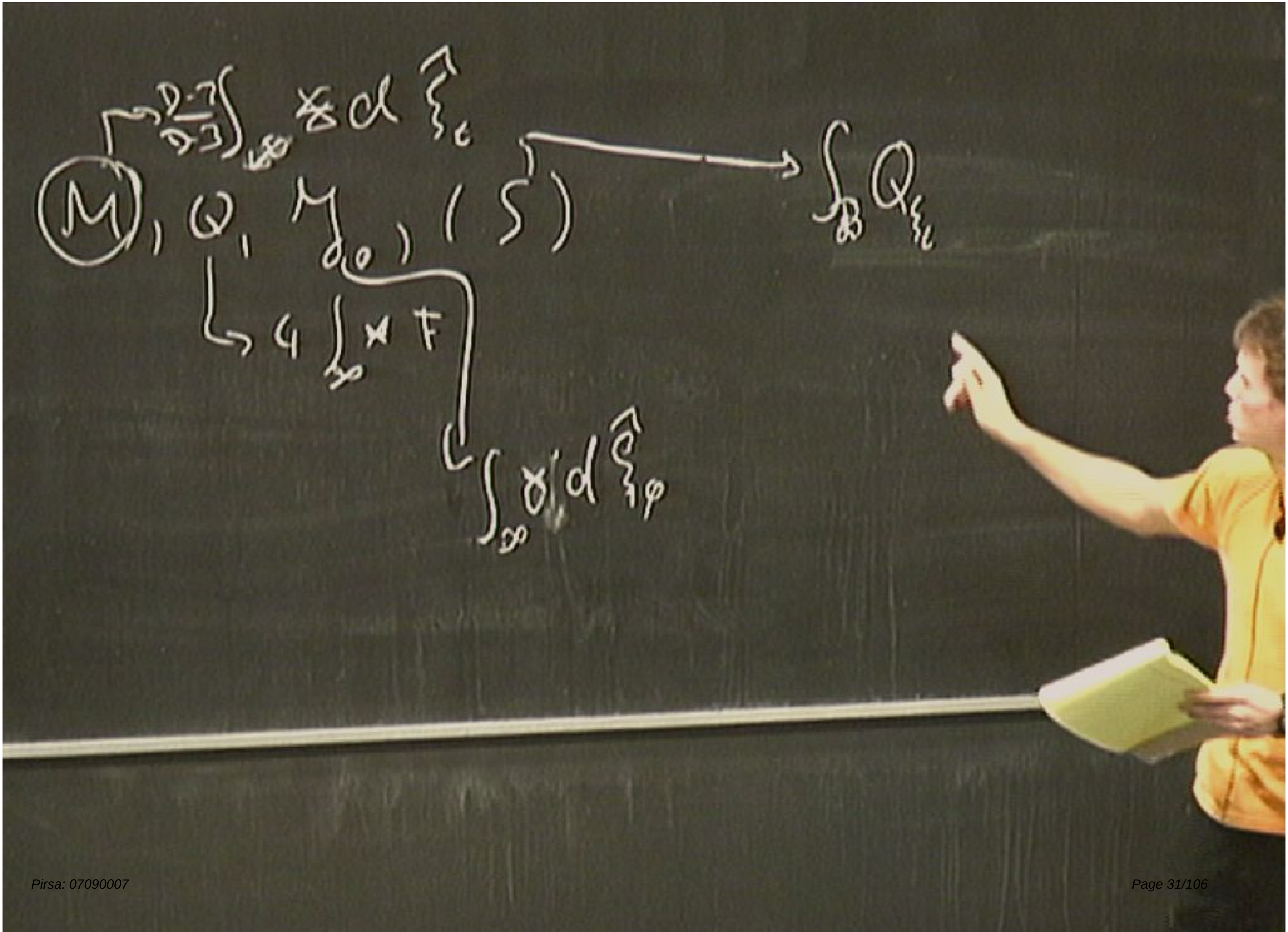


$M \rightarrow \frac{D-2}{D-3}\int_{S_\infty} * d\hat{\xi}_t$

$Q \rightarrow 4\int_{S_\infty} * F$

$J_0 \rightarrow \int_{S_\infty} * d\hat{\xi}_\phi$

$(S) \rightarrow \int_{S_\infty}$


$\frac{D-2}{D-3}\int_{\infty} * d\hat{\xi}_t \longrightarrow \int_{\infty} Q_{\xi_t}$
$M, Q, J_0, (S)$
$4\int_{\infty} * F$
$\int_{\infty} * d\hat{\xi}_\varphi$

$$M,\ Q,\ J_{\phi},\ (S)$$

$$M \rightarrow \frac{D-2}{D-3}\int_{\infty} {*d\hat{\xi}_t}$$

$$Q \rightarrow 4\int_{\infty} {*F}$$

$$J_{\phi} \rightarrow \int_{\infty} {*d\hat{\xi}_{\phi}}$$

$$S \rightarrow \int_{B} Q_{\xi_t} = \frac{\kappa}{2\pi} S$$

$$M \to \frac{D-2}{D-3}\int_{\infty} * d\hat{\xi}_t$$

$$M, \quad Q, \quad J_\varphi, \quad (S)$$

$$Q \to 4\int_{\infty} * F$$

$$J_\varphi \to \int_{\infty} * d\hat{\xi}_\varphi$$

$$S \to \int_{B} Q_{\xi_t} = \frac{\kappa}{4\pi} S$$

$$M,\ Q,\ \Phi_0,\ (S)$$

$$M \to \frac{D-2}{D-3}\int_{S_\infty} *d\hat{\xi}_t$$

$$Q \to 4\int_{S_\infty} *F$$

$$\Phi_0 \to \int_{S_\infty} *d\hat{\xi}_\varphi$$

$$S \to \int_B Q_{\xi_t} = \frac{\varkappa}{4\pi} S$$

$$D_\xi \xi = \varkappa \xi$$

$$\frac{D-2}{D-3}\int_{\infty} \star d\hat{\xi}_t$$

$$M,\ Q,\ M_{\phi},\ (S)$$

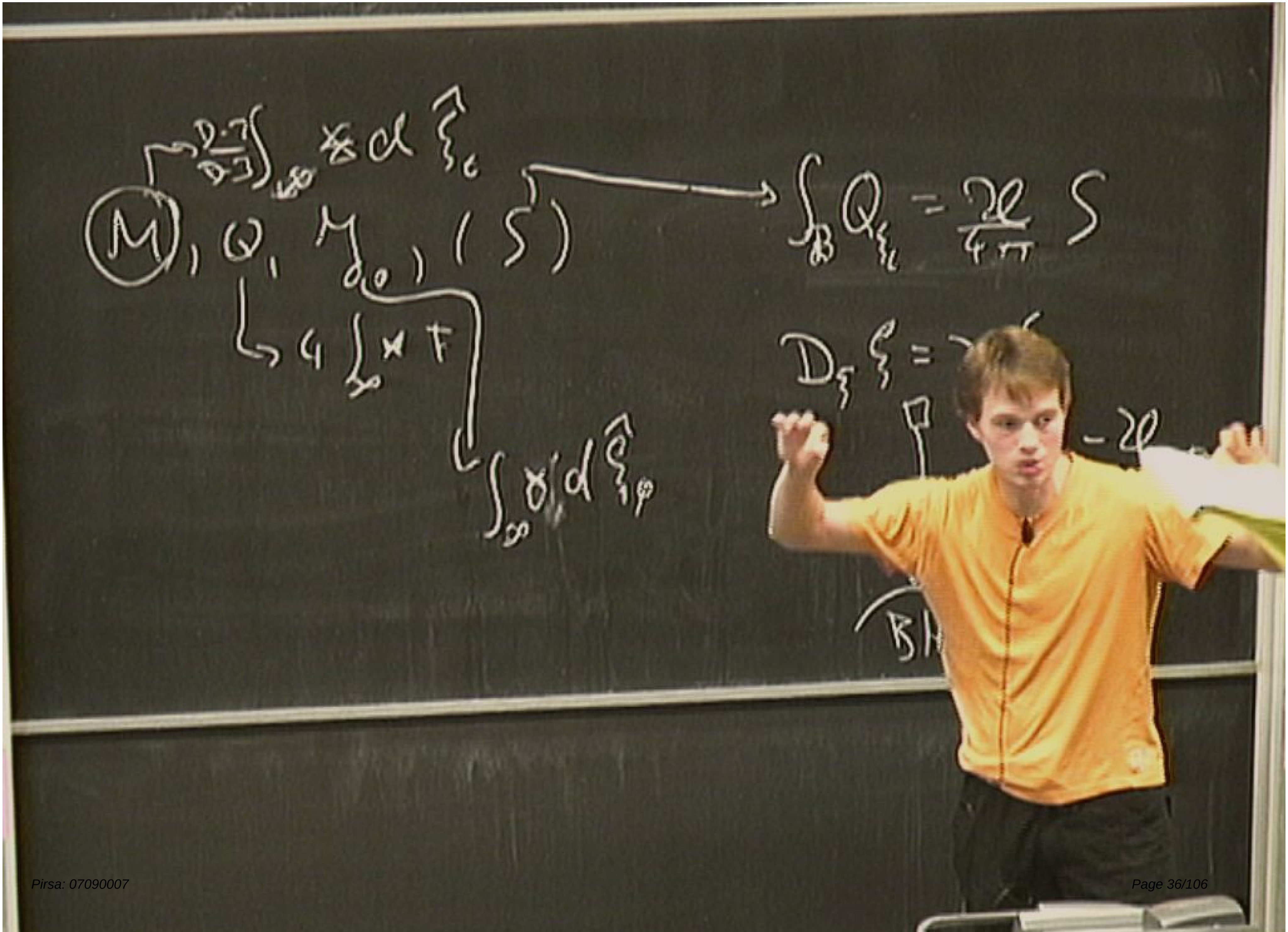
$$\hookrightarrow 4\int_{\infty} \star F$$

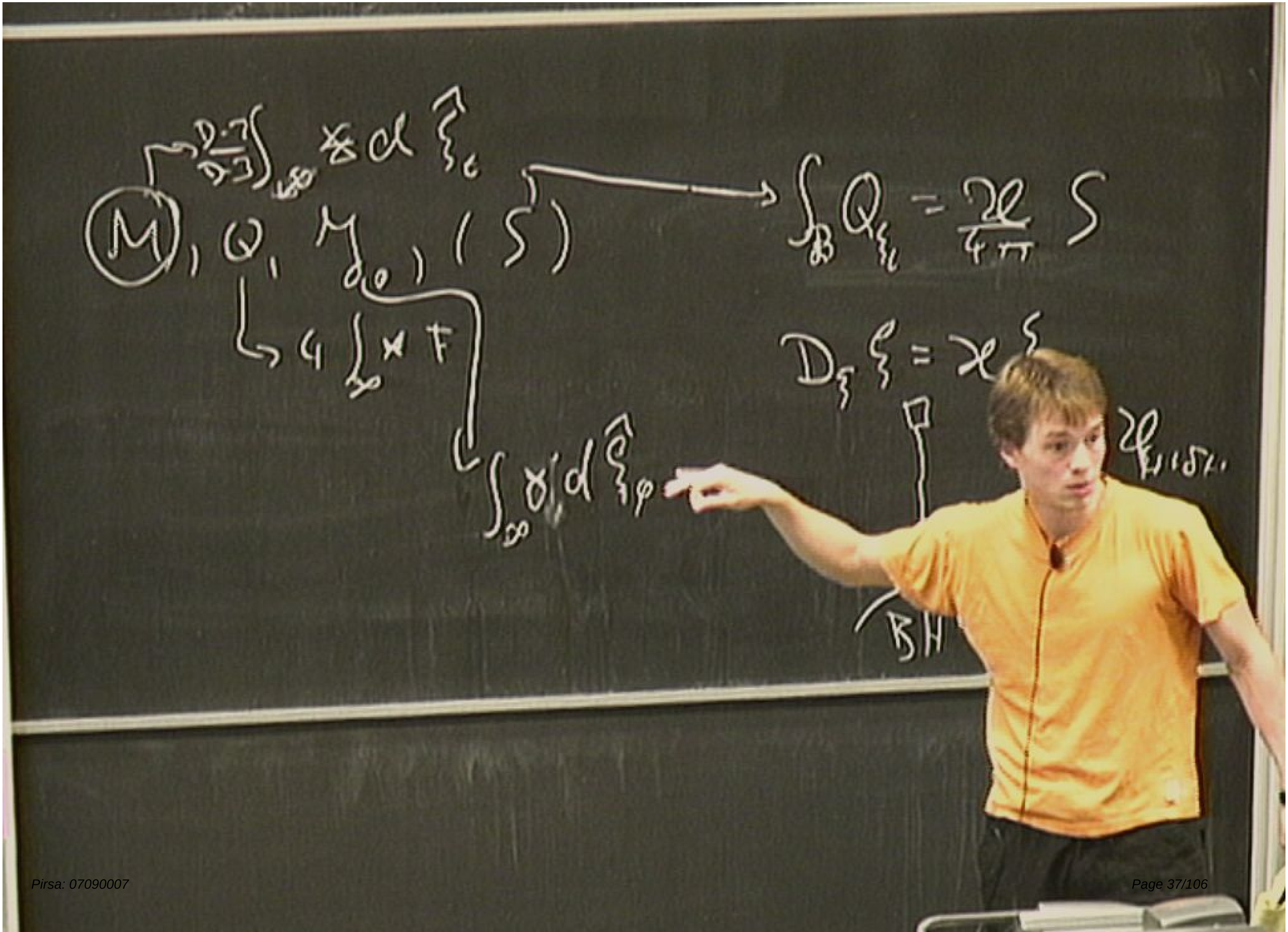
$$\int_{\infty} \star d\hat{\xi}_{\phi}$$

$$\int_{B} Q_{\xi_t} = \frac{\varkappa}{4\pi} S$$

$$D_{\xi}\xi = \varkappa \xi$$

BH


M , Q , J_0 , (S)
BH


$\frac{D-1}{D-3}\int_{S^\infty} \star d\hat{\xi}_t$
$M,\ Q,\ J_\varphi,\ (S)$
$q \int_{S^\infty} \star F$
$\int_{S^\infty} \star d\hat{\xi}_\varphi$
$\int_{B} Q_{\xi_t} = \frac{\kappa}{4\pi} S$
$D_\xi \xi = \kappa \xi$
BH

$$\frac{D-2}{D-3}\int_{\infty} * d\hat{\xi}_t$$

$$\text{M},\ Q,\ \mu_0,\ (S) \qquad \int_B Q_{\xi_t} = \frac{\varkappa}{4\pi} S$$

