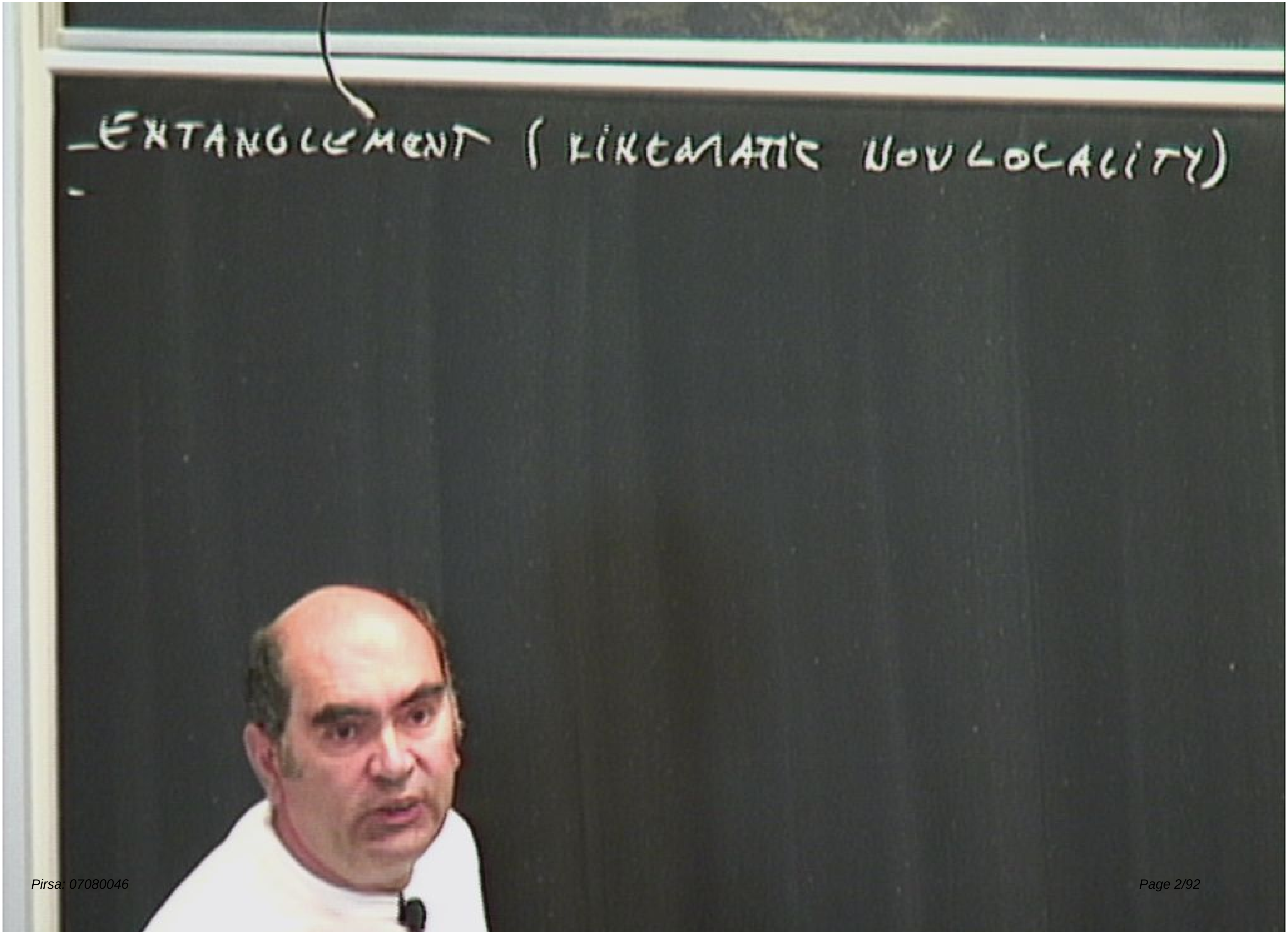
Title: Time symmetric quantum theory, weak measurements, and modular variables (Part 1A)

Date: Aug 28, 2007 03:15 PM

URL: http://pirsa.org/07080046

Abstract:


- ENTANGLEMENT (KINEMATIC NONLOCALITY)
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- ENTANGLEMENT (KINEMATIC NONLOCALITY)
- DYNAMICAL NON-LOCALITY (MODULAR VARIABLES)
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- PRE AND POST SELECTION, WEAK MTS. TIME FLOW

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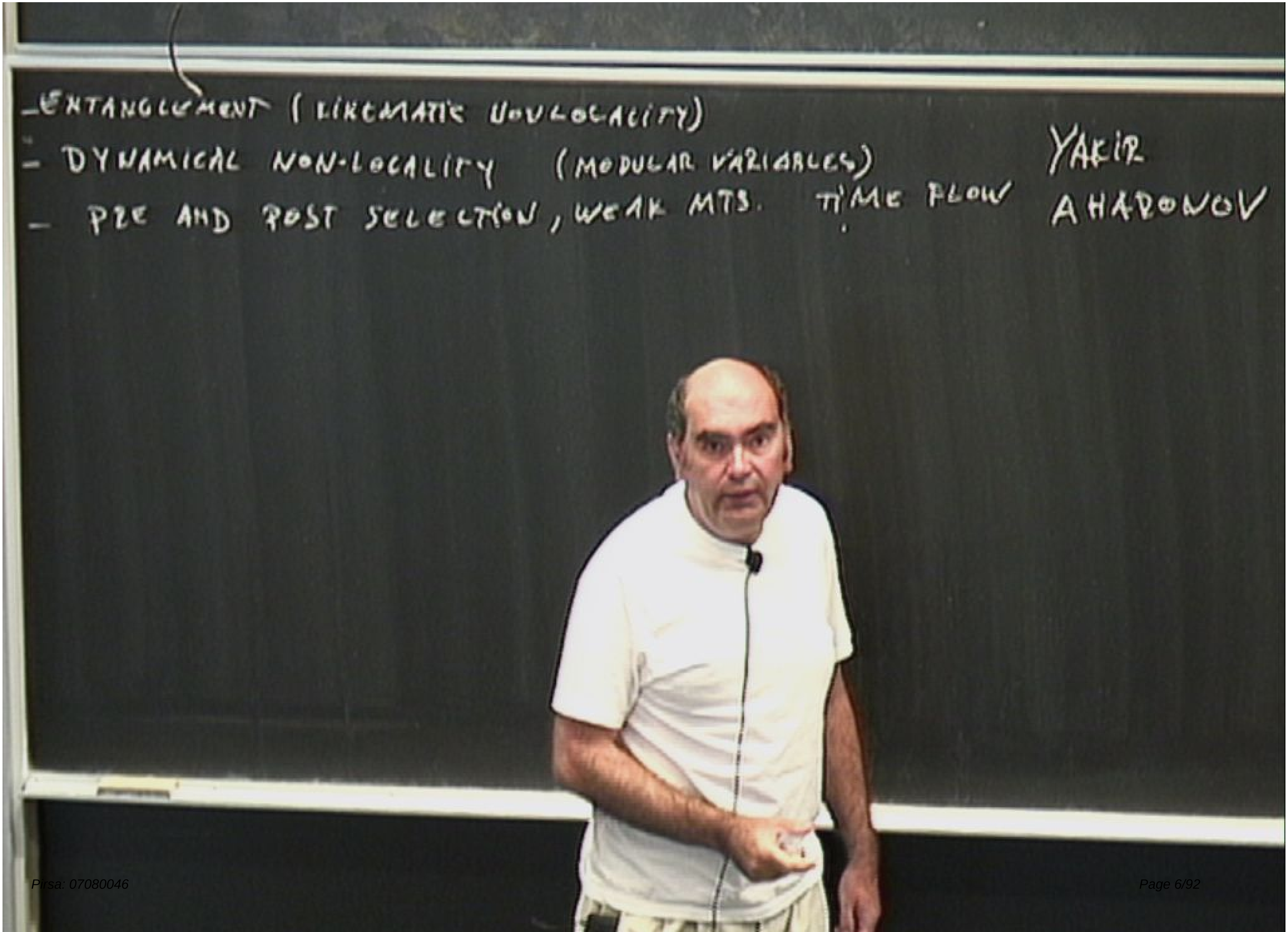
- ENTANGLEMENT (KINEMATIC NONLOCALITY)
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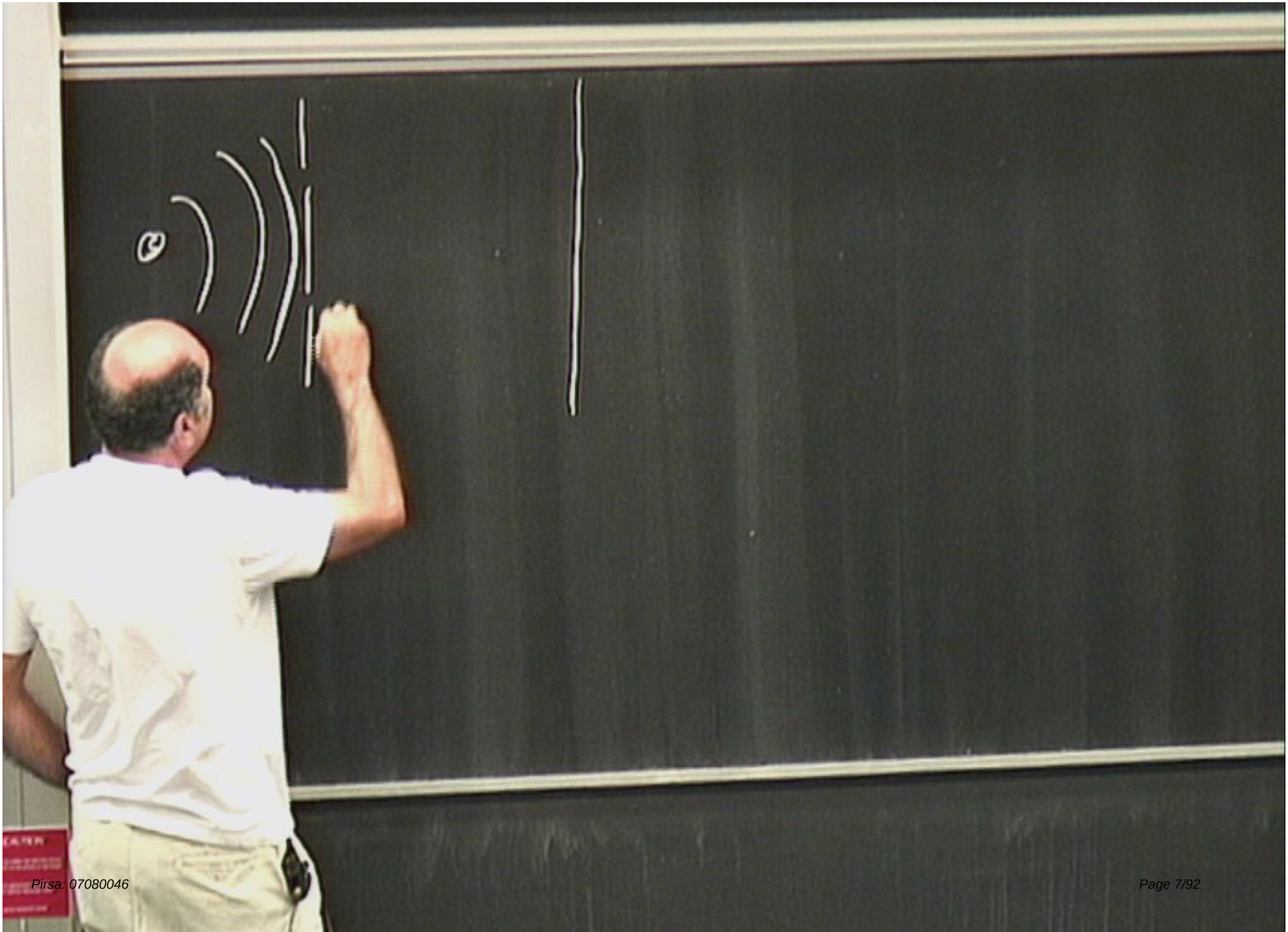
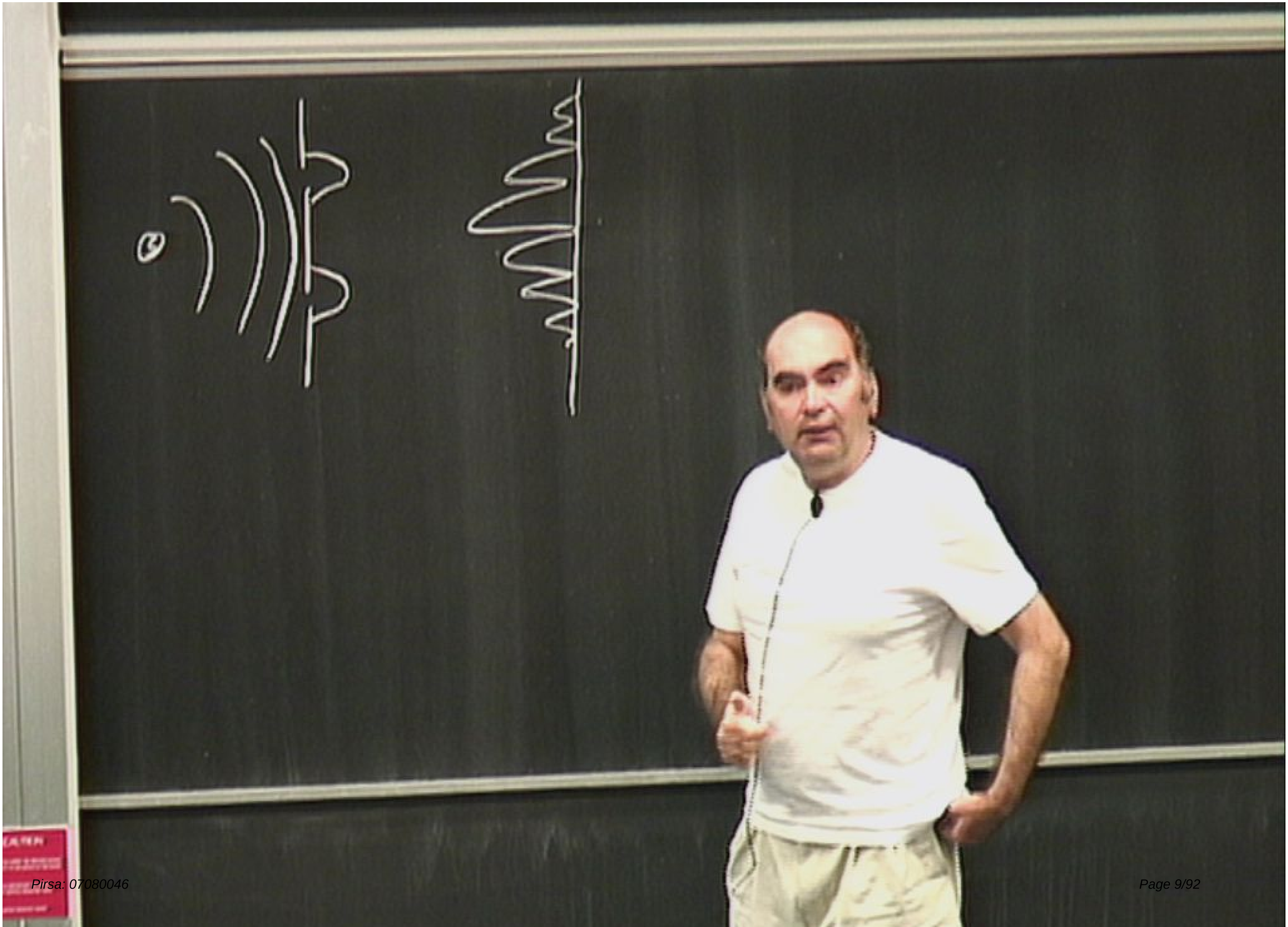
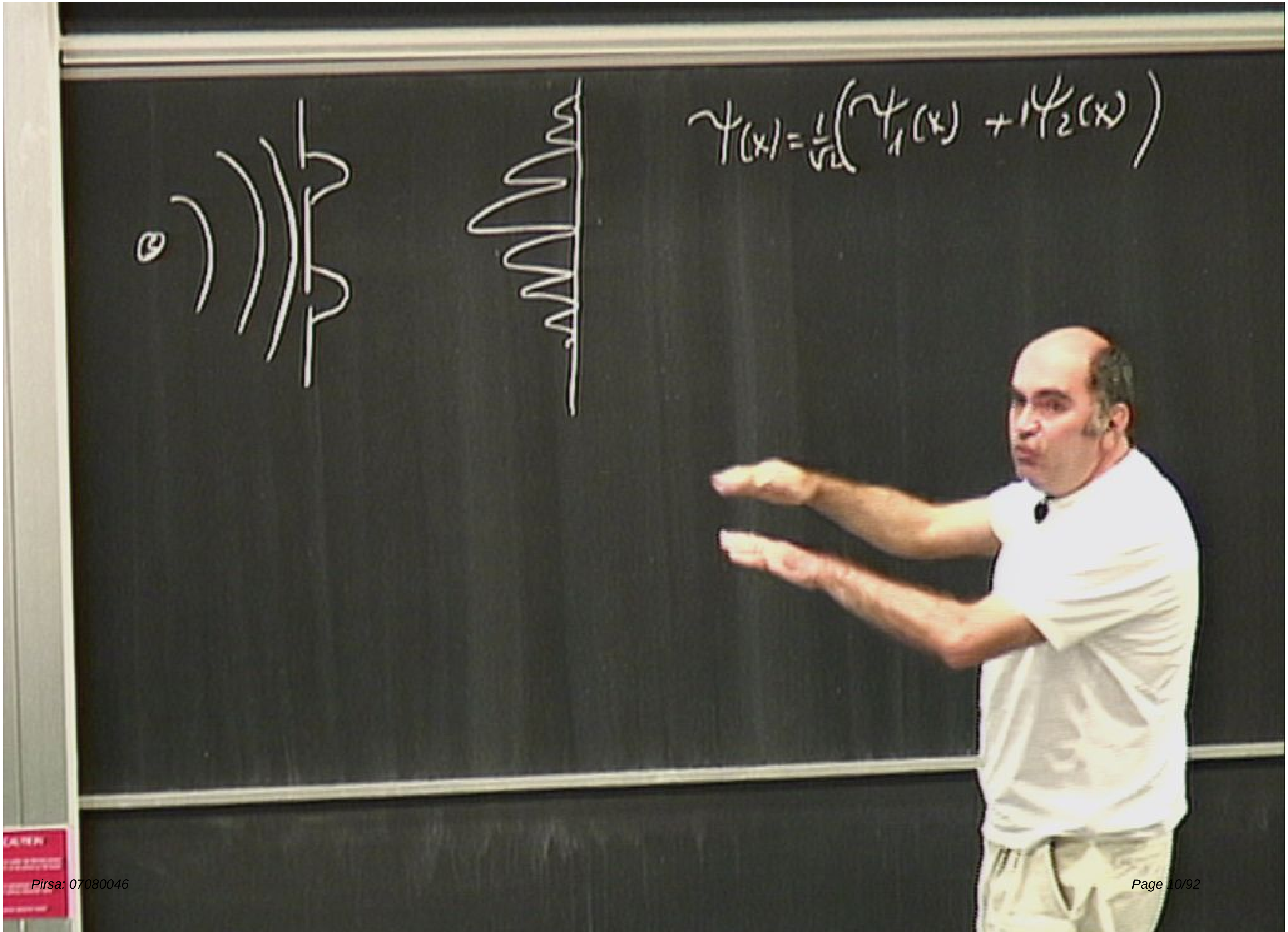
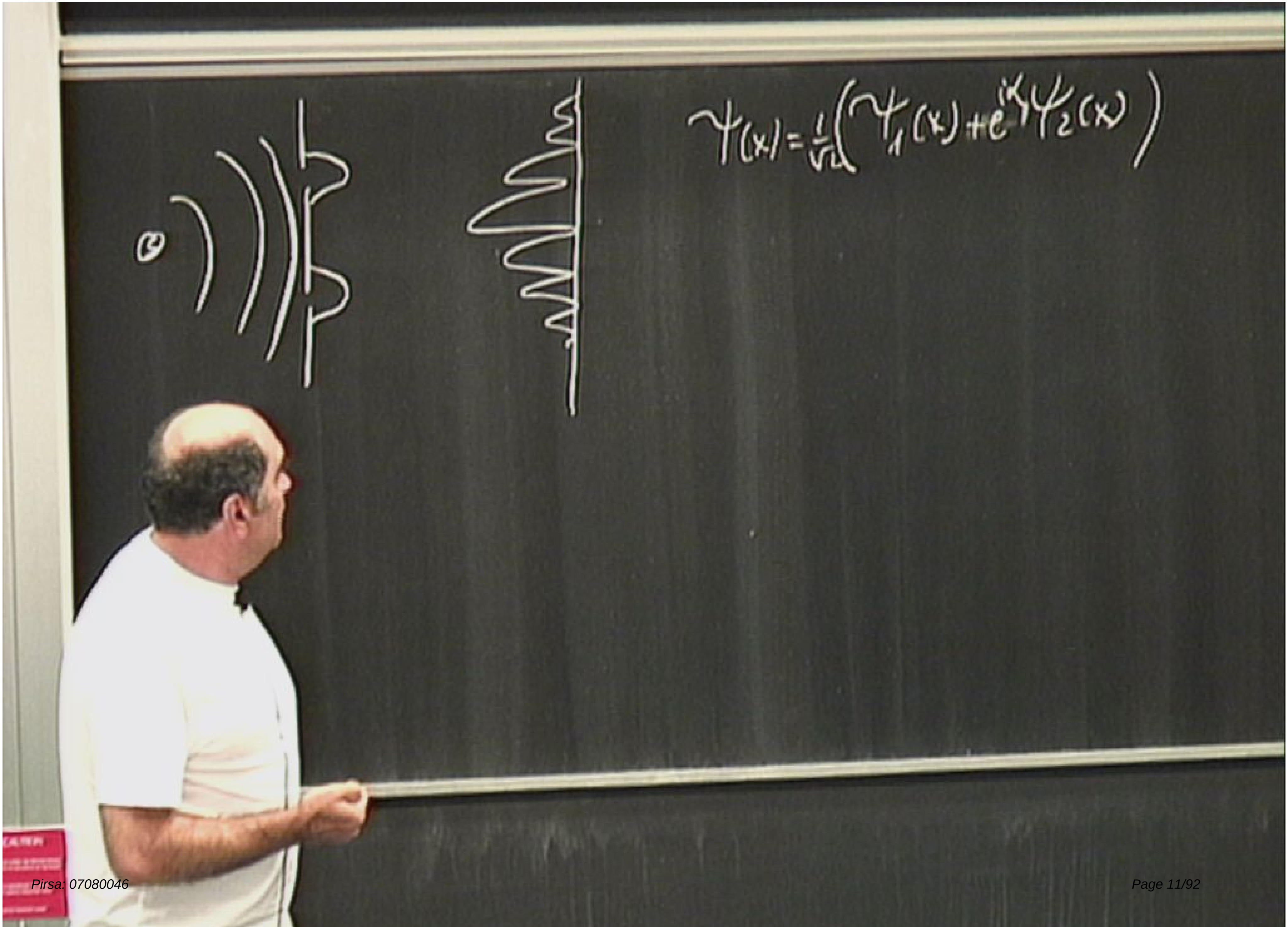
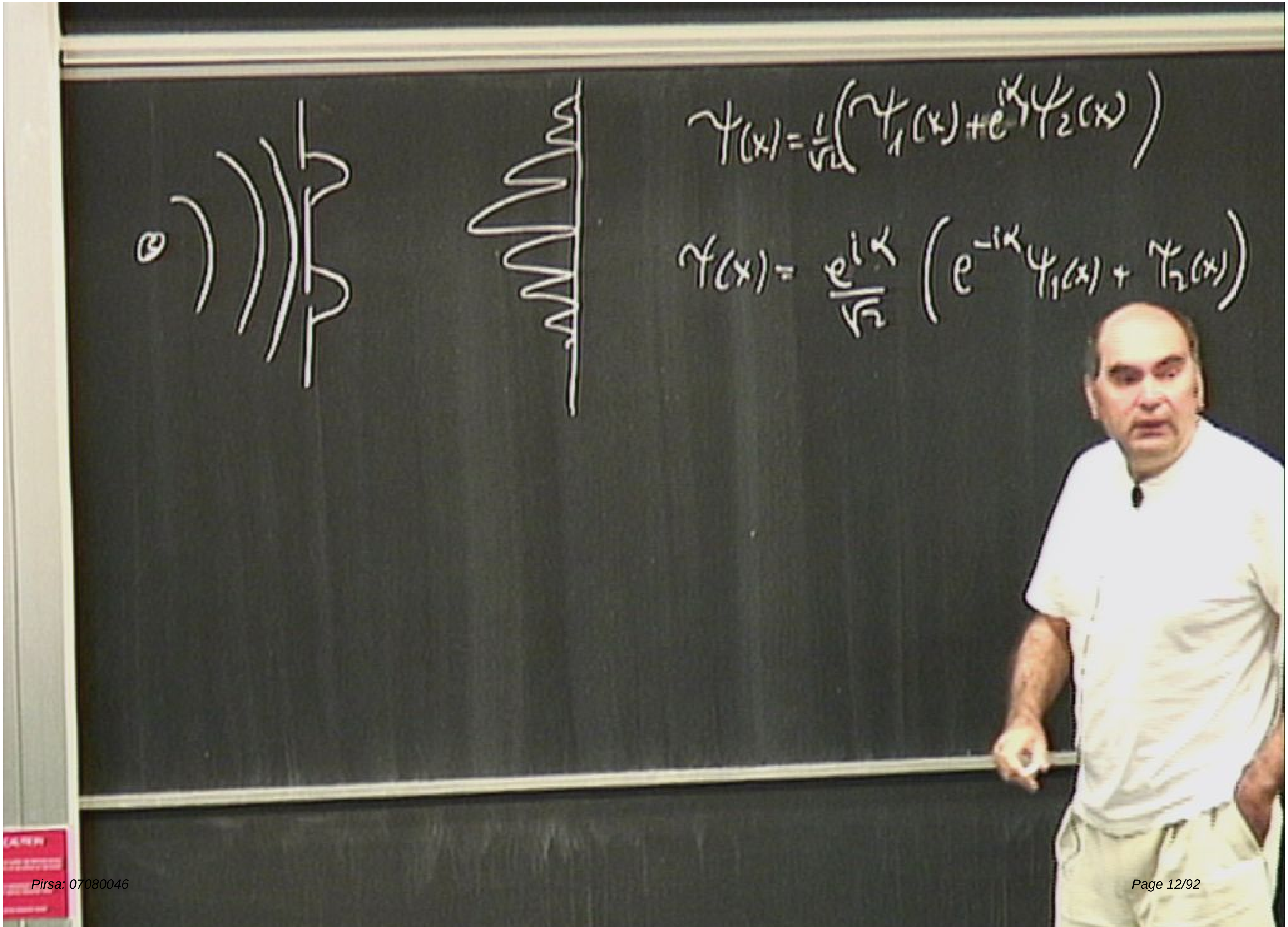
- PRE AND POST SELECTION, WEAK MTS. TIME FLOW
YAKIR
AHARONOV

