Title: Wilson Loops as D-branes

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Abstract:

PLAN

WILSON LOOPS - INTRODUCTION $U(1)$, $SU(N)$

AdS/CFT - INTRODUCTION

- $\mathcal{N}=4$ SYM

- $\frac{1}{2}$ BPS WILSON LOOPS

HOLOGRAPHIC W.L. - D5, D3 PROBES

- LOW ENERGY FIELD THEORY

WILSON LOOPS: NON LOCAL AND GAUGE INVARIANT OPERATORS

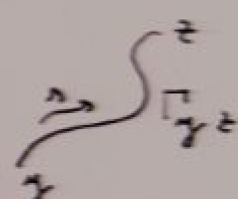
U(1) GAUGE THEORY $[\ A_\mu\ x\ ]$

GAUGE TRANSFORMATION: $A_\mu(x) \xrightarrow{G.T.} A_\mu(x) + \partial_\mu \lambda(x)$

PHASE FACTOR

$$U[\Gamma_{yz}] = \mathrm{EXP}\left[iq \int_{\Gamma_{yz}} ds\, \dot{x}^\mu A_\mu(x) \right]$$

$$U[\Gamma_{yz}] \xrightarrow{G.T} e^{-iq\lambda(y)}\, U[\Gamma_{yz}]\, e^{iq\lambda(z)}$$



PARALLEL TRANSPORTER

IF $\Gamma = C$

A "LOOP"

$$\longrightarrow \quad W_q(C) = \mathrm{EXP}\left[i q \int_C ds\, \dot{x}^\mu A_\mu(x) \right]$$

GAUGE INVARIANT OPERATOR

- q CHARGE [REPRESENTATION OF $U(1)$]
- C CLOSED CURVE

DESCRIBE THE COUPLING OF THE $U(1)$ GAUGE THEORY WITH A NON DYNAMICAL CHARGED PARTICLE.

U(N) YANG-MILLS

$$W_R(C) = \mathrm{Tr}_R\, \mathcal{P}\,\mathrm{Exp}\left[i \int_C ds\, A_\mu(x)\, \dot{x}^\mu \right]$$

- $\mathcal{P}$ PATH ORDERING ($A_\mu = A_\mu^a t^a$
  t^a GENERATORS OF $U(N)$)
- Tr TRACE

W-L IS CHARACTERIZED BY

C A CLOSED CURVE

R REPRESENTATION OF $U(N)$

R IS SUMMARIZED BY A YOUNG TABLEAU

$$R = (n_1, n_2, \dots, n_i, \dots, n_N)$$

n_i: BOXES IN THE i-TH ROW

$$n_1 \geq n_2 \geq \dots \geq n_N$$

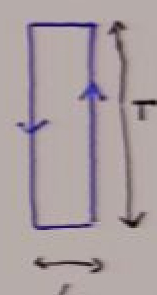
$W_R(C)$ DESCRIBE THE COUPLING OF U(N) YANG-MILLS WITH AN EXTERNAL NON DANYMICAL PARTICLE WITH CHARGE R

E.G. THE EXTERNAL PARTICLE IS A QUARK

$\longrightarrow$ $R = \square$ FUNDAMENTAL

$$\langle W_\square(C_0)\rangle = A(L)\, e^{-T E(L)}$$

$E(L)$: QUARK-ANTIQUARK POTENTIAL

$C_0 =$ 

, $T \to +\infty$

W-L : ORDER PARAMETER FOR CONFINEMENT

WHY HOLOGRAPHIC ?

AdS / CFT DUALITY

STATEMENT:

TYPE IIB STRING ON $AdS_5 \times S_5$ WITH $\int_{S_5} {}^*F_5 = N$ $\overset{\text{EQUIVALENT}}{\Longleftrightarrow}$ $\mathcal{N} = 4$ SYM $U(N)$ ON $\mathbb{R}^{1,3}$

PARAMETERS:

RADIUS: $L^4 = 4\pi g_s N (\alpha')^2$

Y.M. COUPLING: g_{YM}

$\frac{L^2}{\alpha'} \sim \lambda$

$g_s = g_{YM}^2$

T'HOOFT: $\lambda = g_{YM}^2 N$

$N \to +\infty$ LIMIT

NON INTERACTING STRING THEORY

PLANAR GRAPHS

$\lambda \gg 1$ NO α' CORRECTIONS

$\lambda \ll 1$ PERTURBATIVE REGIME

$$\begin{array}{lcccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ N\ D3 & \times & \times & \times & \times & & & & & & \end{array}$$

