Title: Interaction and information flow between quantum systems (Part 1B)

Date: Aug 30, 2007 04:30 PM

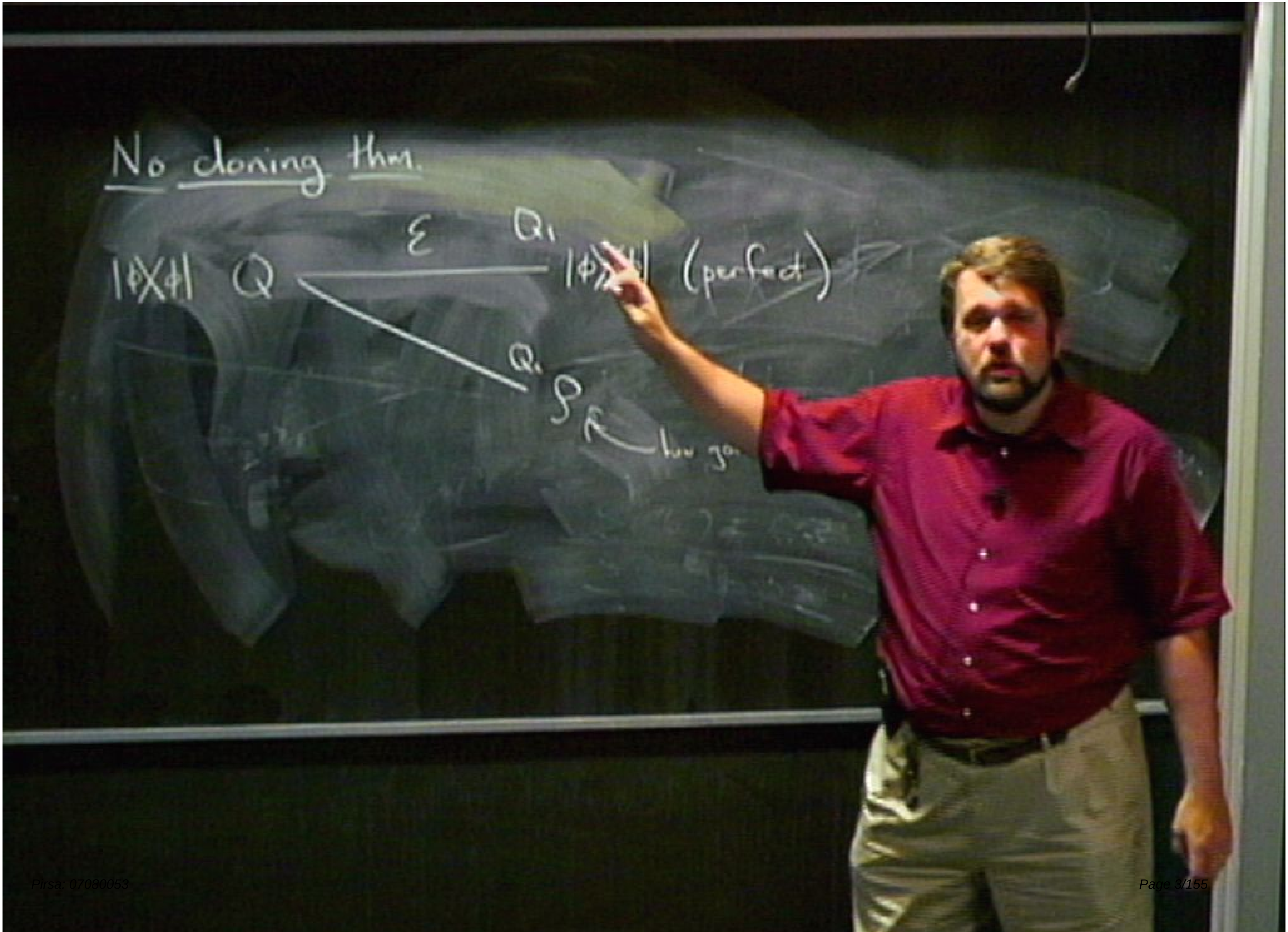
URL: http://pirsa.org/07080053

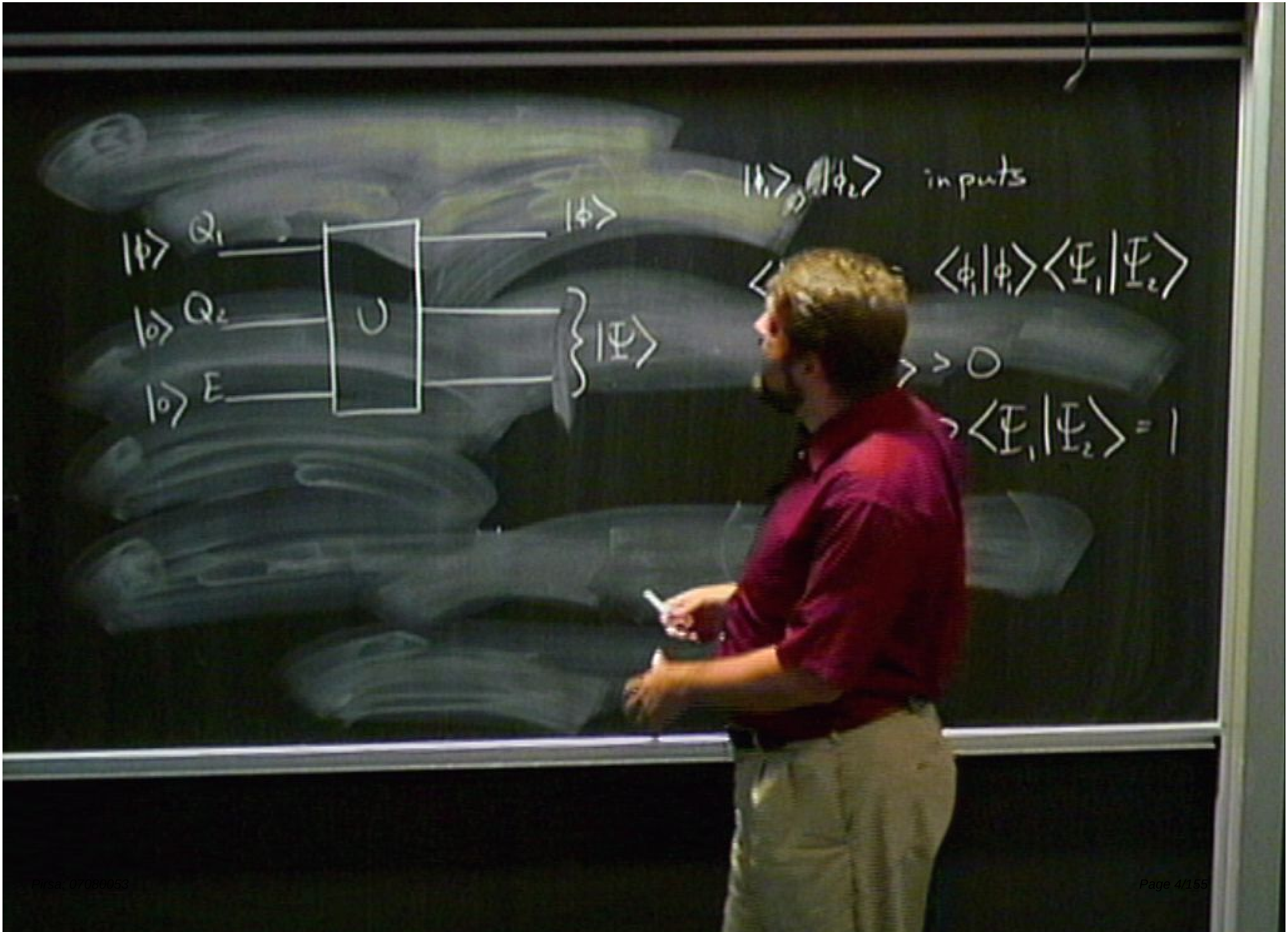
Abstract:

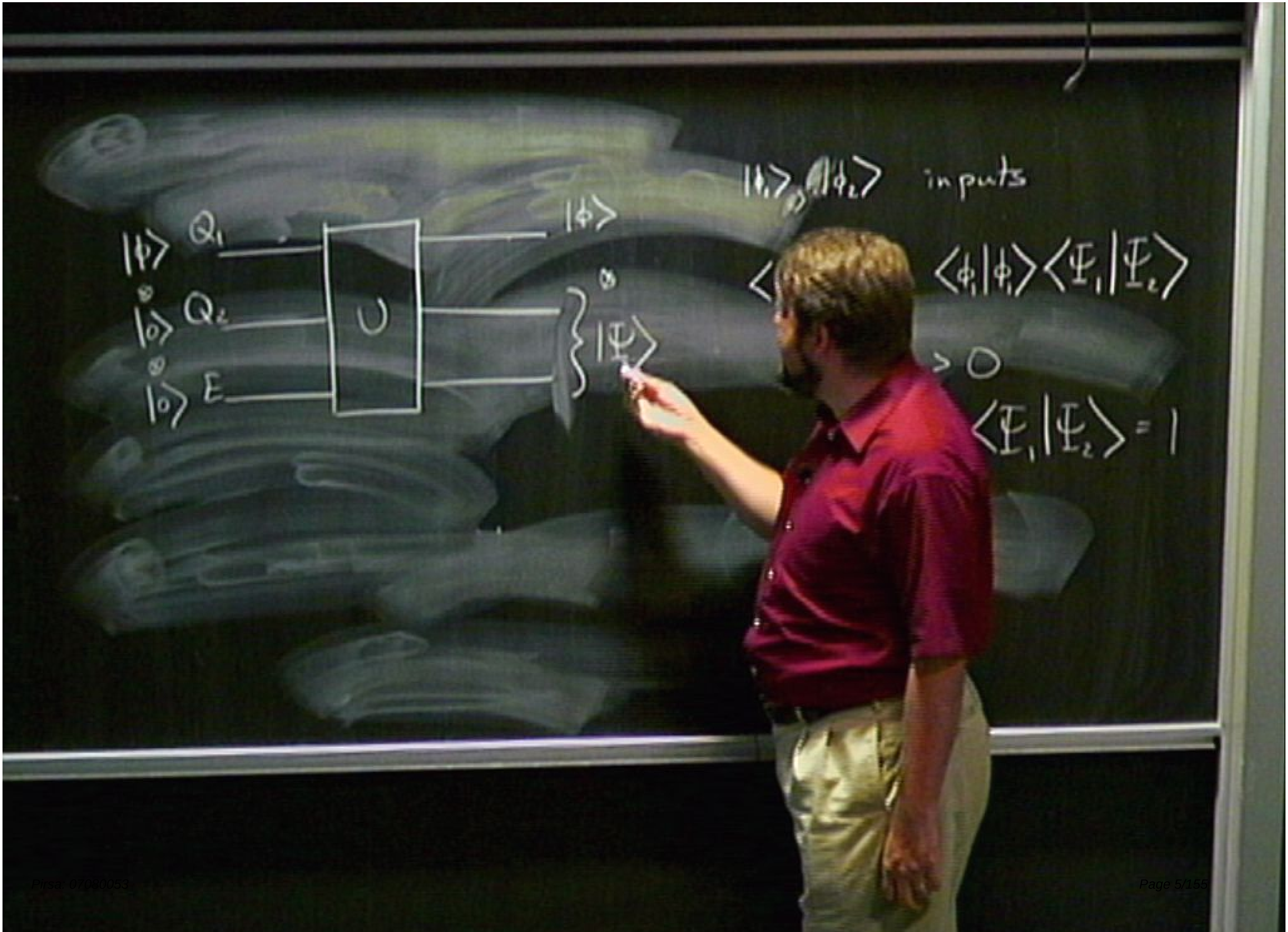
No cloning thm.

$|\phi\rangle\langle\phi| \quad Q \longrightarrow |\phi\rangle\langle\phi| \ \text{(perfect)}$

$\qquad\qquad\quad \searrow \rho$ ← how good?


No cloning thm.
ε
Q1
|φ⟩⟨φ|
Q
|φ⟩⟨φ| (perfect)
Q2
ρ


|ϕ⟩ Q1
|0⟩ Q2
|0⟩ E
U
|ϕ⟩
|Ψ⟩
|ϕ1⟩, |ϕ2⟩ inputs
⟨ϕ1|ϕ1⟩⟨Ψ1|Ψ2⟩
> 0
⟨Ψ1|Ψ2⟩ = 1


|φ⟩ Q1
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⟨φ1|φ1⟩⟨Ψ1|Ψ2⟩
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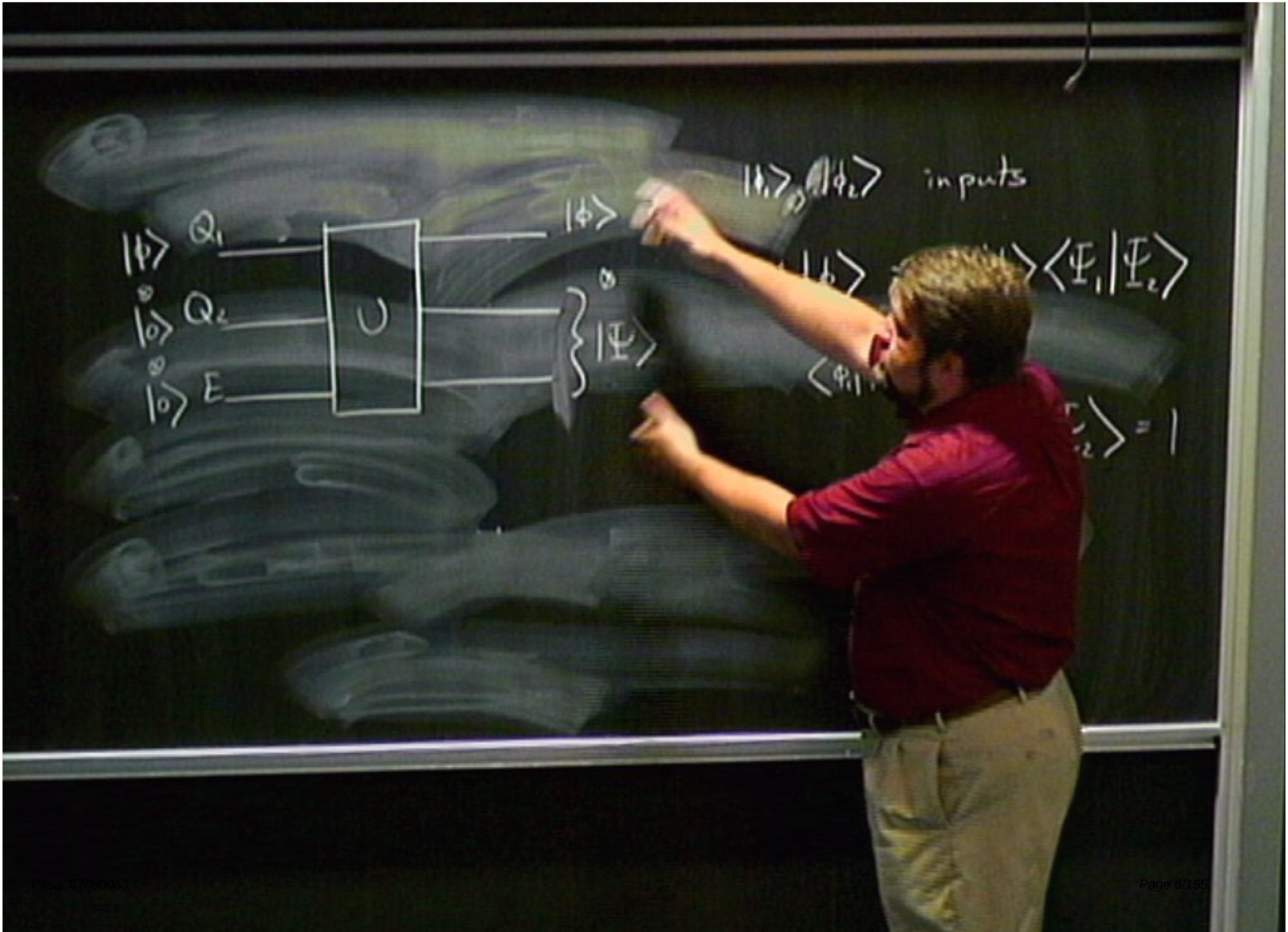
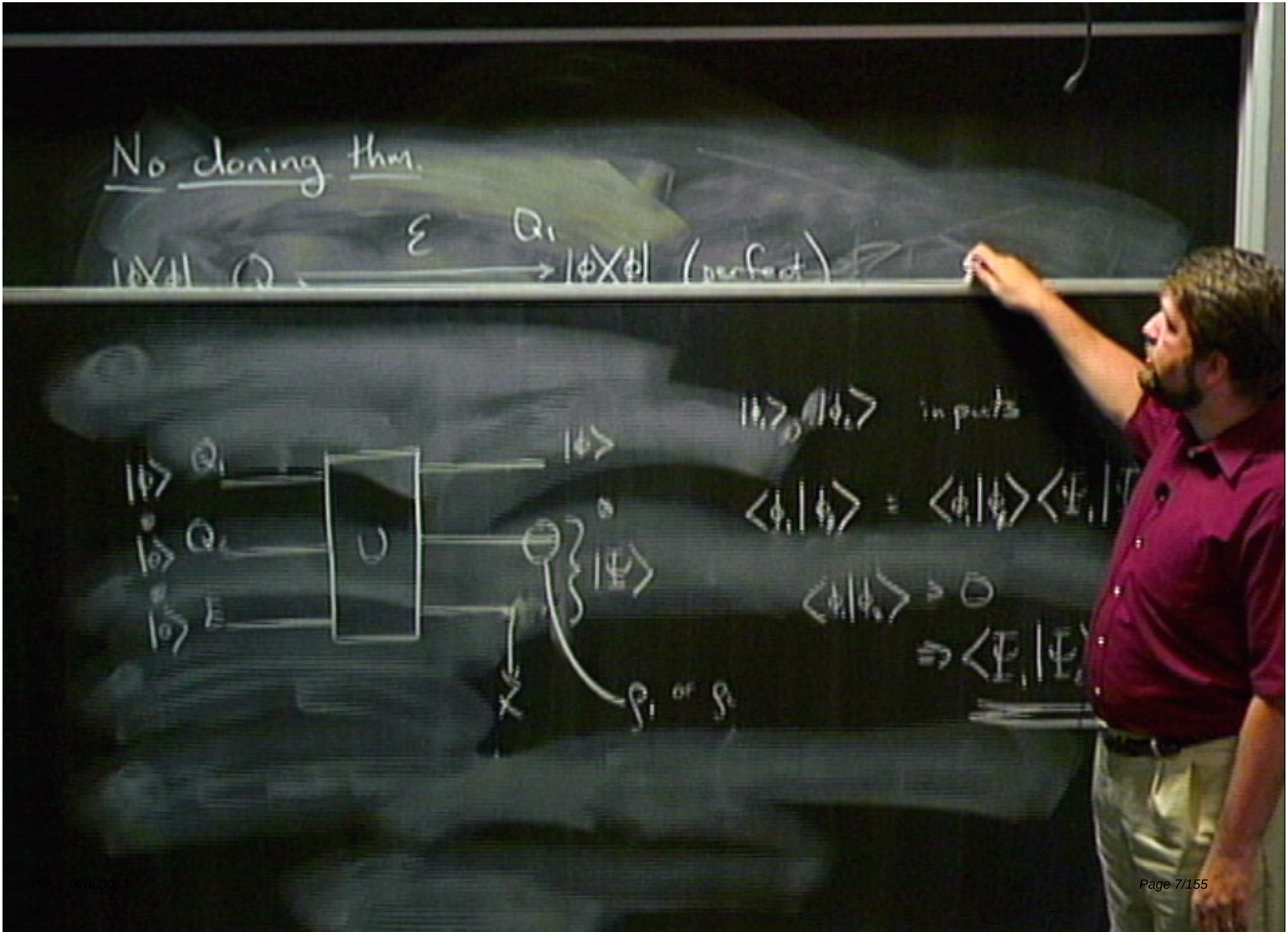
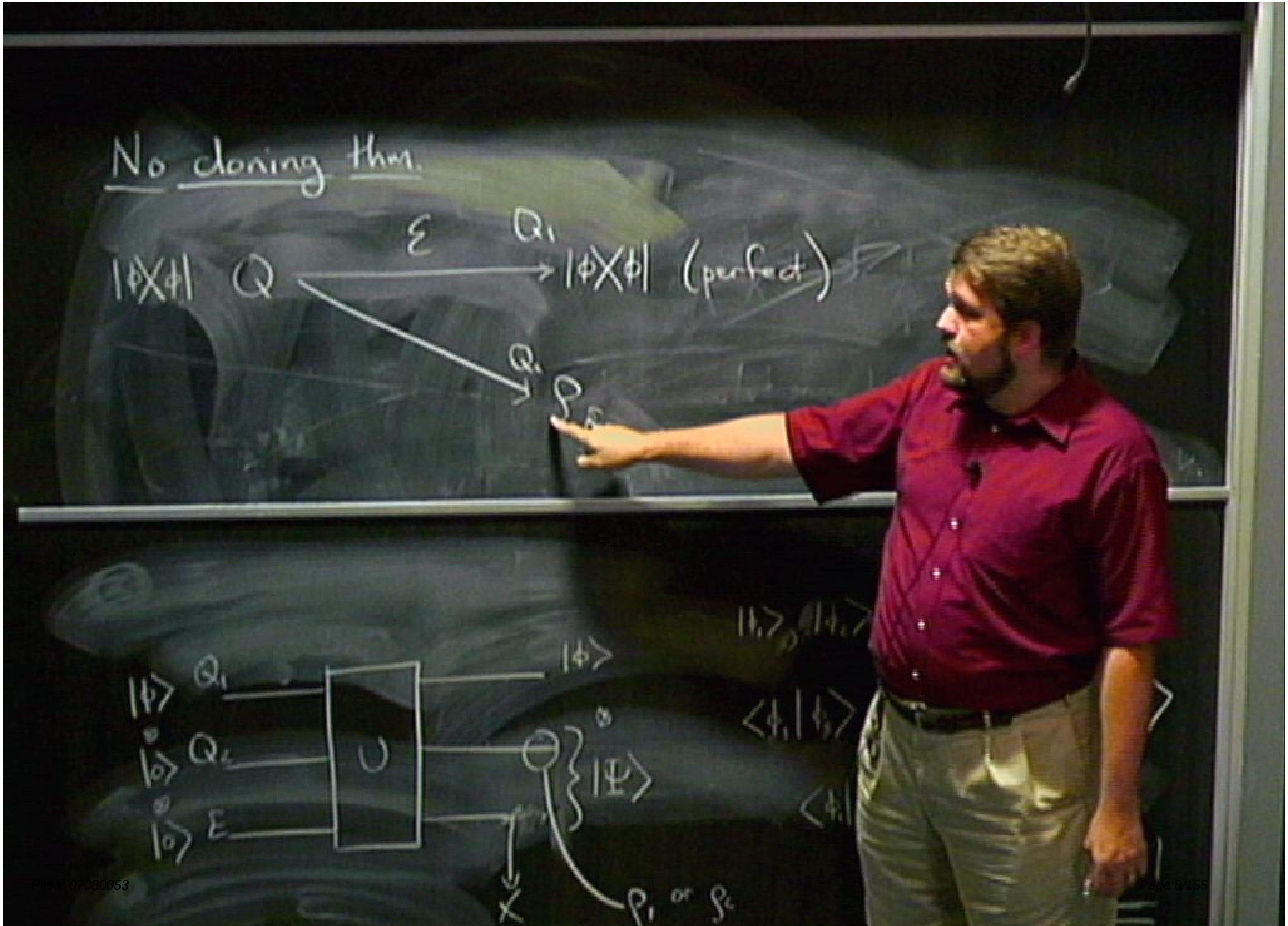
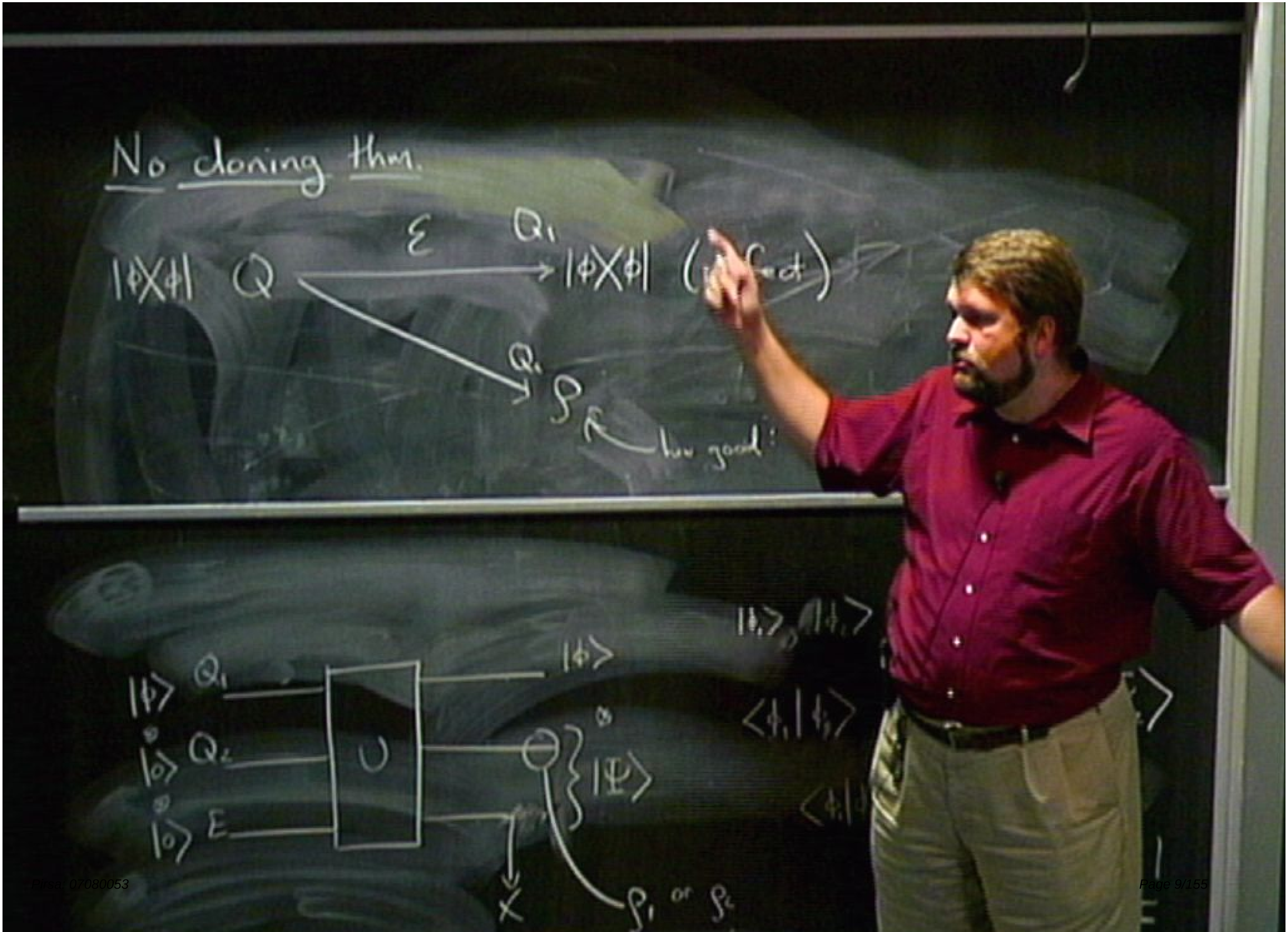

