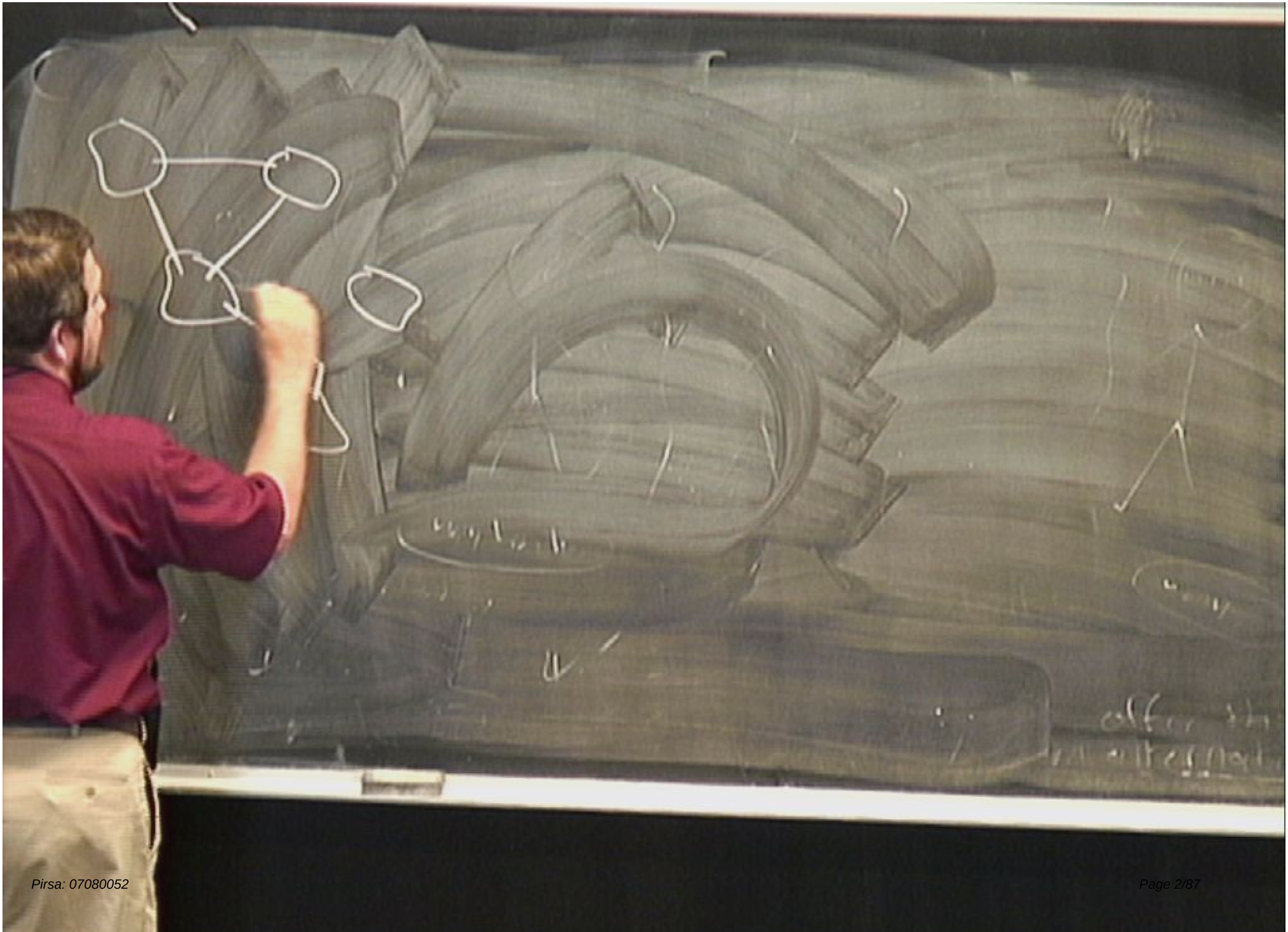
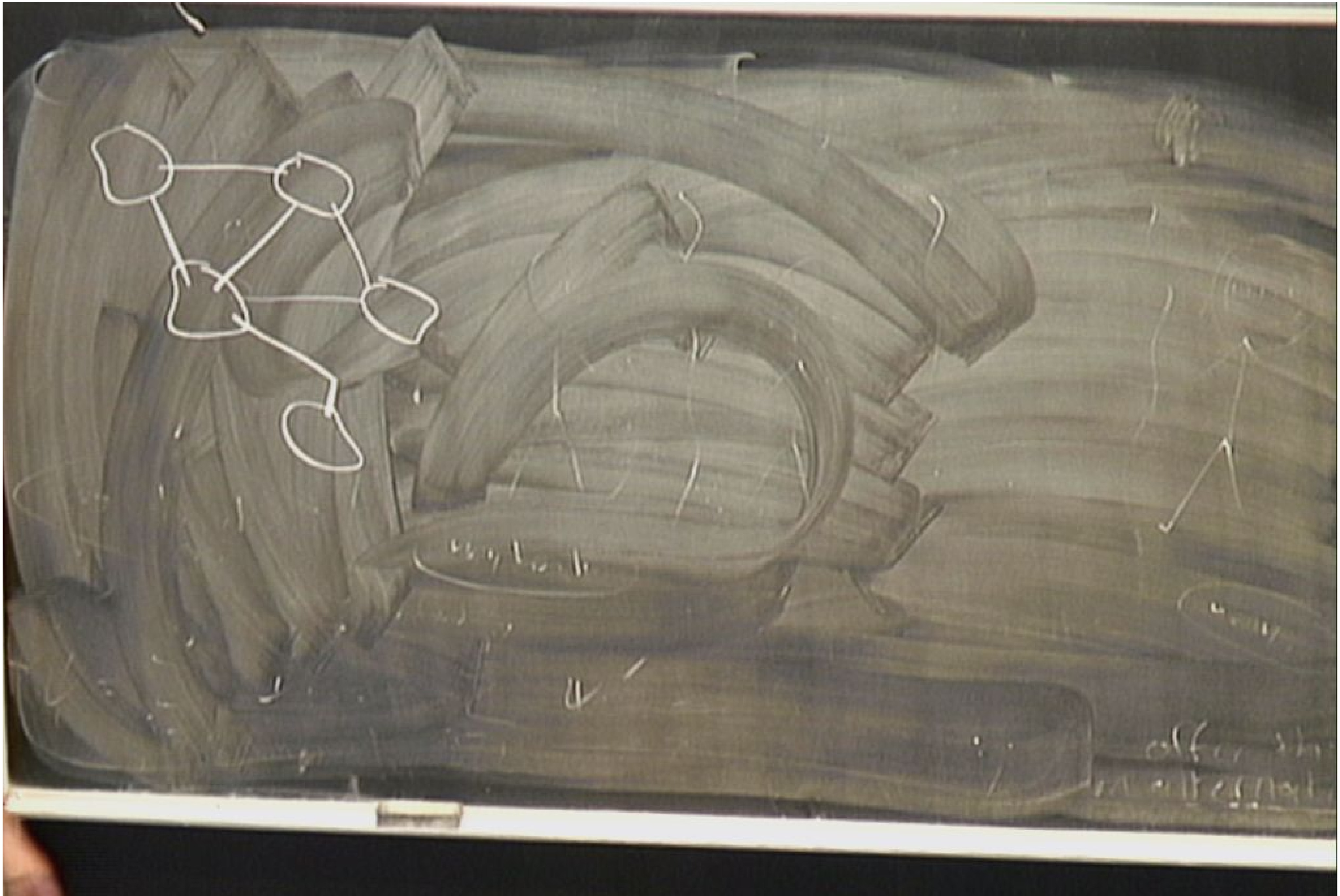
Title: Interaction and information flow between quantum systems (Part 1A)

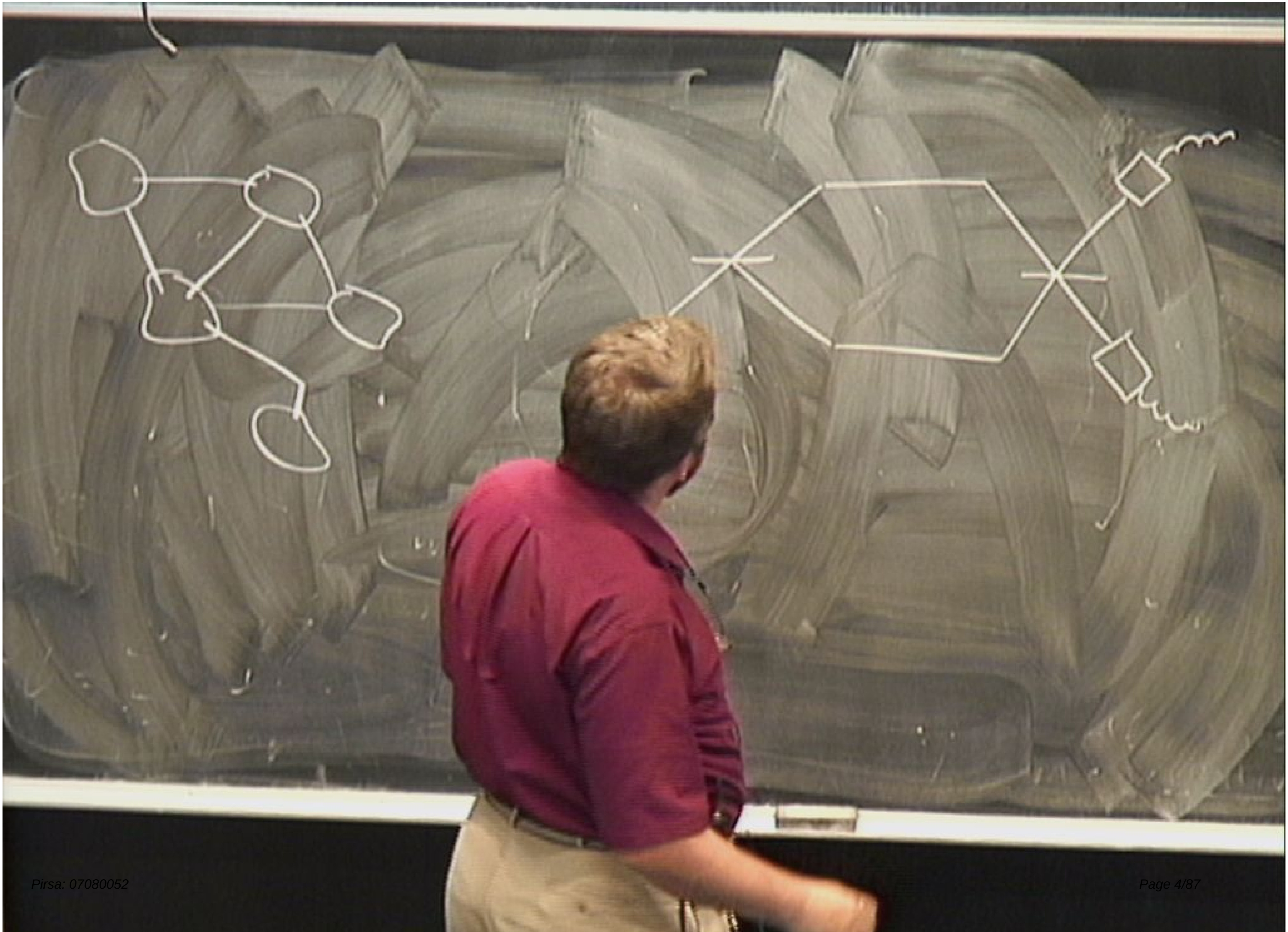
Date: Aug 30, 2007 03:15 PM

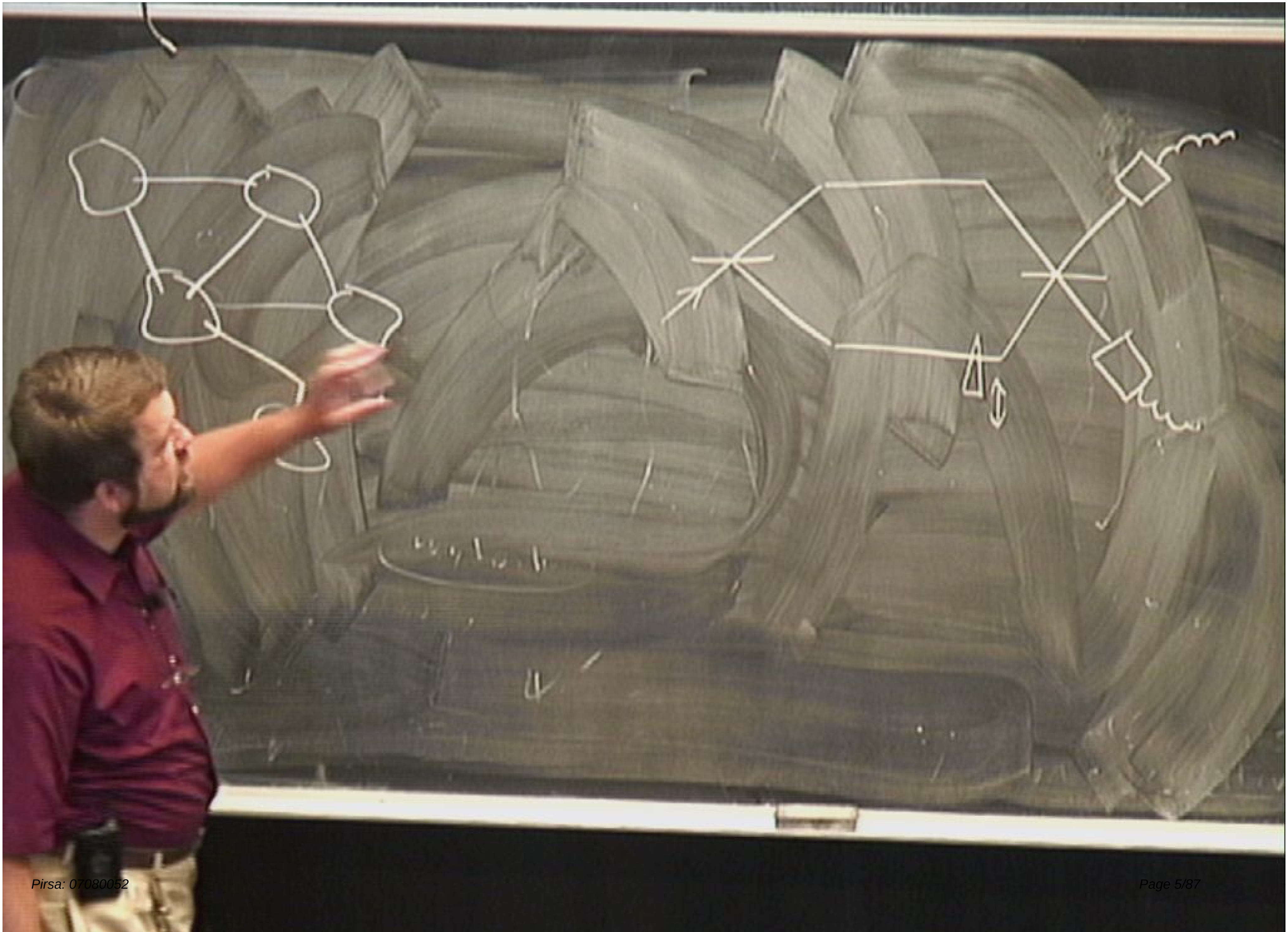
URL: http://pirsa.org/07080052

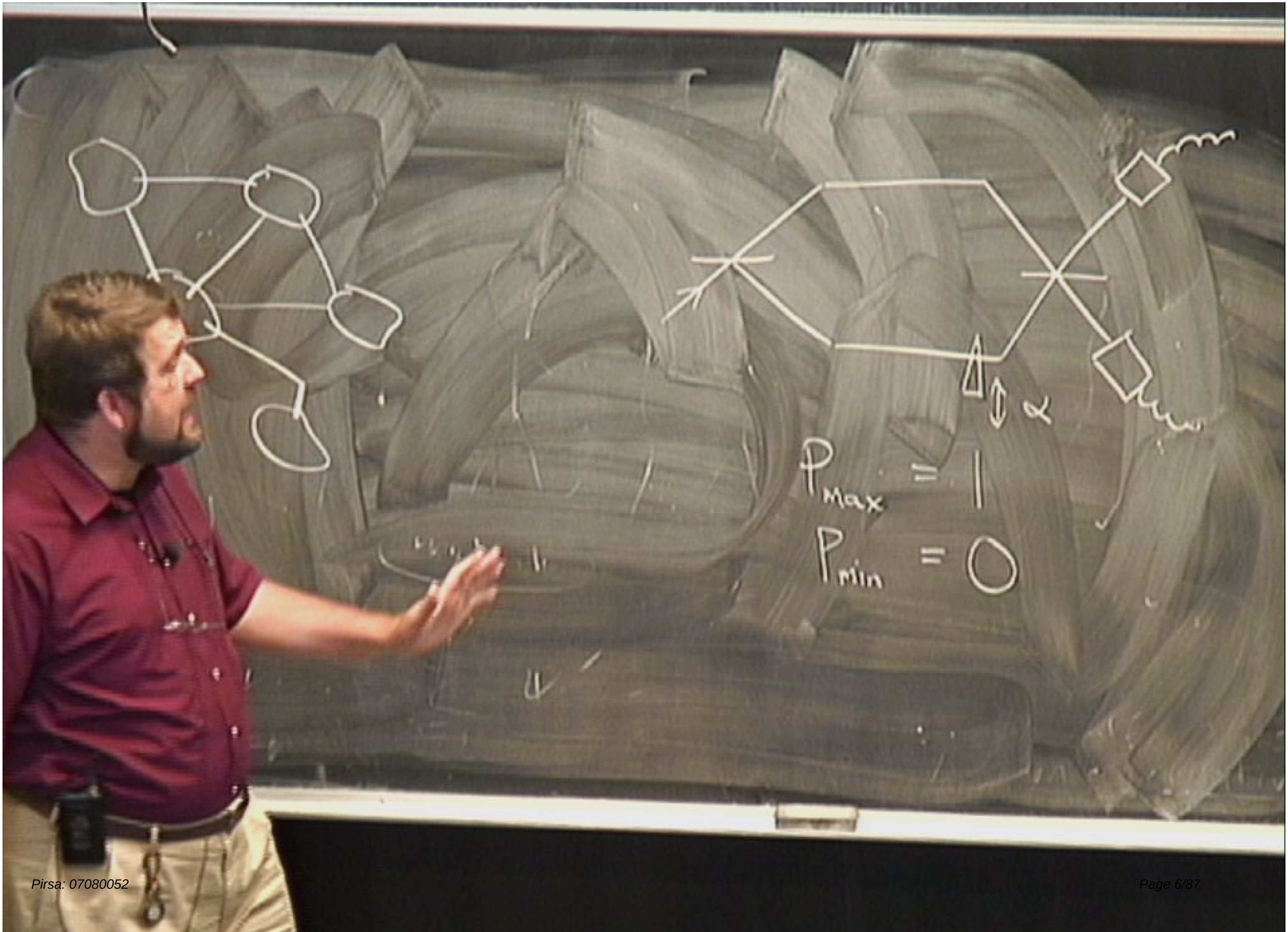
Abstract:

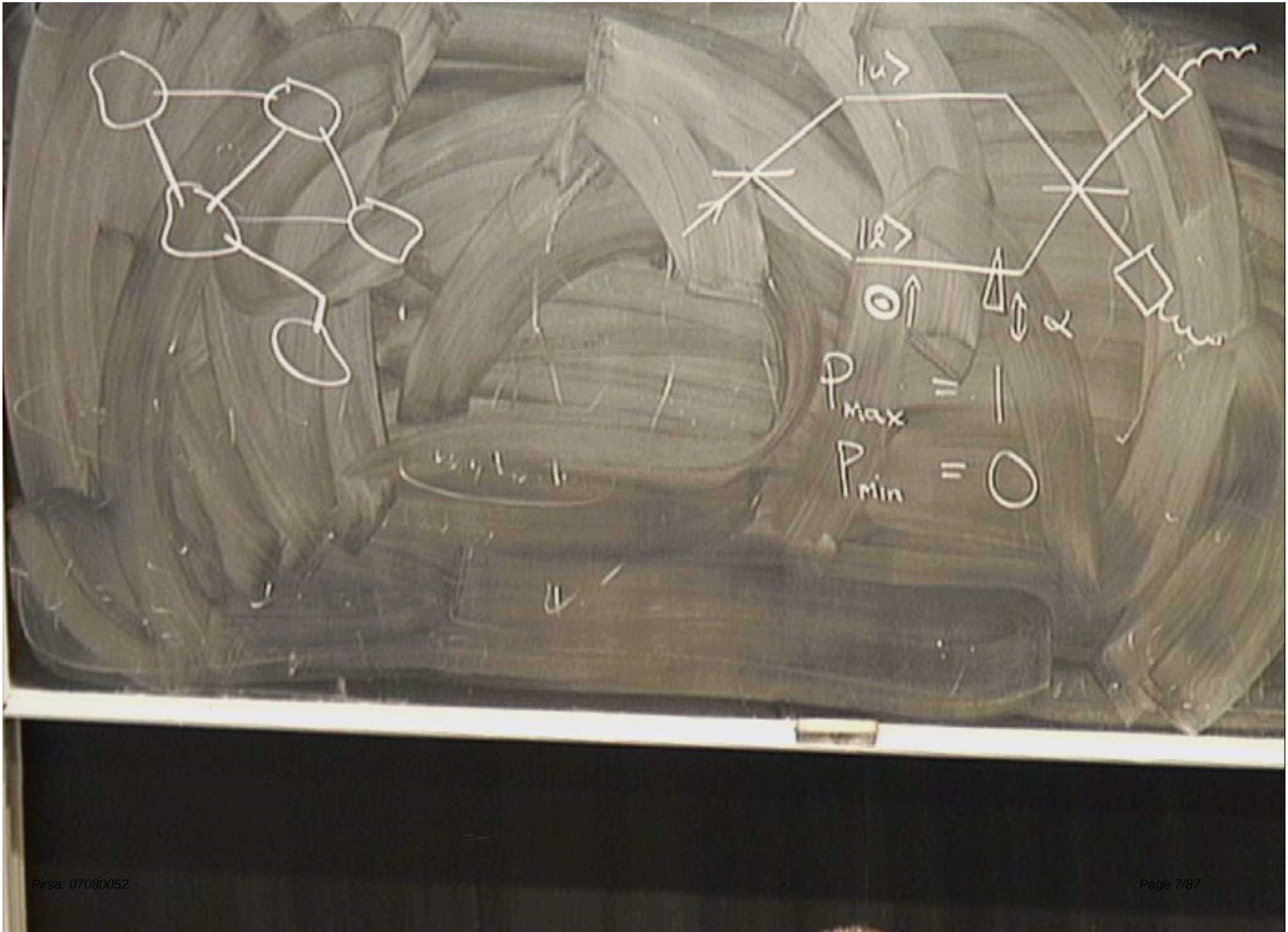


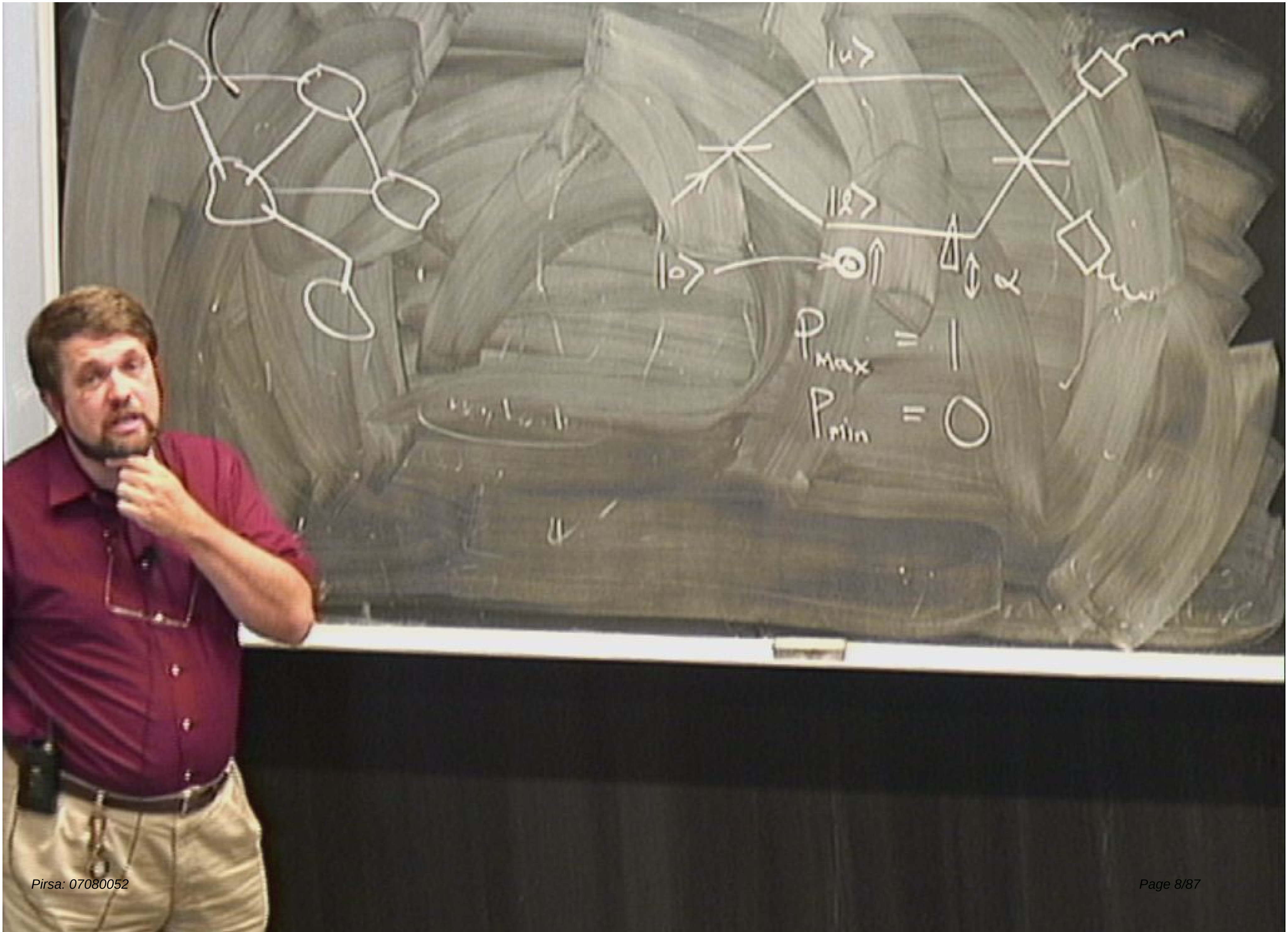


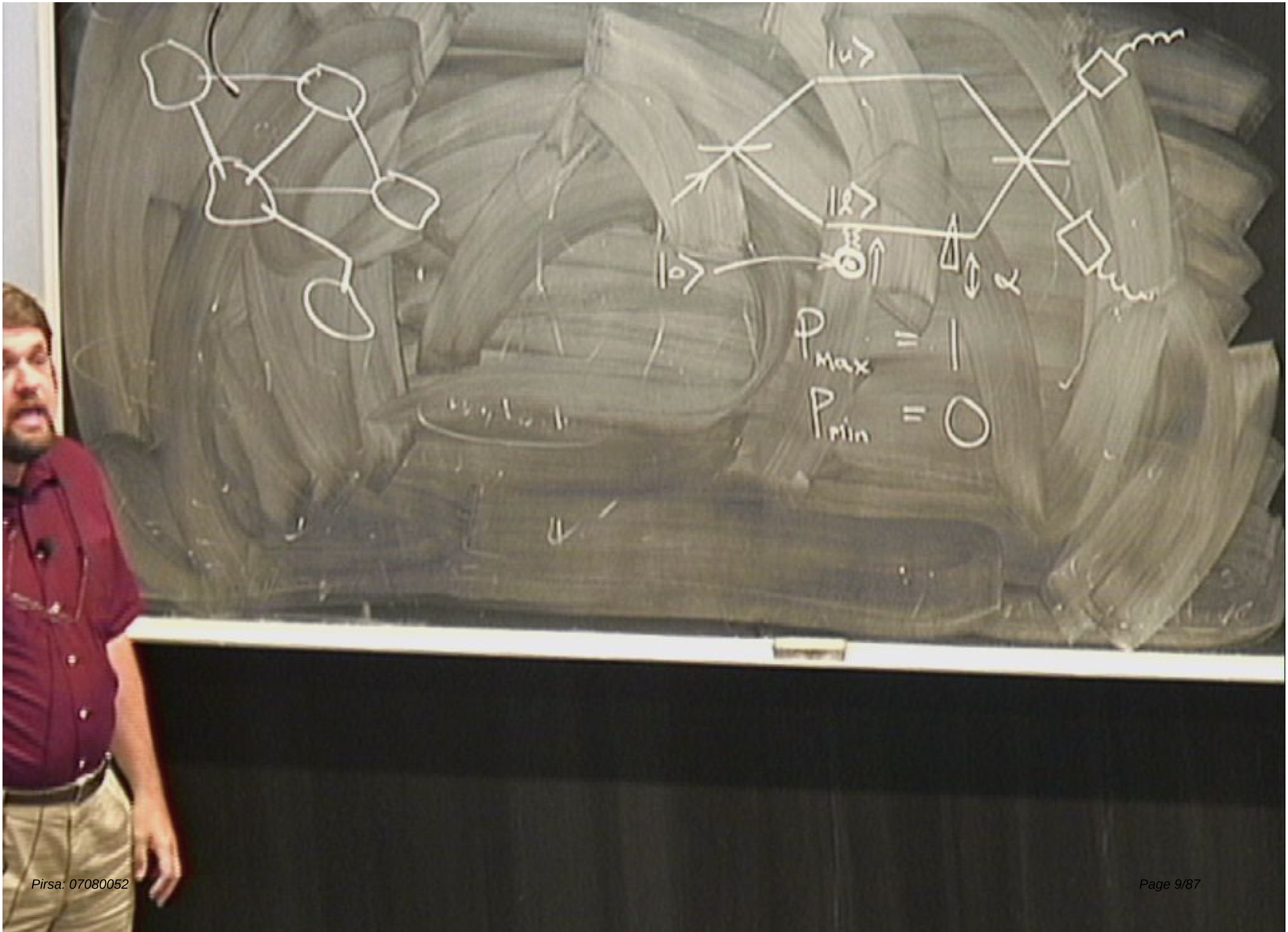


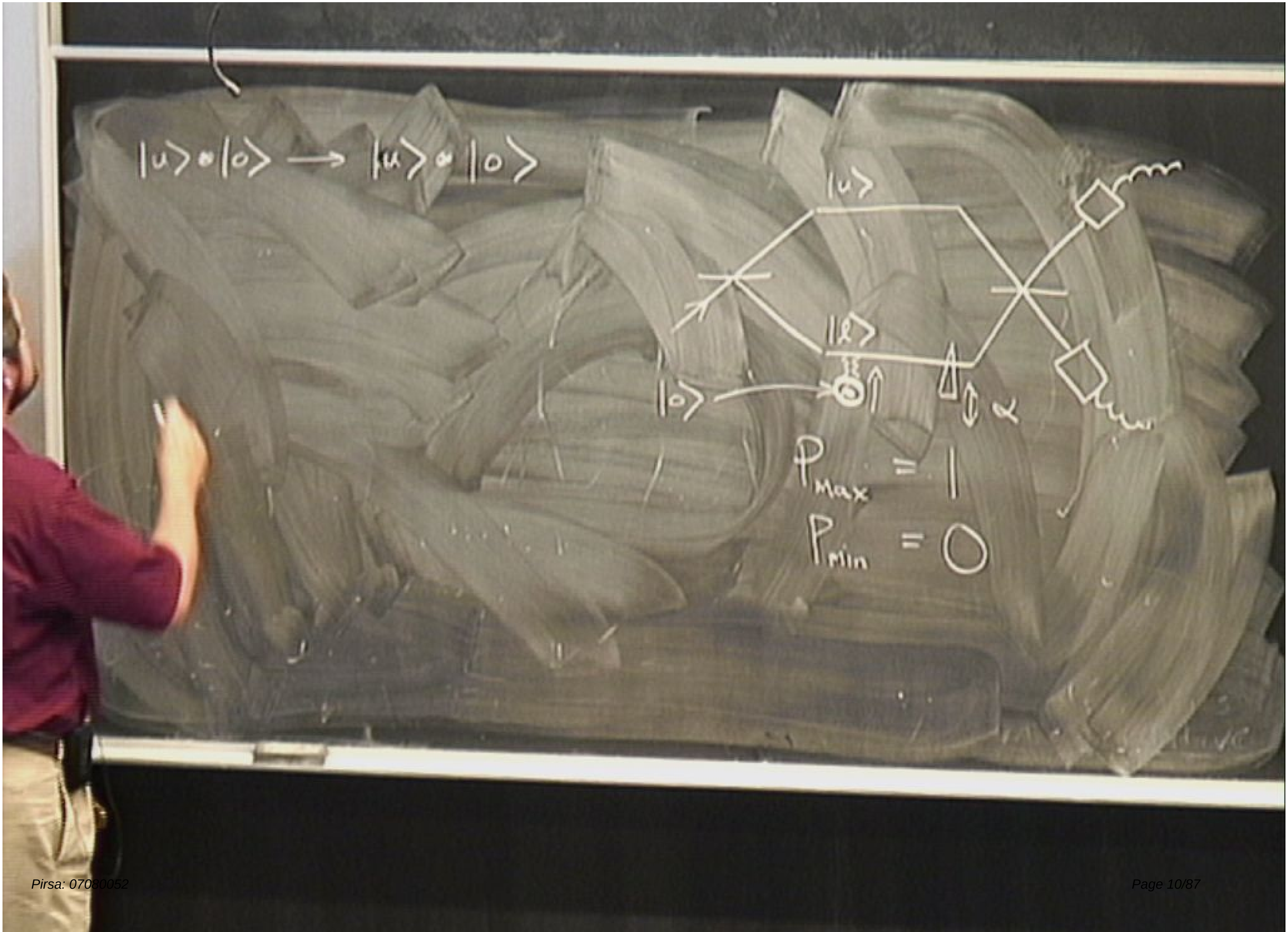


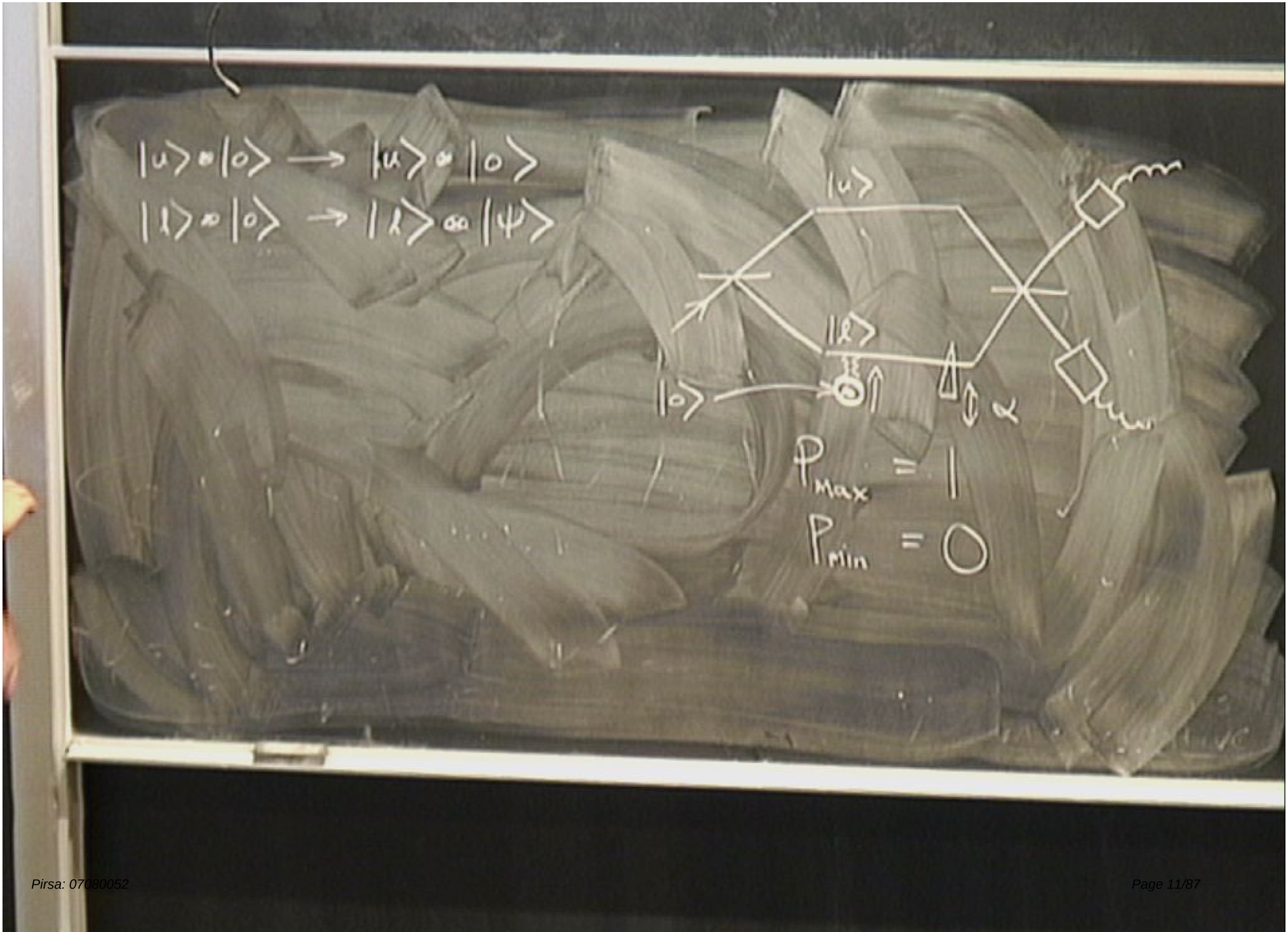

$P_{max} = 1$
$P_{min} = 0$
$\Delta \updownarrow \alpha$

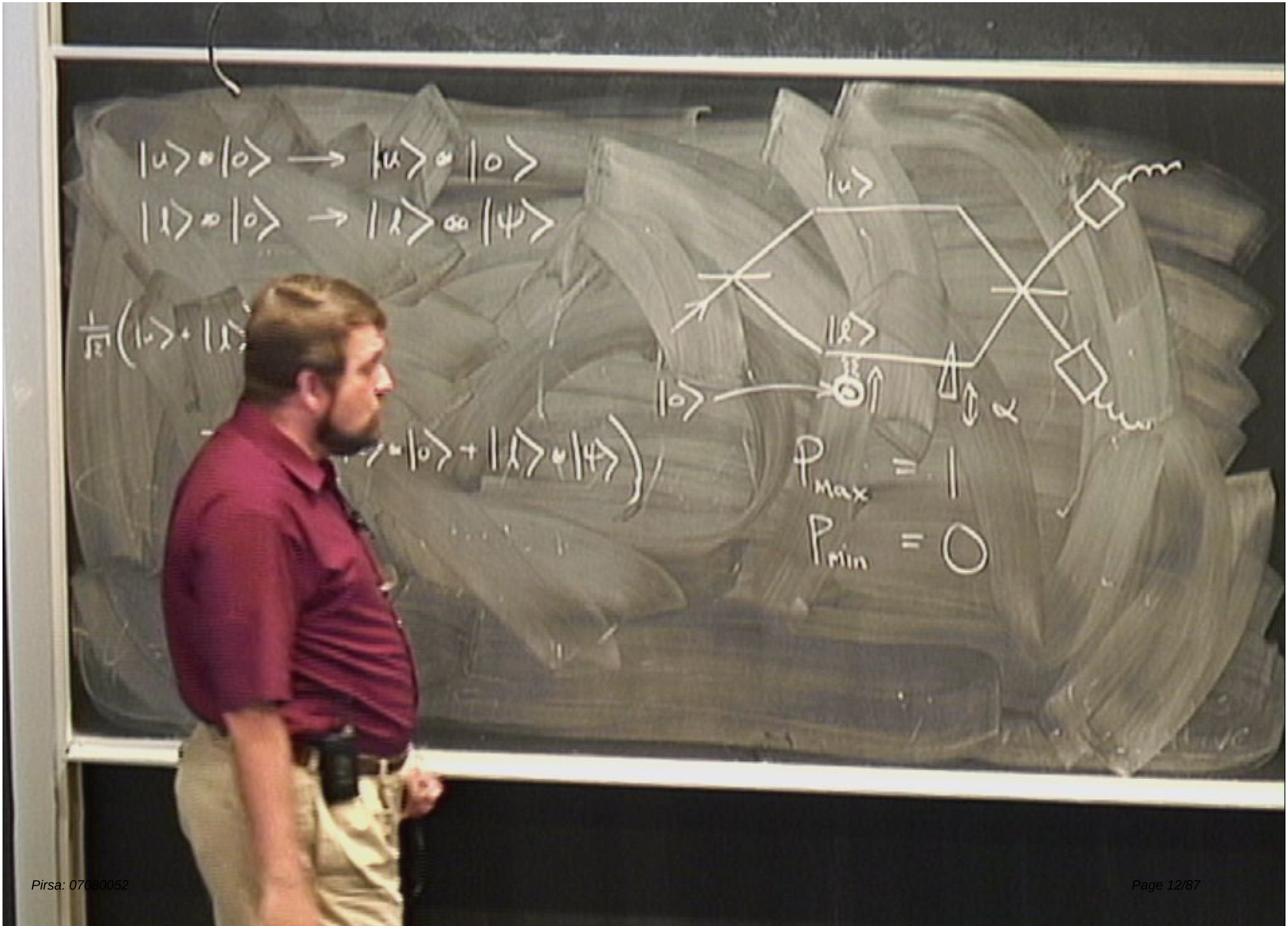

|u>
|ℓ>
α
P_max = 1
P_min = 0

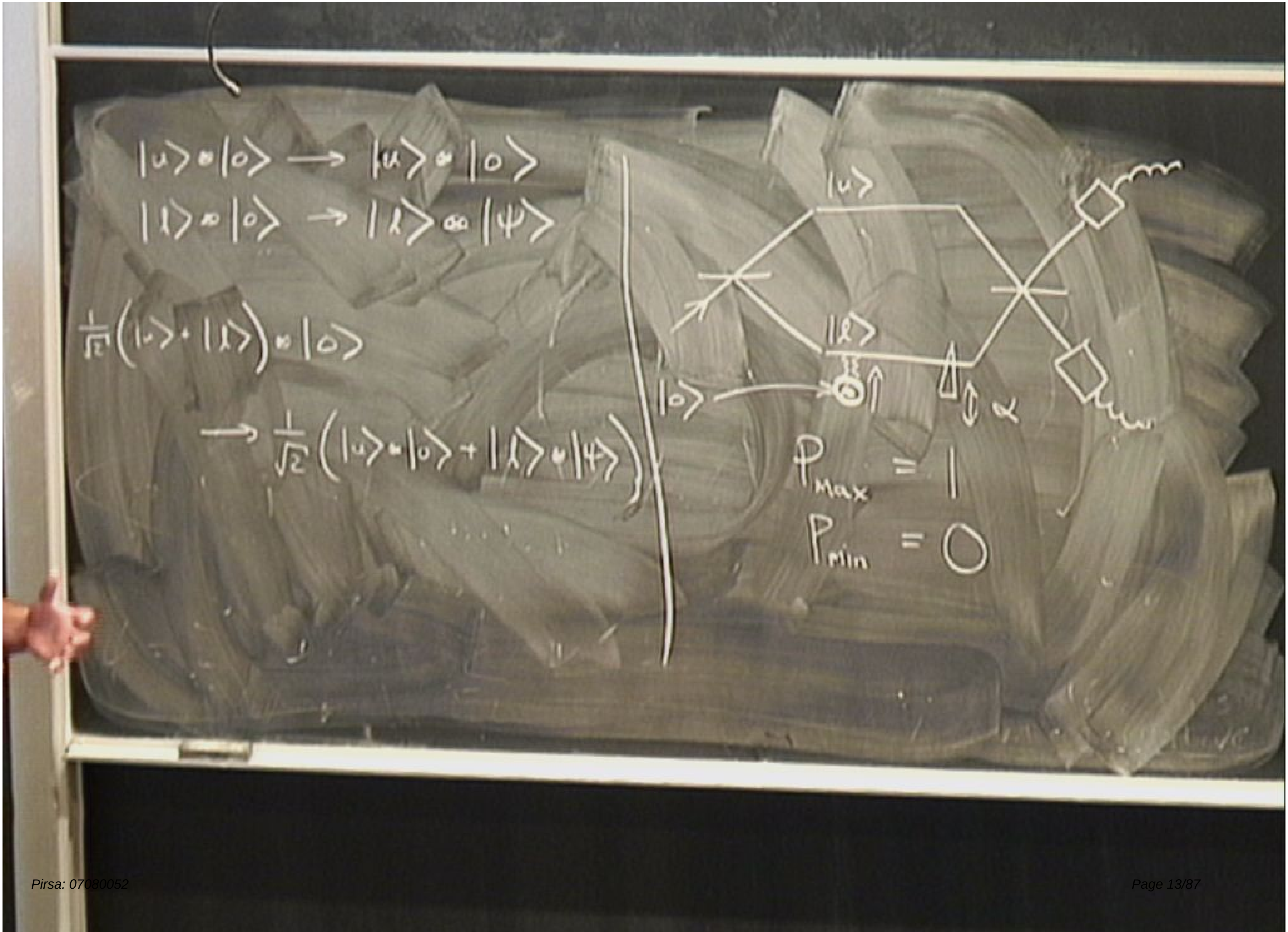

|u⟩
|ℓ⟩
|0⟩
α
Pmax = 1
Pmin = 0

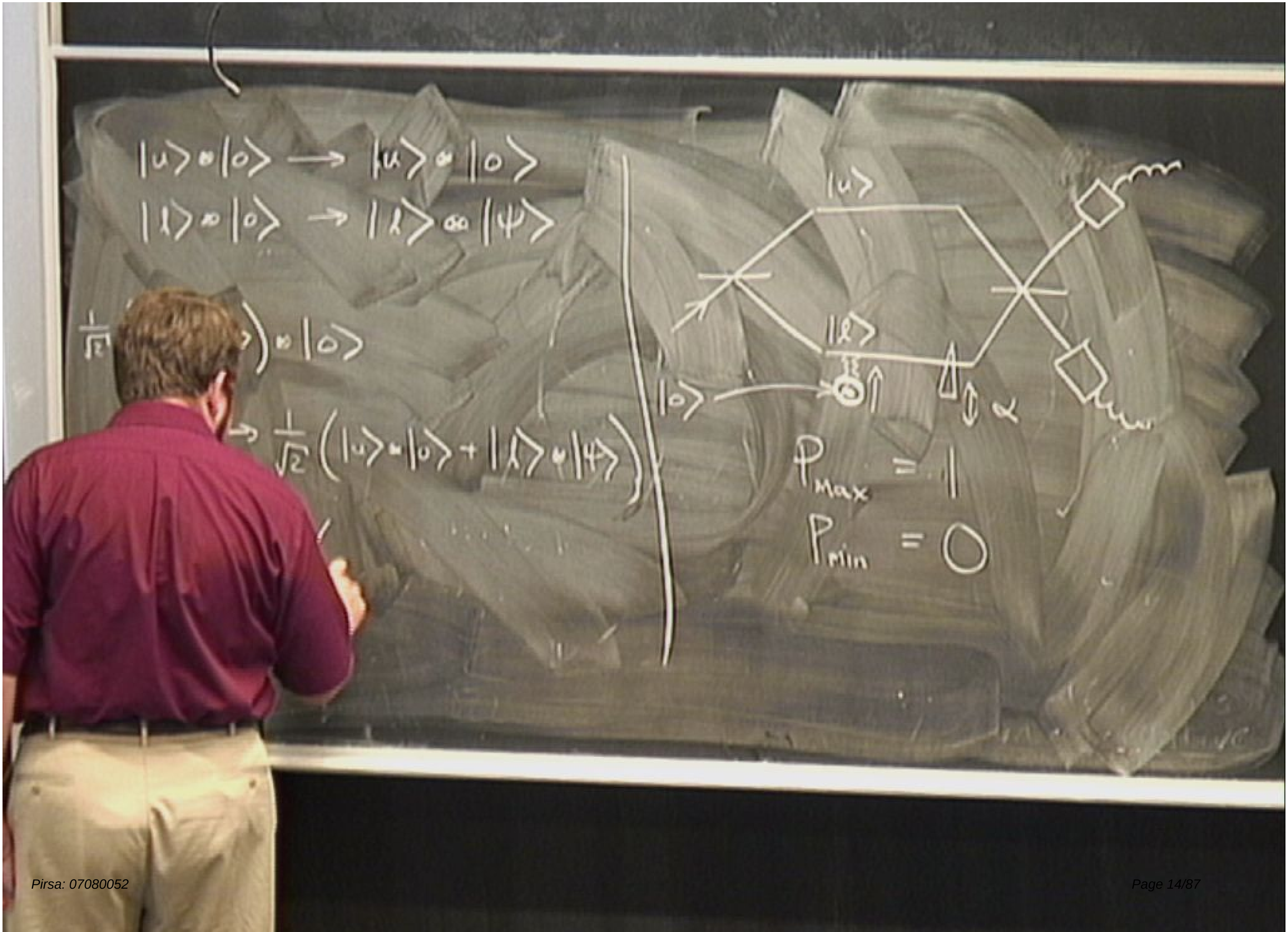

|u⟩
|ℓ⟩
|o⟩
α
$P_{max} = 1$
$P_{min} = 0$

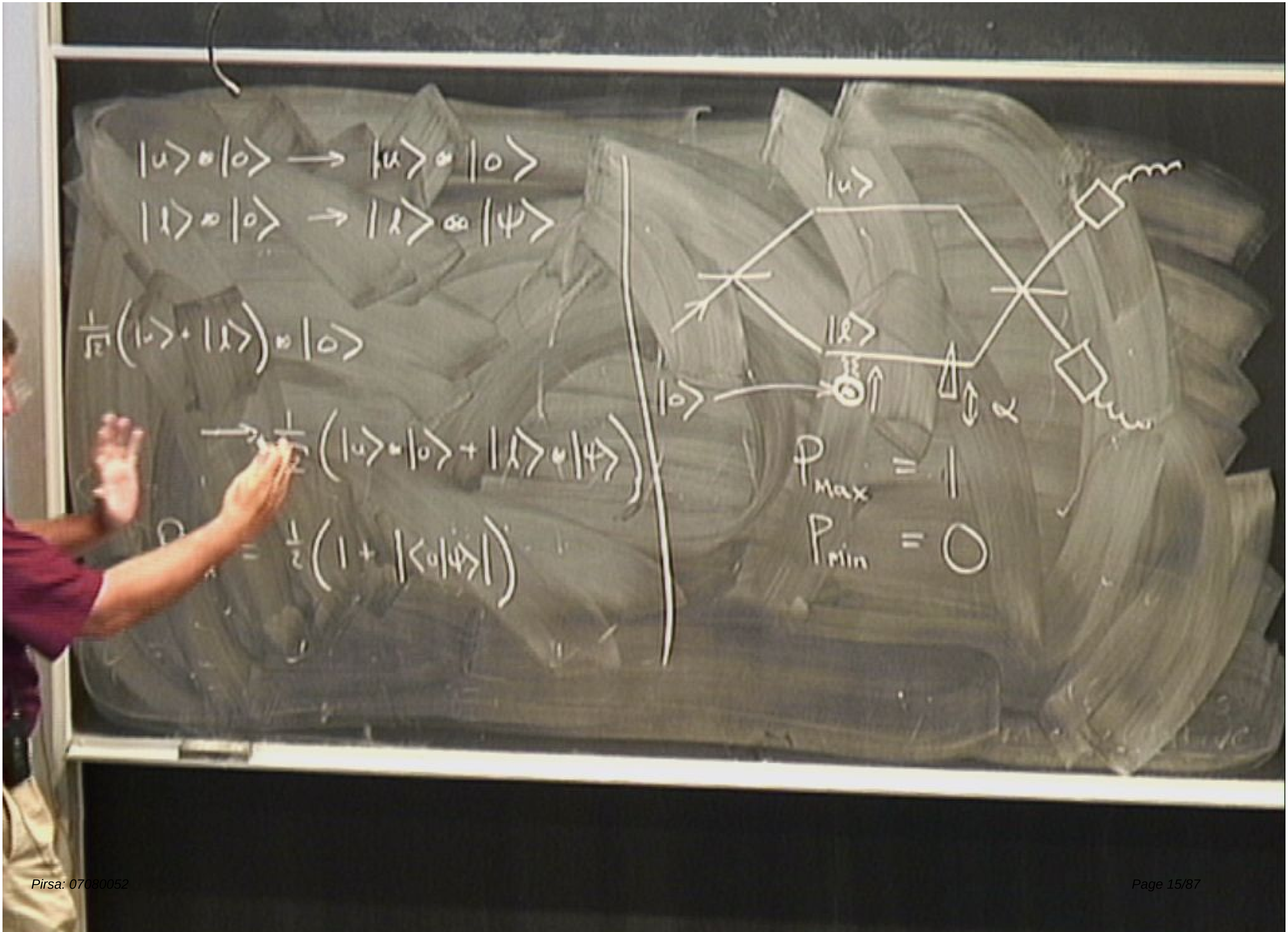

|u> ⊗ |0> → |u> ⊗ |0>
|u>
|ℓ>
|0>
α
P_max = 1
P_min = 0


$|u\rangle \otimes |0\rangle \rightarrow |u\rangle \otimes |0\rangle$
$|\ell\rangle \otimes |0\rangle \rightarrow |\ell\rangle \otimes |\psi\rangle$
$|u\rangle$
$|\ell\rangle$
$|0\rangle$
α
$P_{max} = 1$
$P_{min} = 0$


|u> ⊗ |0> → |u> ⊗ |0>
|λ> ⊗ |0> → |λ> ⊗ |ψ>
|u>
|λ>
|0>
α
P_max = 1
P_min = 0


|u> ⊗ |0> → |u> ⊗ |0>
|λ> ⊗ |0> → |λ> ⊗ |ψ>
1/√2 (|u> + |λ>) ⊗ |0>
→ 1/√2 (|u> ⊗ |0> + |λ> ⊗ |ψ>)
|u>
|λ>
|0>
α
P_max = 1
P_min = 0


$|u\rangle \otimes |0\rangle \longrightarrow |u\rangle \otimes |0\rangle$
$|\lambda\rangle \otimes |0\rangle \rightarrow |\lambda\rangle \otimes |\psi\rangle$
$\rightarrow \frac{1}{\sqrt{2}}\left(|u\rangle \otimes |0\rangle + |\lambda\rangle \otimes |\psi\rangle\right)$
$|u\rangle$
$|\lambda\rangle$
$|0\rangle$
α
$P_{max} = 1$
$P_{min} = 0$


|u⟩ ⊗ |0⟩ → |u⟩ ⊗ |0⟩
|λ⟩ ⊗ |0⟩ → |λ⟩ ⊗ |ψ⟩
1/√2 (|u⟩ + |λ⟩) ⊗ |0⟩
→ 1/√2 (|u⟩ ⊗ |0⟩ + |λ⟩ ⊗ |ψ⟩)
= 1/2 (1 + |⟨0|ψ⟩|)
|u⟩
|ℓ⟩
|0⟩
α
P_max = 1
P_min = 0

$$|u\rangle \otimes |0\rangle \longrightarrow |u\rangle \otimes |0\rangle$$

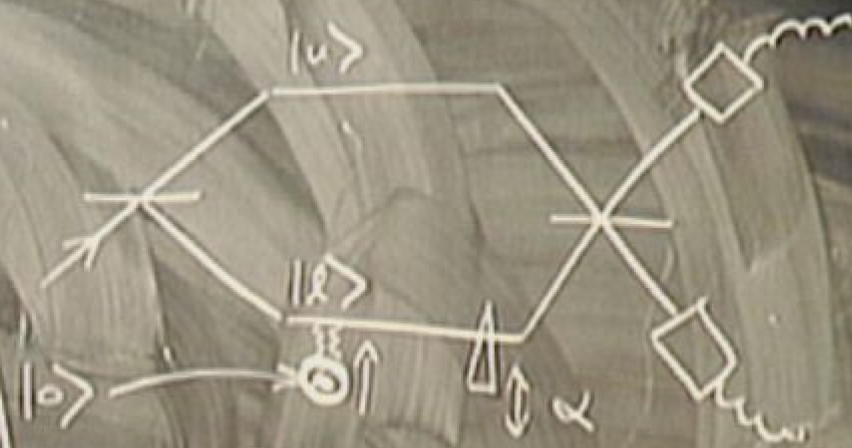
$$|\lambda\rangle \otimes |0\rangle \longrightarrow |\lambda\rangle \otimes |\psi\rangle$$

$$\frac{1}{\sqrt{2}}\left(|u\rangle + |\lambda\rangle\right) \otimes |0\rangle$$

$$\longrightarrow \frac{1}{\sqrt{2}}\left(|u\rangle \otimes |0\rangle + |\lambda\rangle \otimes |\psi\rangle\right)$$

$$P_{max} = \frac{1}{2}\left(1 + |\langle 0|\psi\rangle|\right)$$

$$P_{min} = \frac{1}{2}\left(1 - |\langle 0|\psi\rangle|\right)$$



$$P_{max} = 1$$

$$P_{min} = 0$$

$$|u\rangle \otimes |0\rangle \longrightarrow |u\rangle \otimes |0\rangle$$

$$|\ell\rangle \otimes |0\rangle \longrightarrow |\ell\rangle \otimes |\psi\rangle$$

$$\frac{1}{\sqrt{2}}\left(|u\rangle + |\ell\rangle\right) \otimes |0\rangle$$

$$\longrightarrow \frac{1}{\sqrt{2}}\left(|u\rangle \otimes |0\rangle + |\ell\rangle \otimes |\psi\rangle\right)$$

