Title: Primordial non-Gaussianity

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Abstract: The non-Gaussianity of the primordial cosmological perturbations will be strongly constrained by future observations like Planck. It will provide us with important information about the early universe and will be used to discriminate among models. I will review how different models of the early universe can generate different amount and shapes of non-Gaussianity.

Primordial non-gaussianities

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Perimeter Institute, September 18, 2007

Outline

- Introduction
  - Motivations
  - Observational status
- Computing the non-Gaussianity
- Models
  - Curvaton
  - Single and multi-field inflation
  - No slow-roll models (ghost inflation, DBI...)
  - Ekpyrotic
- Conclusion

Precision physics

- Standard Model: $S[\phi_I] \Rightarrow \langle T\phi_I(x_1)\phi_J(x_2)\rangle,$

$$\langle T\phi_I(x_1)\phi_J(x_2)\phi_K(x_3)\rangle$$

Correlation functions

↓

S-matrix

$$\langle \vec{p}_3, \vec{p}_4 | S | \vec{p}_1, \vec{p}_2 \rangle_{in}$$

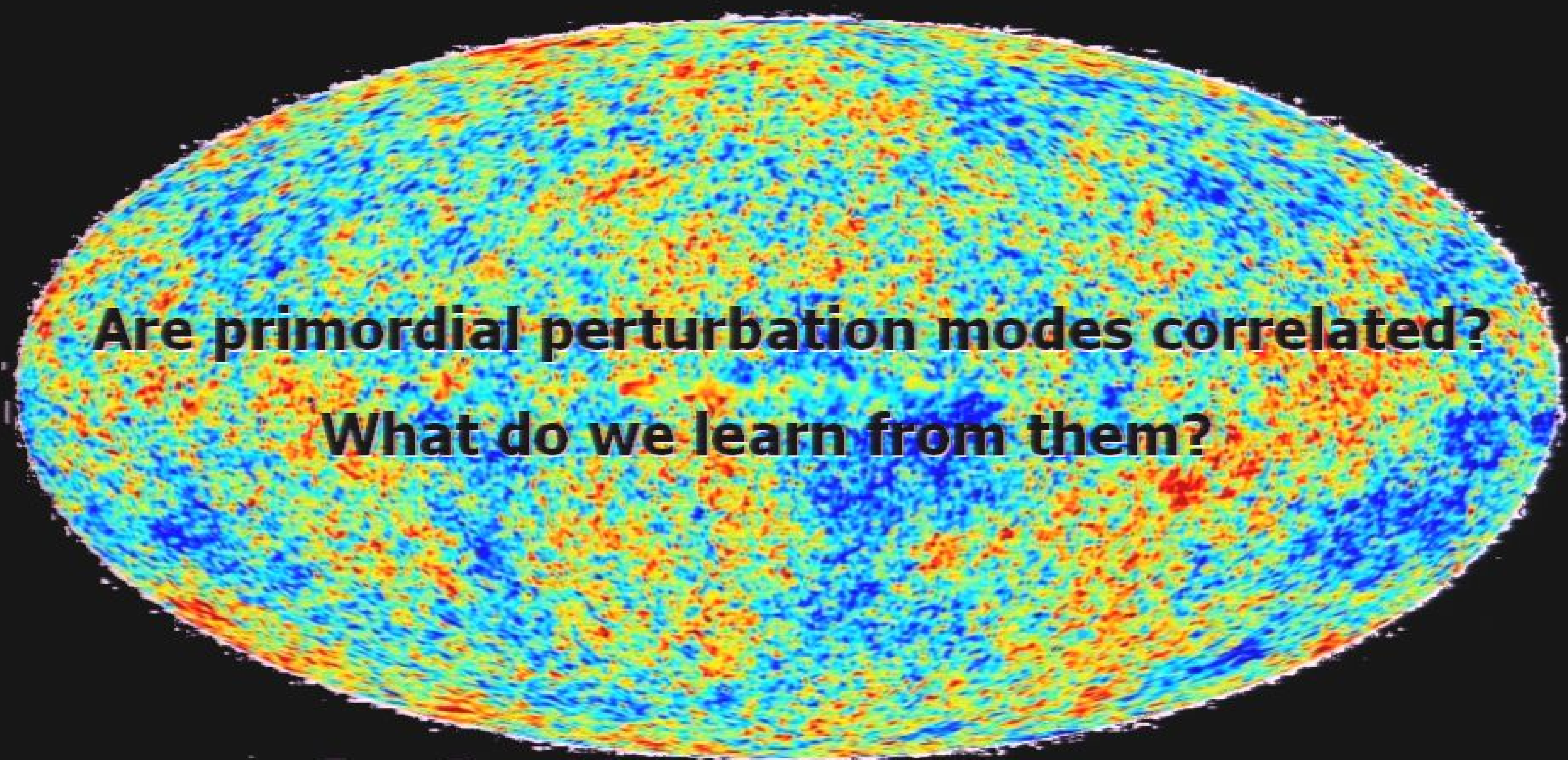
↓

Cross-section

$$d\sigma_{\phi_1\phi_2 \to \phi_3\phi_4}$$

- Accelerator

Precision cosmology?

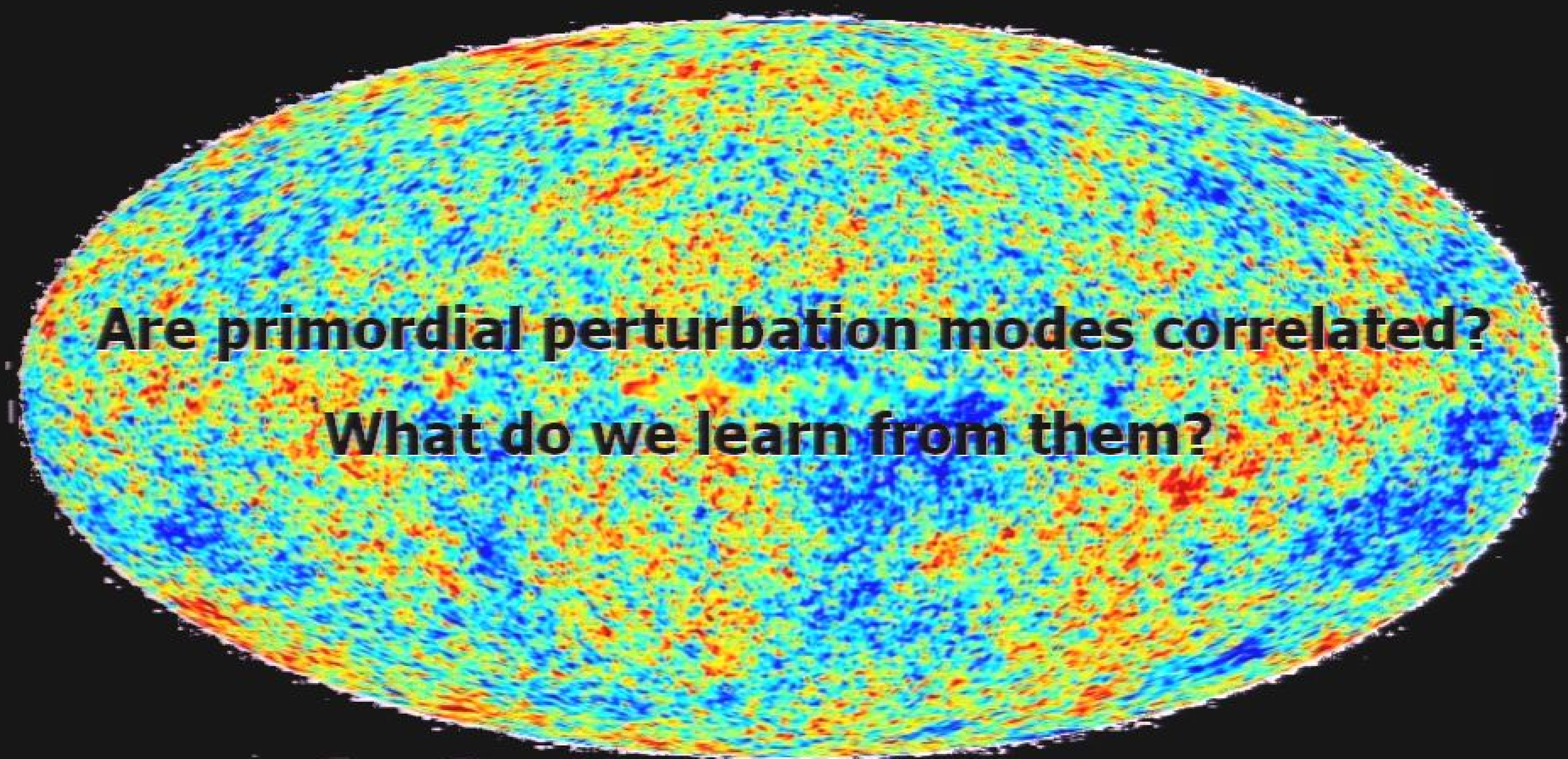


Are primordial perturbation modes correlated?

What do we learn from them?

$$\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\right\rangle, \quad \left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\right\rangle, \quad \left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\zeta_{\vec{k}_4}\right\rangle, \quad \cdots$$

Precision cosmology?



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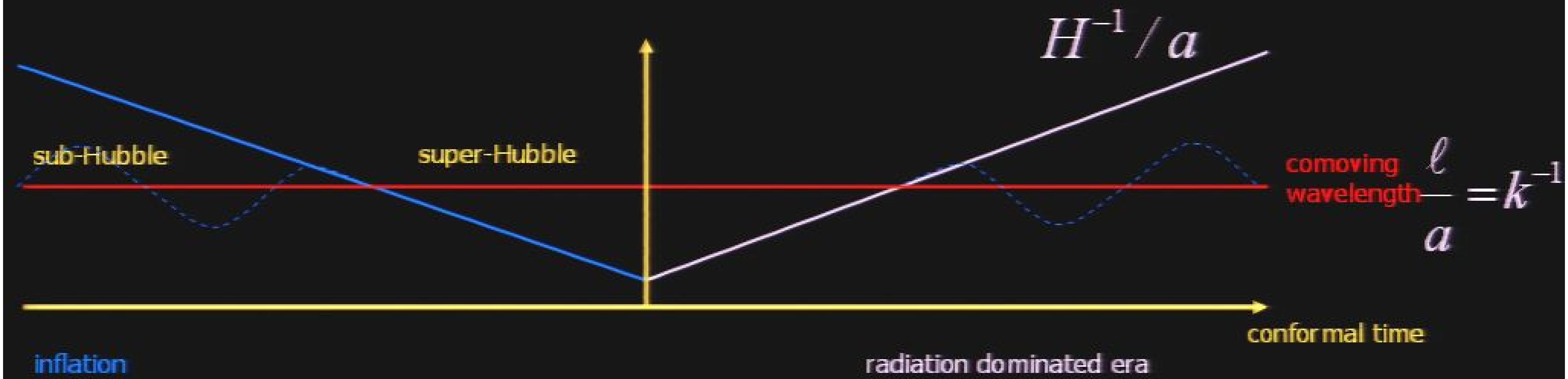
Happy families are all alike; every unhappy family is unhappy in its own way.

Tolstoy 1870, arXiv:87001.4321

"Small" and "large" in the Universe

- "Small" and "large" scales:
  - Hubble radius: $H^{-1}(t) = (\dot{a}/a)^{-1}$



"Small" and "large" in the Universe

- "Small" and "large" scales:
  - Hubble radius: $H^{-1}(t) = (\dot{a}/a)^{-1}$

  - Large scales: $\ell(t) >> H^{-1} \quad \Rightarrow \quad k << Ha$
  - Small scales: $\ell(t) << H^{-1} \quad \Rightarrow \quad k >> Ha$

Curvature perturbation ζ and its conservation

- On super-Hubble scales $\ell >> H^{-1}$ one can neglect spatial gradients

$$\left|\frac{\partial}{\partial x^i} Q\right| \sim \frac{Q}{\ell} << HQ \sim \left|\frac{\partial}{\partial t} Q\right|$$

[Belinski et al. '70; Tomita '72; Salopek Bond '90; ...]