inputs
Q1
Q2
E
U


No cloning thm.
(perfect)
inputs
U
ρ₁ or ρ₂



No cloning thm.
ε
Q1
|φ⟩⟨φ| Q
|φ⟩⟨φ| (perfect)
ρ
|φ⟩ Q1
|0⟩ Q2
|0⟩ E
U
|φ⟩
|Ψ⟩
ρ1 or ρ2

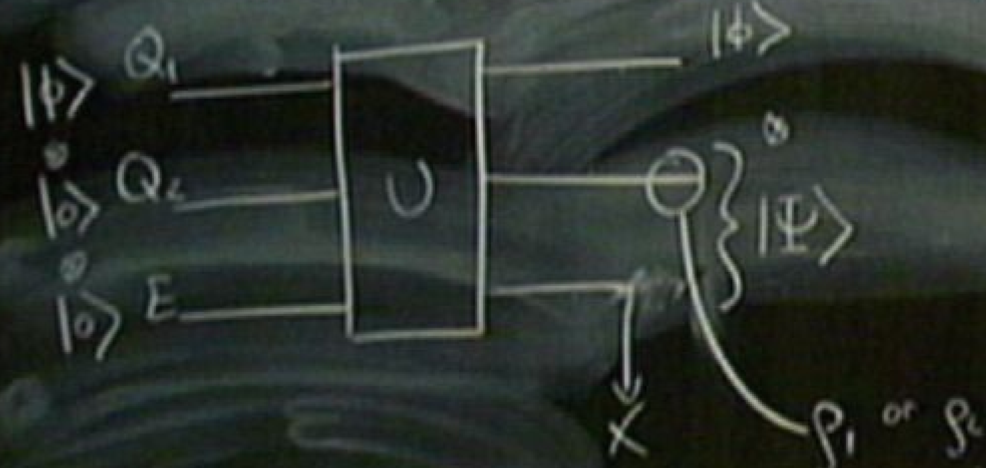

No cloning thm.
how good?

No cloning thm.

$$|\phi\rangle\langle\phi| \quad Q \xrightarrow{\ \mathcal{E}\ } \overset{Q_1}{|\phi\rangle\langle\phi|} \ \text{(perfect)}$$

$$\searrow \overset{Q_2}{\rho}$$

how good?



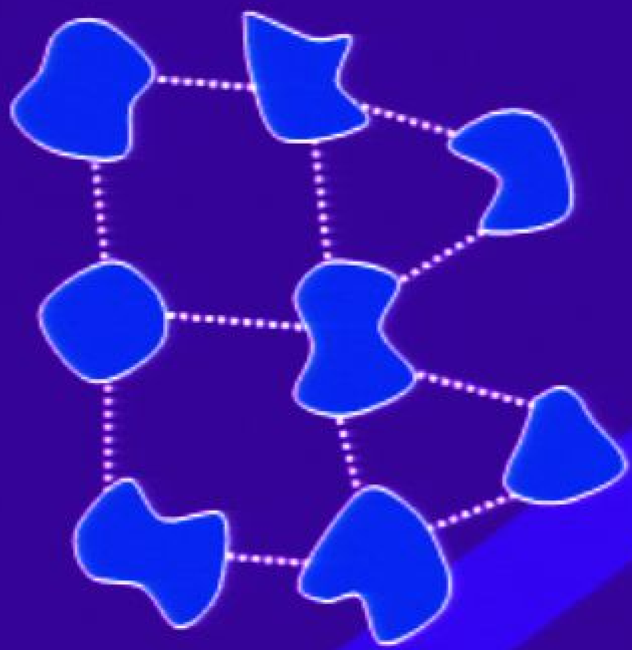
$|\phi_1\rangle, |\phi_2\rangle$ inputs

$$\langle\phi_1|\phi_2\rangle = \langle\phi_1|\phi_2\rangle\langle\Psi_1|\Psi_2\rangle$$

$$\langle\phi_1|\phi_2\rangle > 0$$

$$\Rightarrow \langle\Psi_1|\Psi_2\rangle = 1$$

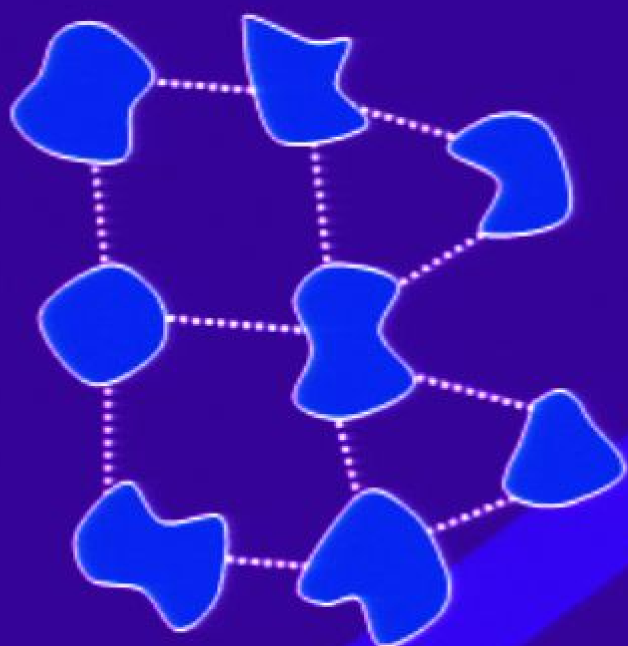
Universe as information network



Universe is divided into subsystems.

Subsystems interact and exchange information.

Universe as information network



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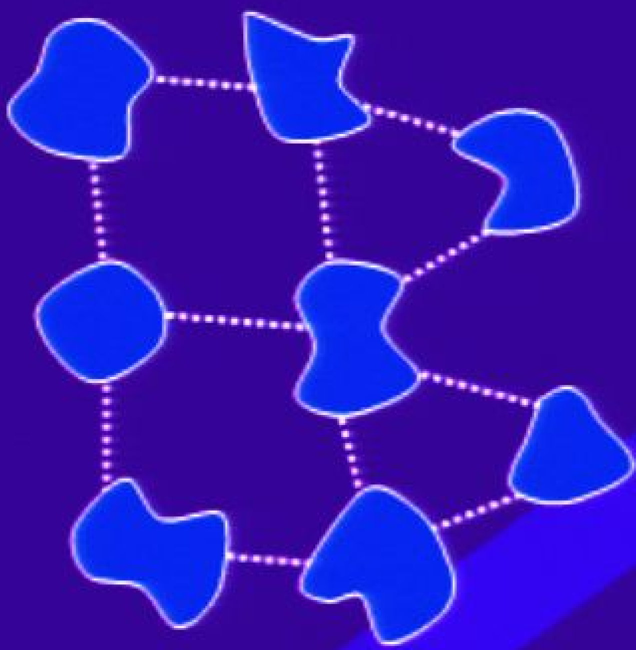
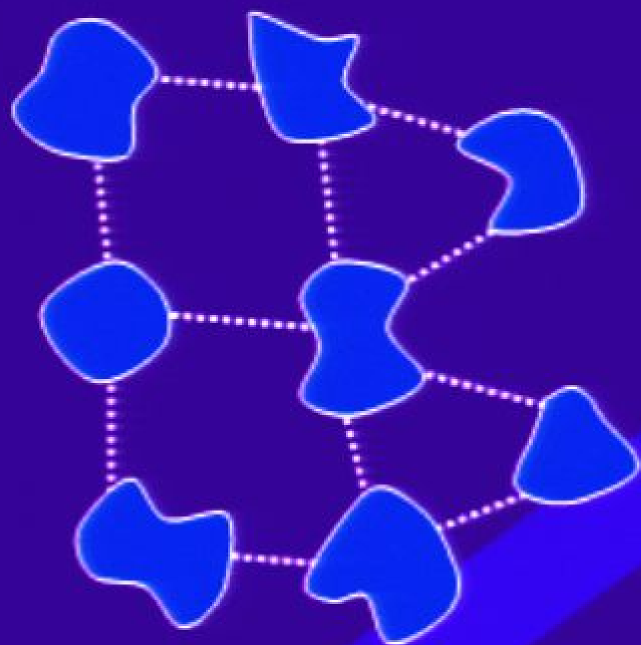


Universe is divided into subsystems.

Subsystems interact and exchange information.

Locality: Not all subsystems exchange information directly.

Universe as information network



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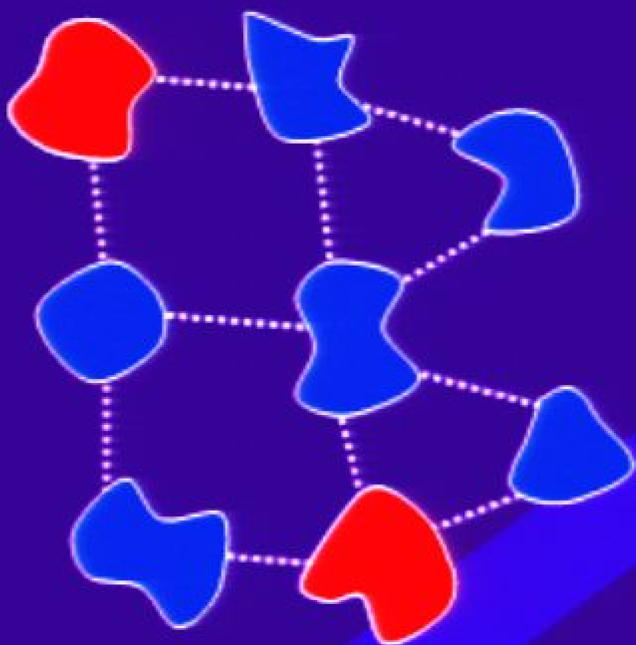

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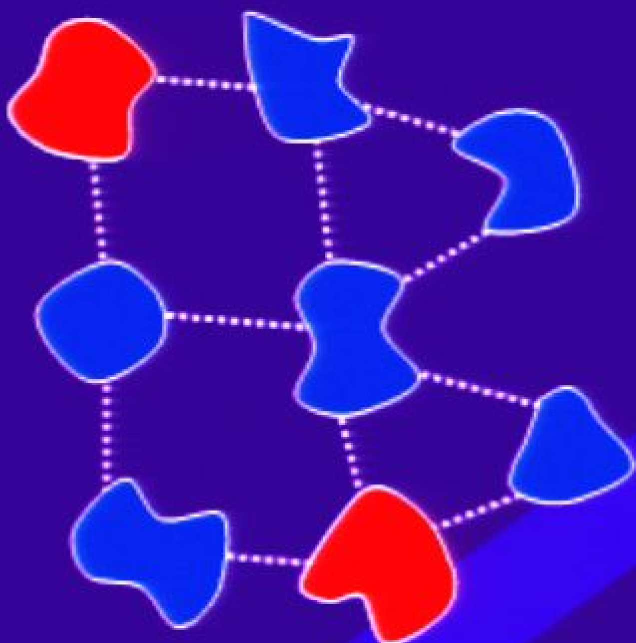

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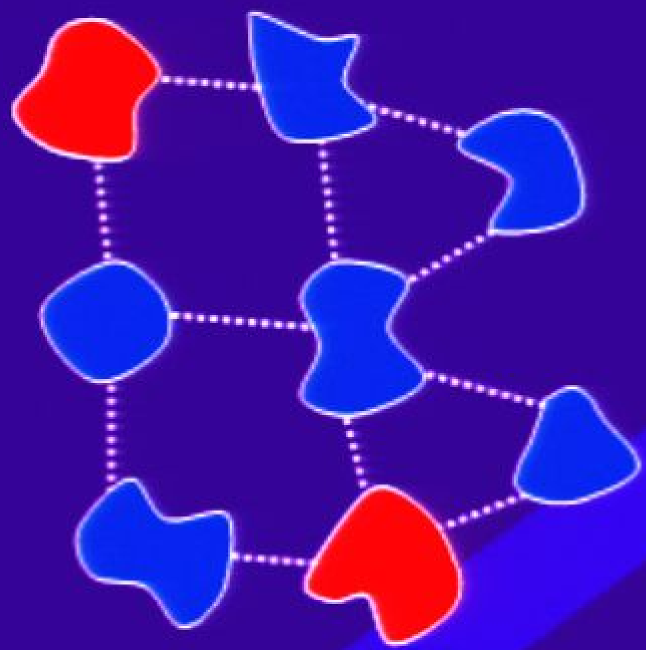
Subsystems interact and exchange information.

Locality: Not all subsystems exchange information directly.

What does quantum mechanics say about the rules of this web?

Universe as information network



Universe is divided into subsystems.

Subsystems interact and exchange information.