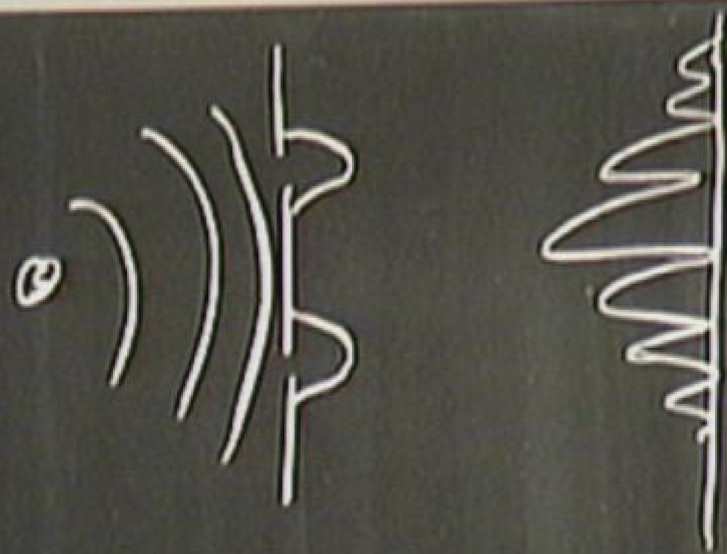





$\psi(x) = \frac{1}{\sqrt{2}}\left(\psi_1(x) + i\psi_2(x)\right)$

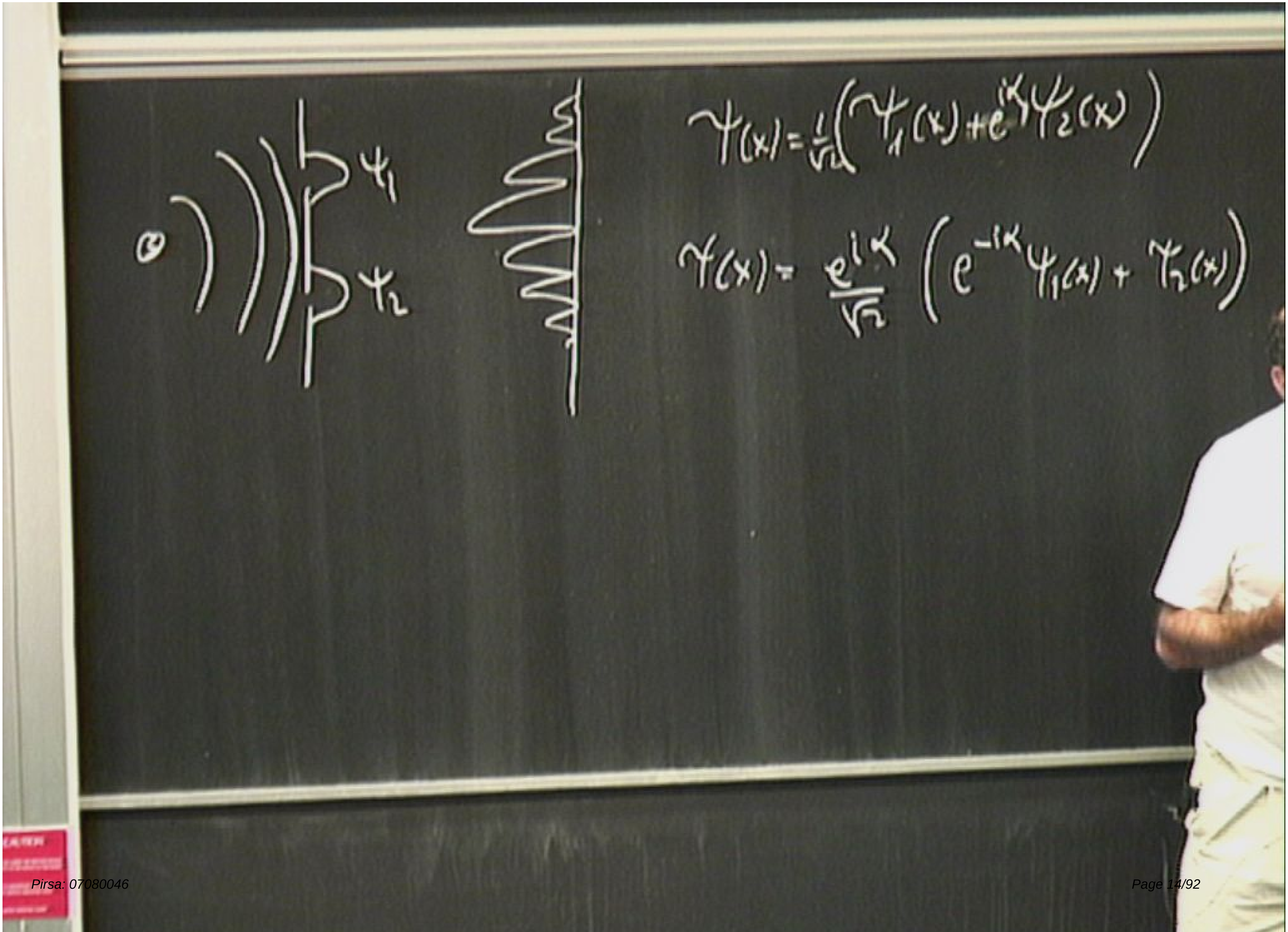

$\psi(x) = \frac{1}{\sqrt{2}}\left(\psi_1(x) + e^{i\chi}\psi_2(x)\right)$

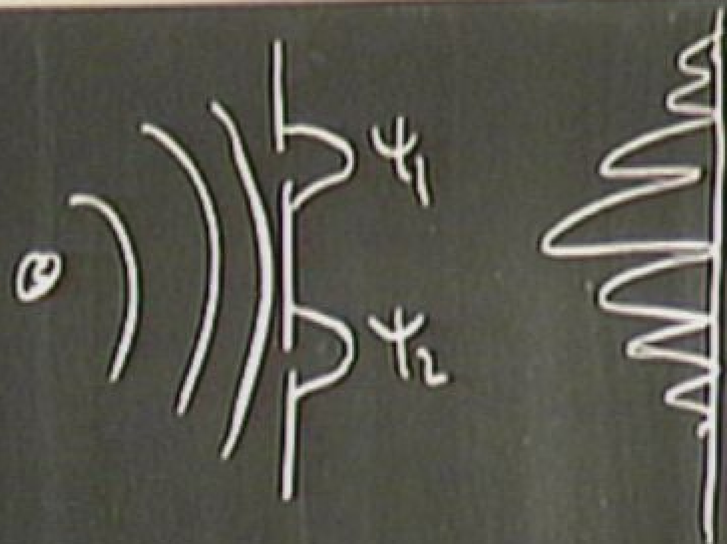

$\psi(x) = \frac{1}{\sqrt{2}}\left(\psi_1(x) + e^{i\alpha}\psi_2(x)\right)$
$\psi(x) = \frac{e^{i\alpha}}{\sqrt{2}}\left(e^{-i\alpha}\psi_1(x) + \psi_2(x)\right)$



$$\psi(x) = \frac{1}{\sqrt{2}}\left(\psi_1(x) + e^{i\kappa}\psi_2(x)\right)$$

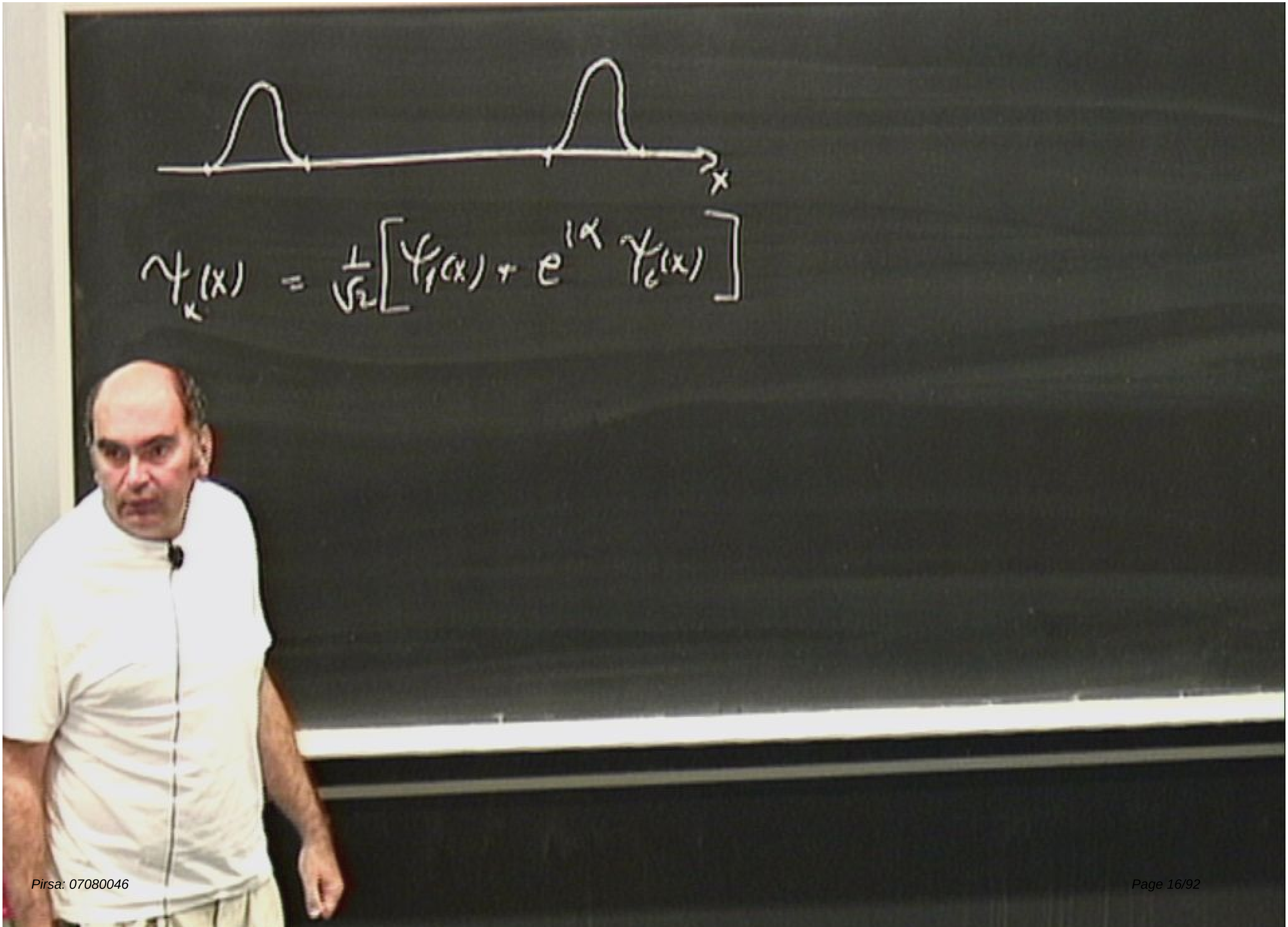
$$\psi(x) = \frac{e^{i\kappa}}{\sqrt{2}}\left(e^{-i\kappa}\psi_1(x) + \psi_2(x)\right)$$

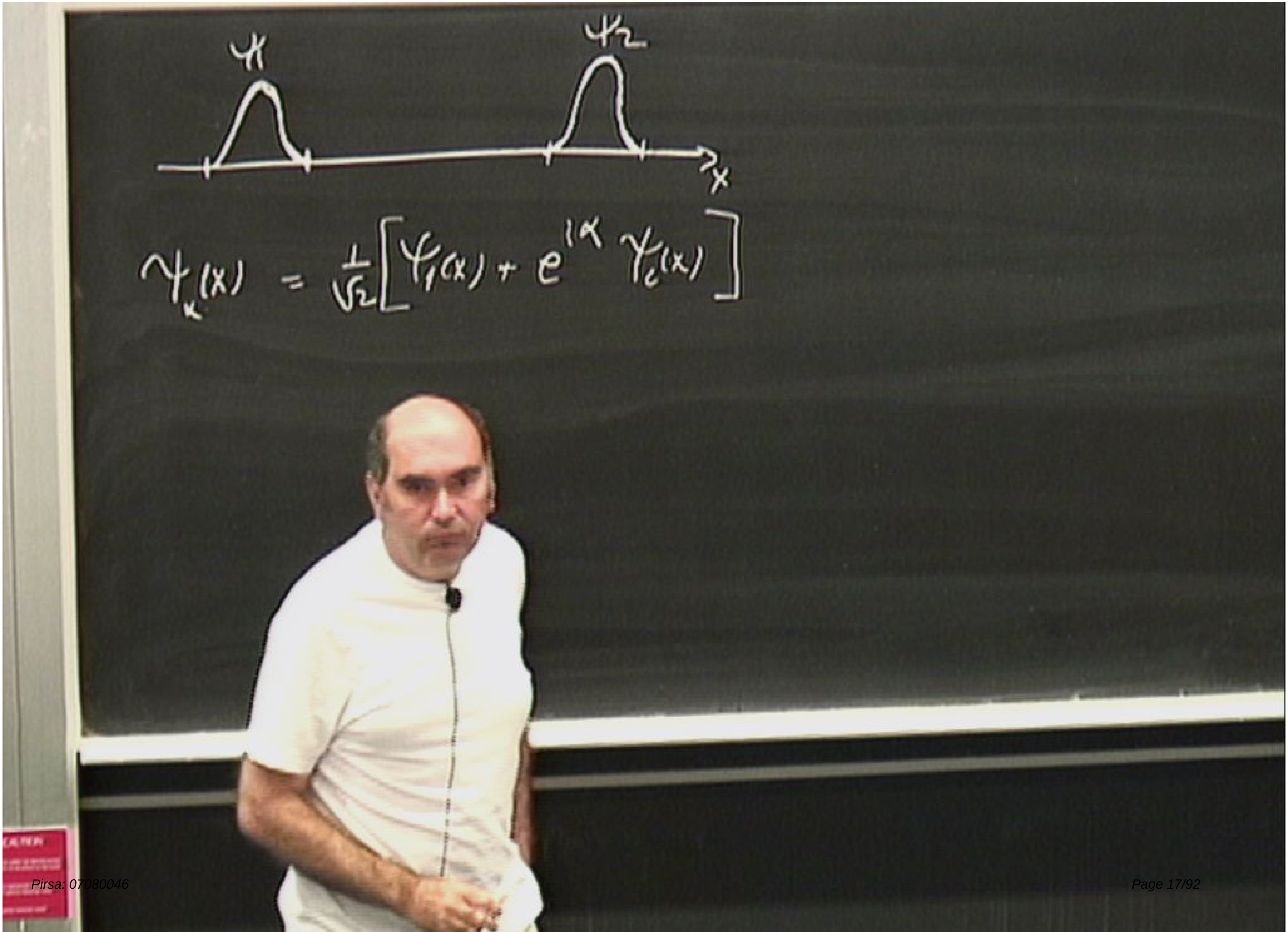

ψ_1
ψ_2
$\psi(x) = \frac{1}{\sqrt{2}}\left(\psi_1(x) + e^{i\kappa}\psi_2(x)\right)$
$\psi(x) = \frac{e^{i\kappa}}{\sqrt{2}}\left(e^{-i\kappa}\psi_1(x) + \psi_2(x)\right)$

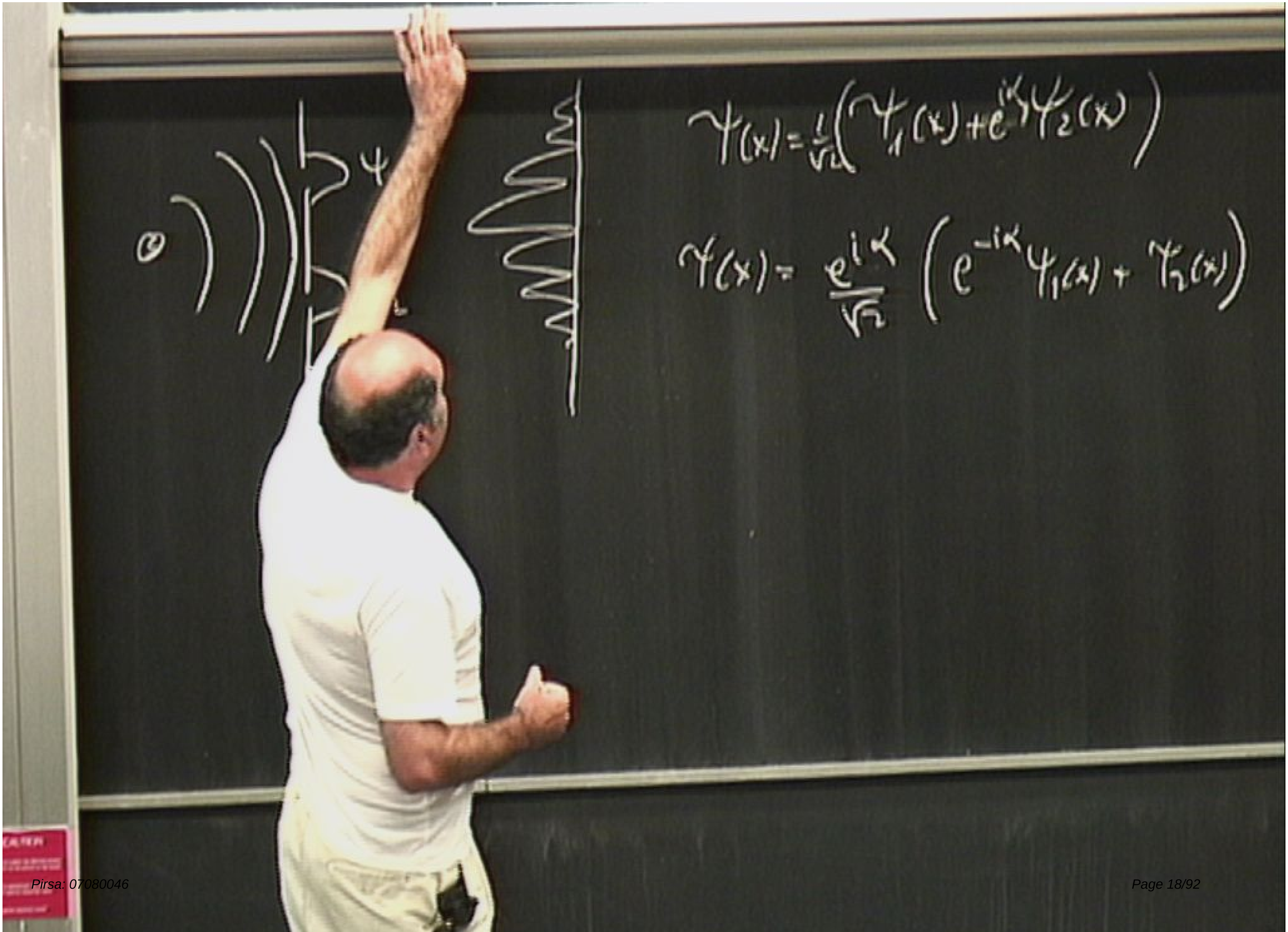


$$\psi(x) = \frac{1}{\sqrt{2}}\left(\psi_1(x) + e^{i\alpha}\psi_2(x)\right)$$

$$\psi(x) = \frac{e^{i\alpha}}{\sqrt{2}}\left(e^{-i\alpha}\psi_1(x) + \psi_2(x)\right)$$


x
$\psi_\alpha(x) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + e^{i\alpha}\psi_2(x)\right]$


ψ_1
ψ_2
x
$\psi_\alpha(x) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + e^{i\alpha}\psi_2(x)\right]$