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- Separate-Universe approach:

$$ds^2 = -dt^2 + \tilde{a}^2(t, \vec{x})\; d\vec{x}^2 \qquad \rho = \bar{\rho}(t)$$

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• Separate-Universe approach:

$$ds^2 = -dt^2 + a^2(t) e^{2\zeta(\vec{x})} d\vec{x}^2 \qquad \rho = \bar{\rho}(t)$$

ζ is *nonlinearly* conserved on super-Hubble scales

Salopek and Bond '91; Rigopoulos and Shellard '03;
Lyth Malik Sasaki '04; Langlois FV '05

Perturbations from power spectrum

$$\left\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \right\rangle = (2\pi)^3 \delta\left(\vec{k} + \vec{k}'\right) P_\zeta(k)$$

- Mostly adiabatic universe model

  $\Rightarrow$ Perturbations generated by single (dominant) component

- Almost scale-invariant

  $\Rightarrow$ Quasi de-Sitter expansion $-\frac{\dot{H}}{H^2} << 1, \quad m << H$

- Almost completely "scalar"

  $\Rightarrow$ No tensor modes observed

- Almost gaussian

  $\Rightarrow$ Weak coupling

Window into the early universe

- Early universe model: 1) $S[\phi] \Rightarrow \langle\phi\phi\rangle, \ \langle\phi\phi\phi\rangle$

  2) Conversion: $\phi \Rightarrow \zeta$

$$\zeta = f_1\phi + f_2\phi^2 \Rightarrow$$

$$\langle\zeta\zeta\zeta\rangle = f_1\langle\phi\phi\phi\rangle + f_2\langle\phi\phi\rangle\langle\phi\phi\rangle + \text{perms}$$

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- Primordial perturbations: $\langle\zeta\zeta\rangle, \ \langle\zeta\zeta\zeta\rangle$

- Observations (ex: CMB anisotropies): $\langle TT\rangle, \ \langle TTT\rangle$

How much gaussian?

$$\zeta(\vec{x}) = \zeta_g(\vec{x}) + \tfrac{3}{5} f_{NL} \zeta_g^2(\vec{x}) \qquad \Rightarrow \quad \langle \zeta\zeta\zeta \rangle \sim f_{NL} \langle \zeta_g \zeta_g \rangle^2$$

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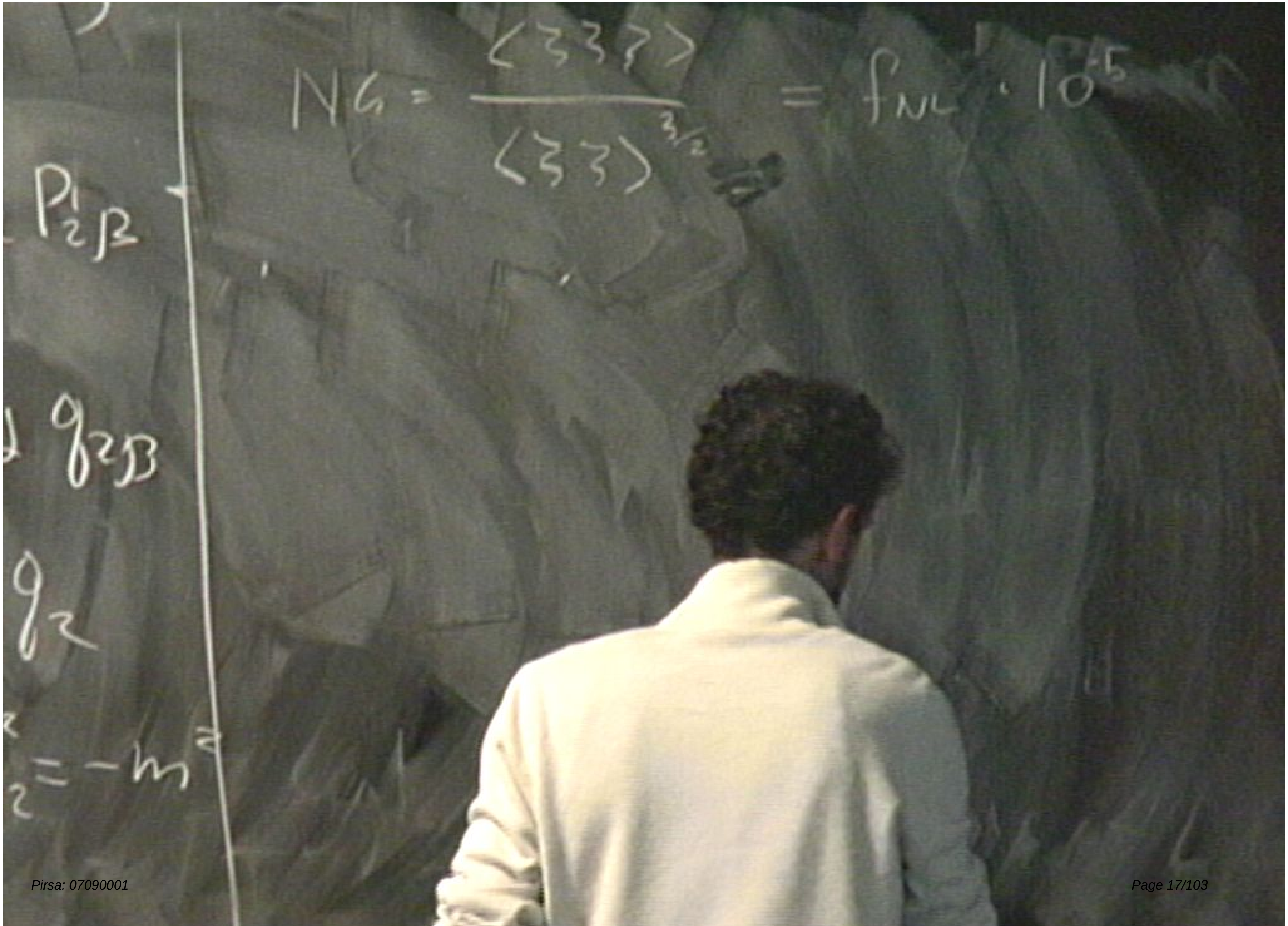
$$NG = \frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta_g \zeta_g \rangle^{3/2}} \sim f_{\rm NL} \langle \zeta_g \zeta_g \rangle^{1/2} \sim f_{\rm NL} 10^{-5} \qquad \zeta_g \sim 10^{-5}$$


NG = ⟨ζζζ⟩ / ⟨ζζ⟩^{3/2} = f_NL · 10^-5

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- Very large NG:

  $$NG \sim 1, \quad f_{\rm NL} \sim 10^5 \;\Rightarrow\; f_{\rm NL} \zeta_g^2 \sim \zeta_g$$

- Small NG:

  $$NG \sim 10^{-5}, \quad f_{\rm NL} \sim 1 \;\Rightarrow\; f_{\rm NL} \zeta_g^2 \sim \zeta_g^2$$

- Very small NG (beyond linear theory):

  $$NG << 10^{-5}, \quad f_{\rm NL} << 1 \;\Rightarrow\; f_{\rm NL} \zeta_g^2 << \zeta_g^2$$

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Shape of non-gaussianity

Babich Creminelli Zaldarriaga '04

$$\left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \right\rangle = (2\pi)^3 \delta(\Sigma_i \vec{k}_i) \tfrac{3}{5} f_{\rm NL}\left(\vec{k}_1, \vec{k}_2, \vec{k}_3\right) P_\zeta(k_1) P_\zeta(k_2) + \text{perms}$$

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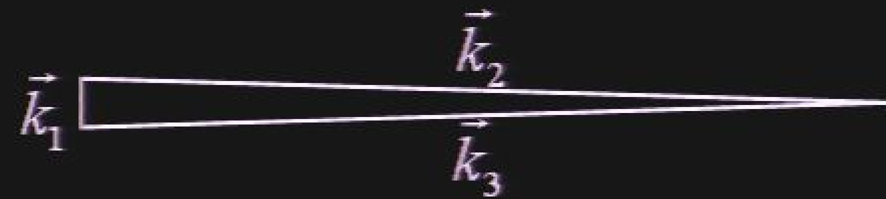
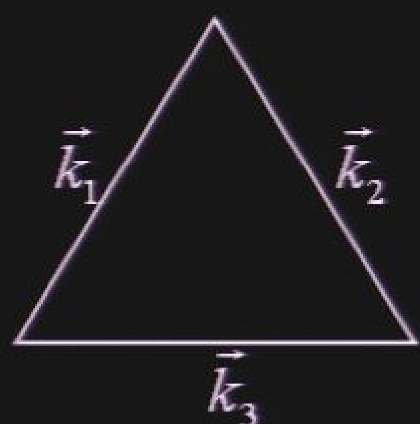
- Local: - maximal signal for $k_1 << k_2, k_3$ squeezed

$$\zeta(\vec{x}) = \zeta_g(\vec{x}) + \tfrac{3}{5} f_{\rm NL} \zeta_g^2(\vec{x})$$



- Equilateral: - maximal signal for $k_1 \sim k_2 \sim k_3$



Current and future bounds (for local shape)

$$\frac{S}{N} \approx \frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^{3/2}} N_{\text{pix}}^{1/2}$$

$$= f_{\text{NL}} 10^{-5} N_{\text{pix}}^{1/2}$$

$$N_{\text{pix}} \approx \ell_{\text{max}}^2 \longrightarrow$$

- WMAP $\left(N_{\text{pix}} = 10^6\right)$

  $|f_{\text{NL}}| \leq 11 \ (\approx 100)$

- Planck $\left(N_{\text{pix}} = 5 \cdot 10^6\right)$

  $|f_{\text{NL}}| \leq 2.9$

- CMB Ideal experiment $\left(N_{\text{pix}} = \infty\right)$

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$$N_{pix} \approx \ell_{max}^{3}$$

- Idealistic experiment: 21-cm

$$|f_{NL}| \leq 0.01 - 1$$

Cooray '06; Pillepich Porciani Matarrese '06

Computing the non-gaussianity

Light field in (quasi) de Sitter

Birrell Davies '82

$$ds = -dt^2 + a^2(t)dx^2, \qquad a = e^{Ht}, \qquad -\dot{H} << H^2$$

- Test field: does not mix with gravity (forget gravity)

$$S = \tfrac{1}{2}\int dx^4 \sqrt{-g}\left[\dot{\phi}^2 - \left(\tfrac{\partial}{a}\phi\right)^2 - m^2\phi^2\right], \qquad m << H$$

Second order in the perturbation ϕ

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Second order in the perturbation ϕ

Amplification of vacuum fluctuations:

- collection of free harmonic oscillators,

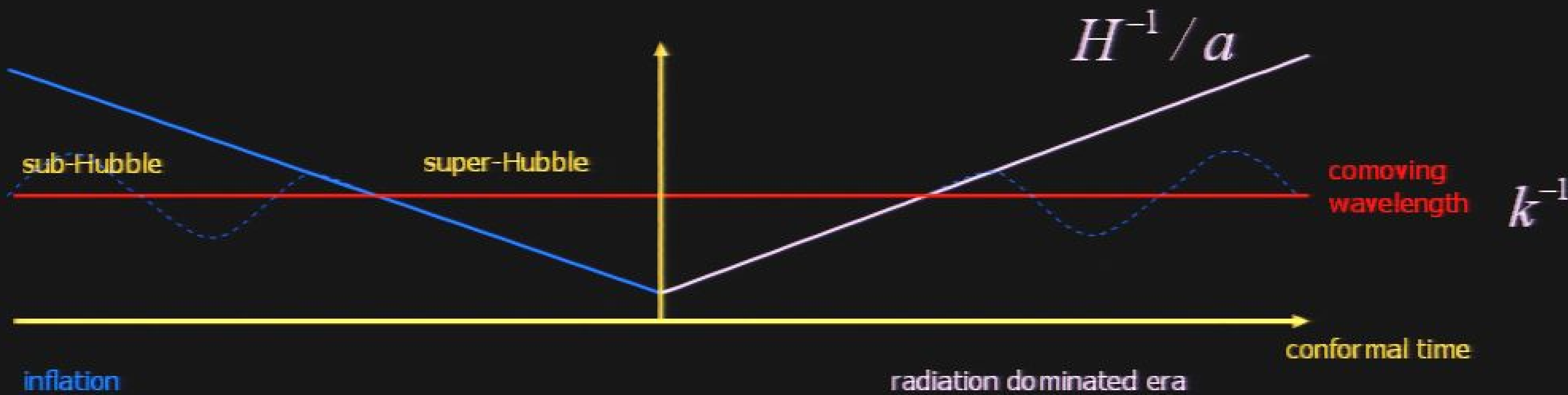
$$\phi_{\vec{k}}(t) = \phi_{\vec{k}}^{cl}(t)a_{\vec{k}}^{+} + \phi_{\vec{k}}^{cl*}(t)a_{-\vec{k}}$$