$$P_{max} = \frac{1}{2}\left(1 + |\langle 0|\psi\rangle|\right)$$

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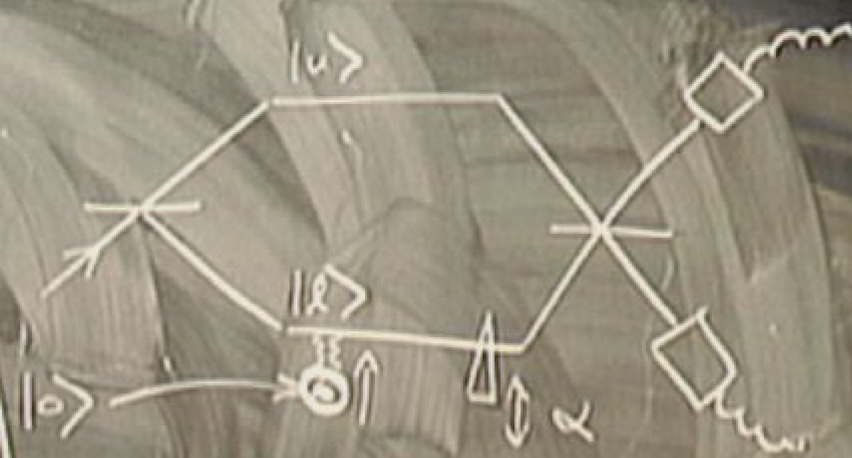
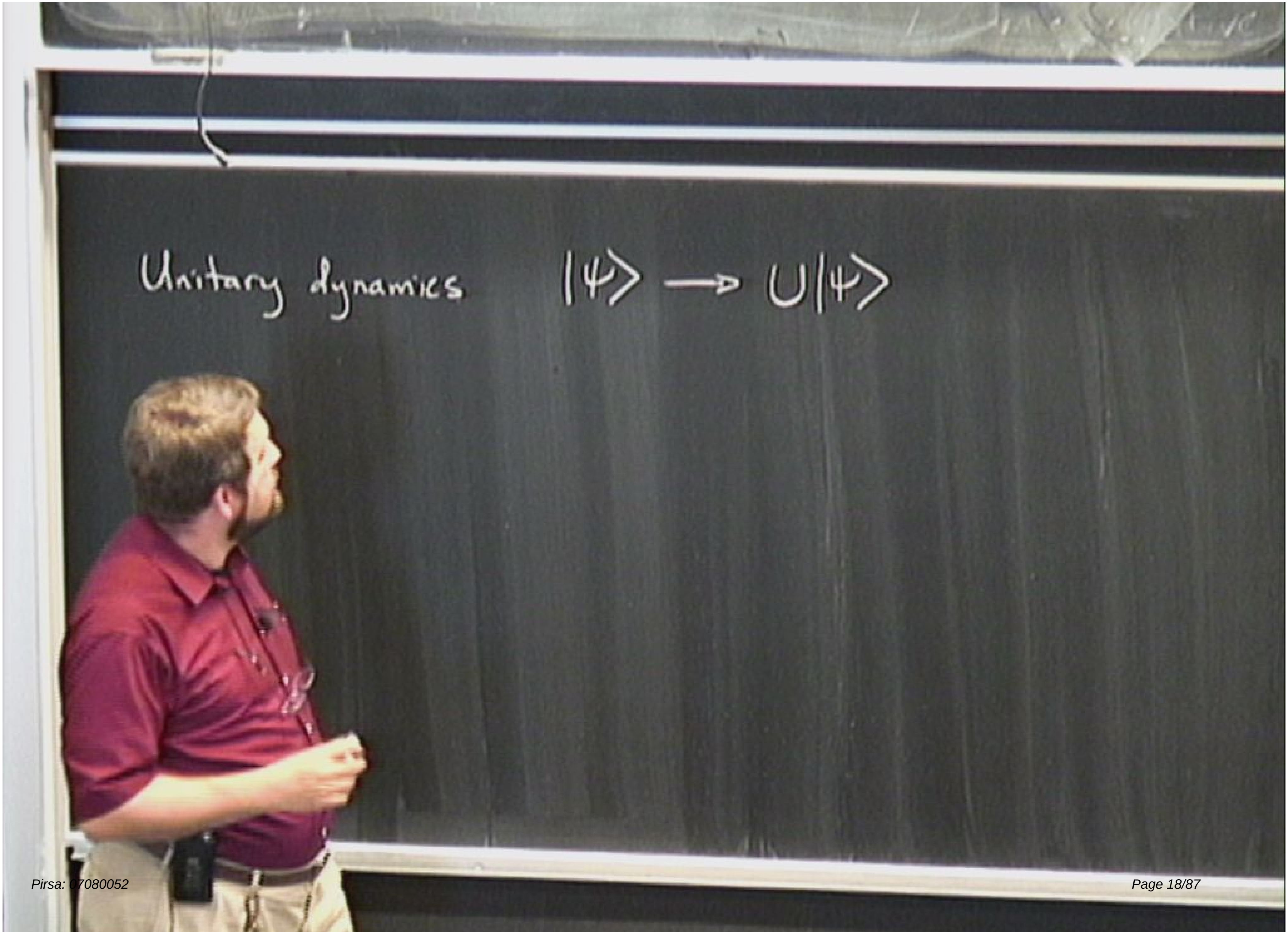
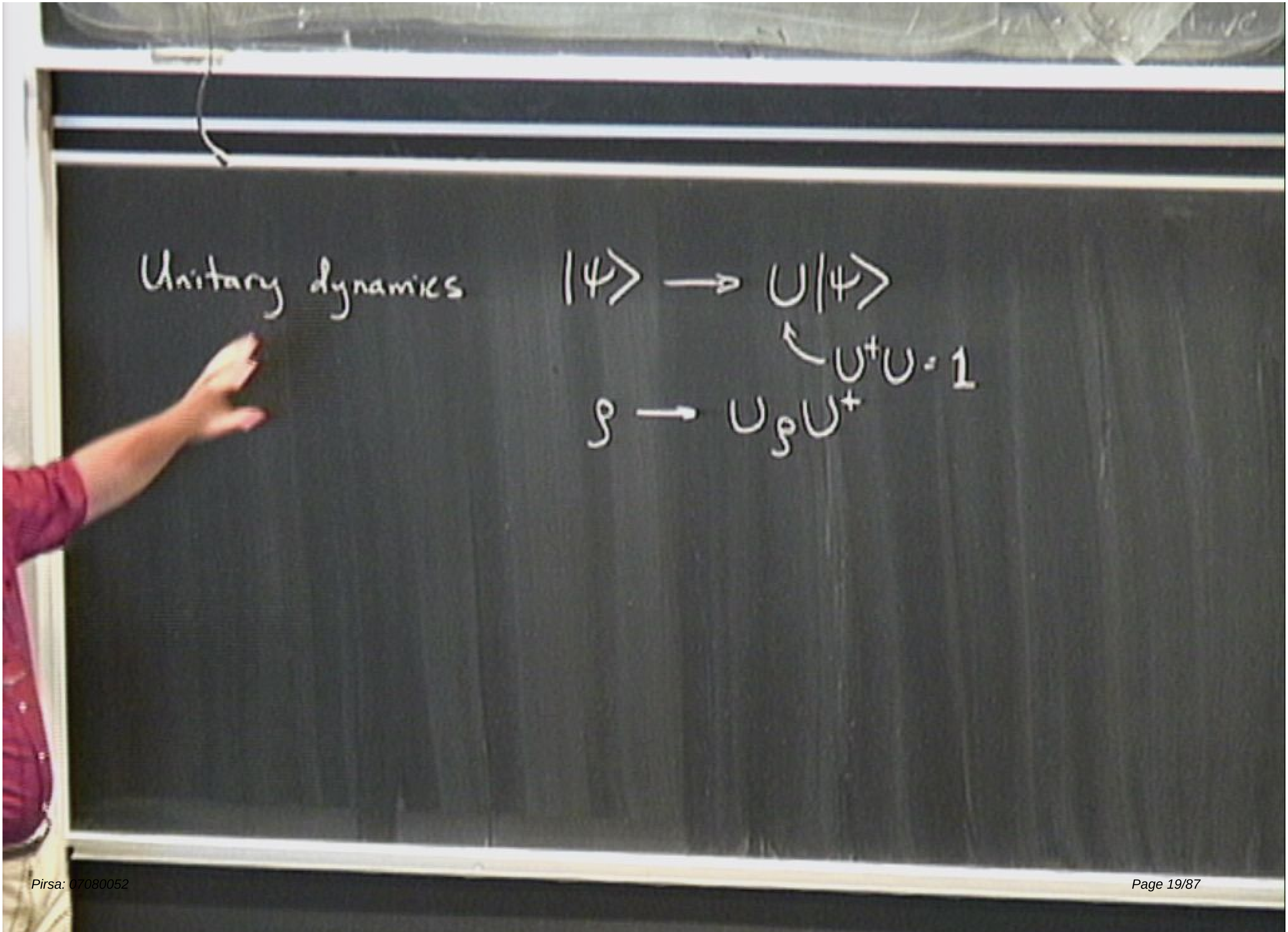


$$P_{max} = 1$$

$$P_{min} = 0$$


Unitary dynamics $|\psi\rangle \rightarrow U|\psi\rangle$


Unitary dynamics
$|\psi\rangle \longrightarrow U|\psi\rangle$
$U^{+}U = 1$
$\rho \longrightarrow U\rho U^{+}$

Unitary dynamics $\quad |\psi\rangle \longrightarrow U|\psi\rangle$

$$U^\dagger U = 1$$

$$\rho \longrightarrow U \rho U^\dagger$$

Open system

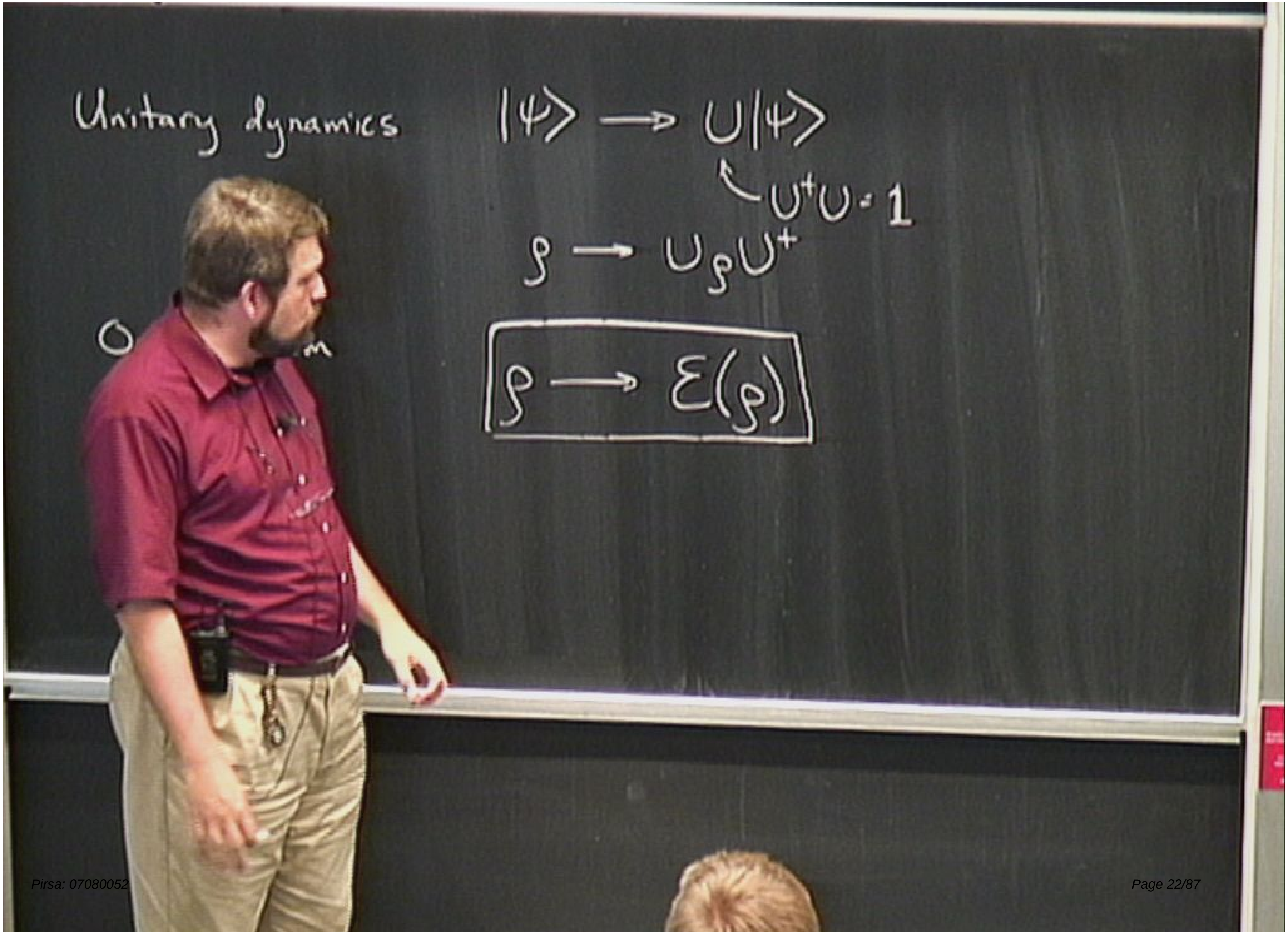
Unitary dynamics $|\psi\rangle \longrightarrow U|\psi\rangle$

$U^\dagger U = 1$

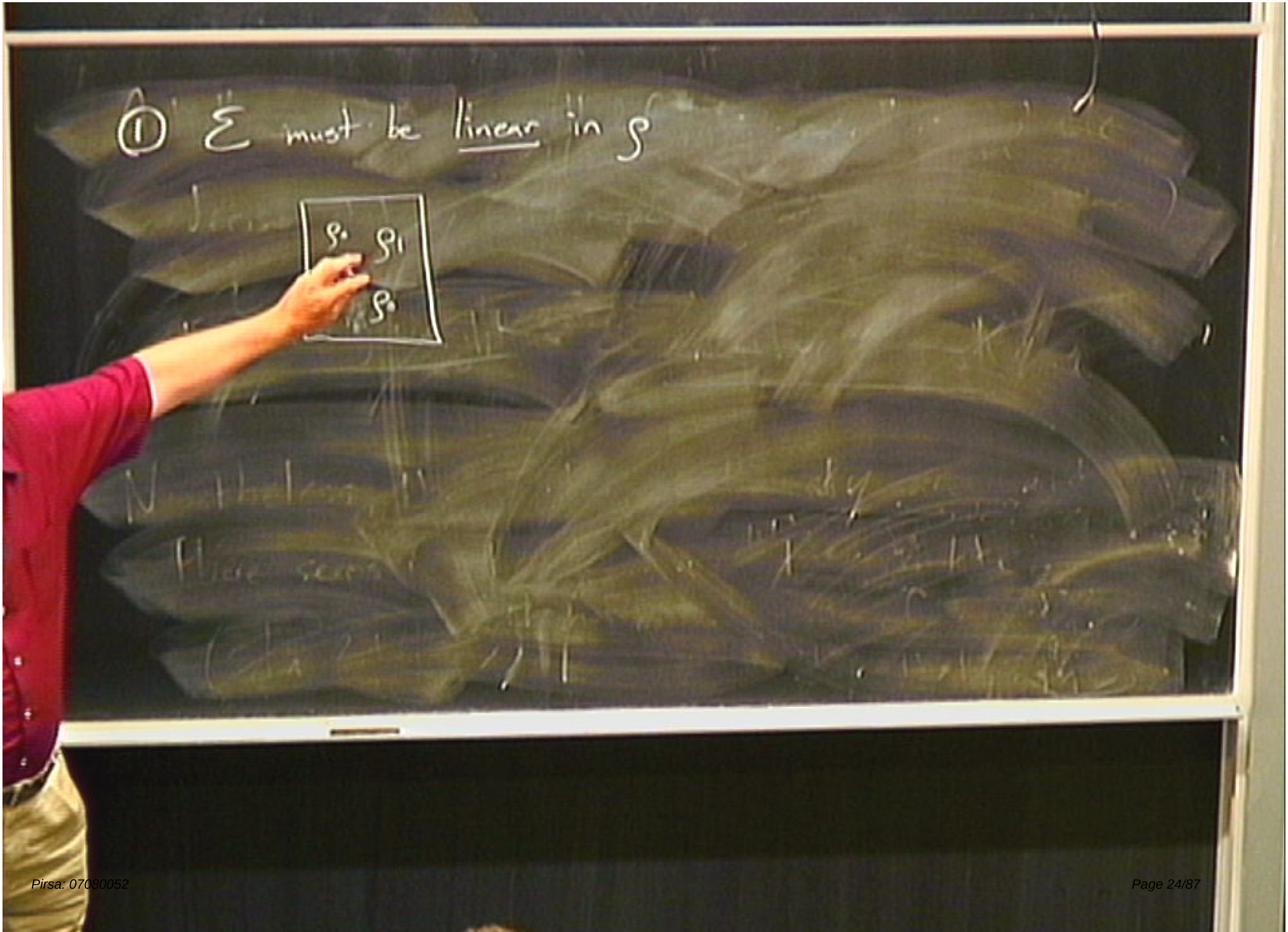
$$\rho \longrightarrow U\rho U^\dagger$$

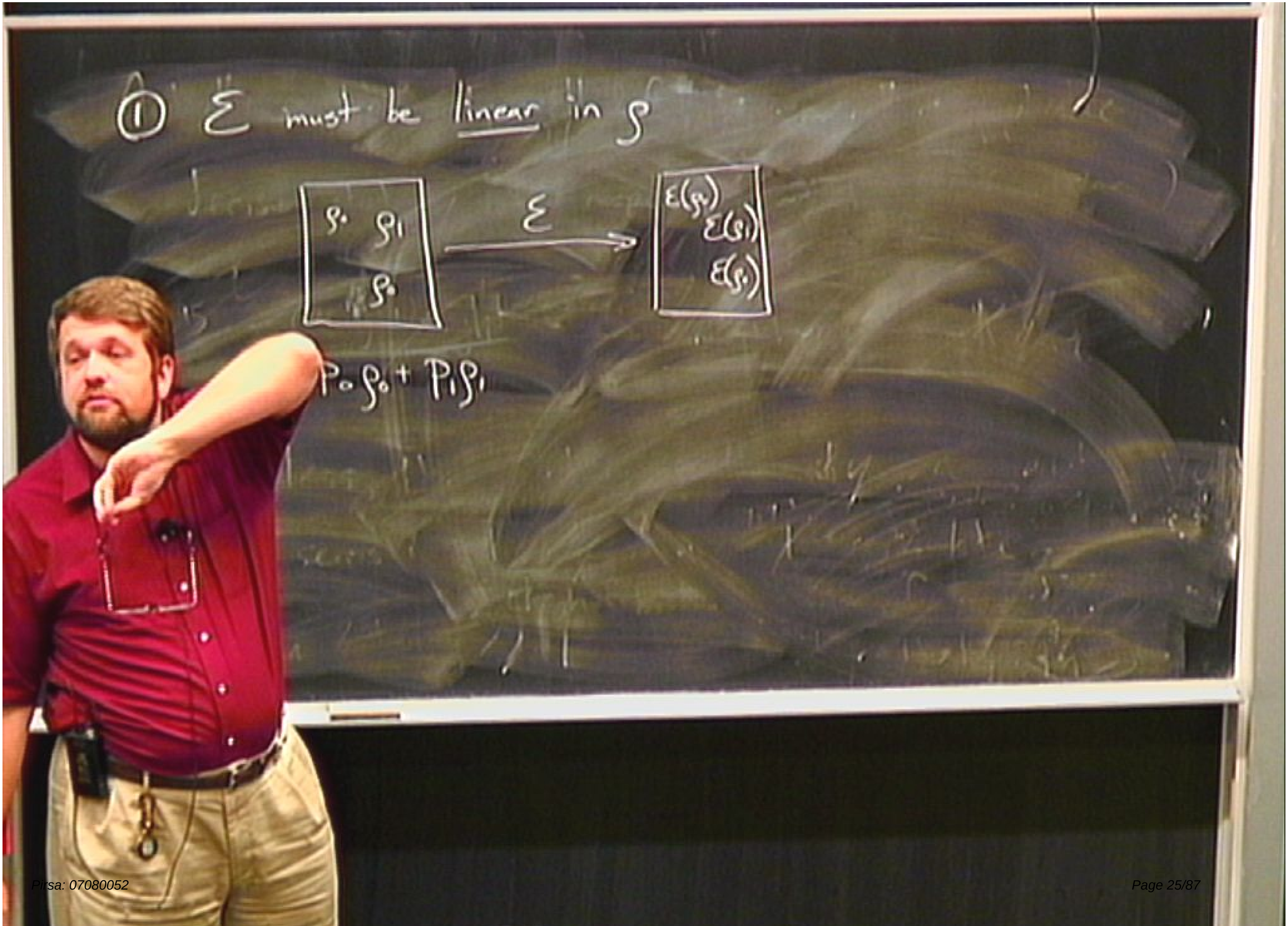
Open system

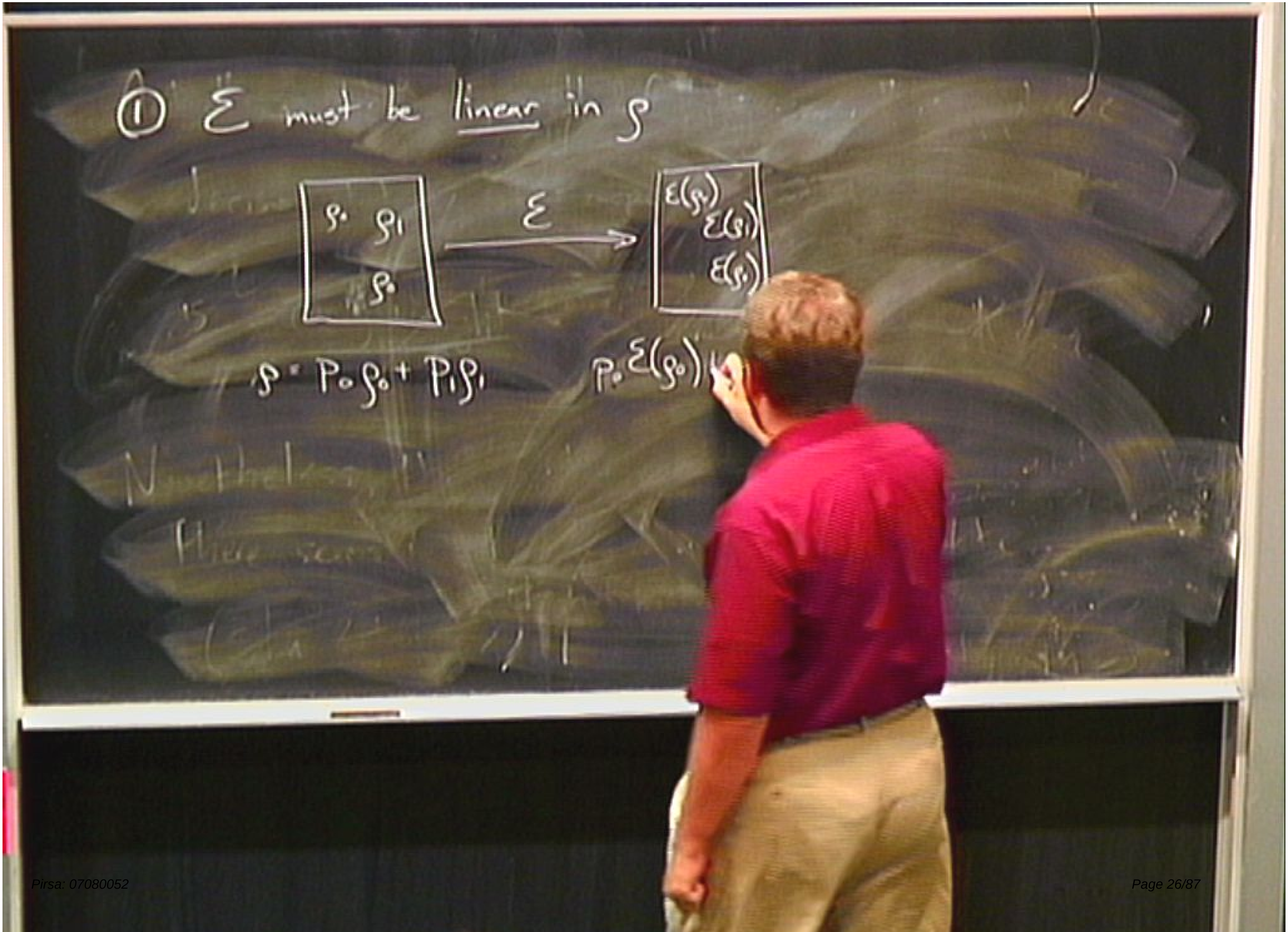
$$\rho \longrightarrow \mathcal{E}(\rho)$$

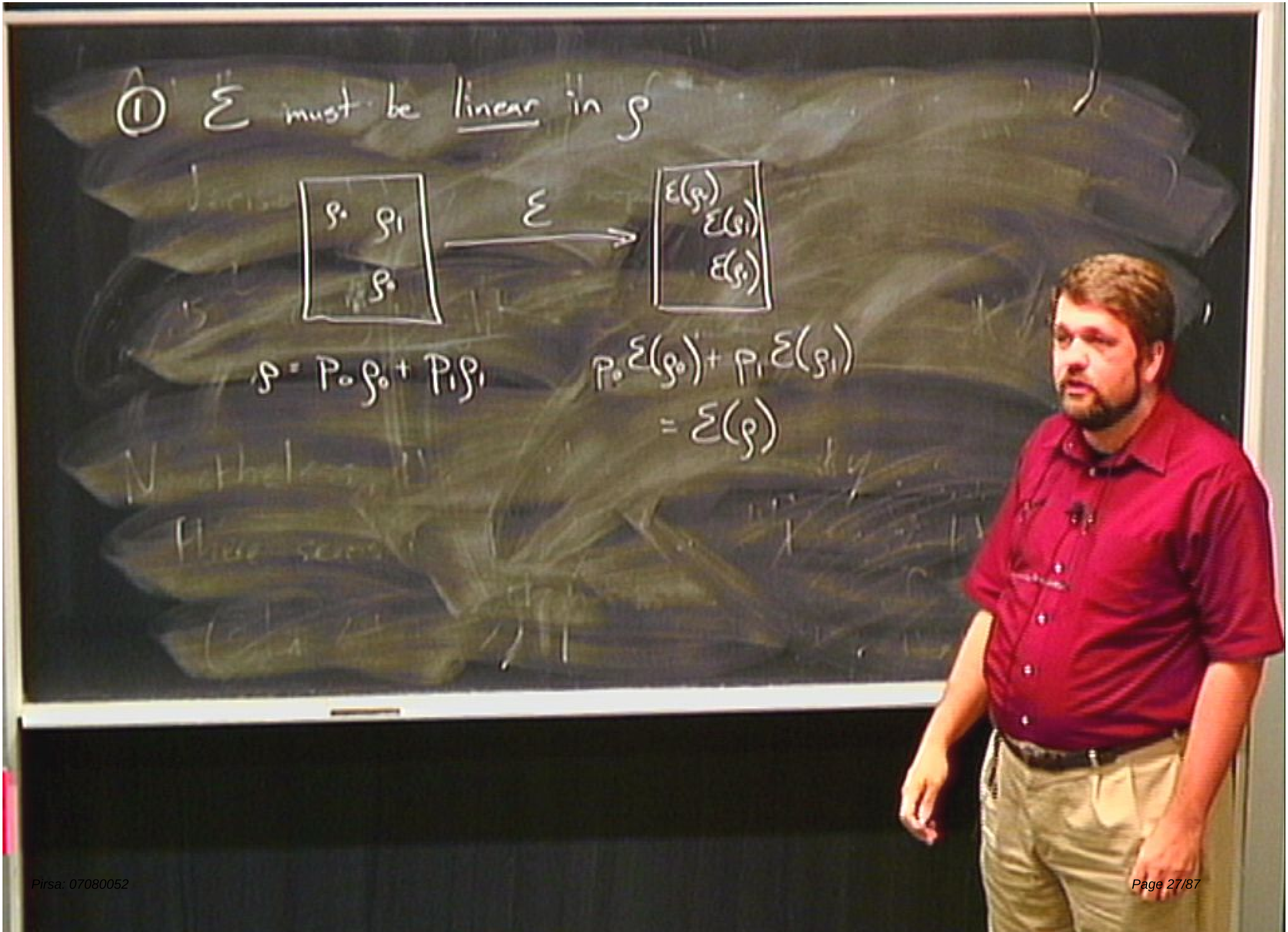

Unitary dynamics
|ψ> → U|ψ>
U†U = 1
ρ → UρU†
ρ → ε(ρ)

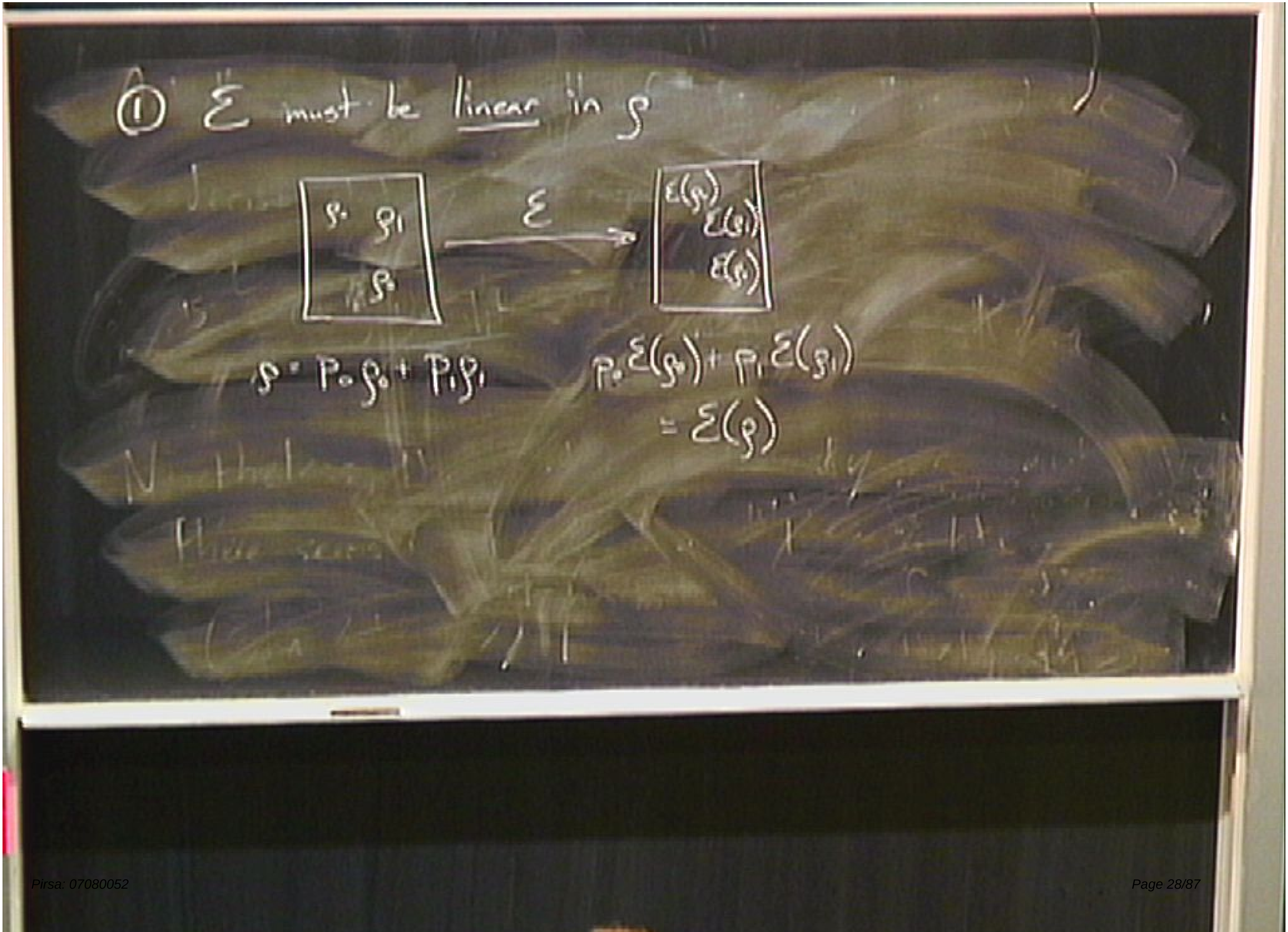
① $\mathcal{E}$ must be linear in ρ

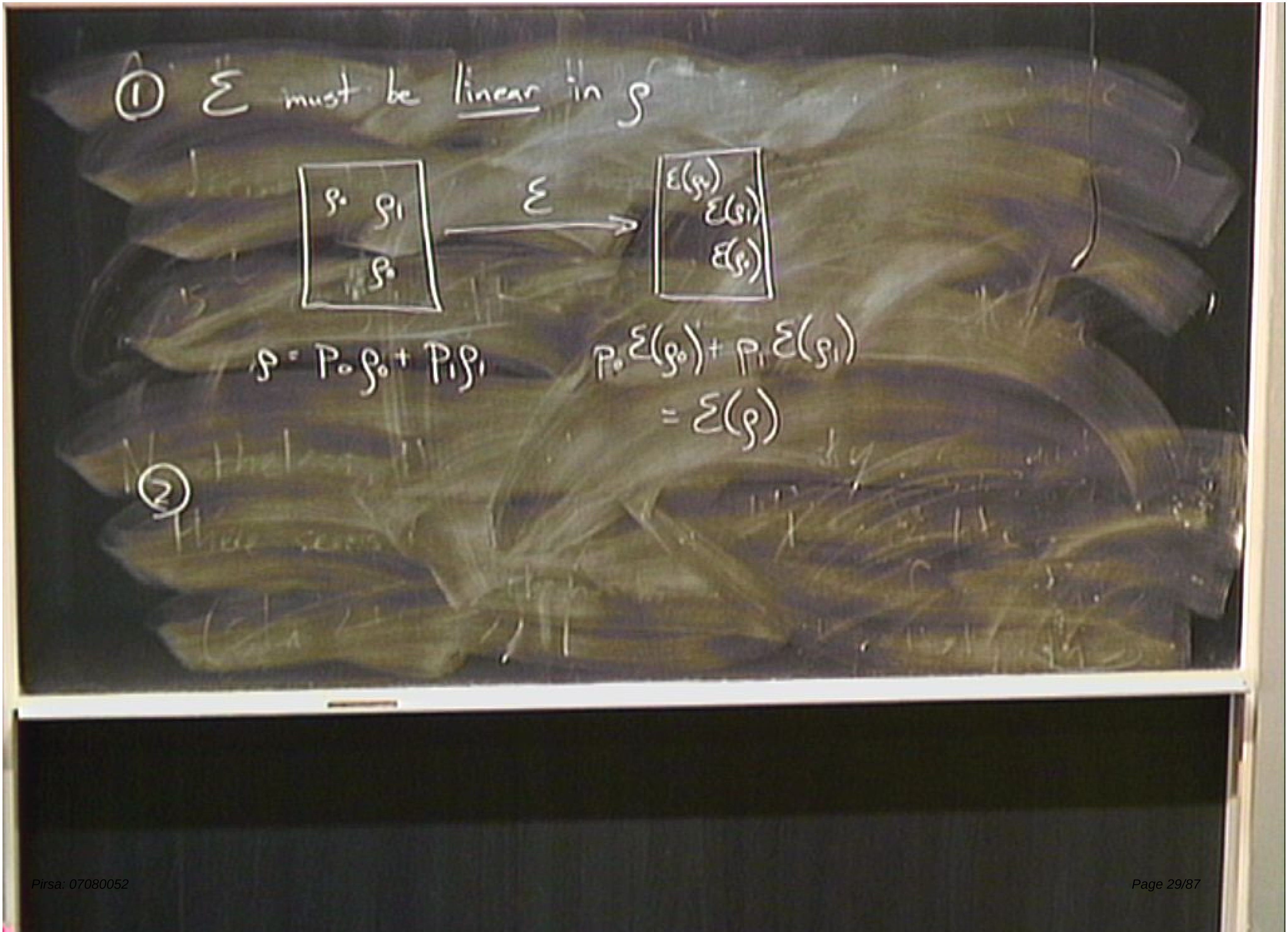

① $\mathcal{E}$ must be linear in ρ
ρ_0
ρ_1


① $\mathcal{E}$ must be linear in ρ
ρ_0 ρ_1
ρ
$\mathcal{E}$
$\mathcal{E}(\rho_0)$
$\mathcal{E}(\rho_1)$
$\mathcal{E}(\rho)$
$p_0\rho_0 + p_1\rho_1$