SPATIAL COORDINATES

N D3

$\lambda = g_s N \gg 1$

SUGRA

CLOSED STRING DESCRIPTION

$\lambda = g_s N \ll 1$

PERT. STRING

OPEN STRING DESCRIPTION

MALDACENA LIMIT

$AdS_5 \otimes S_5$

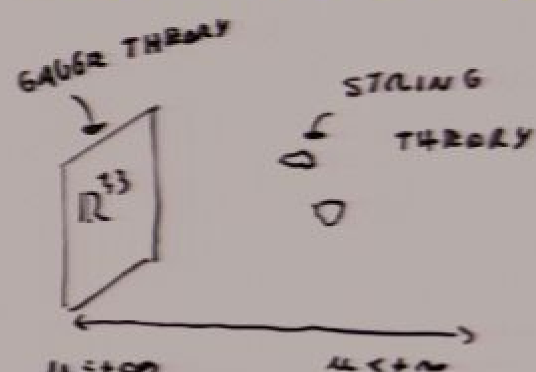
$\mathcal{N}=4$ SYM

HOLOGRAPHY

$$ds^2_{AdS_5 \otimes S^5} = L^2\left(u^2 \eta_{\mu\nu}\, dx^\mu dx^\nu + \frac{du^2}{u^2}\right) + L^2\left(d\theta^2 + \sin^2\theta\, d\Omega_4^2\right)$$

$u \longrightarrow +\infty$ BOUNDARY OF THE SPACE $\mathbb{R}^{1,3}$

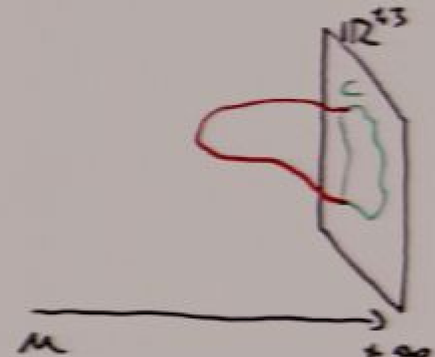
THE PHYSICS OF THE BULK RESULTS FROM A HOLOGRAPHIC IMAGE OF THE PHYSICS ON THE BOUNDARY



GRAVITATIONAL DESCRIPTION OF WILSON LOOPS

REY, YEE
MALDACENA

$W_\square(C)$ IS ASSOCIATED TO A FUNDAMENTAL STRING IN THE BULK ENDING ON C AT THE BOUNDARY OF AdS



→ STRING ACTION, PROPORTIONAL TO THE SURFACE OF THE WORLD-SHEET

$$\langle W_\square(C)\rangle \sim e^{-S}$$

$$\langle W_\square(C)\rangle = \int DX\, e^{-\sqrt{\lambda}\, S[X]}$$

FOR NON-CONFORMAL THEORY

$$E(L) \propto L$$

CONFINEMENT

$\mathcal{N}=4$ SYM $[A_\mu, \phi^I, \lambda]$

W.L. DESCRIBE COUPLING OF A SUPERPARTICLE TO SYM

$\longrightarrow$ BOSONIC WILSON LOOPS

$$C \equiv (x^\mu(s), q^I(s))$$

$$W_R(C) = \mathrm{Tr}_R\, P \exp\left[i \int_C ds \left(A_\mu \dot{x}^\mu + \phi_I \dot{q}^I \right) \right]$$

$\mathcal{N}=4$ SYM INVARIANT UNDER:

POINCARE SUSY: $\delta_{\epsilon_1} A_\mu = i \bar{\epsilon}_1 \gamma_\mu \lambda \quad \delta_{\epsilon_1} \phi_I = i \bar{\epsilon}_1 \gamma_I \lambda$

SUPER CONFORMAL: $\delta_{\epsilon_2} A_\mu = i \bar{\epsilon}_2 x^\nu \gamma_\nu \gamma_\mu \lambda \quad \delta_{\epsilon_2} \phi_I = i \bar{\epsilon}_2 x^\nu \gamma_\nu \cdot \gamma_I \lambda$