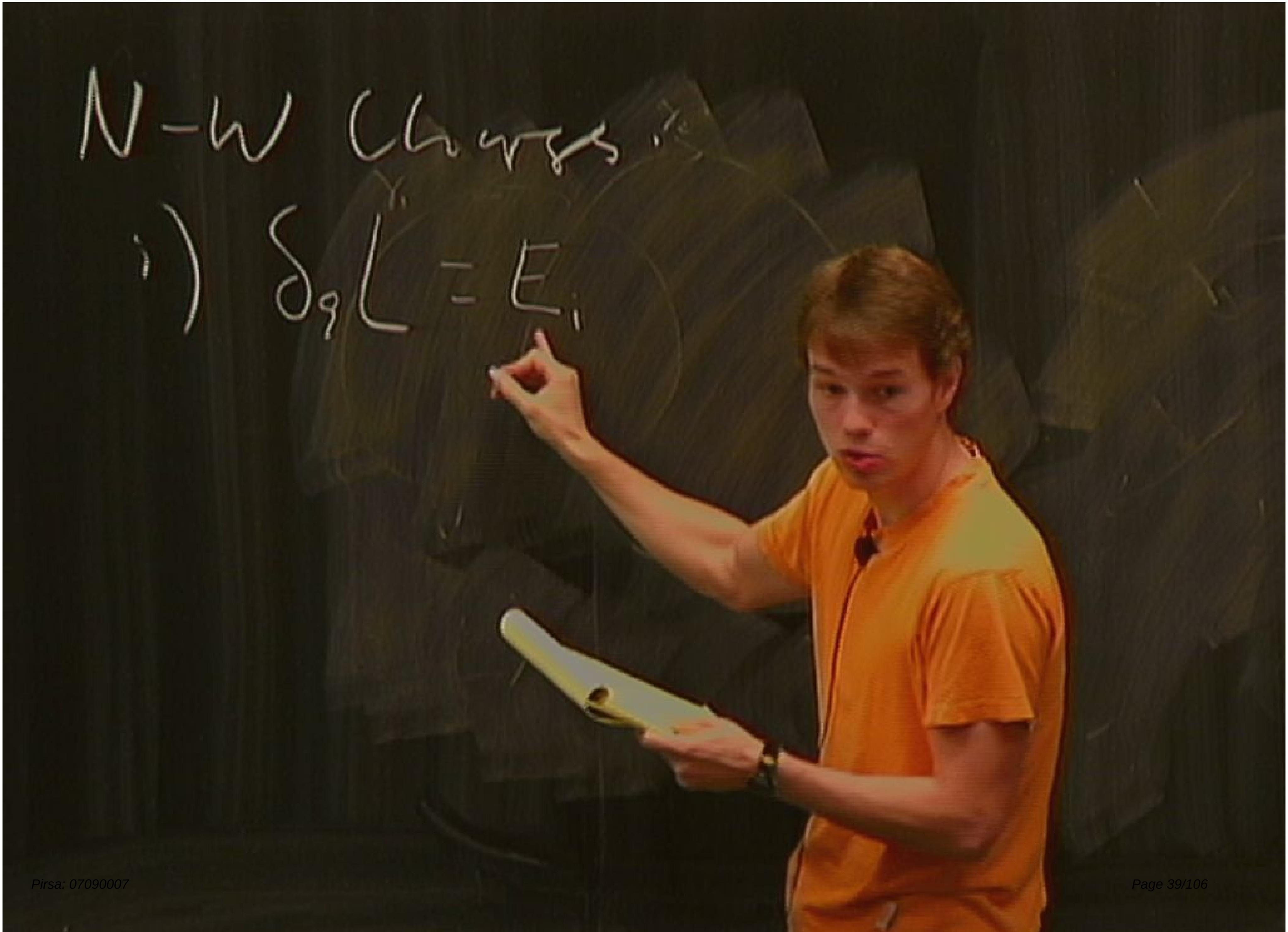
$$4\int_{\infty} * F$$

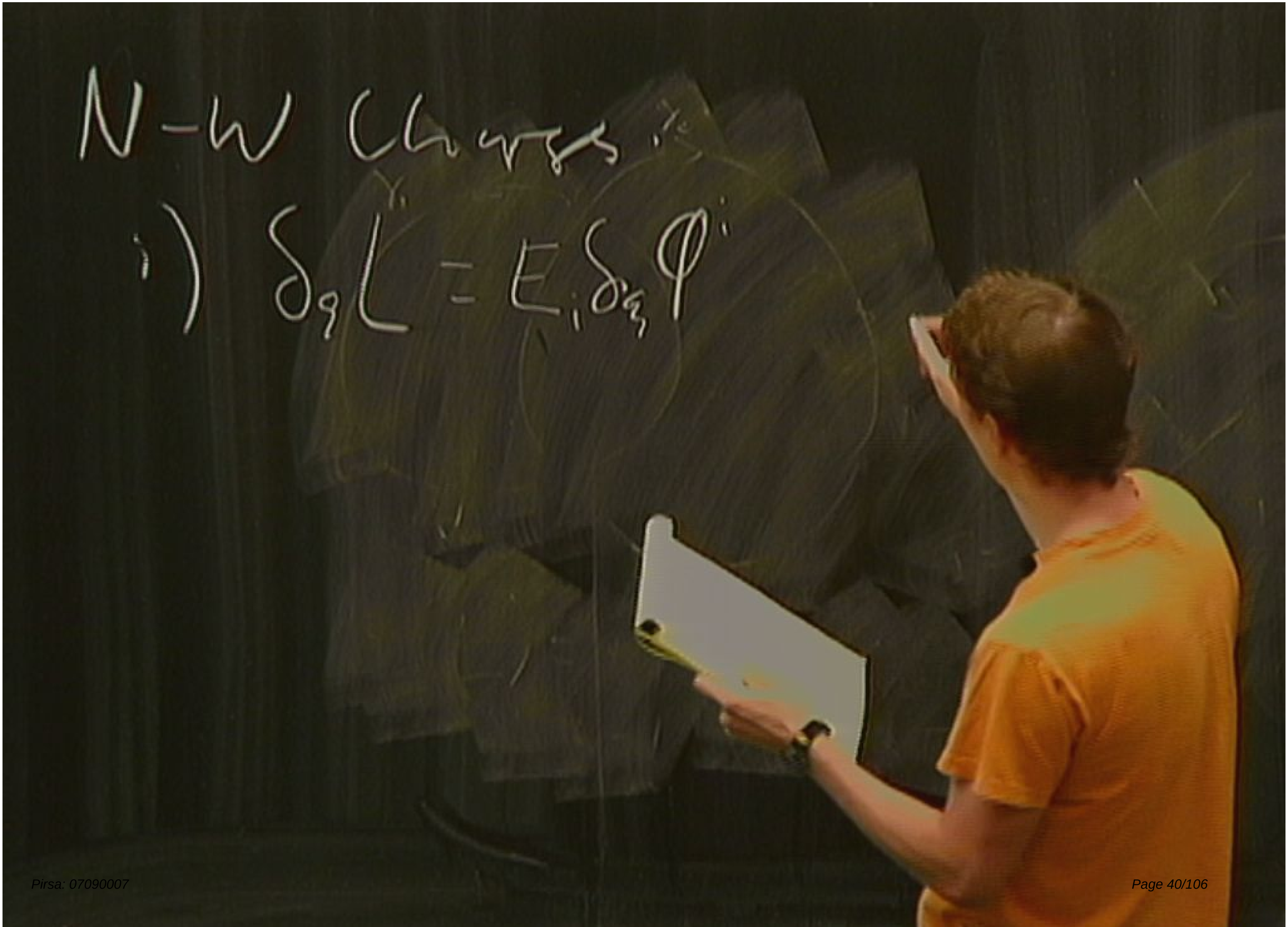
$$D_\xi \xi = \varkappa \xi$$

$$\varkappa_{\xi_t} - \varkappa_{\xi_t + \delta \xi_t}$$

$$\int_{\infty} \gamma\, d\hat{\xi}_\varphi$$

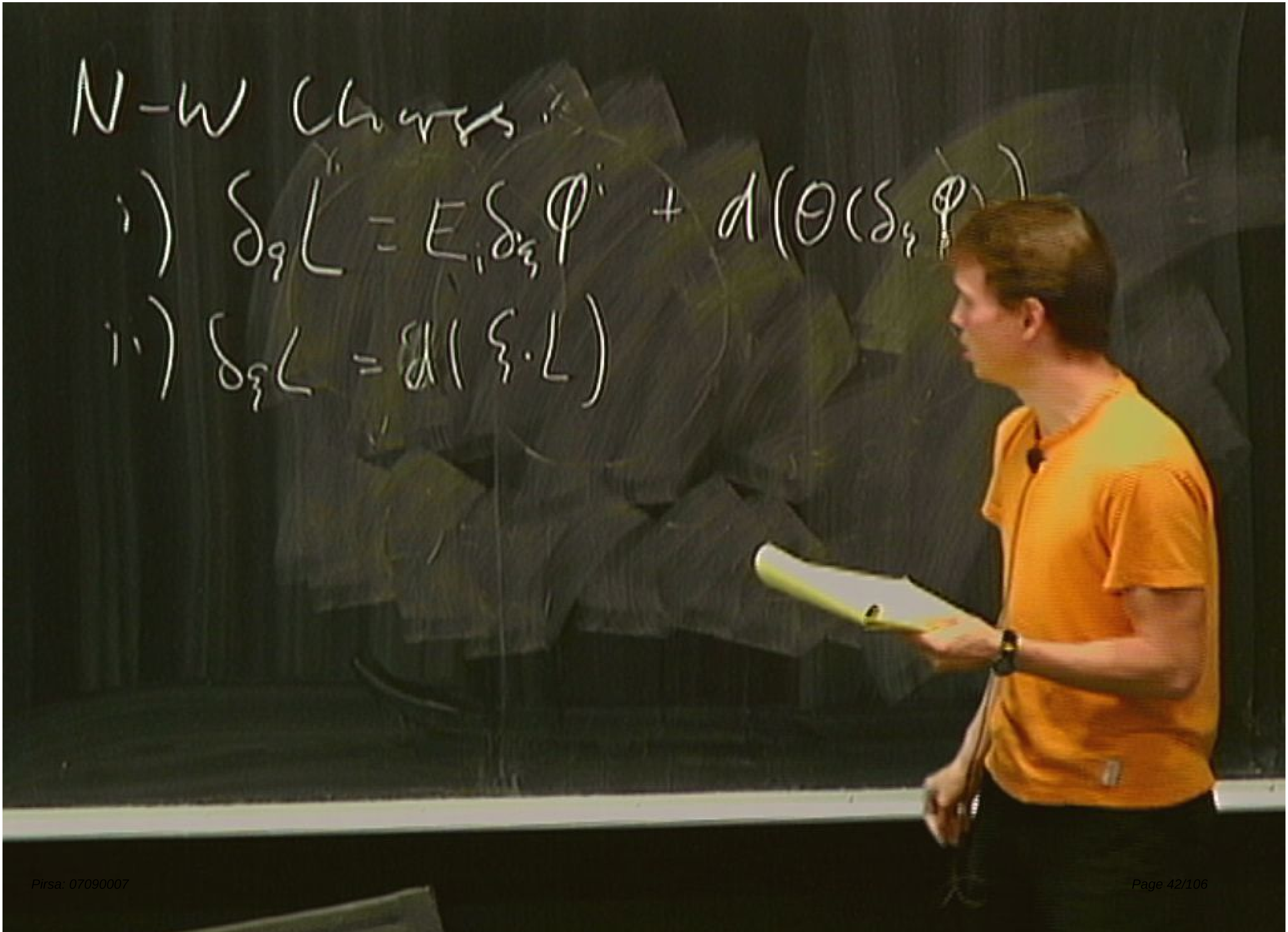
BH

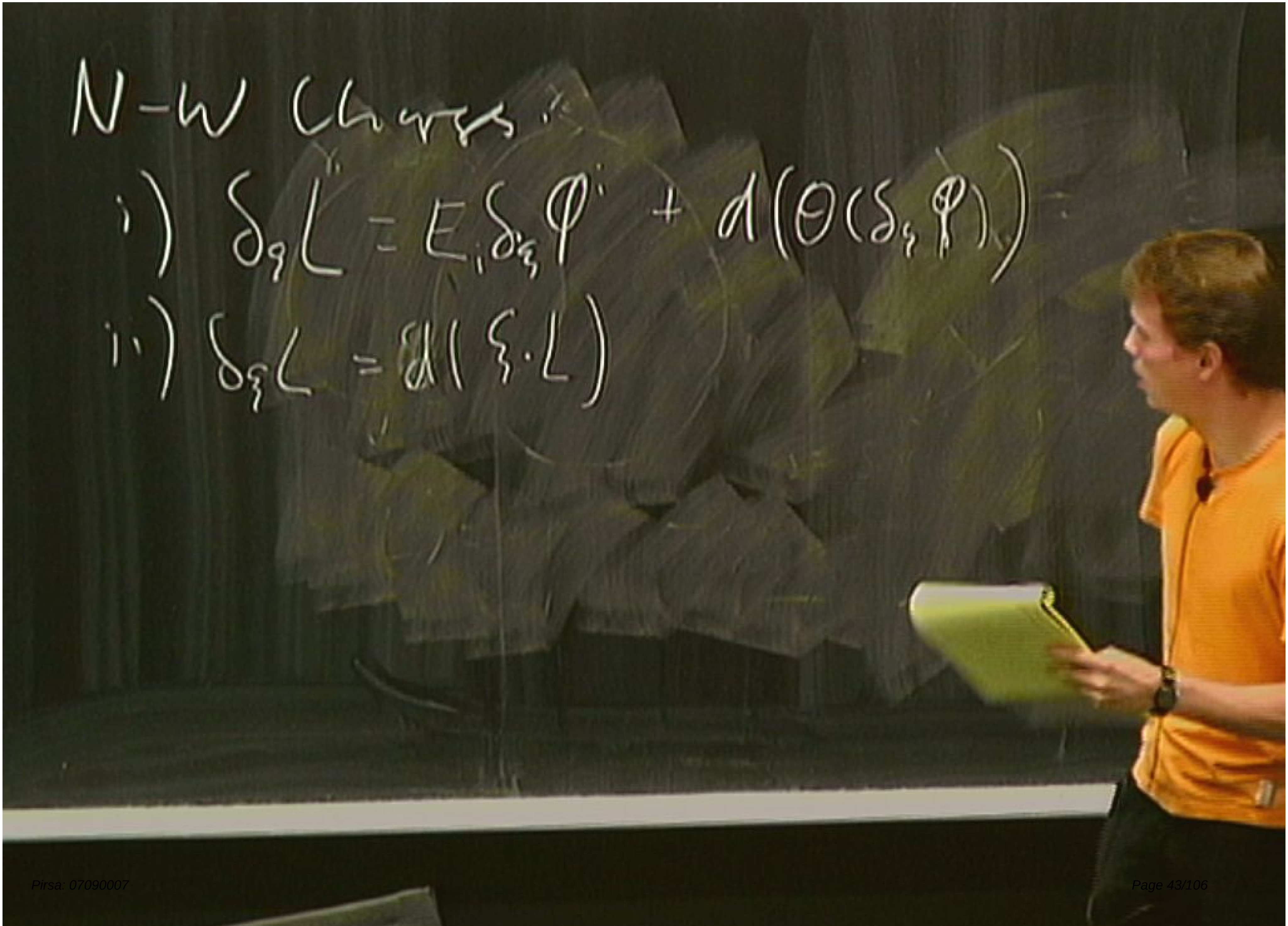

N-W Charges
i) $\delta_q L = E_i$

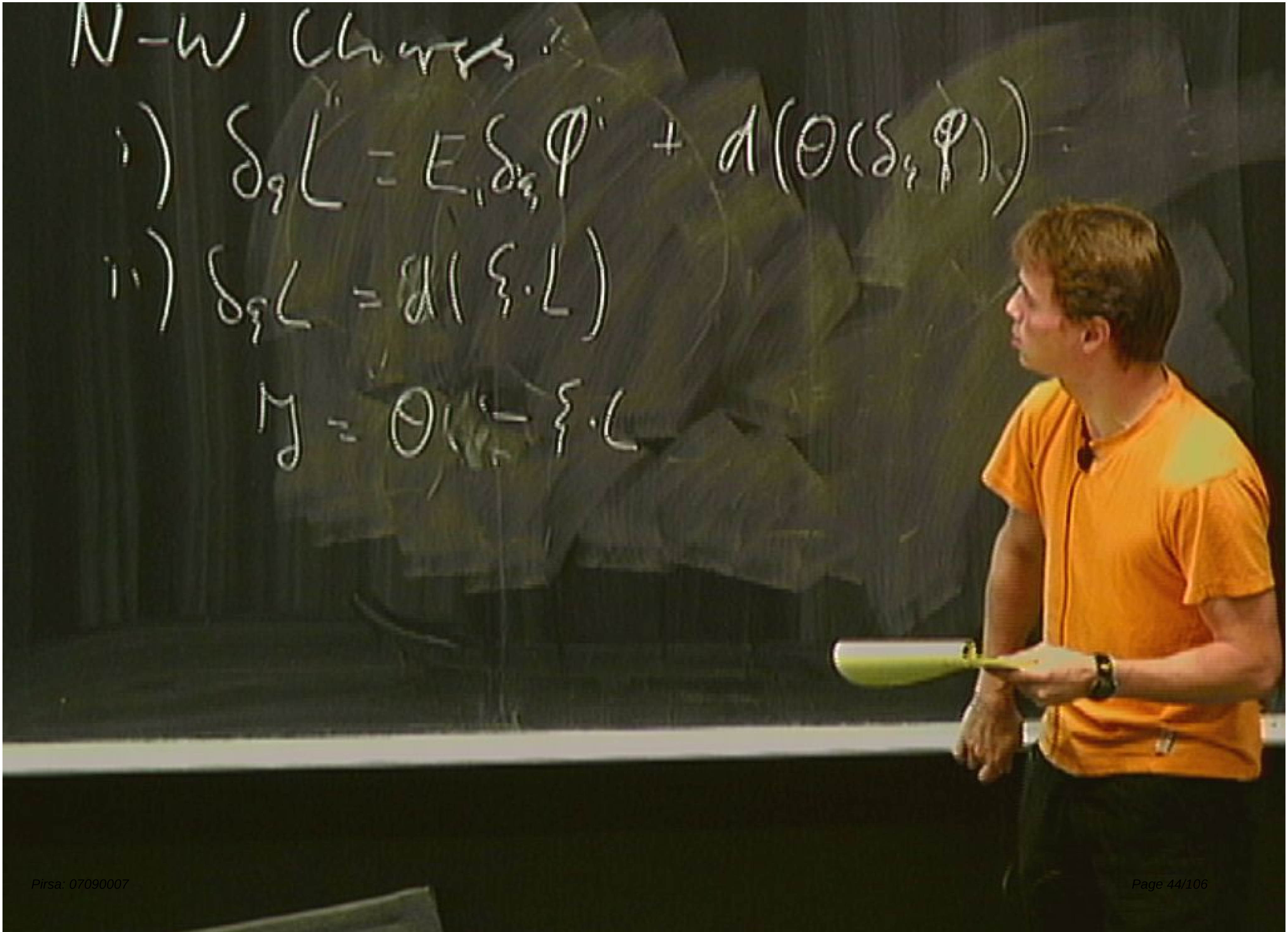

N-W Charges:
i) $\delta_q L = E_i \delta_q \phi^i$

N-W Charges:

i) $\delta_\xi L = E_i \delta_\xi \varphi^i + d(\Theta(\delta_\xi \varphi))$


N-W Charges:
i) $\delta_\xi L = E_i \delta_\xi \varphi^i + d(\Theta(\delta_\xi \varphi))$
ii) $\delta_\xi L = d(\xi \cdot L)$


N-W Charges:
i) $\delta_\xi L = E_i \delta_\xi \varphi^i + d(\Theta(\delta_\xi \varphi))$
ii) $\delta_\xi L = d(\xi \cdot L)$


N-W Charges:
i) $\delta_\xi L = E_i \delta_\xi \varphi^i + d(\Theta(\delta_\xi \varphi))$
ii) $\delta_\xi L = d(\xi \cdot L)$
$J = \Theta( ) - \xi \cdot L$

N-W Charges:

i) $\delta_\xi L = E_i \delta_\xi \varphi^i + d\big(\Theta(\delta_\xi \varphi)\big)$

ii) $\delta_\xi L = d(\xi \cdot L)$

$J = \Theta($[illegible]$) - \xi \cdot L \overset{E_i = 0}{\Rightarrow} dJ = 0$

$J = dQ$

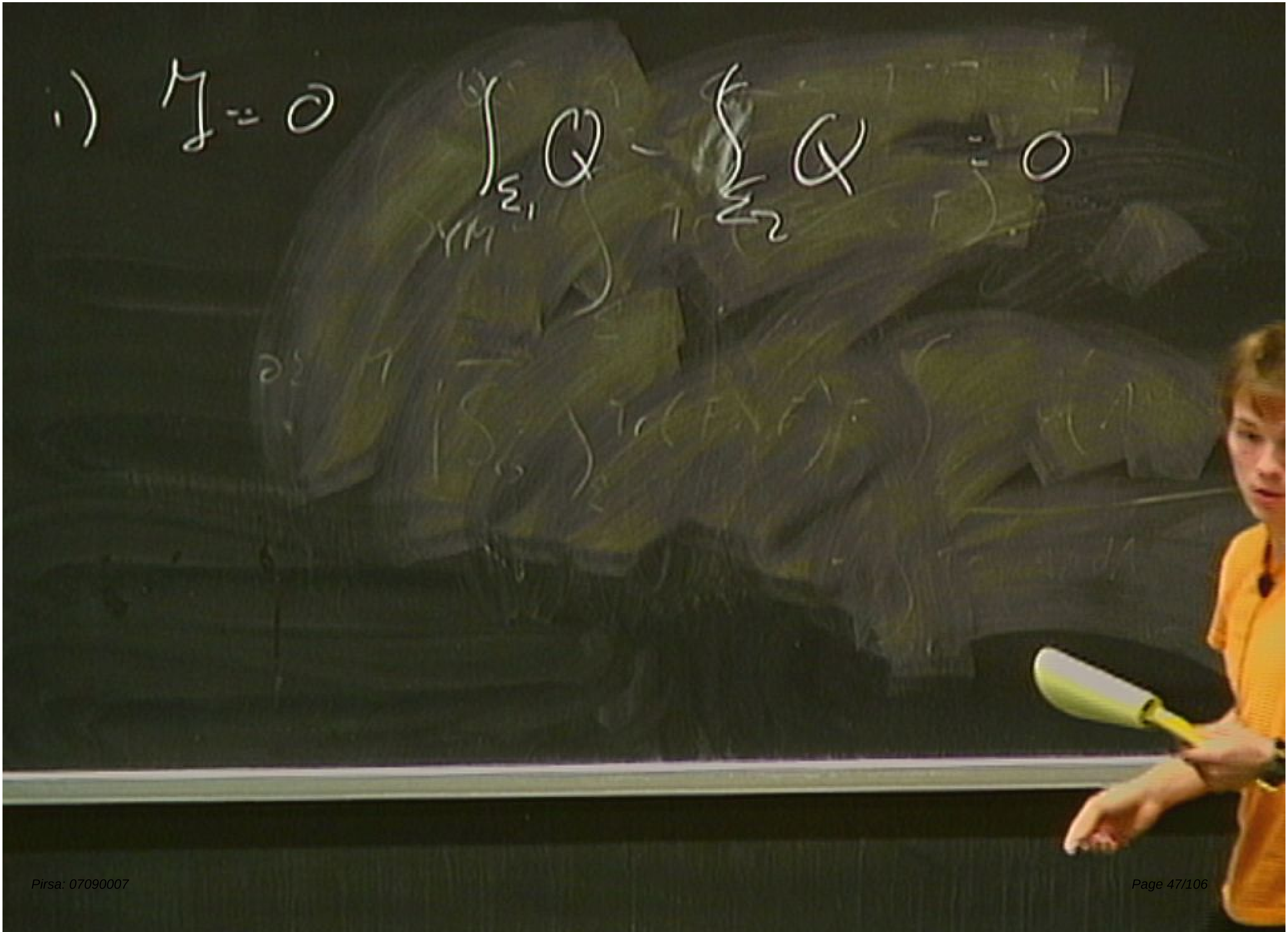
N-W Charges:

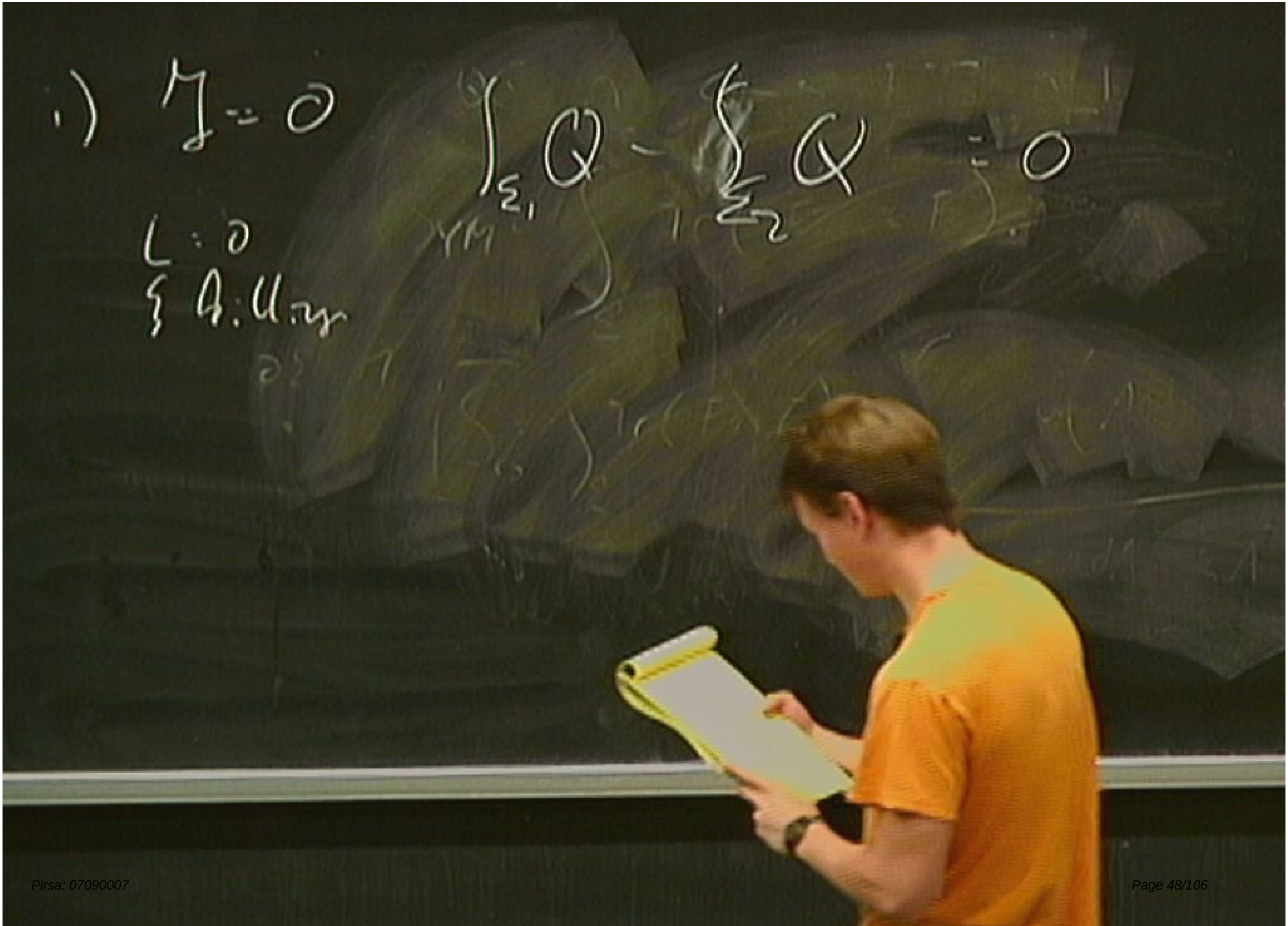
i) $\delta_\xi L = E_i \delta_\xi \varphi^i + d(\Theta(\delta_\xi \varphi))$