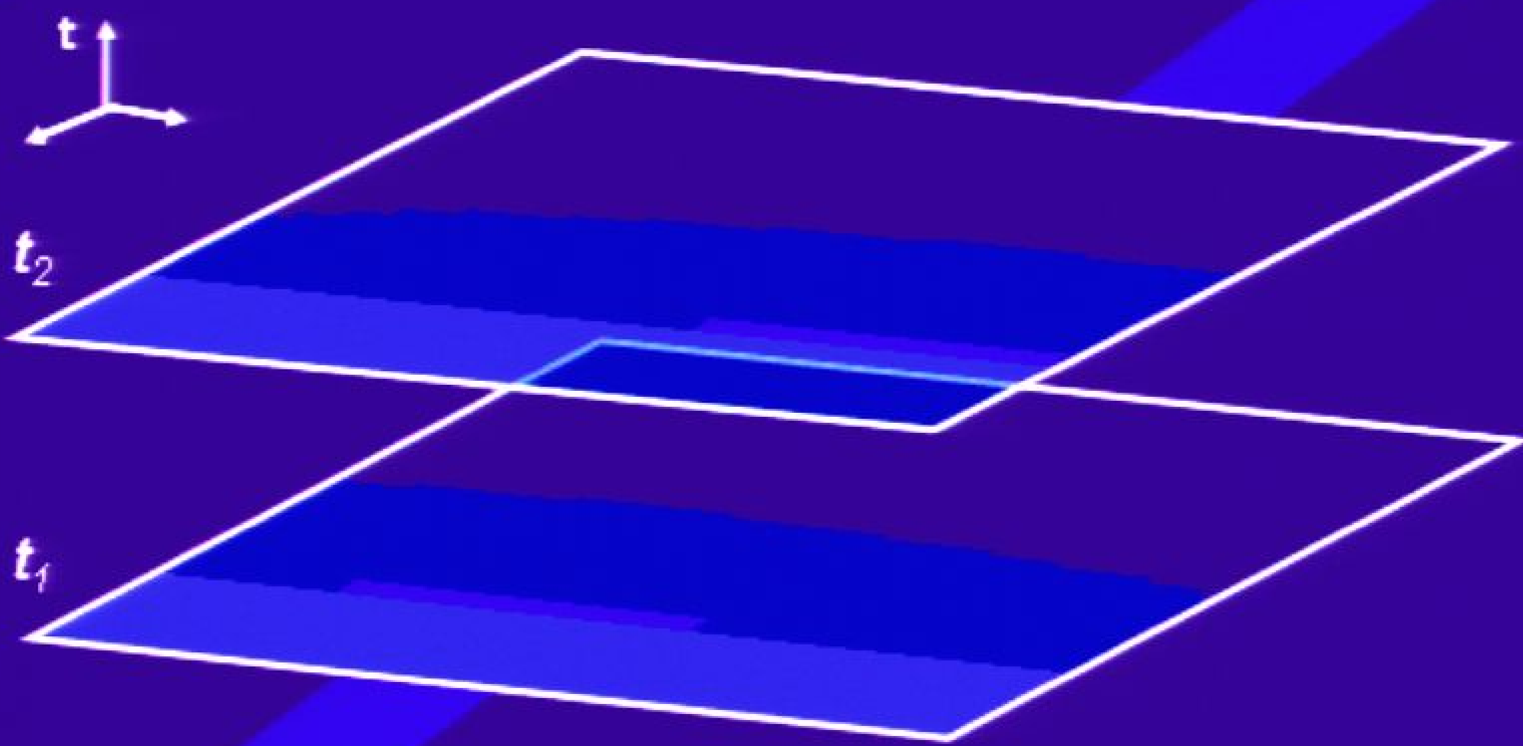
Locality: Not all subsystems exchange information directly.

What does quantum mechanics say about the rules of this web?

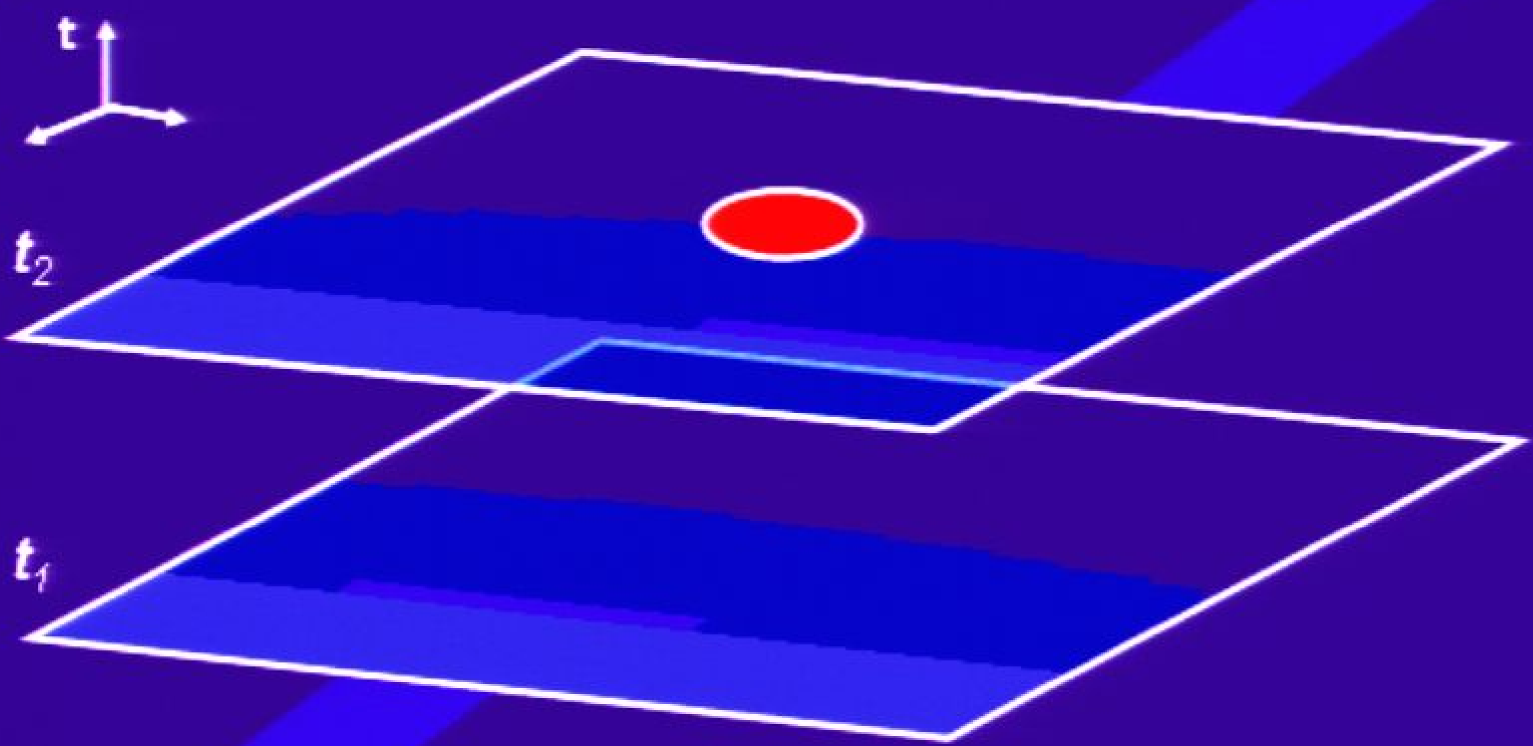
What does quantum mechanics say about locality?

Causal structure

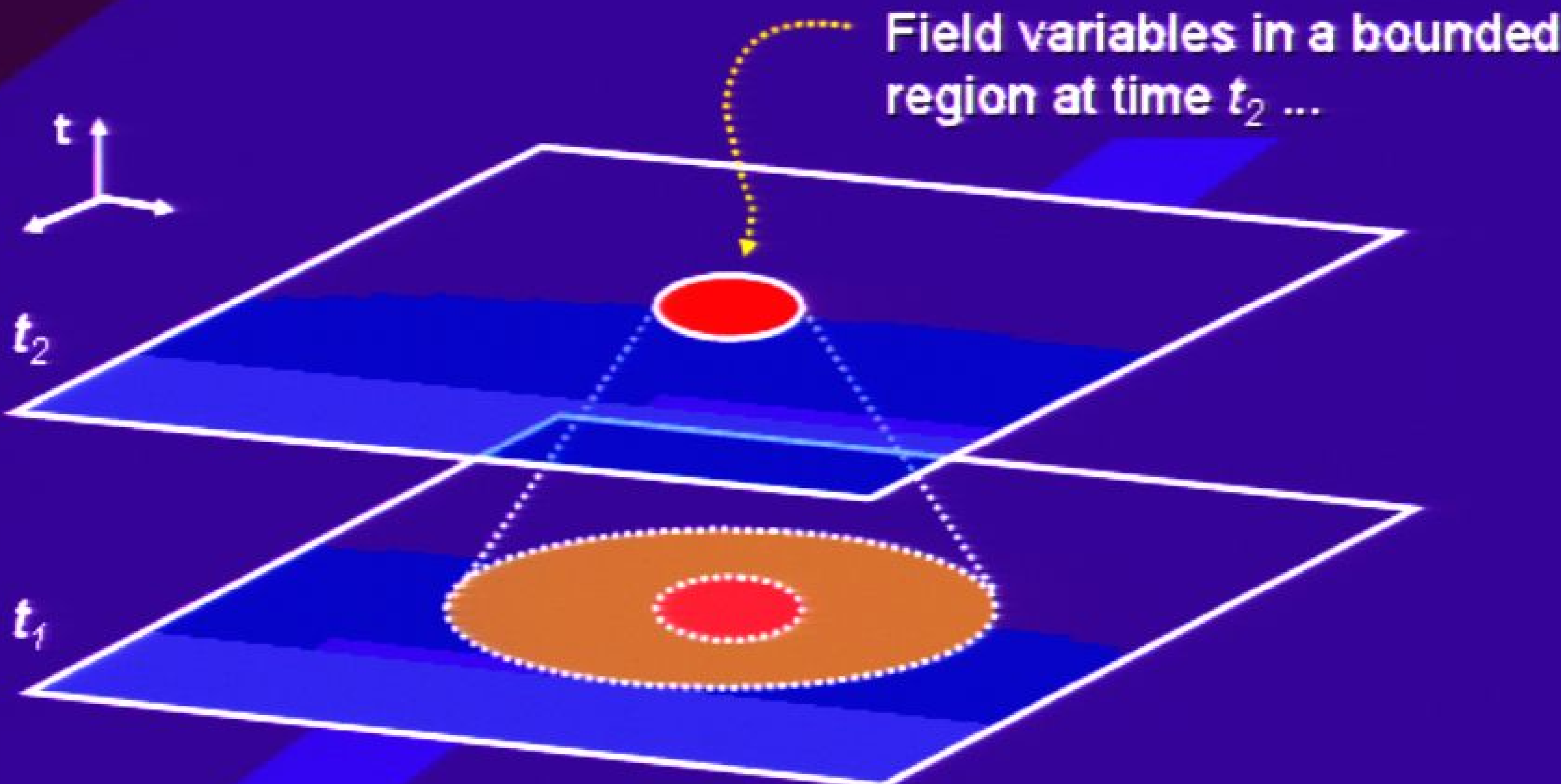
Causal structure



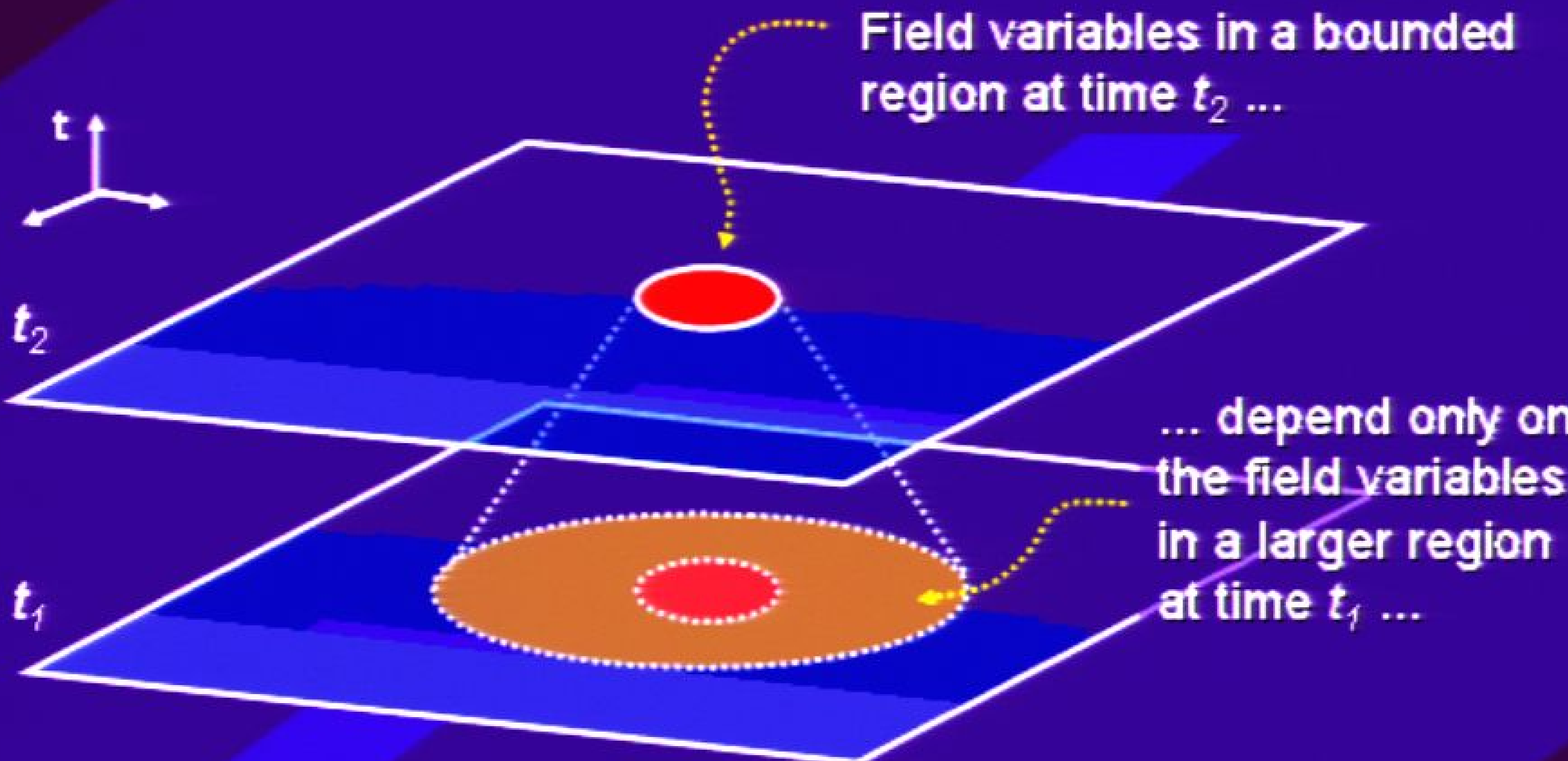
Causal structure



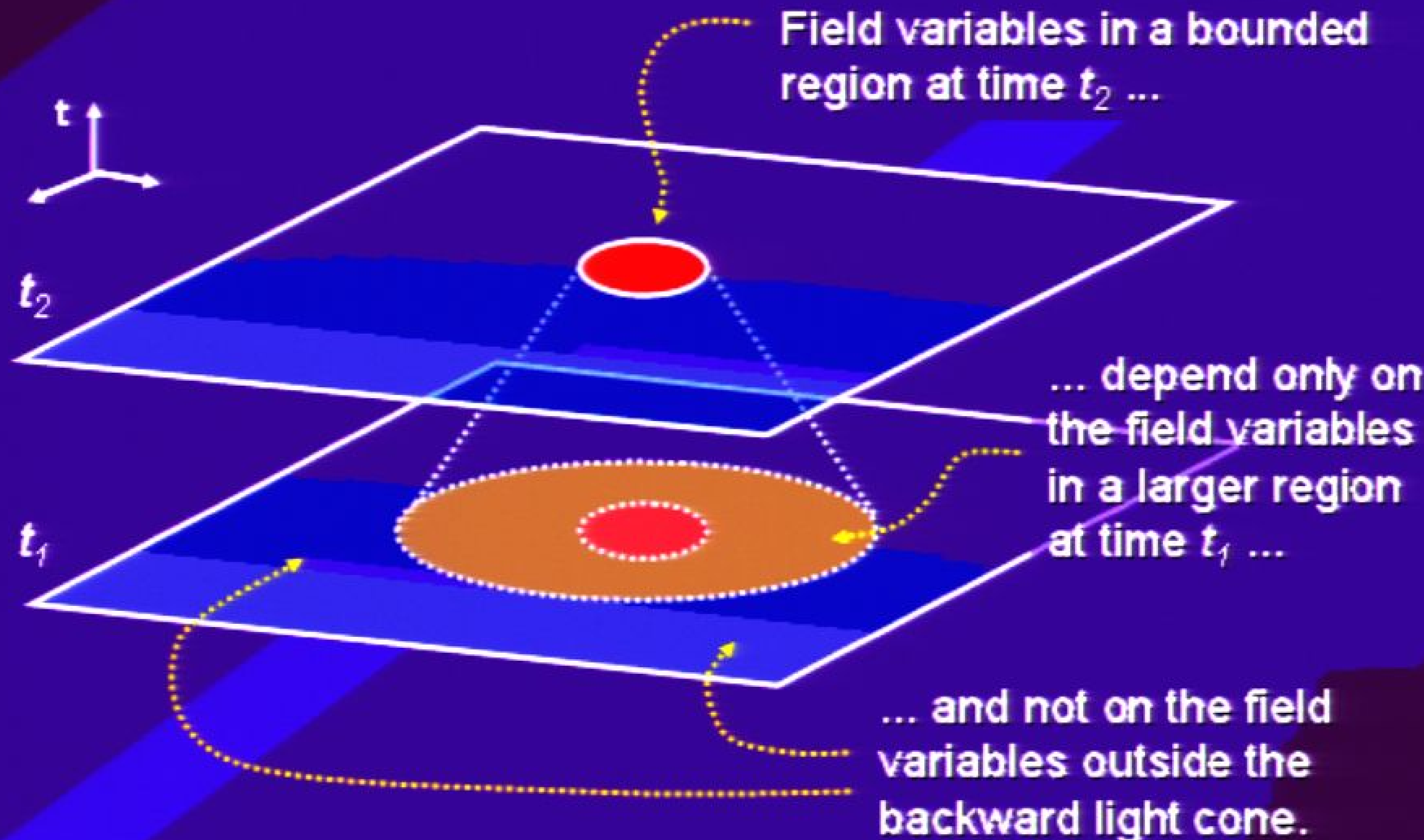
Causal structure



Causal structure



Causal structure



Bulls-eye and chain

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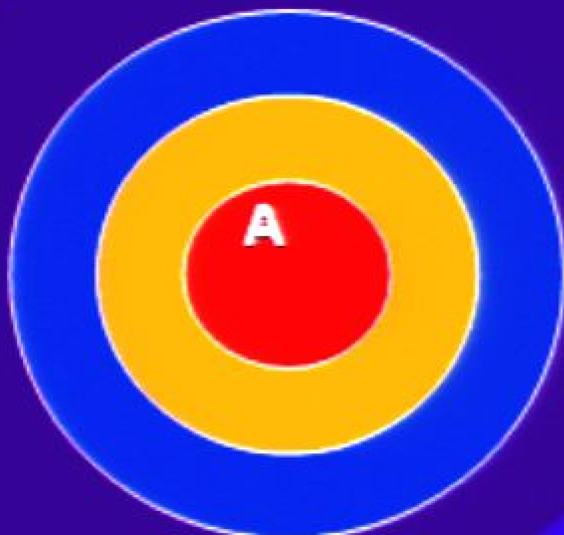
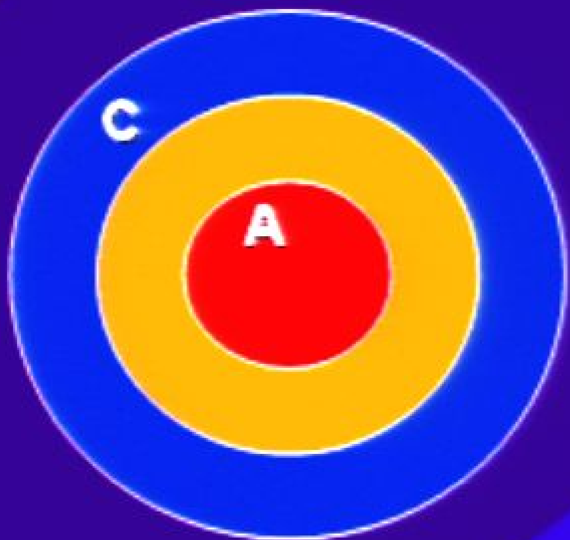


A is the system of interest.

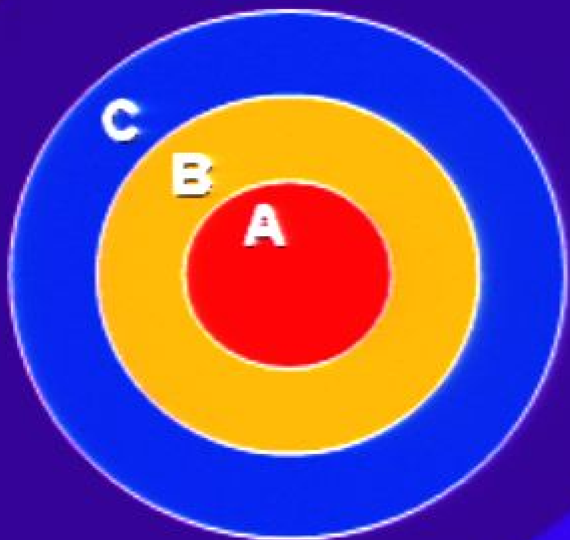
Bulls-eye and chain



A is the system of interest.