$\phi_{\vec{k}}^{cl}(t)$: solution of the classical equation

$a_{\vec{k}}^{+}, a_{\vec{k}}$: creation and annihilation operators $\quad \left[a_{\vec{k}}, a_{\vec{k}'}^{+}\right] = i\delta\left(\vec{k} + \vec{k}'\right)$

Linear gaussian fluctuations

Hawking '82; Starobinsky '82; Guth Pi '82



$$\phi^{cl}_{\vec{k}}(t) = \frac{H}{\sqrt{2k^3}}\left(1 + i\,\frac{k}{aH}\right)e^{-i\frac{k}{aH}}$$

- Small scales: $k >> Ha$ under-damped vacuum quantum fluctuations
- Large scales: $k << Ha$ frozen over-damped fluctuations

$$\langle \phi_{\vec{k}}\phi_{\vec{k}'} \rangle \approx \delta\left(\vec{k} + \vec{k}'\right)\frac{H_*^2}{2k^3} \qquad \left(k = a_* H_*\right)$$

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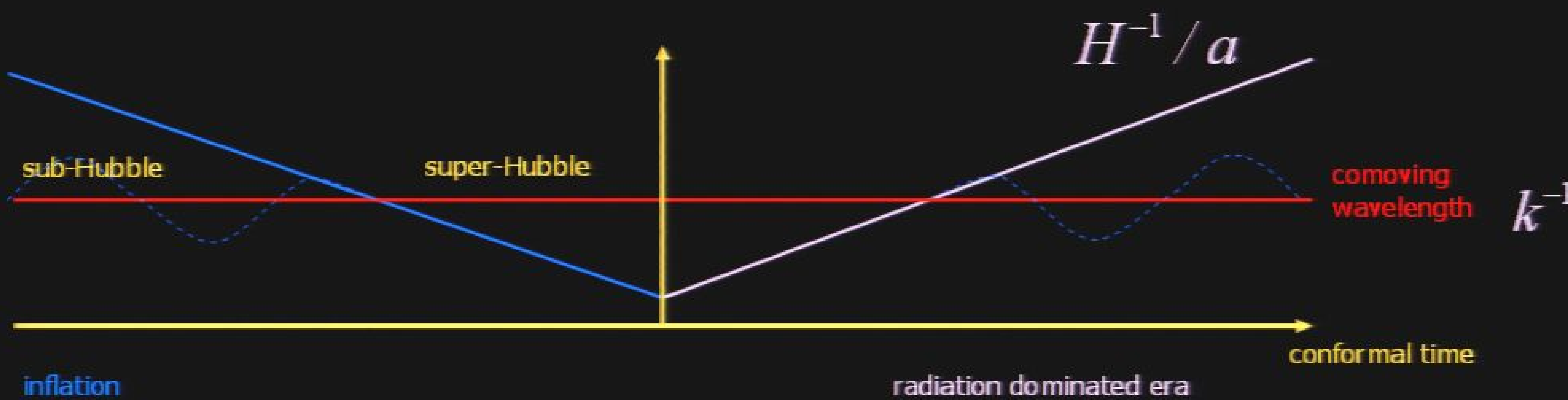

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$$\left\langle \phi_{\vec{k}}\phi_{\vec{k}'}\right\rangle = \left\langle 0\left|\phi_{\vec{k}}\phi_{\vec{k}'}\right|0\right\rangle$$ vacuum of the free theory

Non-gaussian fluctuations

Bernardeau Uzan '02; Zaldarriaga '03

- Nonlinear interaction:

$$S = \frac{1}{2}\int dx^4 \sqrt{-g}\left[\dot{\phi}^2 - \left(\frac{\partial}{a}\phi\right)^2 - m^2\phi^2 - \frac{V^{(3)}}{3}\phi^3\right], \qquad V^{(3)} << H$$

We want to compute:

$$\left\langle \phi^3(t) \right\rangle = \left\langle \Omega \middle| \phi^3(t) \middle| \Omega \right\rangle$$

$\phi(t)$: field operator of the interacting theory $\longrightarrow$ $\phi_g(t)$

$|\Omega\rangle$: ground state of interacting theory $\longrightarrow$ $|0\rangle$

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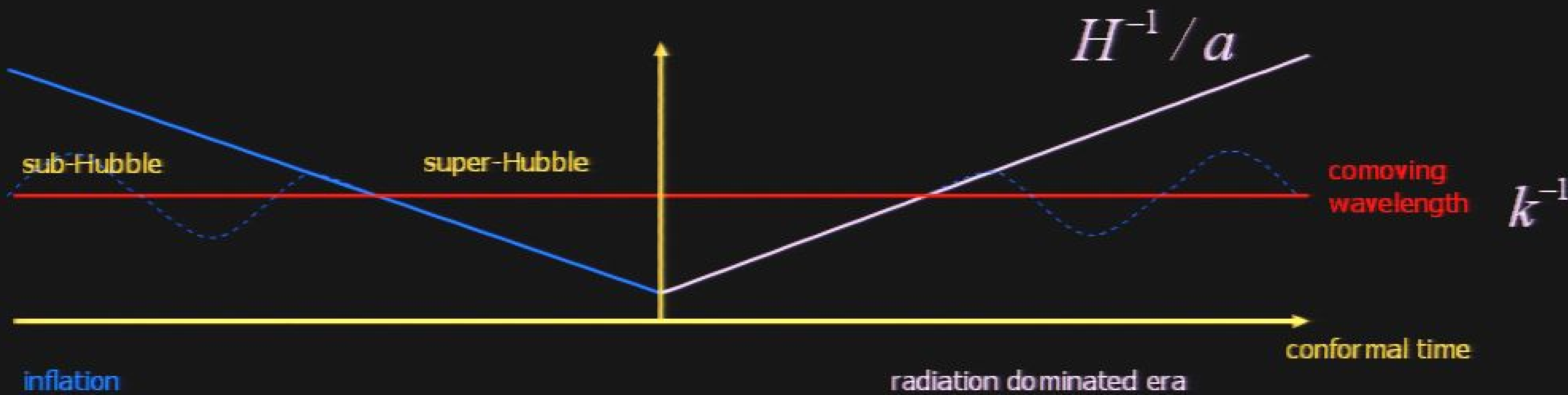

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Free theory = Interacting theory when $t \to -\infty$, $\left(\frac{k}{aH} >> 1\right)$

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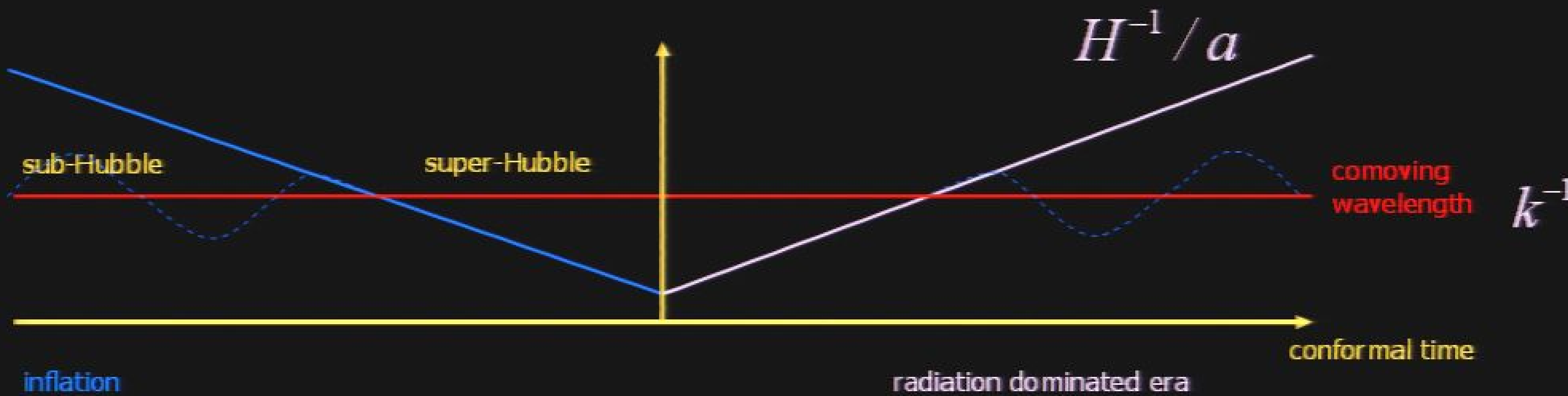

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Expectation value: in-in

$$\langle \Omega | \phi^3(t) | \Omega \rangle = \langle 0 | \bar{T} e^{i\int_{-\infty}^{t} H_{int}} \phi_g^3(t) T e^{-i\int_{-\infty}^{t} H_{int}} | 0 \rangle$$

Weinberg '05
(and vast earlier literature on Schwinger-Keldysh formalism)

$$U_0^{-1}(t,t_0) U(t,t_0) = T e^{-i\int_{t_0}^{t} H_{int}(t')dt'}$$: time evolution operator

T : time-ordered product

$$H_{int}(t) = \int dx^3 a^3 \frac{V^{(3)}}{3!} \phi_g^3$$: interaction Hamiltonian

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This is not a scattering amplitude: in-out

$$\langle \Omega | \hat{Q} | \Omega \rangle \propto \langle 0 | T \hat{Q}_g e^{-i\int_{-\infty}^{\infty} H_{int}} | 0 \rangle$$

Non-gaussianity and nonlinear evolution

- Tree level expression

$$\langle \phi^3(t) \rangle = -i \frac{V^{(3)}}{3!} \int dx^3 \int_{-\infty}^{t} dt' \left\langle \left[\phi_g^3(t), \phi_g^3(t') \right] \right\rangle$$

Maldacena '02

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Maldacena '02

- Nonlinear evolution

Musso '06

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi - \frac{\vec{\nabla}^2}{a^2}\phi = -\frac{V^{(3)}}{2}\phi^2$$

Solution with Green's method:

Growing homogeneous solution (operator): ϕ_g

$$\phi(t,\vec{x}) = \phi_g(t,\vec{x}) - \frac{V^{(3)}}{2} \int dy^3 \int dt' \, a^3(t') G_R(t,\vec{x};t',\vec{y}) \phi_g^2(t',\vec{y})$$

Retarded Green's function

$$\left\langle \phi^3(t) \right\rangle = \left\langle \phi_g^3(t) \right\rangle - i \frac{V^{(3)}}{2} \left\langle \phi_g^2(t) \int dy^3 \int dt' \, a^3(t') \left[\phi_g(t), \phi_g(t') \right] \phi_g^2(t') \right\rangle + \text{perms}$$

Scale dependence, quantum and classical

Maldacena '02

- Three contributions

$$\left\langle \phi_{\vec{k}_1}\phi_{\vec{k}_2}\phi_{\vec{k}_3}\right\rangle = 2(2\pi)^3\delta\left(\Sigma_i \vec{k}_i\right)V^{(3)}\mathrm{Re}\left[-i\phi^*_{g\,\vec{k}_1}\phi^*_{g\,\vec{k}_2}\phi^*_{g\,\vec{k}_3}\int_{-\infty}^{t} dt'\,a^3\phi_{g\,\vec{k}_1}\phi_{g\,\vec{k}_2}\phi_{g\,\vec{k}_3}\right]$$

$$= \underbrace{\int_{k/Ha>>1}}_{(0)} + \underbrace{\int_{k/Ha\sim 1}}_{(1)} + \underbrace{\int_{k/Ha<<1}}_{(2)}$$