$\frac{1}{2}$ BPS WILSON LOOPS

W.L. INVARIANT UNDER $\frac{1}{2}$ POINCARE SUSY AND $\frac{1}{2}$ SUPERCONFORMAL

$\longrightarrow$ SUSY SELECT A PREFERRED CURVE:

$$C = \left(x^0 = t,\ x^i = 0,\ y^I = n^I\right) \quad (n^I)^2 = 1$$

OPERATOR UNDER EXAM

$$W_R = \mathrm{Tr}_R\, P \exp\left[i \int dt \left(A_0 + \phi\right)\right]$$

SYMMETRIES

SYM		$\frac{1}{2}$ BPS W.L.
$SU(2,2) \simeq SO(2,4)$	$\longrightarrow$	$SU(1,1) \times SU(2)$
$SU(4) \simeq SO(6)$	$\longrightarrow$	$SO(5)$
ϵ_1, ϵ_2	$\longrightarrow$	$\begin{cases} \gamma_{0I}\, n^I \epsilon_1 = \epsilon_1 \\ \gamma_{0I}\, n^I \epsilon_2 = -\epsilon_2 \end{cases}$
$PSU(2,2\vert 4)$	$\longrightarrow$	$O_{sp}(4^*\vert 4)$

$\frac{1}{2}$ BPS WILSON LOOPS

W.L. INVARIANT UNDER $\frac{1}{2}$ POINCARE SUSY
AND $\frac{1}{2}$ SUPERCONFORMAL

$\longrightarrow$ SUSY SELECT A PREFERRED CURVE:

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$SU(4) \simeq SO(6)$	$\longrightarrow$	$SO(5)$
$\varepsilon_1, \varepsilon_2$	$\longrightarrow$	$\begin{cases} \gamma_{0I} n^I \varepsilon_1 = \varepsilon_1 \\ \gamma_{0I} n^I \varepsilon_2 = -\varepsilon_2 \end{cases}$
$PSU(2,2\|4)$	$\longrightarrow$	$O_{sp}(4^*\|4)$

GRAVITATIONAL DESCRIPTION OF $\frac{1}{2}$ BPS W.L.

WE LOOK FOR PROBES THAT PRESERVE THE $O_{sp}(4^*|4)$ SYMMETRY, I.E. THE SAME AS $\frac{1}{2}$ BPS W.L.

$R = \square$ FUNDAMENTAL REP.

$\longrightarrow$ FUNDAMENTAL STRING DESCRIBED BY $x^M = x^M(\sigma^0, \sigma^1)$

$x^0 = \sigma^0$ $\quad u = \sigma^1$ $\quad x^i = 0$ $\quad x^I = n^I$

SPANS AdS_2 $\quad$ SITS AT $x^i = 0$ $\quad$ SITS AT A POINT IN S^5

$\rightarrow SU(1,1) \otimes SU(2) \otimes SO(5)$

$\rightarrow$ SAME CONDITION ON $\varepsilon_1, \varepsilon_2$ AS W.L.

$O_{sp}(4^*|4)$ SYMMETRY

$\rightarrow$ AT THE BOUNDARY THE STRING ENDS ON $x^0 = t$

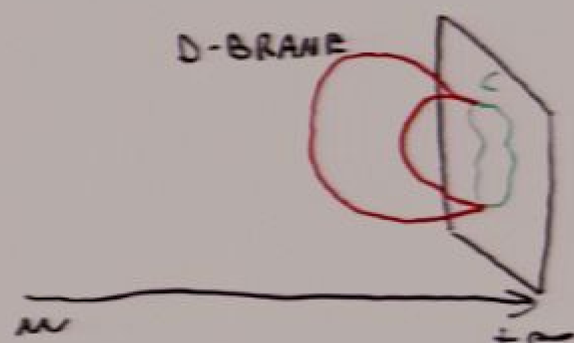
HIGHER REPRESENTATIONS

$\Downarrow$

MULTIPLE (k) COINCIDENT FUNDAMENTAL STRINGS

$\Downarrow$

DESCRIPTION IN TERMS OF A SINGLE D-BRANE WITH (k) UNIT OF FUNDAMENTAL STRING CHARGE



$D5_K$ - BRANE GIANT WILSON LOOP

$X^M = X^M(\sigma^0, \sigma^1, \dots \sigma^5)$ EMBEDDING IN $AdS_5 \otimes S_5$

$$\begin{array}{ccc} AdS_2 & \otimes & S_4 \\ \cap & & \cap \\ AdS_5 & \otimes & S_5 \end{array}$$ EMBEDDING