C is the distant "rest of the world"

Bulls-eye and chain

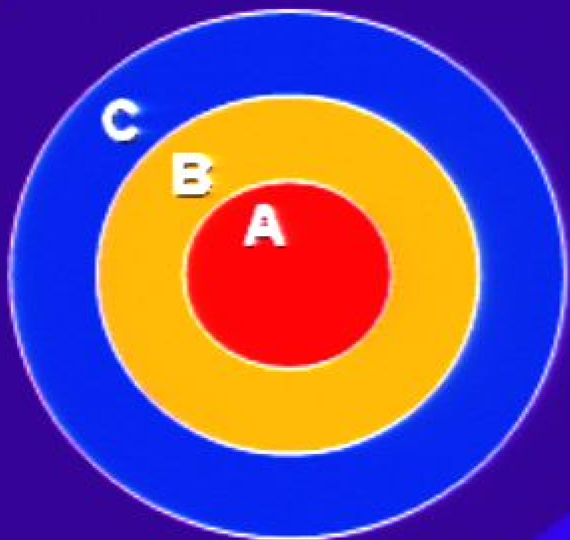


A is the system of interest.

C is the distant “rest of the world”

B is the rest of A’s “neighborhood”

Bulls-eye and chain



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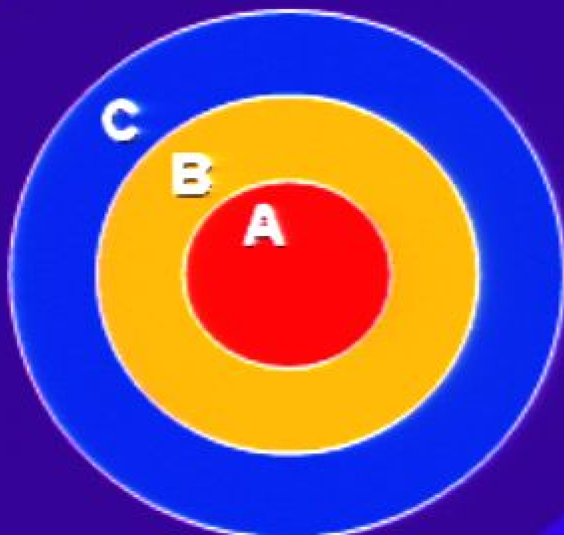


A is the system of interest.

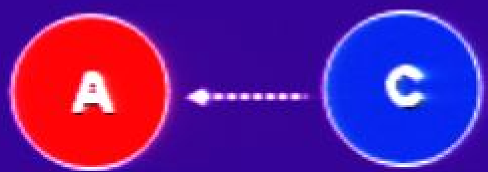
C is the distant "rest of the world"

B is the rest of A's "neighborhood"

Locality: In one time step, there is no information transfer from C to A.

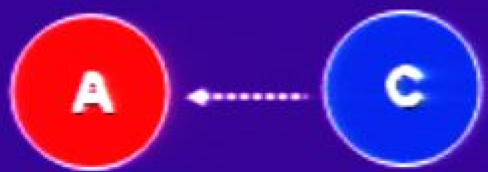
Information flow

Information flow

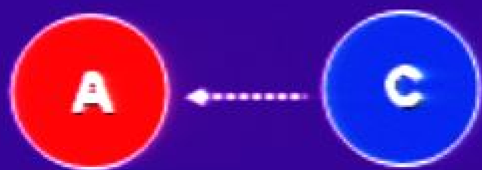


Information flow

When does information “flow” from C to A?



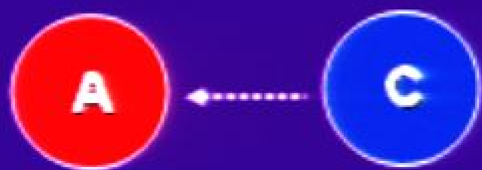
Information flow



When does information "flow" from C to A?

- Information flows from C to A if the final state of A depends on the initial state of C.

Information flow

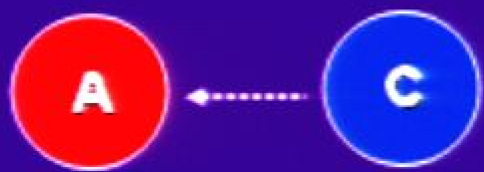


When does information "flow" from C to A?

- Information flows from C to A if the final state of A depends on the initial state of C.
- Information does not flow from C to A if the final state of A does not depend on the initial state of C.

Information flow

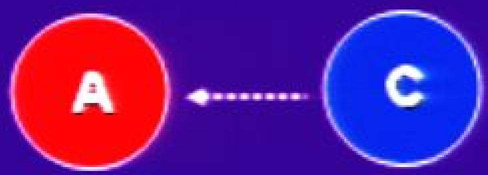
When does information "flow" from C to A?



- Information flows from C to A if the final state of A depends on the initial state of C.
- Information does not flow from C to A if the final state of A does not depend on the initial state of C.

Note: We must consider all possible initial states of A and C.

Information flow



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Quantum difficulties!

When does information "flow" from C to A?

- **Information flows from C to A if the final state of A depends on the initial state of C.**
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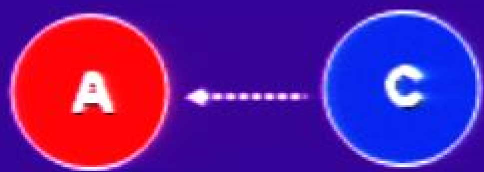
Information flow



Note: We must consider all possible initial states of A and C.

When does information "flow" from C to A?

- **Information flows from C to A if the final state of A depends on the initial state of C.**
- **Information does not flow from C to A if the final state of A does not depend on the initial state of C.**

Quantum difficulties!

Initial state of AC is not determined by the initial states of A and C separately – quantum entanglement.

Two bits (classical)

Two bits (classical)

Two classical bits.

Interaction: Controlled-NOT

C = control bit

T = target bit

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CT	→	CT
0 0	→	0 0
0 1	→	0 1
1 0	→	1 1
1 1	→	1 0

Two bits (classical)

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Note: CNOT operation is reversible

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Final T state does depend on initial C state. There is information flow from C to T.

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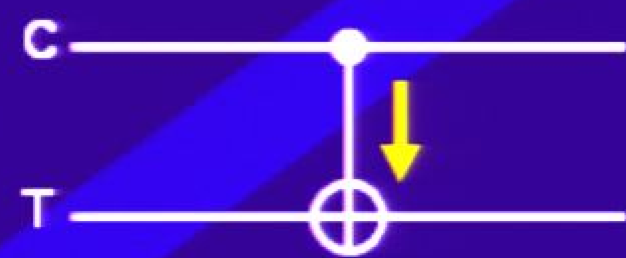

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CT	→	CT
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Final T state does depend on initial C state. There is information flow from C to T.

Final C state does not depend on initial T state. There is no information flow from T to C.

Classical CNOT has one-way information flow from C to T.

Two qubits

Two qubits

Two qubits.

Interaction: quantum CNOT

C = control bit

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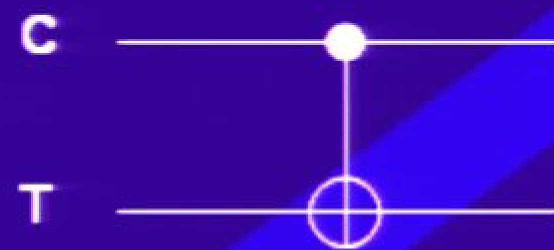

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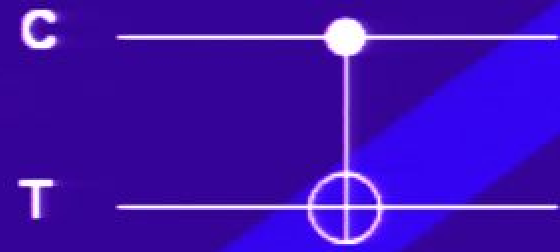

$\vert CT\rangle$	$\rightarrow$	$\vert CT\rangle$
$\vert 0\ 0\rangle$	$\rightarrow$	$\vert 0\ 0\rangle$
$\vert 0\ 1\rangle$	$\rightarrow$	$\vert 0\ 1\rangle$
$\vert 1\ 0\rangle$	$\rightarrow$	$\vert 1\ 1\rangle$
$\vert 1\ 1\rangle$	$\rightarrow$	$\vert 1\ 0\rangle$

Two qubits

Two qubits.

Interaction: quantum CNOT

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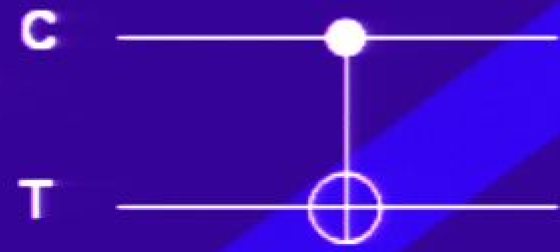

$\lvert CT\rangle$	$\rightarrow$	$\lvert CT\rangle$
$\lvert 0\,0\rangle$	$\rightarrow$	$\lvert 0\,0\rangle$
$\lvert 0\,1\rangle$	$\rightarrow$	$\lvert 0\,1\rangle$
$\lvert 1\,0\rangle$	$\rightarrow$	$\lvert 1\,1\rangle$
$\lvert 1\,1\rangle$	$\rightarrow$	$\lvert 1\,0\rangle$

CNOT is unitary

Two qubits

Two qubits.

Interaction: quantum CNOT

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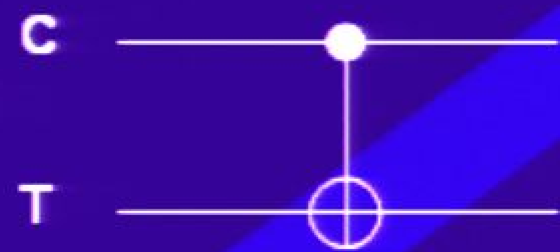

$$|CT\rangle \rightarrow |CT\rangle$$
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One-way information flow? **No!**

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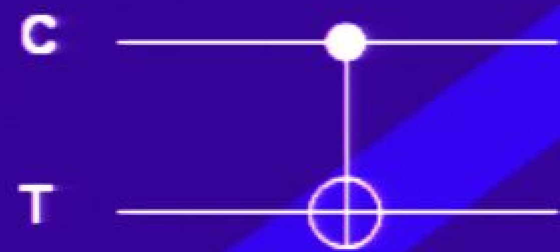

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$\|1\,1\rangle$	$\rightarrow \|1\,0\rangle$

One-way information flow? **No!**

CNOT is unitary

Look at CNOT in a conjugate basis:

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

Two qubits

Two qubits.

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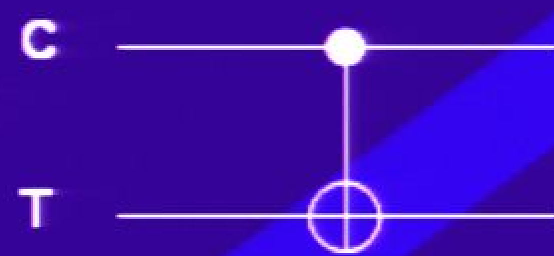

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One-way information flow? **No!**

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$\lvert ++\rangle$	$\rightarrow$	$\lvert ++\rangle$
$\lvert +-\rangle$	$\rightarrow$	$\lvert --\rangle$
$\lvert -+\rangle$	$\rightarrow$	$\lvert -+\rangle$
$\lvert --\rangle$	$\rightarrow$	$\lvert +-\rangle$

Two qubits

Two qubits.

Interaction: quantum CNOT

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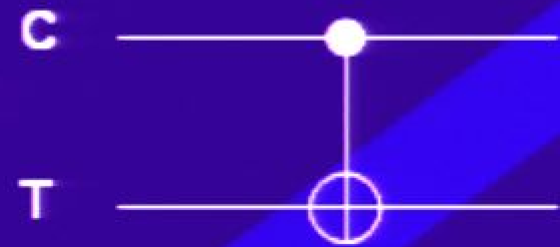

$\lvert CT\rangle$	$\to$	$\lvert CT\rangle$
$\lvert 0\,0\rangle$	$\to$	$\lvert 0\,0\rangle$
$\lvert 0\,1\rangle$	$\to$	$\lvert 0\,1\rangle$
$\lvert 1\,0\rangle$	$\to$	$\lvert 1\,1\rangle$
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$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$\Rightarrow$

$\lvert CT\rangle$	$\to$	$\lvert CT\rangle$
$\lvert + +\rangle$	$\to$	$\lvert + +\rangle$
$\lvert + -\rangle$	$\to$	$\lvert - -\rangle$
$\lvert - +\rangle$	$\to$	$\lvert - +\rangle$
$\lvert - -\rangle$	$\to$	$\lvert + -\rangle$

$\Rightarrow$

In the conjugate basis, control and target qubits switch roles!

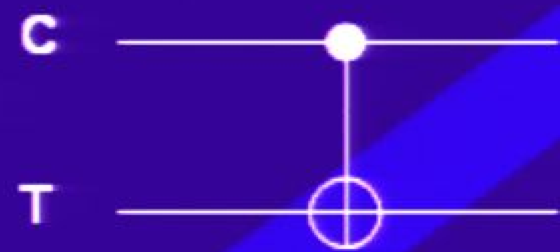
Two qubits

Two qubits.

Interaction: quantum CNOT

C = control bit

T = target bit



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$\lvert 0\,1\rangle$	$\to$	$\lvert 0\,1\rangle$
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$\lvert 1\,1\rangle$	$\to$	$\lvert 1\,0\rangle$

One-way information flow? **No!**

CNOT is **unitary**

Look at CNOT in a conjugate basis:

$$\lvert +\rangle = \tfrac{1}{\sqrt{2}}(\lvert 0\rangle + \lvert 1\rangle)$$

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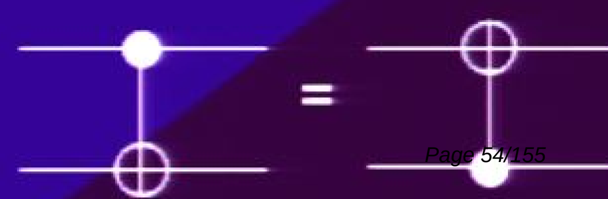
$\lvert CT\rangle$	$\to$	$\lvert CT\rangle$
$\lvert + +\rangle$	$\to$	$\lvert + +\rangle$
$\lvert + -\rangle$	$\to$	$\lvert - -\rangle$
$\lvert - +\rangle$	$\to$	$\lvert - +\rangle$
$\lvert - -\rangle$	$\to$	$\lvert + -\rangle$

In the conjugate basis, control and target qubits switch roles!



No one-way information flow

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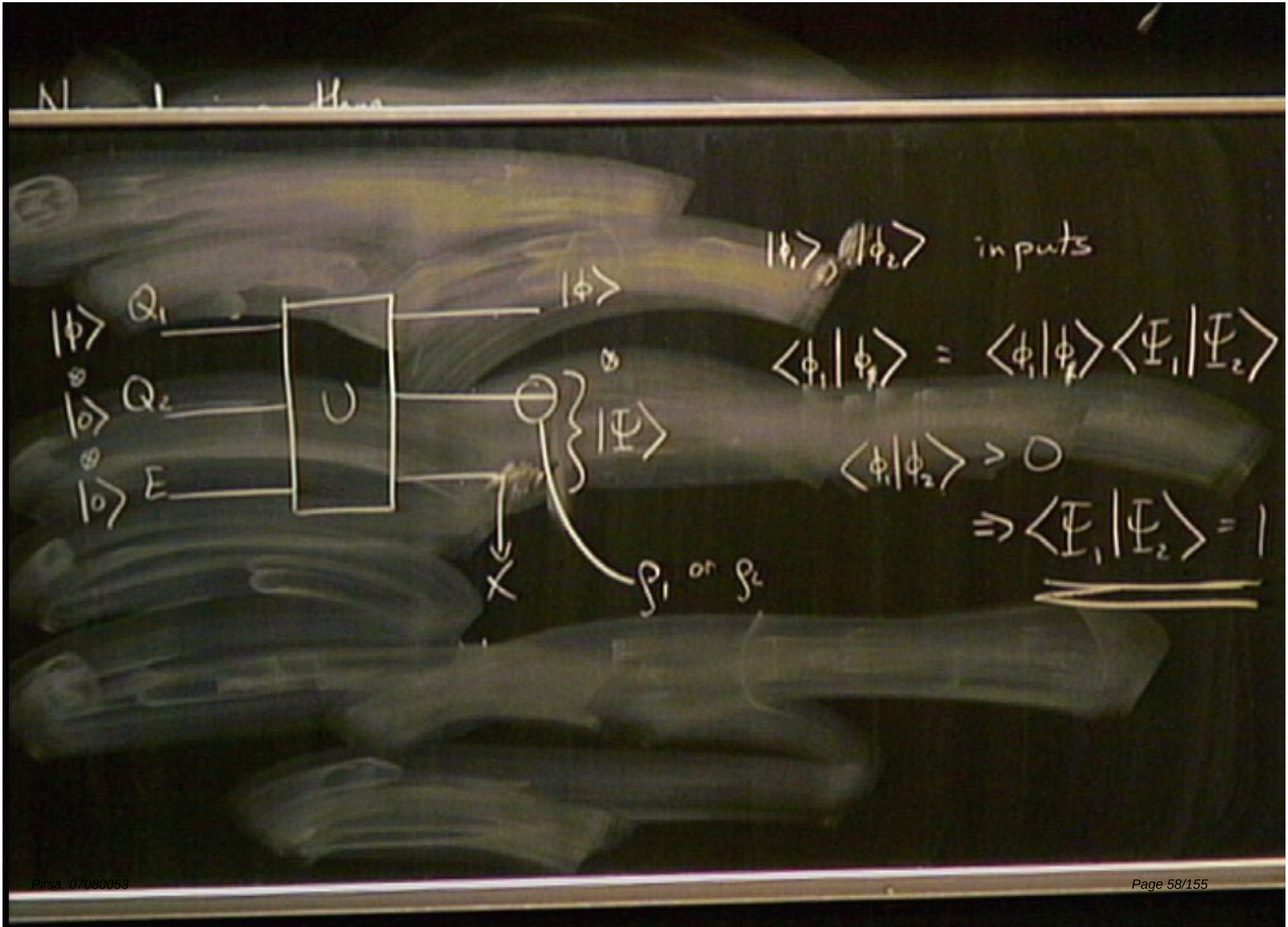
$|\phi_1\rangle$, $|\phi_2\rangle$ inputs
$\langle\phi_1|\phi_2\rangle = \langle\phi_1|\phi_2\rangle\langle\Psi_1|\Psi_2\rangle$
$\langle\phi_1|\phi_2\rangle > 0$
$\Rightarrow \langle\Psi_1|\Psi_2\rangle = 1$
$|\phi\rangle$ Q_1
$|0\rangle$ Q_2
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U
$|\phi\rangle$
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X
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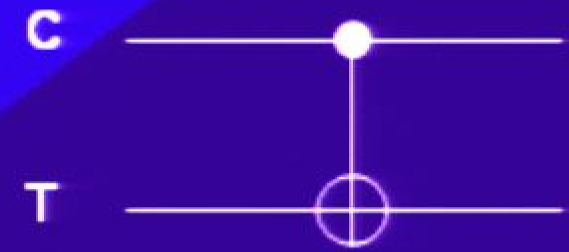

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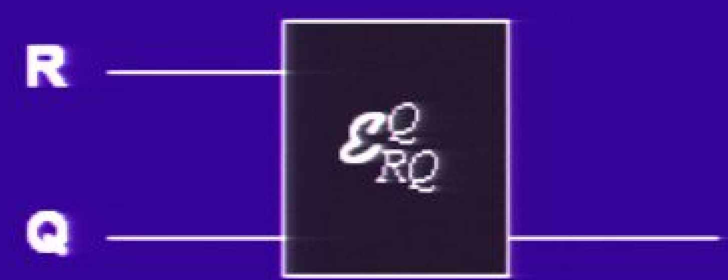
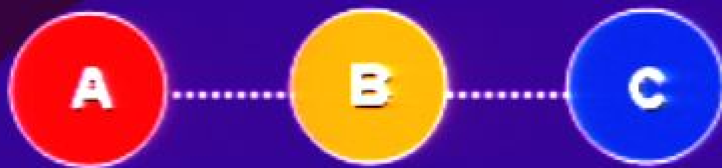


We may also have maps with the final system *larger* than the initial system

Locality × 3

Locality $\times$ 3



Global evolution map $\mathcal{E}^{ABC}$

What does it mean to say "No information flows from C to A"?