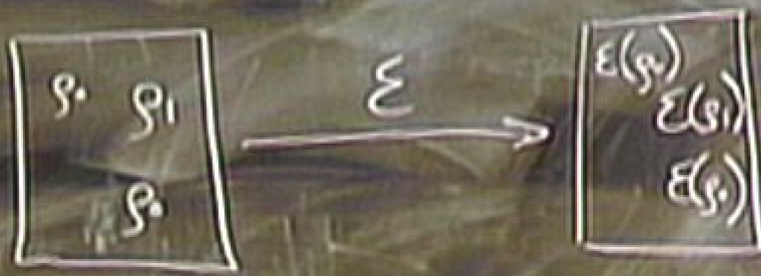

① ℰ must be linear in ρ
ρ₀ ρ₁
ρ₀
ℰ
ℰ(ρ₀)
ℰ(ρ₁)
ℰ(ρ₀)
ρ = p₀ρ₀ + p₁ρ₁
p₀ℰ(ρ₀)


① $\mathcal{E}$ must be linear in ρ
ρ_0 ρ_1
ρ_0
$\mathcal{E}$
$\mathcal{E}(\rho_0)$
$\mathcal{E}(\rho_1)$
$\mathcal{E}(\rho_0)$
$\rho = p_0\rho_0 + p_1\rho_1$
$p_0\mathcal{E}(\rho_0) + p_1\mathcal{E}(\rho_1)$
$= \mathcal{E}(\rho)$


① ε must be linear in ρ
ρ₀ ρ₁
p₀ρ₀
ε
ε(ρ₀)
ε(ρ₁)
ε(ρ₀)
ρ = p₀ρ₀ + p₁ρ₁
p₀ε(ρ₀) + p₁ε(ρ₁)
= ε(ρ)


① ℰ must be linear in ρ
ρ₀ ρ₁ ρ₀
ℰ
ℰ(ρ₀) ℰ(ρ₁) ℰ(ρ₀)
ρ = p₀ρ₀ + p₁ρ₁
p₀ℰ(ρ₀) + p₁ℰ(ρ₁)
= ℰ(ρ)
②

① $\mathcal{E}$ must be linear in ρ

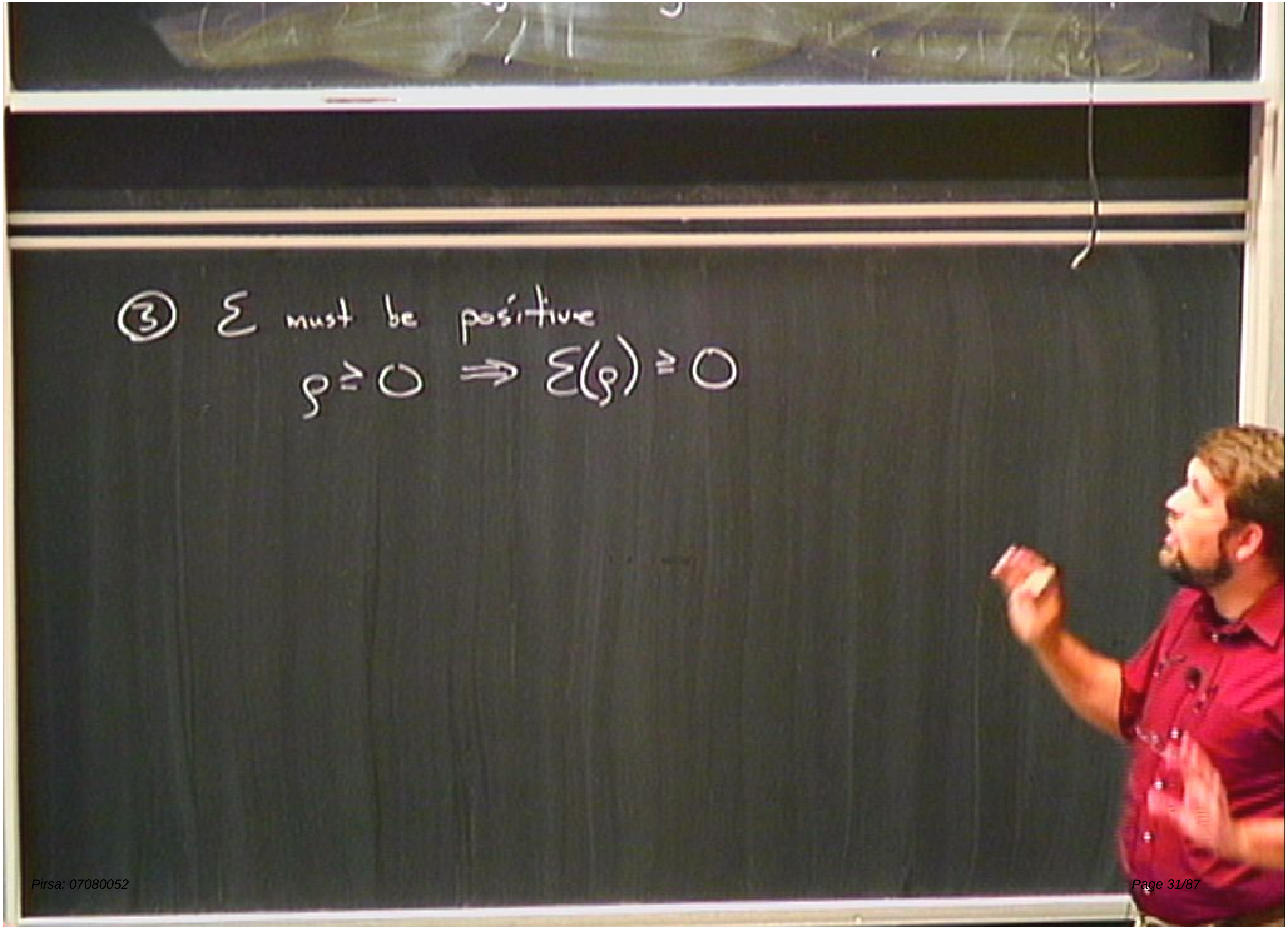


$\rho = p_0 \rho_0 + p_1 \rho_1$ $\qquad$ $p_0 \mathcal{E}(\rho_0) + p_1 \mathcal{E}(\rho_1)$

$= \mathcal{E}(\rho)$

② $\mathcal{E}$ must be trace-preserving

$$\mathrm{tr}\, \mathcal{E}(\rho) = \mathrm{tr}\, \rho$$


③ $\mathcal{E}$ must be positive
$\rho \geq 0 \Rightarrow \mathcal{E}(\rho) \geq 0$

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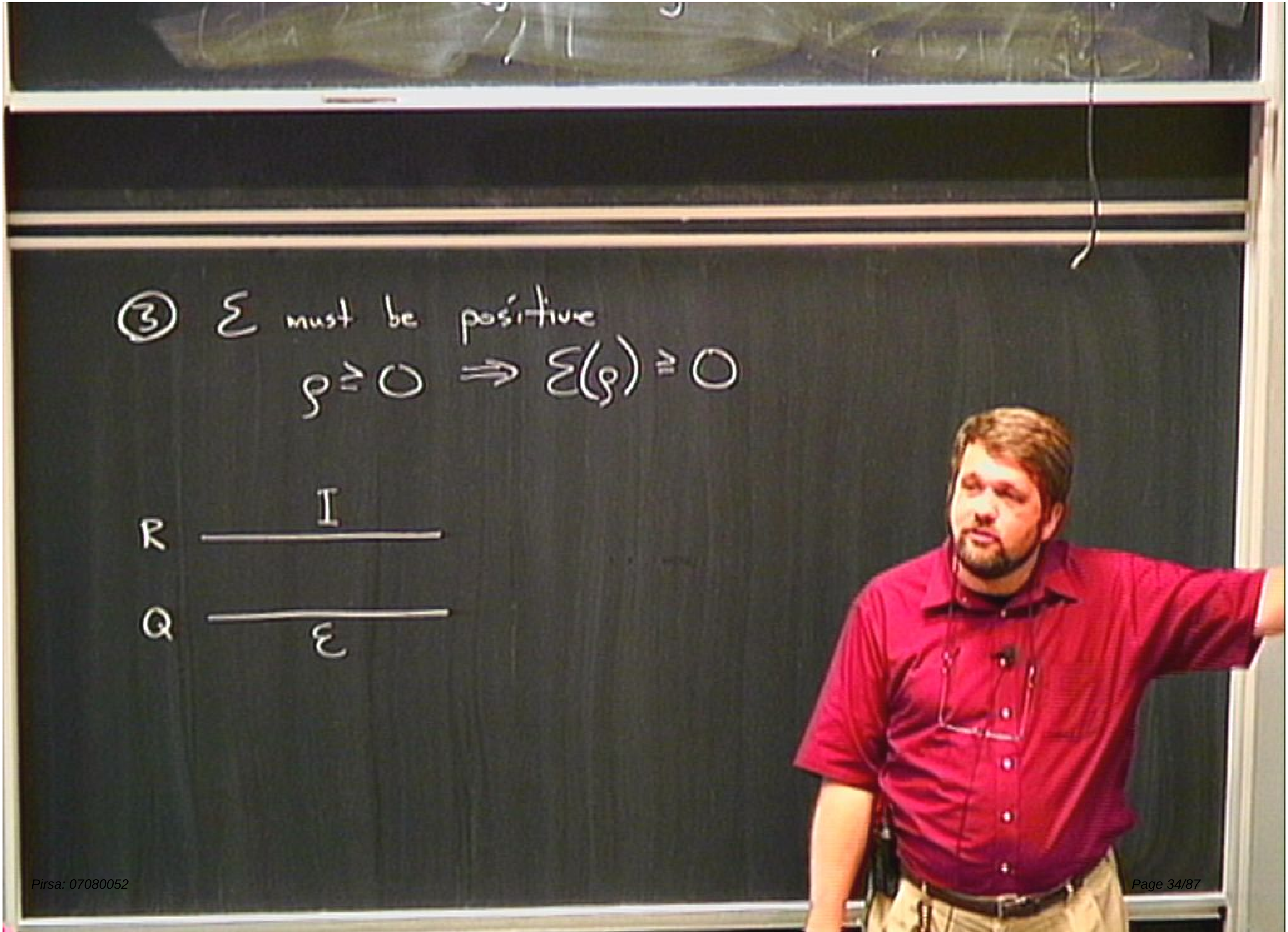
$$\rho \geq 0 \Rightarrow \mathcal{E}(\rho) \geq 0$$

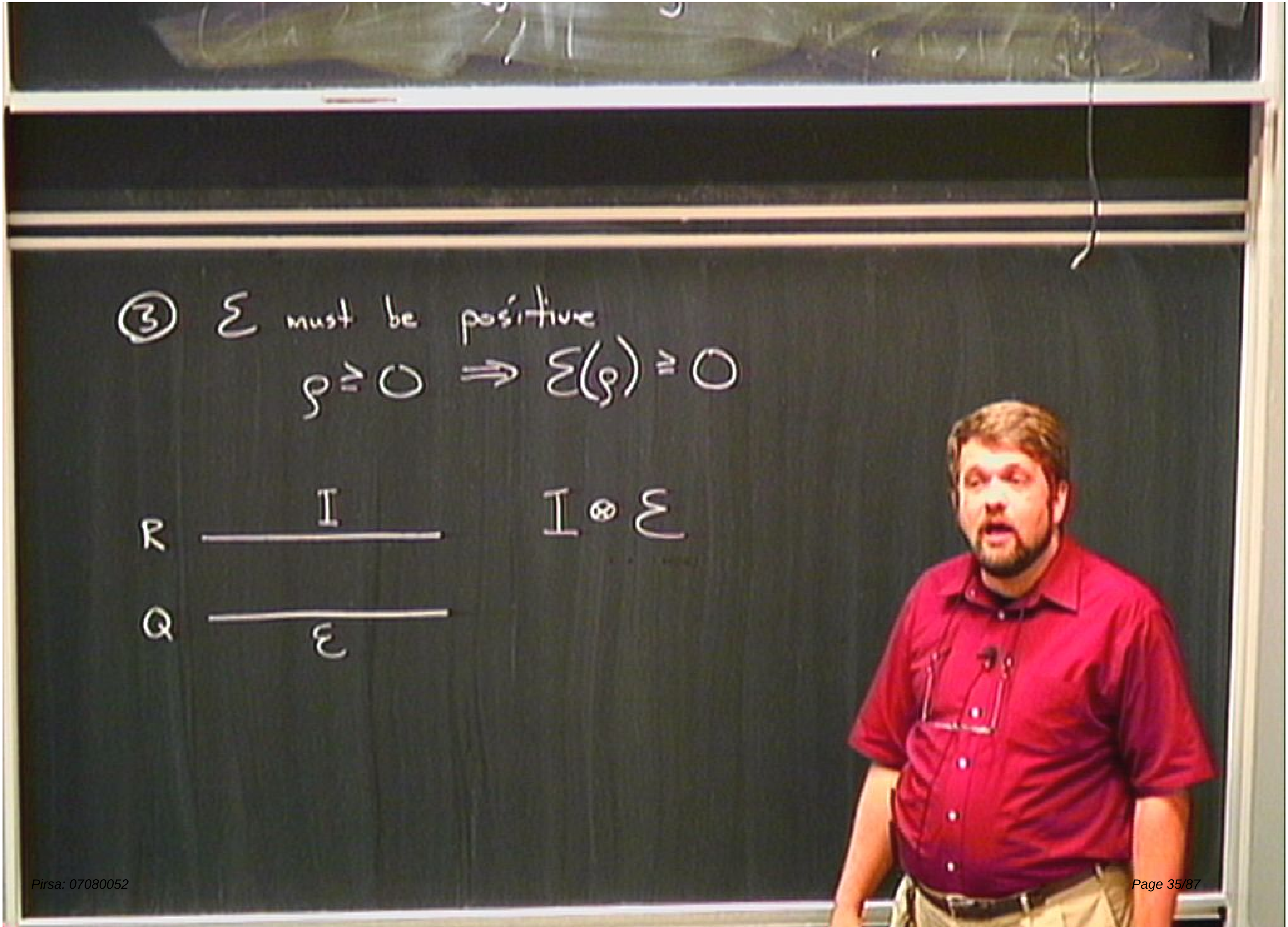
③ $\mathcal{E}$ must be positive

$$\rho \geq 0 \Rightarrow \mathcal{E}(\rho) \geq 0$$

R ______________

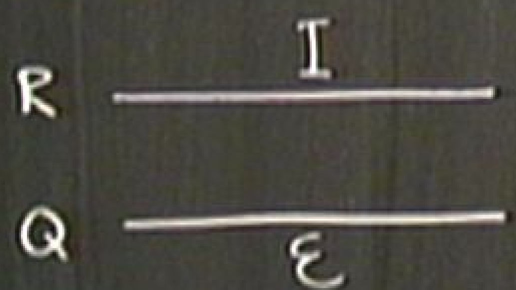
Q ______________


③ $\mathcal{E}$ must be positive
$\rho \geq 0 \Rightarrow \mathcal{E}(\rho) \geq 0$
I
R
Q
$\mathcal{E}$


③ $\mathcal{E}$ must be positive
$\rho \geq 0 \Rightarrow \mathcal{E}(\rho) \geq 0$
R
I
Q
$\mathcal{E}$
$I \otimes \mathcal{E}$

③ $\mathcal{E}$ must be positive

$$\rho \geq 0 \Rightarrow \mathcal{E}(\rho) \geq 0$$



$I \otimes \mathcal{E}$ ← had better be positive

③ $\mathcal{E}$ must be positive

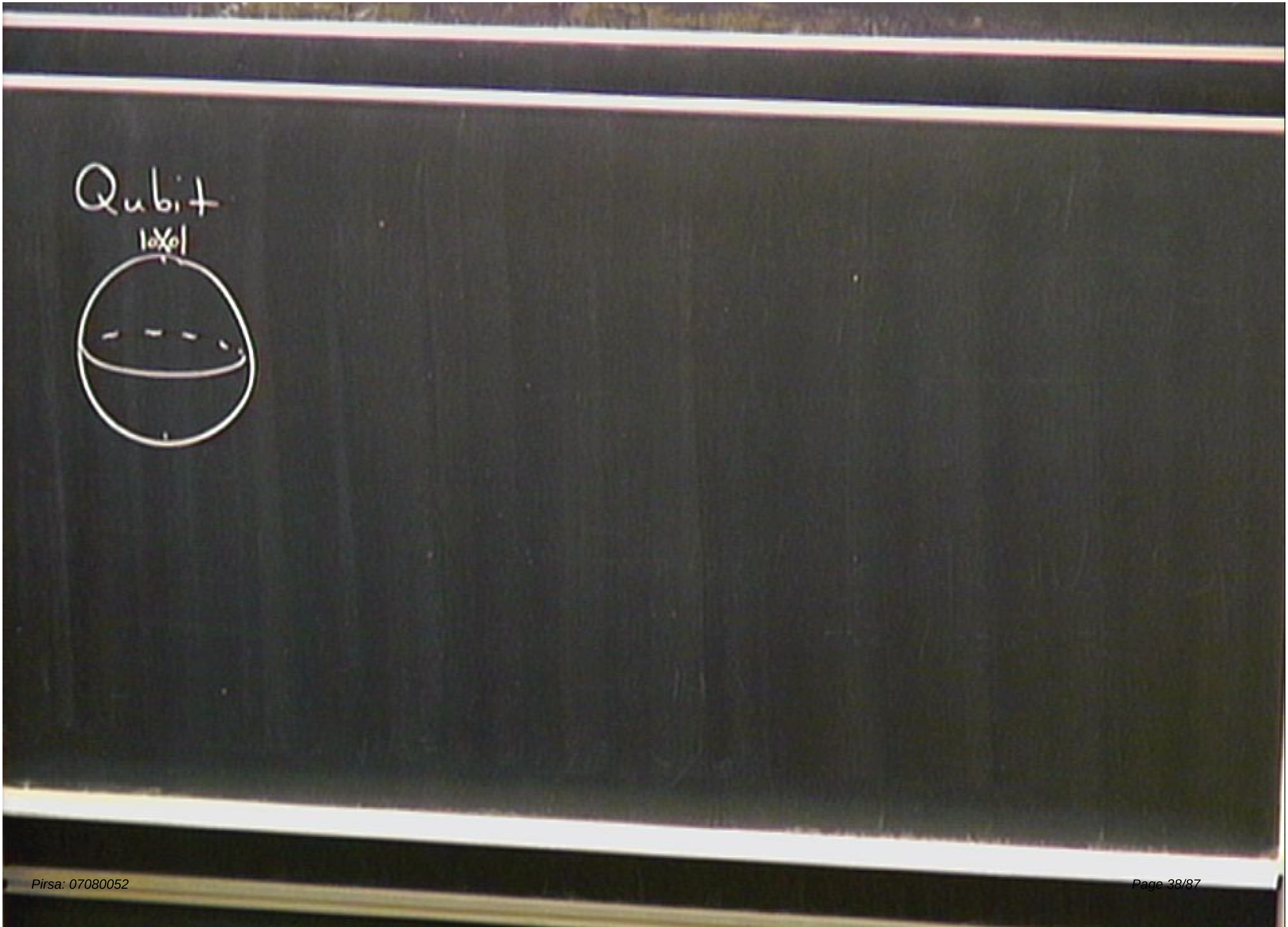
$$\rho \geq 0 \Rightarrow \mathcal{E}(\rho) \geq 0$$

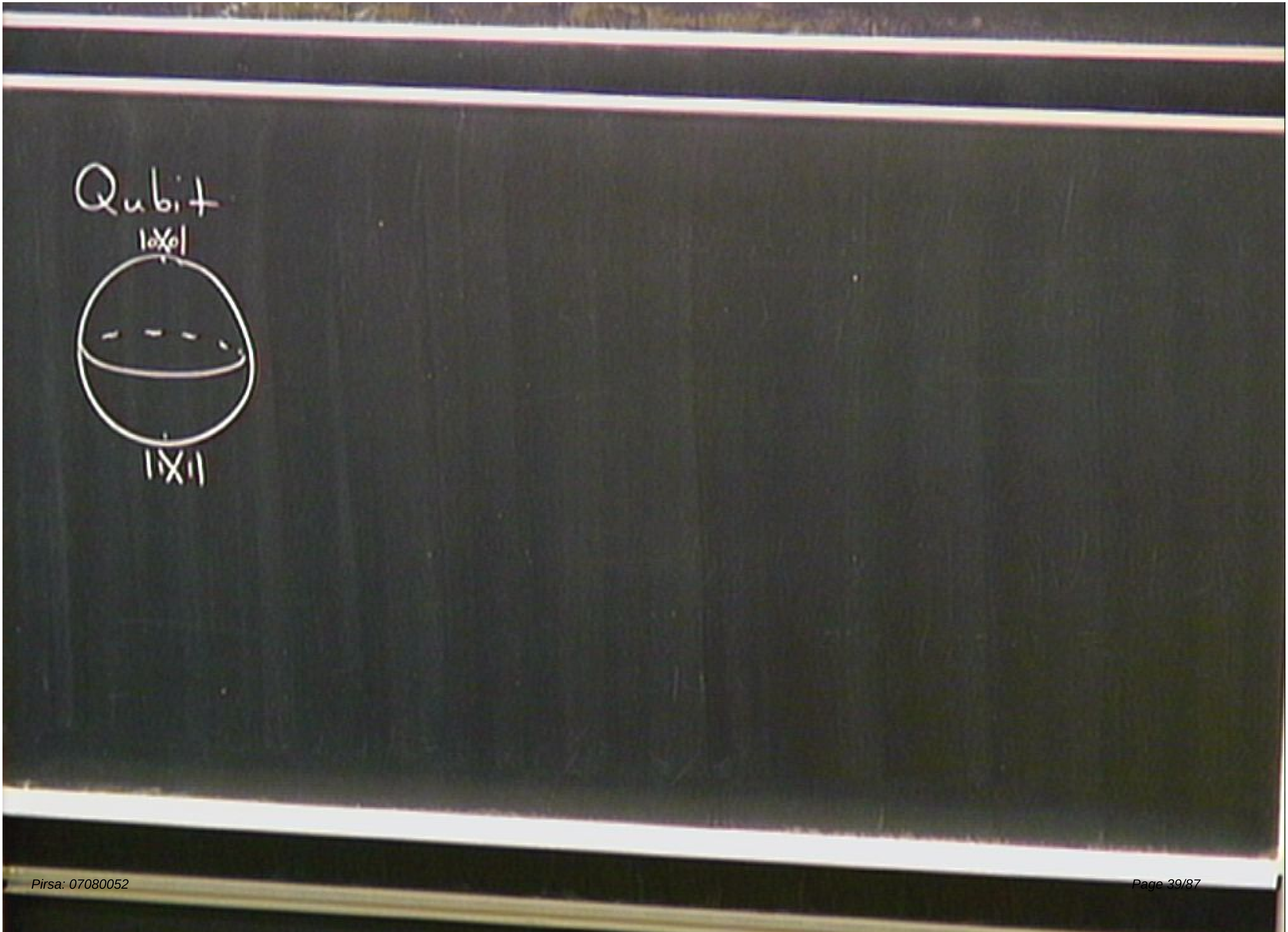
R ——— I ———

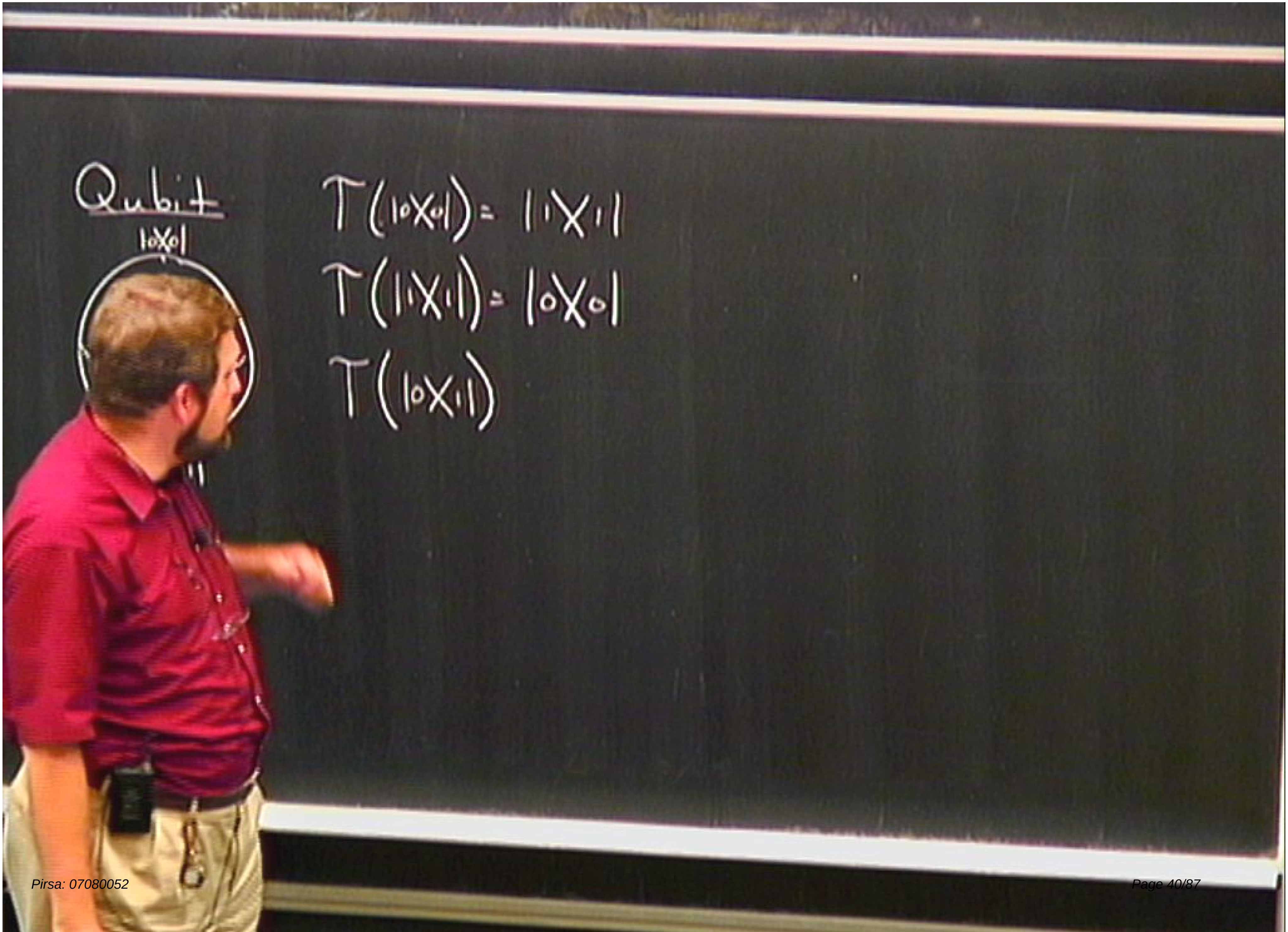
Q ——— $\mathcal{E}$ ———

$I \otimes \mathcal{E}$ ← had better be positive

④ $\mathcal{E}$ must be completely positive ($I \otimes \mathcal{E}$ is positive)


Qubit
|0⟩⟨0|


Qubit
|0⟩⟨0|
|1⟩⟨1|


Qubit
|0⟩⟨0|
T(|0⟩⟨0|) = |1⟩⟨1|
T(|1⟩⟨1|) = |0⟩⟨0|
T(|0⟩⟨1|)