→ $SU(1,1) \times SU(2) \times SO(5)$ SYMMETRY

→ SAME $\frac{1}{2}$ BPS SUSY

EMBEDDING IN S^5

→ θ_K LATITUDE $\in [0, \pi]$

→ $\theta_0 = 0$ $\theta_N = \pi$

→ INCREASE WITH K

$K \leq N$ MAX NUMBER OF FUNDAMENTAL CHARGES

$$D5_K \longleftrightarrow W_{\begin{smallmatrix}\boxed{\ }\\ \boxed{\ }\\ \boxed{\ }\\ \boxed{\ }\end{smallmatrix}} \Big\} K$$

$D3_k$ – BRANE DUAL GIANT WILSON LOOP

$x^M = x^M(\sigma^0, \sigma^1, \sigma^2, \sigma^3)$ EMBEDDING

$$\begin{array}{ccc} AdS_2 \otimes S_2 & & \text{POINT} \\ \cap & & \cap \\ AdS_5 & & S^5 \end{array}$$

$\longrightarrow$ $OSp(4^*|4)$ SYMMETRY

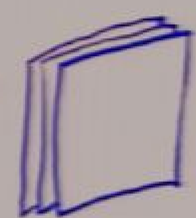
ARBITRARY NUMBER (k) OF FUNDAMENTAL CHARGES

$$D3_k \longleftrightarrow W_{\underbrace{\square\square\square\square\square}_{k}}$$

	0	1	2	3	4	5	6	7	8	9
N D3	x	x	x	x						

SPATIAL COORDINATES

N D3



$\lambda = g_s N >> 1$

$\lambda = g_s N << 1$

SUGRA

PERT. STRING

CLOSED STRING DESCRIPTION

OPEN STRING DESCRIPTION

MALDACENA LIMIT

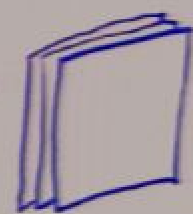
$AdS_5 \otimes S_5$

$\mathcal{N} = 4$ SYM

	0	1	2	3	4	5	6	7	8	9
N D3	x	x	x	x						

SPATIAL COORDINATES

N D3



$\lambda = g_s N >> 1$ $\qquad$ $\lambda = g_s N << 1$

SUGRA $\qquad$ PERT. STRING

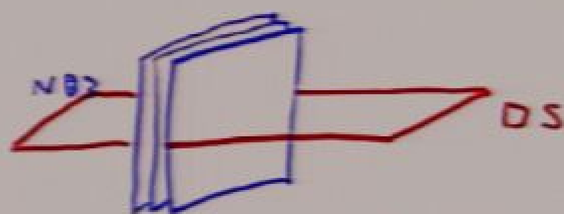
CLOSED STRING DESCRIPTION $\qquad$ OPEN STRING DESCRIPTION

MALDACENA LIMIT

$AdS_5 \otimes S_5$ $\qquad$ $\mathcal{N}=4$ SYM

$D5_k$ GIANT WILSON LOOP

	0	1	2	3	4	5	6	7	8	9 SPATIAL COORDINATES
N D3	x	x	x	x						
D5	x					x	x	x	x	x



$\lambda = g_s N >> 1$ → SUGRA → CLOSED STRING DESCRIPTION

$\lambda = g_s N << 1$ → PERT. STRING → OPEN STRING DESCRIPTION

MALDACENA LIMIT

$AdS_5 \otimes S_5$

⊕

→ $D5_k$ GIANT W.L.

PROBE EMBEDDED IN $AdS_5 \times S_5$

$\mathcal{N}=4$ SYM

⊕

→ DEFECT FIELD THEORY

(NS-D , D-NS)

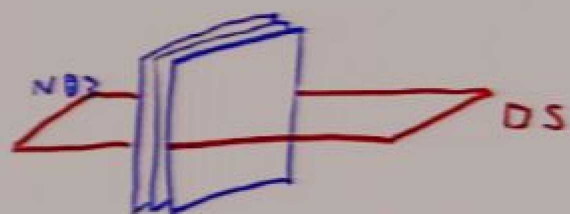
→ NON DYNAMICA

S- D.O.F.