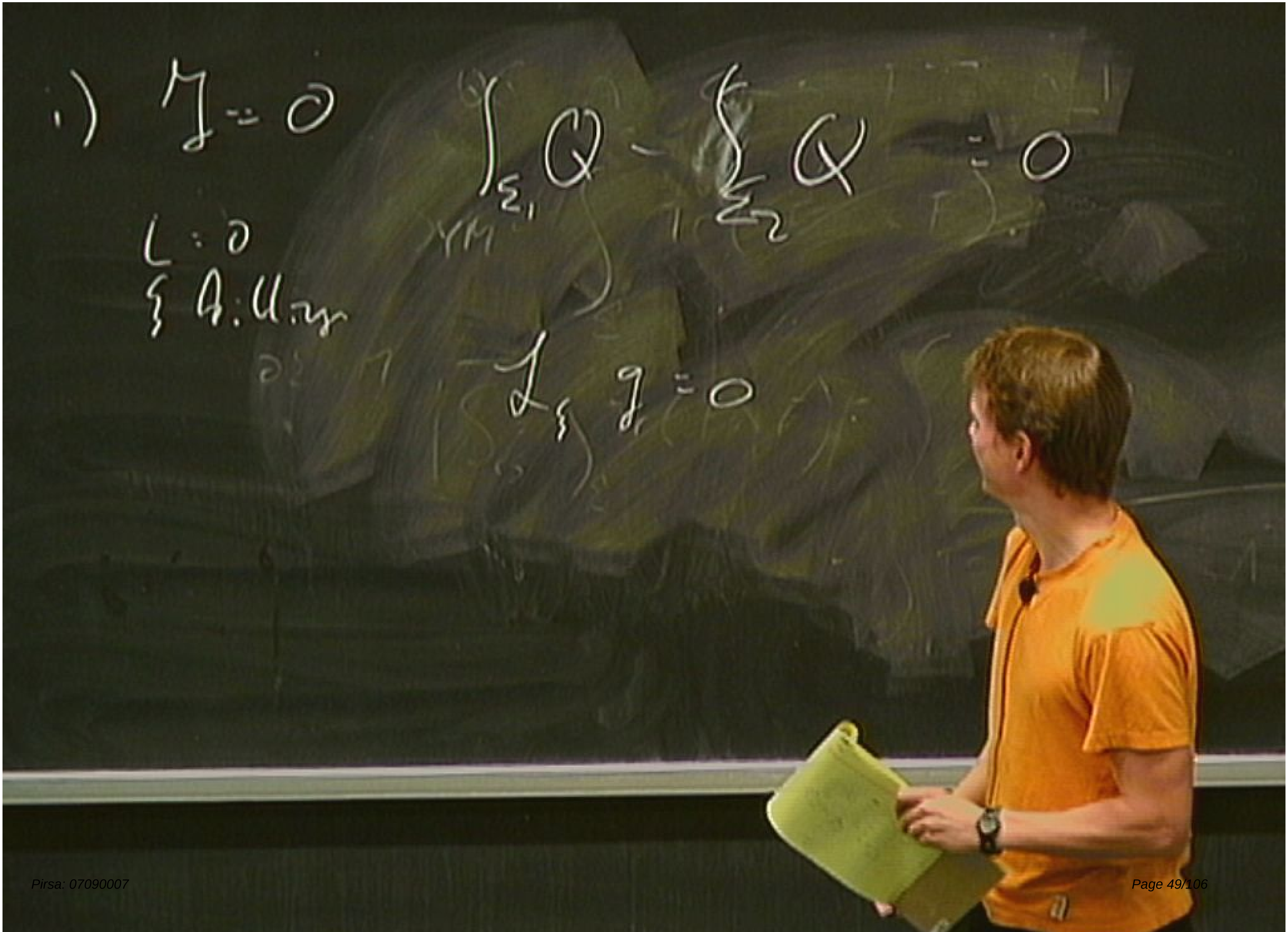
ii) $\delta_\xi L = d(\xi \cdot L)$

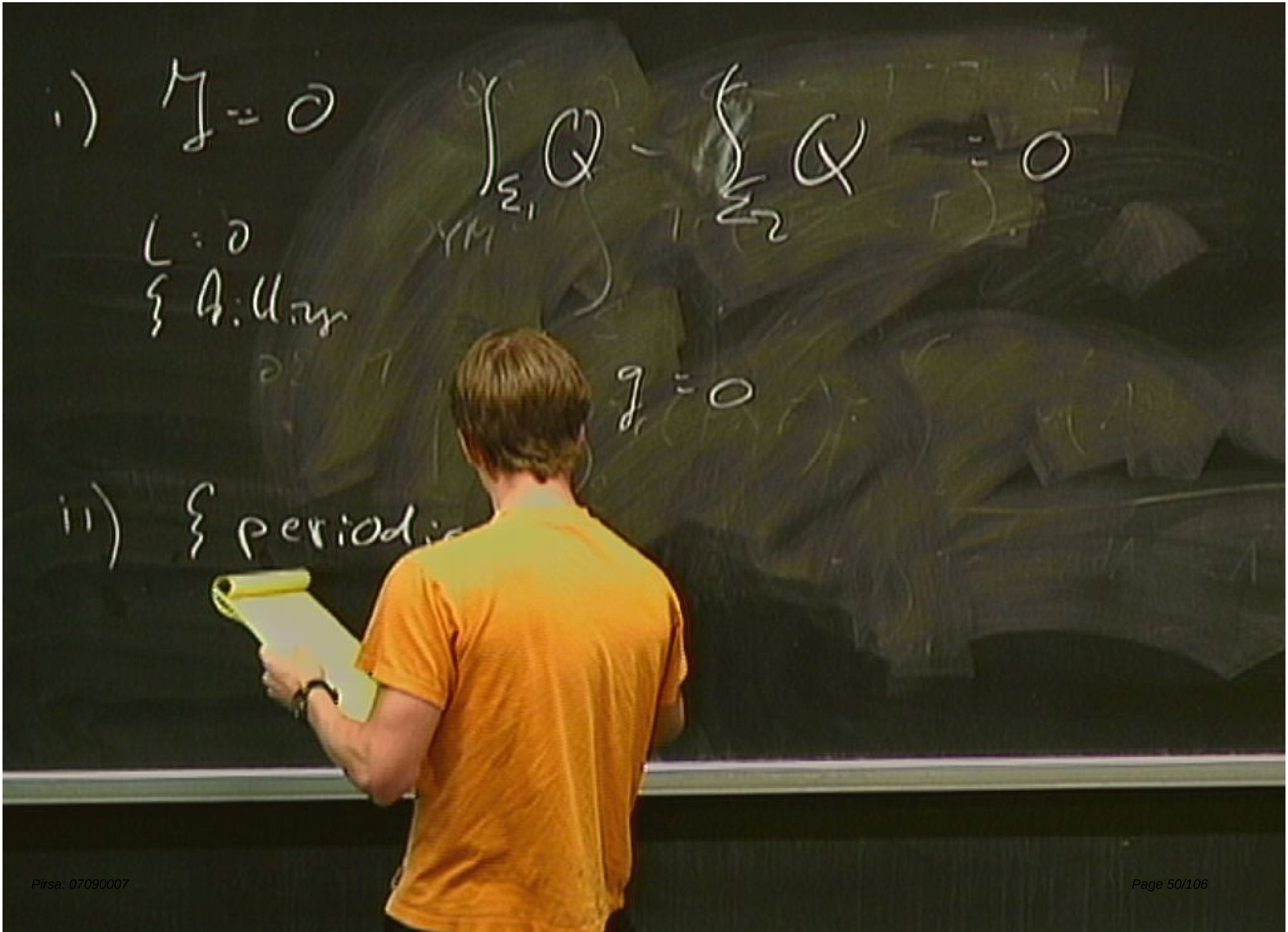
$$J = \Theta(\delta_\xi \varphi) - \xi \cdot L \overset{E_i = 0}{\Longrightarrow} dJ = 0$$

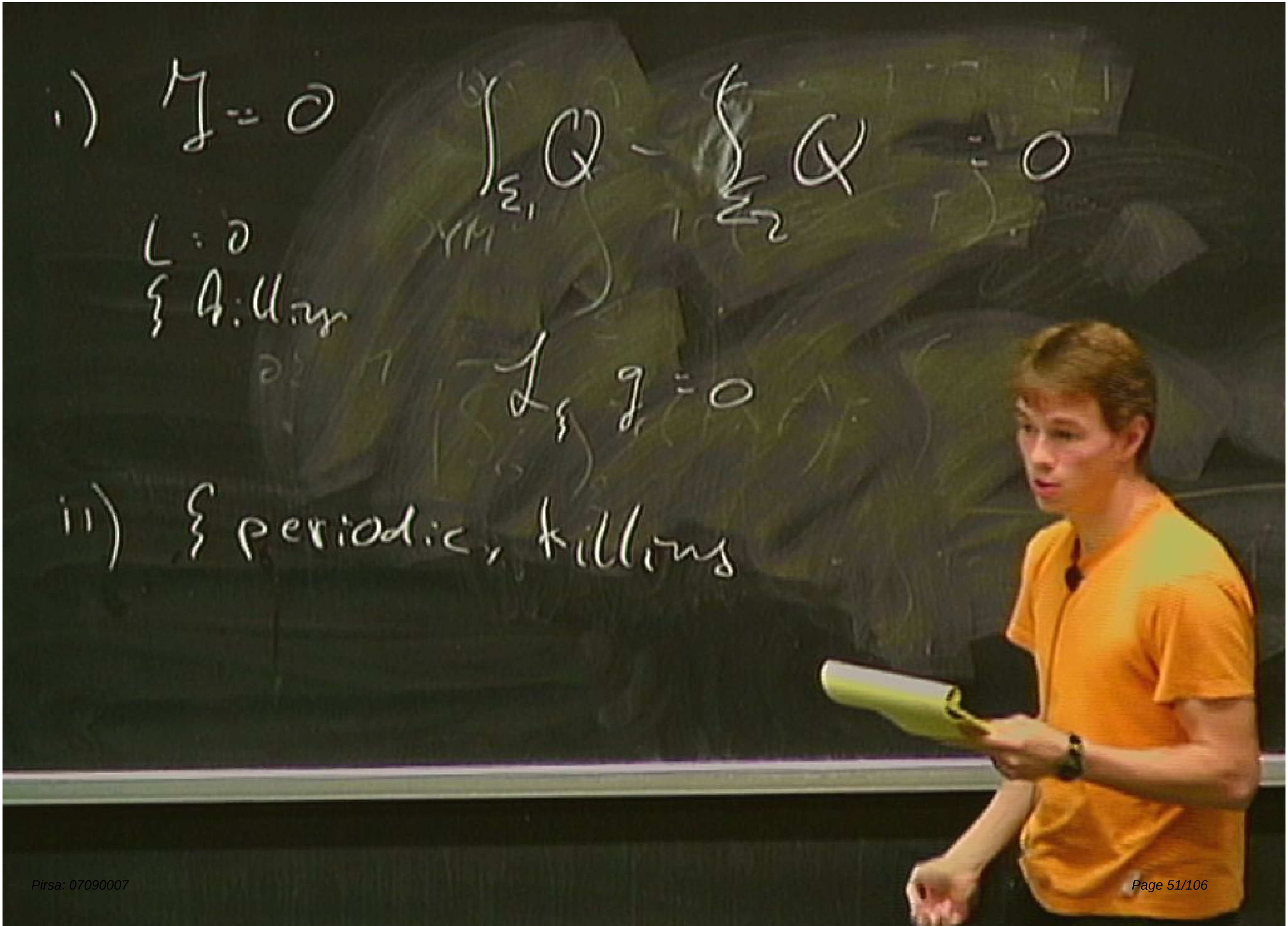
$$J = dQ$$

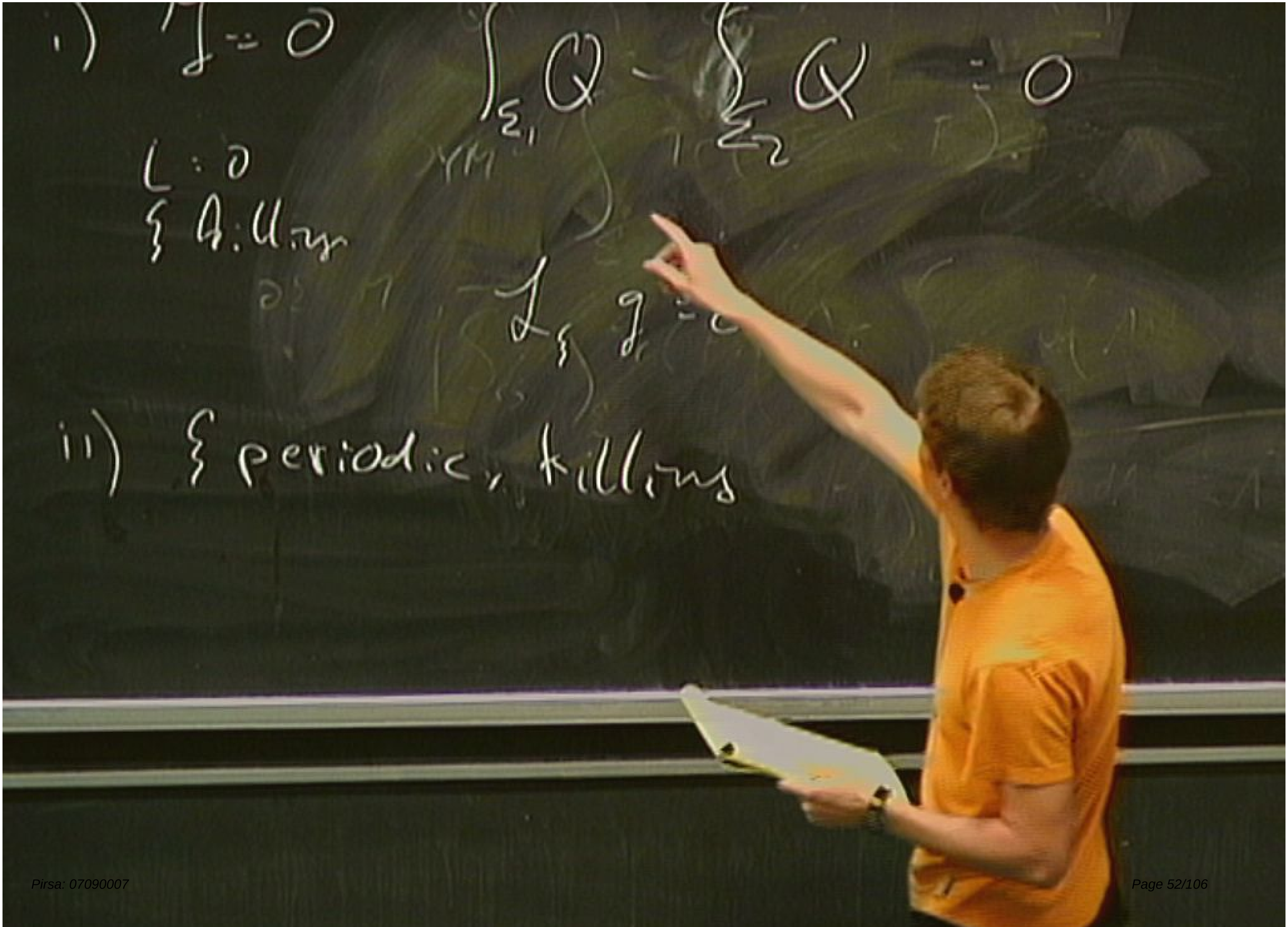

i) $J = 0$ $\int_{\Sigma_1} Q - \int_{\Sigma_2} Q = 0$

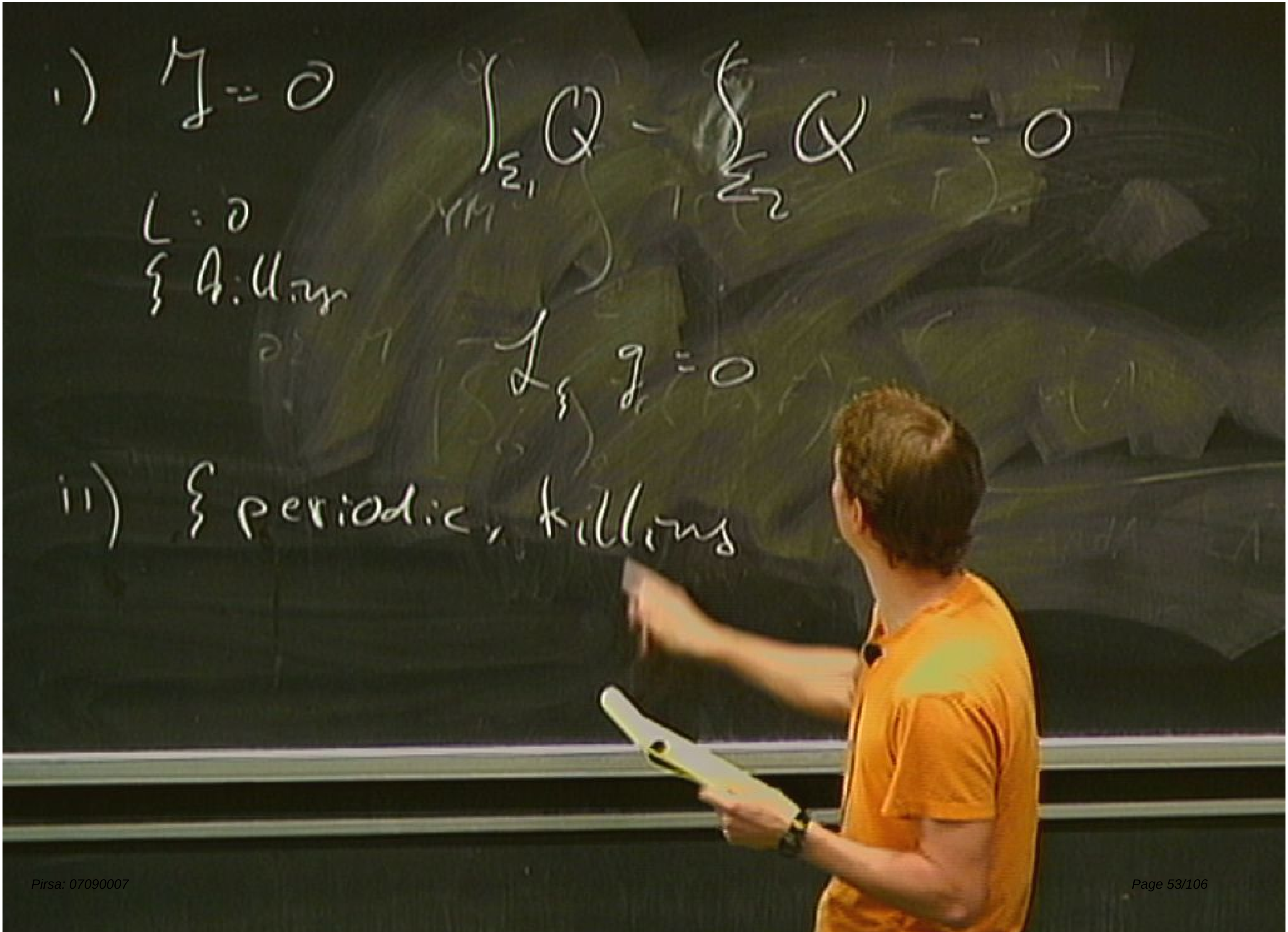

i) F = 0
∫Σ1 Q − ∫Σ2 Q = 0
L = 0

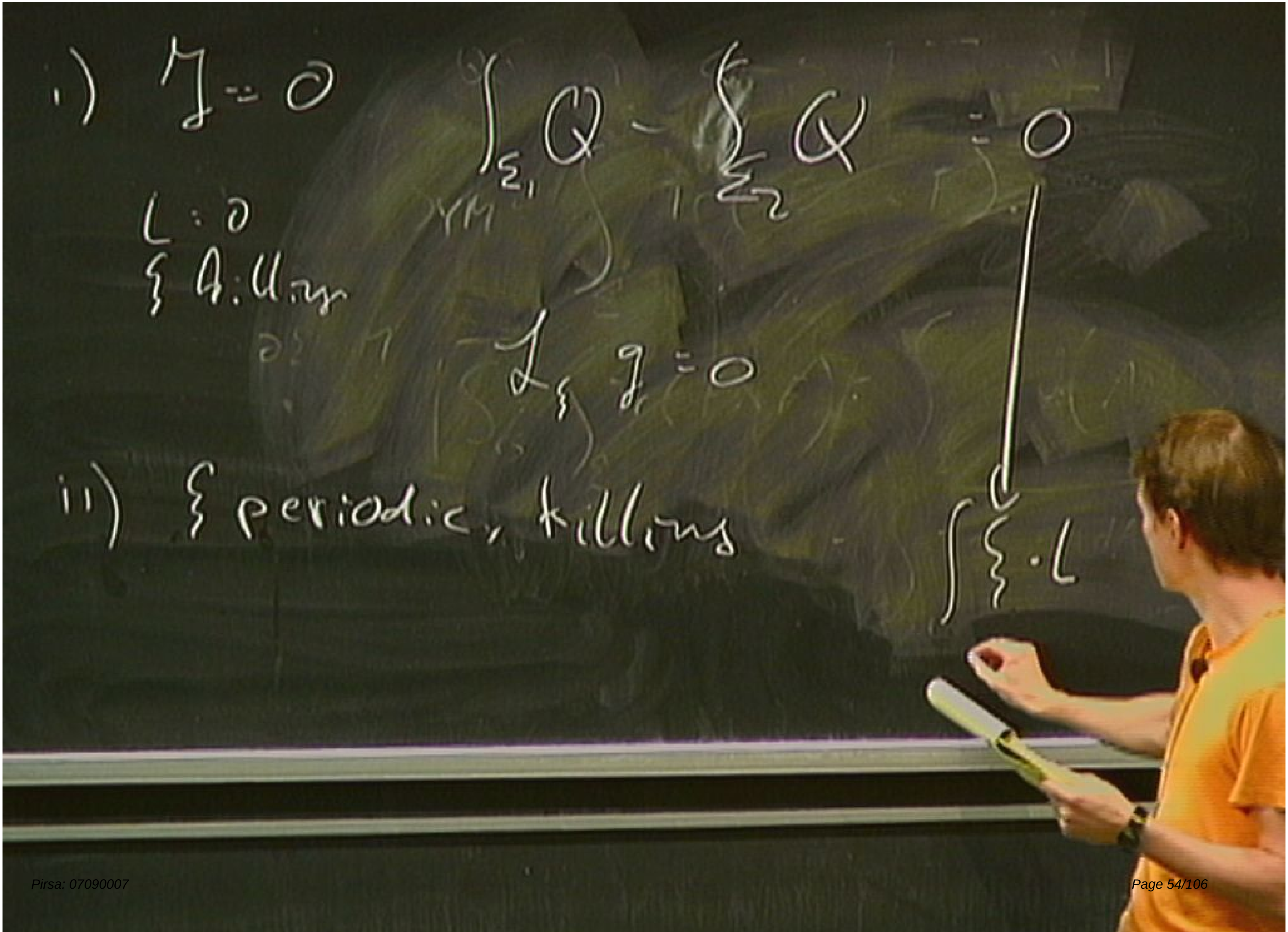

i) $J = 0$
$\int_{\Sigma_1} Q - \int_{\Sigma_2} Q = 0$
$L = 0$
$\mathcal{L}_\xi g = 0$