ψ
$\psi(x) = \frac{1}{\sqrt{2}}\left(\psi_1(x) + e^{i\alpha}\psi_2(x)\right)$
$\psi(x) = \frac{e^{i\alpha}}{\sqrt{2}}\left(e^{-i\alpha}\psi_1(x) + \psi_2(x)\right)$

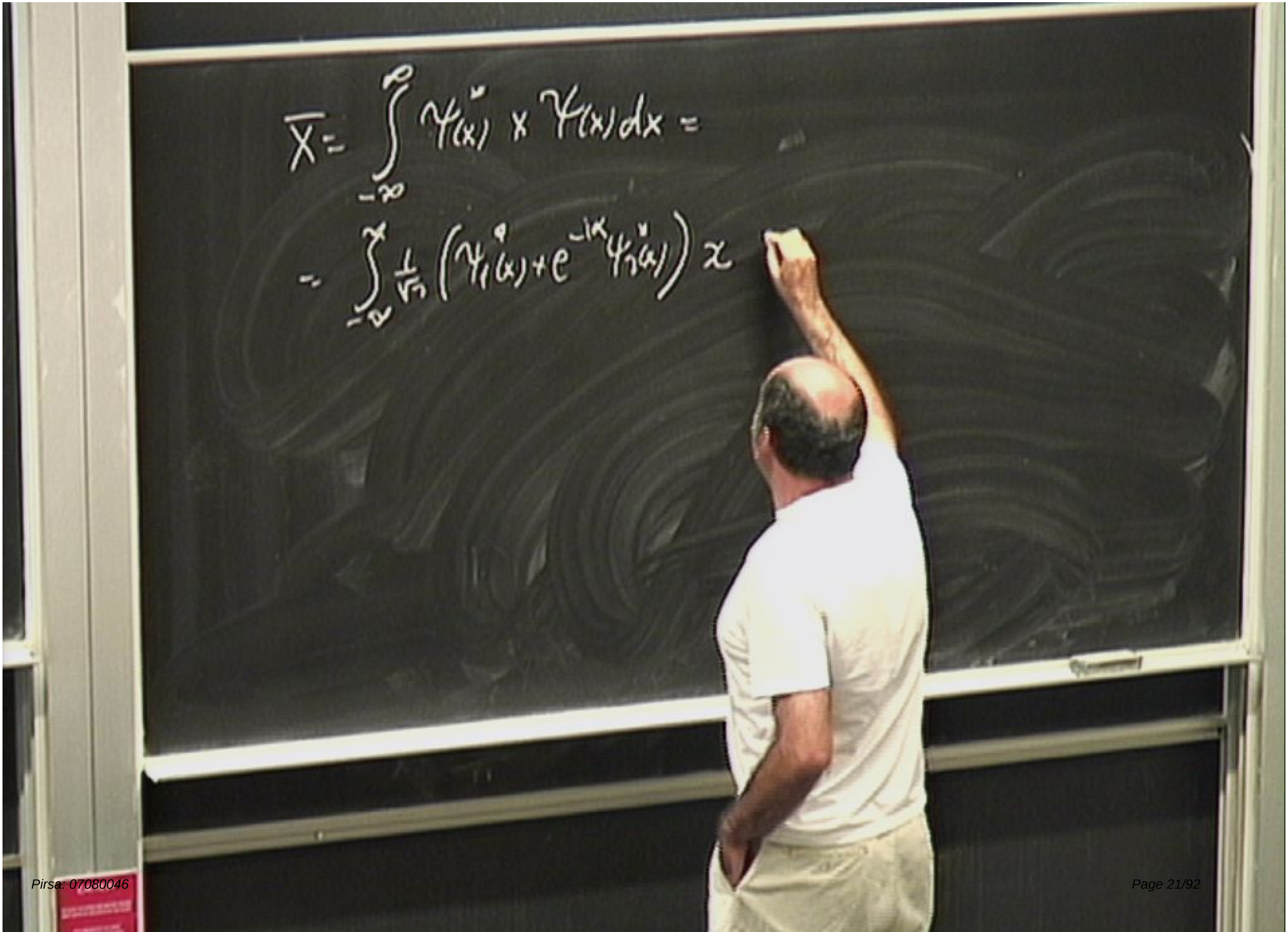
$$\psi_\alpha(x) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + e^{i\alpha}\,\psi_2(x)\right]$$

$$f(\hat{x}, \hat{p})$$

$$\psi_\alpha(x) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + e^{i\alpha}\,\psi_2(x)\right]$$

$$f(\hat{x},\hat{p})$$

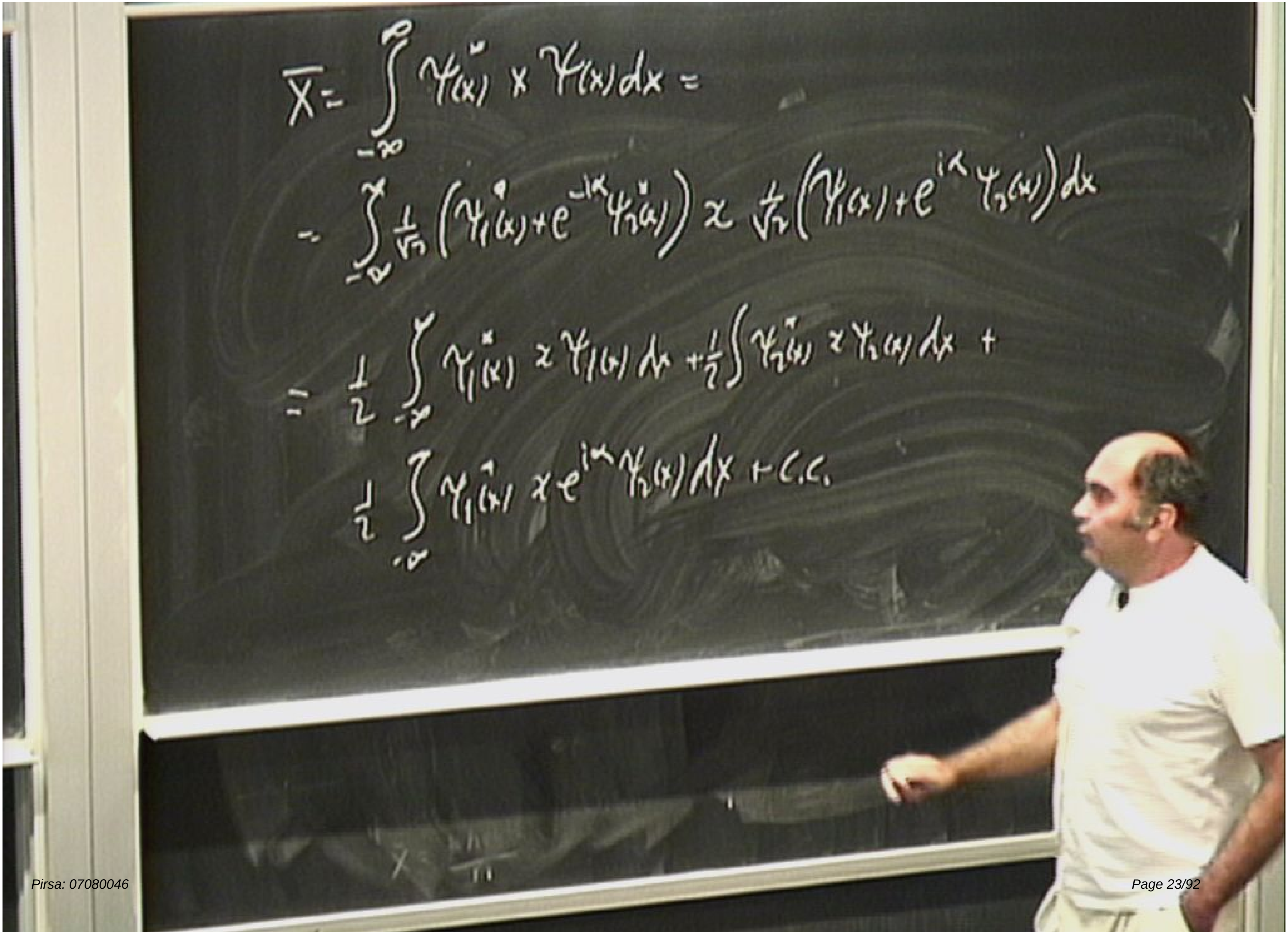
$$\bar{X}$$


$\overline{X} = \int_{-\infty}^{\infty} \psi^*(x)\, x\, \psi(x)\, dx =$
$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2}} \left(\psi_1^*(x) + e^{-i\alpha} \psi_2^*(x) \right) x$

$$\bar{X} = \int_{-\infty}^{\infty} \psi^*(x)\, x\, \psi(x)\, dx =$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2}}\left(\psi_1^*(x) + e^{-i\alpha}\psi_2^*(x)\right) x\, \frac{1}{\sqrt{2}}\left(\psi_1(x) + e^{i\alpha}\psi_2(x)\right) dx$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \psi_1^*(x)\, x\, \psi_1(x)\, dx + \frac{1}{2}\int \psi_2^*(x)\, x\, \psi_2(x)\, dx$$



$\overline{X} = \int_{-\infty}^{\infty} \psi^*(x)\, x\, \psi(x)\, dx =$
$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2}}\left(\psi_1^*(x) + e^{-i\alpha}\psi_2^*(x)\right) x \frac{1}{\sqrt{2}}\left(\psi_1(x)+e^{i\alpha}\psi_2(x)\right)dx$
$= \frac{1}{2}\int_{-\infty}^{\infty}\psi_1^*(x)\, x\, \psi_1(x)\, dx + \frac{1}{2}\int \psi_2^*(x)\, x\, \psi_2(x)\, dx +$
$\frac{1}{2}\int_{-\infty}^{\infty}\psi_1^*(x)\, x\, e^{i\alpha}\psi_2(x)\, dx + c.c.$

$$\bar{X} = \int_{-\infty}^{\infty} \psi^*(x)\, x\, \psi(x)\, dx =$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2}}\left(\psi_1^*(x) + e^{-i\alpha}\psi_2^*(x)\right) x\, \frac{1}{\sqrt{2}}\left(\psi_1(x) + e^{i\alpha}\psi_2(x)\right) dx$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \psi_1^*(x)\, x\, \psi_1(x)\, dx + \frac{1}{2}\int \psi_2^*(x)\, x\, \psi_2(x)\, dx +$$

$$\frac{1}{2}\int_{-\infty}^{\infty} \cancel{\psi_1^*(x)}\, x\, e^{i\alpha}\psi_2(x)\, dx + \cancel{c.c.}$$

$$\psi_\alpha(x) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + e^{i\alpha}\,\psi_2(x)\right]$$

$$f(\hat{x}, \hat{p})$$

$$\overline{X} = \text{INDEPENDENT OF } \alpha$$

$$\overline{x^m} = \int_{-\infty}^{\infty} \psi^*(x)\, x^m \psi(x)\, dx =$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2}}\left(\psi_1^*(x) + e^{-i\alpha}\psi_2^*(x)\right) x^m \frac{1}{\sqrt{2}}\left(\psi_1(x) + e^{i\alpha}\psi_2(x)\right) dx$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \psi_1^*(x)\, x^m \psi_1(x)\, dx + \frac{1}{2}\int \psi_2^*(x)\, x^m \psi_2(x)\, dx +$$

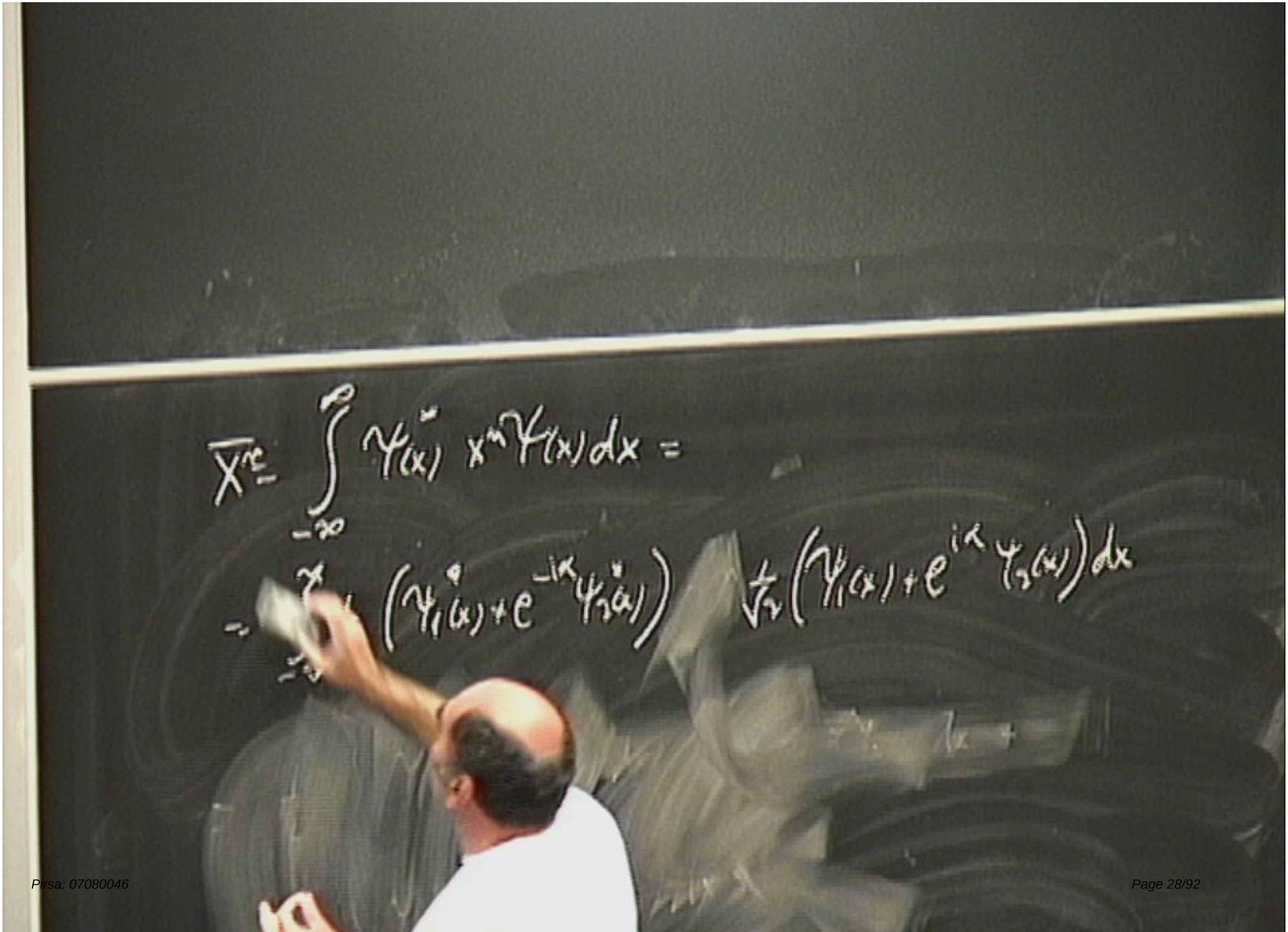
$$\frac{1}{2}\int_{-\infty}^{\infty} \psi_1^*(x)\, x^m e^{i\alpha}\psi_2(x)\, dx + c.c.$$

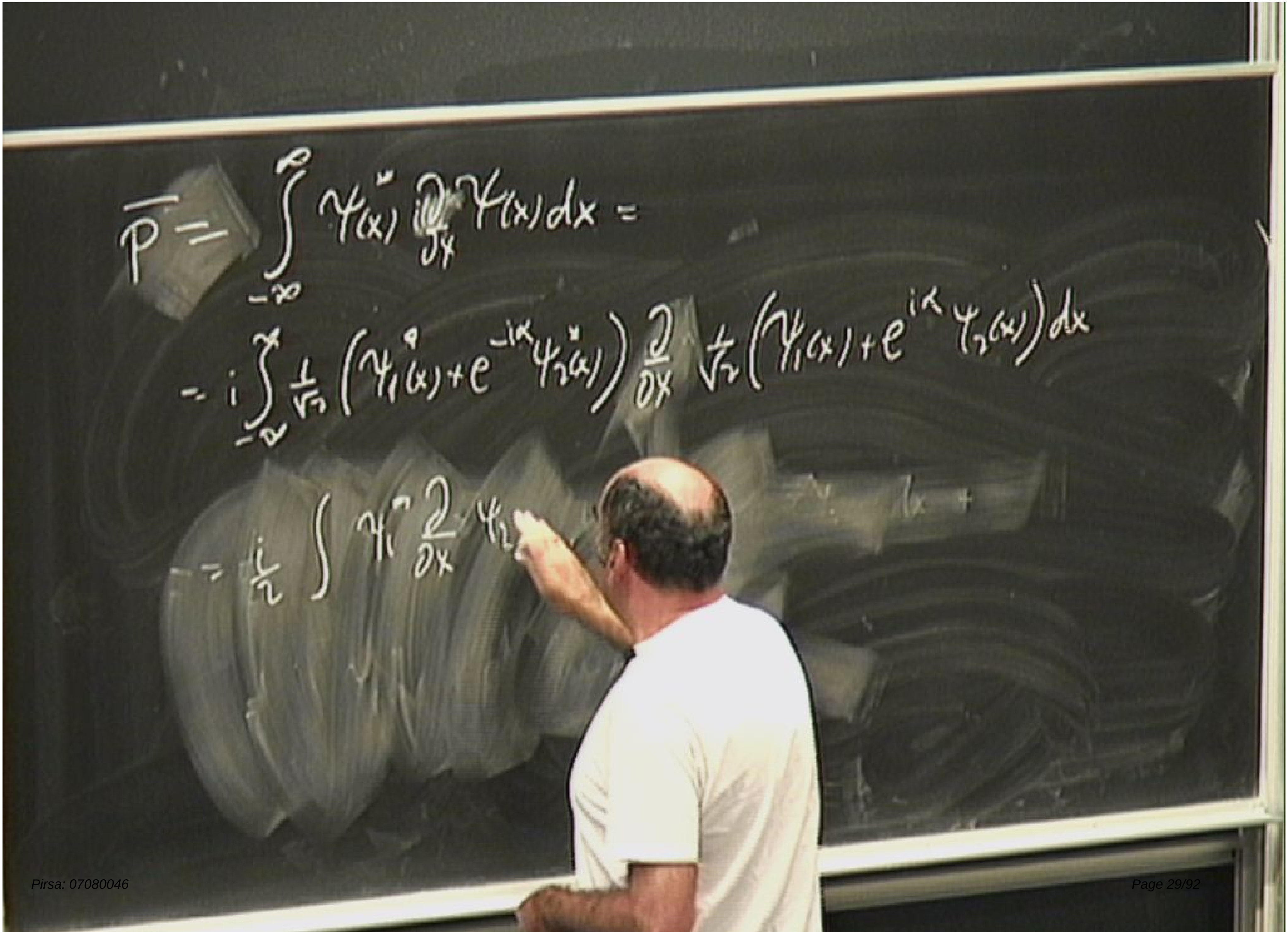
$$\overline{X^n} = \int_{-\infty}^{\infty} \psi^*(x)\, x^n \psi(x)\, dx =$$

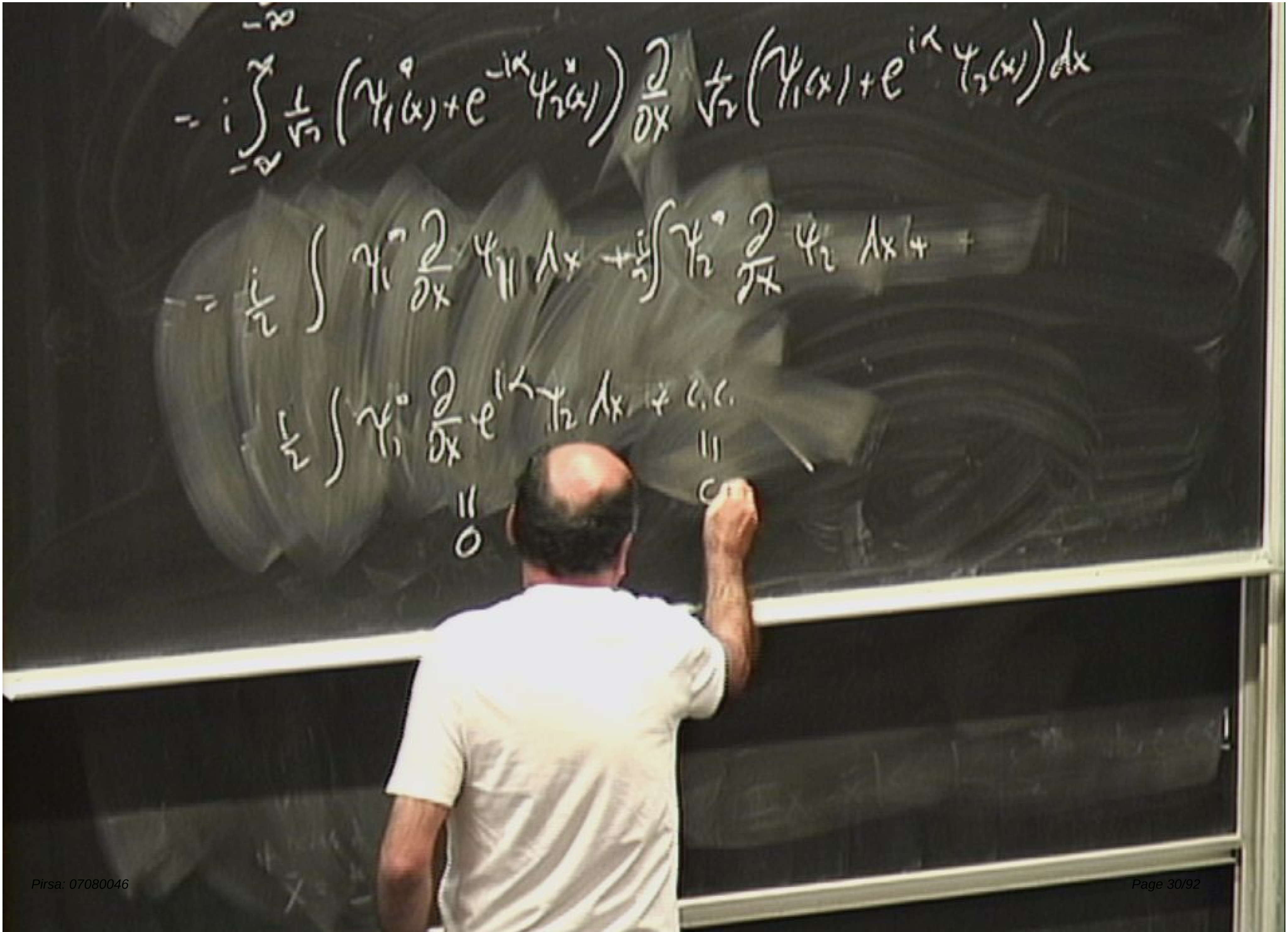
$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2}}\left(\psi_1^*(x) + e^{-i\alpha}\psi_2^*(x)\right) x^n \frac{1}{\sqrt{2}}\left(\psi_1(x) + e^{i\alpha}\psi_2(x)\right) dx$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \psi_1^*(x)\, x^n \psi_1(x)\, dx + \frac{1}{2}\int \psi_2^*(x)\, x^n \psi_2(x)\, dx +$$

$$\frac{1}{2}\int_{-\infty}^{\infty} \cancel{\psi_1^*(x)}\, x^n e^{i\alpha}\psi_2(x)\, dx + \cancel{c.c.}$$