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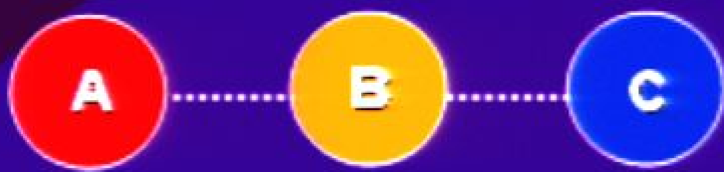

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Good news: All three locality conditions for $\mathcal{E}^{ABC}$ are equivalent.

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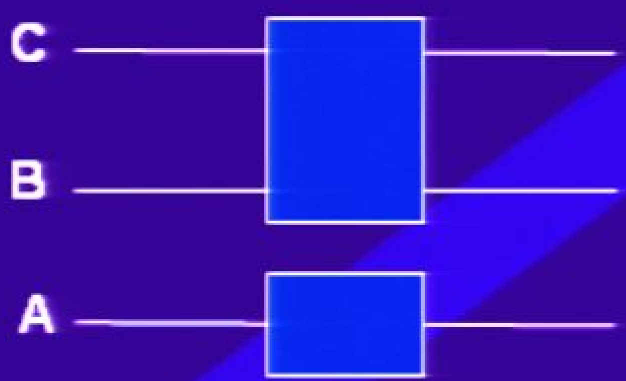

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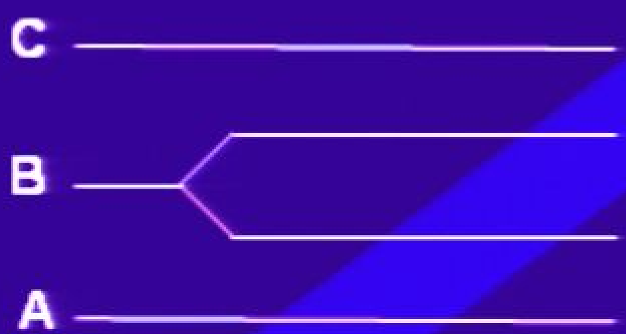

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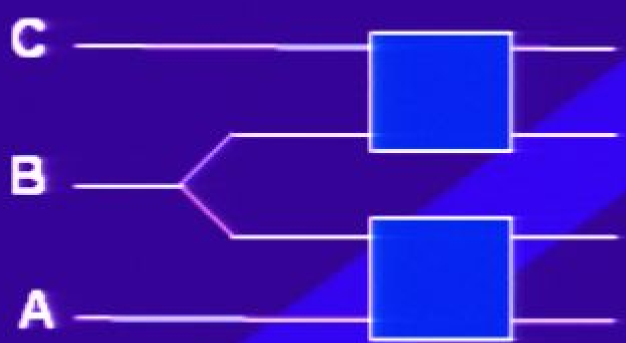

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A and C interact separately with parts of a composite system B.

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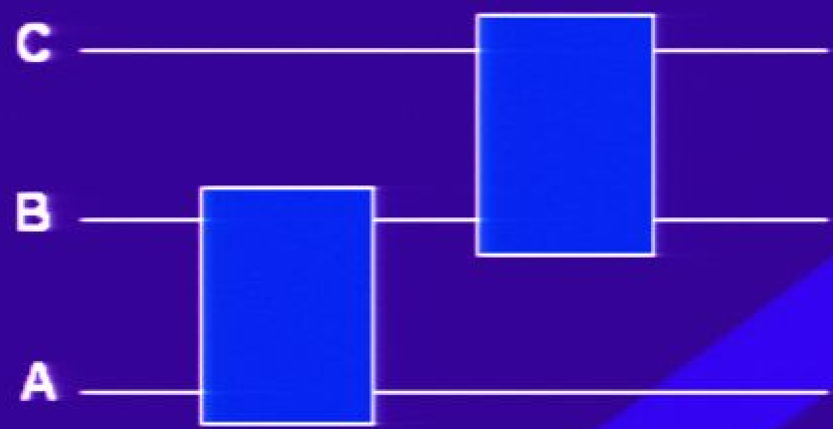

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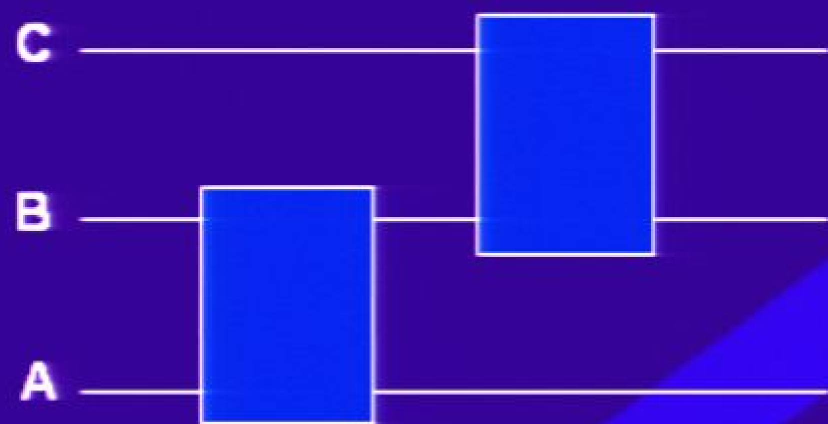

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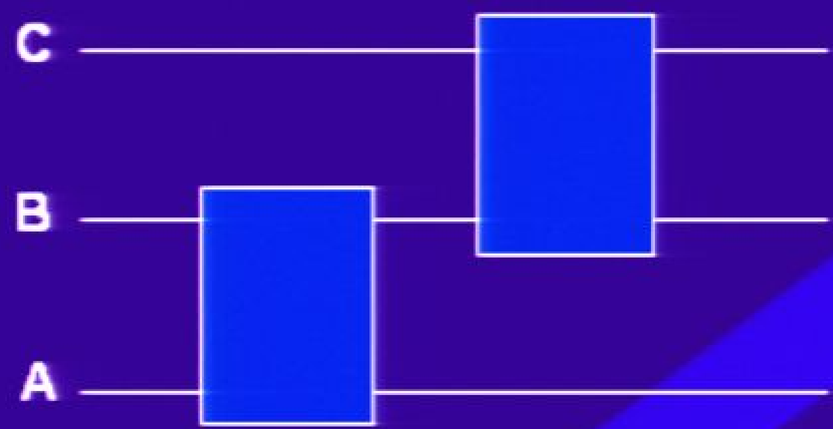

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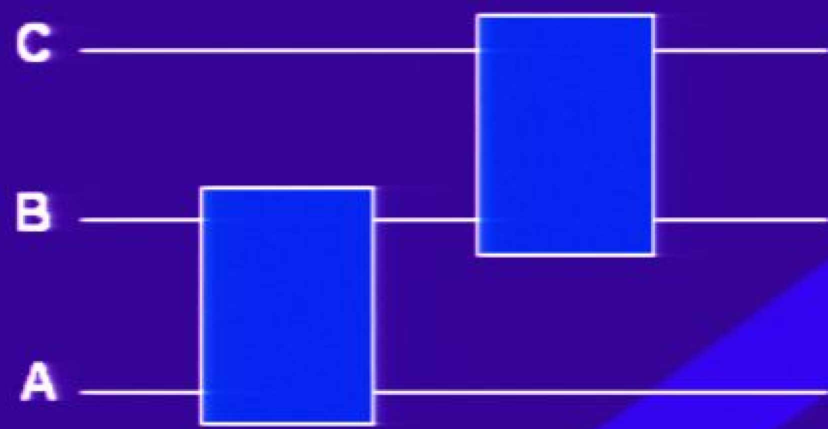

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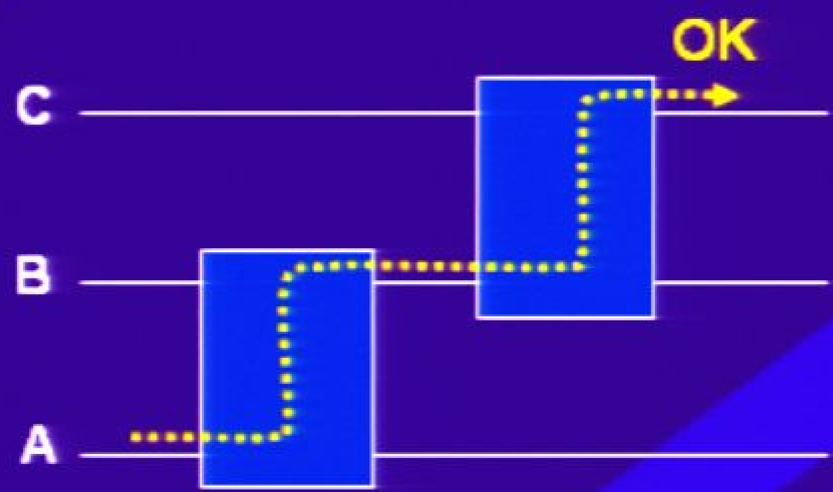

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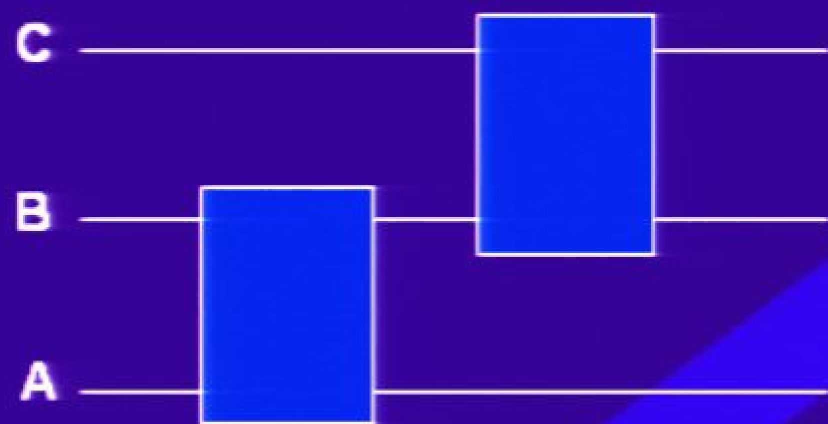

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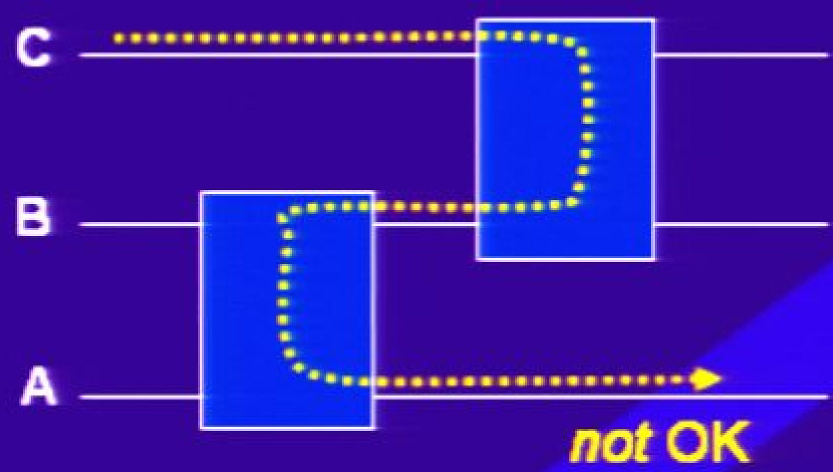

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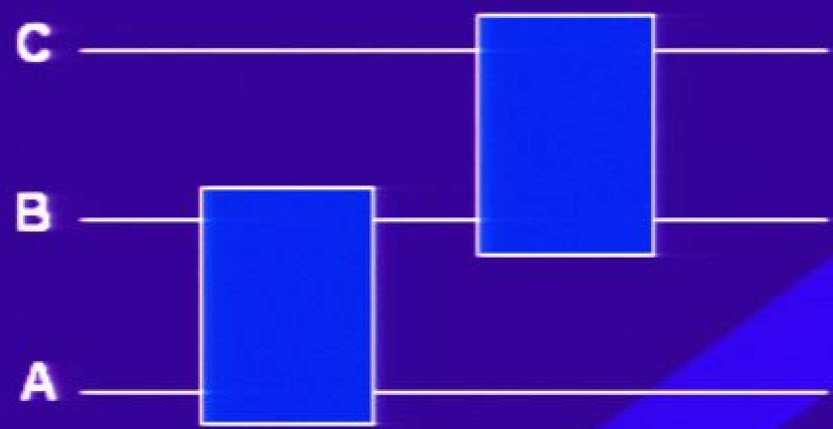

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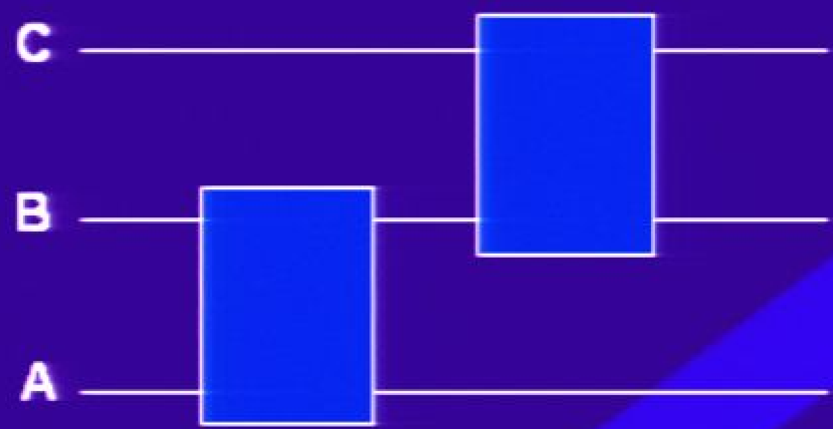

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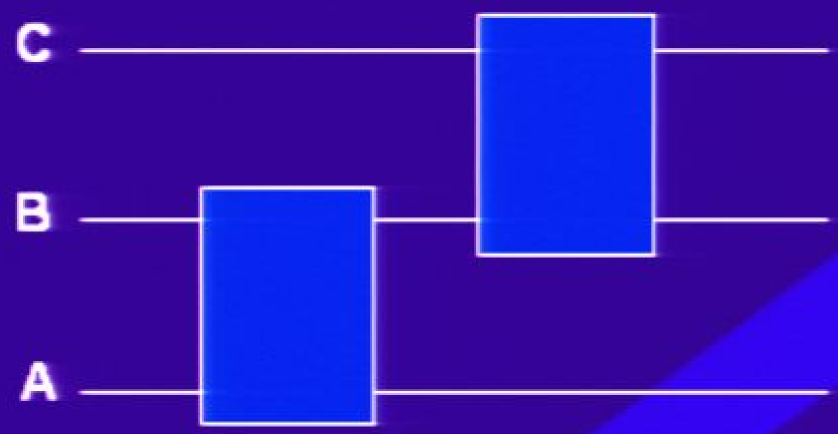

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Remarkable fact: This is the *only* possibility!

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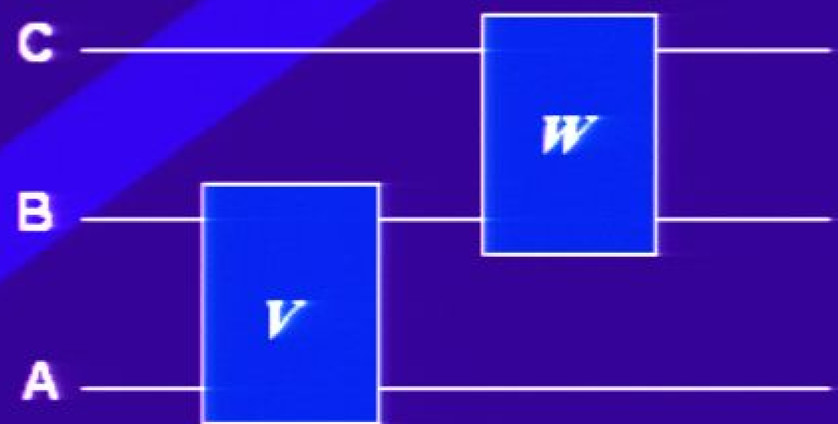
Suppose system ABC evolves via unitary U^{ABC}, such that no information transfer is possible from C to A ("locality"). Then

$$U^{ABC} = \left(1^A \otimes W^{BC}\right)\left(V^{AB} \otimes 1^C\right)$$



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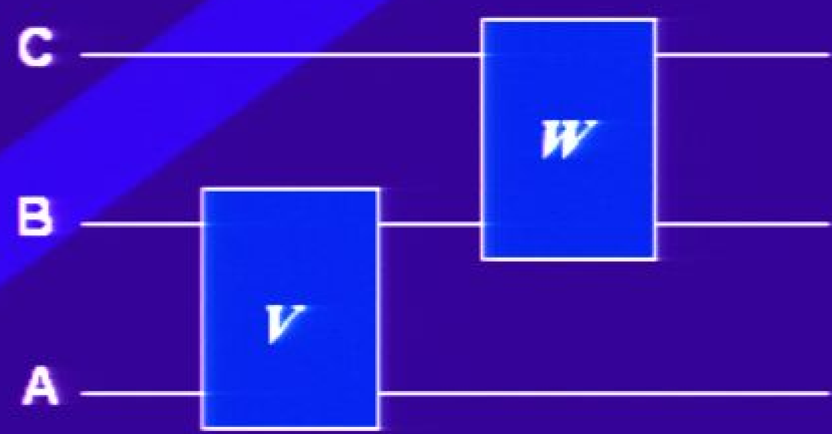

NB – We are not claiming that U actually happened this way ….

How it works

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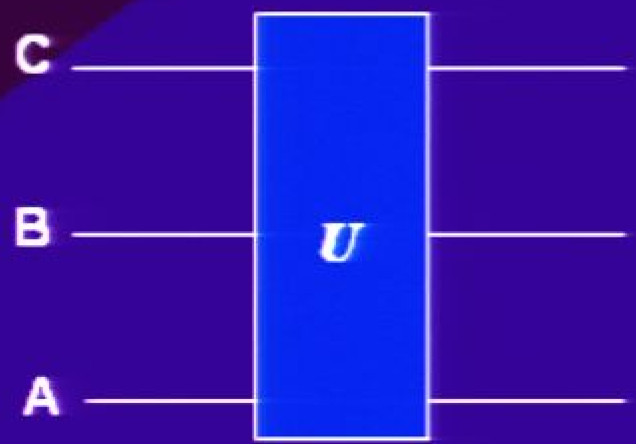

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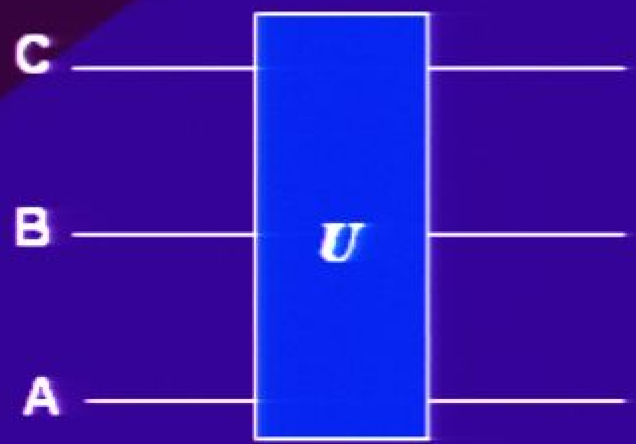

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From these, construct a "natural" tensor product structure in $\mathcal{H}_{AB}$

This is not the same as $\mathcal{H}_A \otimes \mathcal{H}_B$ – it's twisted by a unitary transformation $V^\dagger$

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"Natural" tensor product structure is exactly the subsystem structure here.