0) Small scales, heavy oscillations (vacuum): no contribution

1) Hubble crossing: "quantum" contribution – equilateral configuration

2) Super-Hubble evolution: "classical" contribution – local configuration

3) Nonlinear relation: $\phi \rightarrow \zeta$

Rule of thumb to estimate NG (1)

Creminelli '03

- "Quantum" contribution: $k \sim Ha$

$$S = \frac{1}{2}\int dx^4 \sqrt{-g}\left[\dot{\phi}^2 - \frac{1}{a^2}(\partial\phi)^2 - m^2\phi^2 - \frac{V^{(3)}}{3}\phi^3\right], \qquad V^{(3)} << H$$

Compare the operators in the action at $E \sim H$

$$(\phi \sim H\,,\ \omega \sim H)$$

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$$NG_\phi \sim \frac{V^{(3)} H_*^3}{H_*^4} \sim \frac{V^{(3)}}{H_*}$$

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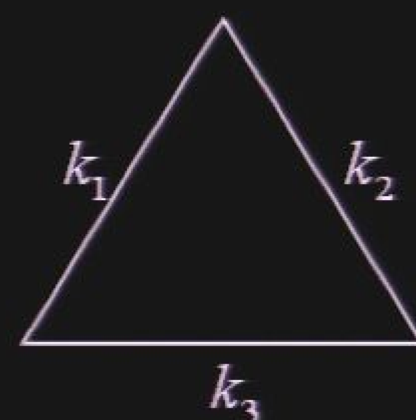

Equilateral: correlation among modes which freeze at the same time is favored

Rule of thumb to estimate NG (2)

Bernardeau Uzan '03

- "Classical" contribution: $k << Ha$

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi \approx -\frac{V^{(3)}}{2}\phi^2, \qquad \ddot{\phi} << 3H\dot{\phi}$$

Large scales classical evolution ($\propto$ number of e-folds N)

$$\phi_{NG} \sim \frac{V^{(3)}}{H_*^2}\phi_g^2 \Delta N \qquad \Delta N = N_{end} - N_* = H_*\left(t_{end} - t_*\right)$$

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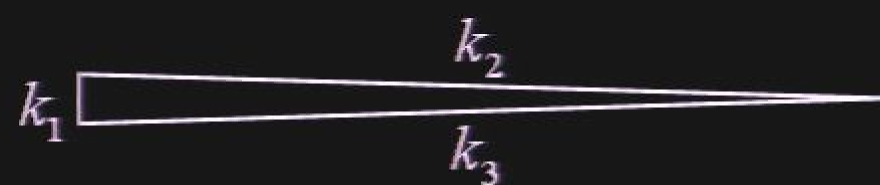

Local: correlation among modes in the squeezed configuration is favored

Relation with curvature perturbation (3)

- Separate-Universe approach:

$$ds^2 = -dt^2 + \underbrace{a^2(t)e^{2\zeta(\vec{x})}}_{\tilde{a}^2(t,\vec{x})}dx^2$$

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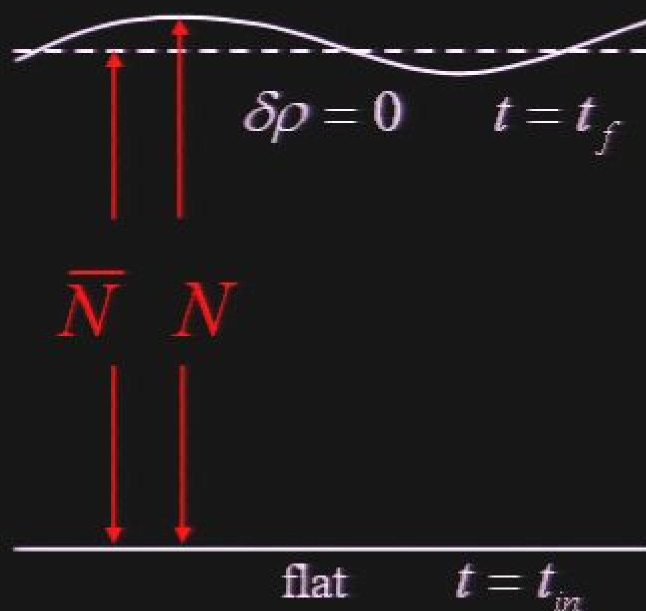
- δN-approach: Starobinsky '85; Sasaki Stewart '95

$$N(t, t_{in}, \vec{x}) = \ln\left(\frac{\tilde{a}(t,\vec{x})}{a(t_{in})}\right)$$: number of e-folds



$$\zeta(\vec{x}) = \delta N = N(t, t_{in}, \vec{x}) - \bar{N}(t, t_{in})$$

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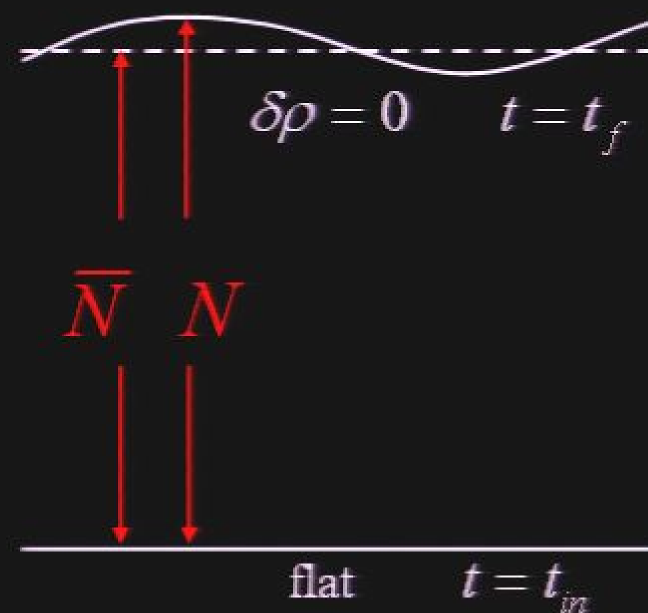


$$\zeta(\vec{x}) = \delta N = N(t, t_{in}, \vec{x}) - \overline{N}(t, t_{in})$$

$$\zeta(t,\vec{x}) = N_{,\phi_{in}}\delta\phi_{in} + \frac{1}{2}N_{,\phi_{in}\phi_{in}}\delta\phi_{in}^2 + \dots$$ Lyth Rodriguez '05

Models generating non-gaussianity

- Curvaton
- Single and multi-field inflation
- Ghost inflation
- Ekpyrotic

Curvaton

Enqvist Sloth; Lyth Wands; Moroi Takahashi '01

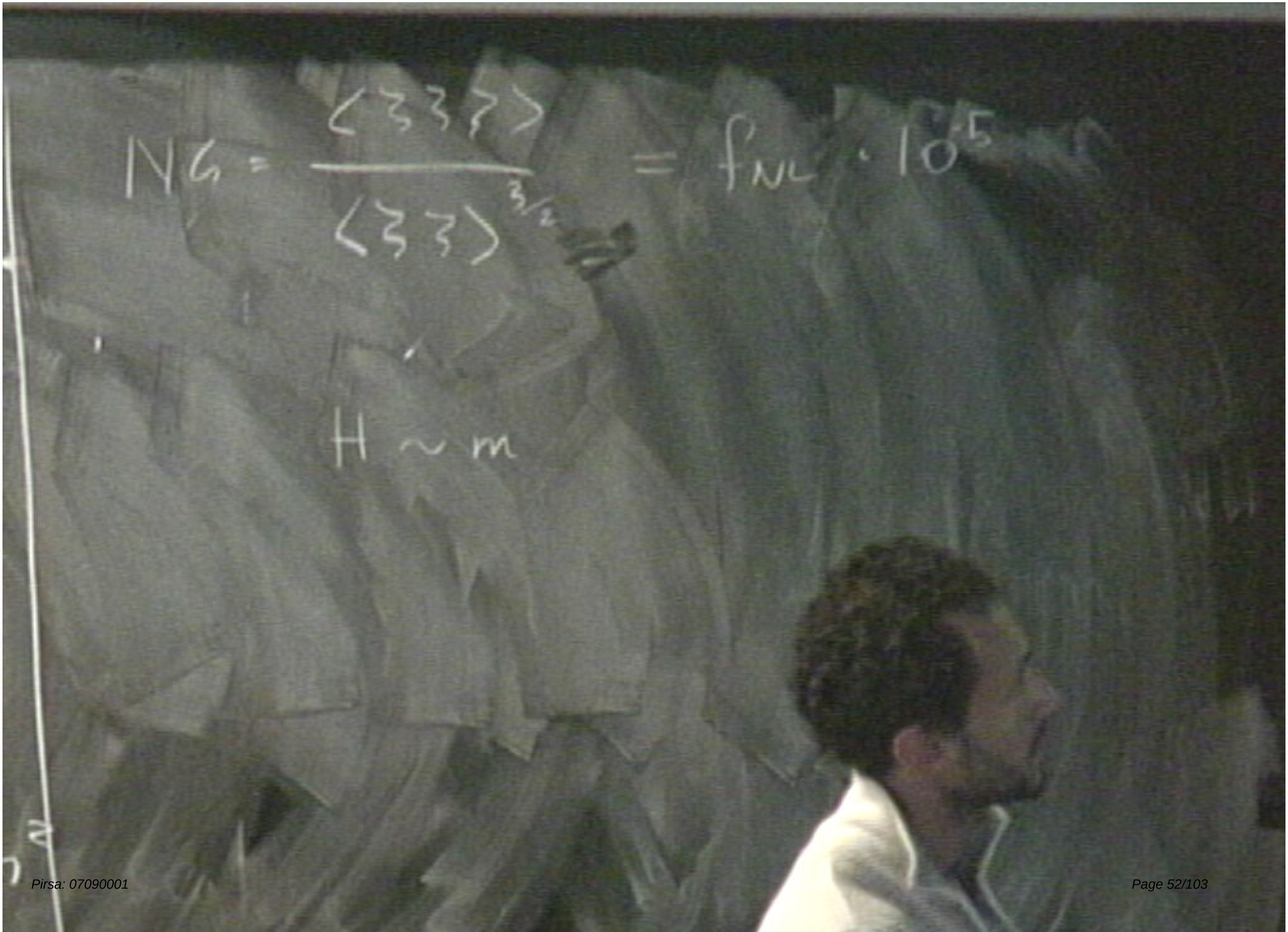
- Light scalar field behaving as test field during inflation
- Dominates the Universe at late time and decays

Three contribution to NG:

1) Hubble crossing contribution due to nonlinear potential
   Bernardeau Uzan '02; Zaldarriaga '03

2) Large scale contribution due to nonlinear potential:
   - during inflation — Bernardeau Uzan '02; Zaldarriaga '03
   - during radiation dominated era (small) — Enqvist Nurmi '05

3) Nonlinear relation: $\sigma \to \zeta$ — Bartolo Matarrese Riotto '03; Lyth Ungarelli Wands '03; Lyth Rodriguez '05


$NG = \frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^{3/2}} = f_{NL} \cdot 10^{-5}$
$H \sim m$

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Contributions to NG

1) Hubble crossing (negligible): $\langle \delta\sigma_* \delta\sigma_* \delta\sigma_* \rangle$

2) Large scale contribution due to nonlinear potential:
- during inflation
- during radiation epoch (neglected)

$$\delta\sigma_{osc} = \delta\sigma_* + \frac{V^{(3)}}{6H^2} \Delta N \delta\sigma_*^2$$