$\overline{X^n} = \int_{-\infty}^{\infty} \psi^*(x)\, x^n \psi(x)\, dx =$
$= \int_{-\infty}^{\infty} \left(\psi_1^*(x) + e^{-i\alpha}\psi_2^*(x)\right) \frac{1}{\sqrt{2}}\left(\psi_1(x) + e^{i\alpha}\psi_2(x)\right) dx$


$\bar{p} = \int_{-\infty}^{\infty} \psi^*(x)\, i\frac{\partial}{\partial x} \psi(x)\, dx =$
$= i\int_{-\infty}^{\infty} \frac{1}{\sqrt{2}}\left(\psi_1^*(x) + e^{-i\alpha}\psi_2^*(x)\right) \frac{\partial}{\partial x} \frac{1}{\sqrt{2}}\left(\psi_1(x)+e^{i\alpha}\psi_2(x)\right)dx$
$= \frac{i}{2}\int \psi_1^* \frac{\partial}{\partial x}\psi_2$

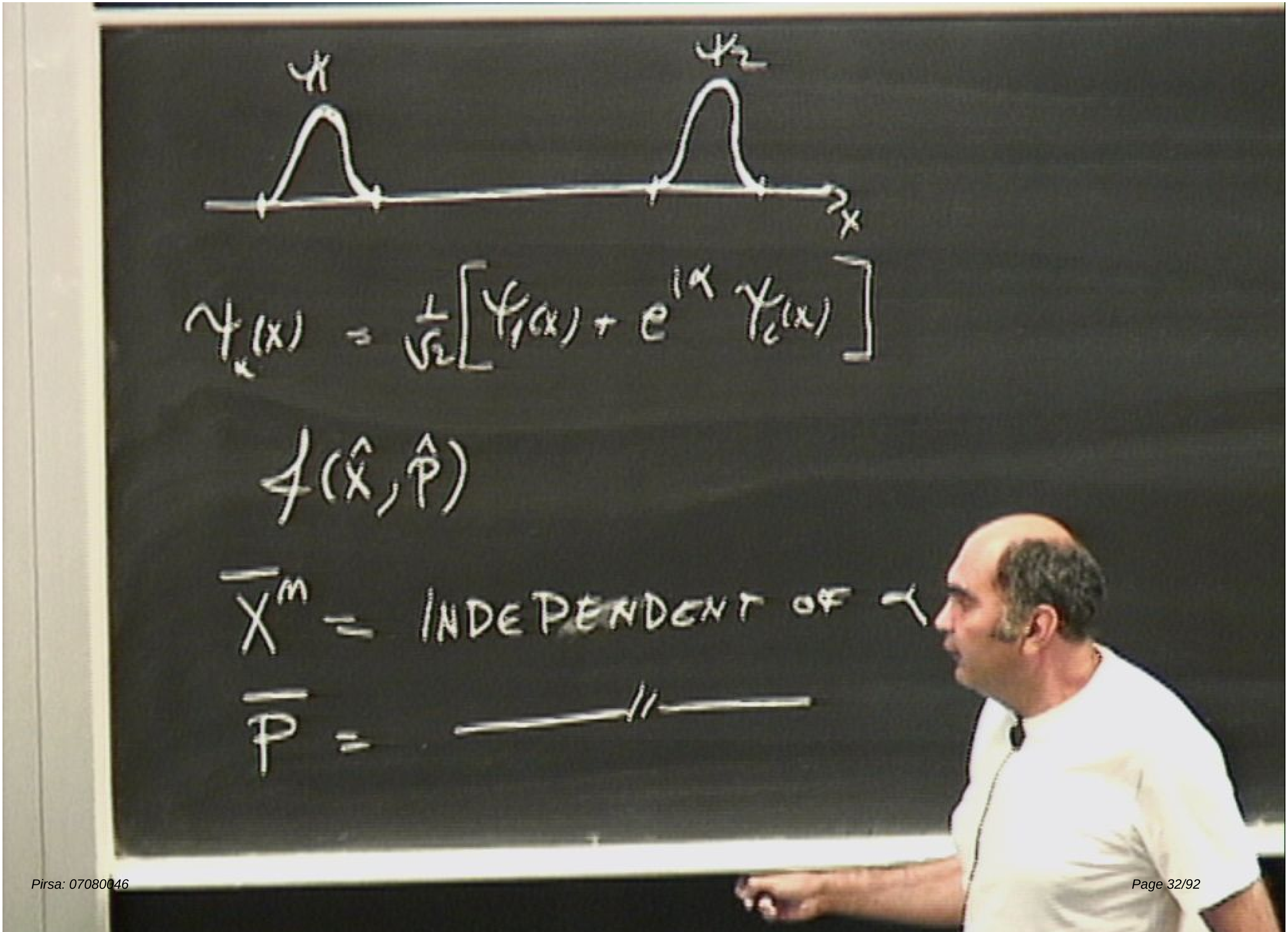

$= i\int_{-\infty}^{\infty} \frac{1}{\sqrt{2}}\left(\psi_1^*(x) + e^{-i\alpha}\psi_2^*(x)\right) \frac{\partial}{\partial x} \frac{1}{\sqrt{2}}\left(\psi_1(x)+e^{i\alpha}\psi_2(x)\right)dx$
$= \frac{i}{2}\int \psi_1^* \frac{\partial}{\partial x}\psi_1 \, dx + \frac{i}{2}\int \psi_2^* \frac{\partial}{\partial x}\psi_2 \, dx +$
$\frac{i}{2}\int \psi_1^* \frac{\partial}{\partial x} e^{i\alpha}\psi_2 \, dx + c.c.$
$= 0$
$= 0$

$$\bar{P} = \int_{-\infty}^{\infty} \psi^*(x)\, i\frac{\partial}{\partial x}\psi(x)\,dx =$$

$$= i\int_{-\infty}^{\infty} \frac{1}{\sqrt{2}}\left(\psi_1^*(x)+e^{-i\alpha}\psi_2^*(x)\right)\frac{\partial}{\partial x}\frac{1}{\sqrt{2}}\left(\psi_1(x)+e^{i\alpha}\psi_2(x)\right)dx$$

$$= \frac{i}{2}\int \psi_1^*\frac{\partial}{\partial x}\psi_1\,dx + \frac{i}{2}\int \psi_2^*\frac{\partial}{\partial x}\psi_2\,dx +$$

$$\underbrace{\frac{i}{2}\int \psi_1^*\frac{\partial}{\partial x}e^{i\alpha}\psi_2\,dx}_{=0} + \underbrace{c.c.}_{=0}$$

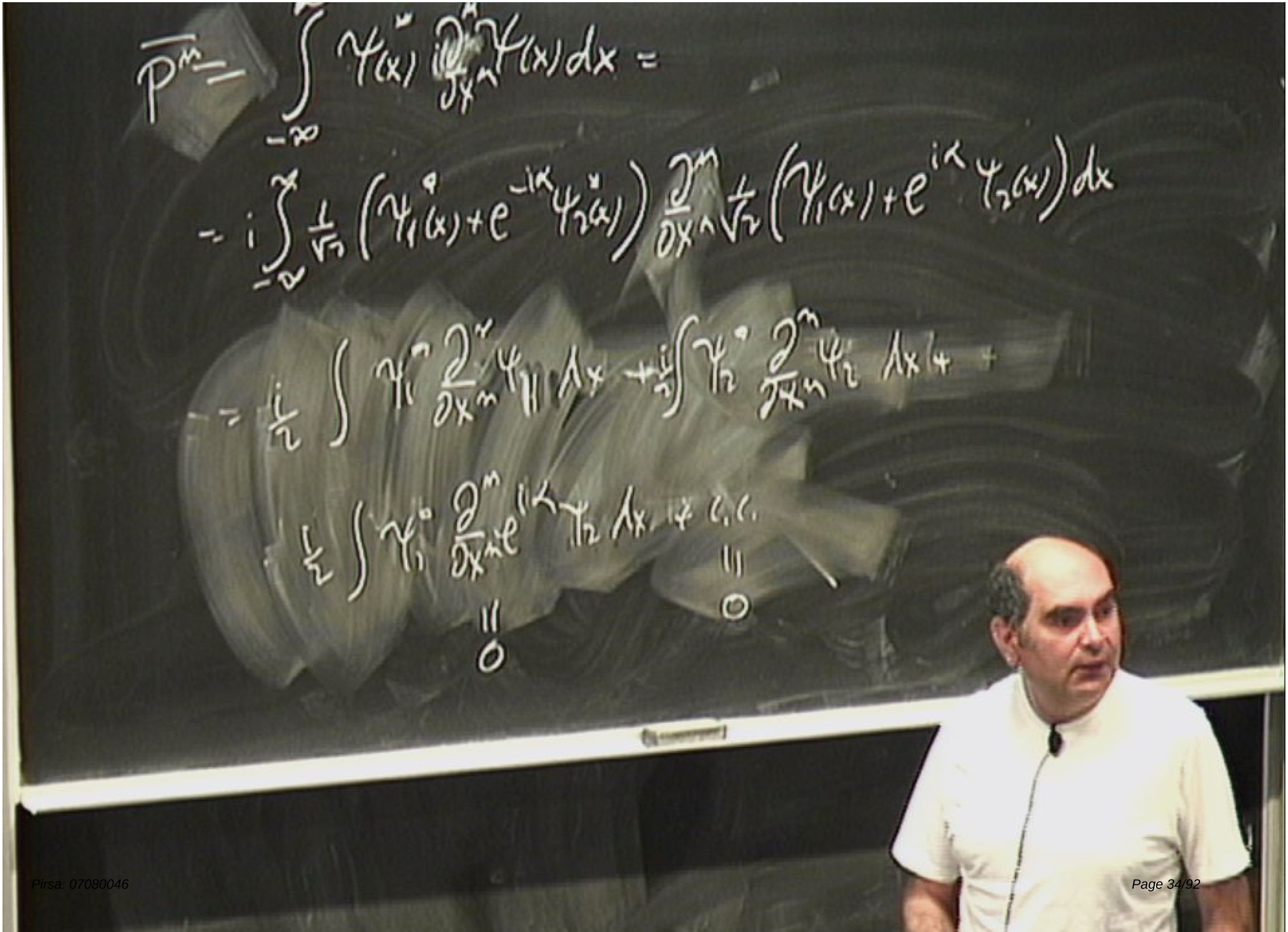

ψ_1
ψ_2
x
$\psi_\alpha(x) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + e^{i\alpha}\psi_2(x)\right]$
$f(\hat{x}, \hat{p})$
$\overline{X^m}$ = INDEPENDENT OF α
$\overline{P}$ = ——//——

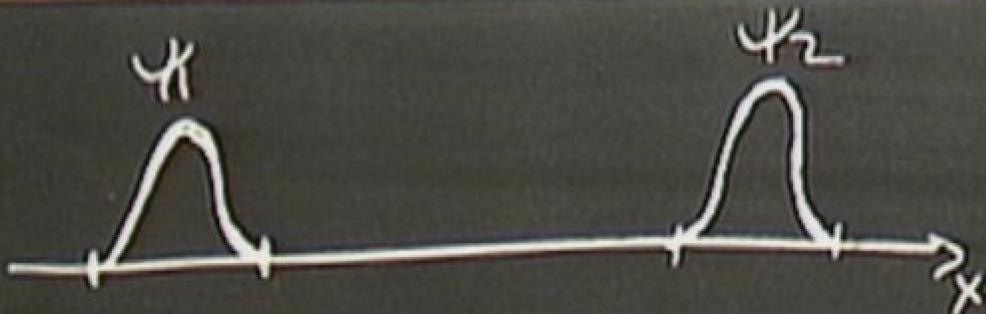
$$\bar{p} = \int_{-\infty}^{\infty} \psi^*(x)\, i\frac{\partial}{\partial x}\psi(x)\,dx =$$

$$= i\int_{-\infty}^{\infty} \frac{1}{\sqrt{2}}\left(\psi_1^*(x) + e^{-i\alpha}\psi_2^*(x)\right)\frac{\partial}{\partial x}\frac{1}{\sqrt{2}}\left(\psi_1(x) + e^{i\alpha}\psi_2(x)\right)dx$$

$$= \frac{i}{2}\int \psi_1^*\frac{\partial}{\partial x}\psi_1\,dx + \frac{i}{2}\int \psi_2^*\frac{\partial}{\partial x}\psi_2\,dx +$$

$$\underset{0}{\underset{\parallel}{\frac{i}{2}\int \psi_1^*\frac{\partial}{\partial x}e^{i\alpha}\psi_2\,dx}} + \underset{0}{\underset{\parallel}{c.c.}}$$


$\overline{p^n} = \int_{-\infty}^{\infty} \psi^*(x)\, i\frac{\partial^n}{\partial x^n}\psi(x)\,dx =$
$= i\int_{-\infty}^{\infty}\frac{1}{\sqrt{2}}\left(\psi_1^*(x)+e^{-i\alpha}\psi_2^*(x)\right)\frac{\partial^n}{\partial x^n}\frac{1}{\sqrt{2}}\left(\psi_1(x)+e^{i\alpha}\psi_2(x)\right)dx$
$= \frac{i}{2}\int \psi_1^*\frac{\partial^n}{\partial x^n}\psi_1\,dx + \frac{i}{2}\int\psi_2^*\frac{\partial^n}{\partial x^n}\psi_2\,dx +$
$\frac{i}{2}\int\psi_1^*\frac{\partial^n}{\partial x^n}e^{i\alpha}\psi_2\,dx + c.c.$
$= 0$
$= 0$



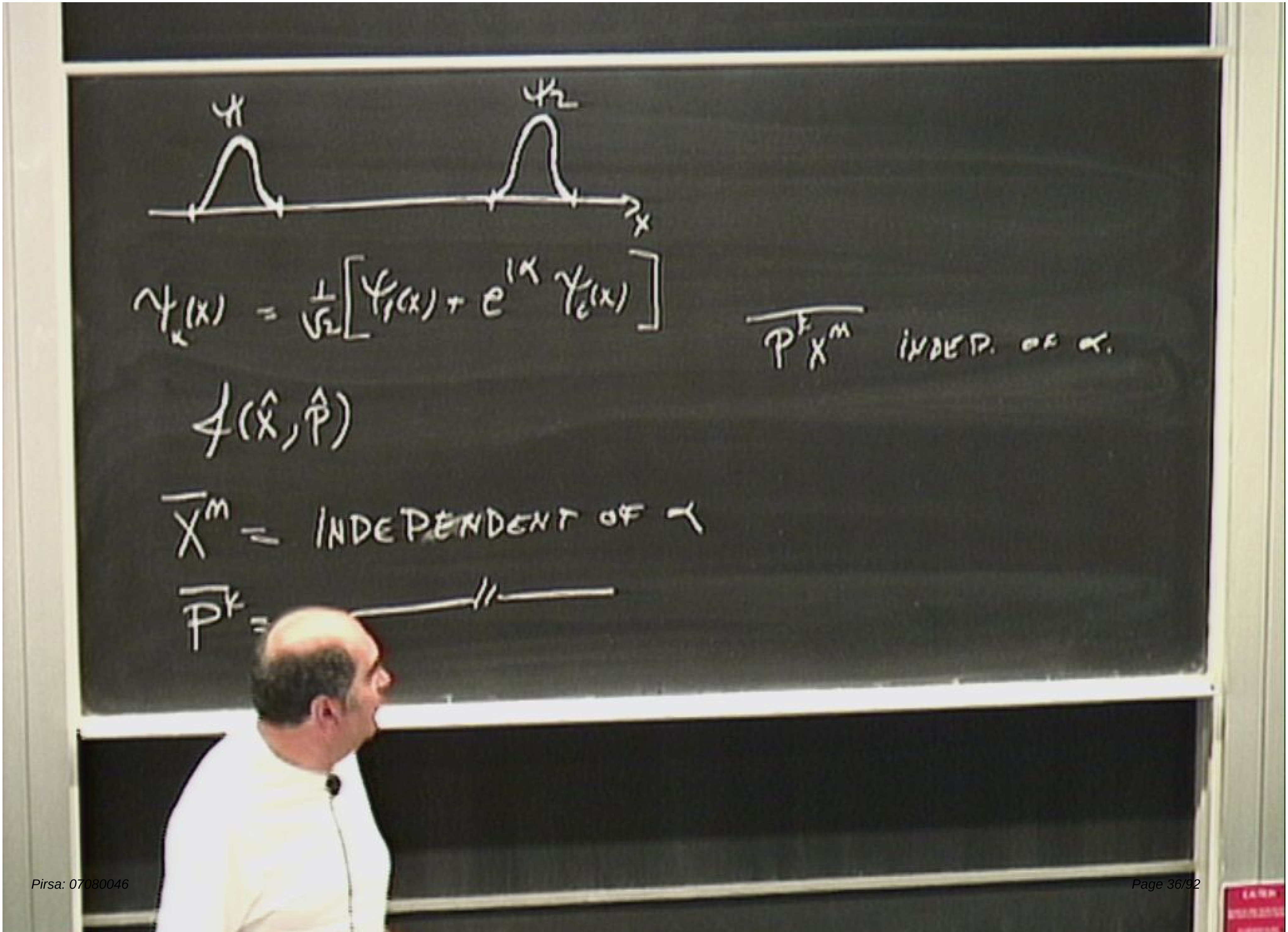
$$\psi_\alpha(x) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + e^{i\alpha}\,\psi_2(x)\right]$$

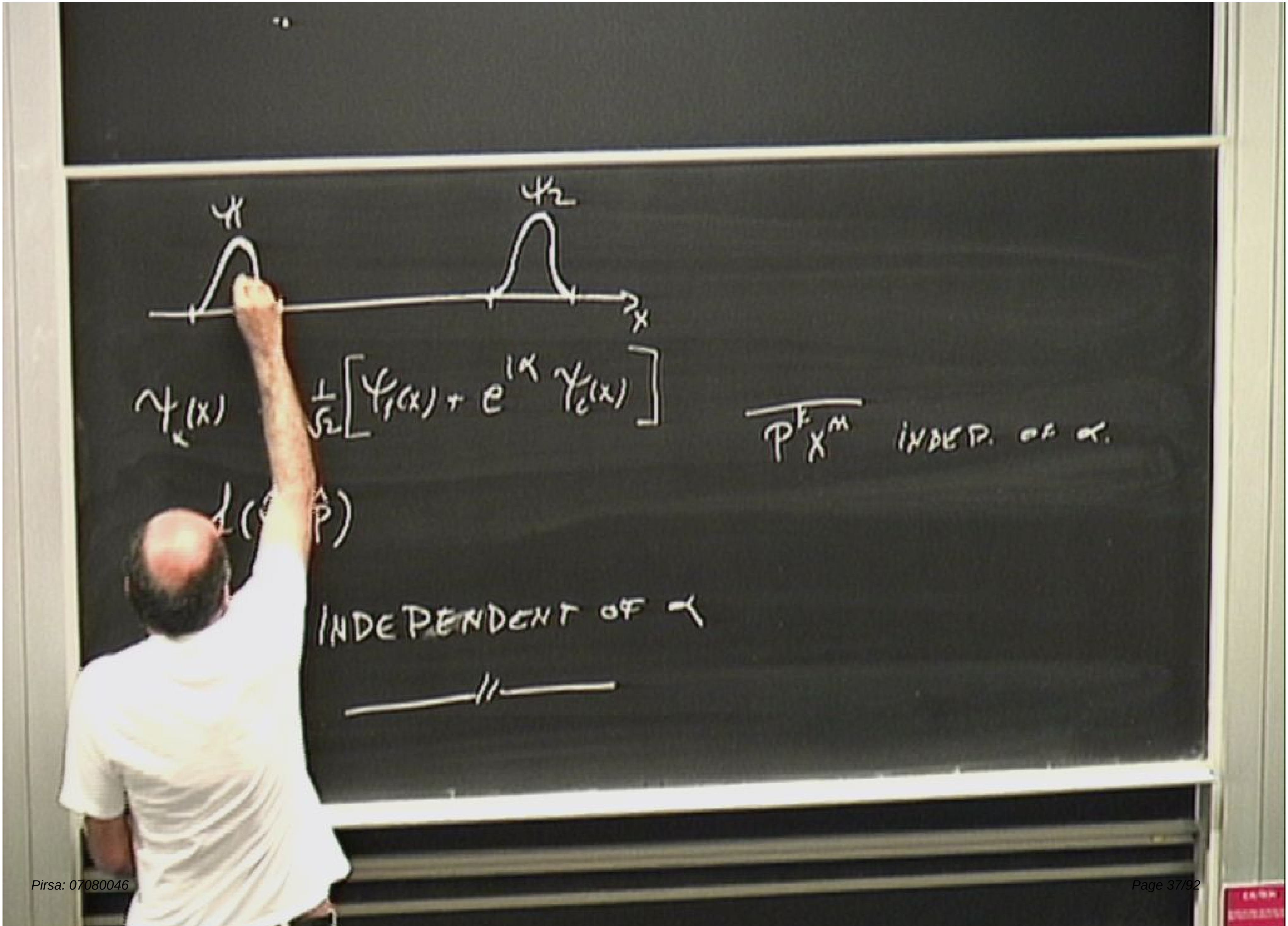
$\overline{P^k X^m}$ INDEP.