i) J = 0
∫Σ1 Q − ∫Σ2 Q = 0
L = 0
g = 0
ii) periodic


i) J = 0
∫Σ1 Q − ∫Σ2 Q = 0
L = 0
ξ
Lξ g = 0
ii) ξ periodic, killing


i) $g \doteq 0$
$\int_{\Sigma_1} Q - \int_{\Sigma_2} Q \doteq 0$
$L \doteq 0$
$\mathcal{L}_\xi g \doteq 0$
ii) ξ periodic, killing


i) $J = 0$
$\int_{\Sigma_1} Q - \int_{\Sigma_2} Q = 0$
$L = 0$
$\mathcal{L}_\xi g = 0$
ii) ξ periodic, killing


i) J = 0
∫Σ1 Q − ∫Σ2 Q = 0
L = 0
ii) ξ periodic, killing
∫ ξ · L

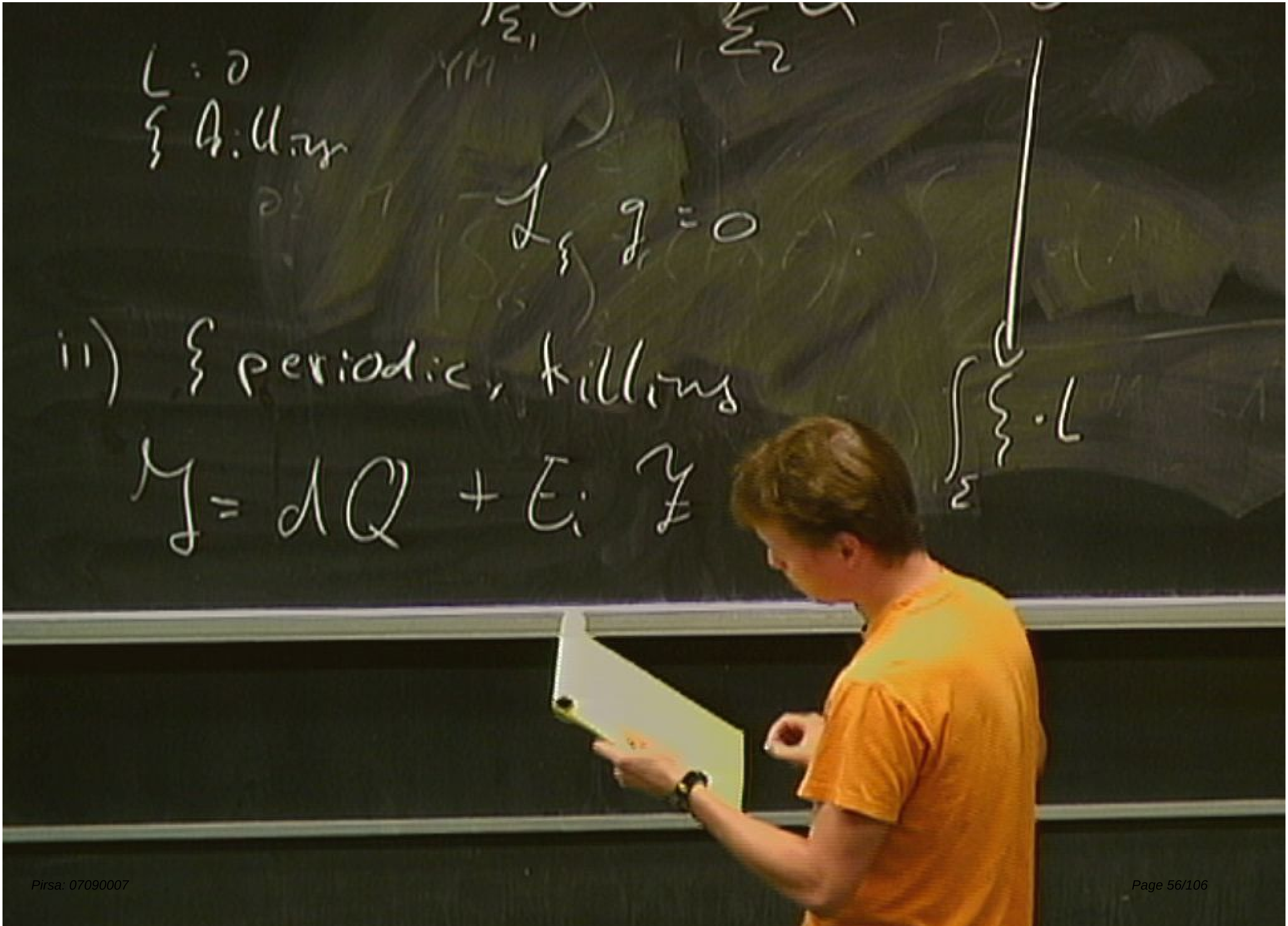
i) $J = 0$ $\quad \int_{\Sigma_1} Q - \int_{\Sigma_2} Q = 0$

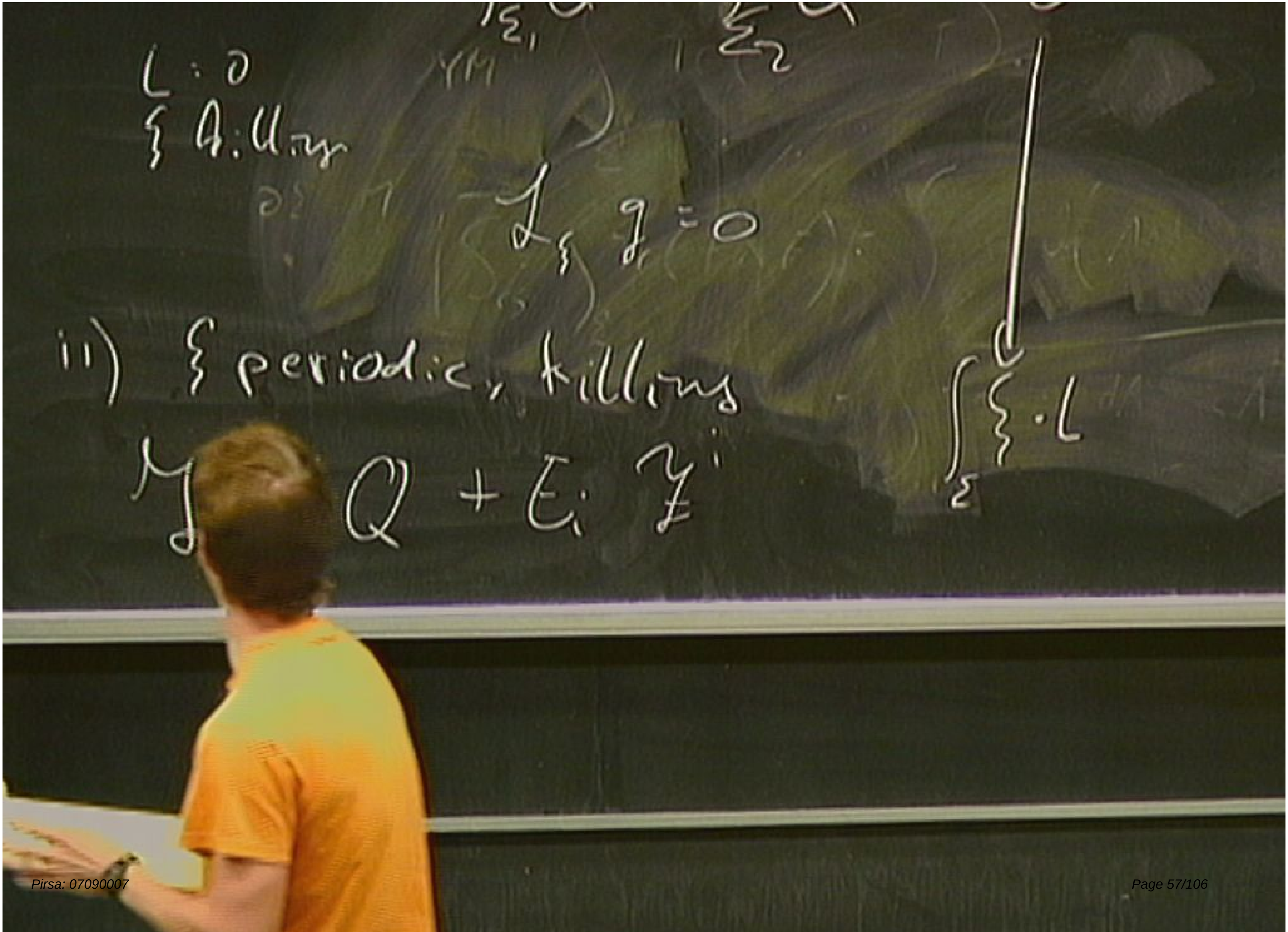
$L = 0$

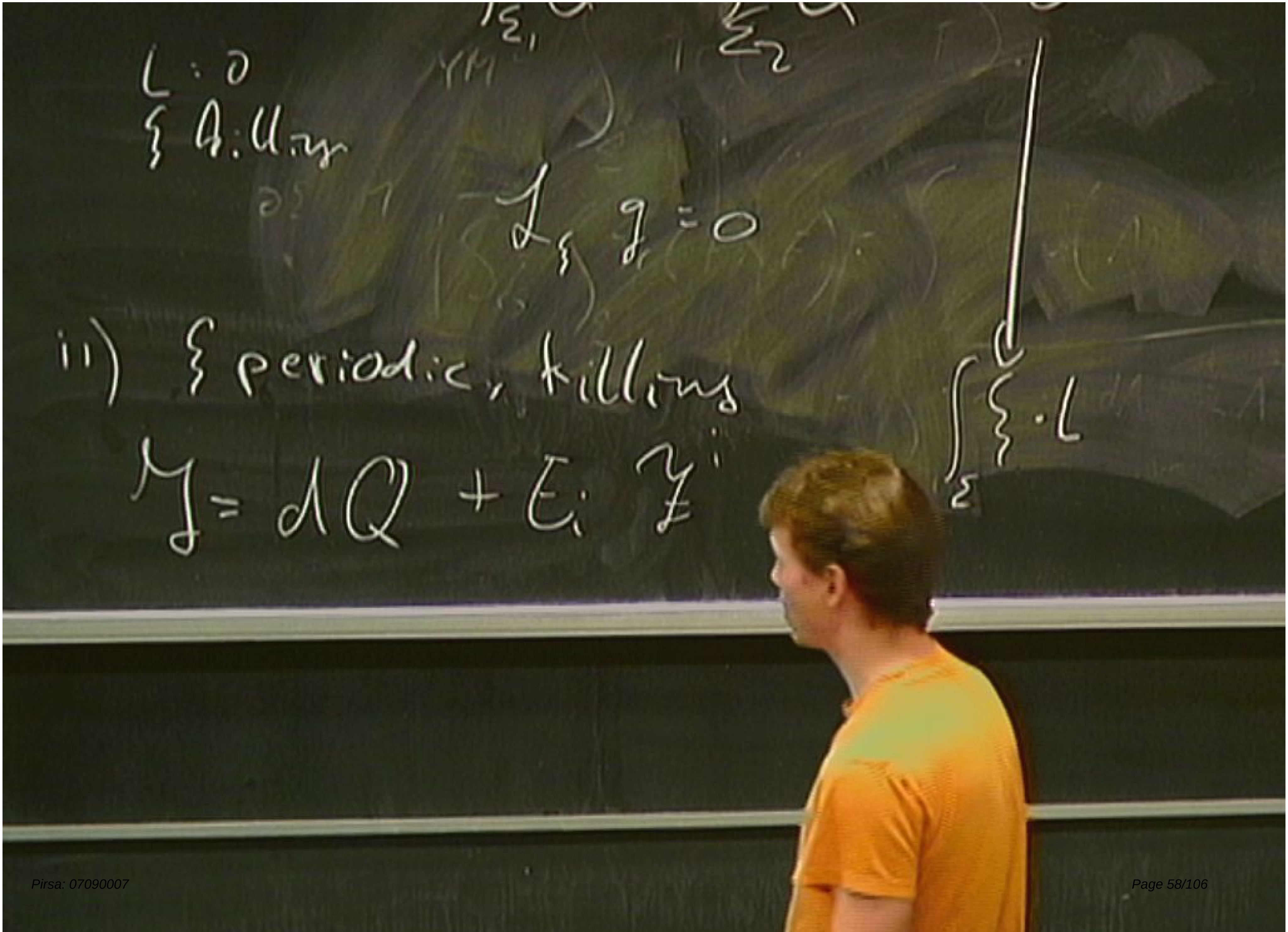
ξ A: killing

$\mathcal{L}_\xi g = 0$

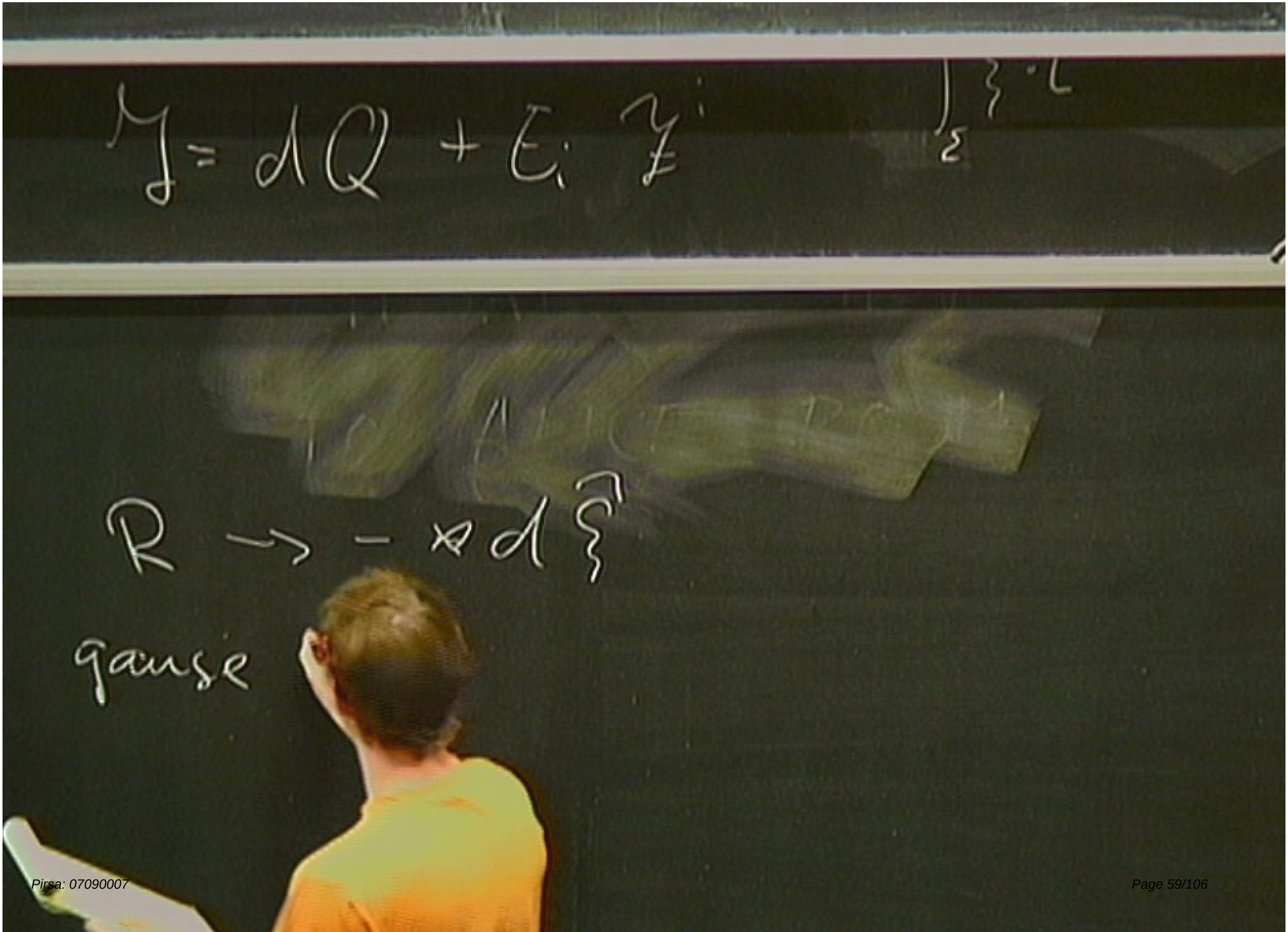
ii) ξ periodic, killing $\quad \int_\Sigma \xi \cdot L$


Σ_1
Σ_2
$L = 0$
$\mathcal{L}_\xi g = 0$
ii) ξ periodic, killing
$J = dQ + E_i Z$
$\int_\Sigma \xi \cdot L$


Σ_1
Σ_2
$L : 0$
$\mathcal{L}_\xi g = 0$
ii) ξ periodic, killing
$Q + E_i Z^i$
$\int_\Sigma \xi \cdot L$



L : 0
$\mathcal{L}_\xi g = 0$
ii) ξ periodic, killing
$J = dQ + E_i Z^i$
$\int_\Sigma \xi \cdot L$


$J = dQ + \epsilon_i Z^i$
$R \to - * d\xi$
gauge

$$J = dQ + \mathcal{E}_i \xi^i \qquad \int_\Sigma \xi \cdot L$$

$$R \longrightarrow - * d\xi$$

gauge $F \wedge * F$ $\longrightarrow$

$A \wedge F \wedge F$

$$R \to -\star d\xi$$

gauge $F \wedge \star F$, $A \wedge F \wedge F$ $\to$ $i_\xi A \dfrac{\delta L}{\delta dA}$

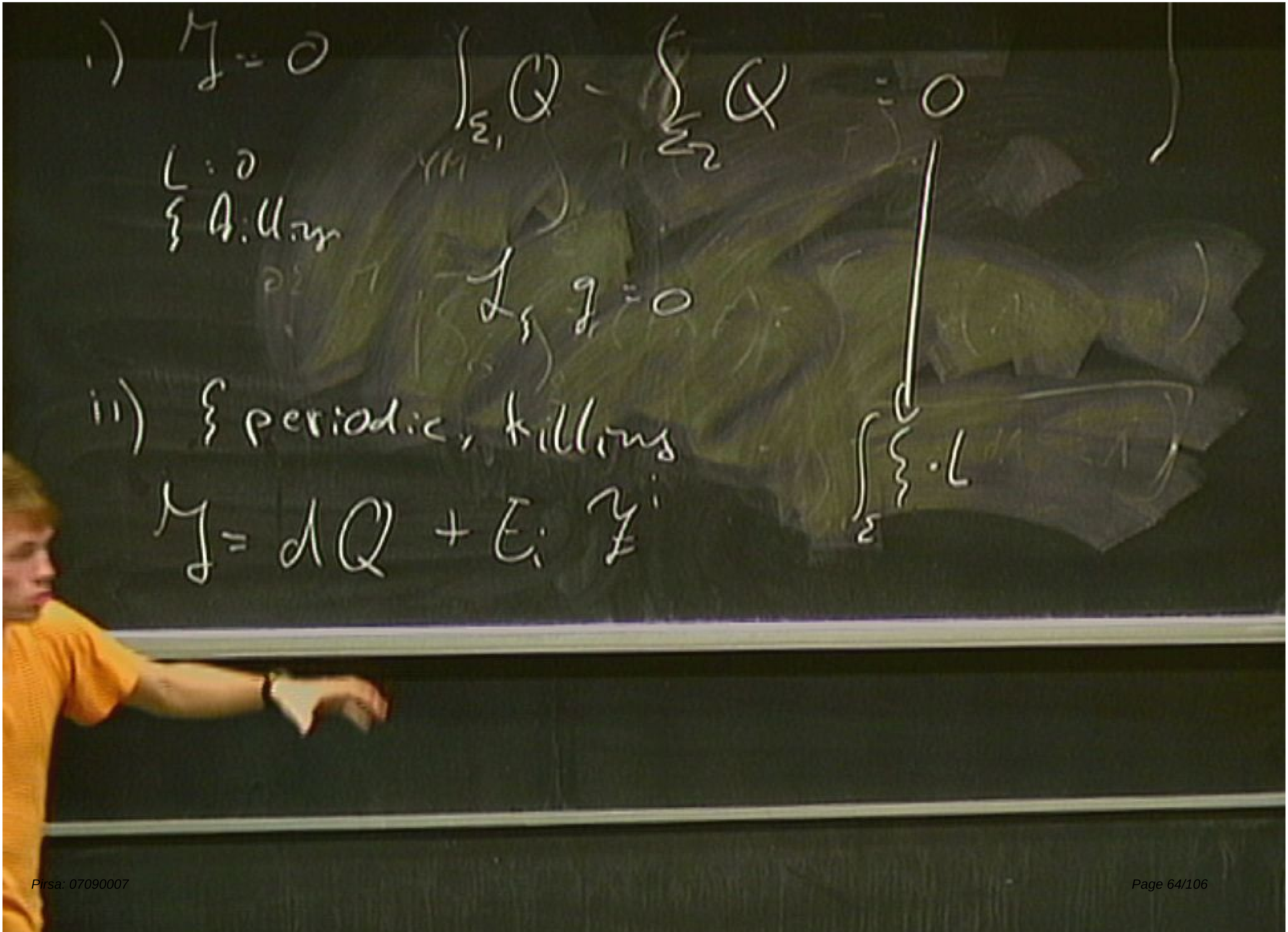
$$R \to -\star d\xi$$

gauge $F \wedge \star F$

$A \wedge F \wedge F \quad \to \quad i_\xi A \frac{\delta L}{\delta dA}$

$R \to - \star d \xi$

gauge $F \wedge \star F$
$A \wedge F \wedge F \to i_\xi A \frac{\delta L}{\delta dA}$


i) $J = 0$ $\int_{\Sigma_1} Q - \int_{\Sigma_2} Q = 0$
$L = 0$
$\mathcal{L}_\xi g = 0$
ii) ξ periodic, killing
$J = dQ + E_i Z^i$
$\int_\Sigma \xi \cdot L$

i) $J = 0$ $\quad \int_{\Sigma_1} Q - \int_{\Sigma_2} Q = 0$

$L = 0$

ξ A: ℓ_{YM}

$\mathcal{L}_\xi g = 0$

ii) ξ periodic, killing

$J = dQ + E_i \, Y^i$

$\int_\Sigma \xi \cdot L$

S^D

$SO(D-1)$

i) $J = 0$ $\quad \int_{\Sigma_1} Q - \int_{\Sigma_2} Q = 0$

$L = 0$

ξ Ai. Un...