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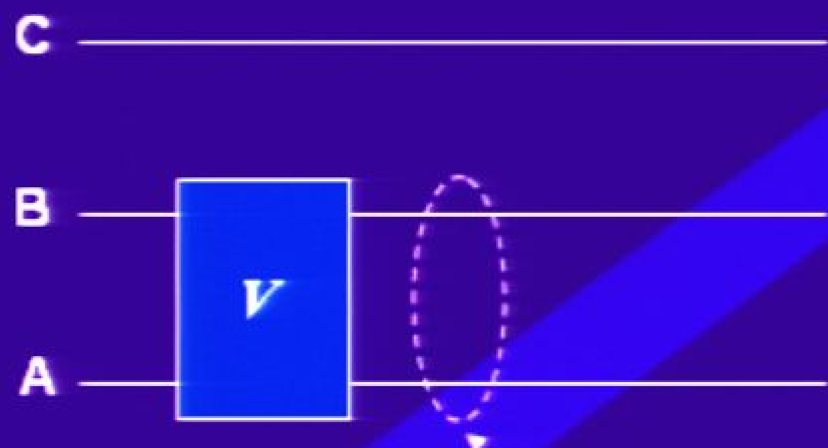

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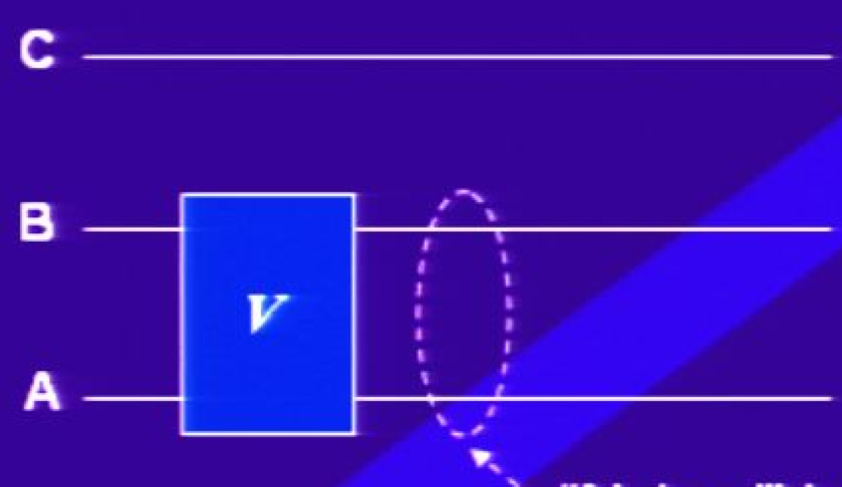

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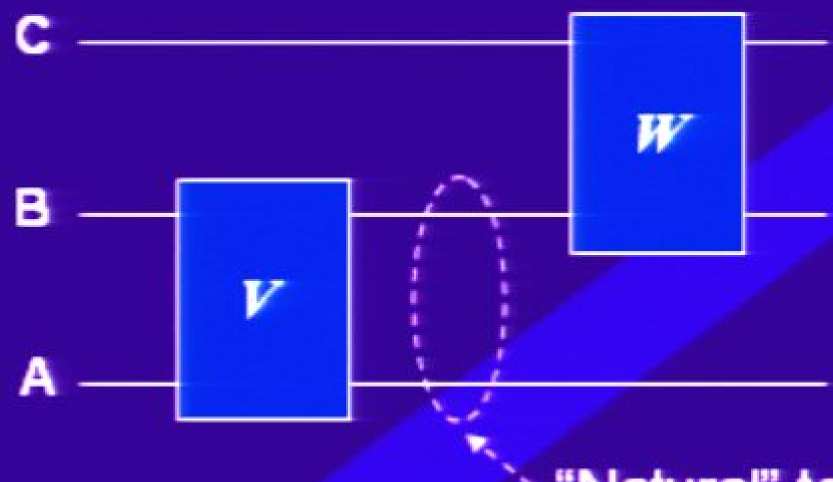

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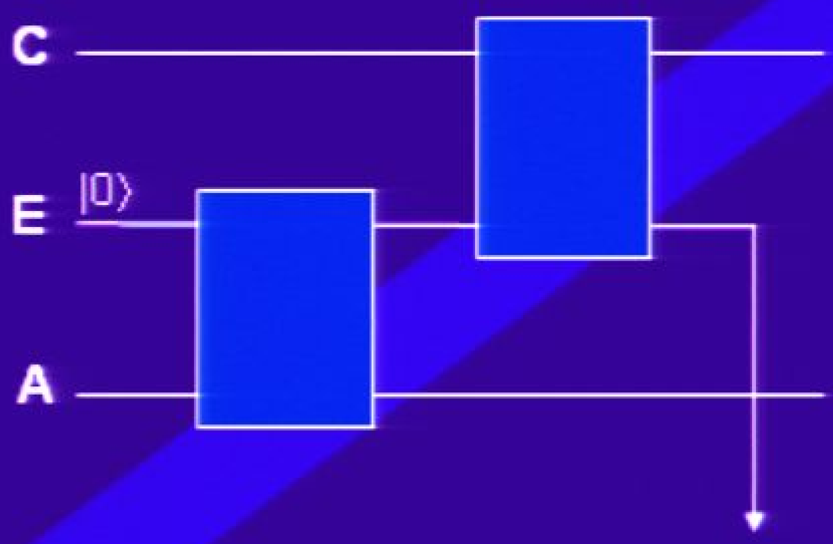

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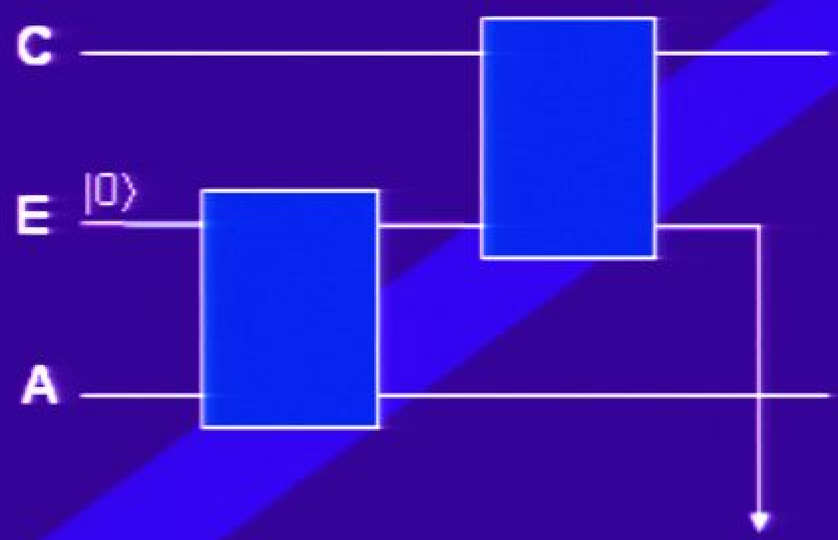

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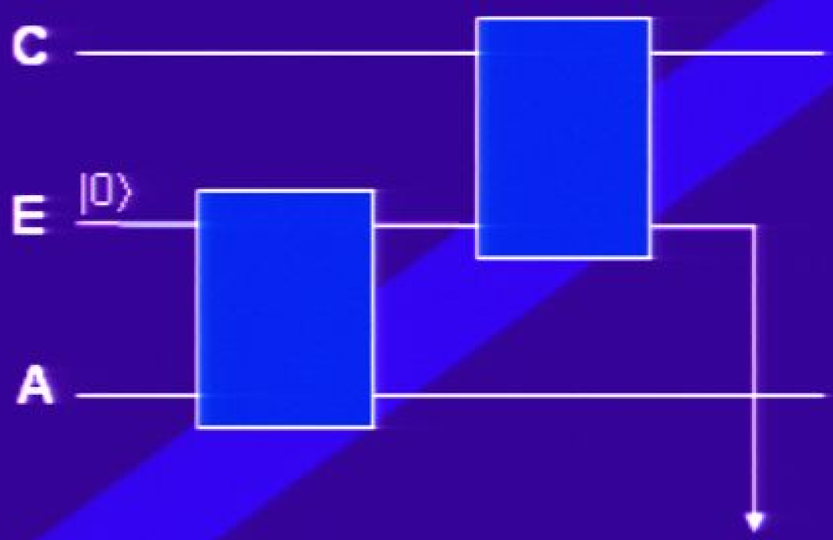

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Semicausal operations are semilocalizable

Beckman et. al. (2001)

Eggeling et al. (2002)

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However . . . .

Locality in general

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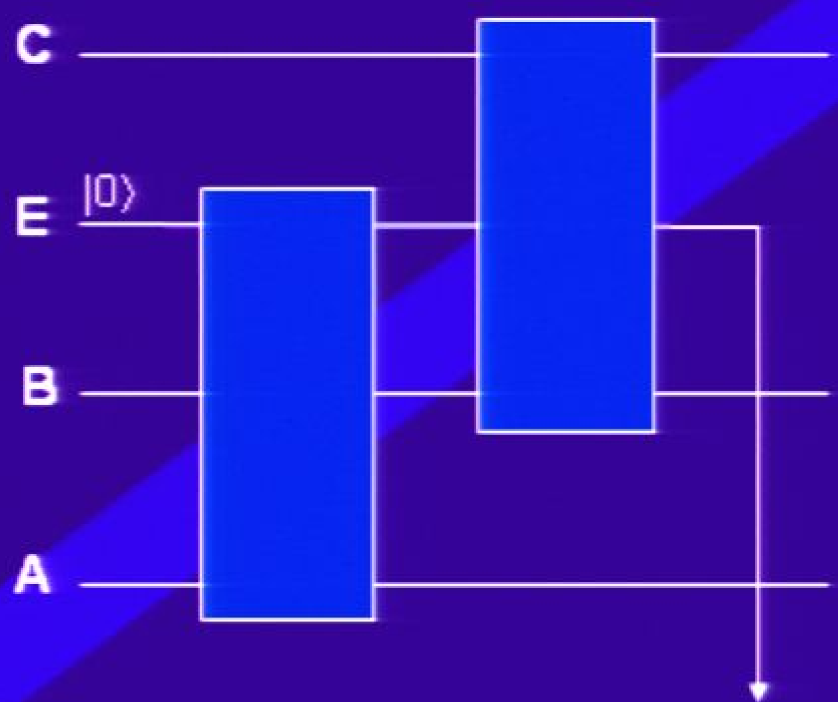

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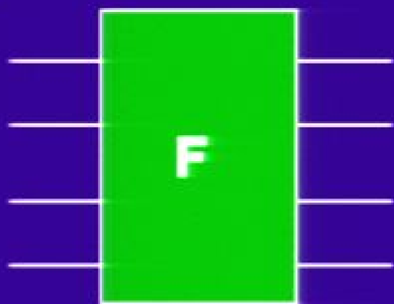


Global update rule for N classical discrete variables:

$$(a,b,\ldots,c) \rightarrow F(a,b,\ldots,c)$$

Reversible map: **F** is 1-1 and onto.

"Quantizing" the classical map

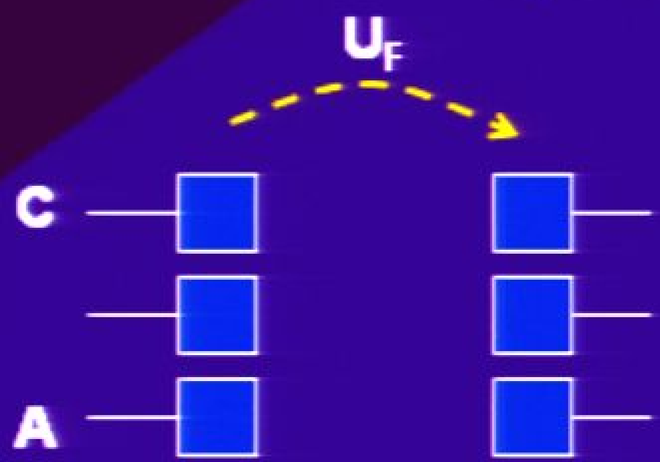
Product basis : $| a,b,\ldots,c \rangle$

Global unitary dynamics:

$$U_F \,| a,b,\ldots,c \rangle = | F(a,b,\ldots,c) \rangle$$

How do the classical locality properties of **F** determine the quantum locality properties of U_F?

Quantum locality



Quantize previous problem (array of N finite quantum systems) with U_F

Is the global evolution U_F "local enough" to prevent information flow from C to A?

N = 3: Even though no information can flow from C to A in the classical case, it sometimes can in the quantum case.

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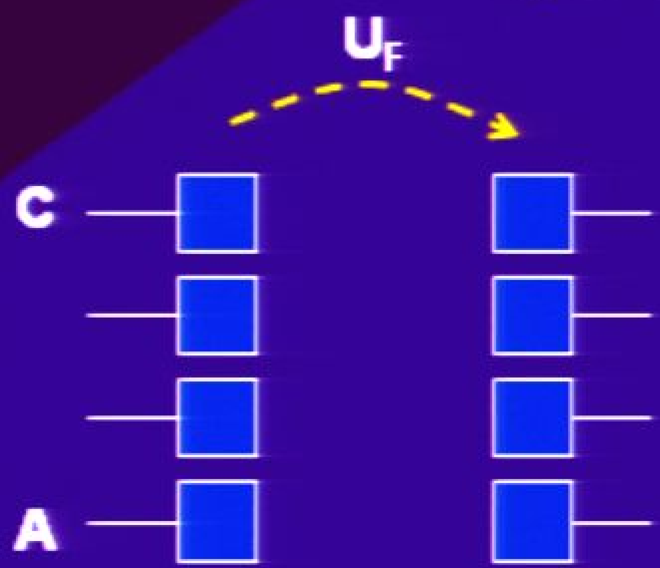

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N = 5: No information can flow even in the quantum case.

N = 4: No information can flow if t
are qubits. We do not know the a
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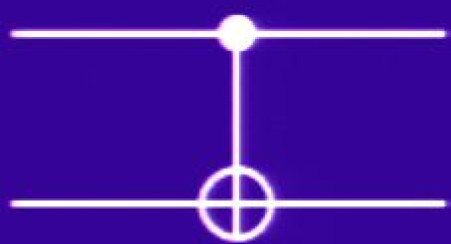
Dissecting CNOT



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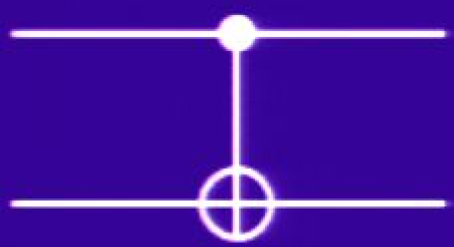

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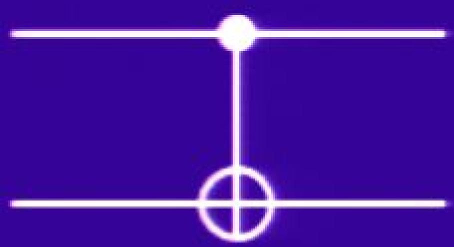

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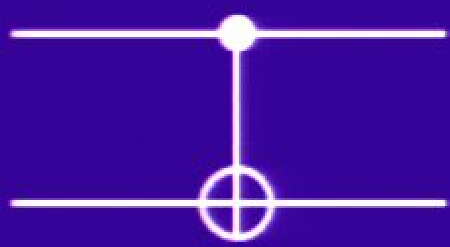

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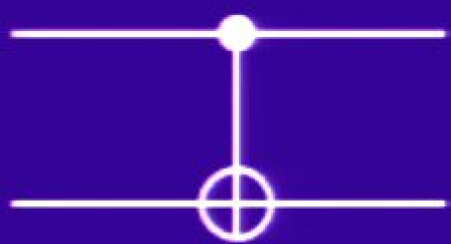
Model: Simple information exchange

Dissecting CNOT



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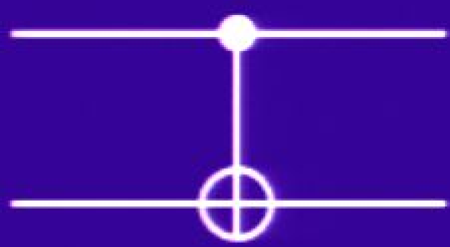
Model: Simple information exchange

Dissecting CNOT



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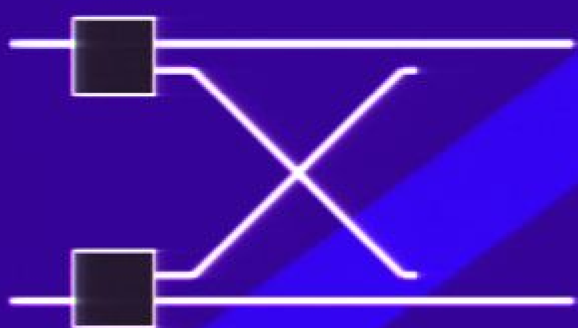

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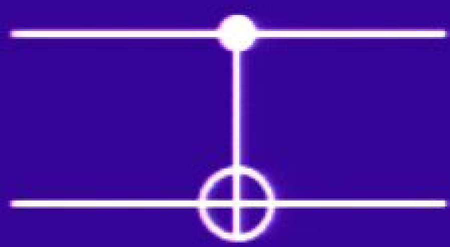

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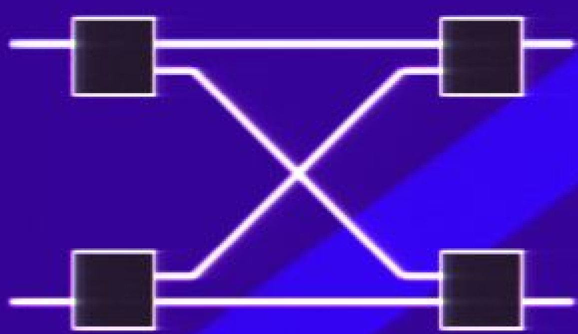

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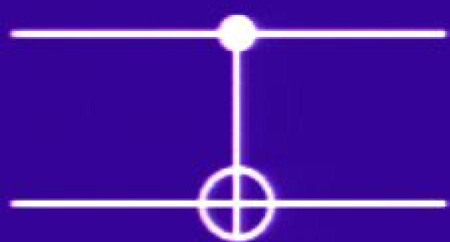

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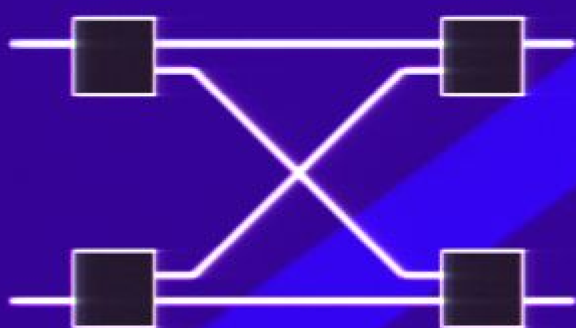

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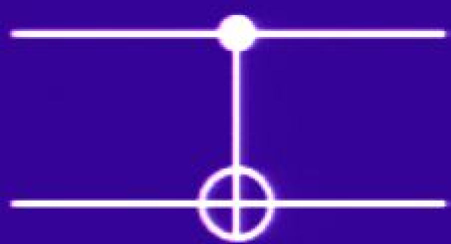

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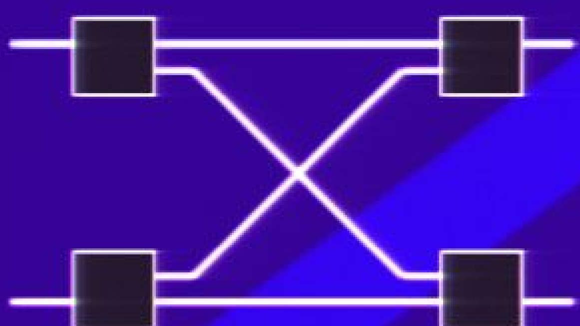