3) Nonlinear relation: $\sigma \to \zeta$

$$N(t_{dec}, t_{osc}) = \ln\left(\frac{a_{dec}}{a_{osc}}\right) = \frac{1}{3} \ln\left(\frac{\rho_{\sigma_{osc}}(\sigma_{osc})}{\rho_{\sigma_{dec}}(\sigma_{osc})}\right)$$

$$\zeta = N_{,\sigma_{osc}} \delta\sigma_{osc} + \frac{1}{2} N_{,\sigma_{osc}\sigma_{osc}} \delta\sigma_{osc}^2$$

$$\langle \xi \xi \rangle^{3/2}$$

$$H \sim m$$

$$\rho \propto \frac{1}{a^3}$$

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Non-gaussianity from the curvaton

$$f_{NL} = \frac{5}{4r}\left(1 + \frac{\sigma_* V^{(3)}}{3H_*^2}\Delta N\right) - \frac{3}{5} - \frac{5r}{6} \qquad r \sim \left.\frac{\rho_\sigma}{\rho}\right|_{dec} \leq 1$$

$$V = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{V^{(3)}}{3!}\sigma^3, \qquad m_{eff}^2 = V'' = m_\sigma^2 + \sigma_* V^{(3)}$$

$$\frac{m_{eff}^2}{3H_*^2} \sim \frac{\sigma_* V^{(3)}}{3H_*^2} = \alpha << 1$$: lightness constraint

back

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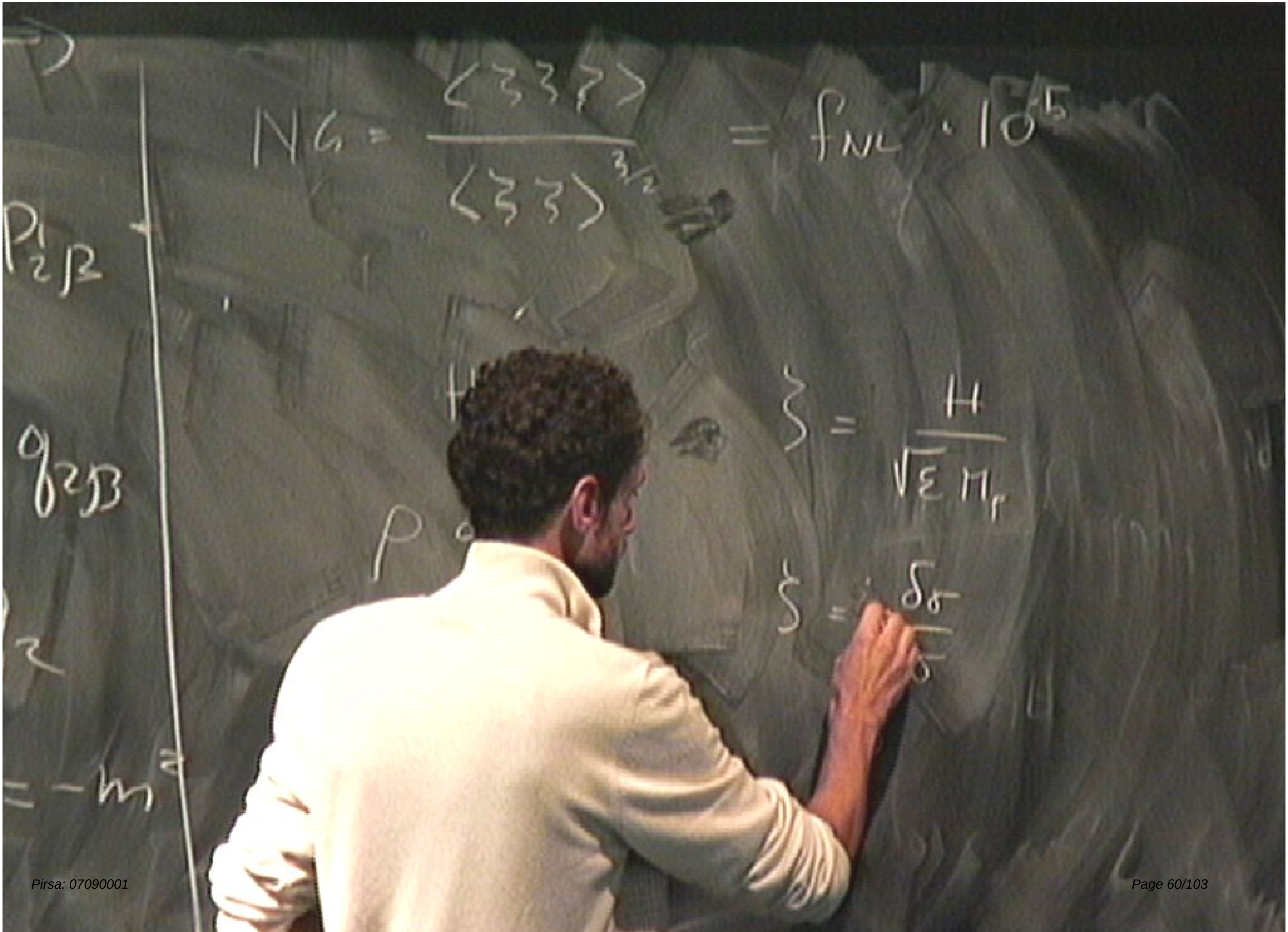
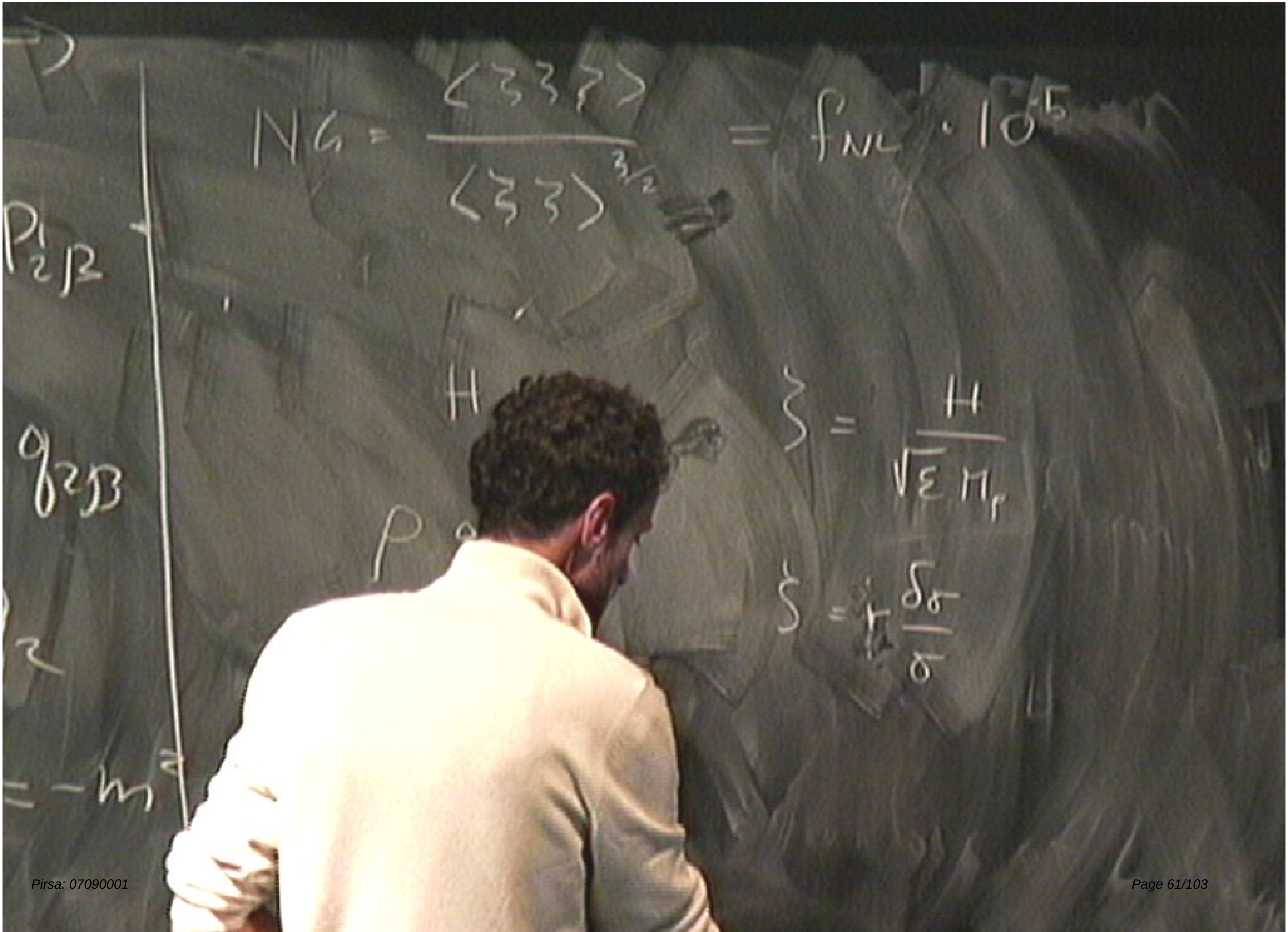

NG = ⟨ζζζ⟩ / ⟨ζζ⟩^3/2 = fNL · 10^-5
ζ = H / √ε MP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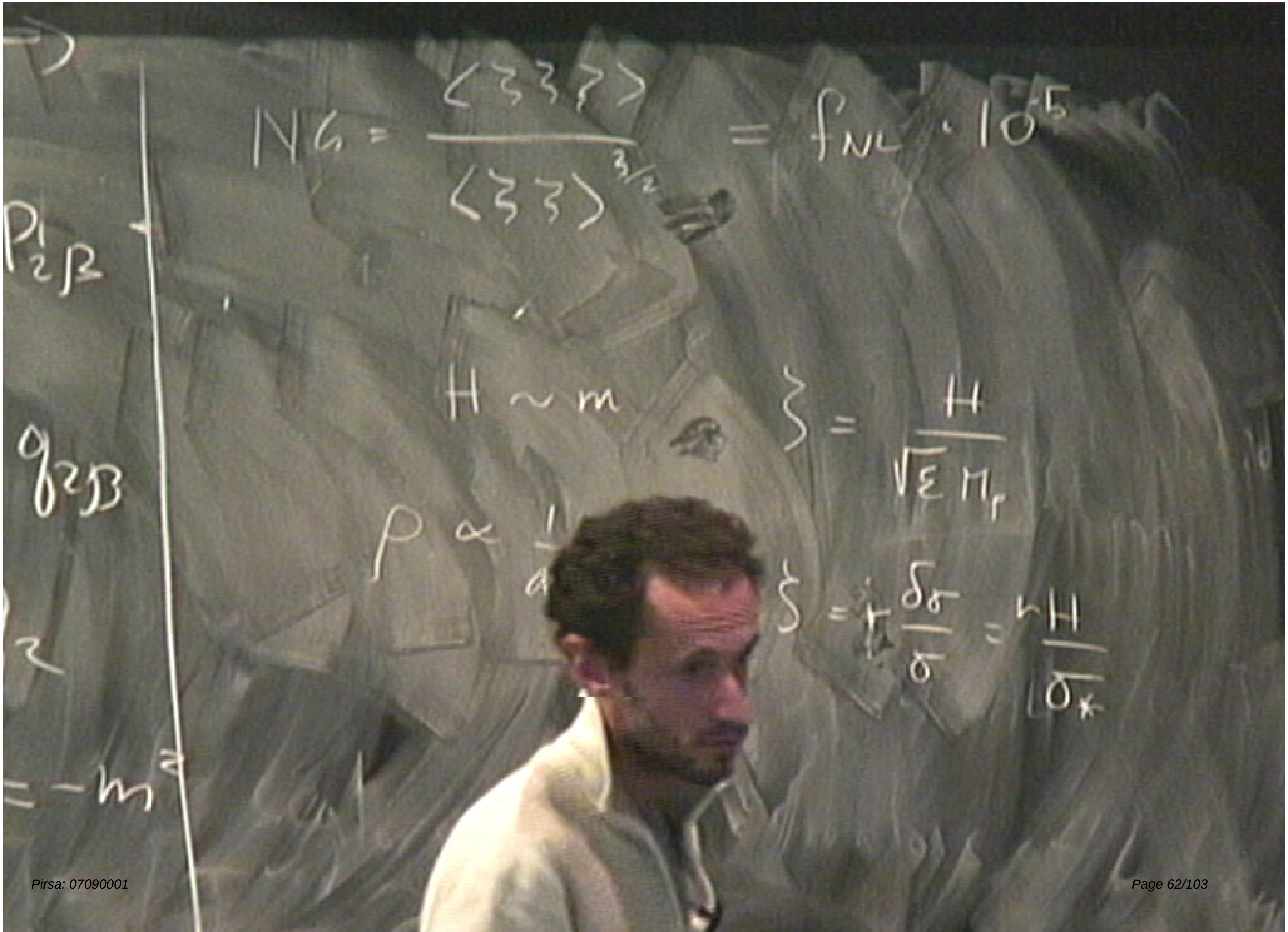
ζ = δσ / σ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H ~ m
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ρ ∝ 1/a
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Single field slow-roll inflation

$$S = \frac{1}{2}\int dx^4 \sqrt{-g}\left[M_P^2 R - (\nabla\phi)^2 - 2V(\phi)\right]$$