$\mathcal{L}_\xi g = 0$

ii) ξ periodic, killing $\quad \int_\Sigma \xi \cdot L$

$J = Q + E_i Z^i$

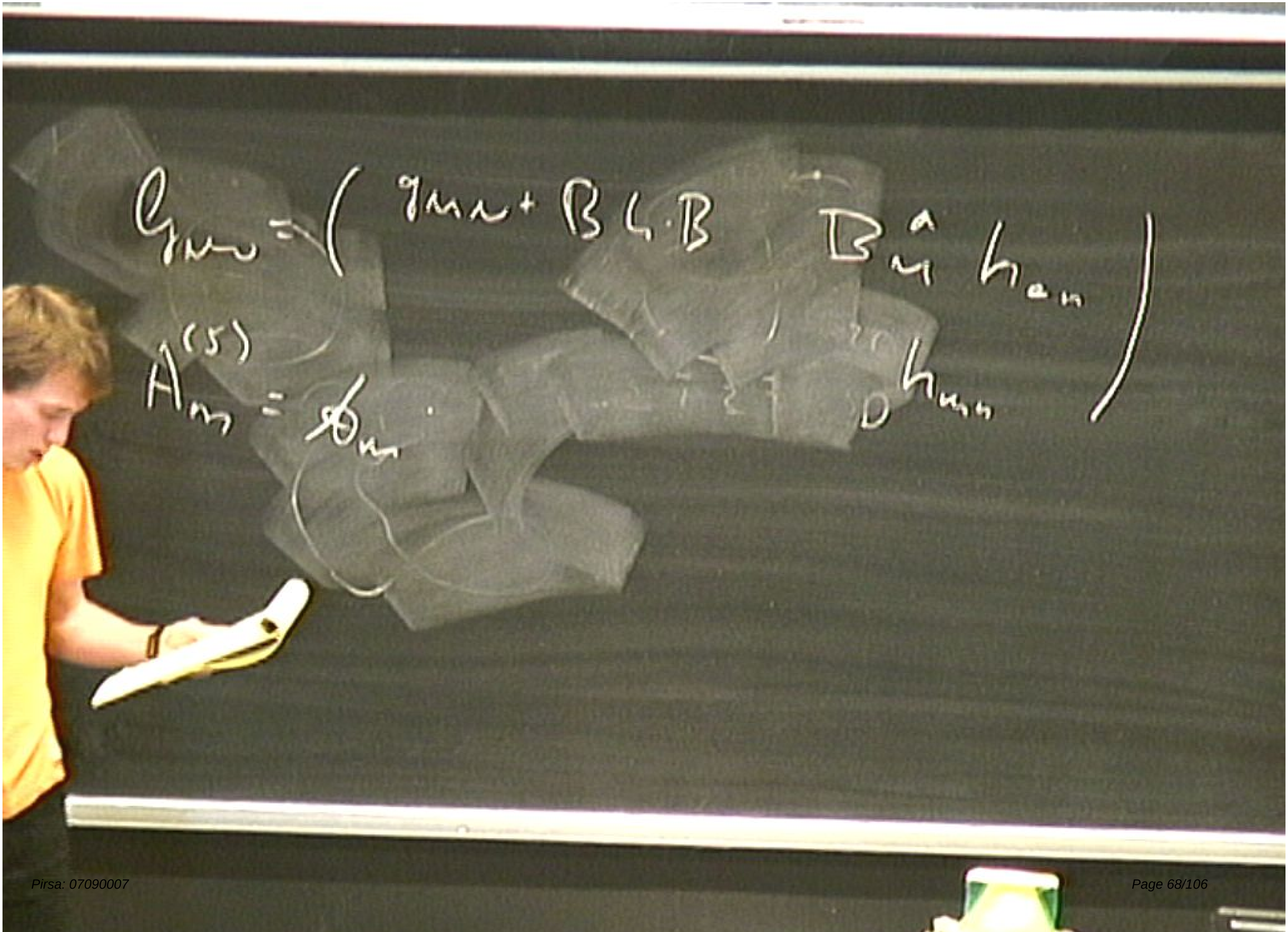
S^D

$SO(D-1)$

$\downarrow$

$U(1)^{\frac{D-1}{2}}$

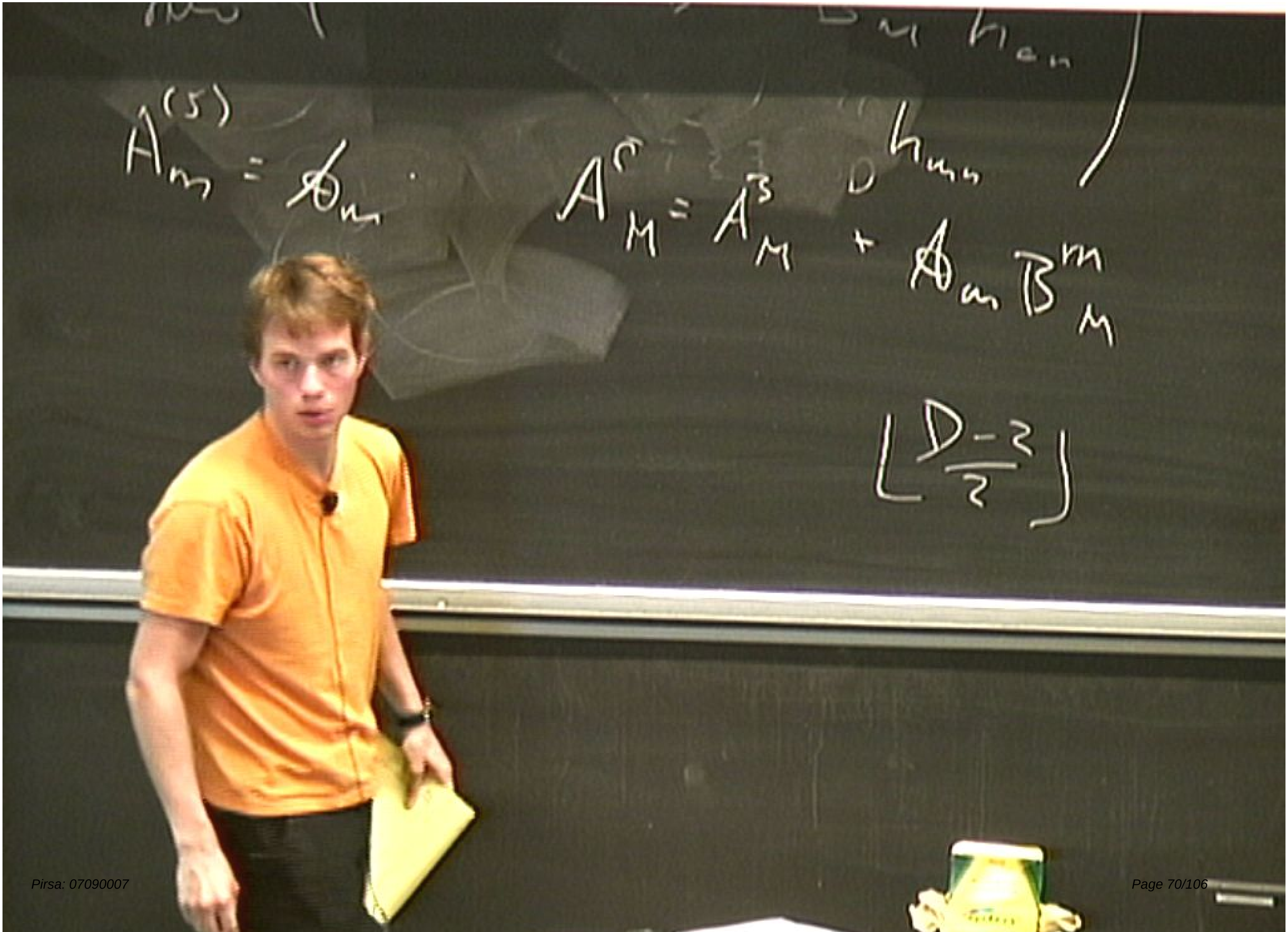
$$g_{\mu\nu} = \begin{pmatrix} g_{mn} + B\,G \cdot B & B_{\mu}^{a}\, h_{an} \\ & h_{mn} \end{pmatrix}$$


g_{MN} = (g_{mn} + B h·B   B^a_m h_{an}
h_{mn})
A^{(5)}_m = φ_m

$$g_{MN} = \begin{pmatrix} g_{mn} + B G \cdot B & B^{\hat{a}}_{\mu} h_{am} \\ & h_{mn} \end{pmatrix}$$

$$A^{(5)}_m = A_m$$

$$A_M = A^3_M + A_m B^m_M$$


$A^{(5)}_m = \phi_m$
h_{mn}
$A^6_M = A^5_M + \phi_m B^m_M$
$\lfloor \frac{D-2}{2} \rfloor$