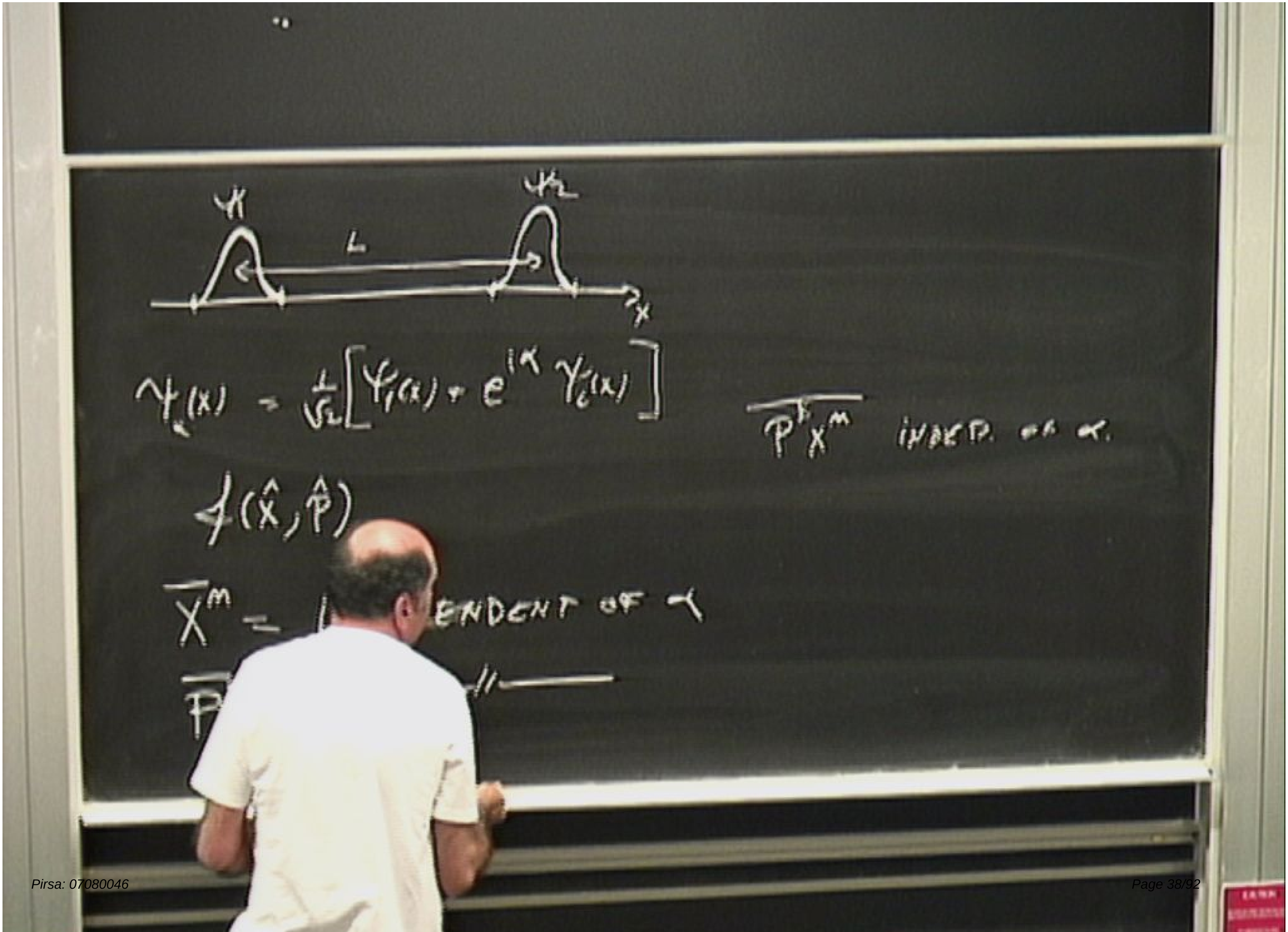
$$f(\hat{x}, \hat{p})$$

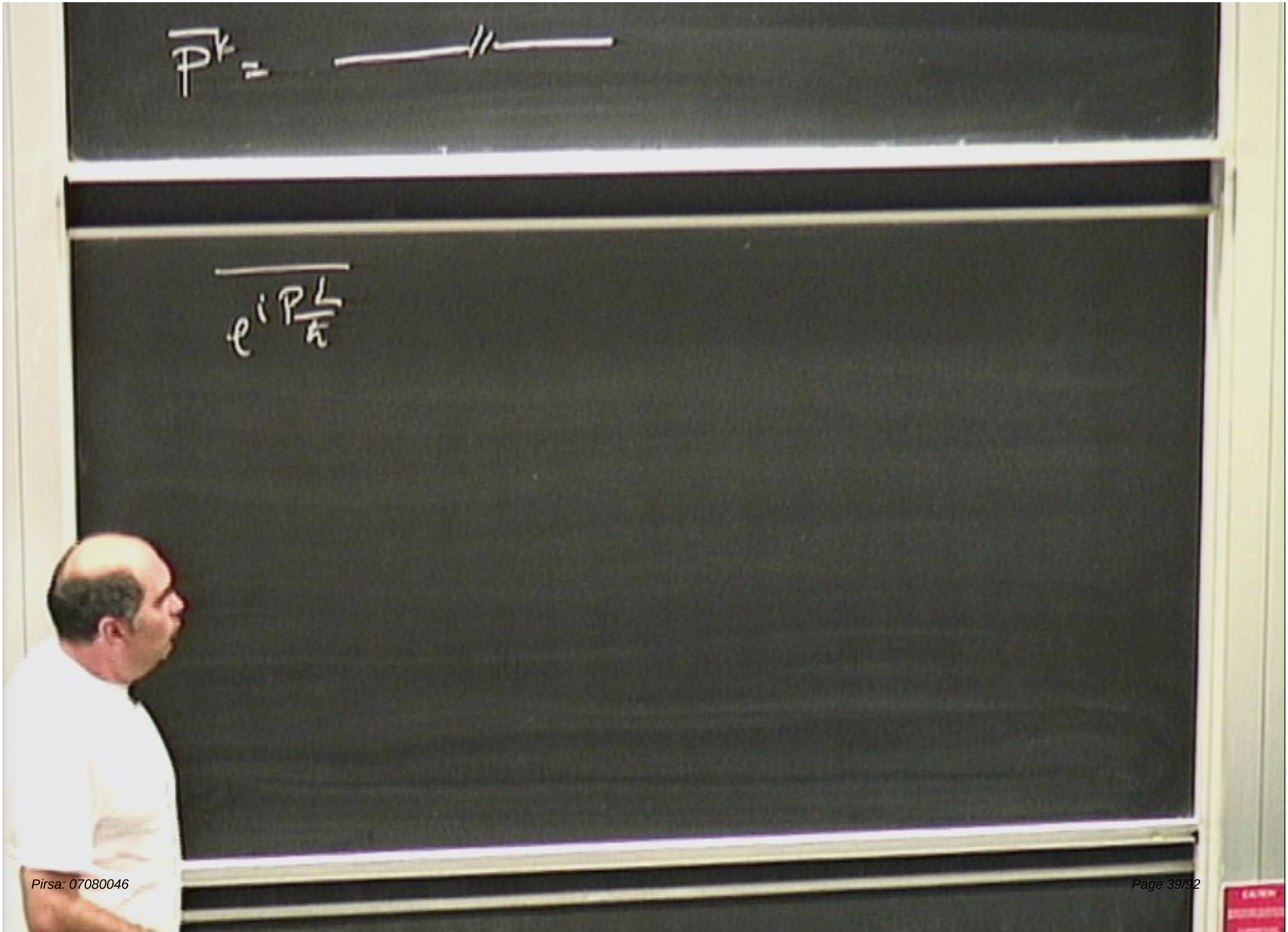
$\overline{X^m}$ = INDEPENDENT OF α

$\overline{P^k}$ = ——— " ———

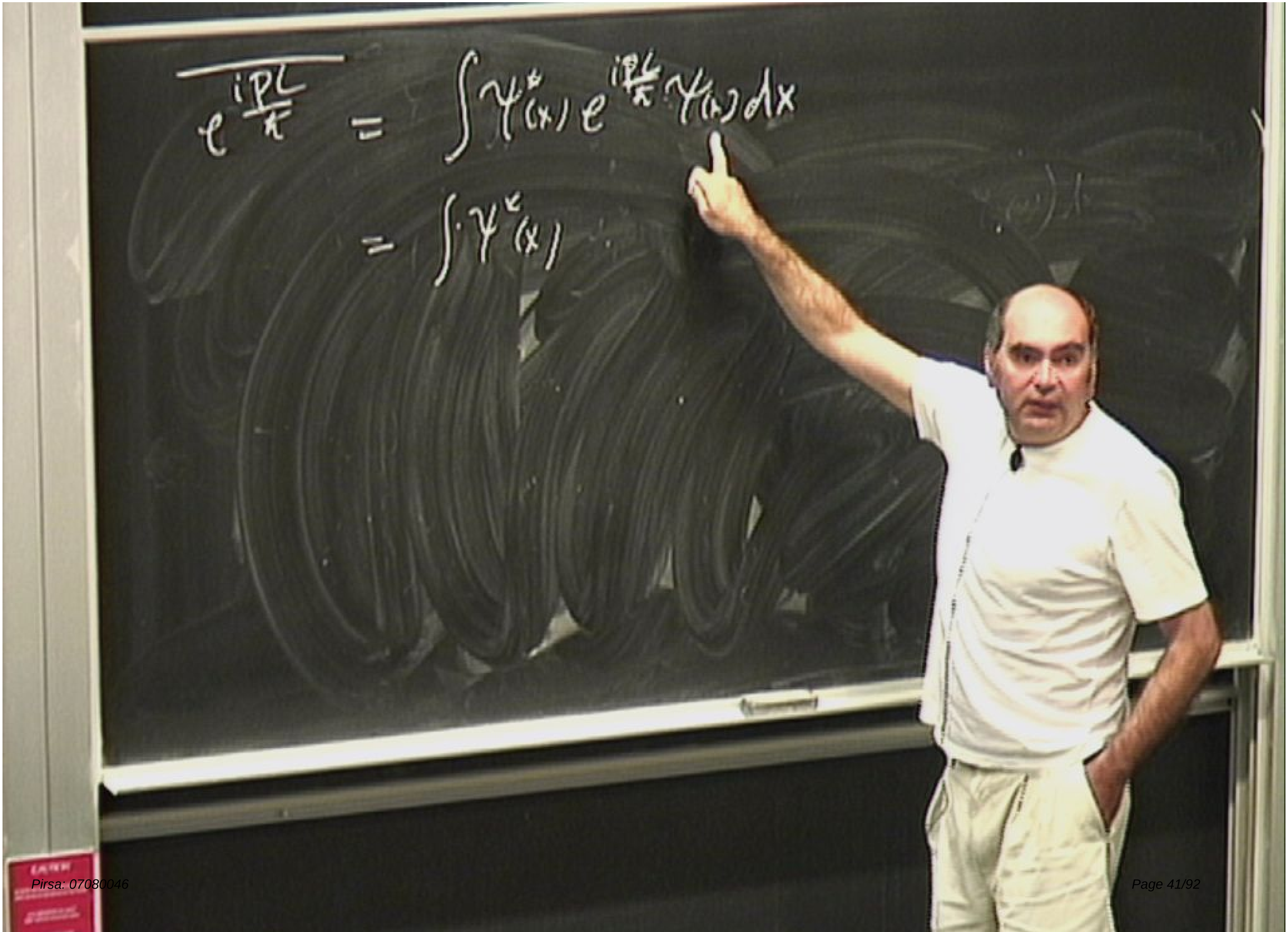

ψ_1
ψ_2
x
$\psi_\alpha(x) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + e^{i\alpha}\,\psi_2(x)\right]$
$\overline{P^k X^m}$ INDEP. OF α.
$f(\hat{X}, \hat{P})$
$\overline{X^m}$ = INDEPENDENT OF α
$\overline{P^k}$ = —— // ——

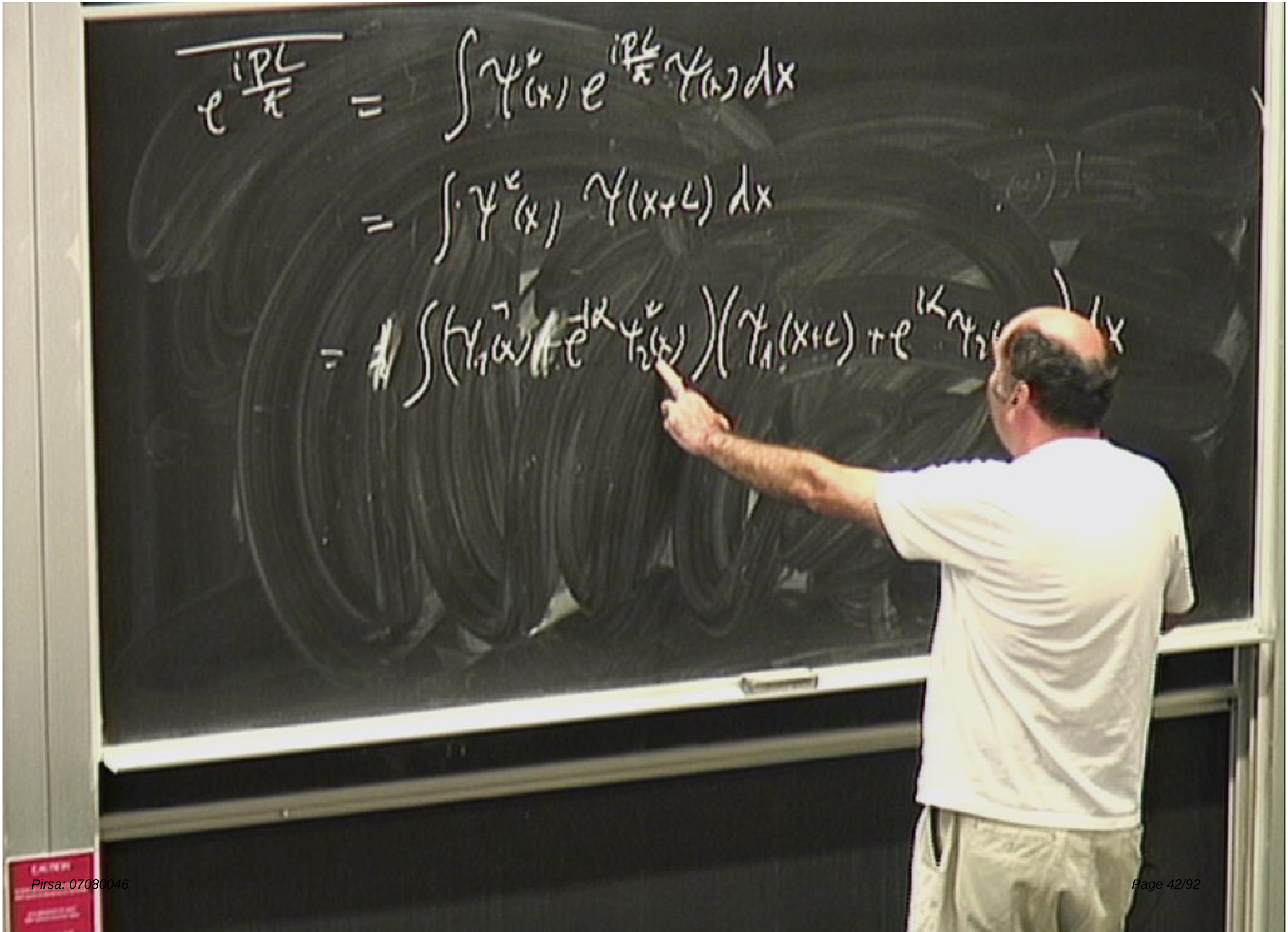

ψ1
ψ2
x
ψα(x) 1/√2 [ψ1(x) + e^iα ψ2(x)]
P^k X^m INDEP. OF α.
INDEPENDENT OF α


ψ_1
ψ_2
L
x
$\psi_\alpha(x) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + e^{i\alpha}\,\psi_2(x)\right]$
$\overline{P^k X^m}$ INDEP. OF α.
$f(\hat{X},\hat{P})$
$\overline{X^m}$ = ENDENT OF α


$\overline{P}^k =$
$\overline{e^{iP\frac{L}{\hbar}}}$

$$\overline{e^{\frac{ipL}{\hbar}}} = \int \psi^*(x)\, e^{\frac{ipL}{\hbar}}\, \psi(x)\, dx$$


$\overline{e^{\frac{iPL}{\hbar}}} = \int \psi^*(x)\, e^{\frac{iPL}{\hbar}}\, \psi(x)\, dx$
$= \int \psi^*(x)$

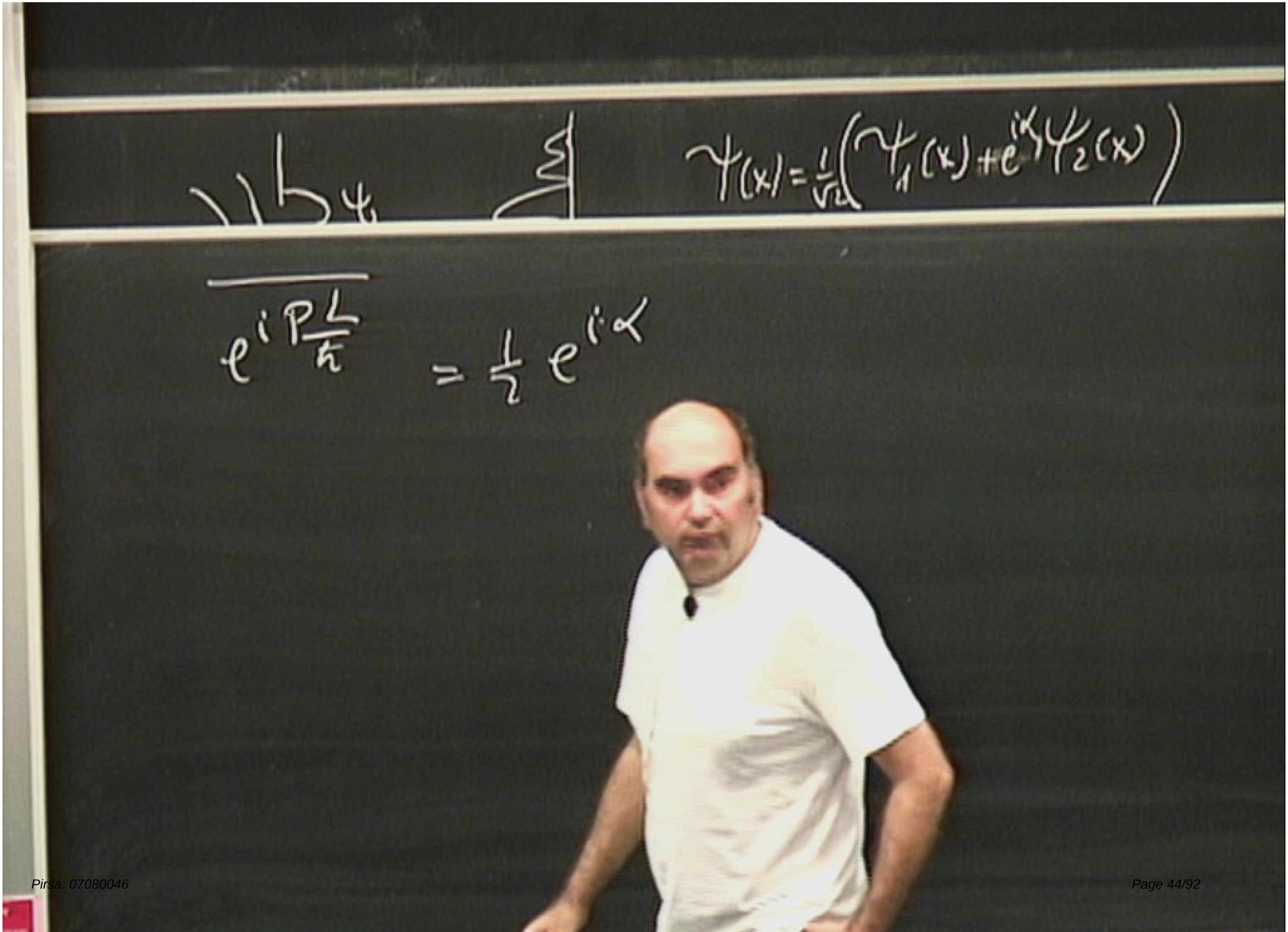

$\overline{e^{\frac{iPL}{\hbar}}} = \int \psi^*(x)\, e^{\frac{iPL}{\hbar}}\, \psi(x)\, dx$
$= \int \psi^*(x)\, \psi(x+L)\, dx$
$= \int (\psi_1^*(x) + e^{-iK}\psi_2^*(x))(\psi_1(x+L) + e^{iK}\psi_2($ … $))\, dx$

$$\overline{e^{\frac{ipL}{\hbar}}} = \int \psi^*(x)\, e^{\frac{ipL}{\hbar}}\, \psi(x)\, dx$$

$$= \int \psi^*(x)\, \psi(x+L)\, dx$$

$$= \int \left(\psi_1^*(x) + e^{-iK}\psi_2^*(x)\right)\left(\psi_1(x+L) + e^{iK}\psi_2(x+L)\right) dx$$

$$\frac{1}{2} \int \psi_1^*(x)\, e^{iK}\, \psi_2(x+L)\, dx$$


$\psi(x) = \frac{1}{\sqrt{2}}\left(\psi_1(x) + e^{i\alpha}\psi_2(x)\right)$
$e^{iP\frac{L}{\hbar}} = \frac{1}{2}e^{i\alpha}$

$$\overline{e^{\frac{iPL}{\hbar}}} = \int \psi^*(x)\, e^{\frac{iPL}{\hbar}}\, \psi(x)\, dx$$

$$= \int \psi^*(x)\, \psi(x+L)\, dx$$

$$= \int \left(\psi_1^*(x) + e^{-i\alpha}\psi_2^*(x)\right)\left(\psi_1(x+L) + e^{i\alpha}\psi_2(x+L)\right) dx$$

$$\frac{1}{2} \int \psi_1^*(x)\, e^{i\alpha}\, \psi_2(x+L)\, dx = \frac{1}{2} e^{i\alpha}$$

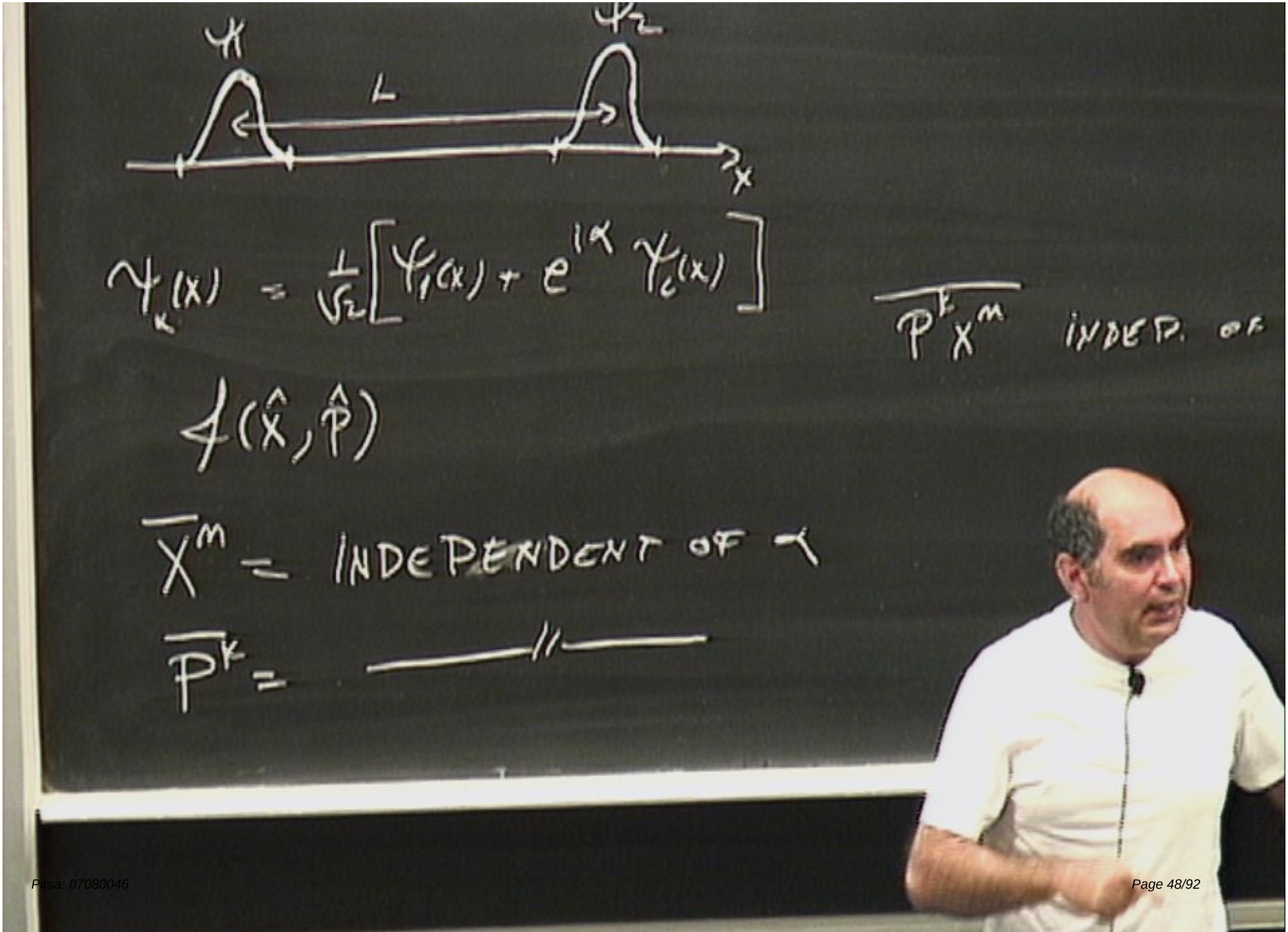
$$\overline{e^{iP\frac{L}{\hbar}}} = \frac{1}{2} e^{i\alpha}$$