Qubit

$$T(|0\rangle\langle 0|) = |1\rangle\langle 1|$$

$$T(|1\rangle\langle 1|) = |0\rangle\langle 0|$$

$$T(|0\rangle\langle 1|) = |0\rangle\langle 1|$$

$$T(|1\rangle\langle 0|) = |1\rangle\langle 0|$$

Qubit

$$T(|0\rangle\langle 0|) = |1\rangle\langle 1|$$

$$T(|1\rangle\langle 1|) = |0\rangle\langle 0|$$

$$T(|0\rangle\langle 1|) = |0\rangle\langle 1|$$

$$T(|1\rangle\langle 0|) = |1\rangle\langle 0|$$

Two qubits

$I \otimes T$ is <u>not</u> positive

Qubit

$$T(|0\rangle\langle 0|) = |1\rangle\langle 1|$$

$$T(|1\rangle\langle 1|) = |0\rangle\langle 0|$$

$$T(|0\rangle\langle 1|) = |0\rangle\langle 1|$$

$$T(|1\rangle\langle 0|) = |1\rangle\langle 0|$$

Two qubits: $I \otimes T$ is <u>not</u> positive

Qubit

$$T(|0\rangle\langle 0|) = |1\rangle\langle 1|$$

$$T(|1\rangle\langle 1|) = |0\rangle\langle 0|$$

$$T(|0\rangle\langle 1|) = |0\rangle\langle 1|$$

$$T(|1\rangle\langle 0|) = |1\rangle\langle 0|$$

bits: $I \otimes T$ is <u>not</u> positive

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Qubit

$$T(|0\rangle\langle 0|) = |1\rangle\langle 1|$$

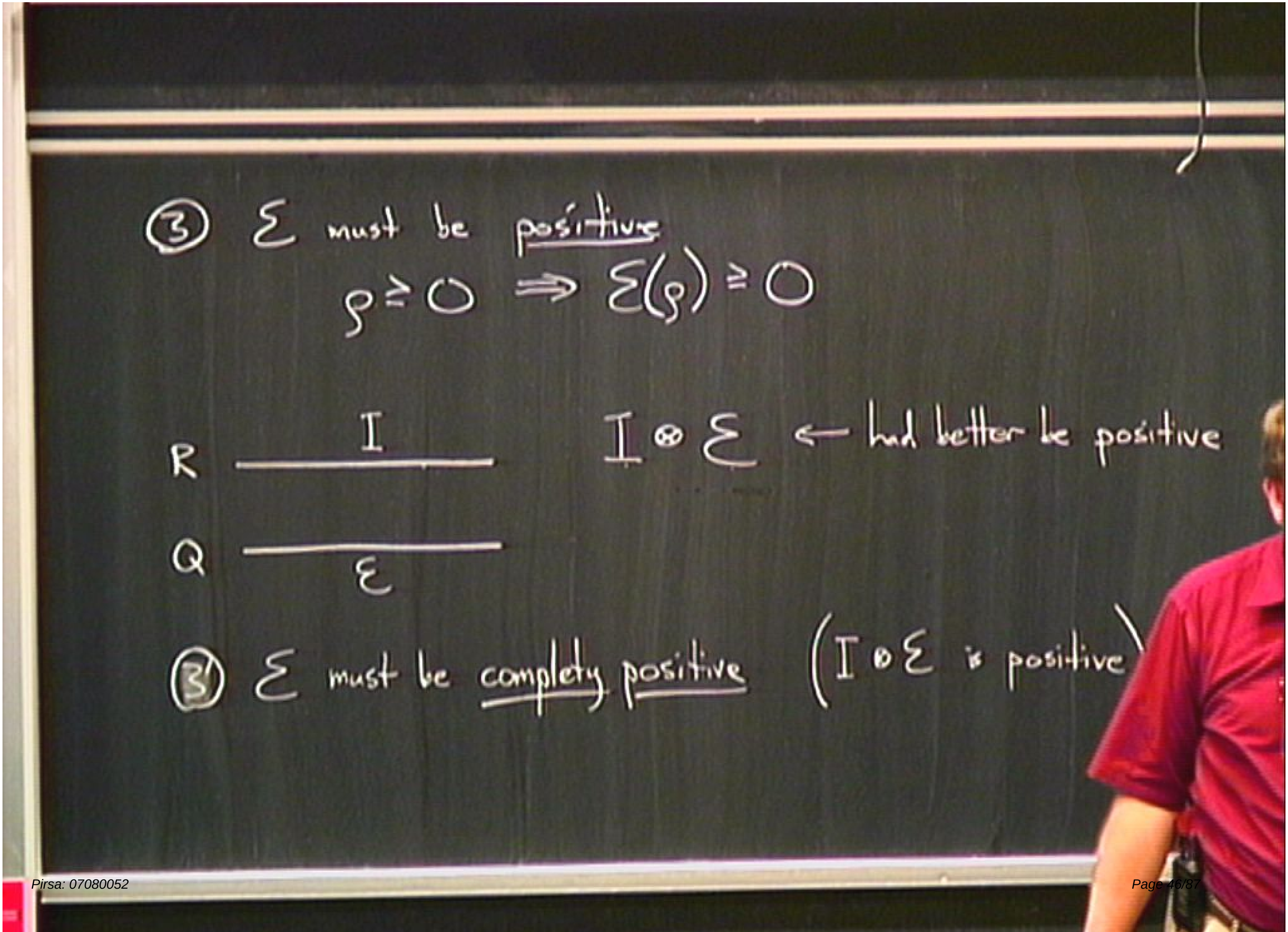
$$T(|1\rangle\langle 1|) = |0\rangle\langle 0|$$

$$T(|0\rangle\langle 1|) = |0\rangle\langle 1|$$

$$T(|1\rangle\langle 0|) = |1\rangle\langle 0|$$

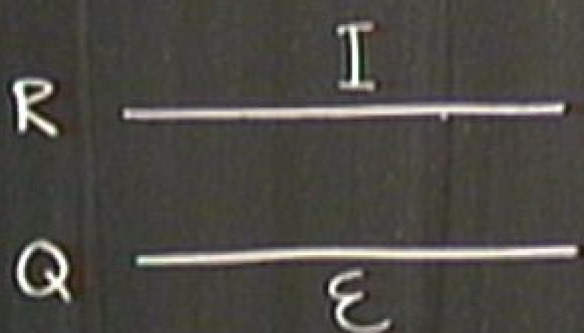
Two qubits: $I \otimes T$ is <u>not</u> positive

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$


③ ℰ must be positive
ρ ≥ 0 ⇒ ℰ(ρ) ≥ 0
R I
Q ℰ
I ⊗ ℰ ← had better be positive
③' ℰ must be completely positive (I ⊗ ℰ is positive)

③ $\mathcal{E}$ must be positive

$$\rho \geq 0 \Rightarrow \mathcal{E}(\rho) \geq 0$$



$I \otimes \mathcal{E}$ ← had better be positive

③' $\mathcal{E}$ must be completly positive ($I \otimes \mathcal{E}$ is positive)

Unitary dynamics $|\psi\rangle \longrightarrow U|\psi\rangle$

$U^\dagger U = 1$

$\rho \longrightarrow U \rho U^\dagger$

Open system $\boxed{\rho \longrightarrow \mathcal{E}(\rho)}$

③ $\mathcal{E}$ must be <u>positive</u>

$$\rho \geq 0 \Rightarrow \mathcal{E}(\rho) \geq 0$$

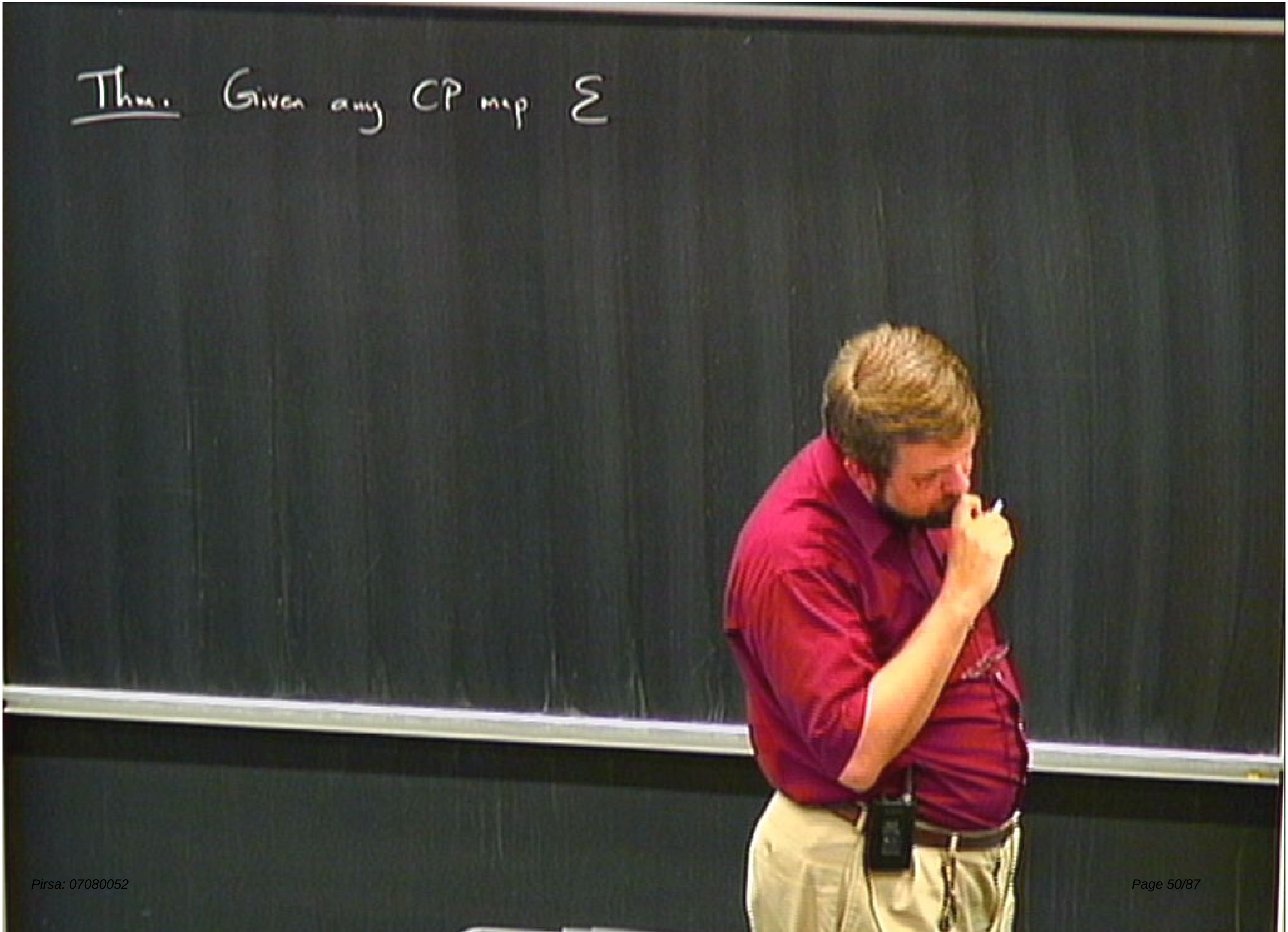
R ——— I ———

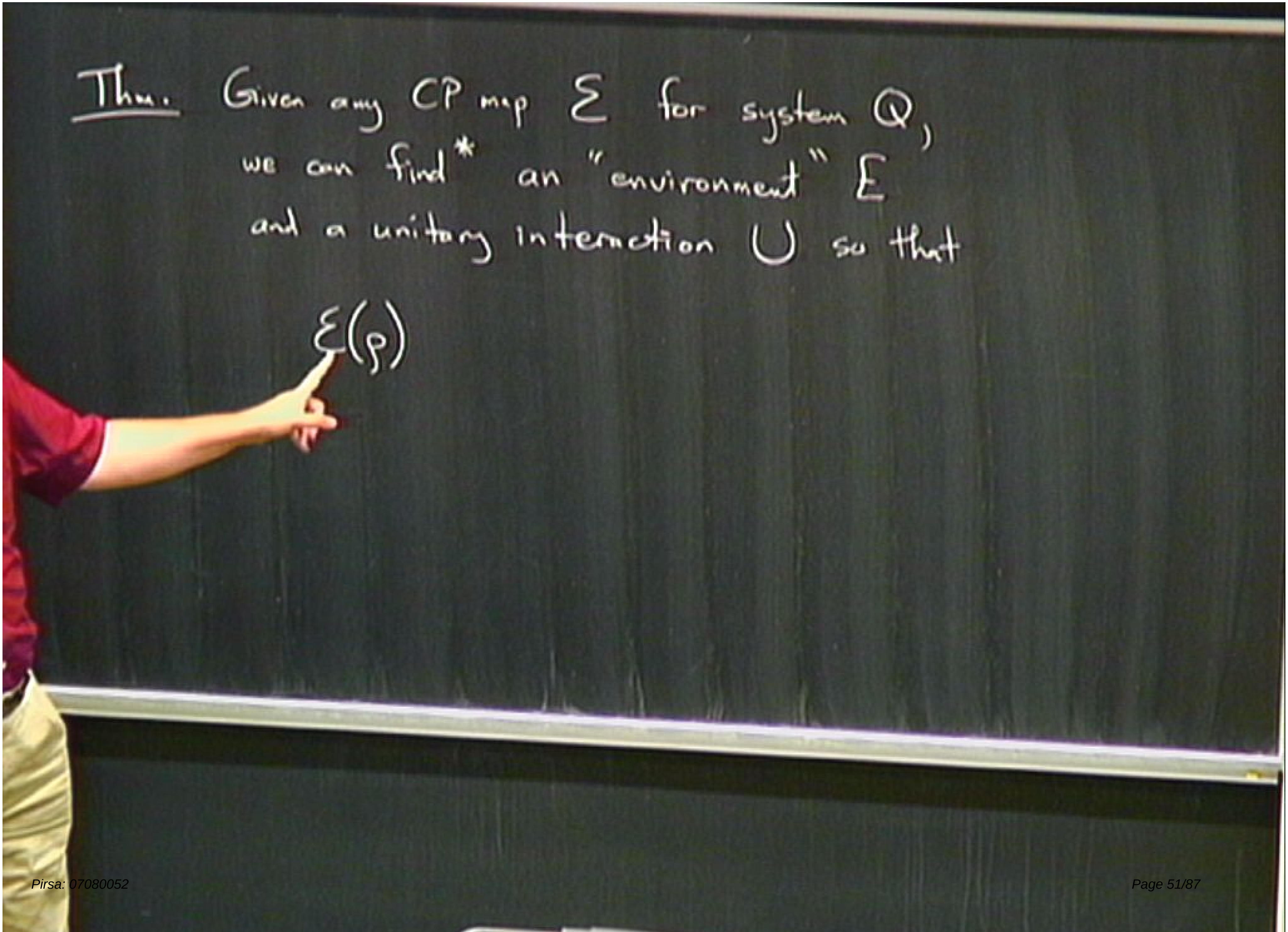
Q ——— $\mathcal{E}$ ———

$I \otimes \mathcal{E}$ ← had better be positive

③' $\mathcal{E}$ must be <u>completely positive</u> ($I \otimes \mathcal{E}$ is positive)

<u>CP maps</u>


Thm. Given any CP map $\mathcal{E}$


Thm. Given any CP map $\mathcal{E}$ for system Q,
we can find* an "environment" E
and a unitary interaction U so that
$\mathcal{E}(\rho)$

Thm. Given any CP map $\mathcal{E}$ for system Q, we can find* an "environment" E and a unitary interaction U so that

$$\mathcal{E}(\rho) = \mathrm{tr}_E\, U\left(\rho \otimes |0\rangle\langle 0|\right)U^{+}$$

Thm. Given any CP map $\mathcal{E}$ for system Q, we can find* an "environment" E and a unitary interaction U so that

$$\mathcal{E}(\rho) = \mathrm{tr}_E U(\rho \otimes |0\rangle\langle 0|)U^{+}$$

I can make $\mathcal{E}$ by

- Appending E in $|0\rangle$
- Evolving by U
- Discarding E

Thm. Given any CP map $\mathcal{E}$ for system Q, we can find* an "environment" E and a unitary interaction U so that

$$\mathcal{E}(\rho) = \mathrm{tr}_E\, U\left(\rho \otimes |0\rangle\langle 0|\right)U^{+}$$

*mathematically

I can make $\mathcal{E}$ by

- Appending E in $|0\rangle$
- Evolving by U
- Discarding E

Qubit

$$T(|0\rangle\langle 0|) = |1\rangle\langle 1|$$

$$T(|1\rangle\langle 1|) = |0\rangle\langle 0|$$

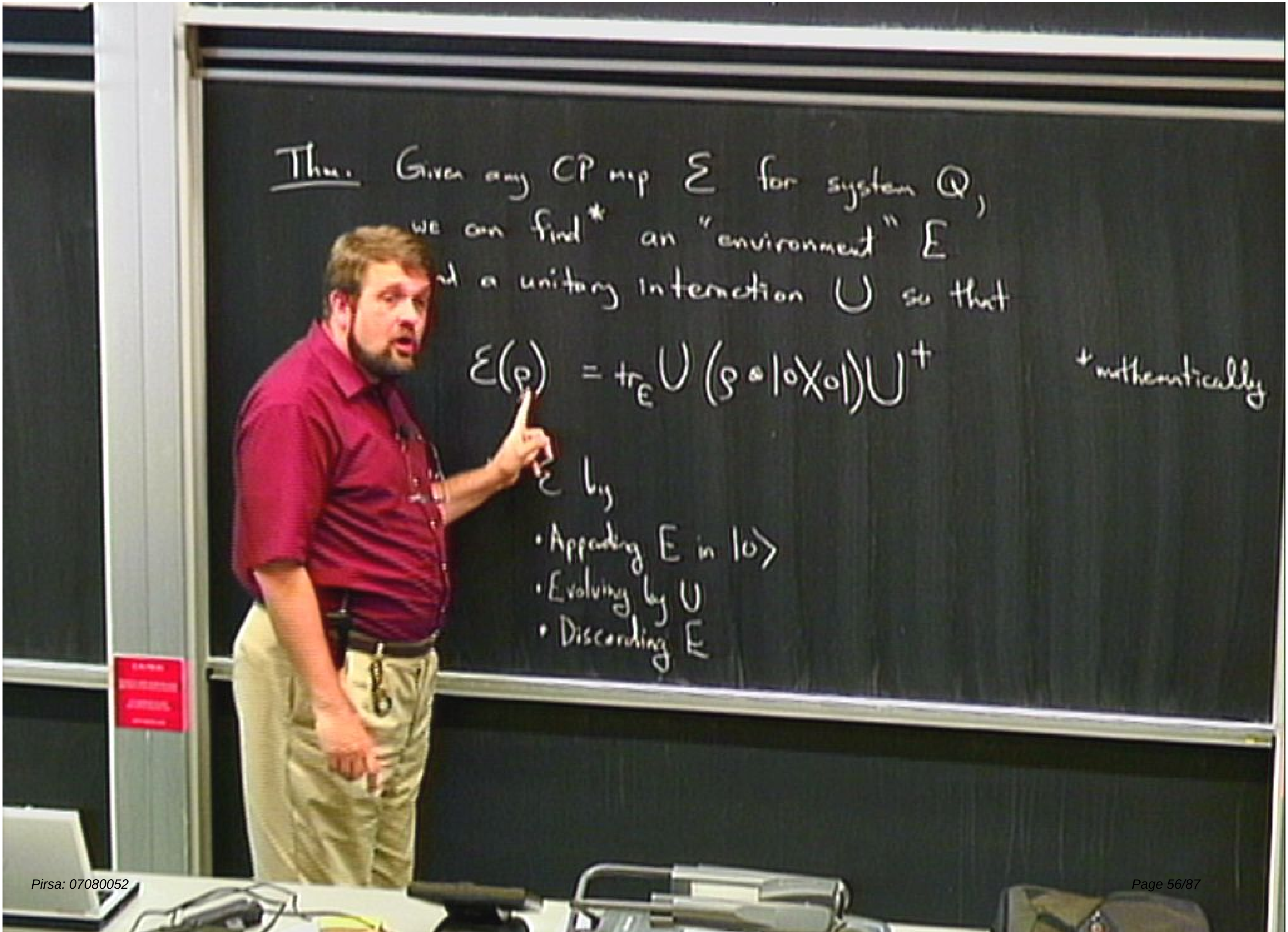
$$T(|0\rangle\langle 1|) = |0\rangle\langle 1|$$

$$T(|1\rangle\langle 0|) = |1\rangle\langle 0|$$

Two qubits: $I \otimes T$ is <u>not</u> positive

$$\tfrac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

<u>Thm.</u> Given any CP map $\mathcal{E}$ for system Q, we can find* an "environment" E


Thm. Given any CP map ℰ for system Q,
we can find* an "environment" E
a unitary interaction U so that
ℰ(ρ) = tr_E U (ρ ⊗ |0⟩⟨0|) U†
*mathematically
ℰ by
• Appending E in |0⟩
• Evolving by U
• Discarding E

Thm. Given any CP map $\mathcal{E}$ for system Q, we can find* an "environment" E and a unitary interaction U so that

$$\mathcal{E}(\rho) = \mathrm{tr}_E\, U\left(\rho \otimes |0\rangle\langle 0|\right)U^{\dagger}$$

*mathematically

I can make $\mathcal{E}$ by

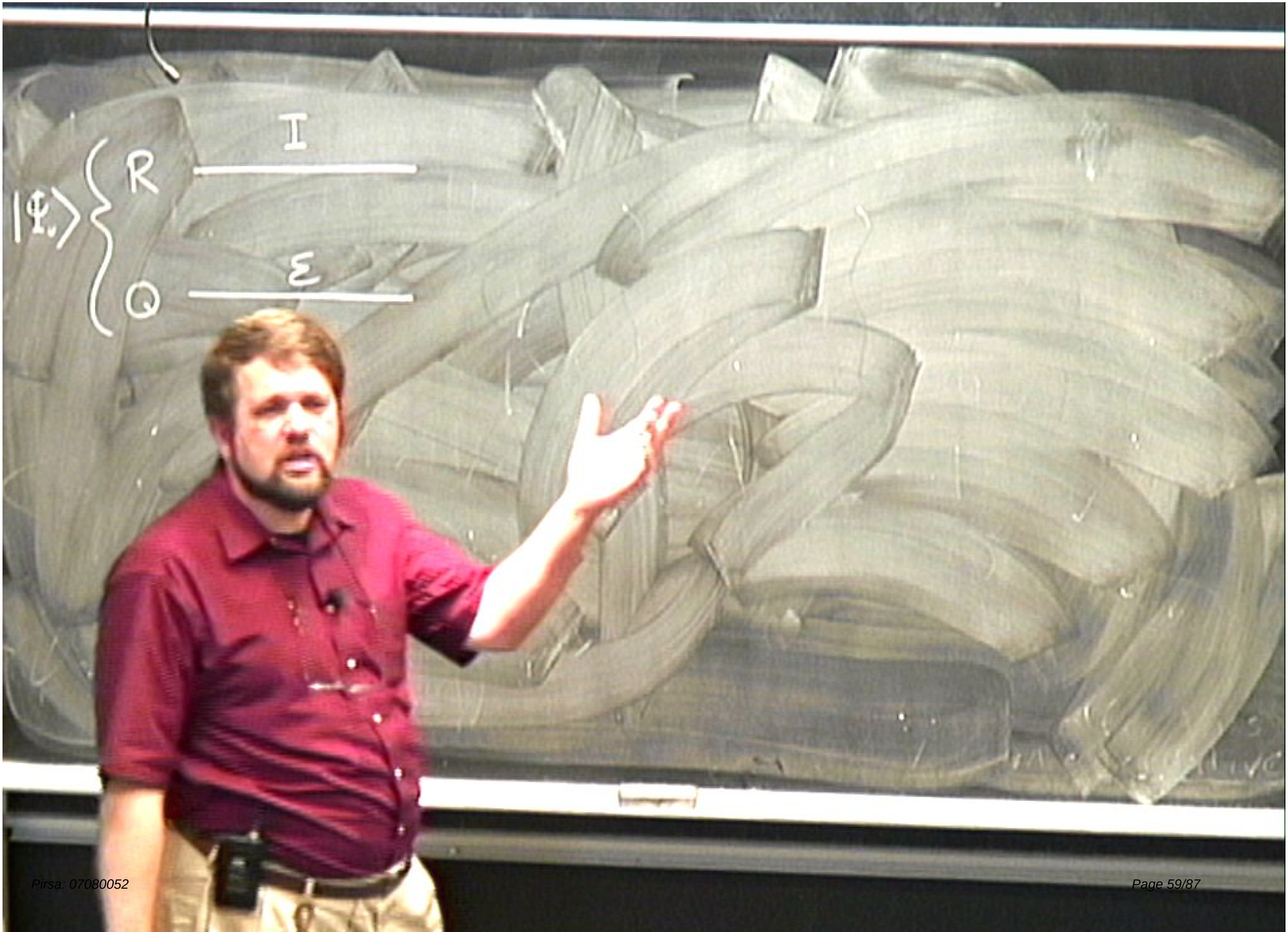
- Appending E in $|0\rangle$
- Evolving by U
- Discarding E

Unitary dynamics $|\psi\rangle \longrightarrow U|\psi\rangle$

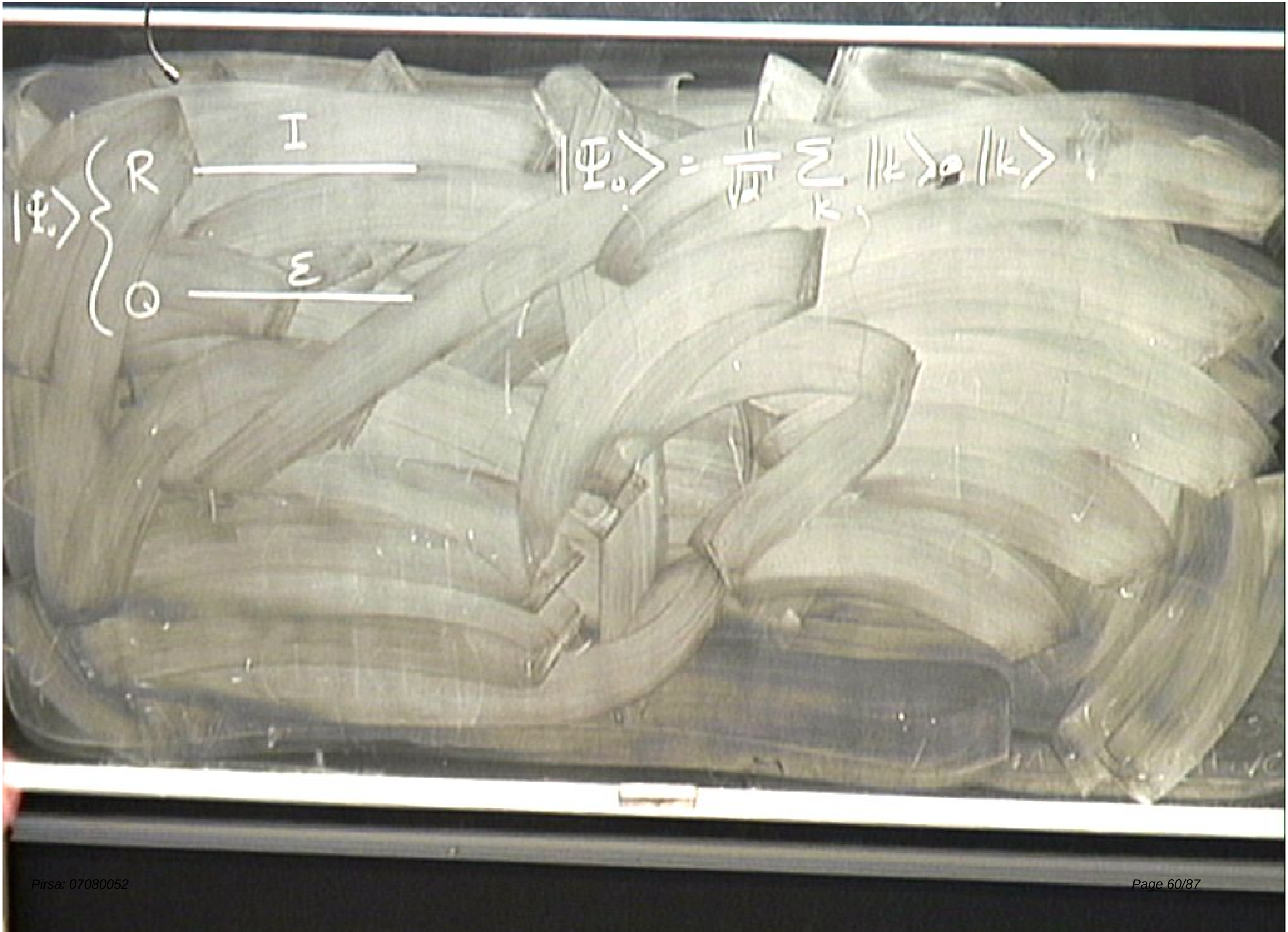
$U^\dagger U = 1$

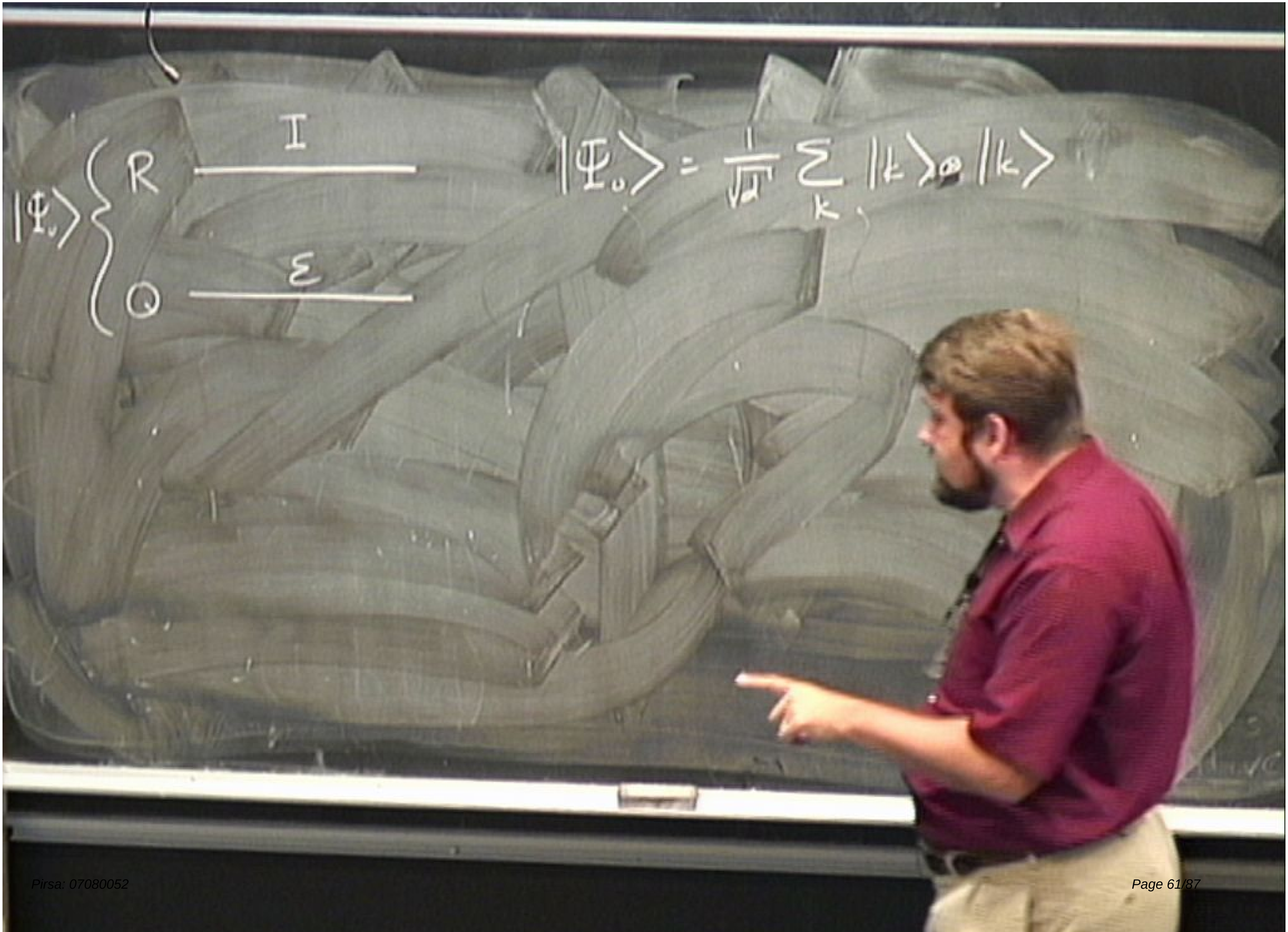
$\rho \longrightarrow U\rho U^\dagger$

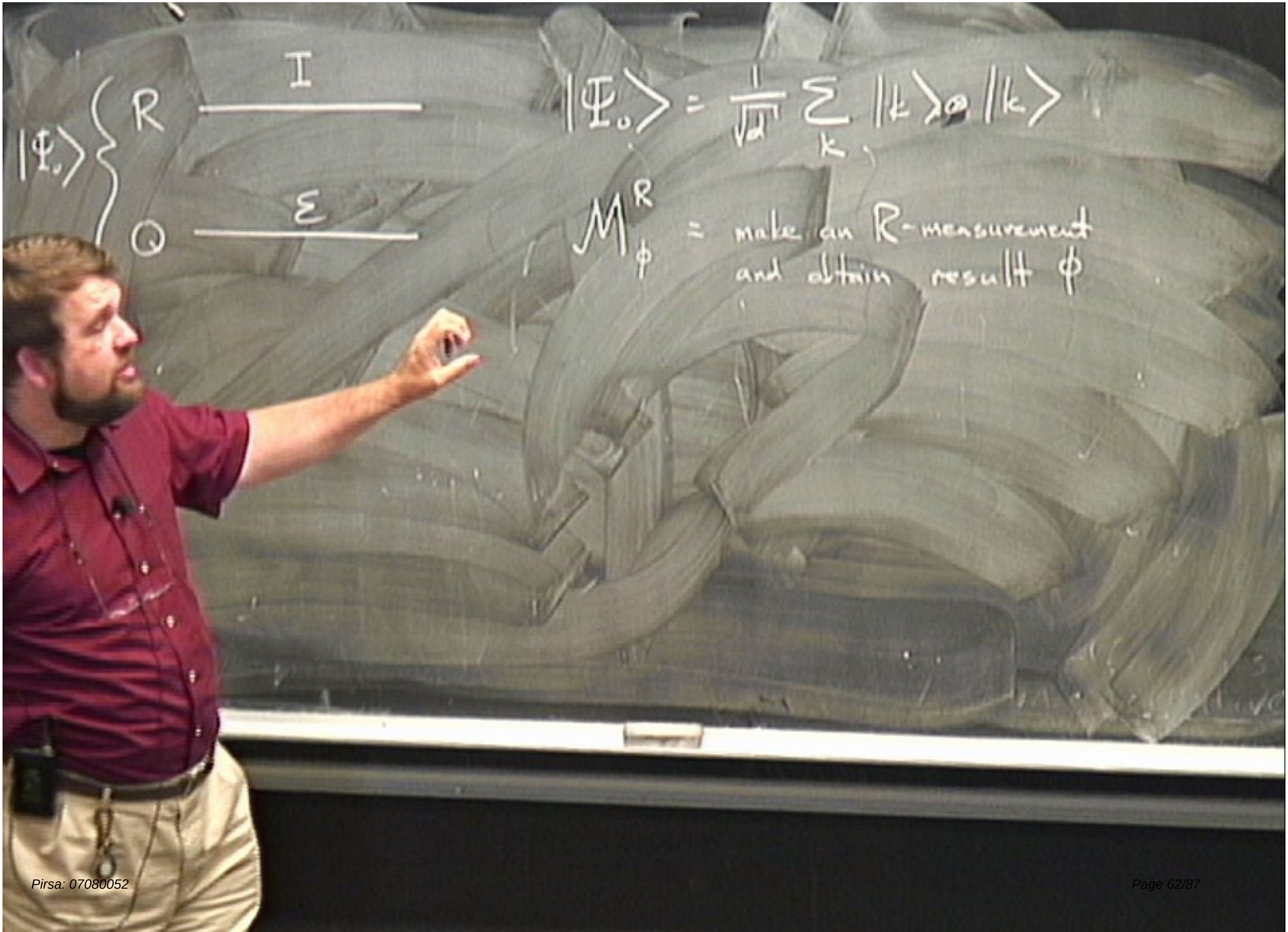
Open system $\boxed{\rho \longrightarrow \mathcal{E}(\rho)}$