- Slow-roll = weak coupling

$$\varepsilon \equiv M_P^2 \left(\frac{V'}{V}\right)^2 << 1,$$

$$M_P^2 \frac{V''}{V} << 1$$

$$\Rightarrow \quad \frac{V^{(3)}}{H} \sim \varepsilon^2 \frac{H}{\sqrt{\varepsilon} M_P} = \varepsilon^2 \zeta$$

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- Mixing with gravity

$$(\nabla\phi)^2 \supset g^{00}\left(\dot{\bar{\phi}}^2 + 2\dot{\bar{\phi}}\delta\dot{\phi} + \delta\phi^2\right) = \left(1 - \frac{\dot{\bar{\phi}}}{2H}\delta\phi + \ldots\right)\left(\dot{\bar{\phi}}^2 + 2\dot{\bar{\phi}}\delta\dot{\phi} + \delta\phi^2\right)$$

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2nd order expansion: free theory $\Rightarrow$ two-point function

3rd order expansion: interacting theory $\Rightarrow$ three-point function

Linear fluctuations in slow-roll inflation

Mukhanov '90

- Gauge choice

$$ds^2 = -dt^2 + a^2(t)e^{2\zeta(\vec{x})}d\vec{x}^2 \qquad \phi = \bar{\phi}(t)$$

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$$S_2 = \frac{1}{2}\int dx^4 a^3 \varepsilon\left[\dot{\zeta}^2 - \left(\frac{\partial}{a}\zeta\right)^2\right]$$

- Amplified vacuum fluctuations (like test field in de Sitter)

$$\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \rangle \approx \delta\left(\vec{k} + \vec{k}'\right) \frac{1}{\varepsilon M_P^2} \frac{H_*^2}{2k^3}$$

Non-gaussian fluctuations in slow-roll

Maldacena '02

- Third order action

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+ other terms suppressed by slow-roll   Ex: $\propto V^{(3)}$

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$\tfrac{6}{5} f_{\mathrm{NL}} = n_s - 1 \sim 0.03$ in the squeezed limit

model independent

Non-gaussian fluctuations in slow-roll

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in the squeezed limit

model independent

- Multi-field at Hubble-crossing

$$\left\langle \delta\phi^I_{\vec{k}_1 *}\delta\phi^J_{\vec{k}_1 *}\delta\phi^K_{\vec{k}_1 *}\right\rangle,\ I,J,K = \text{fields}$$

Seery Lidsey '05

Non-gaussianity from multi-field inflation

$$\zeta = N_{,I}\delta\phi_*^I + \frac{1}{2}N_{,IJ}\delta\phi_*^I\delta\phi_*^J$$

- General expression:

$$\frac{6}{5}f_{\rm NL} = \frac{1}{16}r + \frac{N_{,IJ}N_{,I}N_{,J}}{\left(N_{,I}^2\right)^2}$$

FV Wands '06

$r = 8\frac{P_T}{P_\zeta} = O(\varepsilon) << 1$: tensor to scalar ratio

back

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FV Wands '06

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- Separable potential: $V(\phi_1,\phi_2) = V_1(\phi_1) + V_2(\phi_2)$

$$V(\phi_1,\phi_2) = \frac{1}{2}m_1^2\phi_1^2 + \frac{1}{2}m_2^2\phi_2^2 \qquad \frac{6}{5}f_{\rm NL} = \frac{1}{8}r \sim 0.01$$

Slow-roll inflation = small NG

back

Ghost inflation

$$\mathcal{L} = (\partial\phi)^2 - \frac{1}{M^4}(\partial\phi)^4$$

Arkani-Hamed et al '03;
Creminelli et al '04

wrong kinetic sign

$\phi = M^2 t + \pi(t, \vec{x})$: ghost condensate

- Effective theory of fluctuations:

$$\mathcal{L} = \frac{1}{2}\dot{\pi}^2 - \frac{\alpha}{2M^2}(\nabla^2\pi)^2 - \frac{\beta}{2M^2}\dot{\pi}(\nabla\pi)^2 + \ldots$$

$\alpha \sim 1, \;\; \beta \sim 1, \;\; H << M$ $\qquad \omega \sim \frac{k^2}{M} \Rightarrow \pi \sim M\left(\frac{H}{M}\right)^{1/4}$

back

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$$NG_\phi = \frac{\langle \pi\pi\pi \rangle}{\langle \pi\pi \rangle^{3/2}} \sim \beta\left(\frac{H}{M}\right)^{1/4} \sim 10^{-3} \quad \Rightarrow \quad f_{NL} \sim 100$$

back

Old ekpyrotic: single field

Khoury Ovrut Steinhardt Turok '01

$$V = -V_0 e^{-c_0 \varphi}$$

$$a = (-t)^{\frac{2}{c_0^2} << 1}$$

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$$\alpha \sim 1, \;\; \beta \sim 1, \;\; H << M \qquad \omega \sim \frac{k^2}{M} \Rightarrow \pi \sim M\left(\frac{H}{M}\right)^{1/4}$$

$$NG_\phi = \frac{\langle \pi\pi\pi \rangle}{\langle \pi\pi \rangle^{3/2}} \sim \beta\left(\frac{H}{M}\right)^{1/4} \sim 10^{-3} \quad \Rightarrow \quad f_{NL} \sim 100$$

back

Old ekpyrotic: single field

Khoury Ovrut Steinhardt Turok '01

$$V = -V_0 e^{-c_0 \varphi}$$

$$a = (-t)^{\frac{2}{c_0^2} << 1}$$

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- Amplified vacuum fluctuations:

$$\zeta = A_\zeta + \frac{B_\zeta}{Ha^2}$$

$\Rightarrow$ growing mode transferred to decaying mode in the expanding universe (thus unobservable)

$\Rightarrow$ constant mode transferred to constant mode in the expanding universe: blue (thus incompatible with observations)

Lyth '03; Creminelli Nicolis Zaldarriaga '04

New ekpyrotic: two fields

Lehners McFadden Turok Steinhardt;
Buchbinder Khoury Ovrut;
Creminelli Senatore `07

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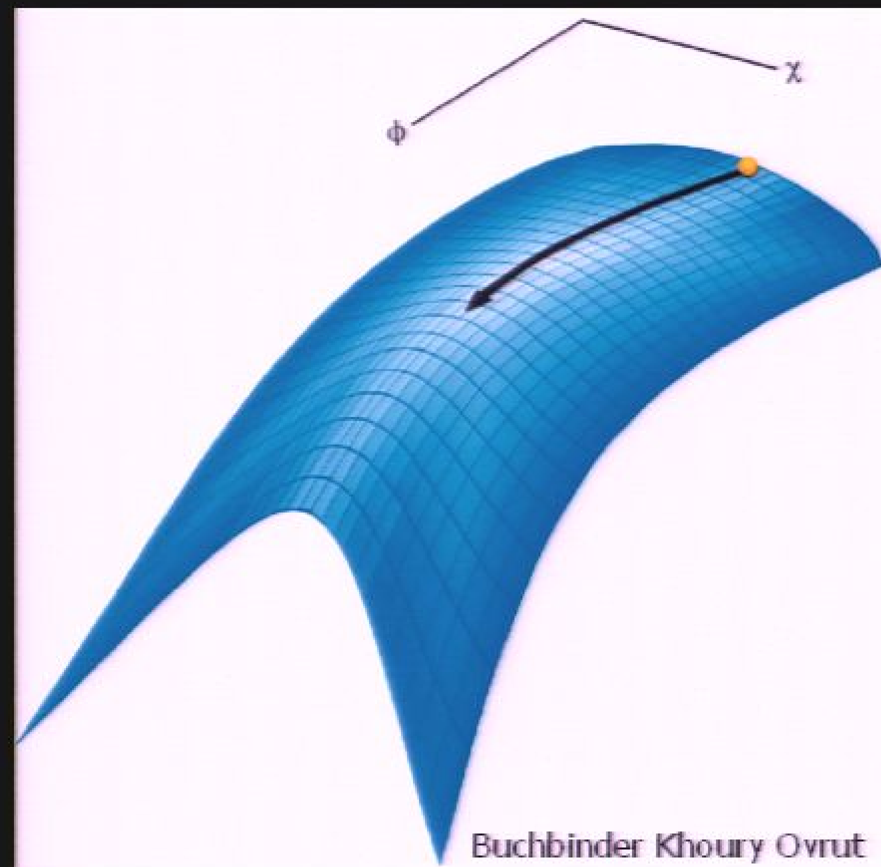

- Rotation in field space:

Koyama Wands '07

$$\phi = \frac{c_2\varphi_1 + c_1\varphi_2}{\sqrt{c_1^2 + c_2^2}}; \quad \chi = \frac{c_1\varphi_1 - c_2\varphi_2}{\sqrt{c_1^2 + c_2^2}}$$

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Scaling $\qquad$ φ_2 -dominated

Linear fluctuations

Koyama Mizuno Wands '07

- Linear orthogonal field evolution:

$$\delta\ddot{\chi}_{\vec{k}} + \frac{6}{tc^2}\delta\dot{\chi}_{\vec{k}} + \left(\frac{k^2}{t^{4/c^2}} - \frac{2\left(1-\frac{6}{c^2}\right)}{t^2}\right)\delta\chi_{\vec{k}} = 0$$

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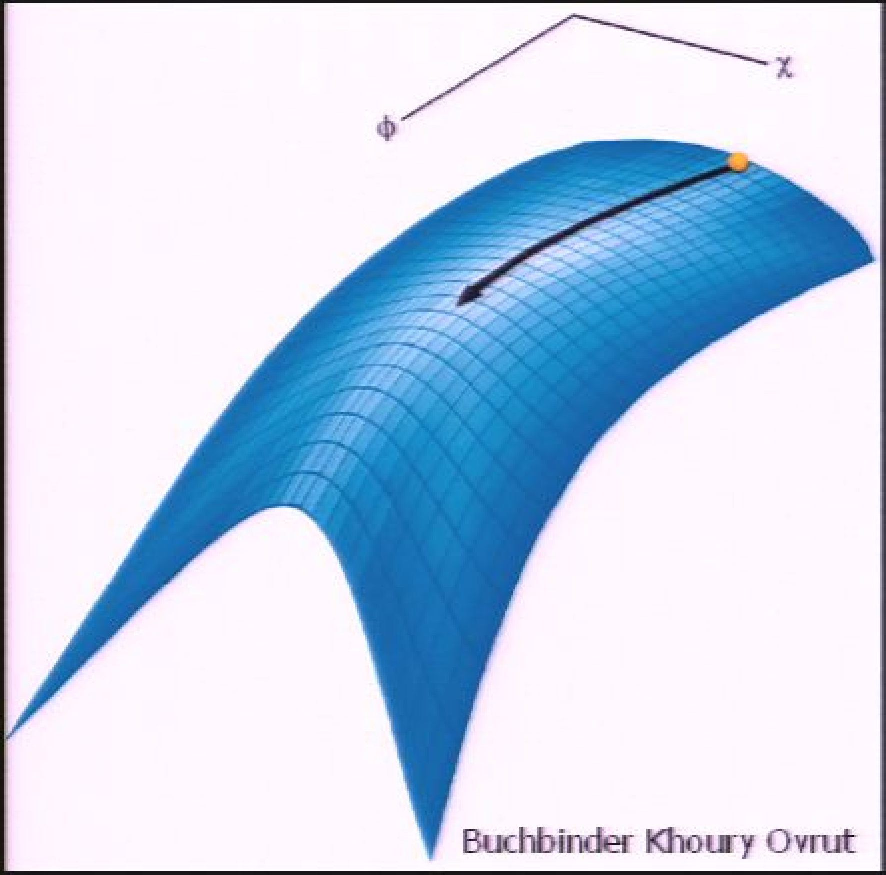