$D5_k$ GIANT WILSON LOOP

	0	1	2	3	4	5	6	7	8	9
N D3	x	x	x	x						
D5	x					x	x	x	x	x

SPATIAL COORDINATES



$\lambda = g_s N >> 1$ → SUGRA → CLOSED STRING DESCRIPTION

$\lambda = g_s N << 1$ → PERT. STRING → OPEN STRING DESCRIPTION

MALDACENA LIMIT

$AdS_5 \otimes S_5$ ⊕ → $D5_k$ GIANT W.L.

PROBE EMBEDDED IN $AdS_5 \times S_5$

$\mathcal{N}=4$ SYM ⊕ → DEFECT FIELD THEORY (NS-D, D-NS)

→ NON DYNAMICAL S-S D.O.F.

LOW ENERGY FIELD THEORY D-S

DEFECT FIELD THEORY

$$S = S_{\mathcal{N}=4} + \int dt \left(i\chi^{+} \partial_t \chi + \chi^{+}(A_0+\phi)\chi \right)$$

∈ THE FUND. REP. OF U(N)

$$+ \; S_{EXTRA}$$

NON DYNAMICAL D.O.F $(\widetilde{A})$

$$S_{EXTRA} = \int dt \left(\chi^{+} \widetilde{A}_0 \chi - k \widetilde{A}_0 \right)$$

ENCODE THE CHOICE OF K

WE NOW INTEGRATE OUT THE D.O.F. ASSOCIATED TO DS_k.

GAUGE CHOICE : $A_0 + \phi = \mathrm{DIAG}(\omega_1, \cdots \omega_N)$

E.O.M. FOR χ's : $(i\partial_t + \omega_i)\chi_i = 0$

$$Z = \int [D\chi][D\chi^{+}][D\widetilde{A}_0] \, e^{i(S + S_{EXTRA})}$$

INTEGRATING ONLY THE χ, χ^{+} $0<t<\beta$
AND IGNORING S_{EXTRA}

$$Z^{*} = e^{iS_{\mathcal{N}=4}} \cdot \prod_{i=1}^{N} (1 + x_i) \qquad x_i = e^{i\beta\omega_i}$$

EXPANDING

$$E_\ell(x_1,\dots x_\ell)=\sum_{i_1<i_2<\dots<i_\ell} x_{i_1}\cdots x_{i_\ell}$$

$$\prod_{i=1}^{N}(1+x_i)=\sum_{\ell=0}^{N}E_\ell(x_1,\dots x_\ell)$$

WHERE $$E_\ell = \mathrm{Tr}_{\left.\begin{smallmatrix}\square\\ \vdots\\ \square\end{smallmatrix}\right\}\ell}\, P\exp\left[i\int dt\,(A_0+\phi)\right]$$

$$= W_{\left.\begin{smallmatrix}\square\\ \vdots\\ \square\end{smallmatrix}\right\}\ell}$$

THUS:

$$Z^{*} = e^{iS_{N=4}}\sum_{\ell=0}^{N} W_{\left.\begin{smallmatrix}\square\\ \vdots\\ \square\end{smallmatrix}\right\}\ell}$$

S_{EXTRA}

INTEGRATING $\tilde{A}_0$ $\rightsquigarrow$ $\sum_{i=1}^{N}\chi_i^{+}\chi_i = K$

$$\longrightarrow \ell = K$$

$$DS_K \longleftrightarrow Z = e^{iS_{N=4}}\, W_{\left.\begin{smallmatrix}\square\\ \vdots\\ \square\end{smallmatrix}\right\}K}$$