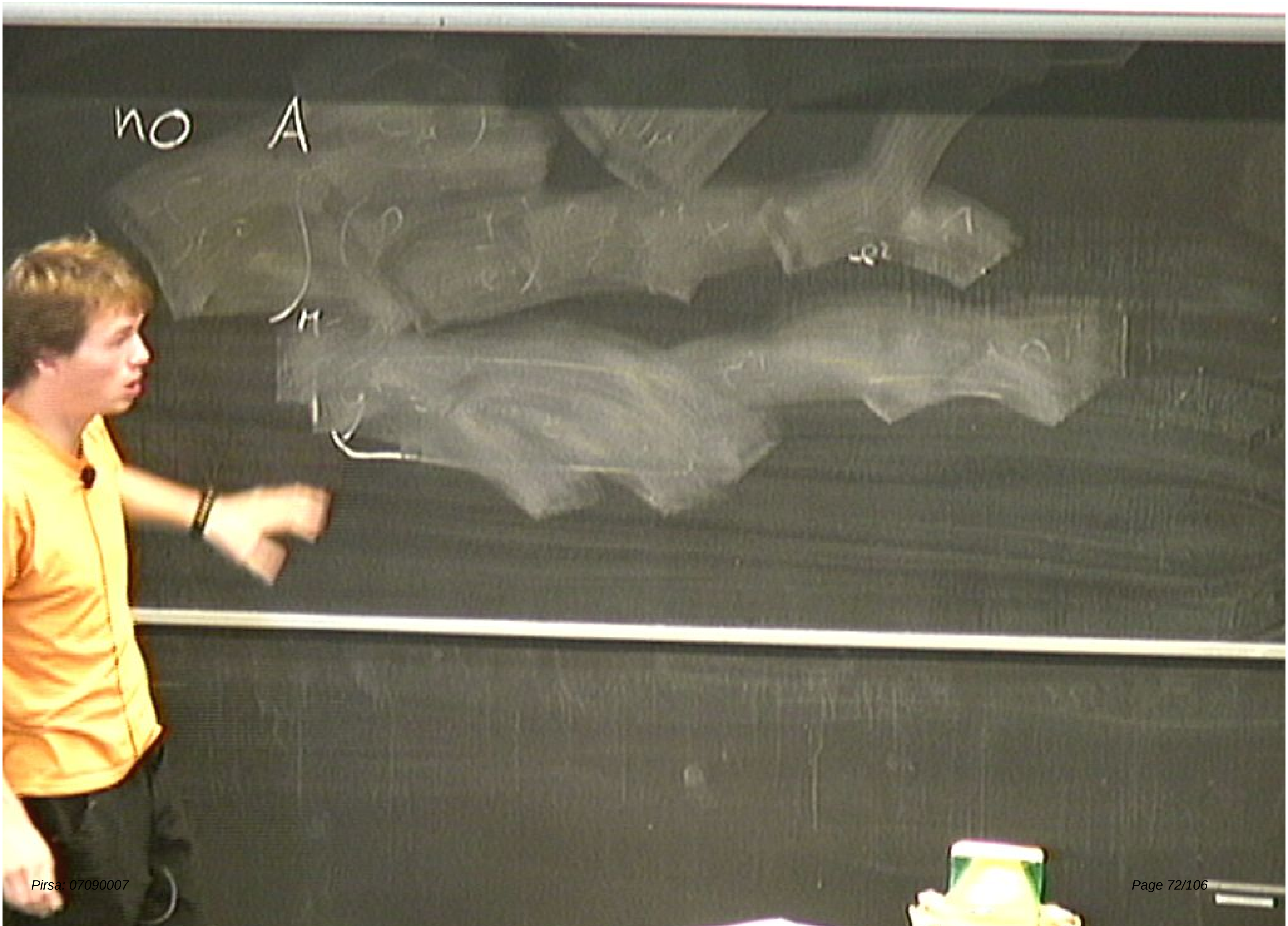
$B_M\, h_{en}$

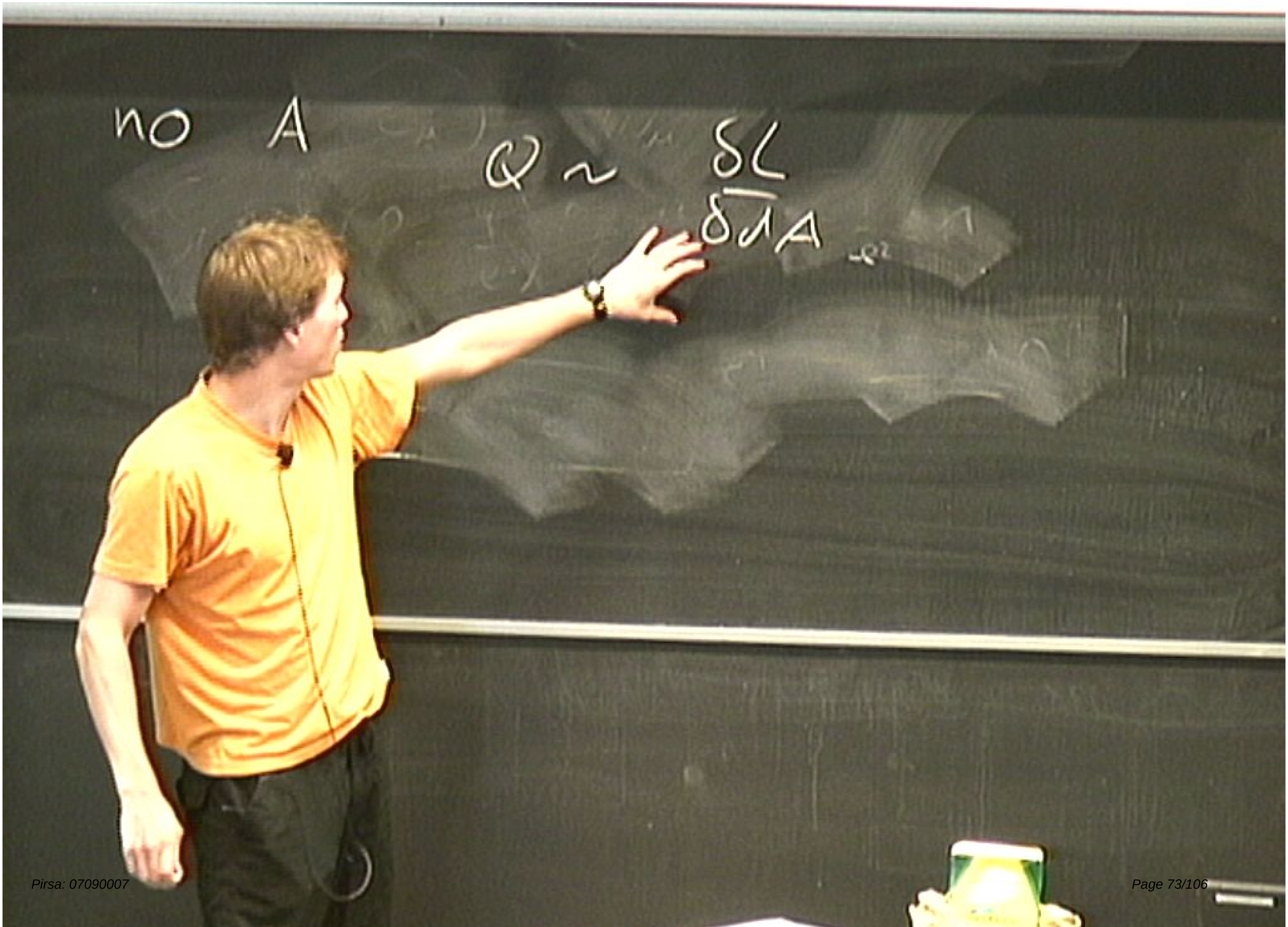
h_{mn}

$A^{(5)}_m = \phi_m \qquad A^r_M = A^3_M + \phi_m B^m_M$

$Q_{Bi} = g^i = \int_\Sigma Q_{\Sigma p i}$

$\left\lfloor \frac{D-2}{2} \right\rfloor$

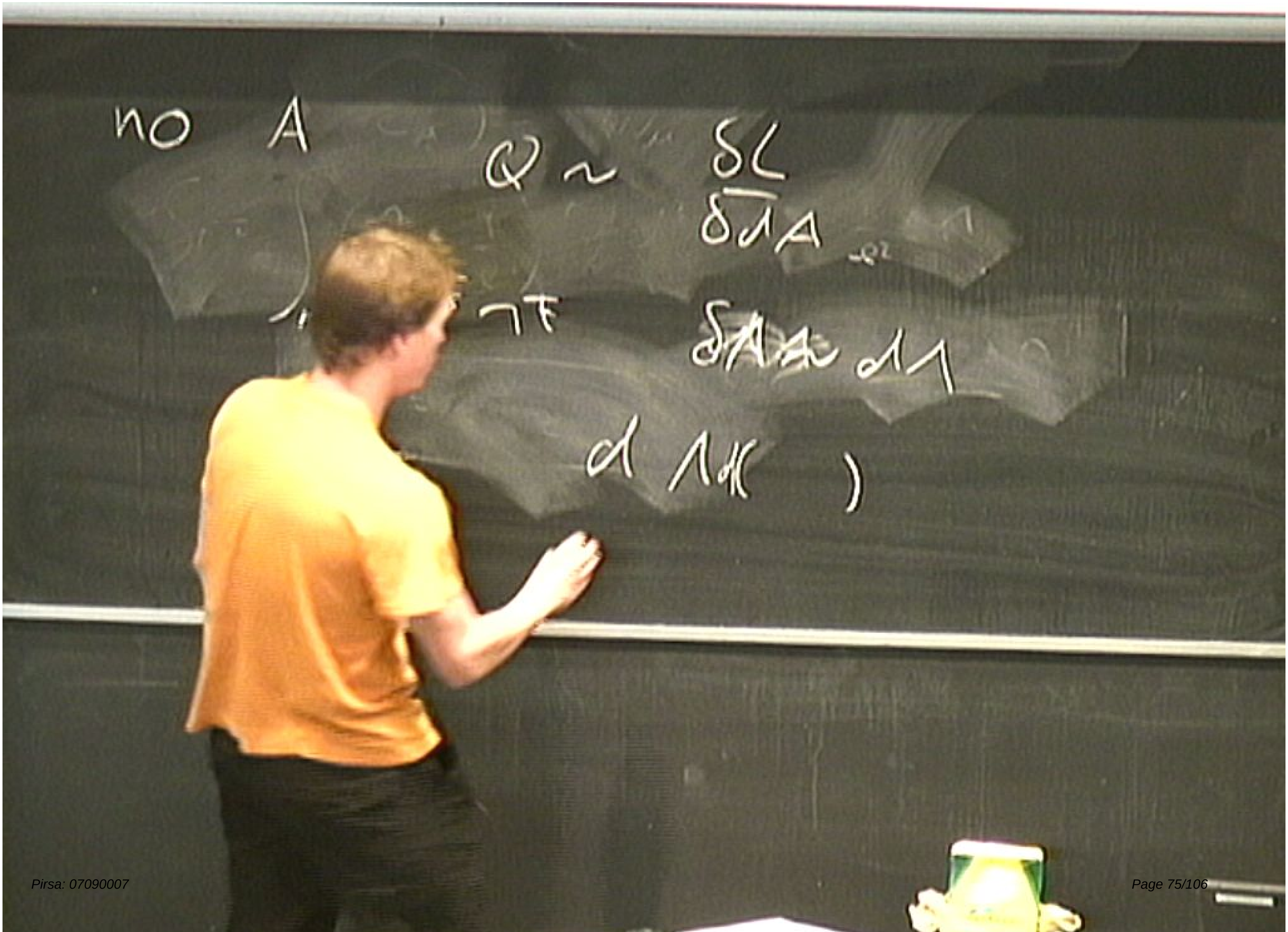

no A

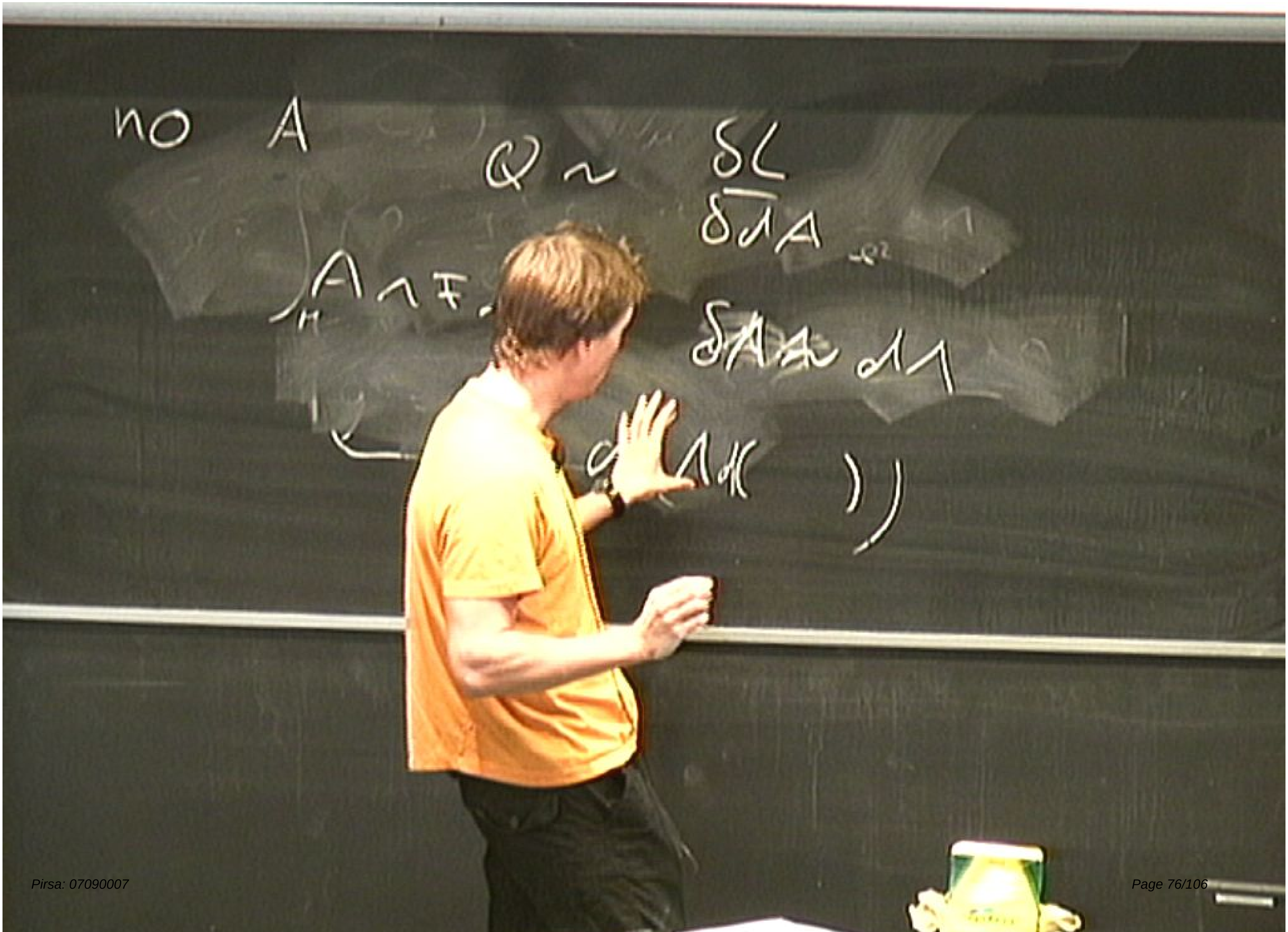

no A
$Q \sim \frac{\delta L}{\delta \partial A}$

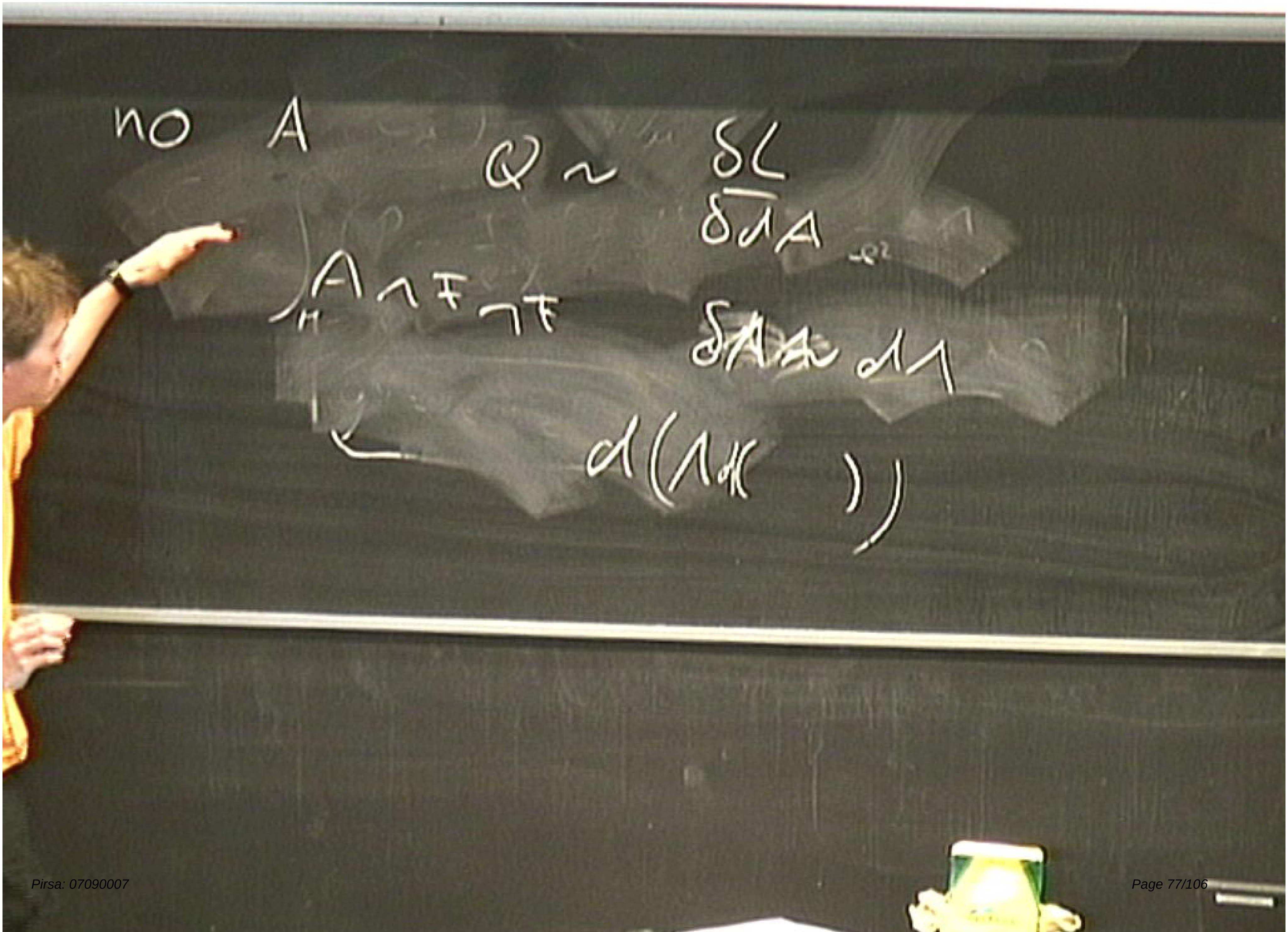
no A

$Q \sim \frac{\delta \mathcal{L}}{\delta \partial A}$

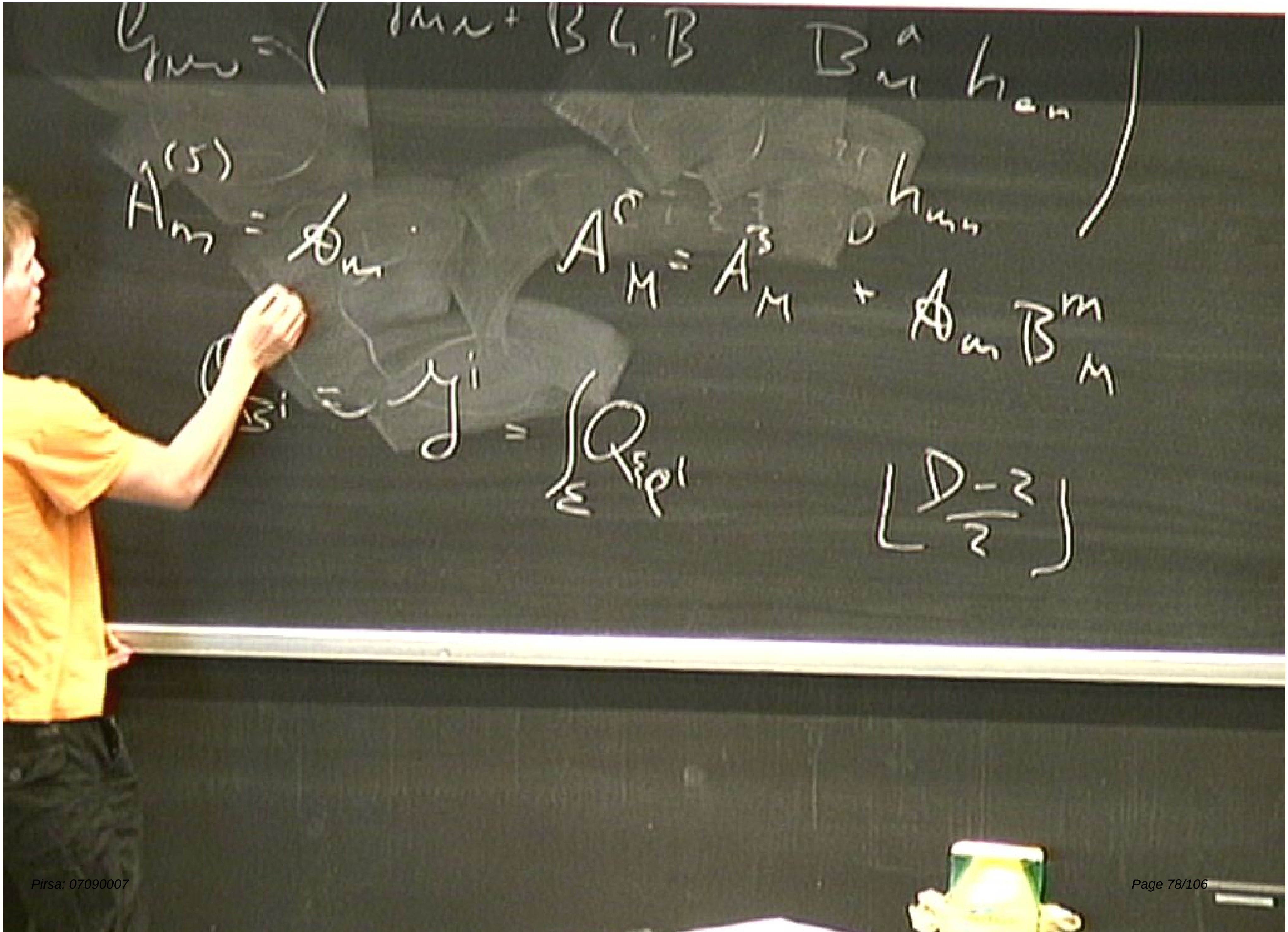
$A \wedge F \quad \wedge F$


no A
Q ~ δL

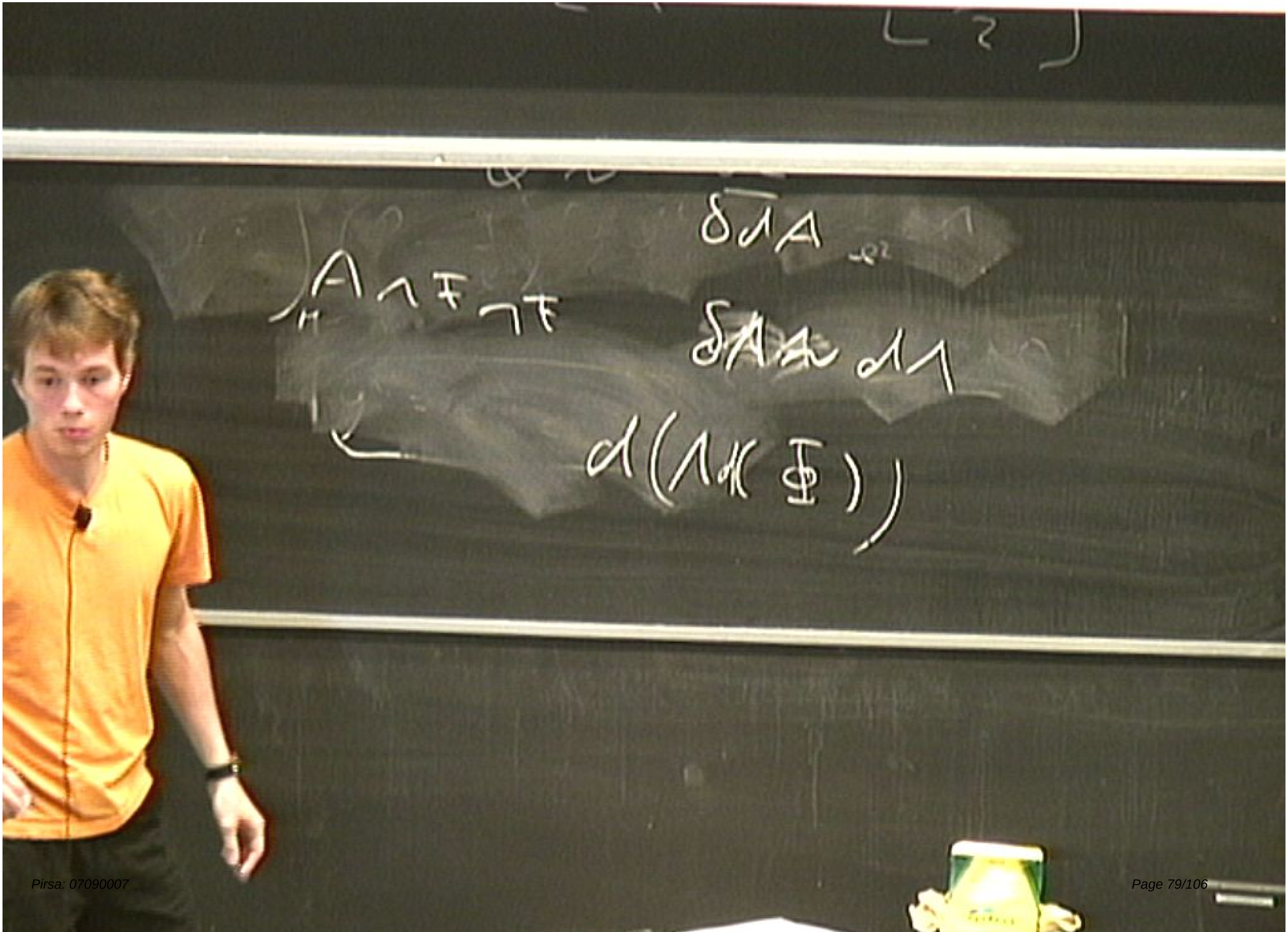

no A
Q ~ δL
δdA
A∧F
δA∧dΛ

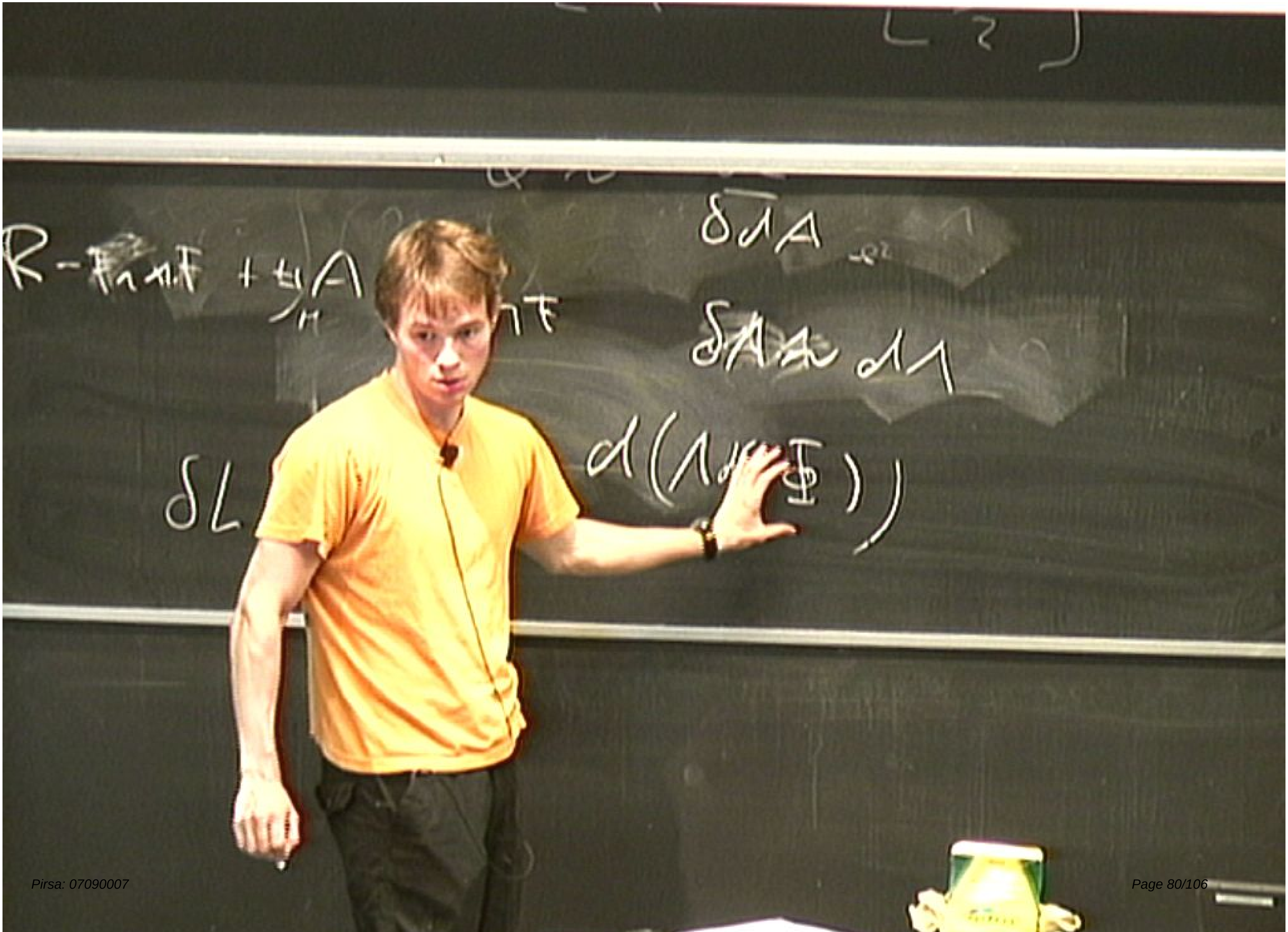


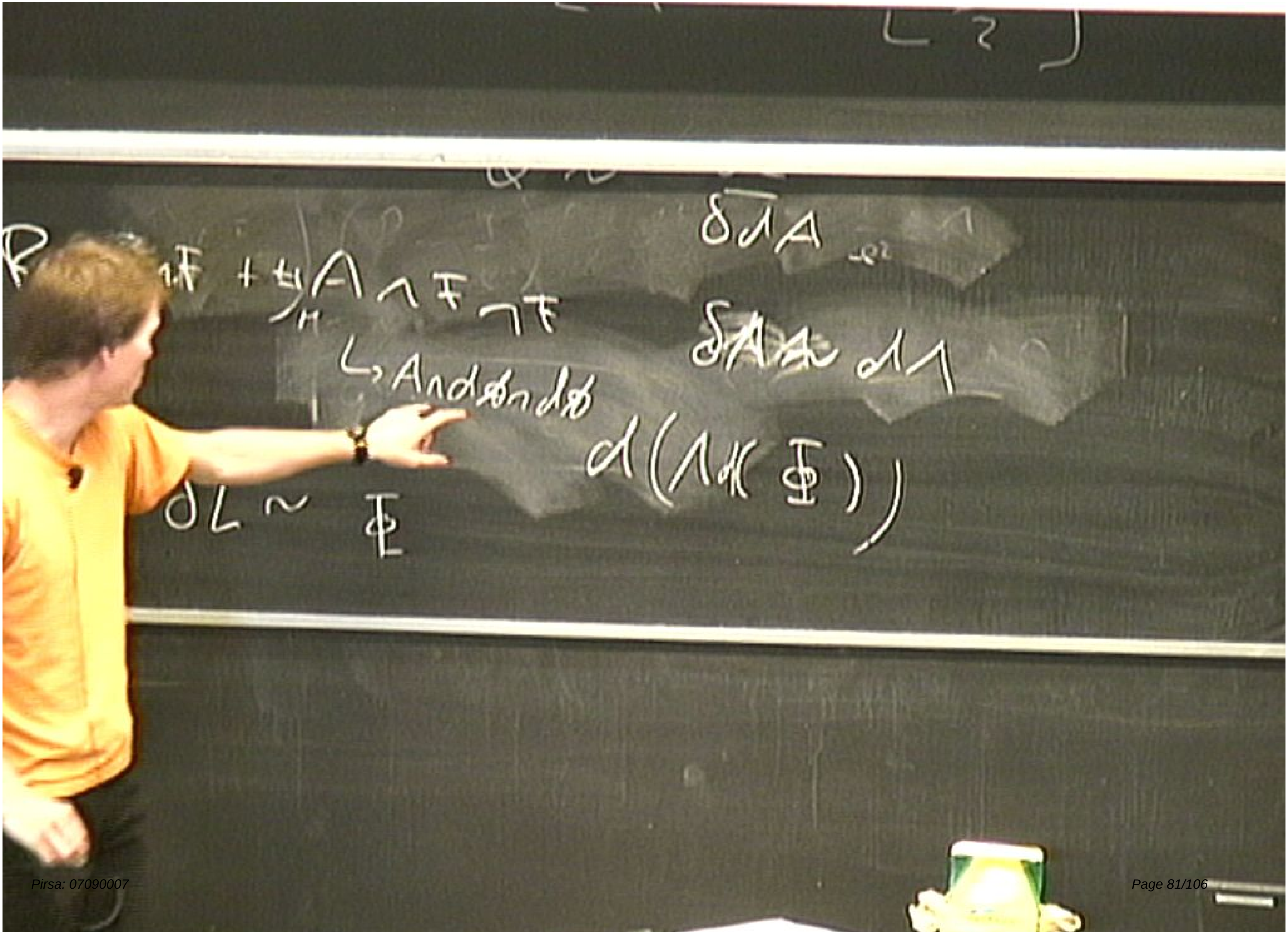
no A
Q ~ δL
δdA
A∧F∧F
δA∧dΛ
d(Λ∧( ))



A^(5)_m = A_m
A^6_M = A^5_M + A_m B^m_M
h_mn
J^i = ∫_Σ Q_{S^p}
⌊(D-2)/2⌋


δΛA
AΛF ΛF
δAΛ dΛ
d(Λ*(Φ))




δL ~ Φ

$$\delta \Lambda A$$

$$R - \cancel{F \wedge F} + A \wedge F \wedge F$$

$$\delta A \quad d\Lambda$$

$$\hookrightarrow A \wedge d\phi \wedge d\phi$$

$$\downarrow$$

$$\phi \wedge d\phi \quad d(\Lambda K(\Phi)))$$

$$\delta L \sim \Phi$$

• Susy

⇒ anholonomic commutator

⇒

$$Q_{\xi_t} \sim d\xi + Q\,\xi\cdot A - g\,\xi\cdot B$$

- Susy
  → on Isagr commut.