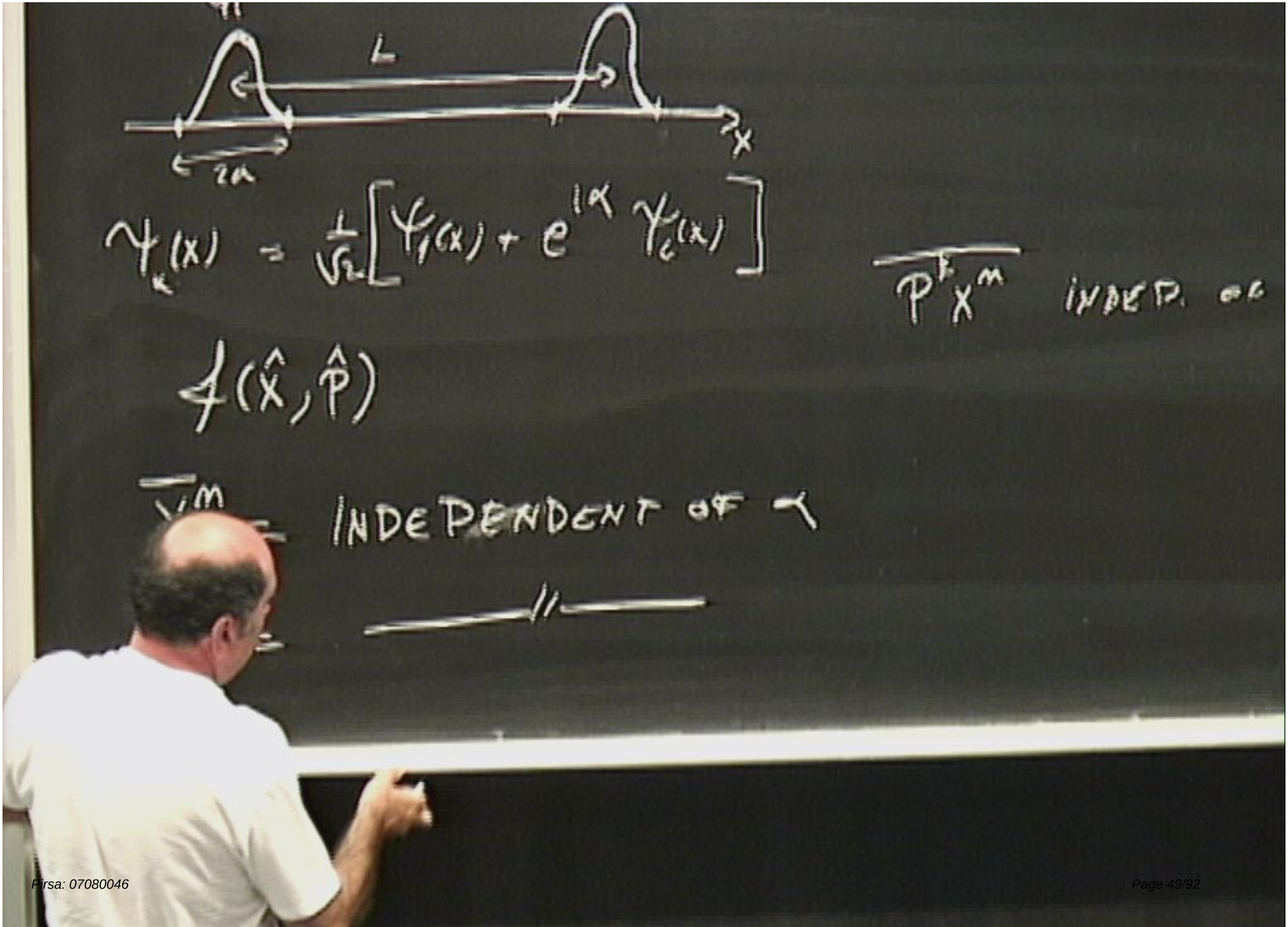
$$① \quad e^{i\frac{PL}{\hbar}} = \sum_n \frac{\left(\frac{iPL}{\hbar}\right)^n}{n!} = \sum \left(\frac{iL}{\hbar}\right)^n \frac{1}{n!} P^n$$

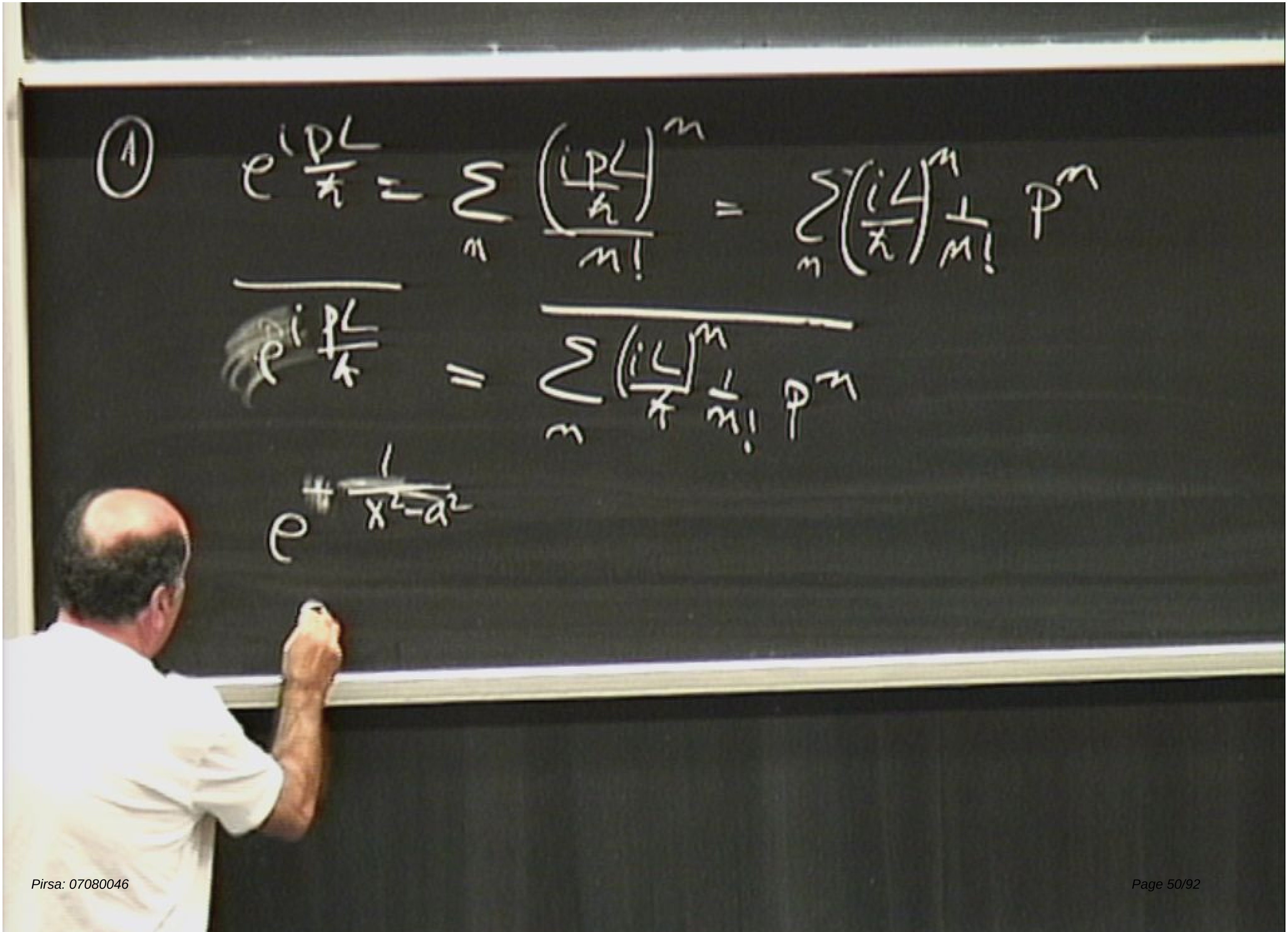
$$\overline{e^{iP\frac{L}{\hbar}}} = \frac{1}{2} e^{i\alpha}$$

(1) $$e^{iP\frac{L}{\hbar}} = \sum_n \frac{\left(\frac{iPL}{\hbar}\right)^n}{n!} = \sum_n \left(\frac{iL}{\hbar}\right)^n \frac{1}{n!} P^n$$

$$\overline{e^{iP\frac{L}{\hbar}}} = \overline{\sum_n \left(\frac{iL}{\hbar}\right)^n \frac{1}{n!} P^n}$$


ψ_1
ψ_2
L
x
$\psi_\alpha(x) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + e^{i\alpha}\,\psi_2(x)\right]$
$f(\hat{x}, \hat{p})$
$\overline{X^m}$ = INDEPENDENT OF α
$\overline{P^k}$ = —//—
$\overline{P^k X^m}$ INDEP. OF


L
x
2a
$\psi_\alpha(x) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + e^{i\alpha}\,\psi_2(x)\right]$
$\overline{P^k X^m}$ INDEP. OF
$f(\hat{X}, \hat{P})$
INDEPENDENT OF α


1
$e^{i\frac{pL}{\hbar}} = \sum_n \frac{\left(\frac{ipL}{\hbar}\right)^n}{n!} = \sum_n \left(\frac{iL}{\hbar}\right)^n \frac{1}{n!} p^n$
$e^{i\frac{pL}{\hbar}} = \sum_n \left(\frac{iL}{\hbar}\right)^n \frac{1}{n!} p^n$
$e^{\frac{1}{x^2-a^2}}$

(1) $$e^{i\frac{pL}{\hbar}} = \sum_n \frac{\left(\frac{ipL}{\hbar}\right)^n}{n!} = \sum_n \left(\frac{iL}{\hbar}\right)^n \frac{1}{m!}\, p^n$$

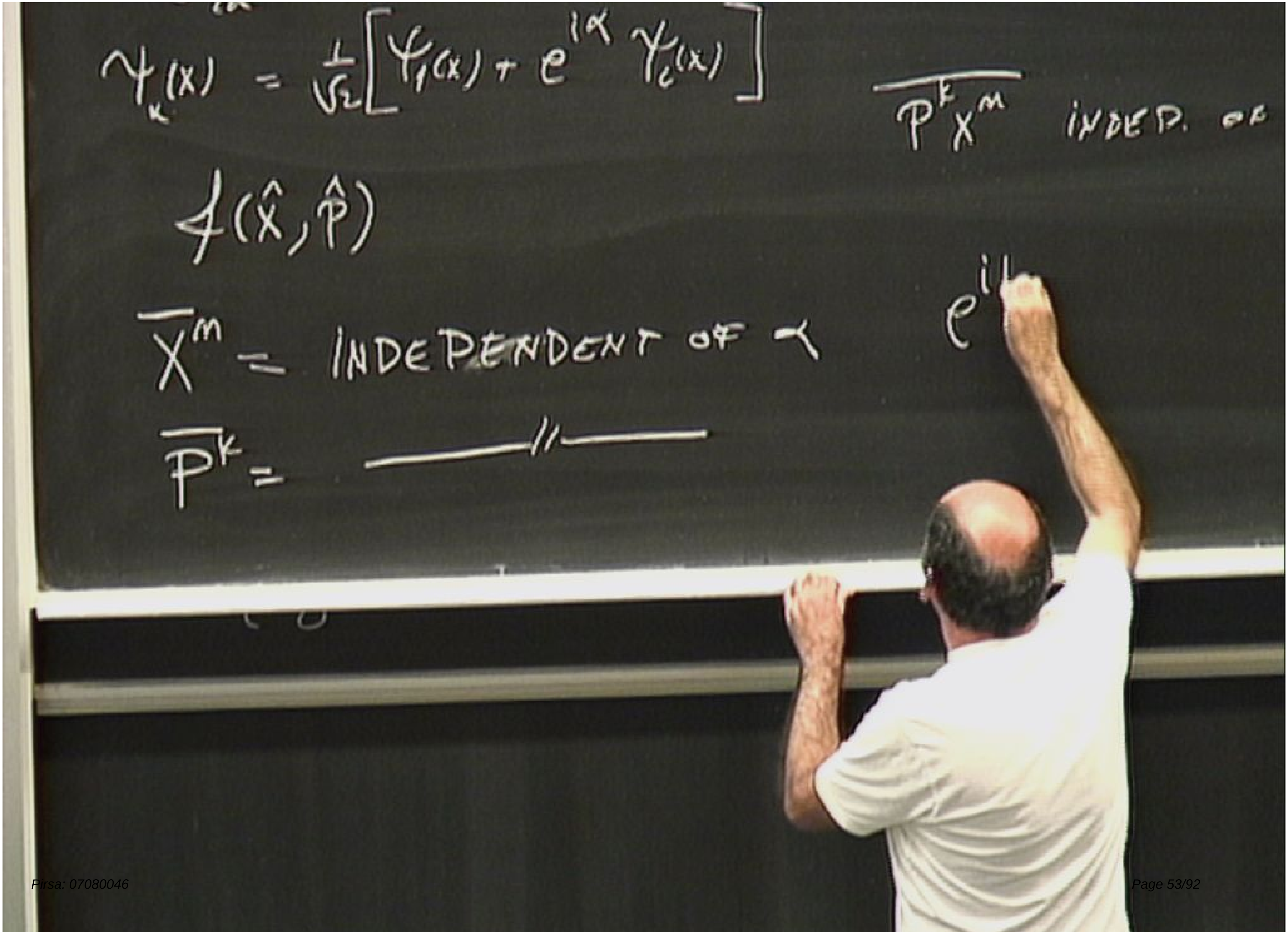
$$\overline{e^{i\frac{pL}{\hbar}}} = \overline{\sum_n \left(\frac{iL}{\hbar}\right)^n \frac{1}{n!}\, p^n}$$

(2) $$\begin{cases} e^{\frac{1}{x^2-a^2}} & |x|<a \\ 0 & \end{cases}$$

① $e^{i\frac{pL}{\hbar}} = \sum_n \frac{\left(\frac{ipL}{\hbar}\right)^n}{n!} = \sum_n \left(\frac{iL}{\hbar}\right)^n \frac{1}{n!} p^n$

$\overline{e^{i\frac{pL}{\hbar}}} = \overline{\sum_n \left(\frac{iL}{\hbar}\right)^n \frac{1}{n!} p^n}$

② $\begin{cases} e^{\frac{1}{x^2-a^2}} & |x|<a \\ 0 \end{cases}$


$\psi_k(x) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + e^{i\alpha}\psi_2(x)\right]$
$\overline{P^k X^m}$ INDEP. OF
$f(\hat{X}, \hat{P})$
$\overline{X^m}$ = INDEPENDENT OF α
$\overline{P^k}$ = —— // ——

$$\psi_\alpha(x) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + e^{i\alpha}\,\psi_2(x)\right]$$

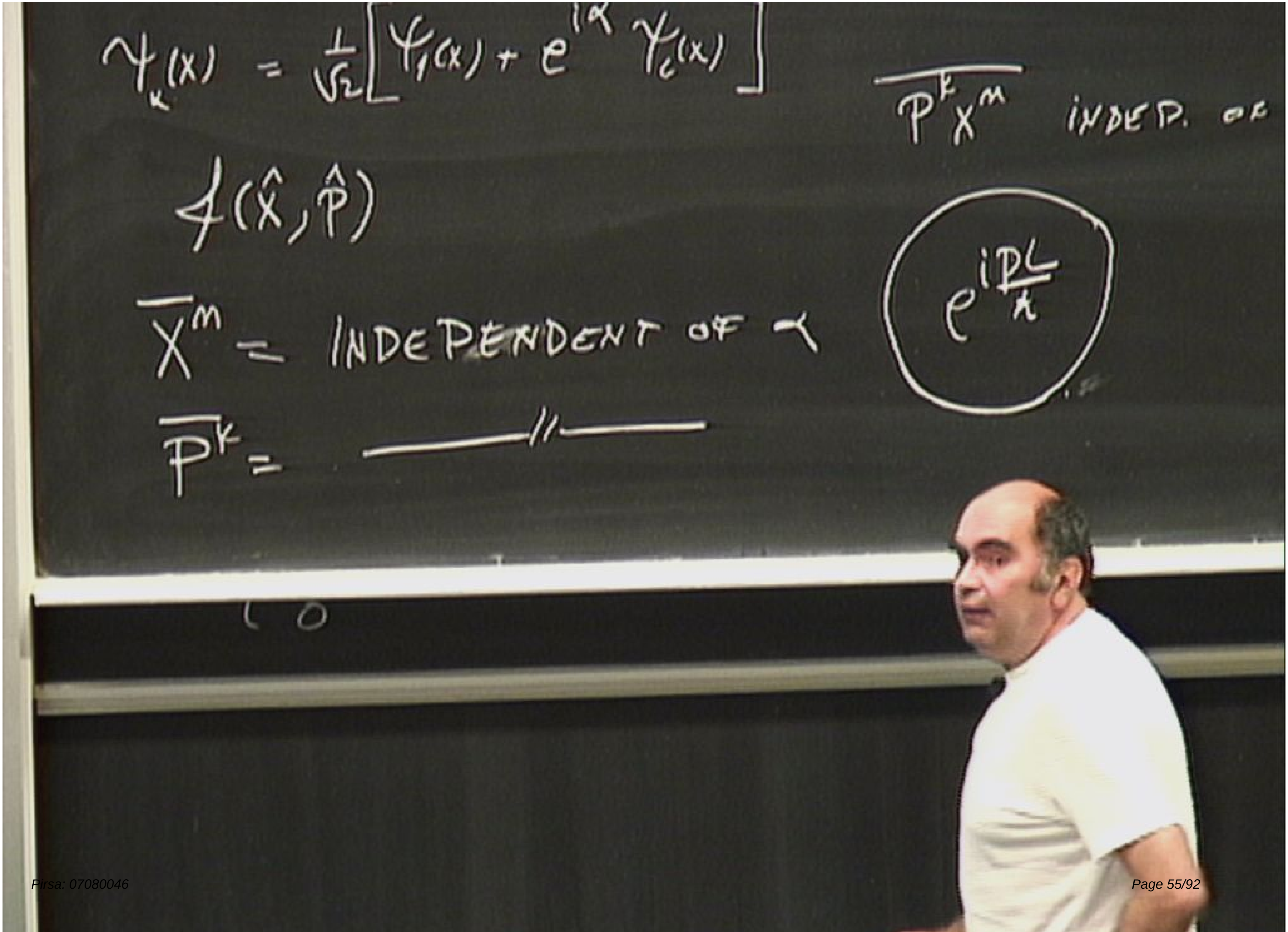
$$f(\hat{x}, \hat{p})$$

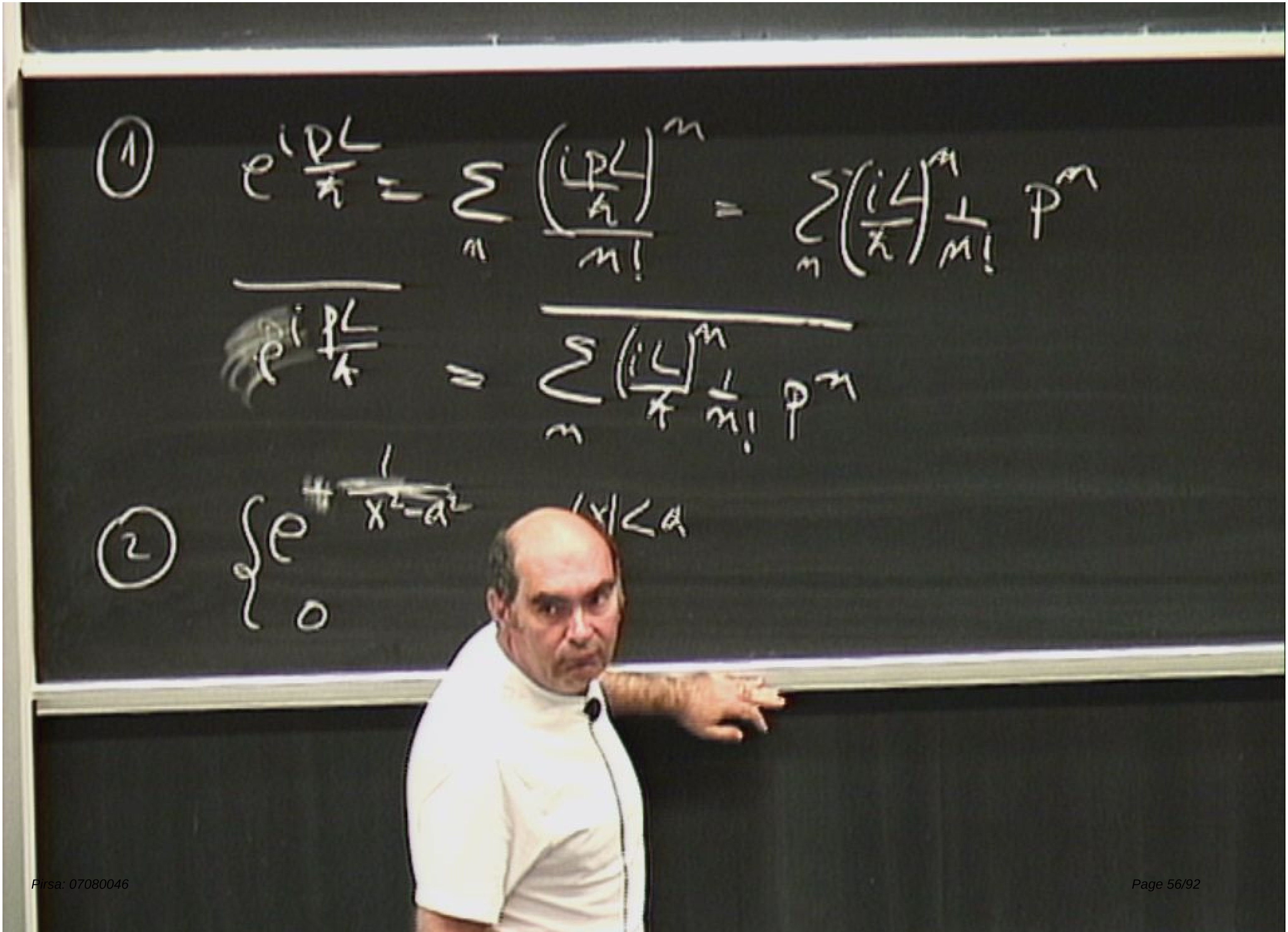
$\overline{X^m}$ = INDEPENDENT OF α

$\overline{P^k}$ = ——— // ———

$\overline{P^k X^m}$ INDEP. OF

$$e^{\frac{ipL}{\hbar}}$$


$\psi_\alpha(x) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + e^{i\alpha}\psi_2(x)\right]$
$\overline{P^k X^m}$ INDEP. OF
$f(\hat{X}, \hat{P})$
$e^{\frac{iPL}{\hbar}}$
$\overline{X^m}$ = INDEPENDENT OF α
$\overline{P^k}$ = —//—