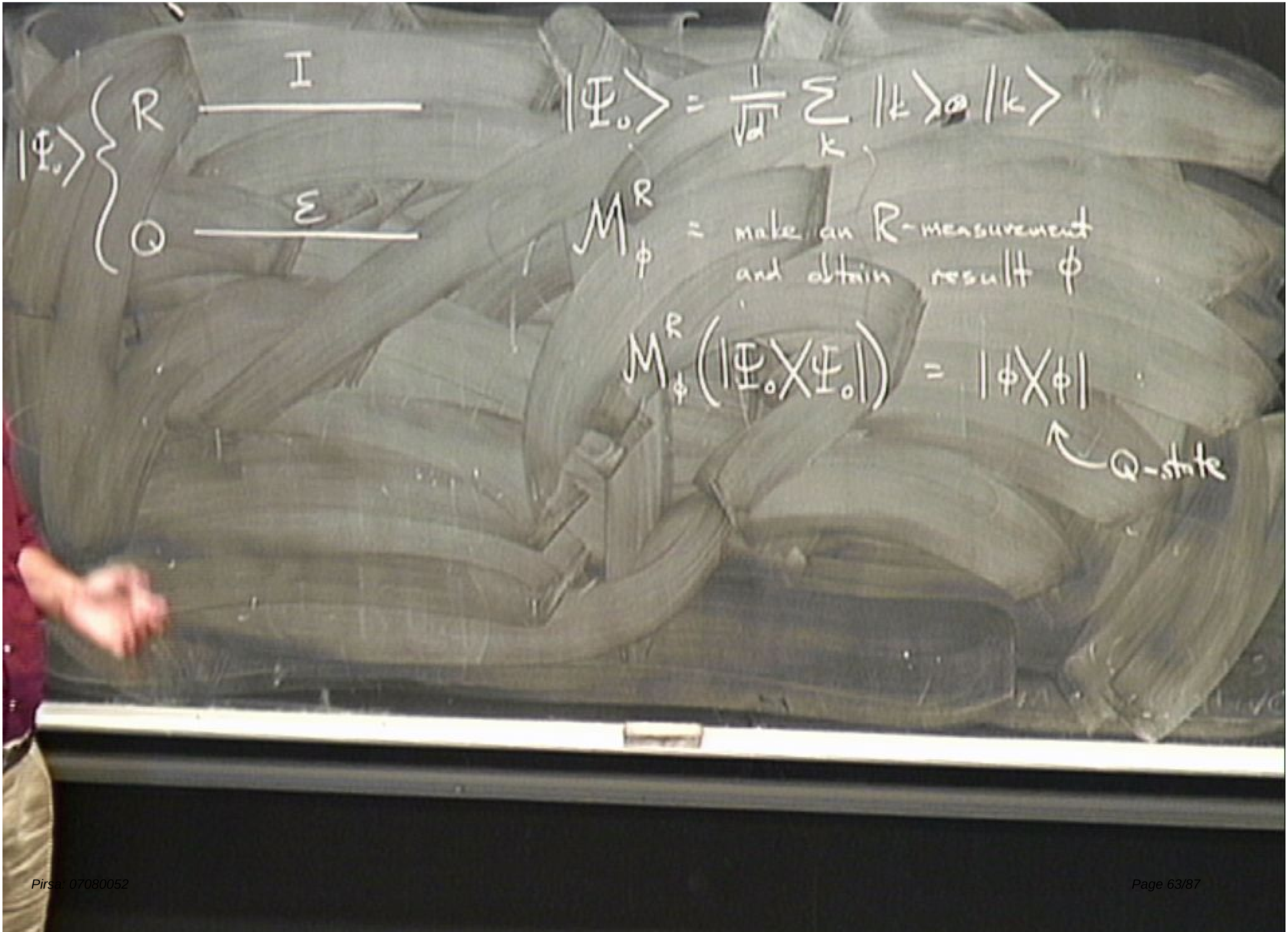


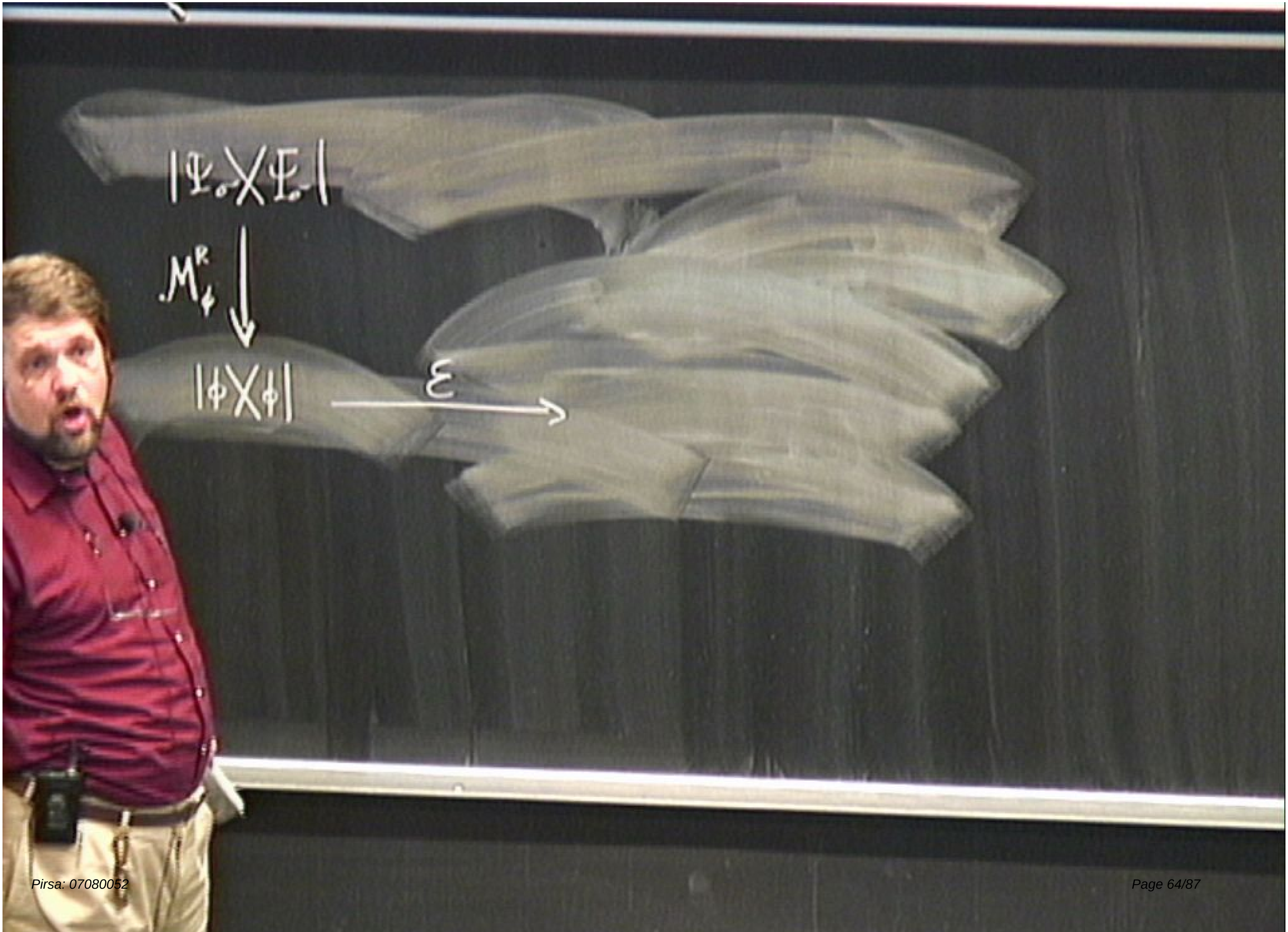
$|\Psi_0\rangle \begin{cases} R & \overline{\quad I \quad} \\ Q & \overline{\quad \mathcal{E} \quad} \end{cases}$


|Ψ₀⟩
R
I
Q
E
|Ψ₀⟩ = 1/√d Σ_k |k⟩ |k⟩

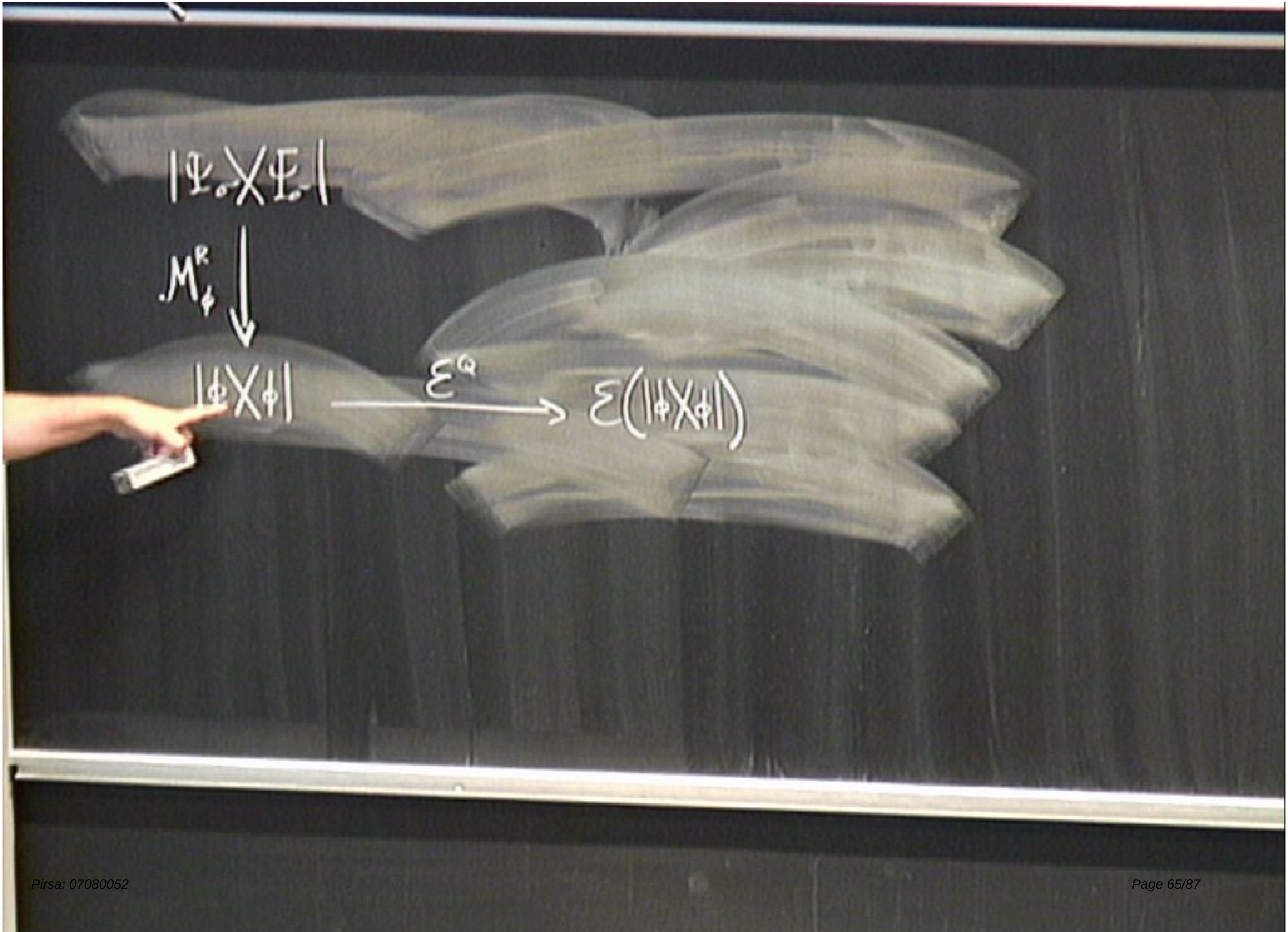

$|\Psi_0\rangle$
R
I
Q
ε
$|\Psi_0\rangle = \frac{1}{\sqrt{d}} \sum_k |k\rangle \otimes |k\rangle$

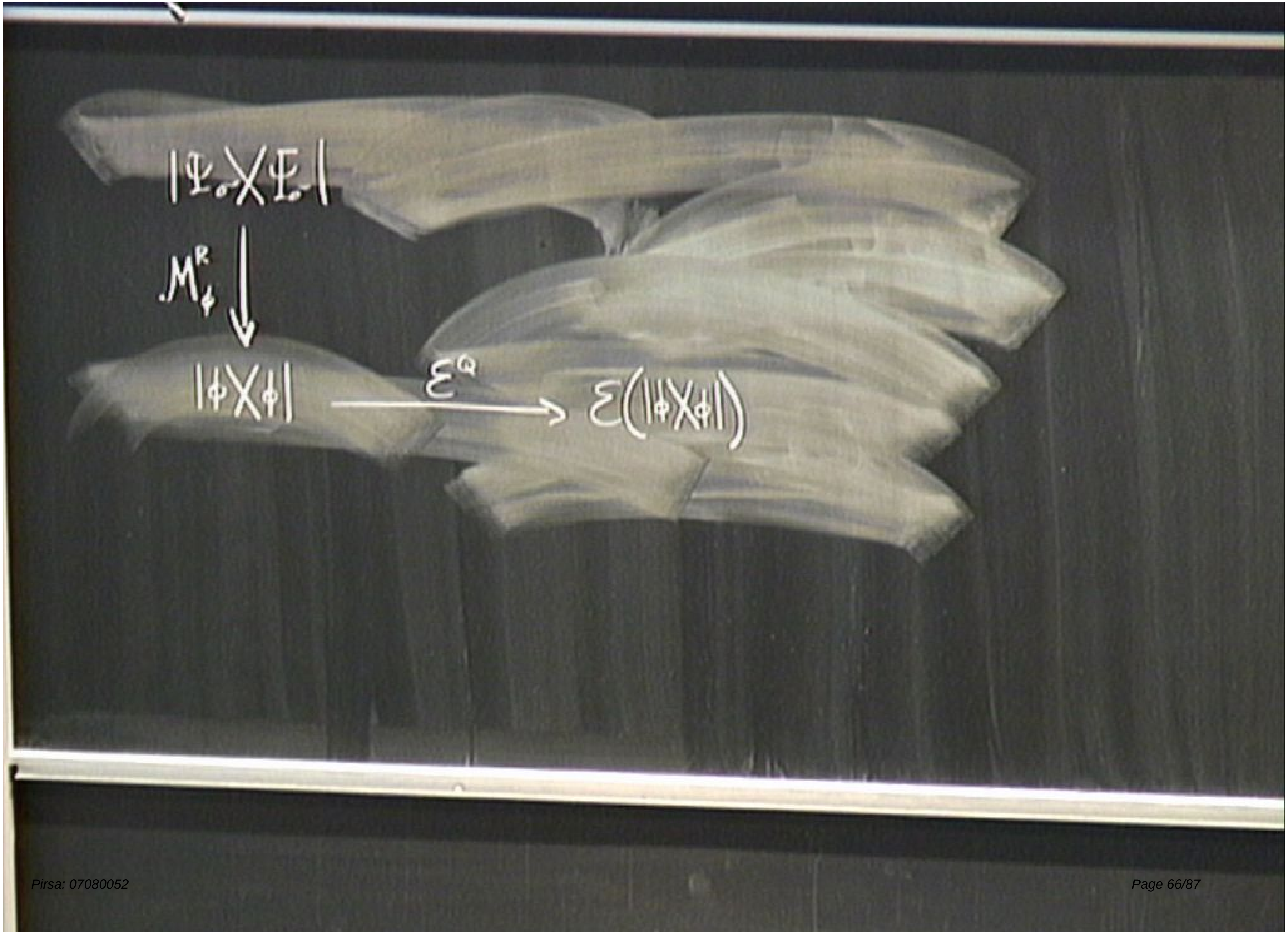

$|\Psi_0\rangle$
R
I
Q
$\mathcal{E}$
$|\Psi_0\rangle = \frac{1}{\sqrt{d}} \sum_k |k\rangle \otimes |k\rangle$
M^R_ϕ = make an R-measurement
and obtain result ϕ

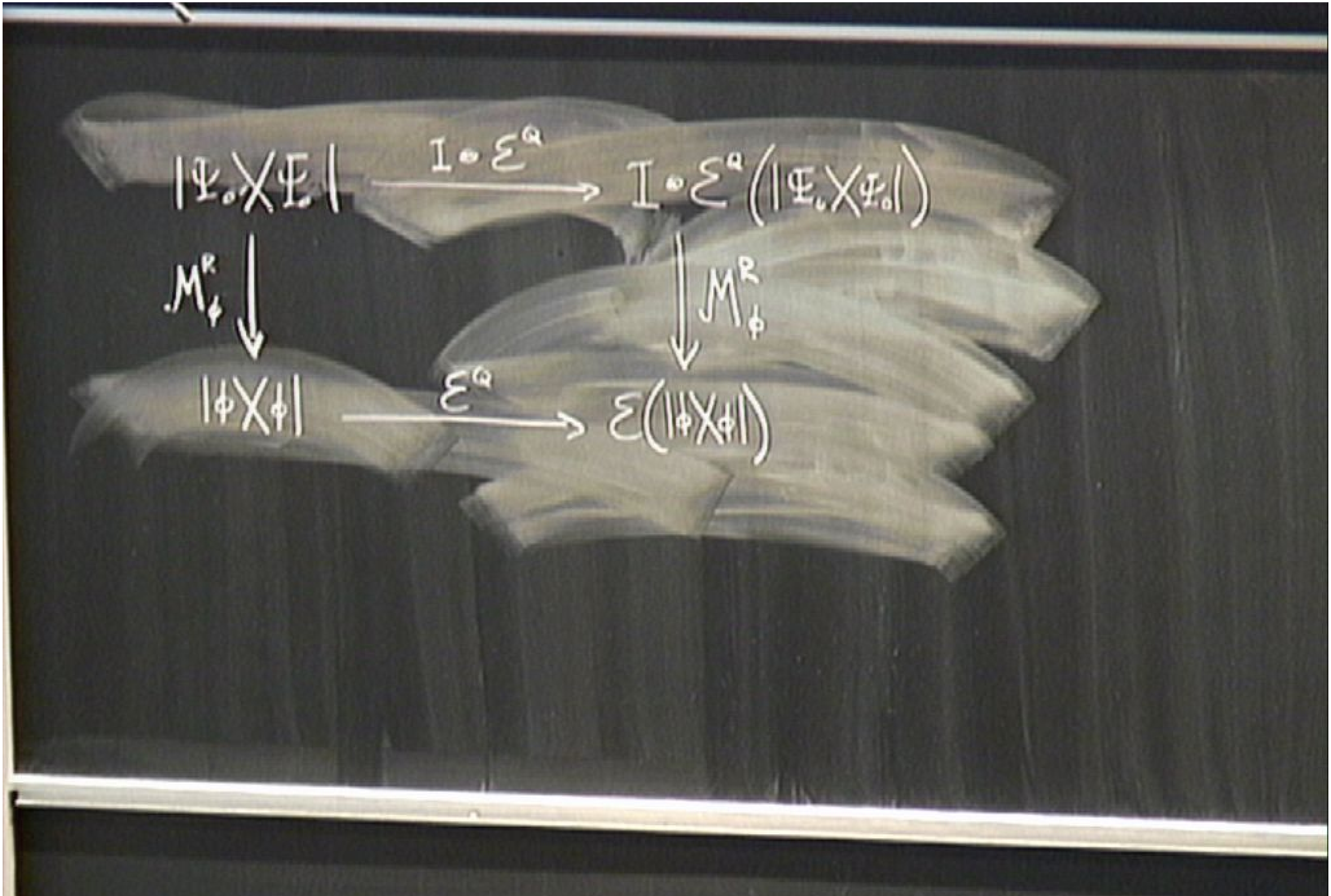

$|\Phi_0\rangle \begin{cases} R \ \text{———} \ I \\ Q \ \text{———} \ \mathcal{E} \end{cases}$
$|\Phi_0\rangle = \frac{1}{\sqrt{d}} \sum_k |k\rangle \otimes |k\rangle$
M^R_ϕ = make an R-measurement and obtain result ϕ
$M^R_\phi(|\Phi_0\rangle\langle\Phi_0|) = |\phi\rangle\langle\phi|$
Q-state

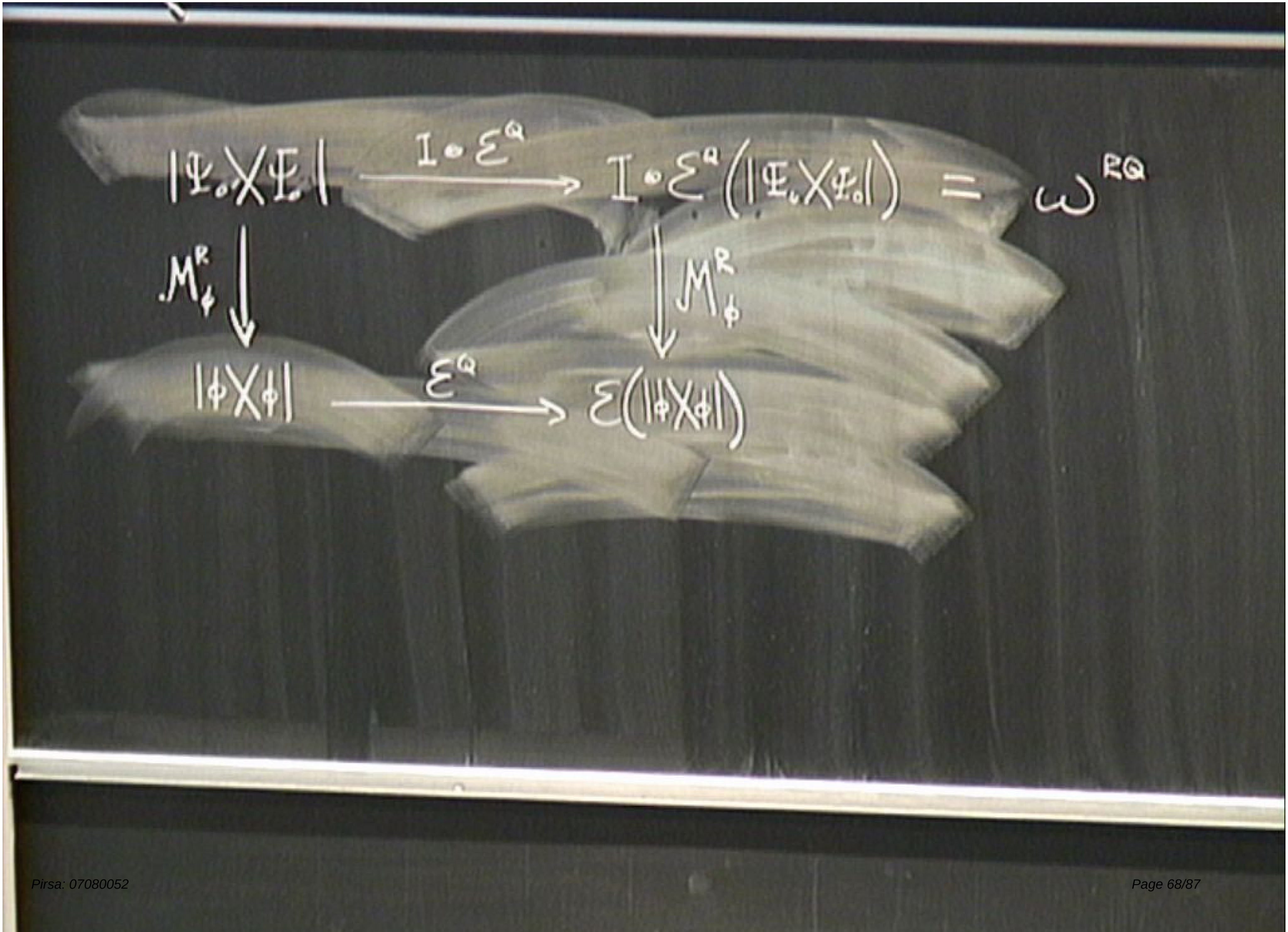


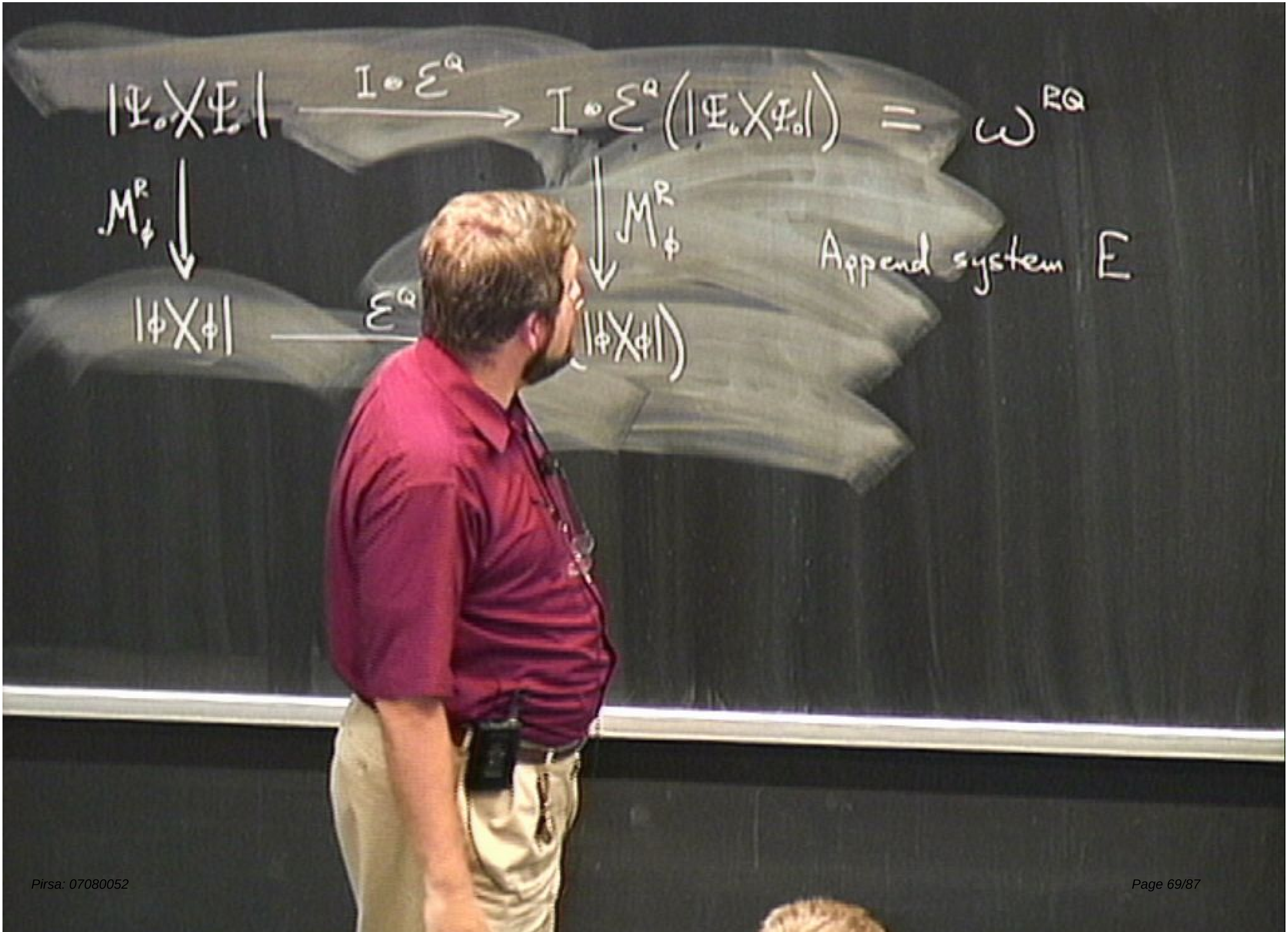
|Ψ₀⟩⟨Ψ₀|
M^R_φ
|φ⟩⟨φ|
ε

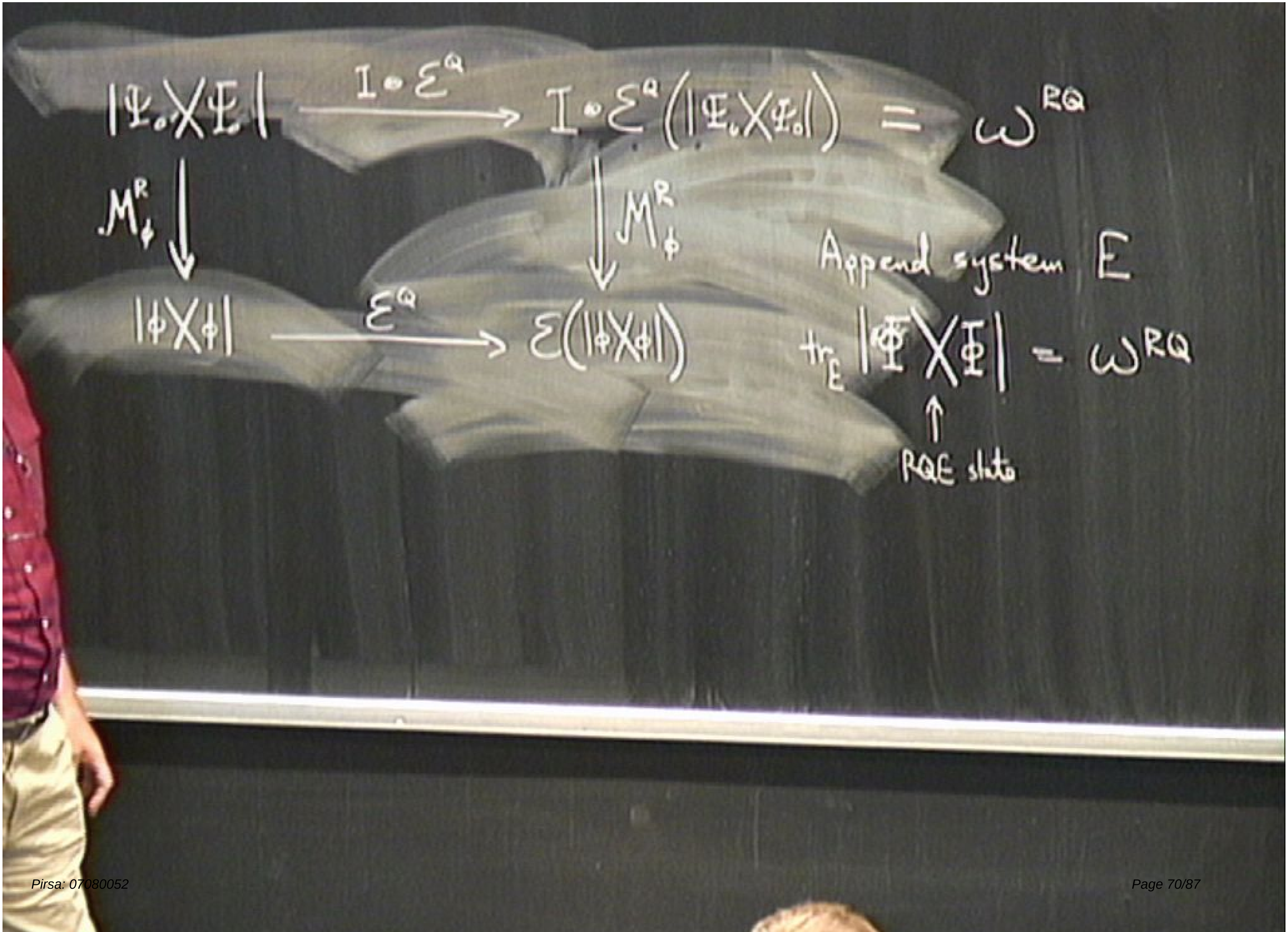

$|\Psi_0\rangle\langle\Psi_0|$
M^R_ϕ
$|\phi\rangle\langle\phi|$
$\mathcal{E}^Q$
$\mathcal{E}(|\phi\rangle\langle\phi|)$