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Non-gaussianity

Koyama Mizuno FV Wands '07

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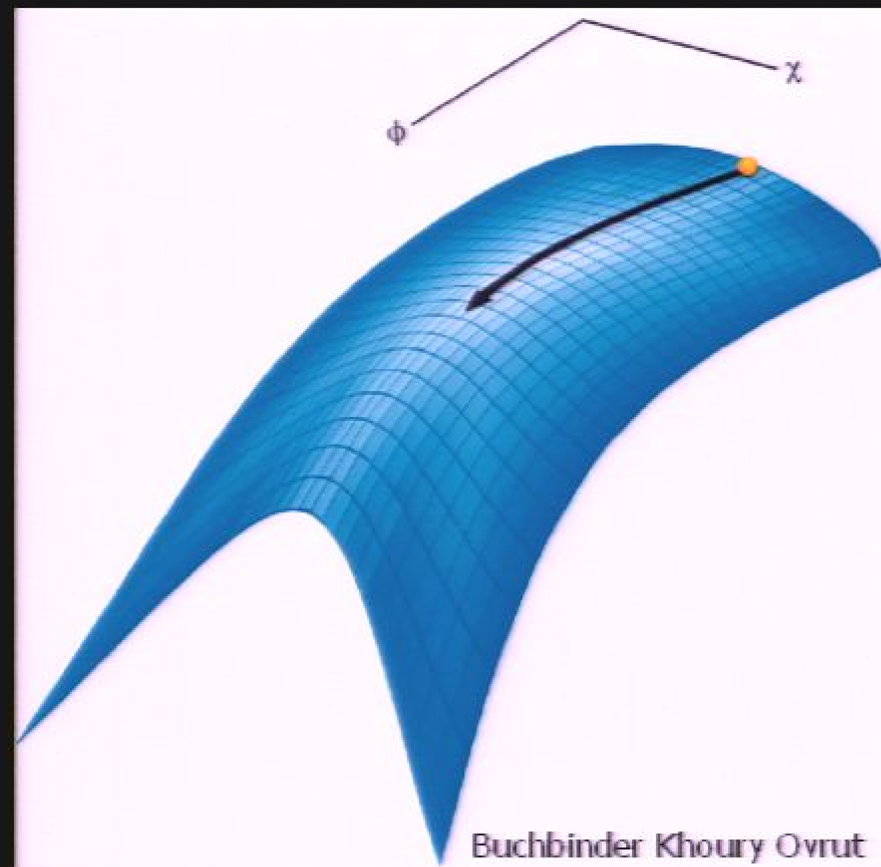

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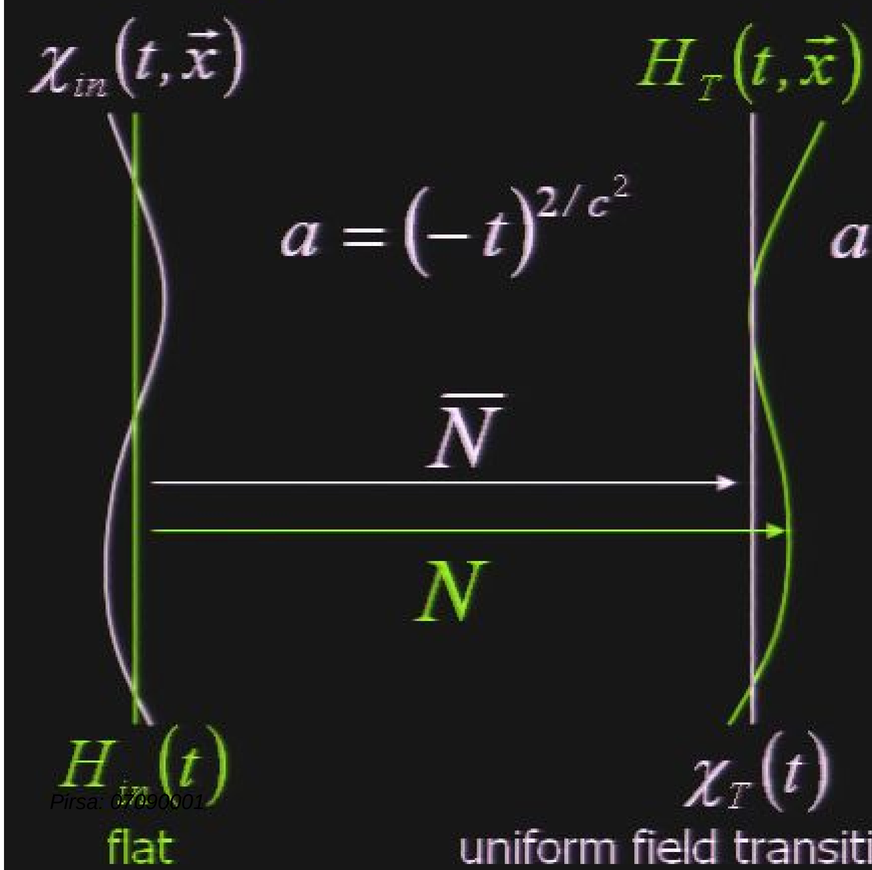

$$a = (-t)^{2/c^2} \qquad a = (-t)^{2/c_2^2} \qquad \Rightarrow N = \frac{2}{c_1^2}\ln|H_T| + \text{const}$$

back

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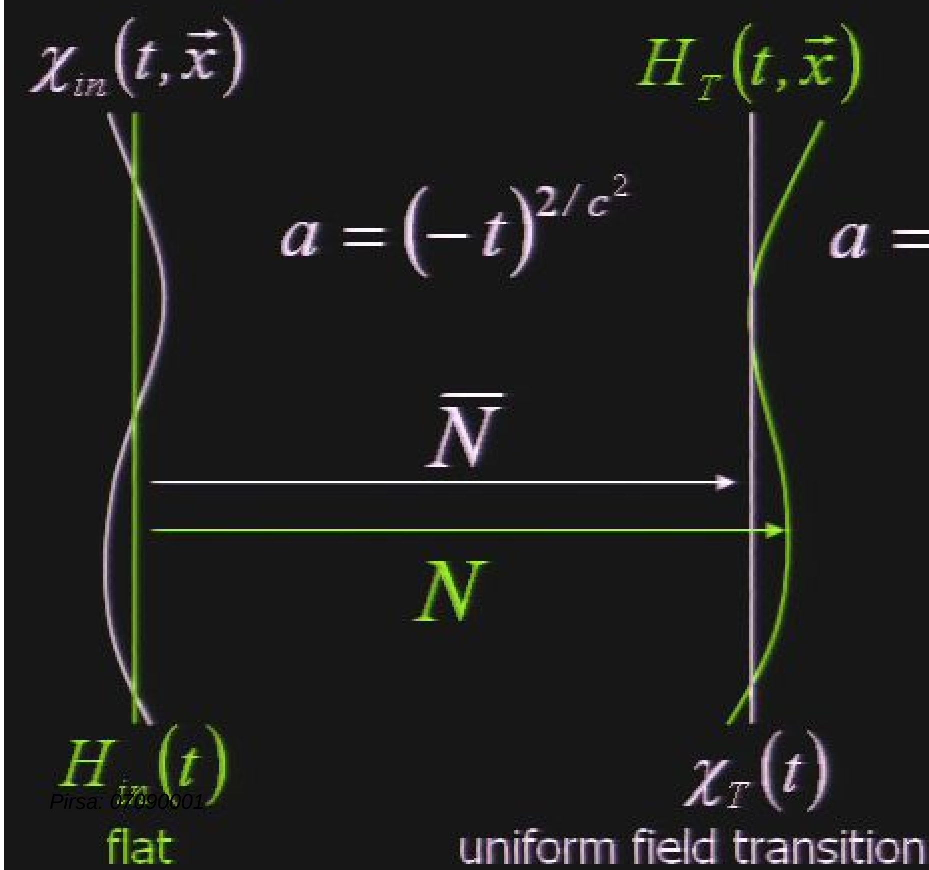

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$$f_{\rm NL} = \frac{5}{12}c_1^2$$

back

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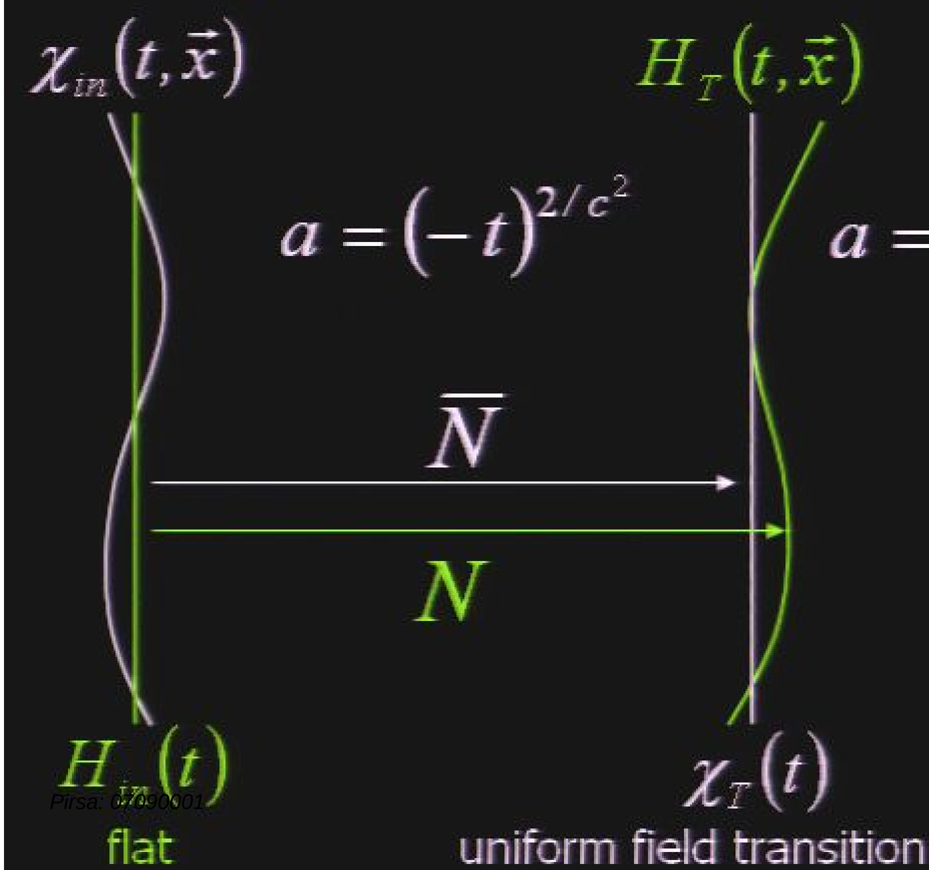

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back

Conclusion

Non-Gaussianity can be used as a discriminator among early universe models:

1) Slow-roll models: small NG (model independent signature)

2) No slow-roll models: NG can be larger than 1 and in the ballpark of current observations (equilateral configuration); however they are model dependent

3) Curvaton-like models: NG can be large (local configuration)

4) Ekpyrotic model: NG is large and local; tension with spectral index

Non-gaussianity from multi-field inflation

$$\zeta = N_{,I}\delta\phi_*^I + \frac{1}{2}N_{,IJ}\delta\phi_*^I\delta\phi_*^J$$

- General expression:

$$\frac{6}{5}f_{\rm NL} = \frac{1}{16}r + \frac{N_{,IJ}N_{,I}N_{,J}}{\left(N_{,I}^2\right)^2}$$