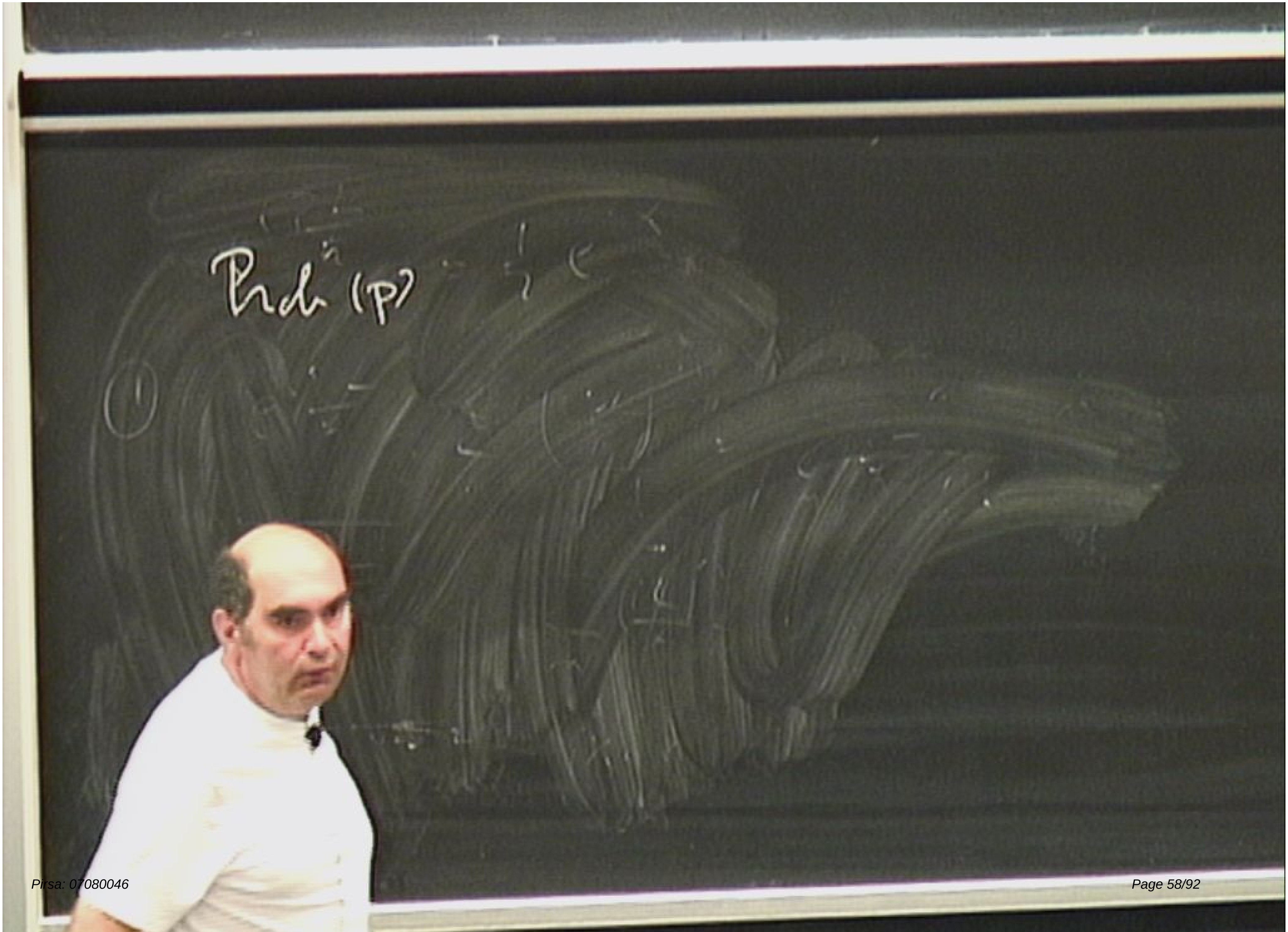
1 $e^{i\frac{pL}{\hbar}} = \sum_n \frac{\left(\frac{ipL}{\hbar}\right)^n}{n!} = \sum_n \left(\frac{iL}{\hbar}\right)^n \frac{1}{n!} p^n$
$e^{i\frac{pL}{\hbar}} = \sum_n \left(\frac{iL}{\hbar}\right)^n \frac{1}{n!} p^n$
2 $\begin{cases} e^{\frac{1}{x^2-a^2}} & |x|<a \\ 0 & \end{cases}$

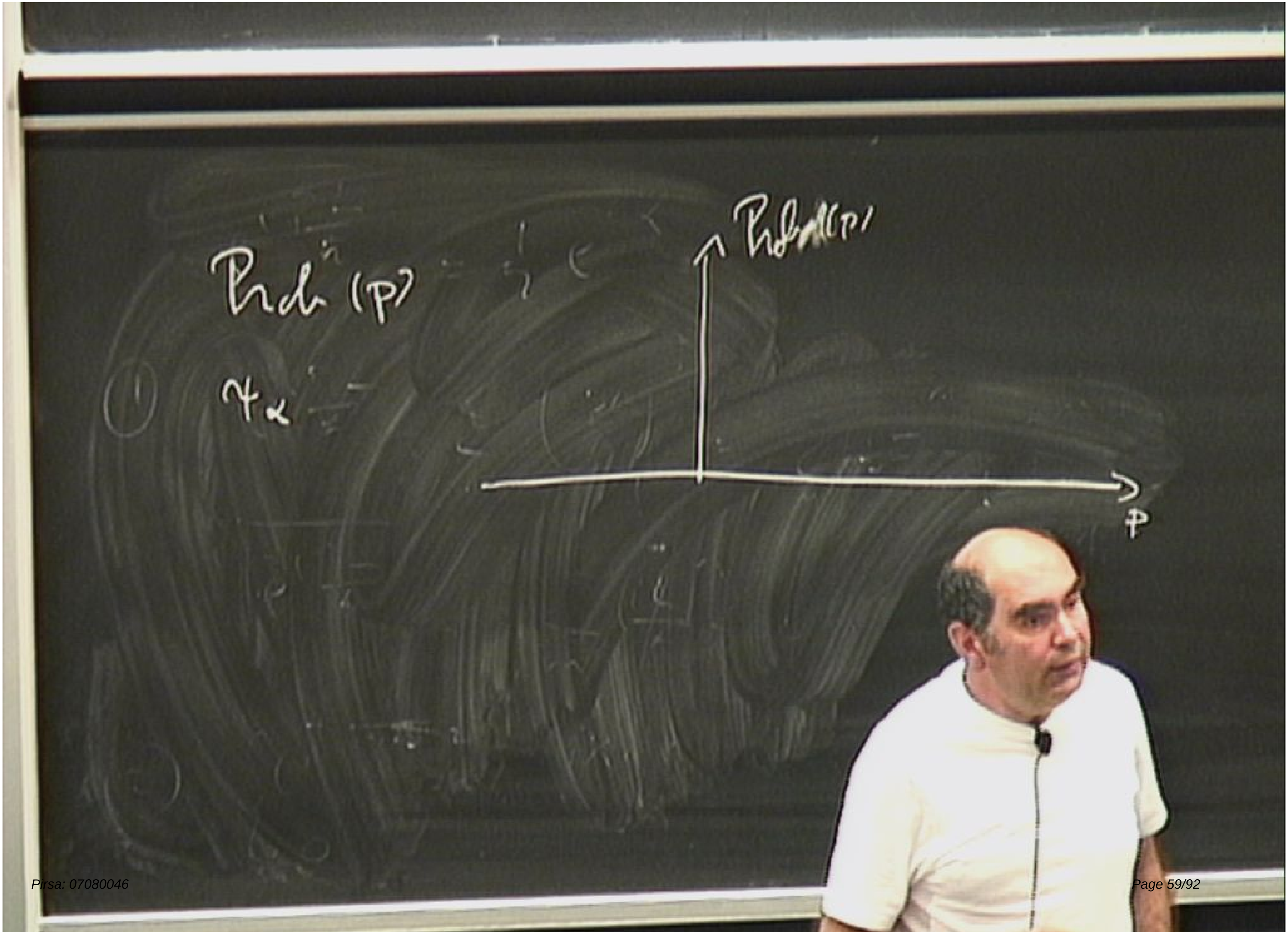
$$\overline{e^{i\frac{PL}{\hbar}}} = \frac{1}{2} e^{i\alpha}$$

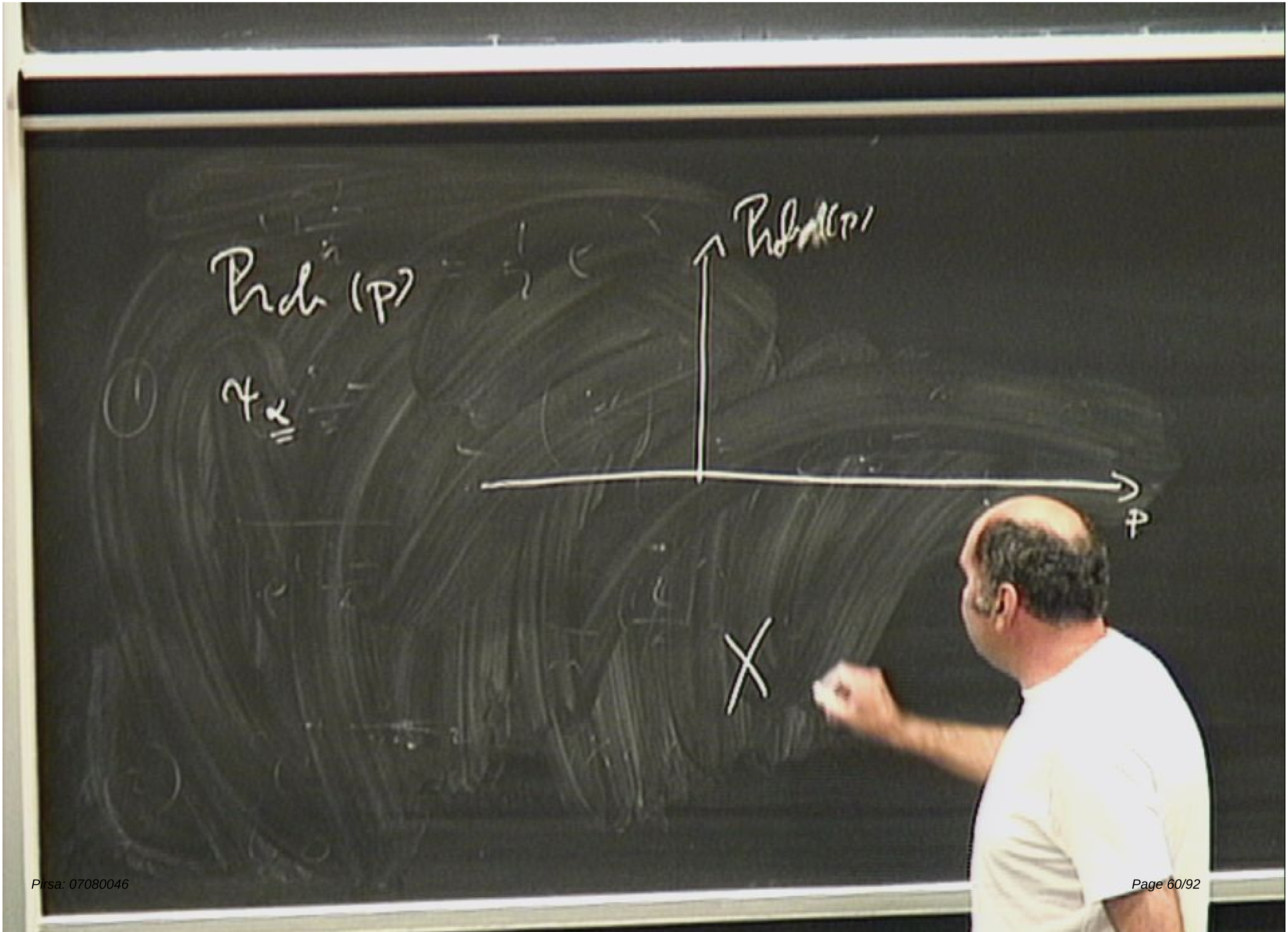
(1) $$e^{i\frac{PL}{\hbar}} = \sum_n \frac{\left(\frac{iPL}{\hbar}\right)^n}{n!} = \sum_n \left(\frac{iL}{\hbar}\right)^n \frac{1}{n!} P^n$$

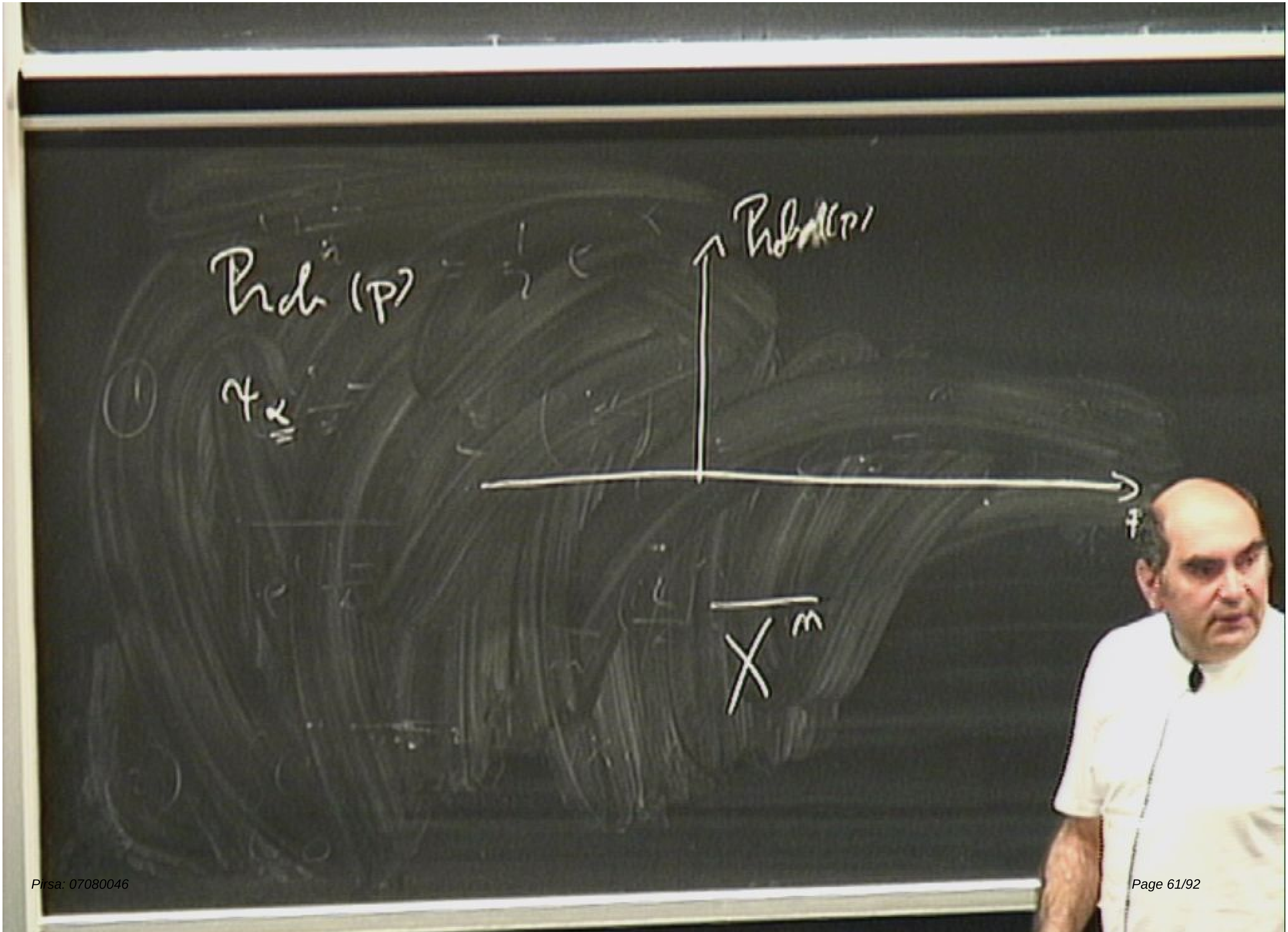
$$\overline{e^{i\frac{PL}{\hbar}}} = \sum_n \left(\frac{iL}{\hbar}\right)^n \frac{1}{n!} \overline{P^n}$$

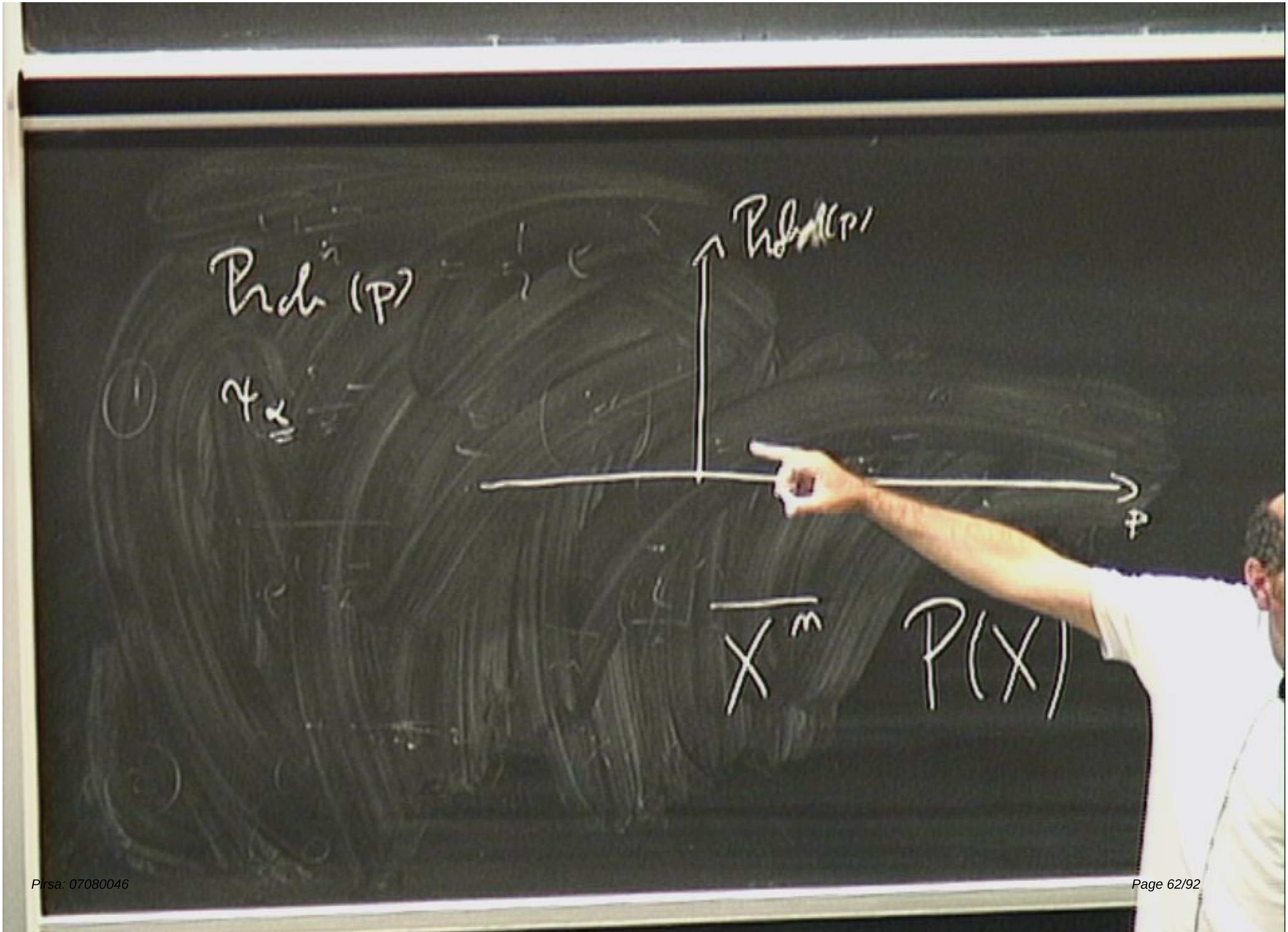
(2) $$\begin{cases} e^{\frac{1}{x^2-a^2}} & |x|<a \\ 0 \end{cases}$$

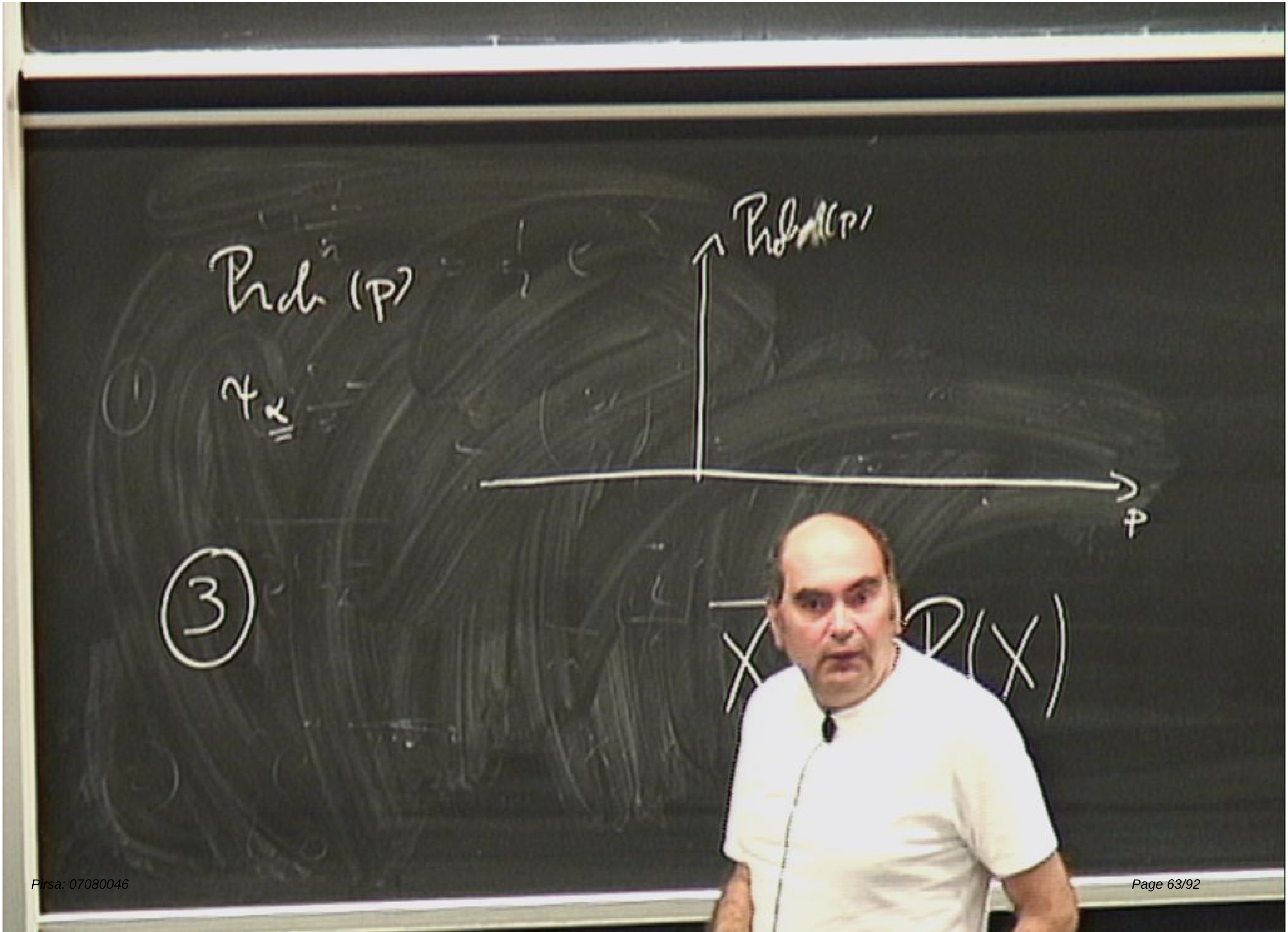

Prob (p)