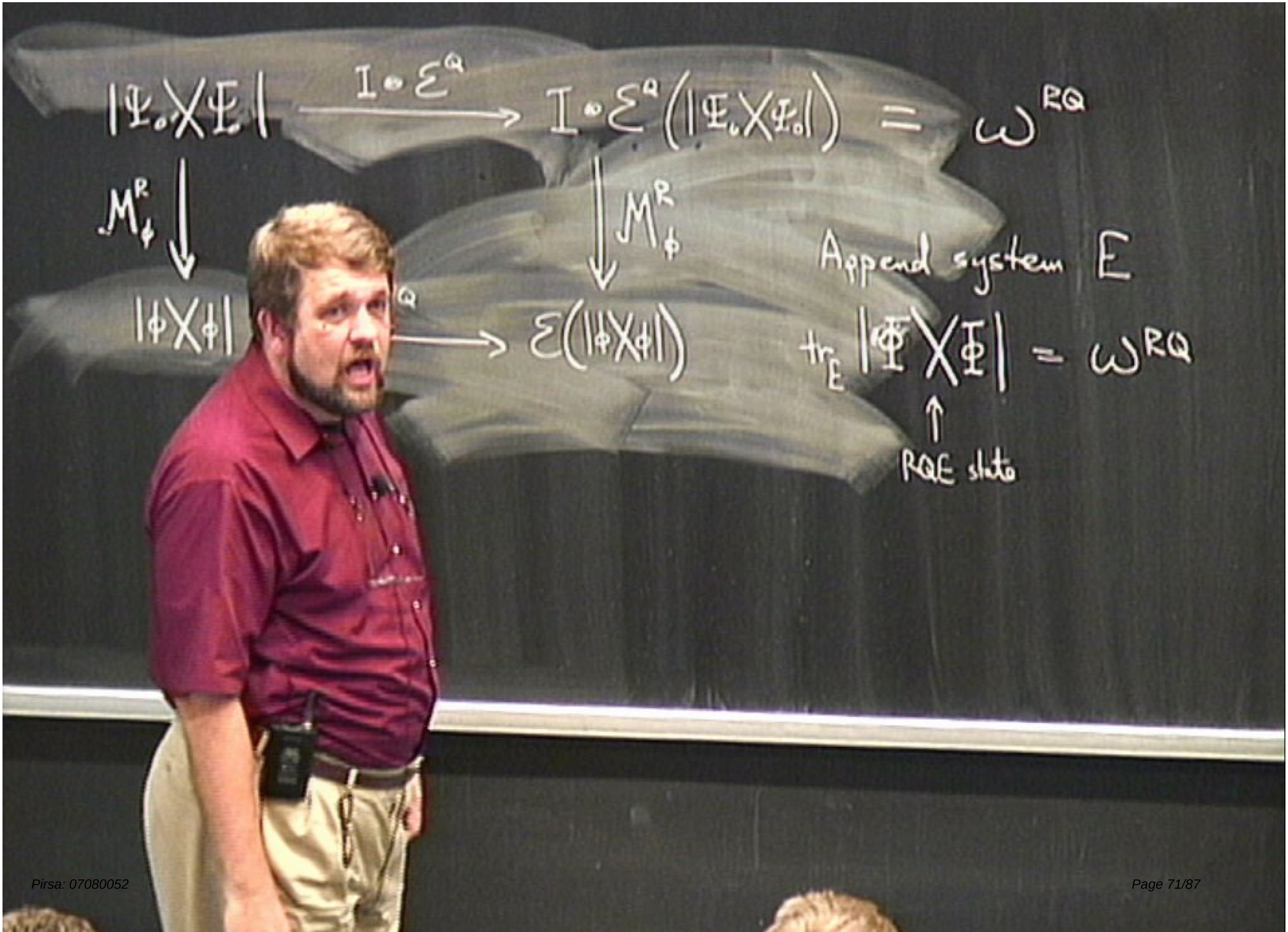

$|\Psi_0\rangle\langle\Psi_0|$
$\mathcal{M}^R_\phi$
$|\phi\rangle\langle\phi|$
$\mathcal{E}^Q$
$\mathcal{E}(|\phi\rangle\langle\phi|)$

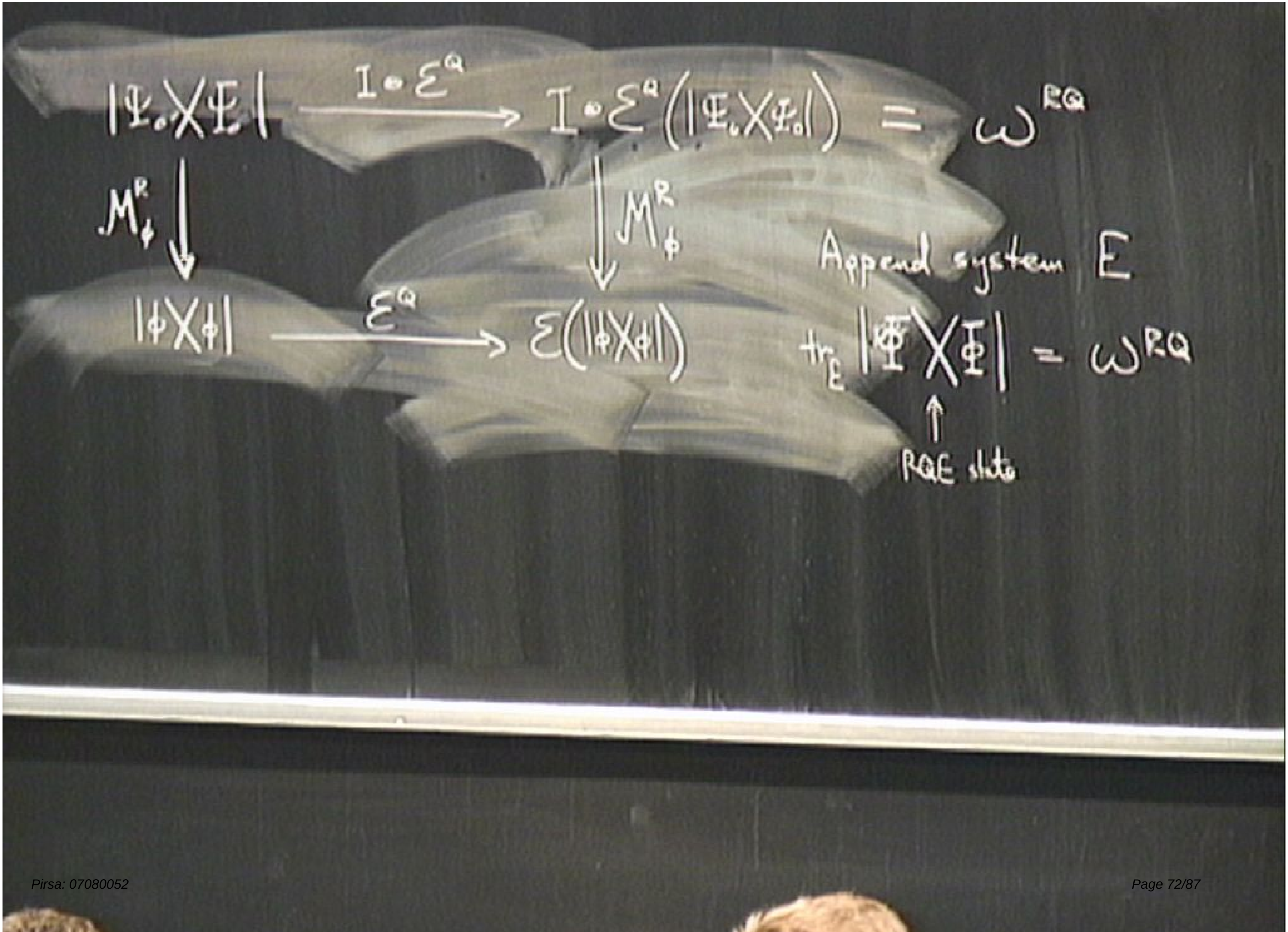

$|\Psi_0\rangle\langle\Psi_0|$
$I \otimes \mathcal{E}^Q$
$I \otimes \mathcal{E}^Q(|\Psi_0\rangle\langle\Psi_0|)$
M_ϕ^R
M_ϕ^R
$|\phi\rangle\langle\phi|$
$\mathcal{E}^Q$
$\mathcal{E}(|\phi\rangle\langle\phi|)$


$|\Psi_0\rangle\langle\Psi_0| \xrightarrow{I\otimes\mathcal{E}^Q} I\otimes\mathcal{E}^Q(|\Psi_0\rangle\langle\Psi_0|) = \omega^{RQ}$
M^R_ϕ
M^R_ϕ
$|\phi\rangle\langle\phi| \xrightarrow{\mathcal{E}^Q} \mathcal{E}(|\phi\rangle\langle\phi|)$


$|\Psi_0\rangle\langle\Psi_0| \xrightarrow{I \otimes \mathcal{E}^Q} I \otimes \mathcal{E}^Q(|\Psi_0\rangle\langle\Psi_0|) = \omega^{RQ}$
M^R_ϕ
$|\phi\rangle\langle\phi|$
$\mathcal{E}^Q$
Append system E


$|\Psi_0\rangle\langle\Psi_0| \xrightarrow{I\otimes\mathcal{E}^Q} I\otimes\mathcal{E}^Q(|\Psi_0\rangle\langle\Psi_0|) = \omega^{RQ}$
$\mathcal{M}^R_\phi$
$|\phi\rangle\langle\phi| \xrightarrow{\mathcal{E}^Q} \mathcal{E}(|\phi\rangle\langle\phi|)$
Append system E
$\mathrm{tr}_E |\Phi\rangle\langle\Phi| = \omega^{RQ}$
RQE state


$|\Psi_0\rangle\langle\Psi_0| \xrightarrow{I \otimes \mathcal{E}^Q} I \otimes \mathcal{E}^Q(|\Psi_0\rangle\langle\Psi_0|) = \omega^{RQ}$
M^R_ϕ
$|\phi\rangle\langle\phi|$
$\mathcal{E}(|\phi\rangle\langle\phi|)$
Append system E
$tr_E |\Phi\rangle\langle\Phi| = \omega^{RQ}$
RQE state


$|\Psi_0\rangle\langle\Psi_0| \xrightarrow{I \otimes \mathcal{E}^Q} I \otimes \mathcal{E}^Q(|\Psi_0\rangle\langle\Psi_0|) = \omega^{RQ}$
M_ϕ^R
$|\phi\rangle\langle\phi| \xrightarrow{\mathcal{E}^Q} \mathcal{E}(|\phi\rangle\langle\phi|)$
Append system E
$tr_E |\Phi\rangle\langle\Phi| = \omega^{RQ}$
RQE state

$$|\Psi_0\rangle\langle\Psi_0| \xrightarrow{I\otimes\mathcal{E}^Q} I\otimes\mathcal{E}^Q\left(|\Psi_0\rangle\langle\Psi_0|\right) = \omega^{RQ}$$

$M^R_\phi \downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow M^R_\phi$

$$|\phi\rangle\langle\phi| \xrightarrow{\mathcal{E}^Q} \mathcal{E}\left(|\phi\rangle\langle\phi|\right)$$

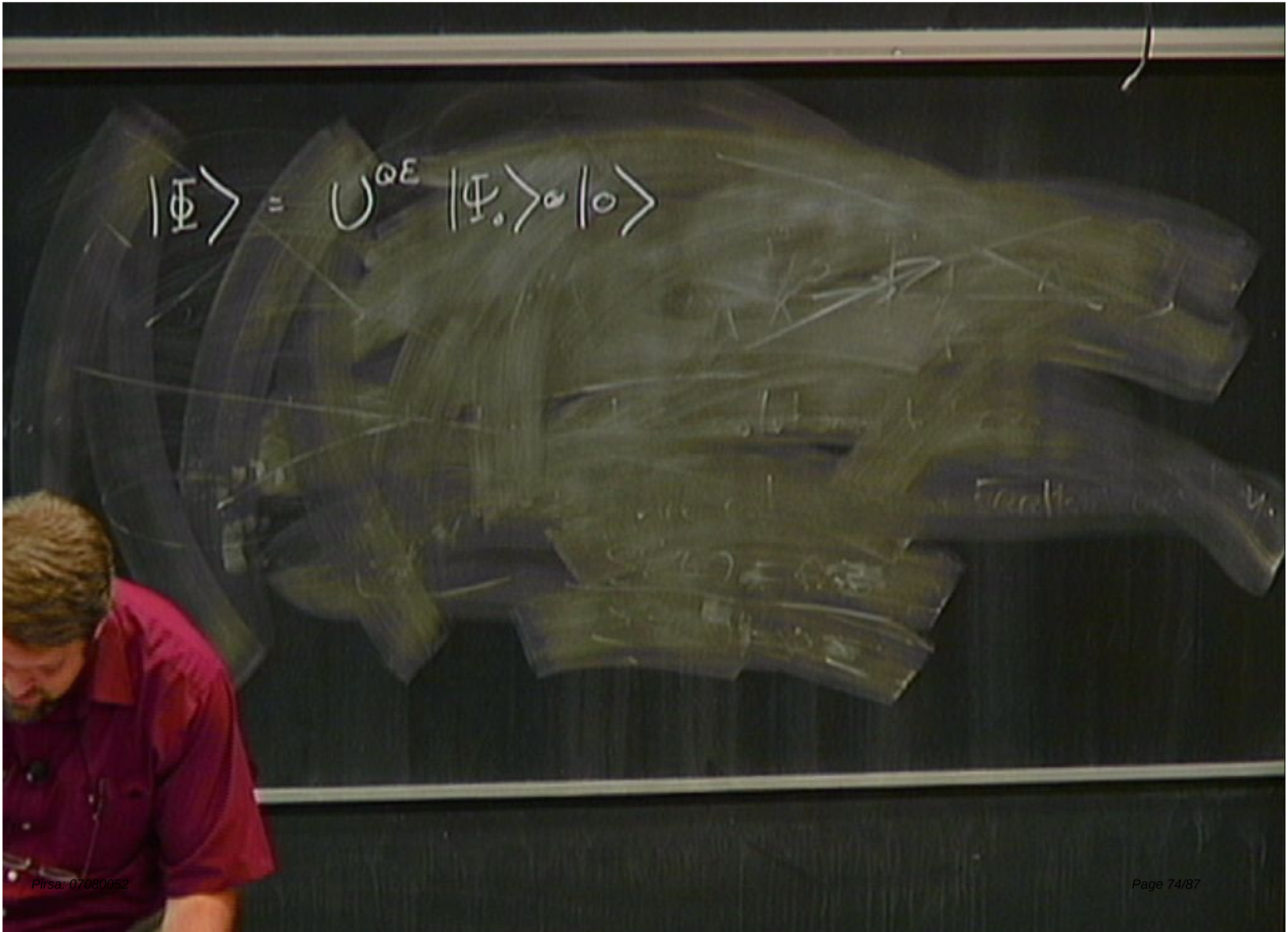
Append system E

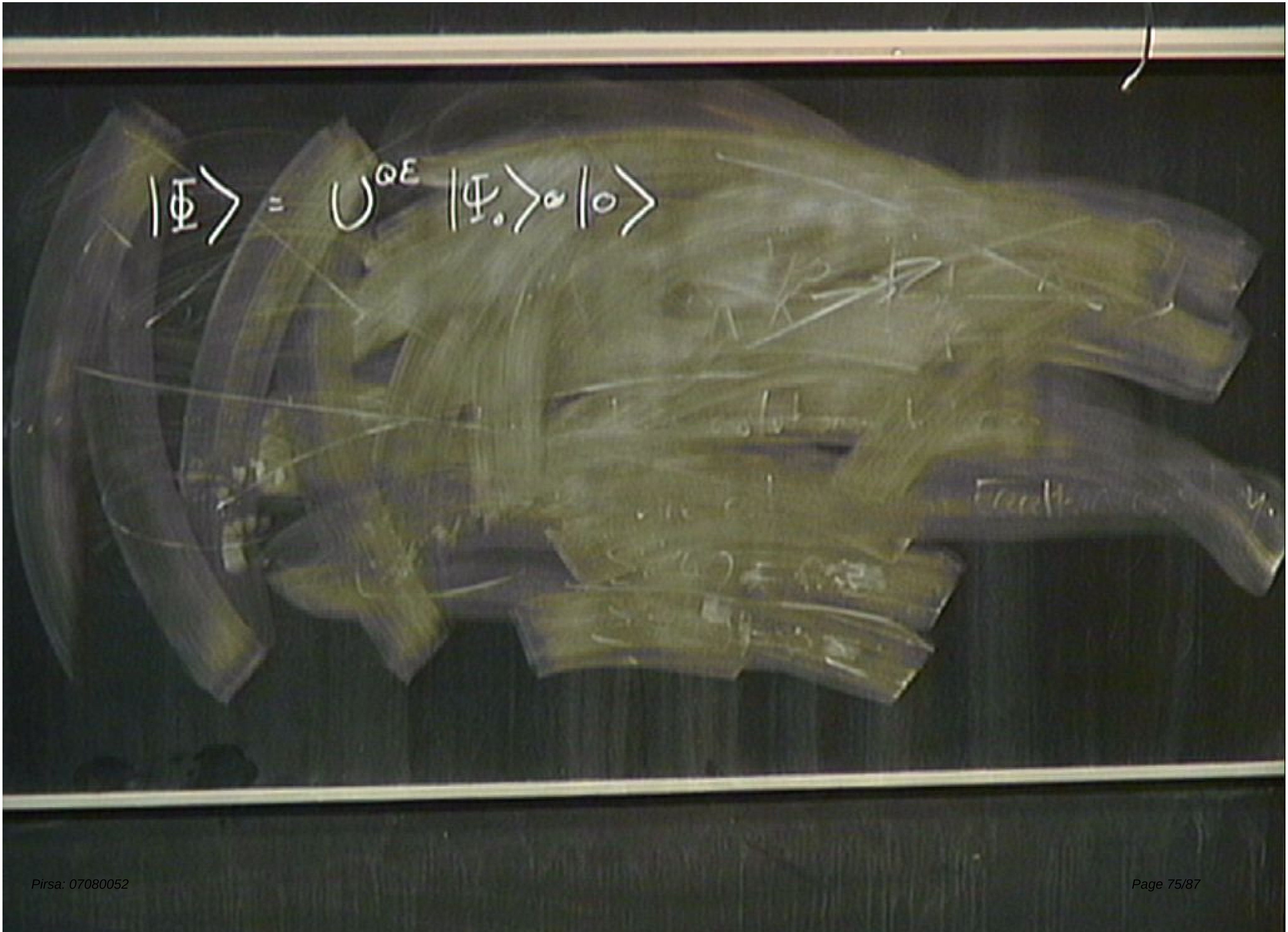
$$\mathrm{tr}_E\, |\Phi\rangle\langle\Phi| = \omega^{RQ}$$