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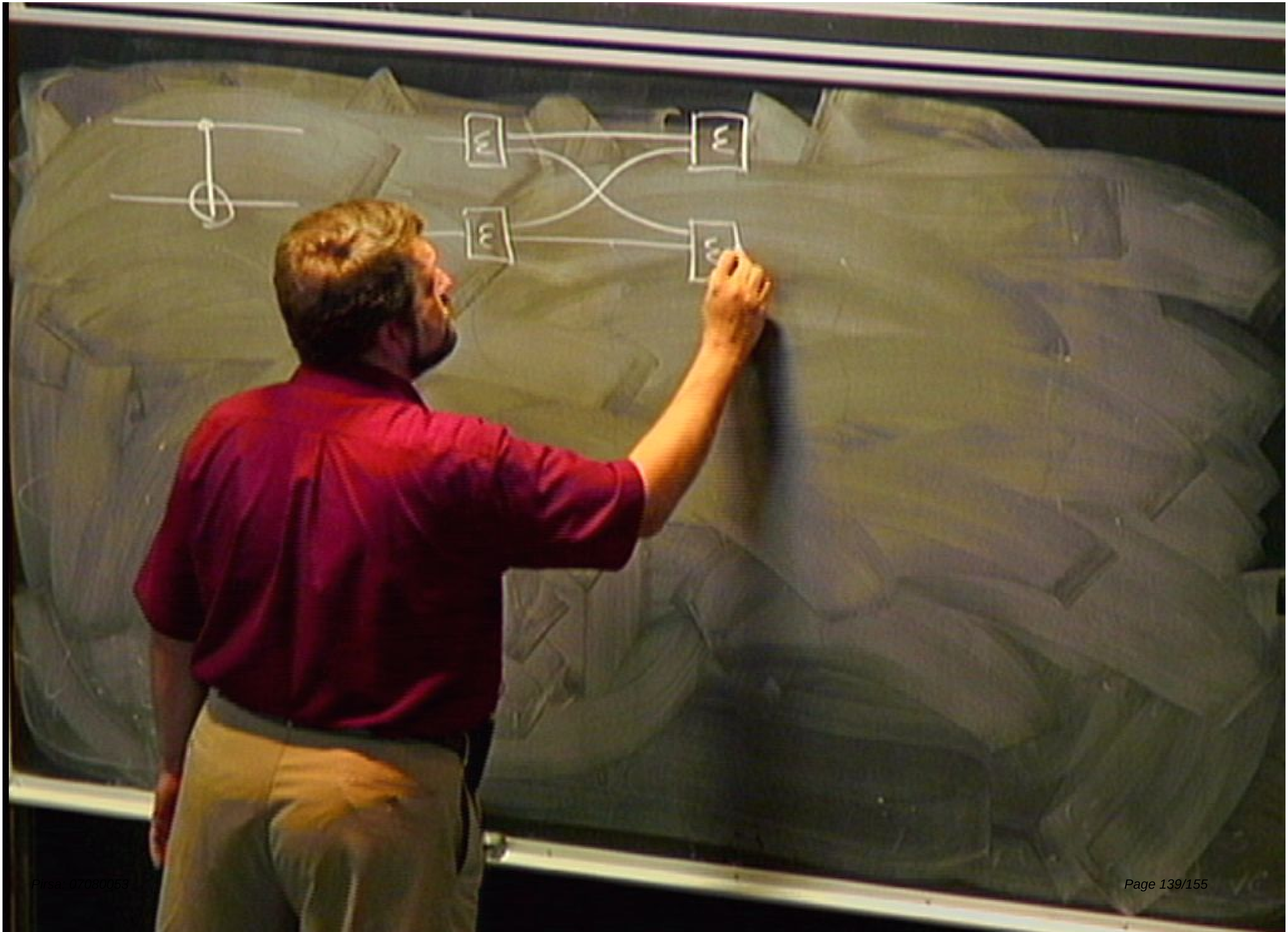
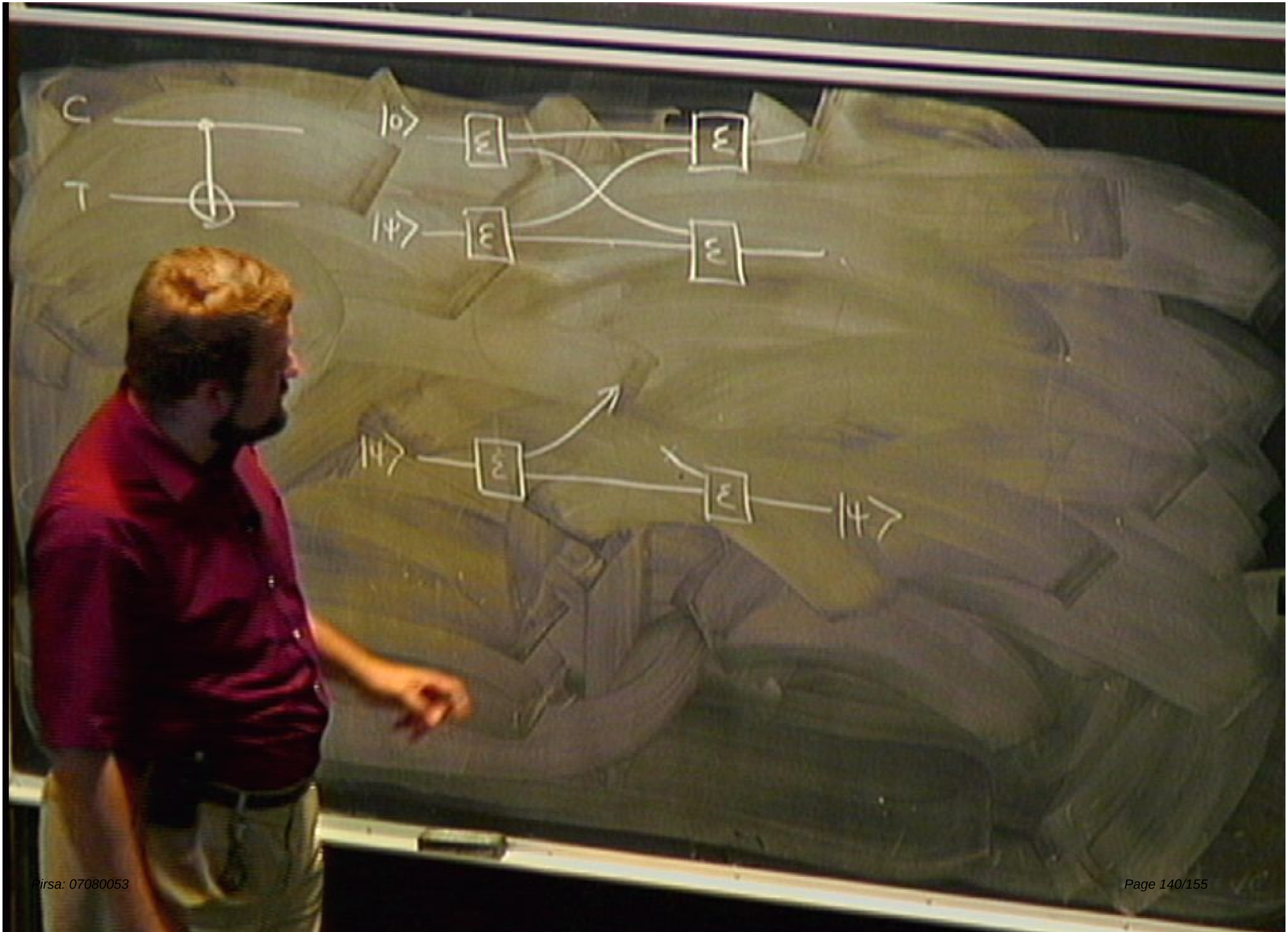
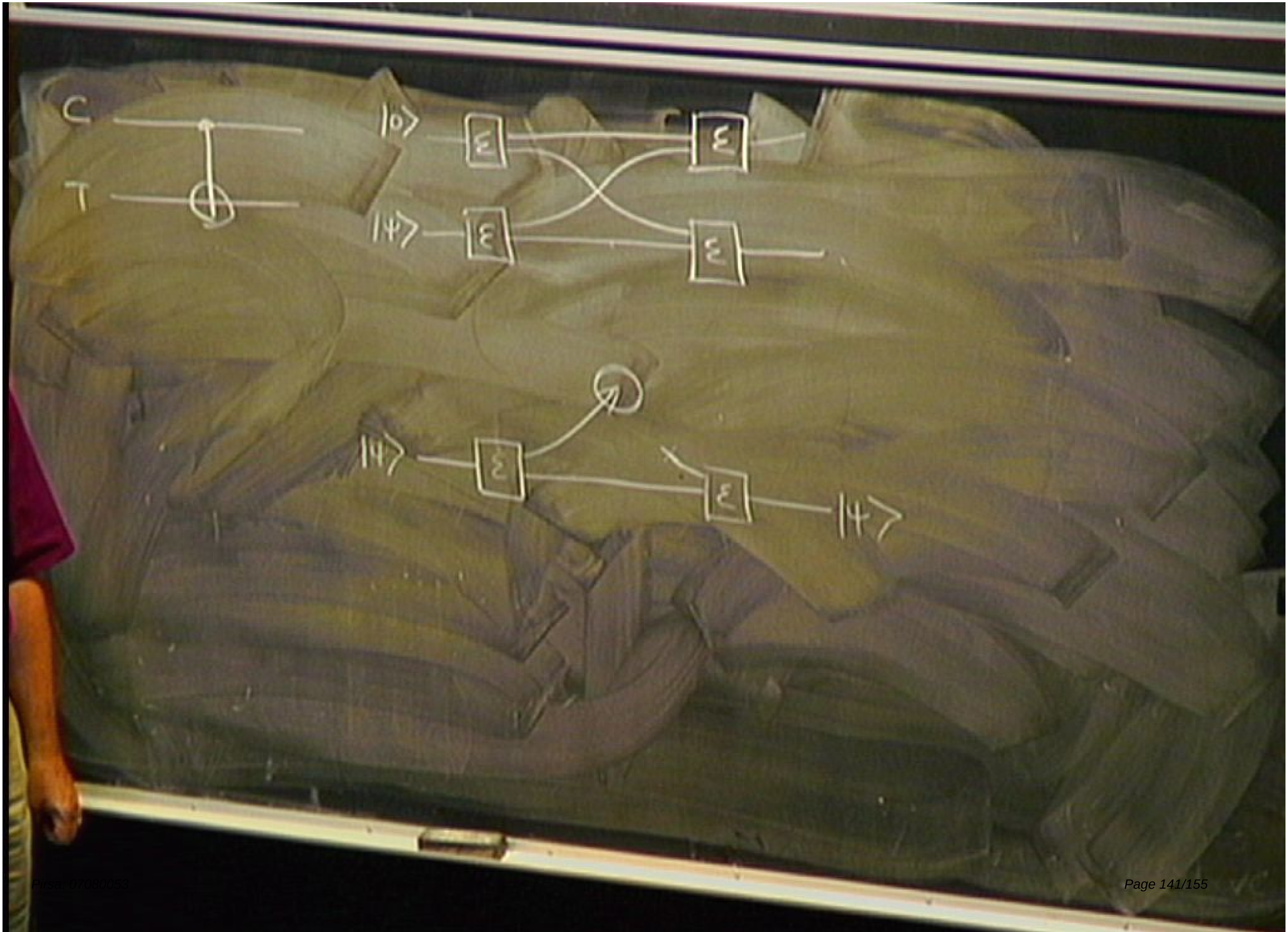
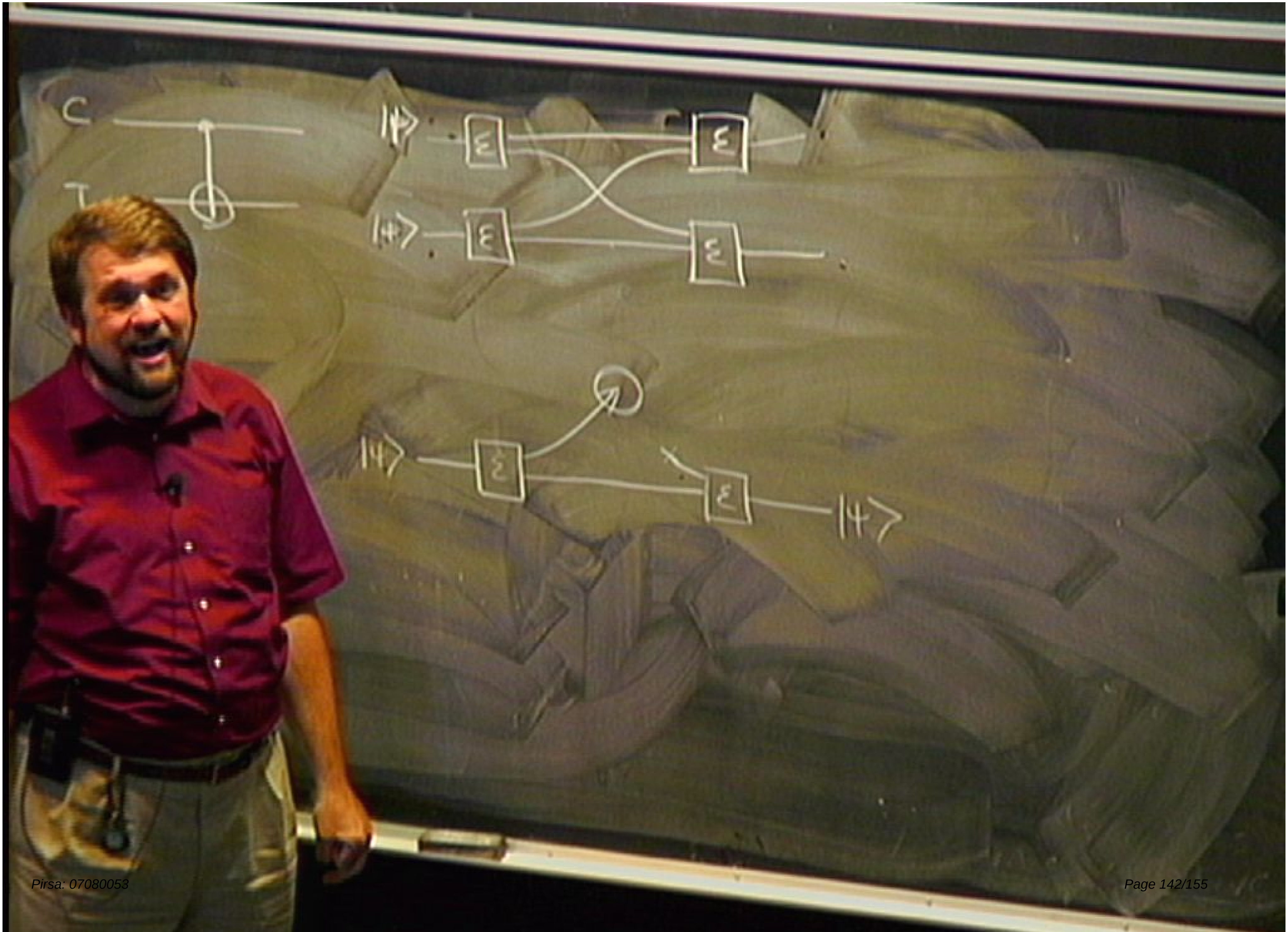
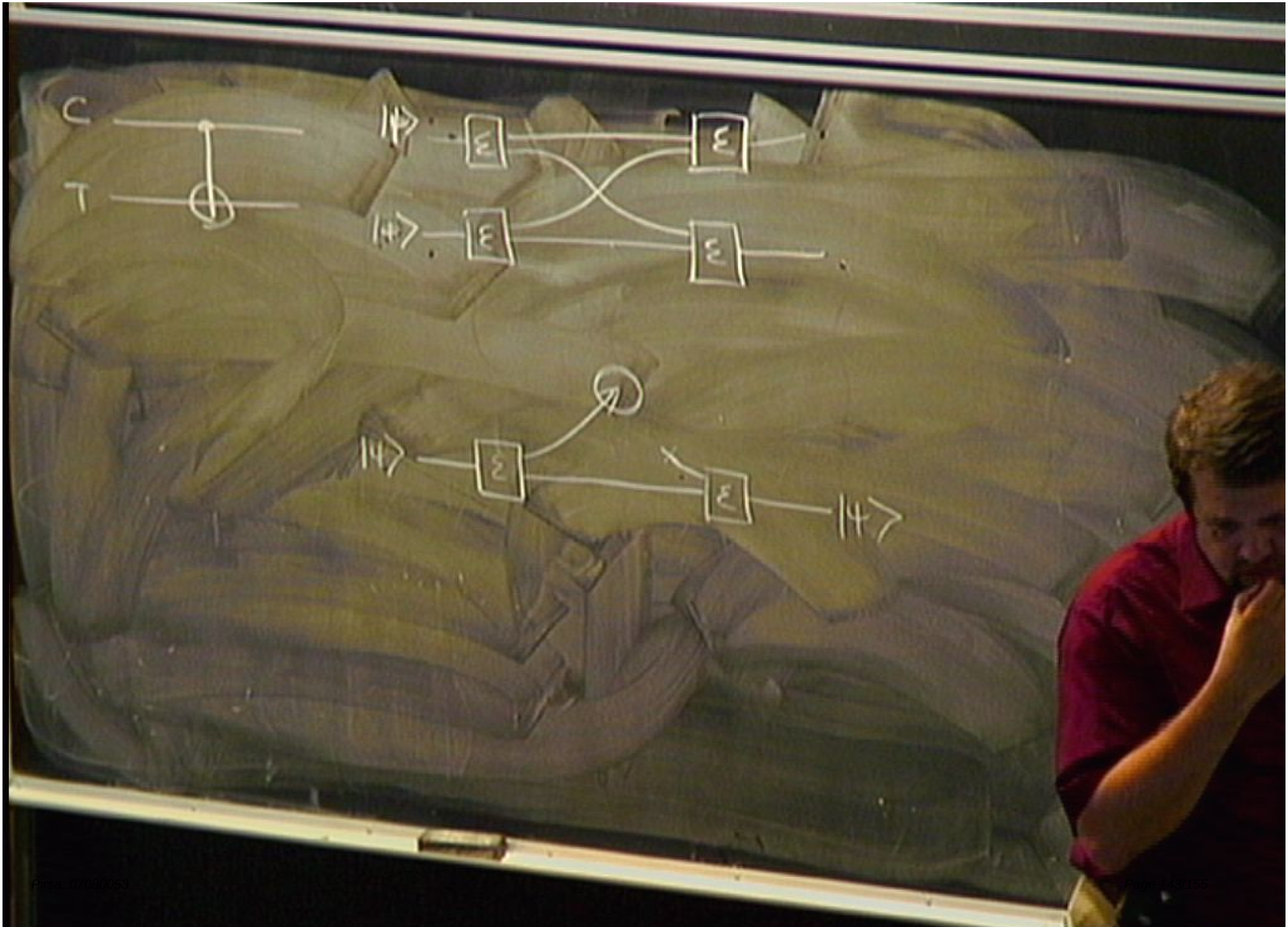
Can CNOT be modeled by local CP maps and simple information exchange?


ε
ε
ε
ε


C
T
|0>
|ψ>
ε
ε
ε
ε
|ψ>
ε
ε
|ψ>


C
T
|0⟩
|ψ⟩
ε
|ψ⟩
ε
|ψ⟩


C
T
|ψ⟩
E
E
|ψ⟩
E
E
|ψ⟩
E
E
|ψ⟩


C
T
|ψ>
ε
ε
|ψ>
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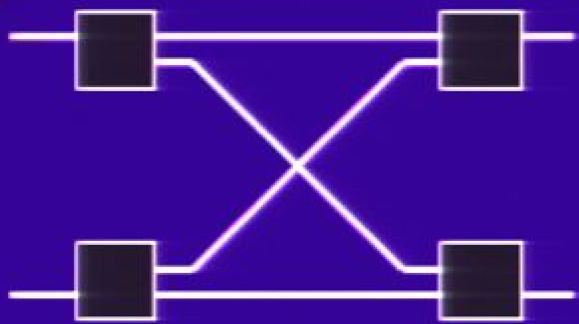
ε
|ψ>
ε
ε
|ψ>

Dissecting CNOT



M. Nathanson: No entangling unitary two-qubit gate can be modeled by local CP maps and simple information exchange

(This holds only for qubit gates!)