	0	1	2	3	4	5	6	7	8	9 SPATIAL COORDINATES
N D3	x	x	x	x						

N D3

$\lambda = g_s N >> 1$

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SUGRA

PERT. STRING

CLOSED STRING DESCRIPTION

OPEN STRING DESCRIPTION

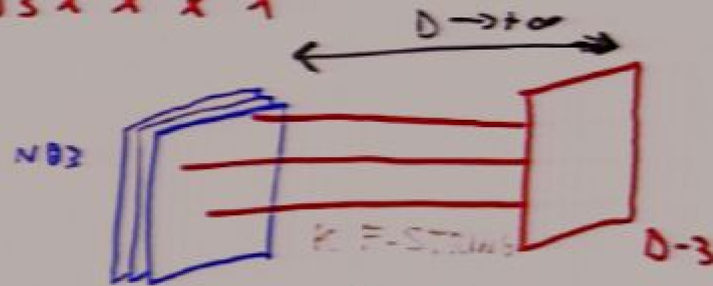
MALDACENA LIMIT

$AdS_5 \otimes S_5$

$\mathcal{N}=4$ SYM

$D3_k$ DUAL GIANT WILSON LOOP

	0	1	2	3	4	5	6	7	8	9	SPATIAL COORDINATES
N D3	x	x	x	x							
D3	x	x	x	x							



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$AdS_5 \otimes S_5$

$\mathcal{N} = 4$ SYM

→ $AdS_2 \otimes S_2 \subset AdS_5$

PROBE D-3

→ NON RELATIVISTIC W-BOSONS

→ W-BOSONS INSERTIONS IN THE PATH INTEGRAL

LOW ENERGY FIELD THEORY D3

CONSIDER $U(N+1)$ SYM IN THE COULOMB BRANCH

$$\langle \phi \rangle = \begin{vmatrix} 0 & 0 \\ 0 & D \end{vmatrix}$$

↳ DISTANCE BETWEEN TWO STACKS OF BRANE

$$U(N+1) \longrightarrow U(N) \times U(1)$$

$$\hat{A}_\mu = \begin{vmatrix} A_\mu & w_\mu \\ w_\mu^+ & \tilde{A}_\mu \end{vmatrix}$$

↳ W-BOSONS, TRANSFORM IN THE FUNDAMENTAL OF $U(N)$

D IS THE MASS OF W-BOSONS

$D \to +\infty$ IS A NON RELATIVISTIC LIMIT

$$S = S_{N=4} + \int i\chi^+ \partial_t \chi + \chi^+ (A_0 + \phi)\chi$$

$$w = \frac{1}{\sqrt{D}} e^{-itD} \chi$$

LOW ENERGY FIELD THEORY D3

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$$w = \frac{1}{\sqrt{D}} e^{-itD} \chi$$

K STRETCHED STRINGS

$$= \begin{cases} K \ \text{W-BOSON CREATION AT } t=-\infty \\ K \ \text{W-BOSON ANNIHILATION AT } t=+\infty \end{cases}$$

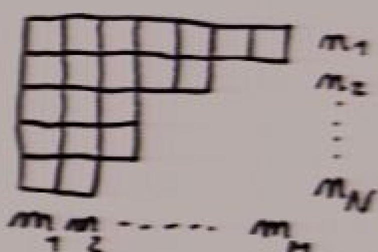
$$Z = e^{iS_{N=4}} \int [D\chi][D\chi^\dagger]\, e^{iS_\chi} \cdot$$

$$\cdot \frac{1}{K!} \sum_{i_1,\dots i_K} \chi_{i_1}(\infty) \cdots \chi_{i_K}(\infty)\, \chi^\dagger_{i_1}(-\infty) \cdots \chi^\dagger_{i_K}(-\infty)$$

$$= e^{iS_{N=4}}\, W_{\underbrace{\square\square\square\square\square}_{K}}$$

$$D3_K \longleftrightarrow W_{\underbrace{\square\square\square\square\square}_{K}}$$

$\frac{1}{2}$ BPS WILSON LOOP IN A GENERIC REPRESENTATION



IS ASSOCIATED TO

$$\left(D5_{m_1}, D5_{m_2}, \ldots, D5_{m_M} \right)$$

OR

$$\left(D3_{m_1}, D3_{m_2}, \ldots \; D3_{m_N} \right)$$

W. L. EXPECTATION VALUE

ANTISYMMETRIC REPRESENTATION

$$\langle W_{\square_K} \rangle = e^{-S_{DS_K}}$$

CONSIDERING $C = C_0$

$$\rightsquigarrow E_{\square} = \frac{2N}{3\pi} \sin^3\theta_K \cdot E_0(L)$$

$$K = N \Rightarrow \theta_N = \pi \Rightarrow E_{\square\}N} = 0$$

SINGULET STATES DO NOT INTERACT

CONCLUSION

- ANY $\frac{1}{2}$ BPS WILSON LOOPS HAS A DUAL DESCRIPTION IN TERMS OF D5 OR ALTERNATIVELY IN TERMS OF D3

- COMPUTATION OF WILSON LOOPS EXPECTATION VALUE USING BRANES