⇒ $$Q_{\xi_{int}} \sim d\xi + Q\,\xi\cdot A - \delta\,\xi\cdot B$$

$$\delta S = \int_{\Sigma}$$

- Sn > 4
  - → on horizon commute
  - →

$$Q_{\xi_{H}} \sim d\tilde{\xi} + Q\,\tilde{\xi}\cdot\tilde{A} - g\,\xi\cdot B$$

$$\not{\alpha}\, S = \int_{\Sigma_H} Q_{\xi_{H}}$$

- Sn>y
  - → on horizon cross-section
  - →

$$Q_{\xi_H} \sim d\tilde{\xi} + Q\,\xi\cdot \underbrace{\overset{=0}{\tilde{A}}}_{\dot{A}=0} - \delta\,\xi\cdot B$$

$$\kappa S = \int_{\Sigma_H} Q_{\xi_H}$$

- Sn > 4
  → on horizon consumption

$$Q_{\xi_H} \sim d\,\xi + Q\, \underset{\dot{A}=0}{\overset{\overset{=0}{\sim}}{\xi \cdot A}} - g\, \xi \cdot \underset{=0}{B}$$

$$\mathcal{H} S = \int_{\Sigma_H} Q_{\xi_H}$$

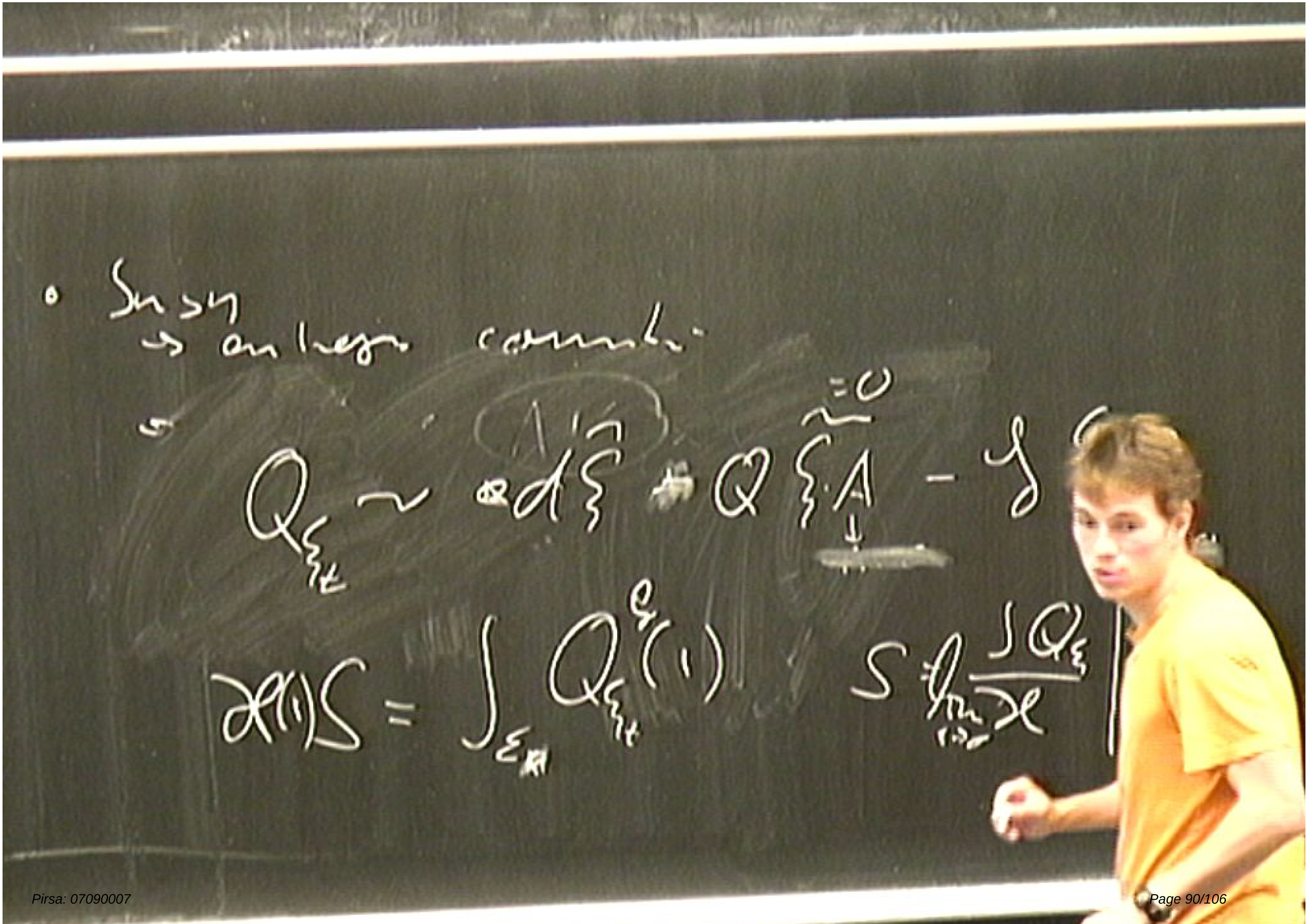
- Sn > 4
  → on horizon commute
- → $$Q_{\xi_H} \sim \oint d\xi + Q \, \overbrace{\xi \cdot \underset{\dot{A}=0}{A}}^{=0} - \oint \xi \cdot \underset{=0}{B}$$

$$\mathcal{H}(1) S = \int_{\Sigma_H} Q^{\xi}_{\xi_H}(1) \qquad S = \frac{\int Q_\xi}{\varkappa}\Bigg|_{\Sigma\to 0}$$

- Sn > 4
  $\Rightarrow$ on horizon commute
  $\Rightarrow$

$$Q_{\xi_H} \sim d\xi + Q\,\xi \cdot \underset{\tilde{A}=0}{\overset{=0}{\tilde{A}}} - g\,\xi \cdot \underset{=0}{B}$$

$$\mathcal{H}(1)S = \int_{\mathcal{E}_H} Q^{\xi}_{\xi_H}(1) \qquad S = \lim_{r\to\infty} \frac{\delta Q_\xi}{\delta\mathcal{H}}\bigg|_{r=0}$$



◦ Susy
→ on large commute

→ $Q_{\xi_{1t}} \sim \underbrace{d\overset{1/r}{\xi}}_{\partial\mathcal{E}S} + Q\,\xi\cdot\overset{=0}{\tilde{A}} - \delta\,\xi\cdot B$

$\partial\mathcal{E}(1)S = \int_{\Sigma_H} Q^{\xi}_{\xi_{1t}}(1) \qquad S = \lim_{r\to\infty} \frac{\delta Q_\xi}{\partial\mathcal{E}}\Big|_{r}$

- Susy
  → on brane commute Φ_{+i} Ω_{+i}

$$\Phi_{+i} = 0$$

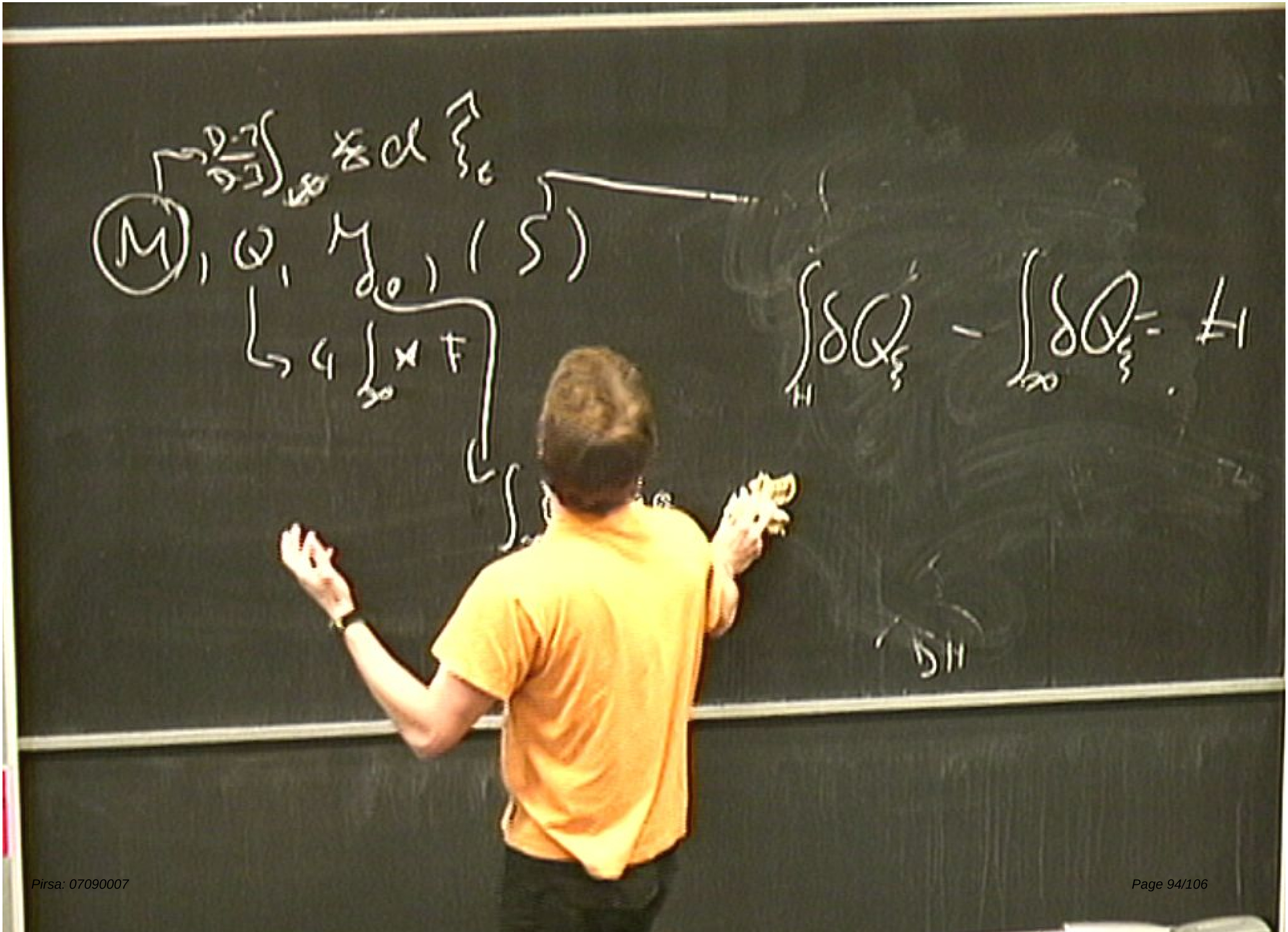
$$Q_{\xi_{+}} \sim \underbrace{d\xi}_{\partial S} + Q\,\xi \cdot A \;\; \& \;\; \xi \cdot B$$

$$\mathcal{H}(1)S = \int_{\Sigma_{\pi}} Q^{\xi}_{\xi_{+}}(1) \;\;\; S \neq \ldots \neq 0$$

∘ Susy
→ on large cornrub

$$\delta Q_{\xi_{it}} \sim g\left(d\underbrace{\xi}_{\partial e S} + Q\,\xi\cdot \underset{\Downarrow}{\tilde{A}}^{\phi_{\mu\nu}=0} - g\,\xi\cdot \underset{\Downarrow}{B}^{\Omega_{\mu\nu}}\right)$$

$$\mathcal{H}^{(1)} S = \int_{\Sigma_{\infty}} Q^{e\,(1)}_{\xi_{it}} \qquad S \neq \lim_{r\to\infty} \frac{\delta Q_\xi}{\mathcal{H}}\Big|_{\Gamma\neq 0}$$



$$\frac{D-2}{D-3}\int_{S_\infty} * d\xi_0$$

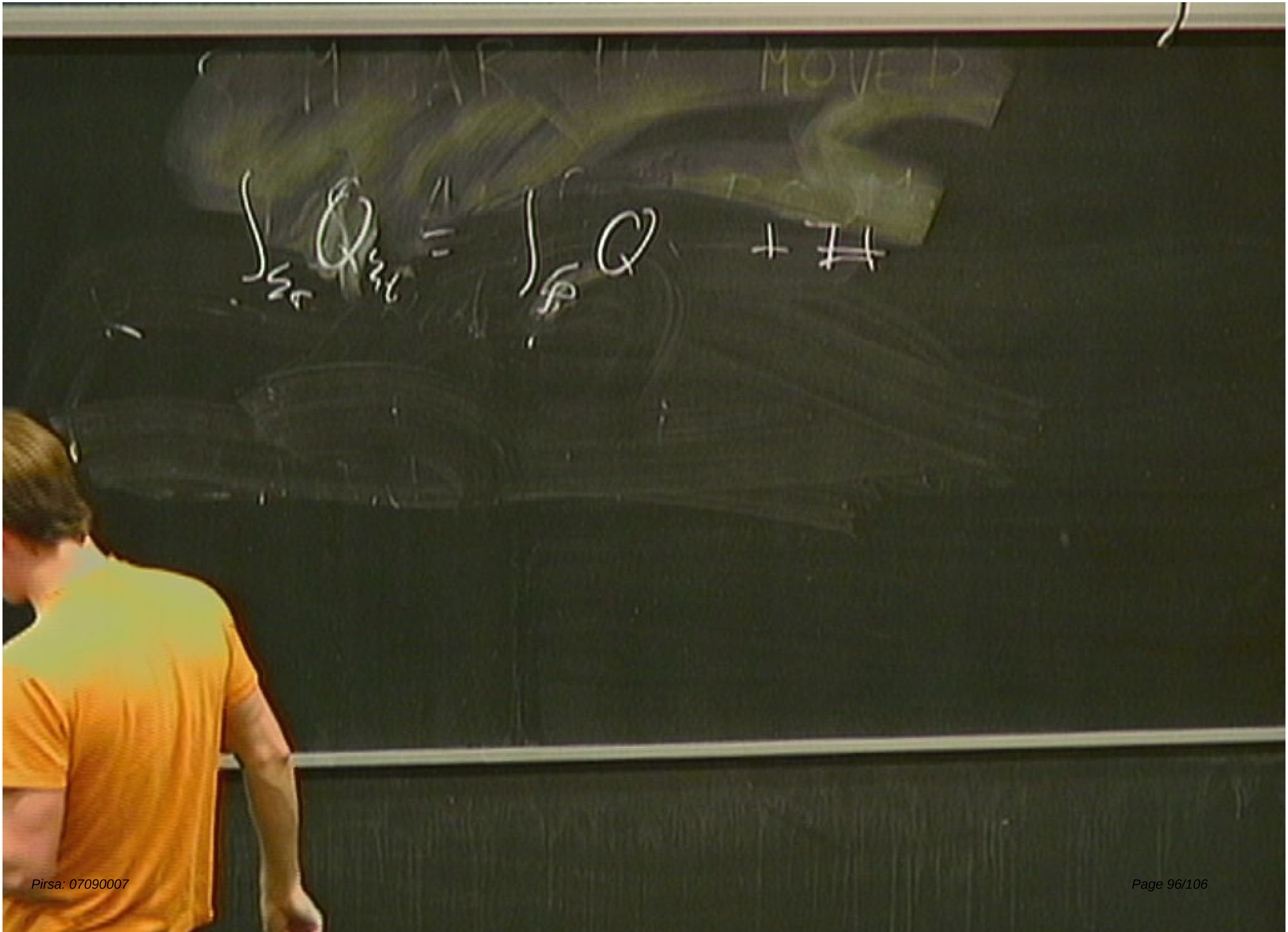
$$M,\ Q,\ \mu_0,\ (S)$$

$$\hookrightarrow q\int_{S_\infty} * F$$

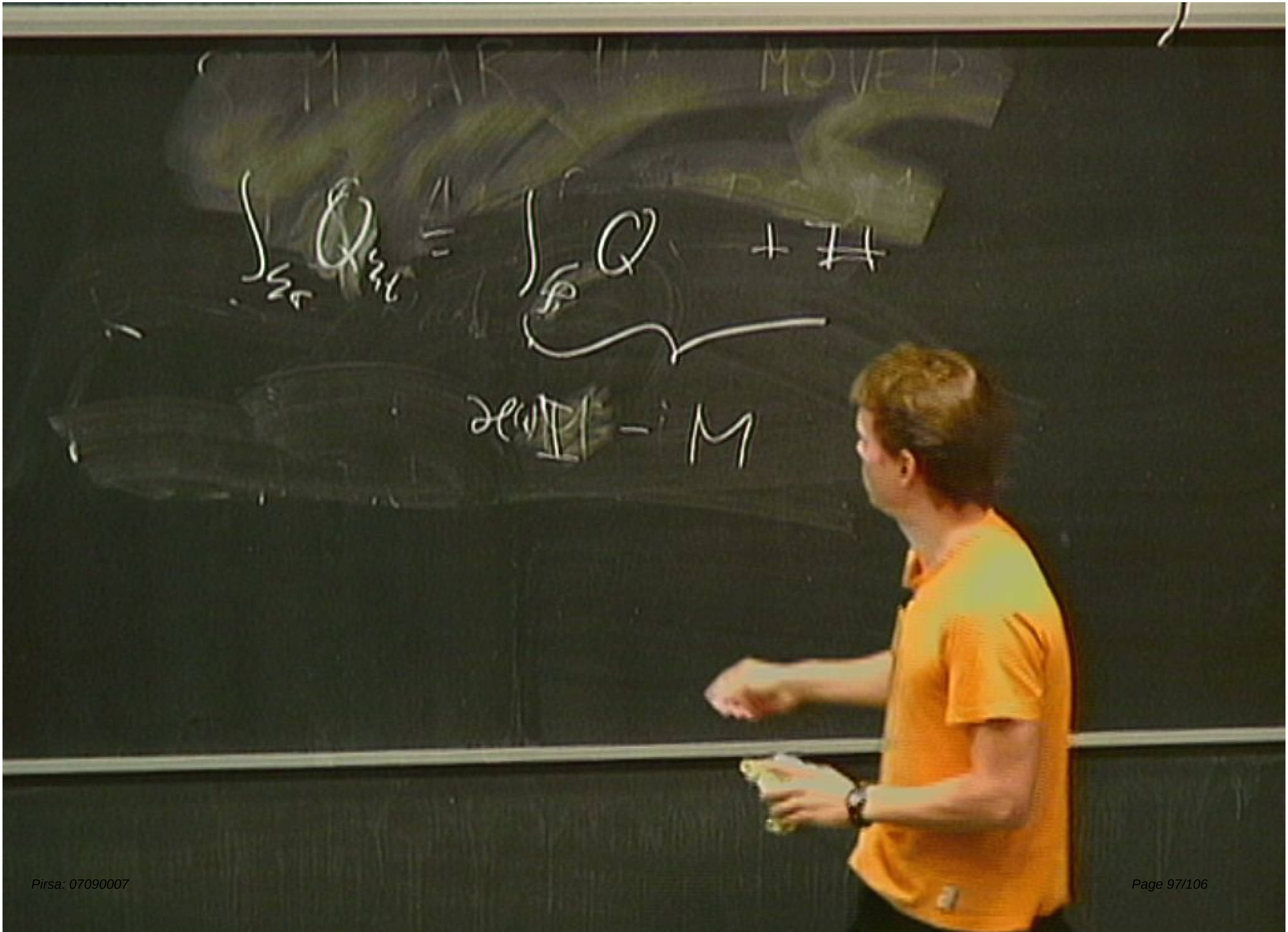
$$\int \delta Q_\xi - \int_{\infty} \delta Q_\xi = \pm 1$$