FV Wands '06

$r = 8\frac{P_T}{P_\zeta} = O(\varepsilon) << 1$: tensor to scalar ratio

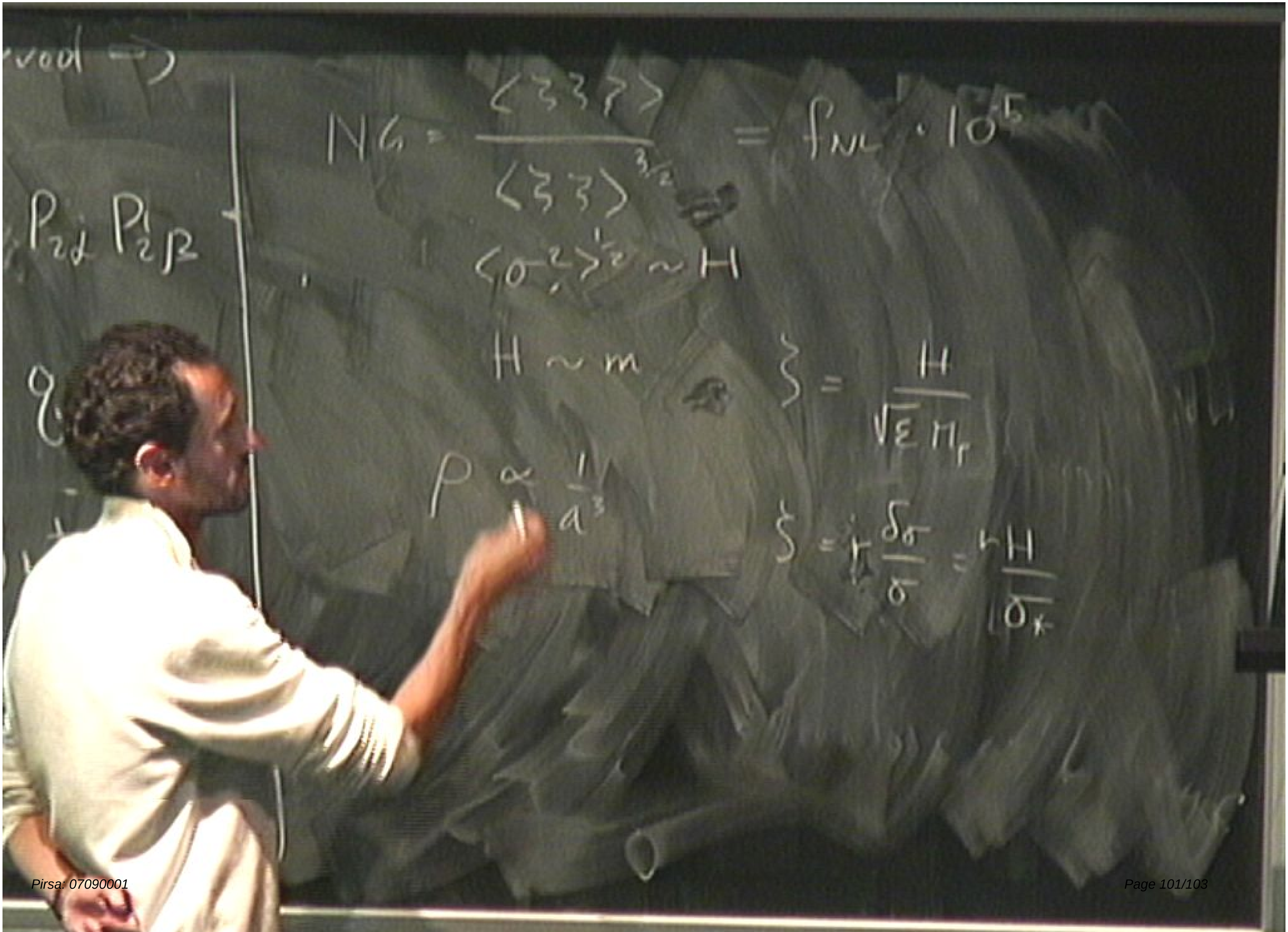
- Separable potential: $V(\phi_1,\phi_2) = V_1(\phi_1) + V_2(\phi_2)$

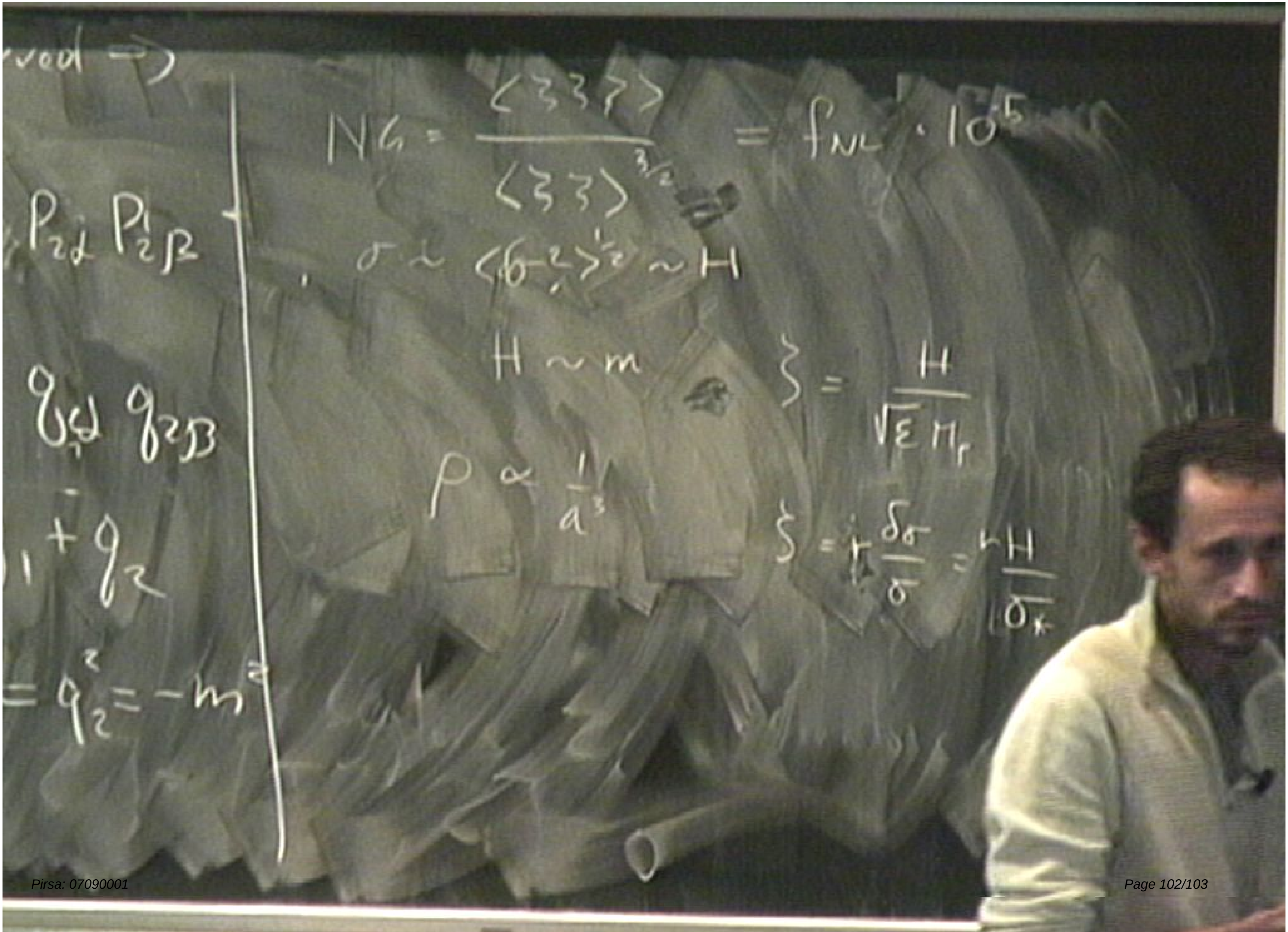
$$V(\phi_1,\phi_2) = \frac{1}{2}m_1^2\phi_1^2 + \frac{1}{2}m_2^2\phi_2^2$$

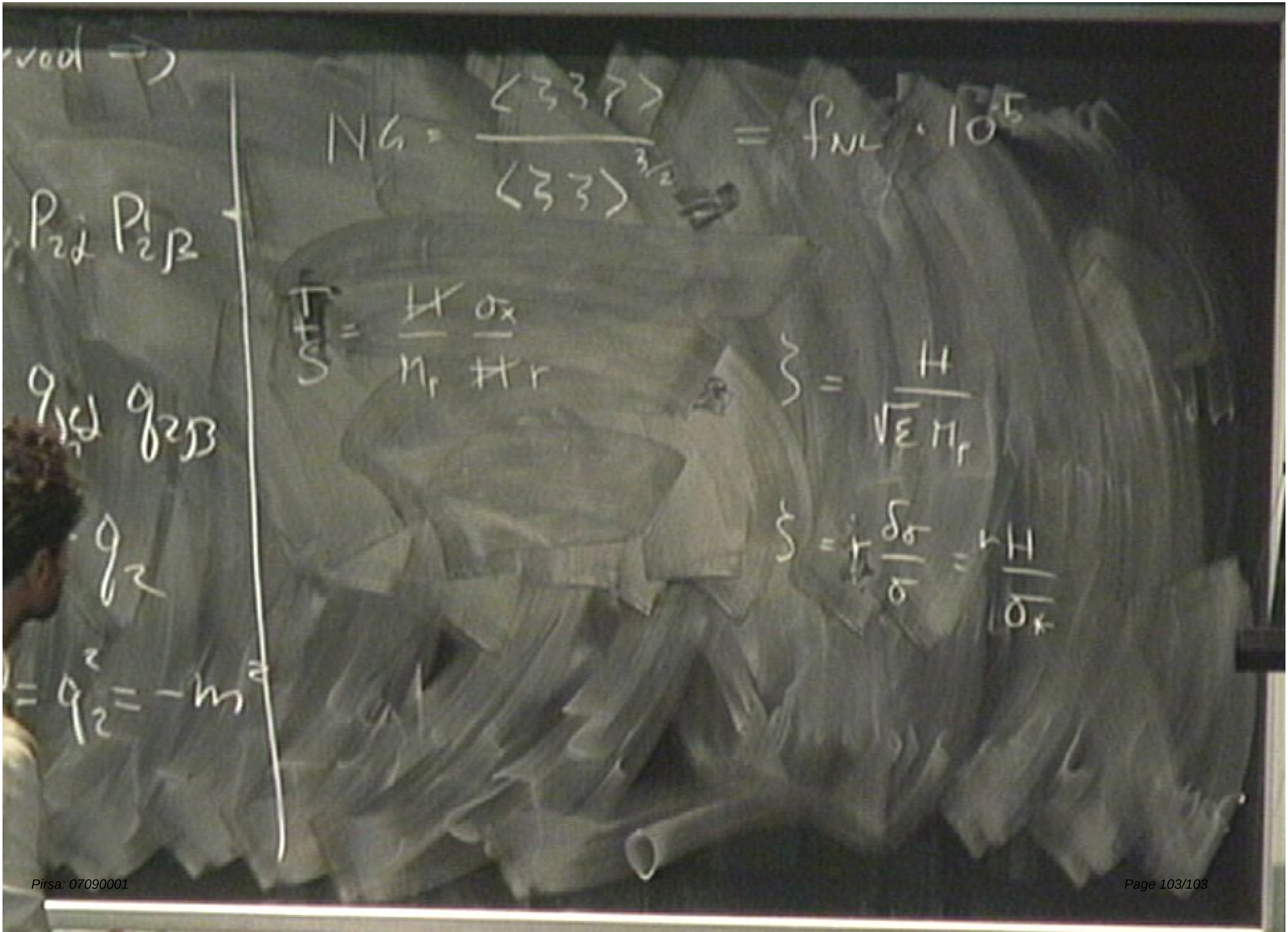
$$\frac{6}{5}f_{\rm NL} = \frac{1}{8}r \sim 0.01$$

Slow-roll inflation = small NG

back


$NG = \frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^{3/2}} = f_{NL} \cdot 10^{-5}$
$\langle \sigma^2 \rangle^{1/2} \sim H$
$H \sim m$
$\zeta = \frac{H}{\sqrt{\epsilon} M_P}$
$\rho \propto \frac{1}{a^3}$
$\zeta = r \frac{\delta\sigma}{\sigma} = r \frac{H}{\sigma_*}$
$P_{2\alpha} P_{2\beta}$


NG = ⟨ξξξ⟩ / ⟨ξξ⟩^{3/2} = f_NL · 10^-5
σ ~ ⟨δσ^2⟩^{1/2} ~ H
H ~ m
ξ = H / (√ε M_P)
ρ ∝ 1/a^3
ξ = δσ/σ = H/σ_*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