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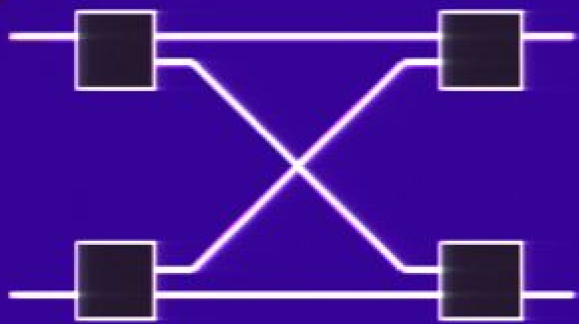

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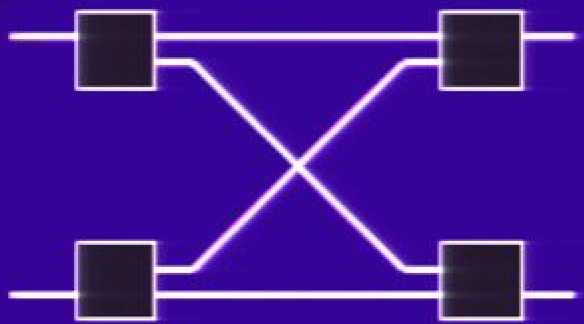

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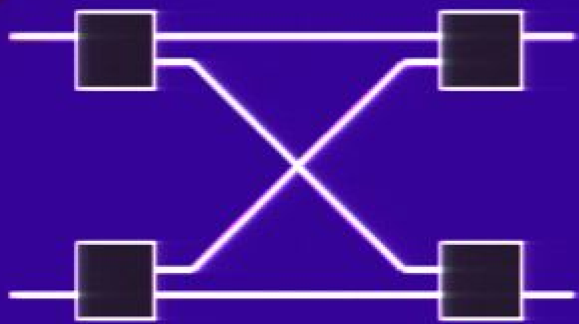
Dissecting CNOT



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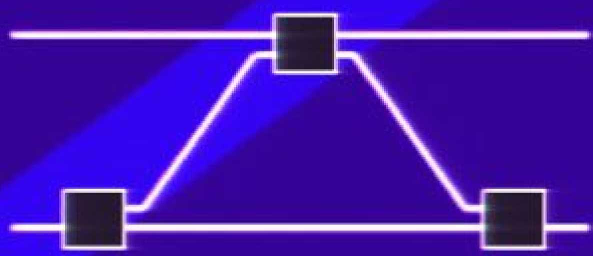

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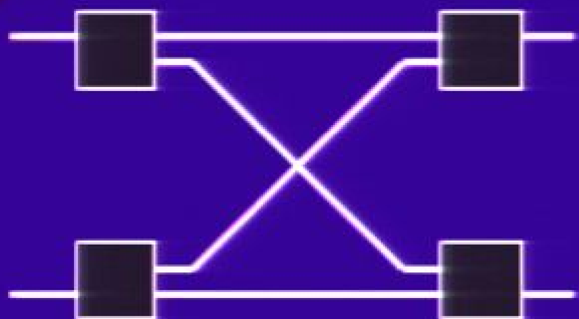

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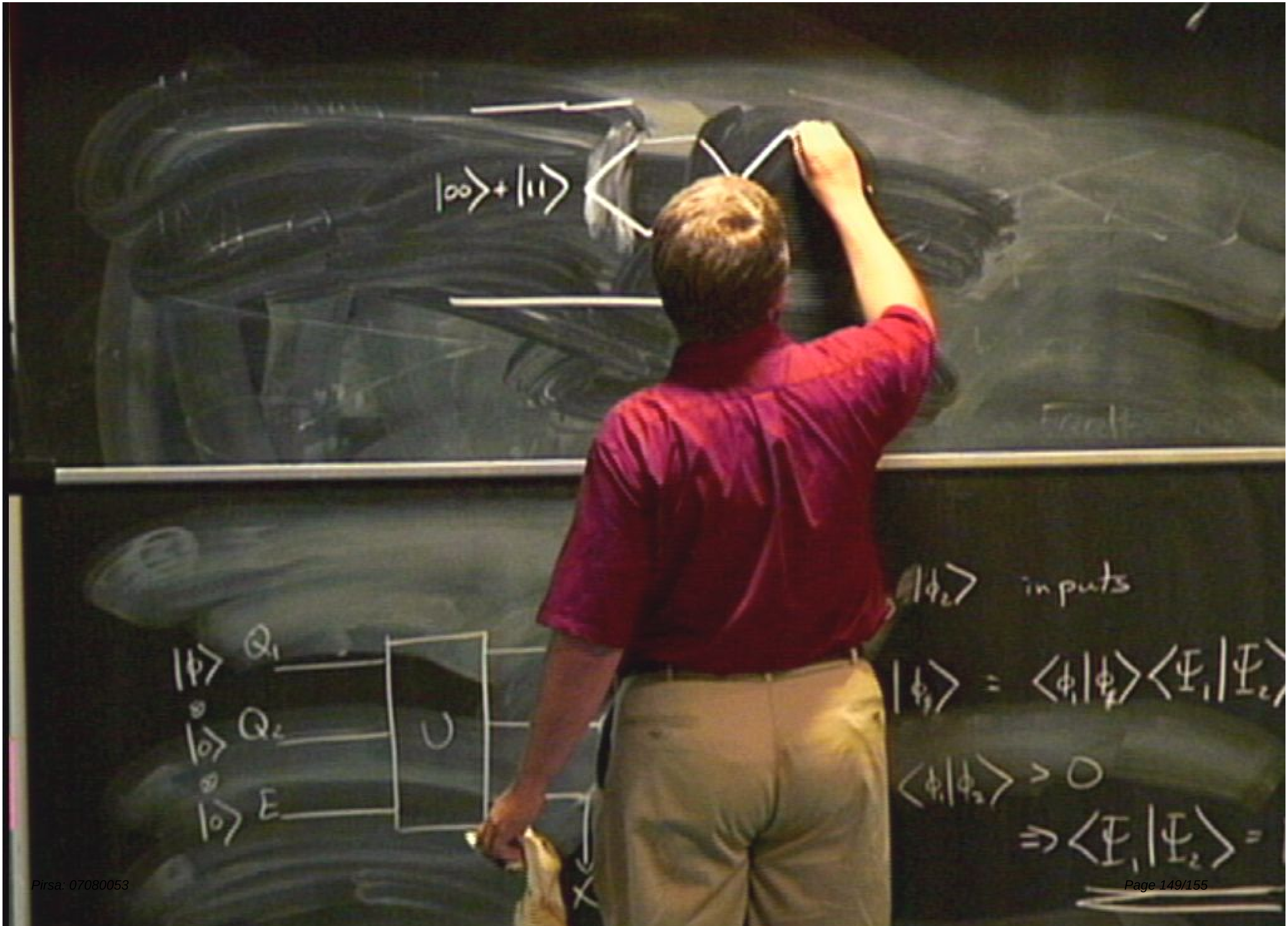
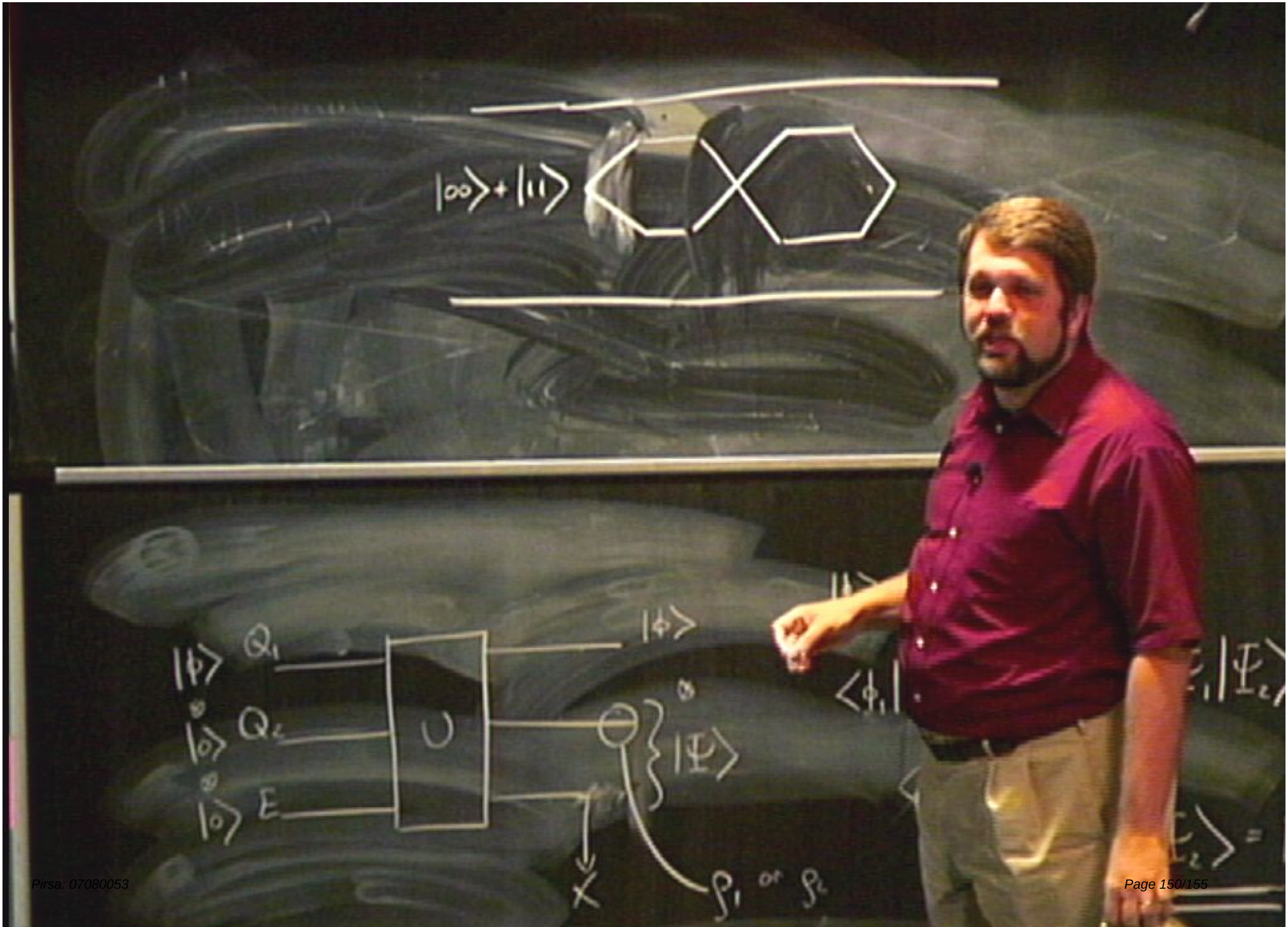
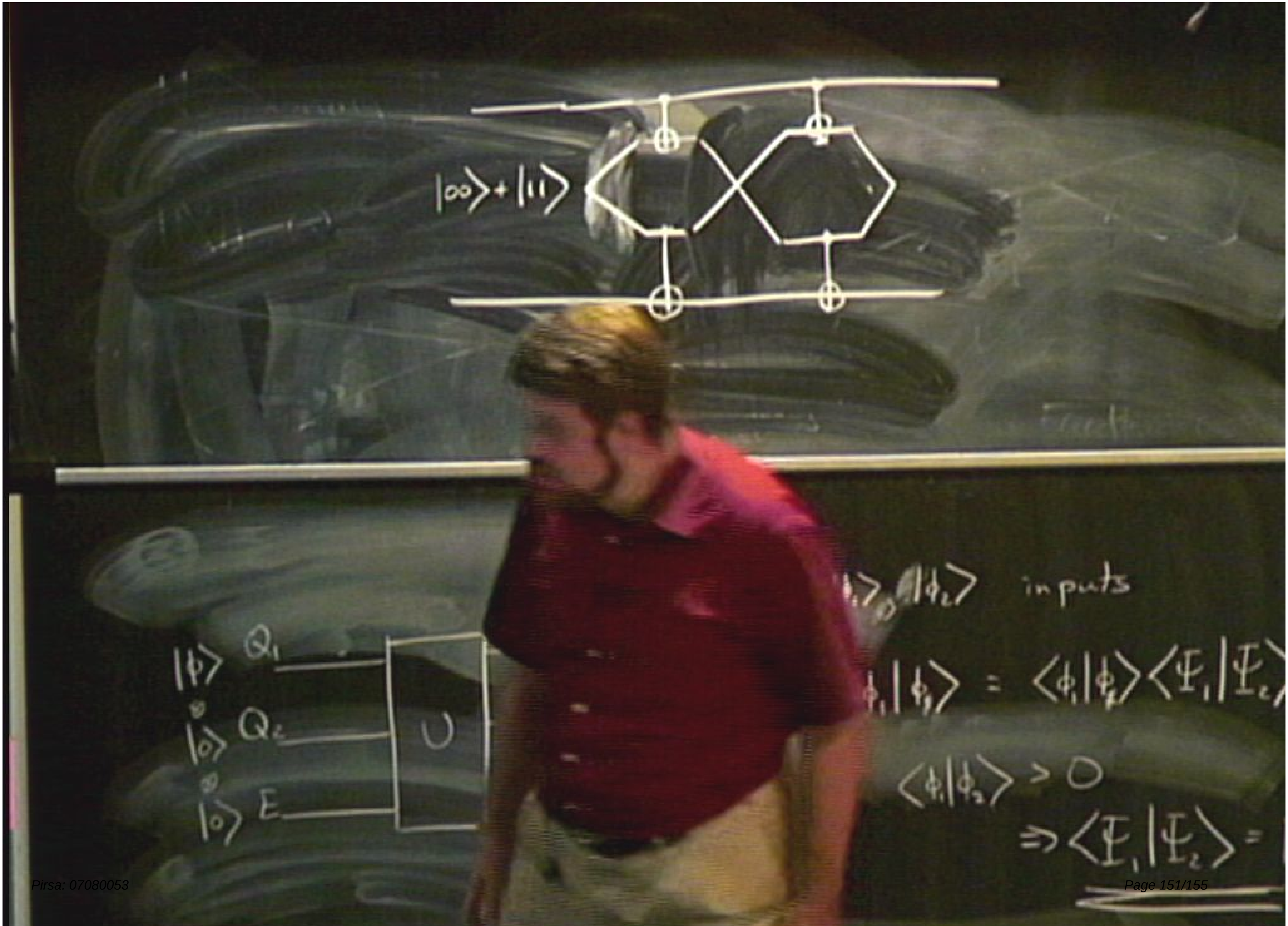
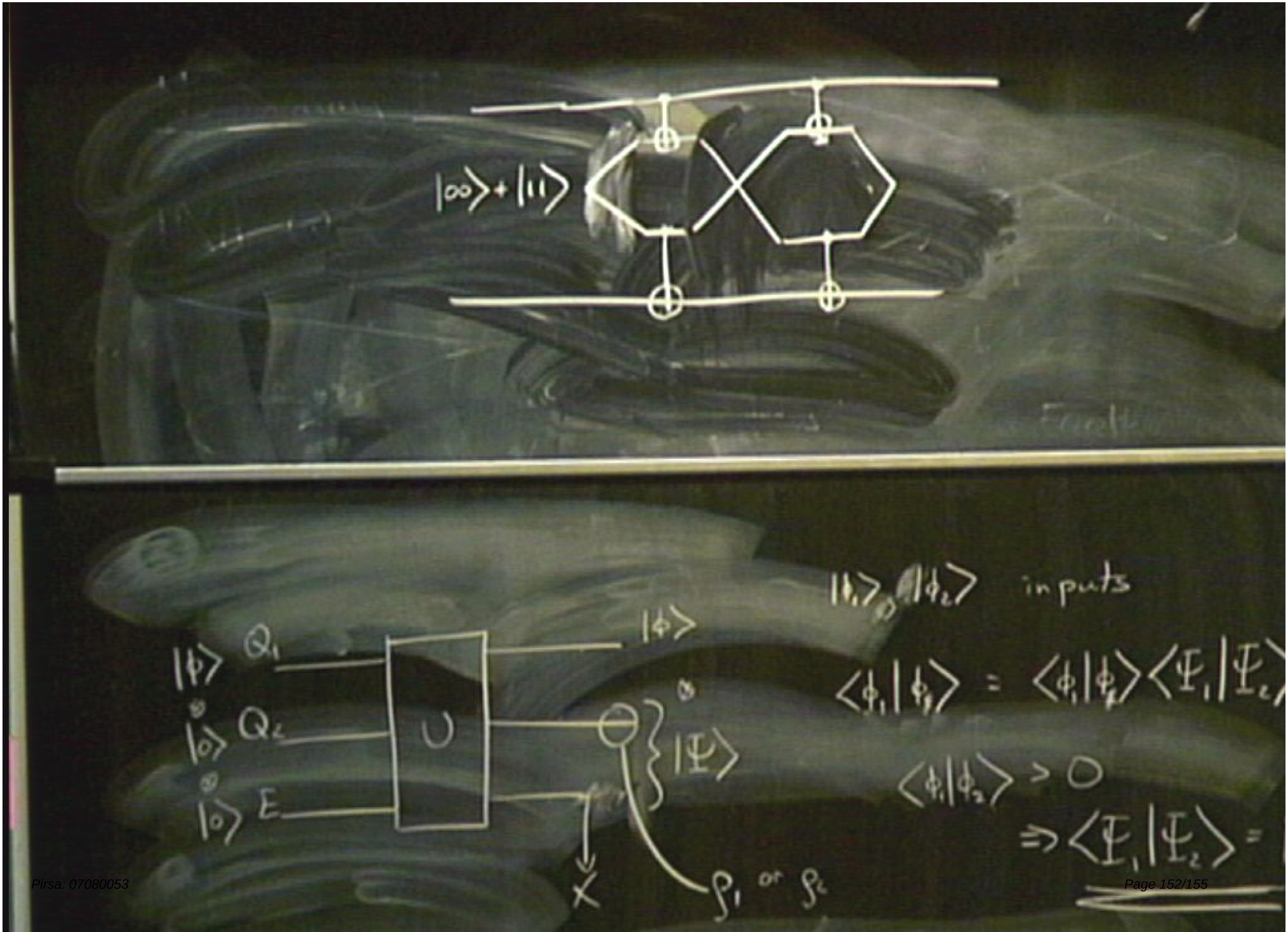
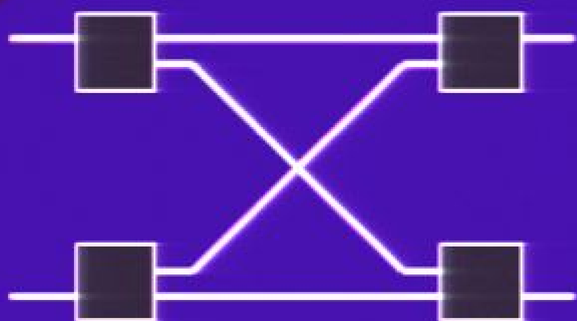

|00⟩+|11⟩
|φ⟩ Q1
|0⟩ Q2
|0⟩ E
U
|φ2⟩ inputs
⟨φ1|φ2⟩ > 0


|00> + |11>
|φ>
Q1
|0>
Q2
|0>
E
U
|ψ>
|Ψ>
X
ρ1 or ρ2


|00⟩+|11⟩
inputs
|φ⟩ Q1
|0⟩ Q2
U
|0⟩ E
⟨φ1|φ2⟩ > 0


$|00\rangle + |11\rangle$
$|\phi_1\rangle, |\phi_2\rangle$ inputs
$|\phi\rangle$
Q_1
Q_2
E
U
$|\Psi\rangle$
X
ρ_1 or ρ_2
$\langle\phi_1|\phi_2\rangle = \langle\phi_1|\phi_2\rangle\langle\Psi_1|\Psi_2\rangle$
$\langle\phi_1|\phi_2\rangle > 0$
$\Rightarrow \langle\Psi_1|\Psi_2\rangle =$

Dissecting CNOT



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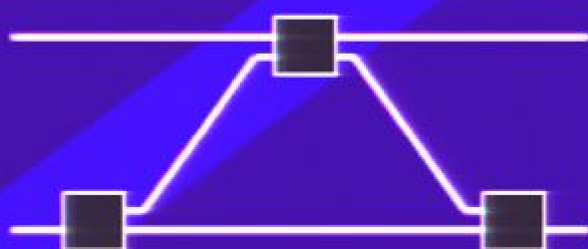

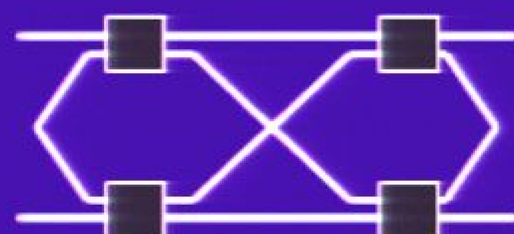

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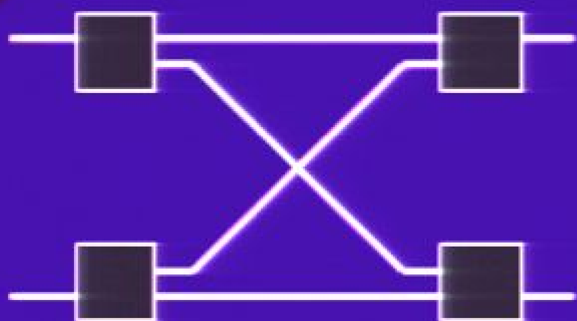

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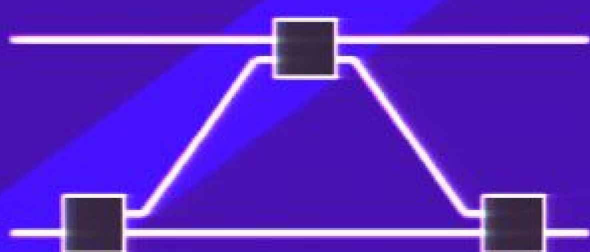

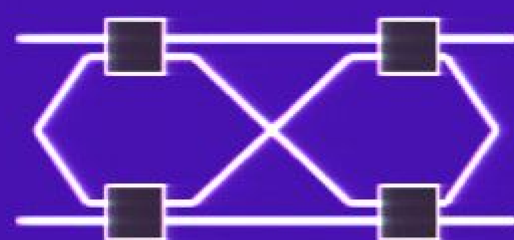

What is the essential difference between these information flow patterns and simple information exchange?

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