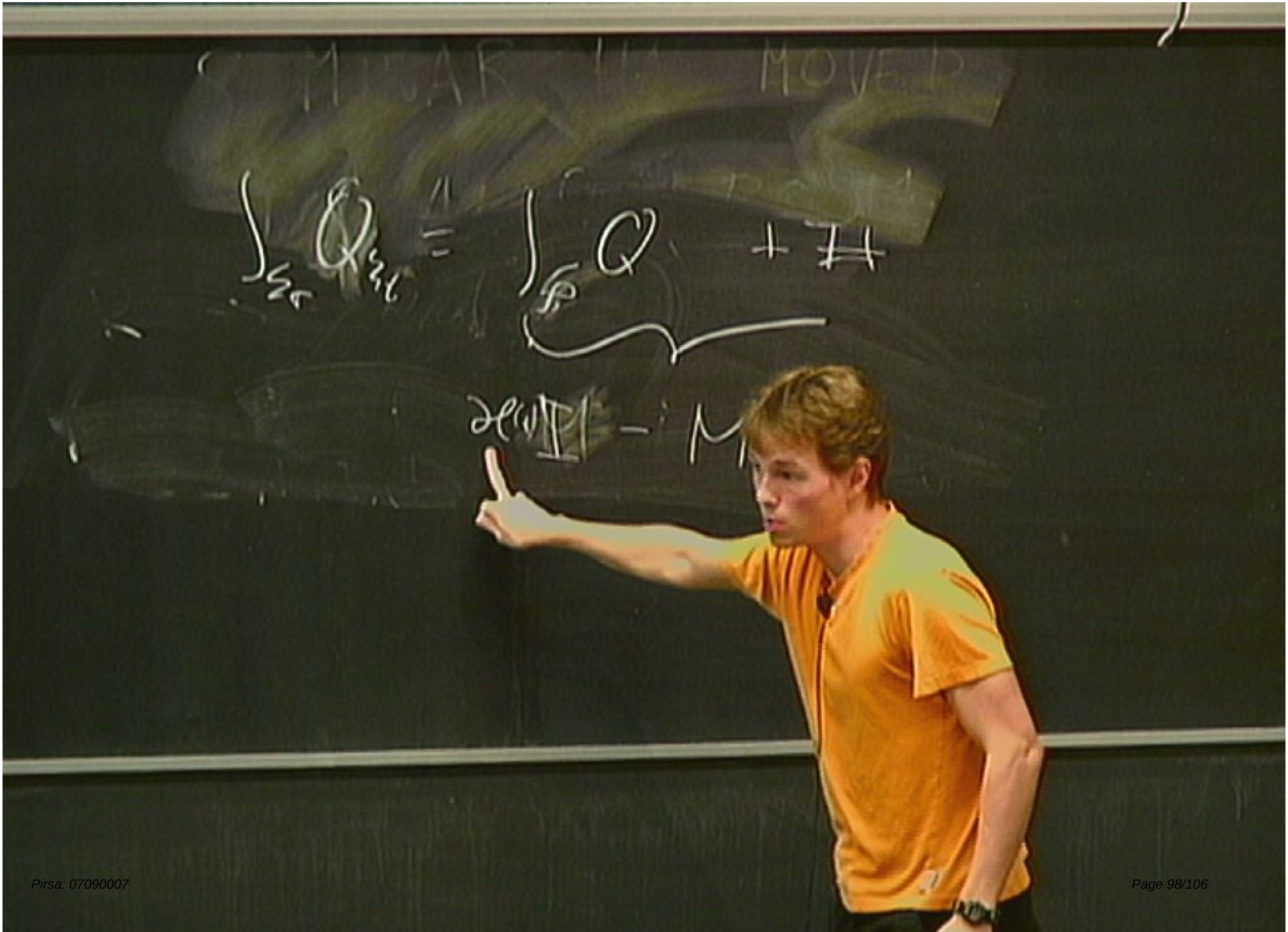
$$\hookrightarrow \delta M(S, r) = T(r)\,\delta S + \phi(r)\,\delta Q + \Omega(r)\,\delta J$$

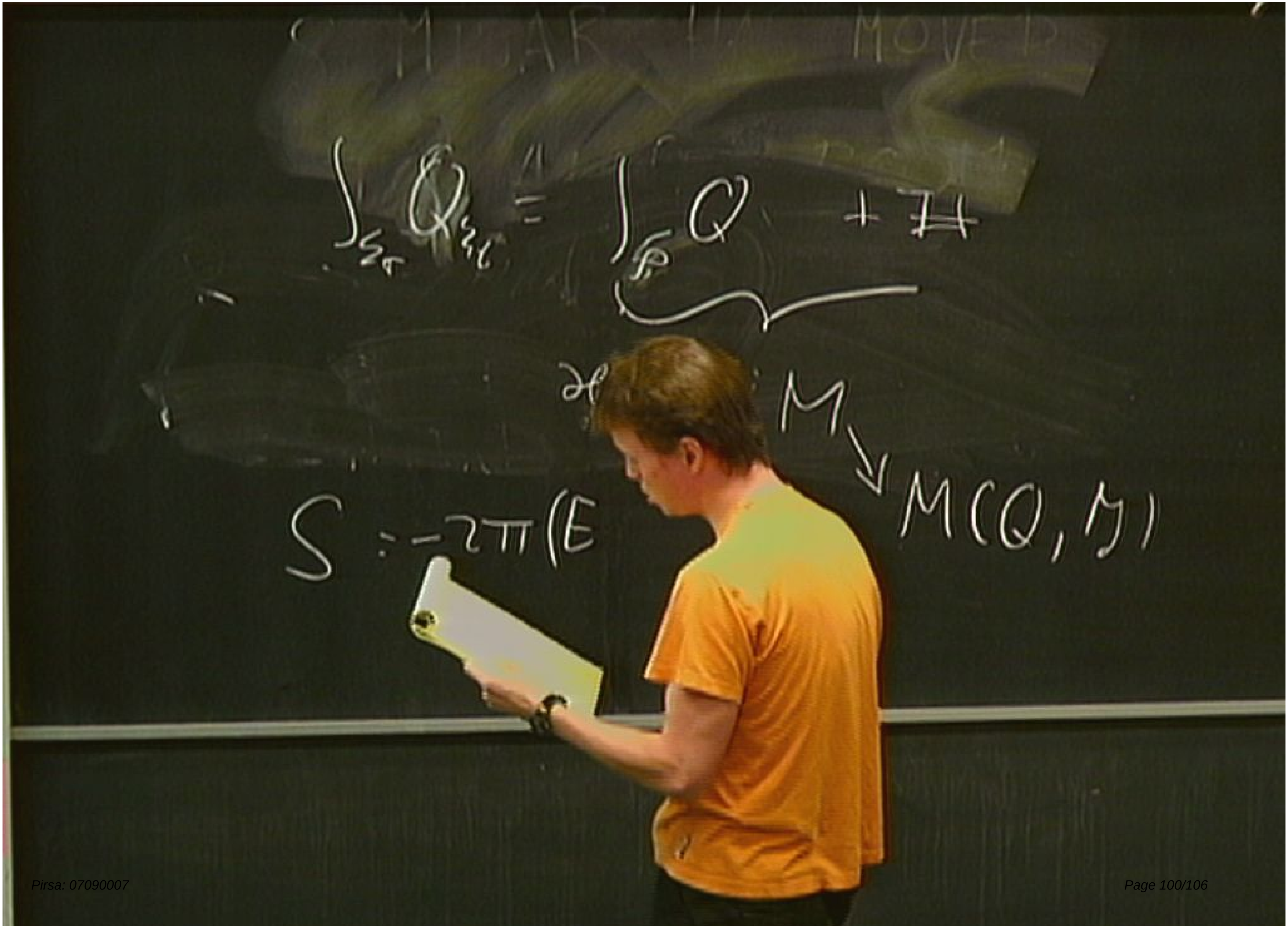
DH

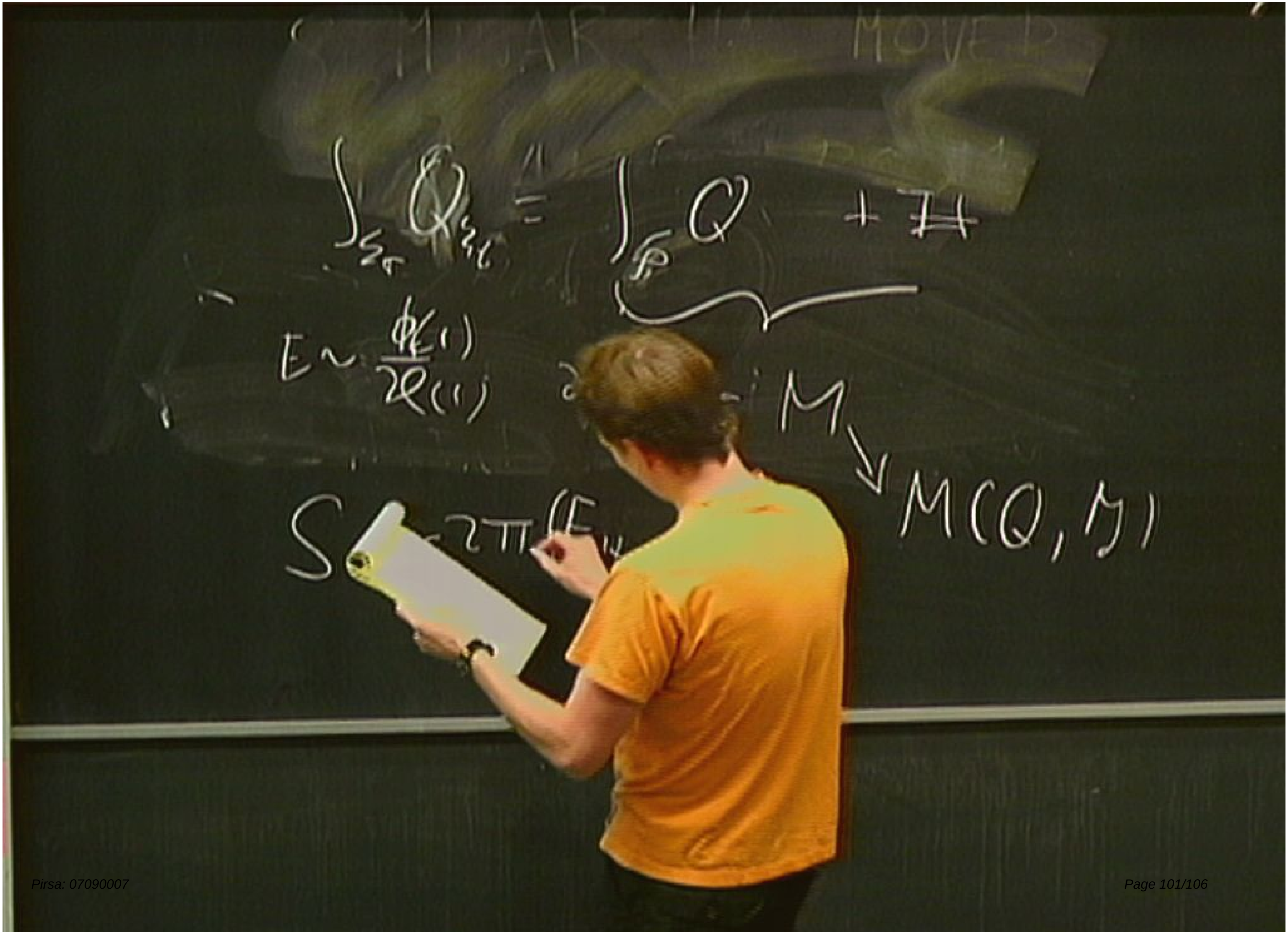


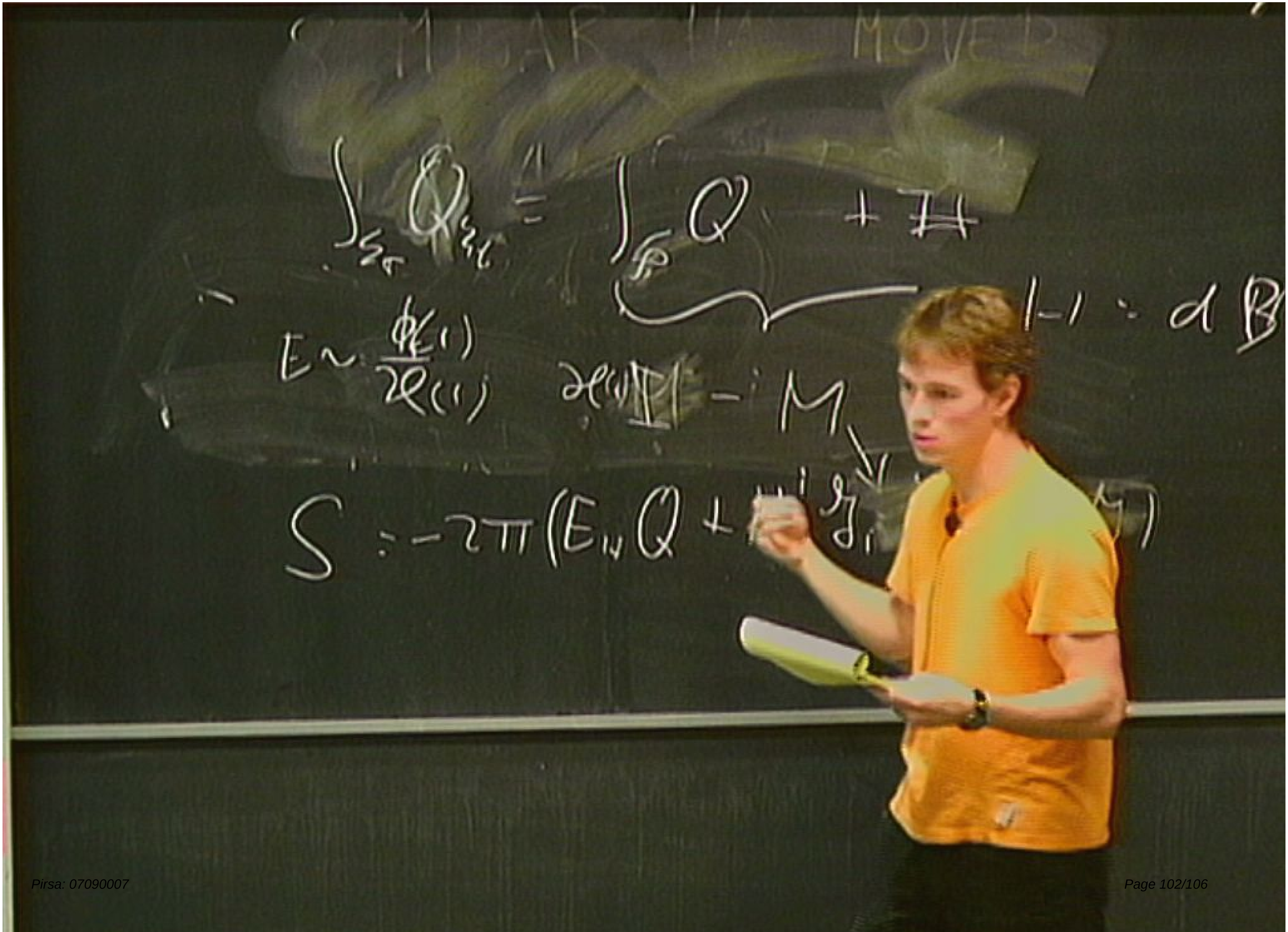
$\int_{\Sigma_\sigma} Q_{\Sigma_t} = \int_{\Sigma} Q + \#$

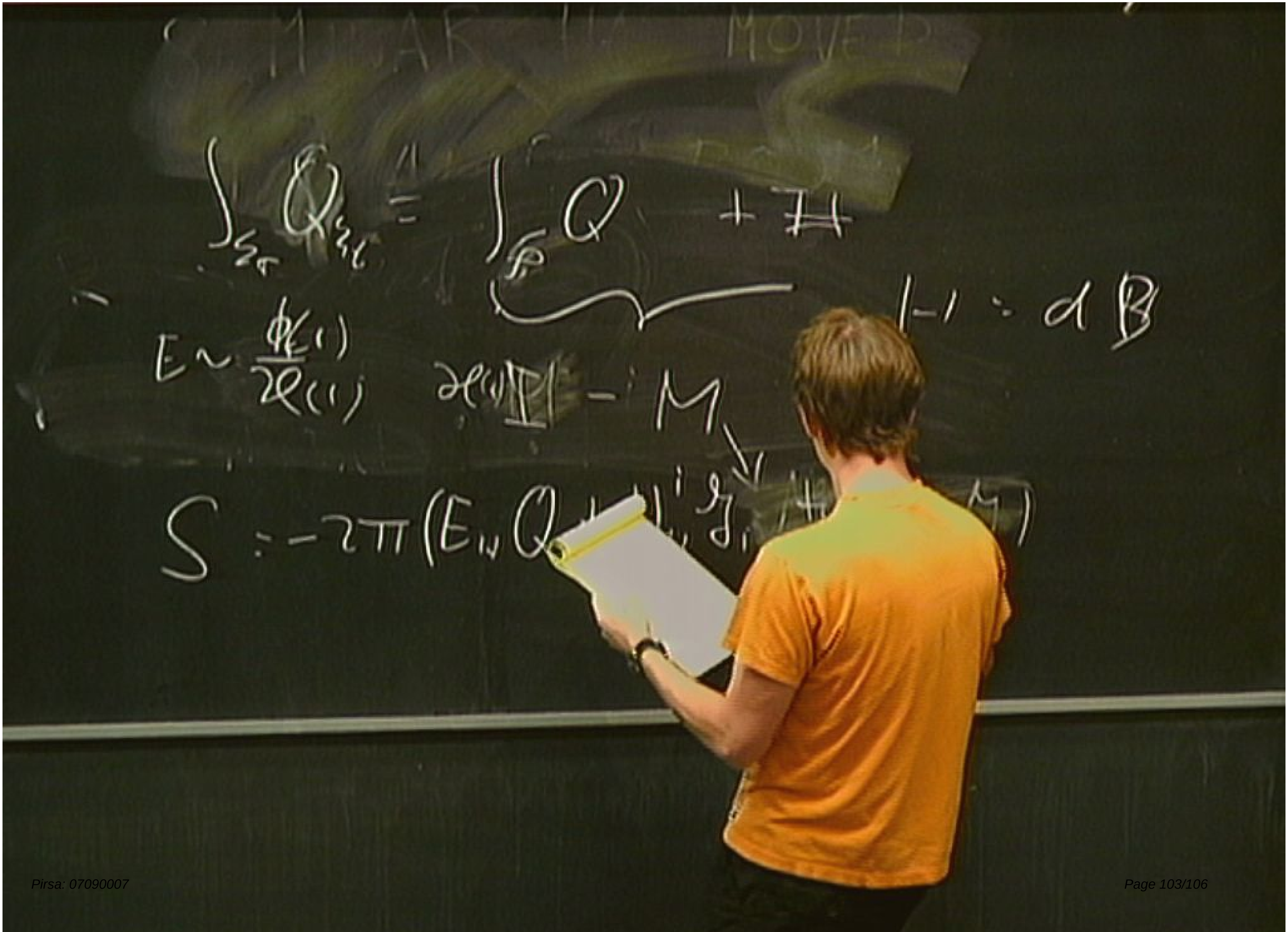

M

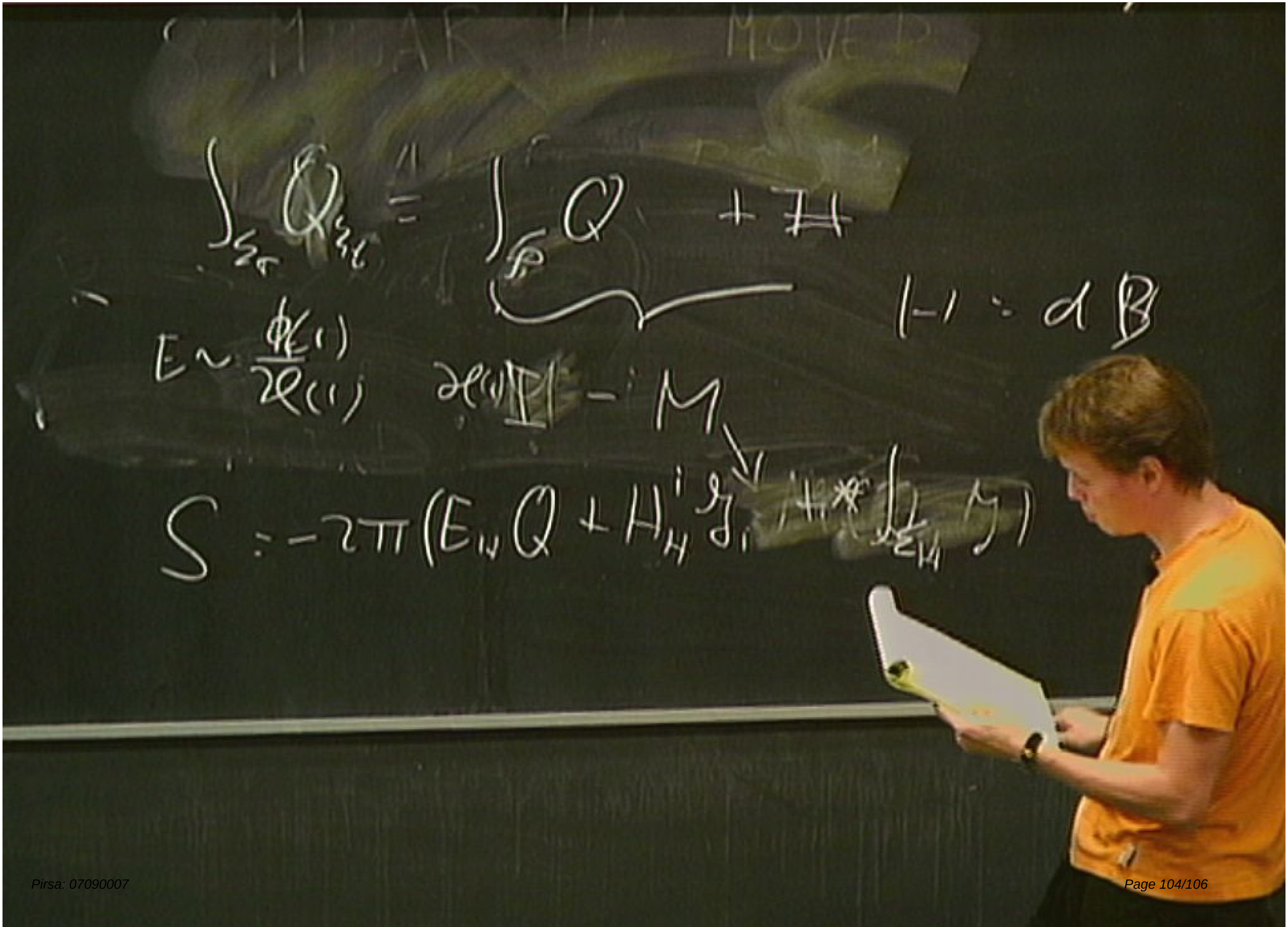



M
M(Q, J)
S = -2π(E


M(Q, J)







$$\int_{\Sigma_\sigma} Q_{\Sigma_t} = \int_{S^2} Q \; + \; \#$$

$$H = dB$$

$$E \sim \frac{\Phi(1)}{\partial e(1)} \qquad \partial e(1) \; - \; M$$

$$S = -2\pi \left(E_H Q + H_H^i \, g_i \, \left(\int_{\Sigma_H} * \right) \right)$$

$$\int_{\Sigma_\sigma} Q_{\Sigma_t} = \int_{\Sigma} Q + \underbrace{\#}$$

$$H = dB$$

$$E \sim \frac{\phi_E(1)}{\mathcal{H}(1)} \quad \mathcal{H}(1)\,\mathbb{I} - M$$

$$S = -2\pi\left(E_H Q + H_H\, g_1\, \ast\left(\int_{\Sigma_H} \omega\right)\right)$$