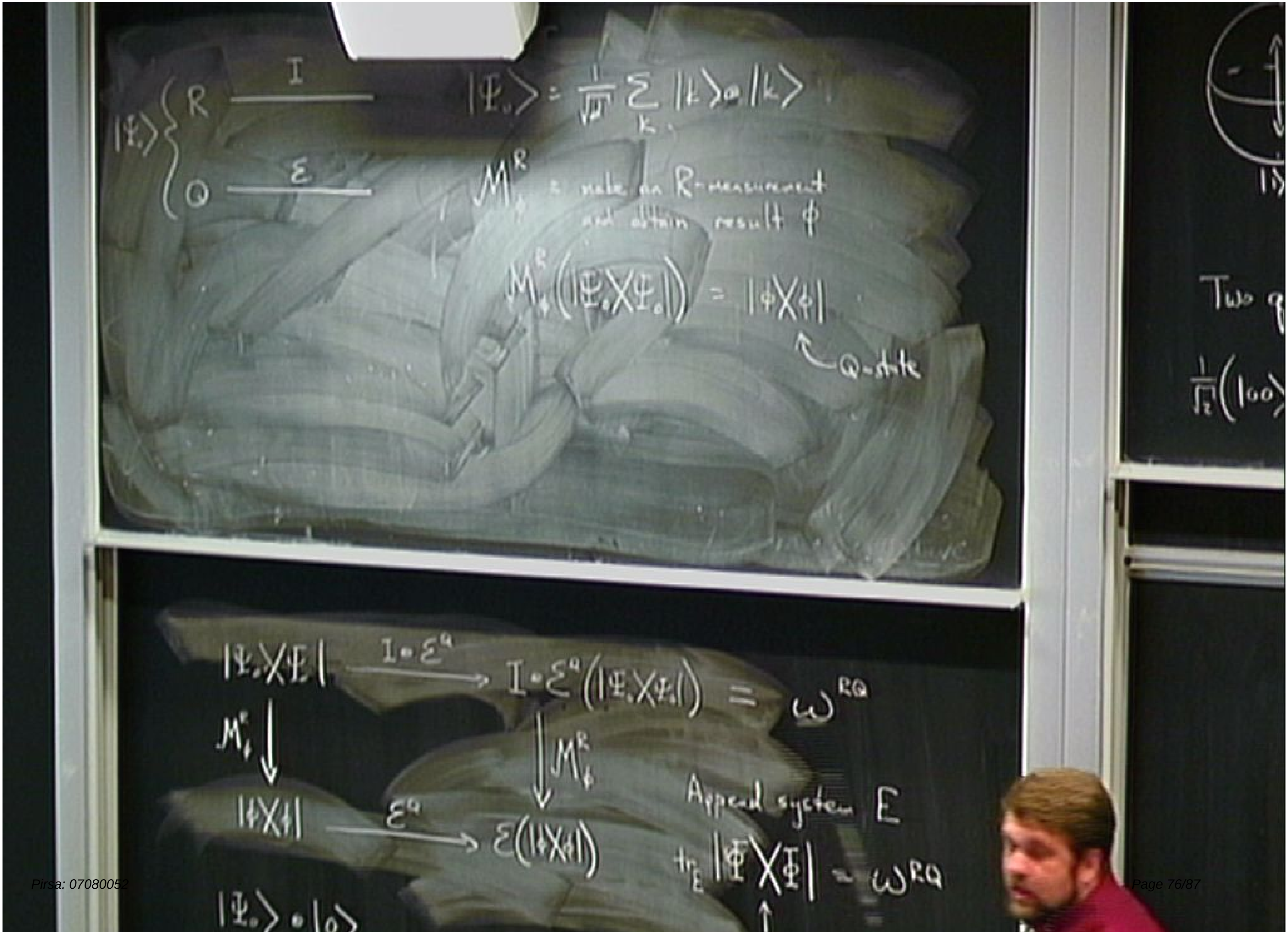
$\uparrow$ RQE state

$$|\Psi_0\rangle \otimes |0\rangle$$

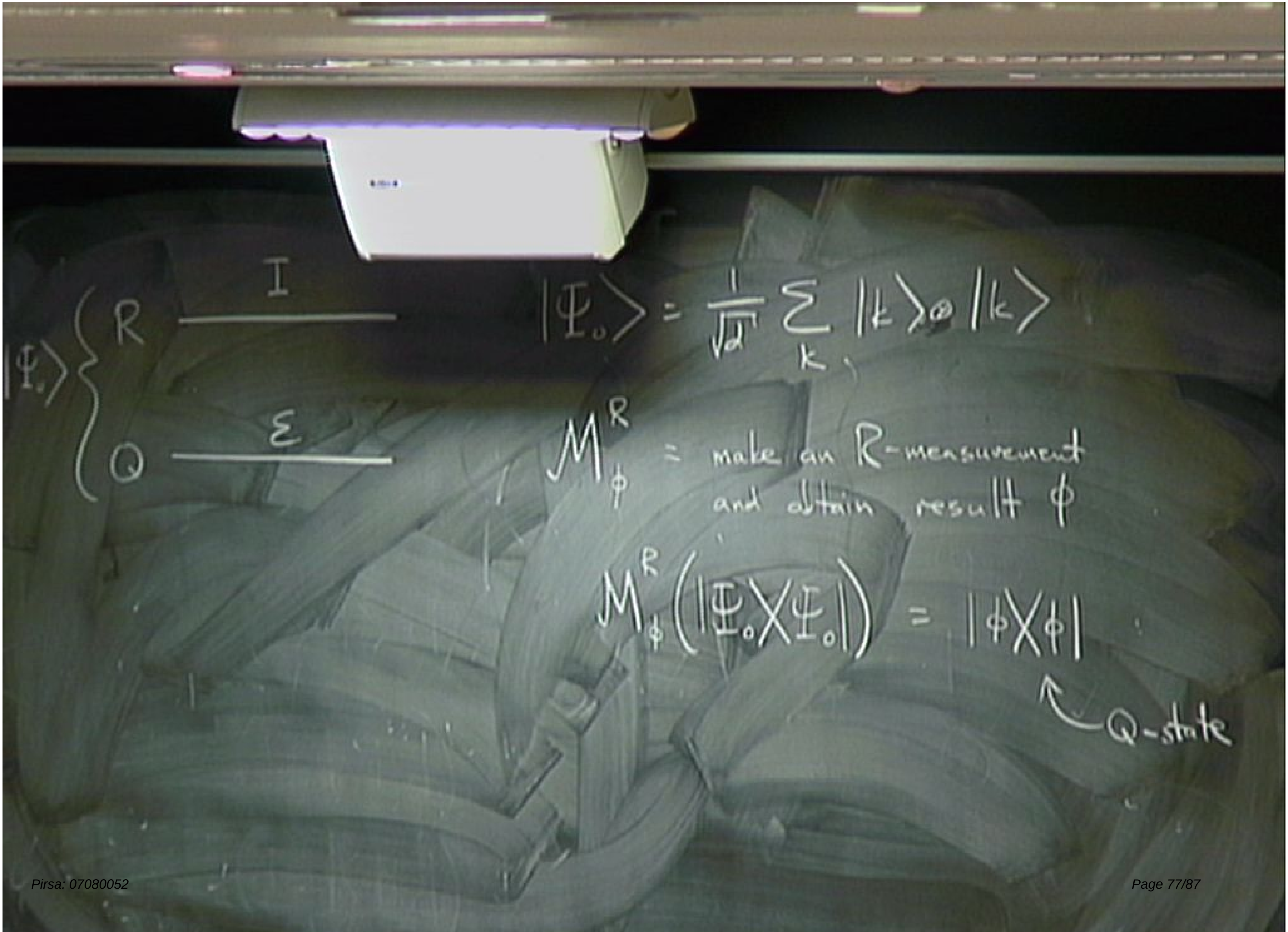
$$|\Phi\rangle$$


$|\Phi\rangle = U^{QE} |\Phi_0\rangle \otimes |0\rangle$


|Φ〉 = U^QE |Ψ₀〉⊗|0〉

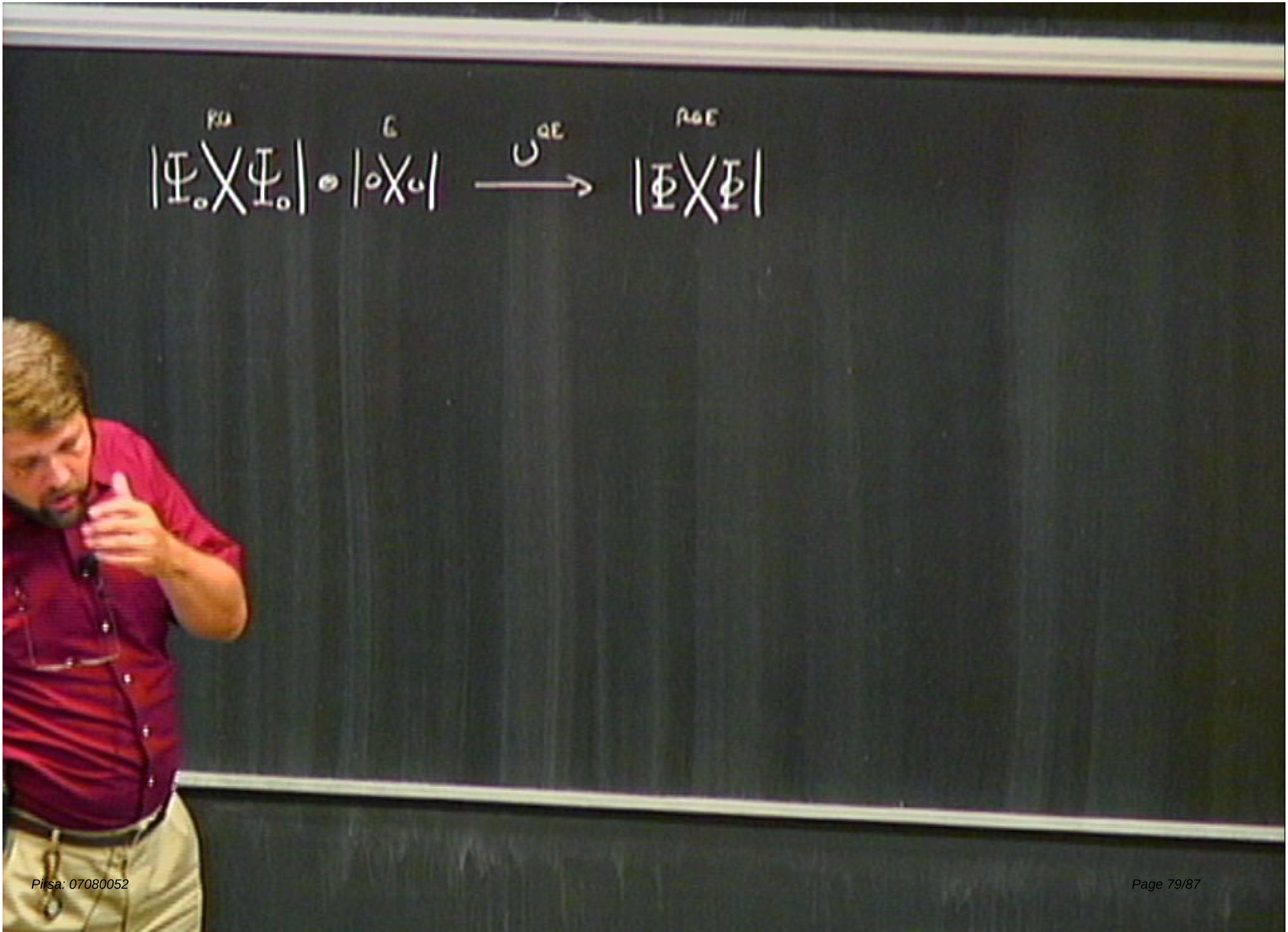


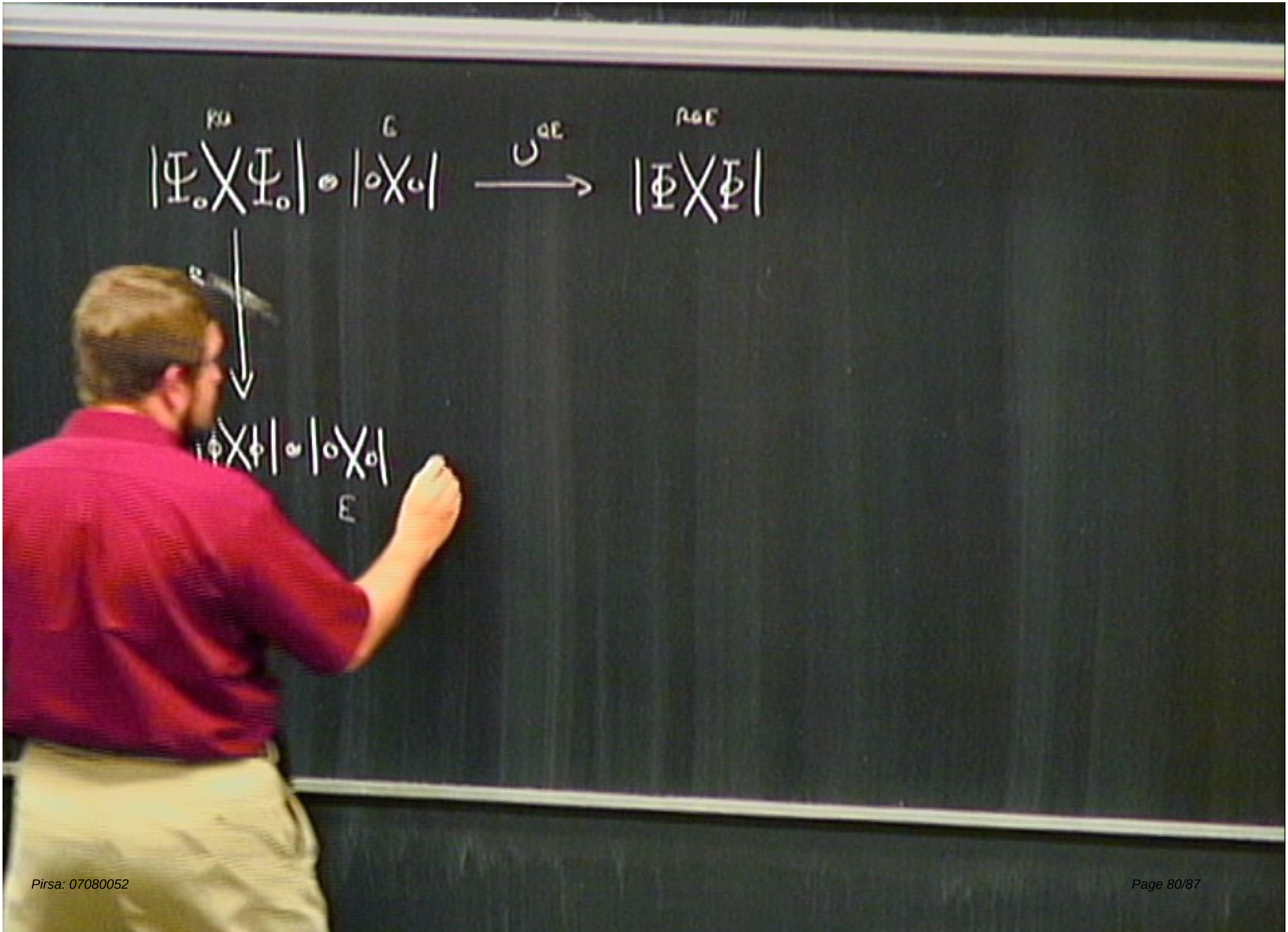
R I
Q ℰ
|Ψ₀⟩ = 1/√d Σ_k |k⟩⊗|k⟩
M_φ^R = make an R-measurement and obtain result φ
M_φ^R(|Ψ₀⟩⟨Ψ₀|) = |φ⟩⟨φ|
Q-state
Two q
|Ψ₀⟩⟨Ψ₀| → I⊗ℰ^Q → I⊗ℰ^Q(|Ψ₀⟩⟨Ψ₀|) = ω^RQ
M_φ^R
|φ⟩⟨φ| → ℰ^Q → ℰ(|φ⟩⟨φ|)
Append system E
tr_E |Φ⟩⟨Φ| = ω^RQ
|Ψ₀⟩⊗|0⟩

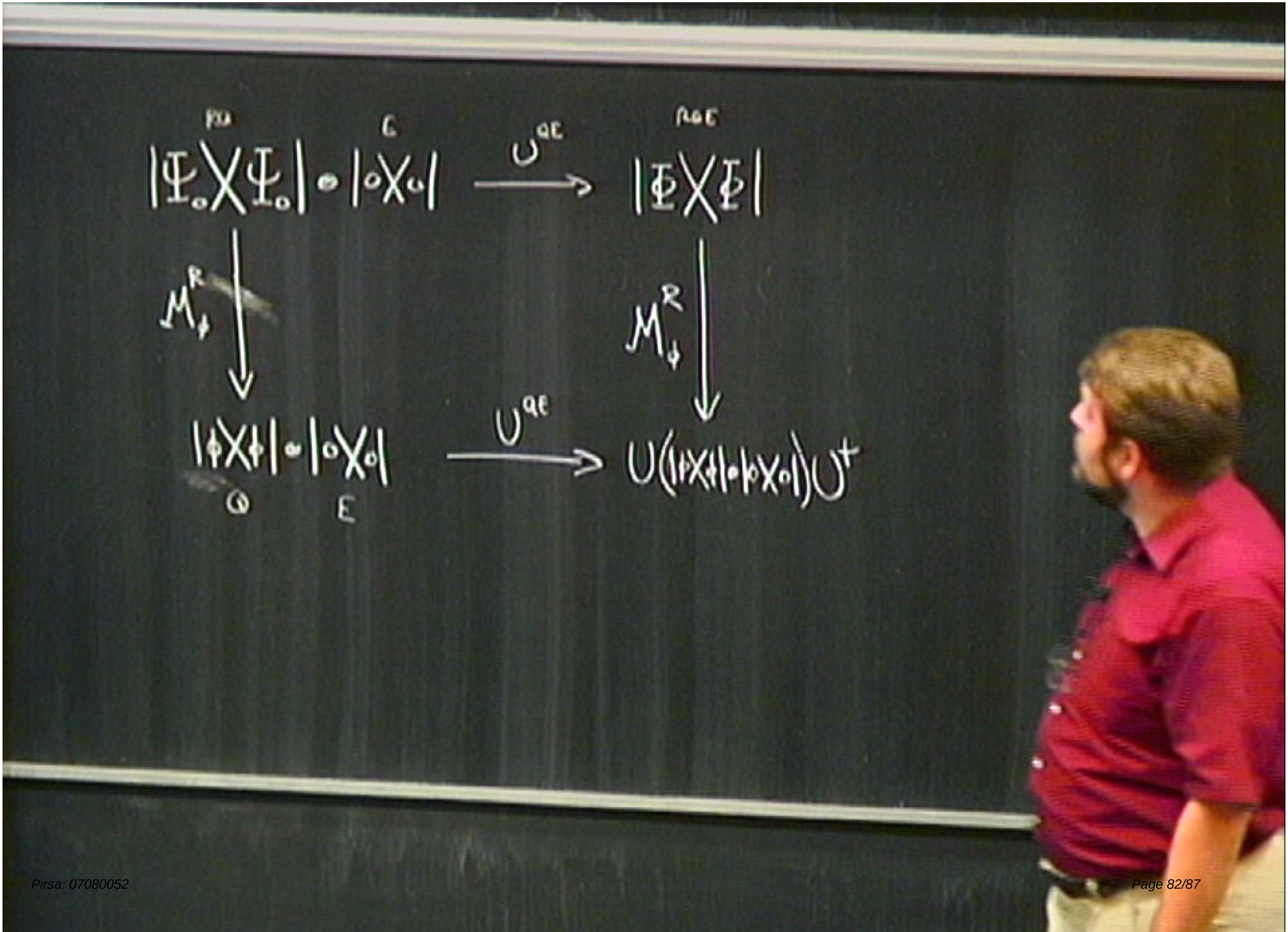


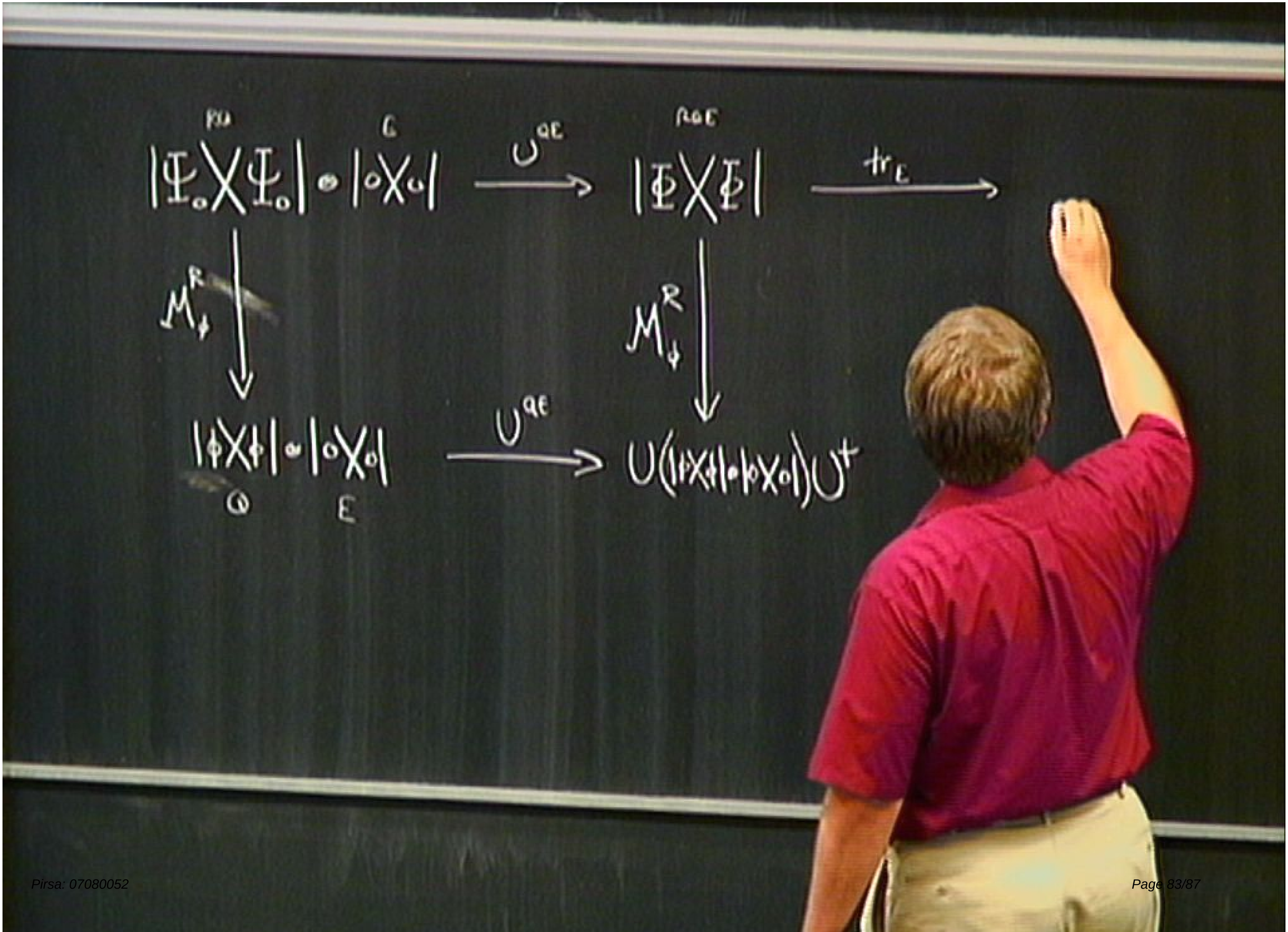
$|\Psi_0\rangle$ R I Q E
$|\Psi_0\rangle = \frac{1}{\sqrt{d}} \sum_k |k\rangle \otimes |k\rangle$
M^R_ϕ = make an R-measurement and obtain result ϕ
$M^R_\phi(|\Psi_0\rangle\langle\Psi_0|) = |\phi\rangle\langle\phi|$
Q-state

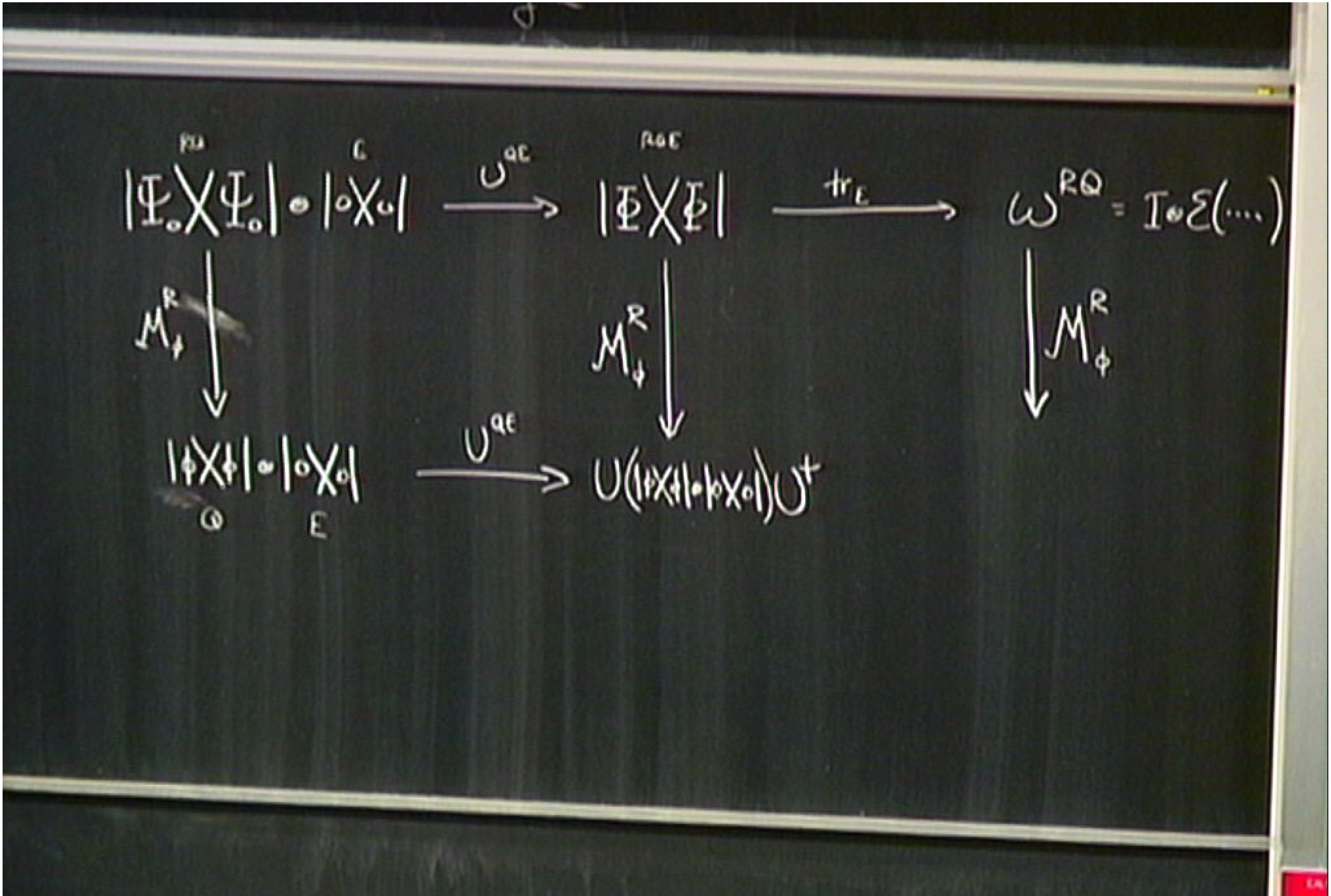
$$|\Psi_0\rangle\langle\Psi_0|^{PQ} \otimes |0\rangle\langle 0|^{G}$$

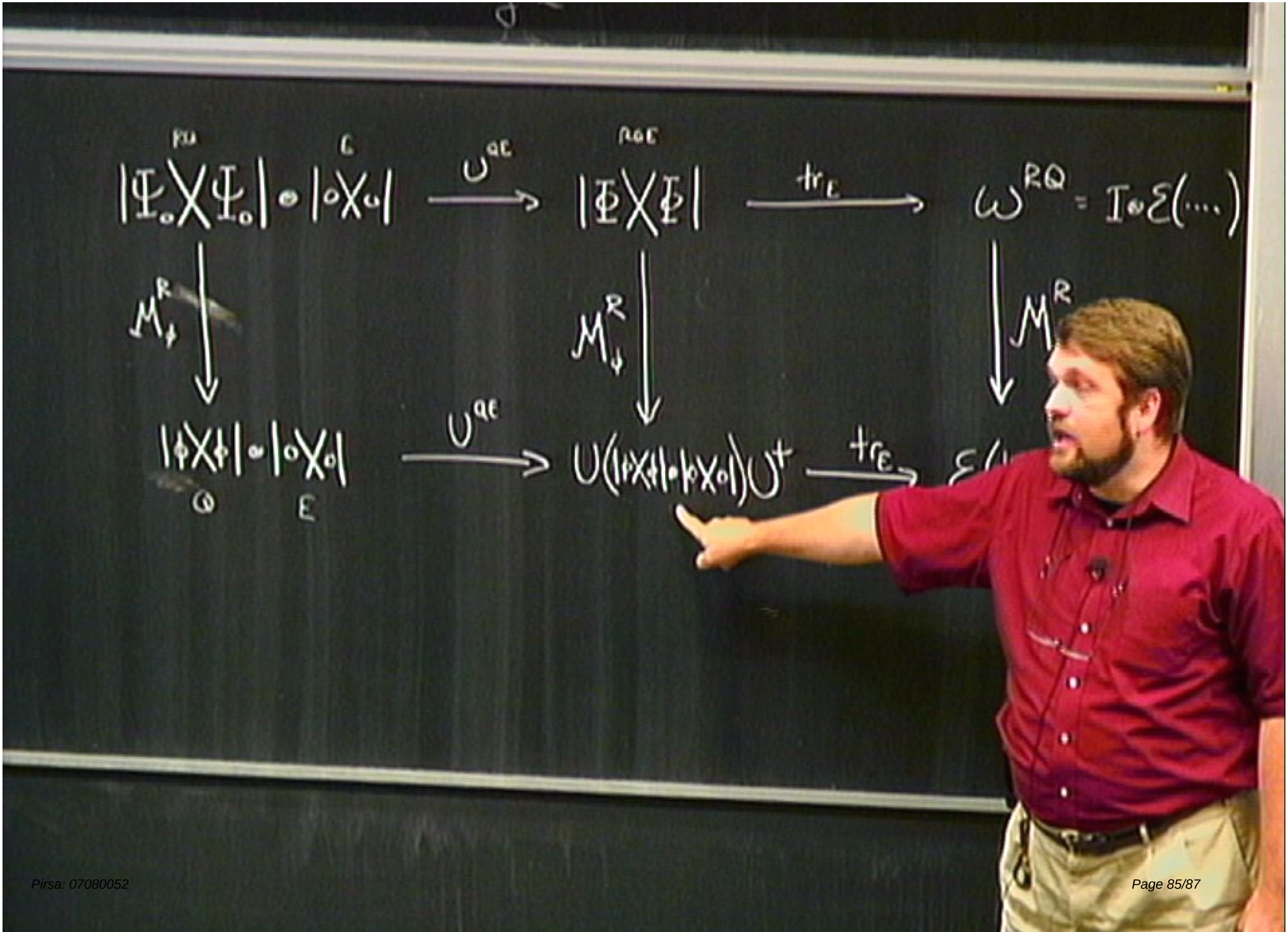

$|\Psi_0\rangle\langle\Psi_0|^{RA} \otimes |0\rangle\langle 0|^{E} \xrightarrow{U^{AE}} |\Phi\rangle\langle\Phi|^{RAE}$



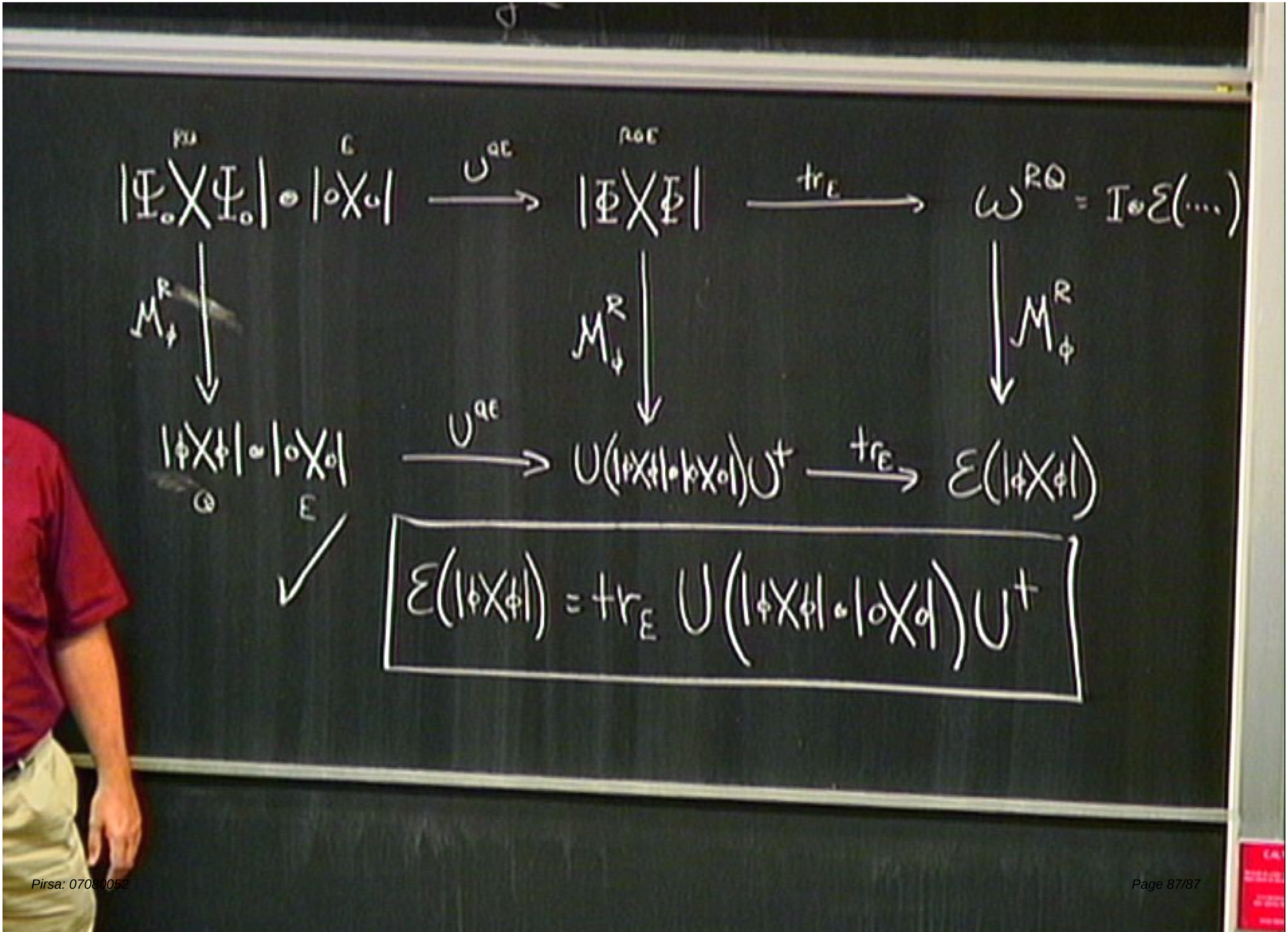

RQ
E
$|\Psi_0\rangle\langle\Psi_0| \otimes |0\rangle\langle 0|$
U^{QE}
RQE
$|\Phi\rangle\langle\Phi|$
M_ϕ^R
M_ϕ^R
$|\phi\rangle\langle\phi| \otimes |0\rangle\langle 0|$
Q
E
U^{QE}
$U(|\phi\rangle\langle\phi| \otimes |0\rangle\langle 0|)U^\dagger$




$|\Phi_0\rangle\langle\Phi_0|^{RQ} \otimes |0\rangle\langle 0|^{E} \xrightarrow{U^{QE}} |\Phi\rangle\langle\Phi|^{RQE} \xrightarrow{tr_E} \omega^{RQ} = I \otimes \mathcal{E}(\ldots)$
$\mathcal{M}^R_\phi$
$\mathcal{M}^R_\phi$
$\mathcal{M}^R_\phi$
$|\phi\rangle\langle\phi|^{Q} \otimes |0\rangle\langle 0|^{E} \xrightarrow{U^{QE}} U(|\phi\rangle\langle\phi| \otimes |0\rangle\langle 0|)U^{\dagger}$


|Ψ₀⟩⟨Ψ₀| ⊗ |0⟩⟨0|
U^QE
|Φ⟩⟨Φ|
tr_E
ω^RQ = I⊗E(....)
M_φ^R
|φ⟩⟨φ| ⊗ |0⟩⟨0|
Q
E
U^QE
U(|φ⟩⟨φ|⊗|0⟩⟨0|)U†
tr_E

$$
\begin{array}{ccccc}
|\Psi_0\rangle\langle\Psi_0|^{RQ} \otimes |0\rangle\langle 0|^{E} & \xrightarrow{U^{QE}} & |\Phi\rangle\langle\Phi|^{RQE} & \xrightarrow{tr_E} & \omega^{RQ} = I \otimes \mathcal{E}(\dots) \\
\Big\downarrow \mathcal{M}^{R}_{\phi} & & \Big\downarrow \mathcal{M}^{R}_{\phi} & & \Big\downarrow \mathcal{M}^{R}_{\phi} \\
|\phi\rangle\langle\phi|^{Q} \otimes |0\rangle\langle 0|^{E} & \xrightarrow{U^{QE}} & U(|\phi\rangle\langle\phi| \otimes |0\rangle\langle 0|)U^{\dagger} & \xrightarrow{tr_E} & \mathcal{E}(|\phi\rangle\langle\phi|)
\end{array}
$$


$|\Psi_0\rangle\langle\Psi_0| \otimes |0\rangle\langle 0| \xrightarrow{U^{QE}} |\Phi\rangle\langle\Phi| \xrightarrow{tr_E} \omega^{RQ} = I \otimes \mathcal{E}(\dots)$
M^R_ϕ
$|\phi\rangle\langle\phi| \otimes |0\rangle\langle 0| \xrightarrow{U^{QE}} U(|\phi\rangle\langle\phi| \otimes |0\rangle\langle 0|)U^\dagger \xrightarrow{tr_E} \mathcal{E}(|\phi\rangle\langle\phi|)$
$\mathcal{E}(|\phi\rangle\langle\phi|) = tr_E\, U(|\phi\rangle\langle\phi| \otimes |0\rangle\langle 0|)U^\dagger$