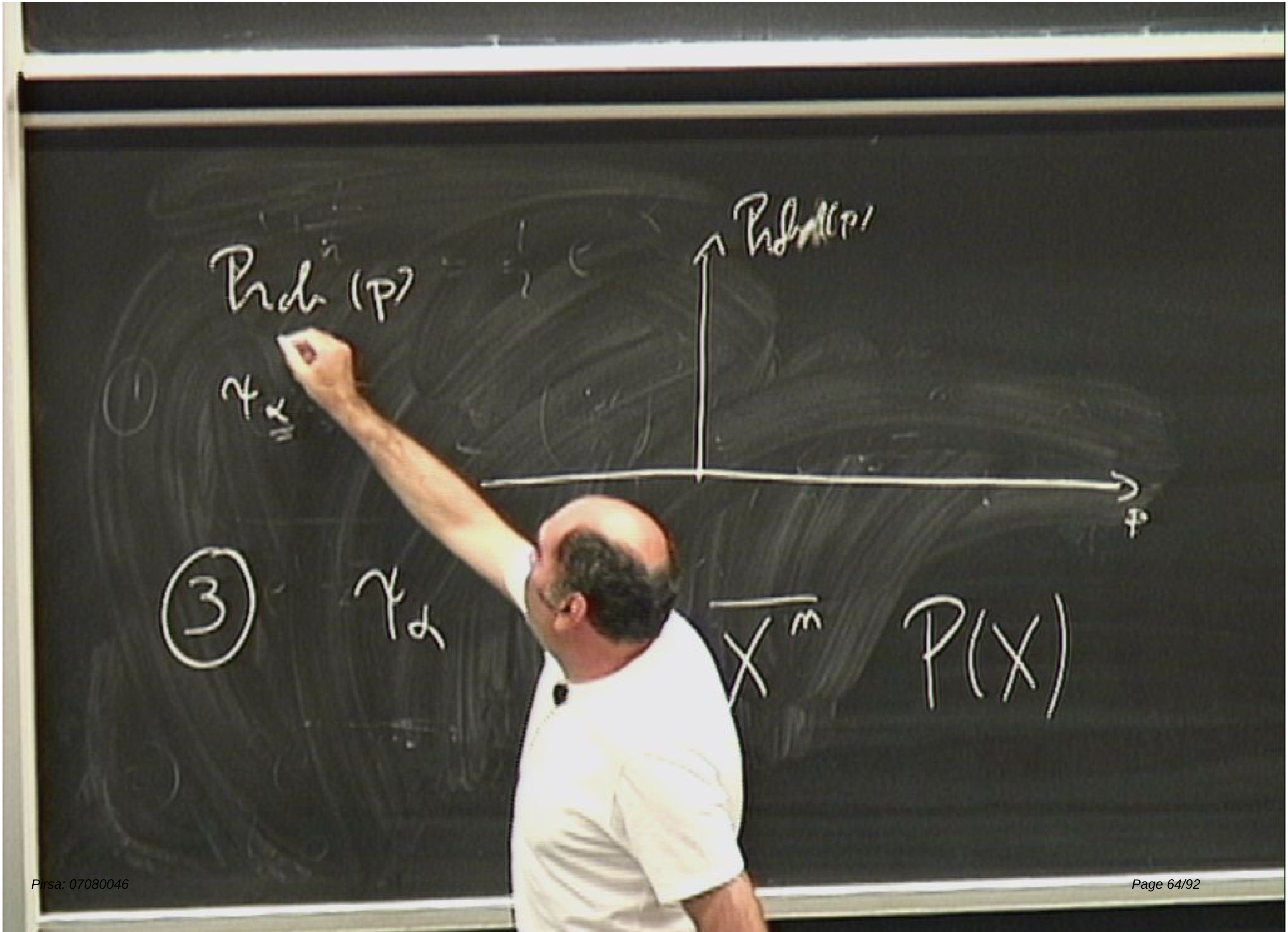

Prob (p)
p


Prob (p)
p

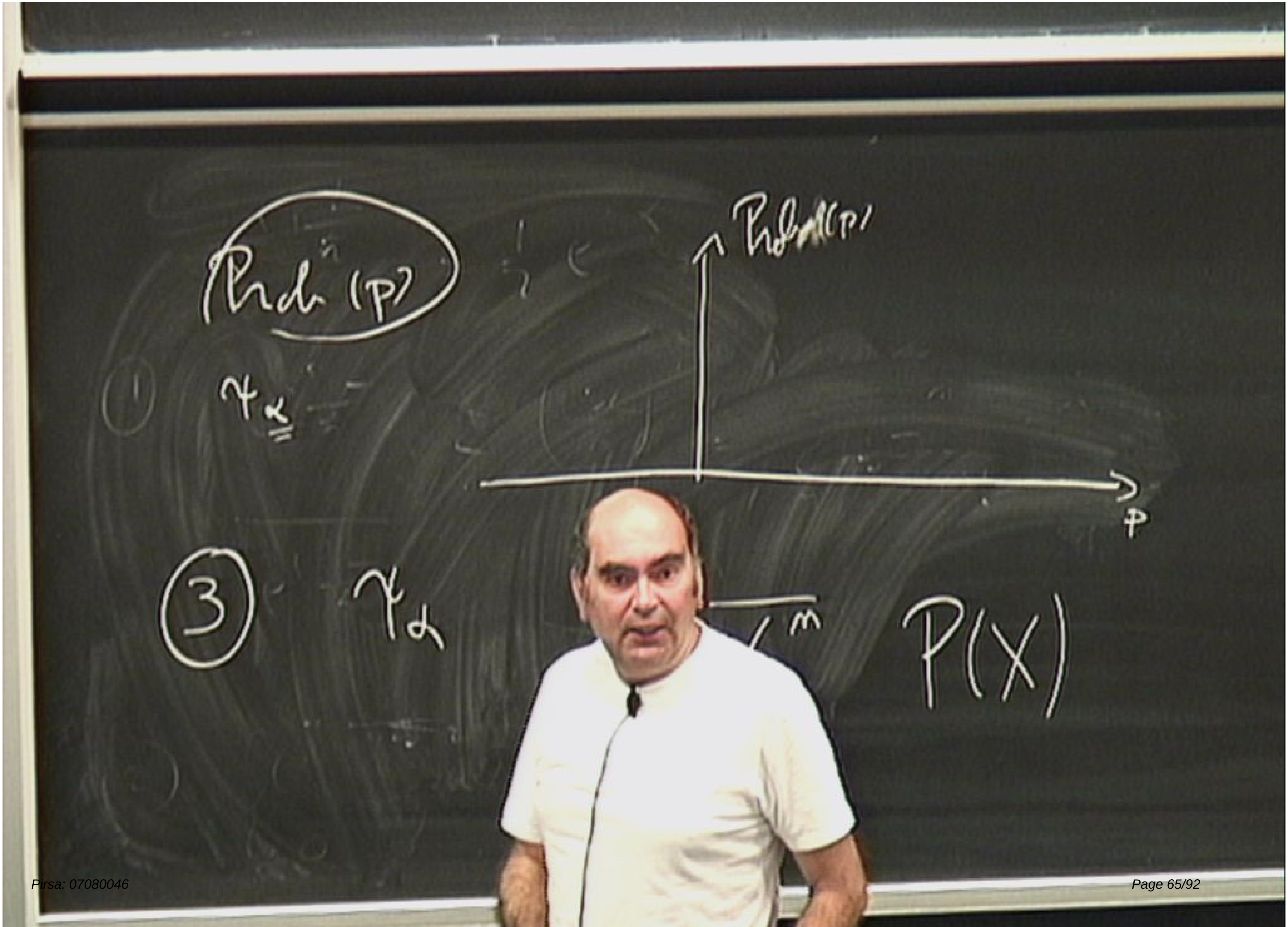

Prob (p)
Prob(p)
X^m

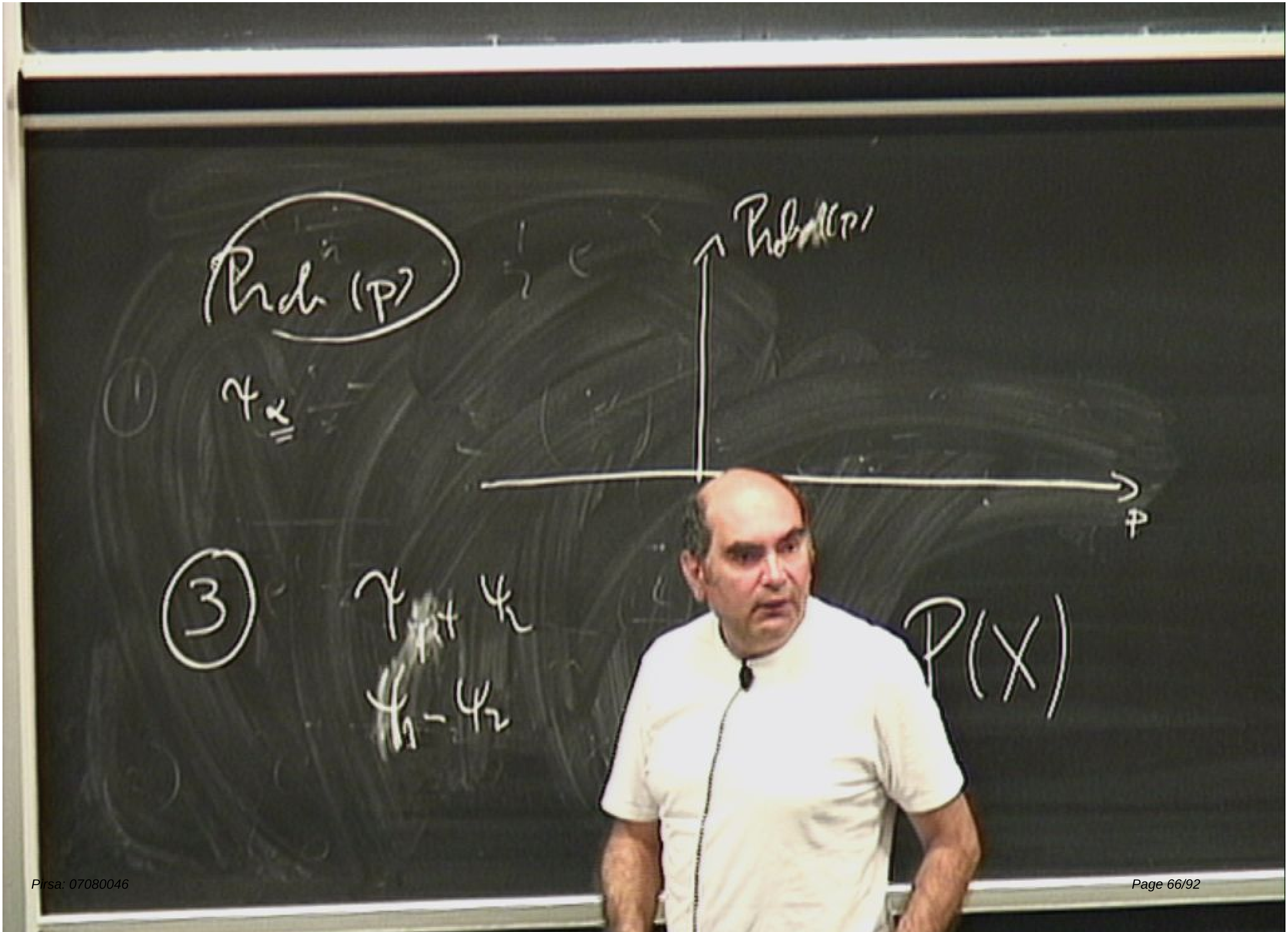

Prob (p)
$\overline{X^m}$
P(X)
p

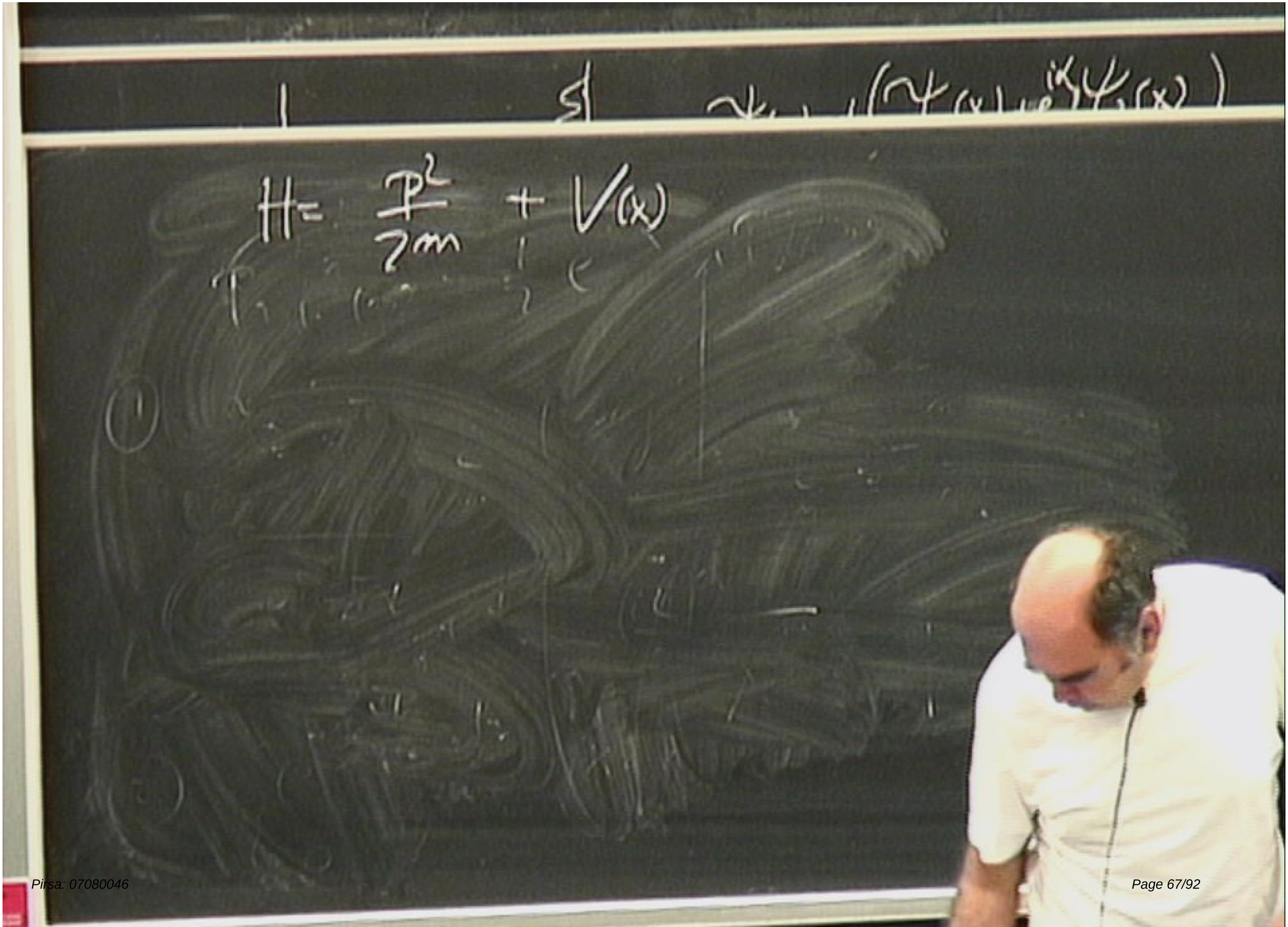

Prob (p)
p
3

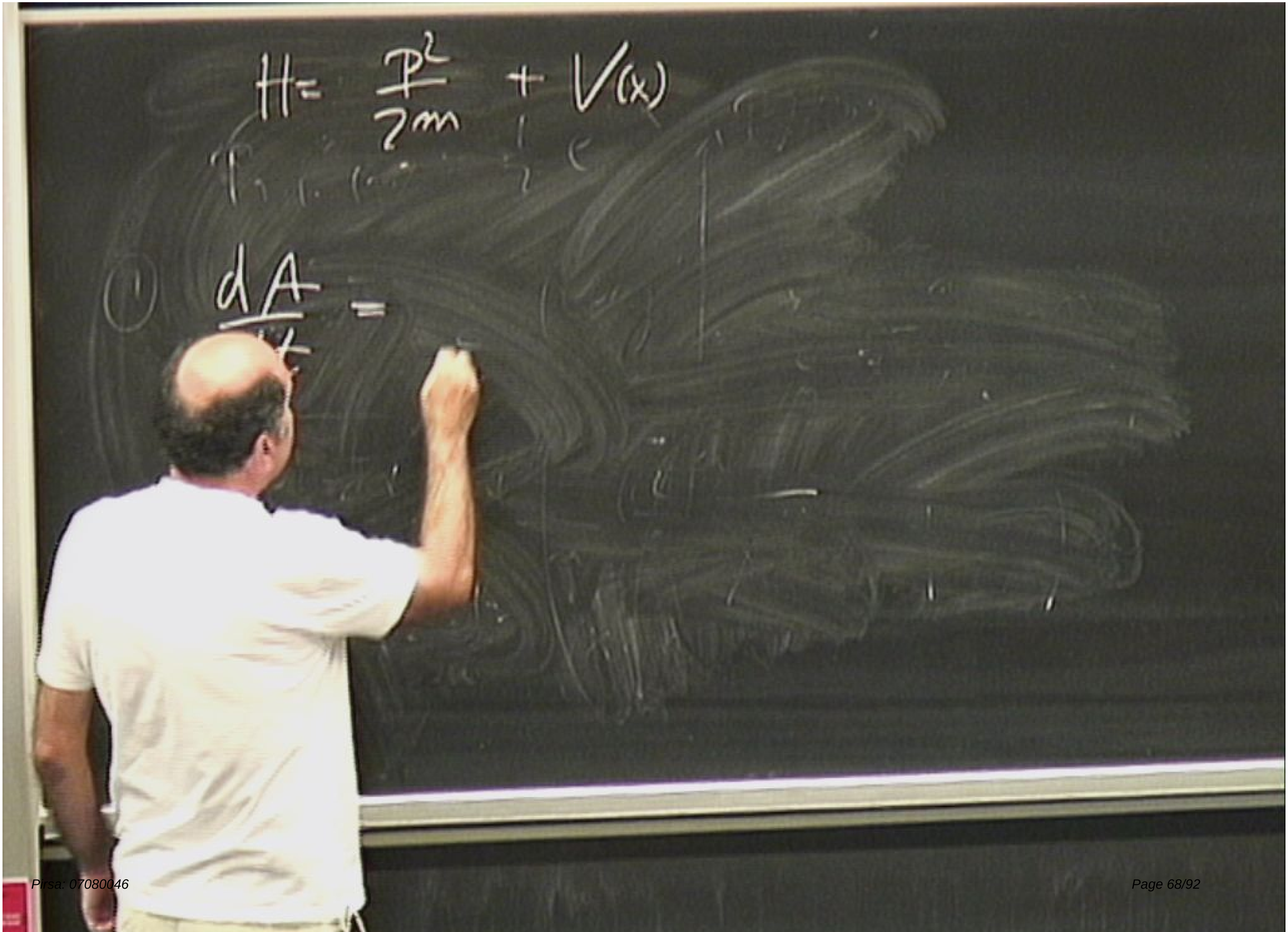


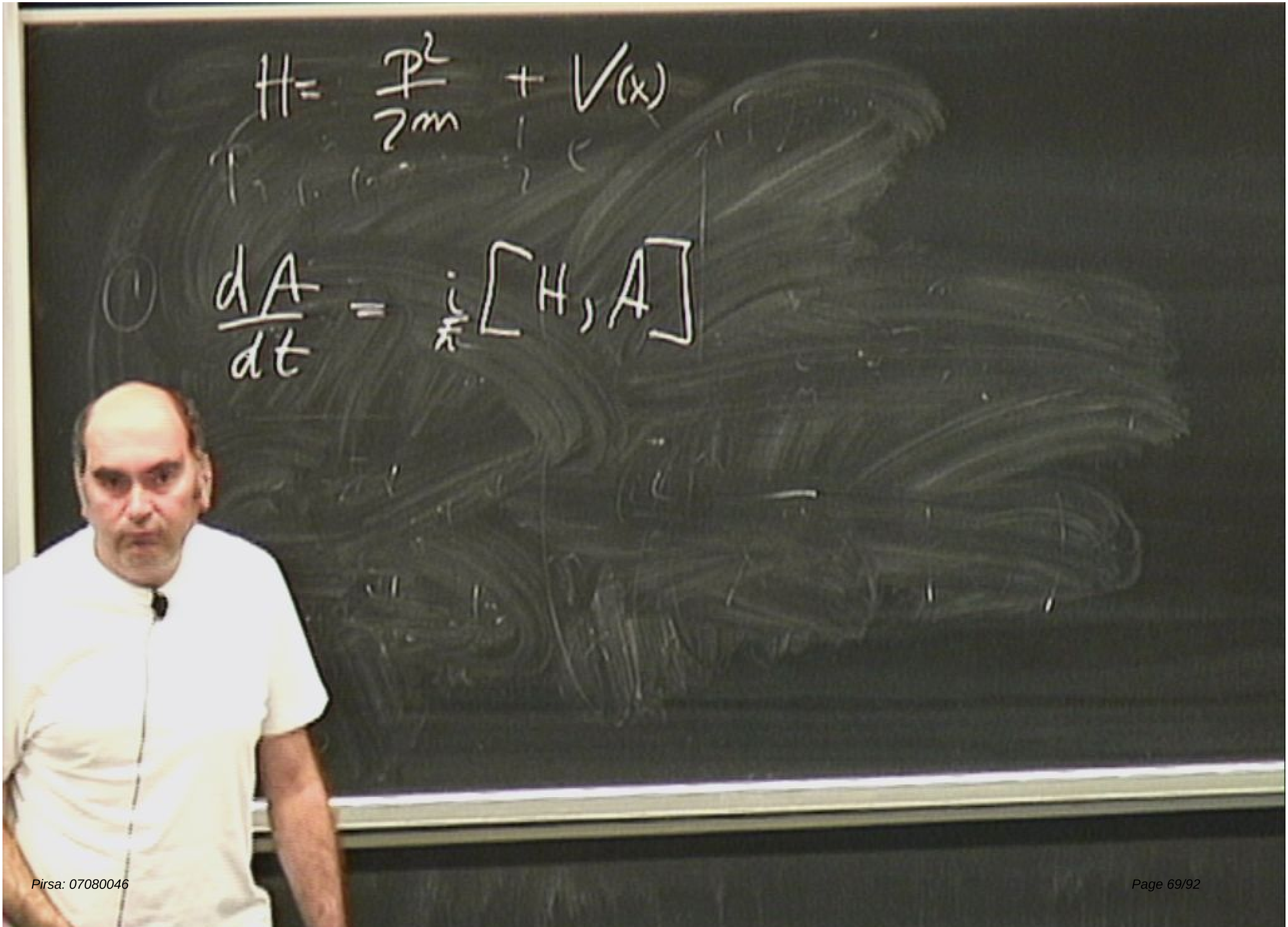
Prob (p)
Prob(p)
p
3
P(X)

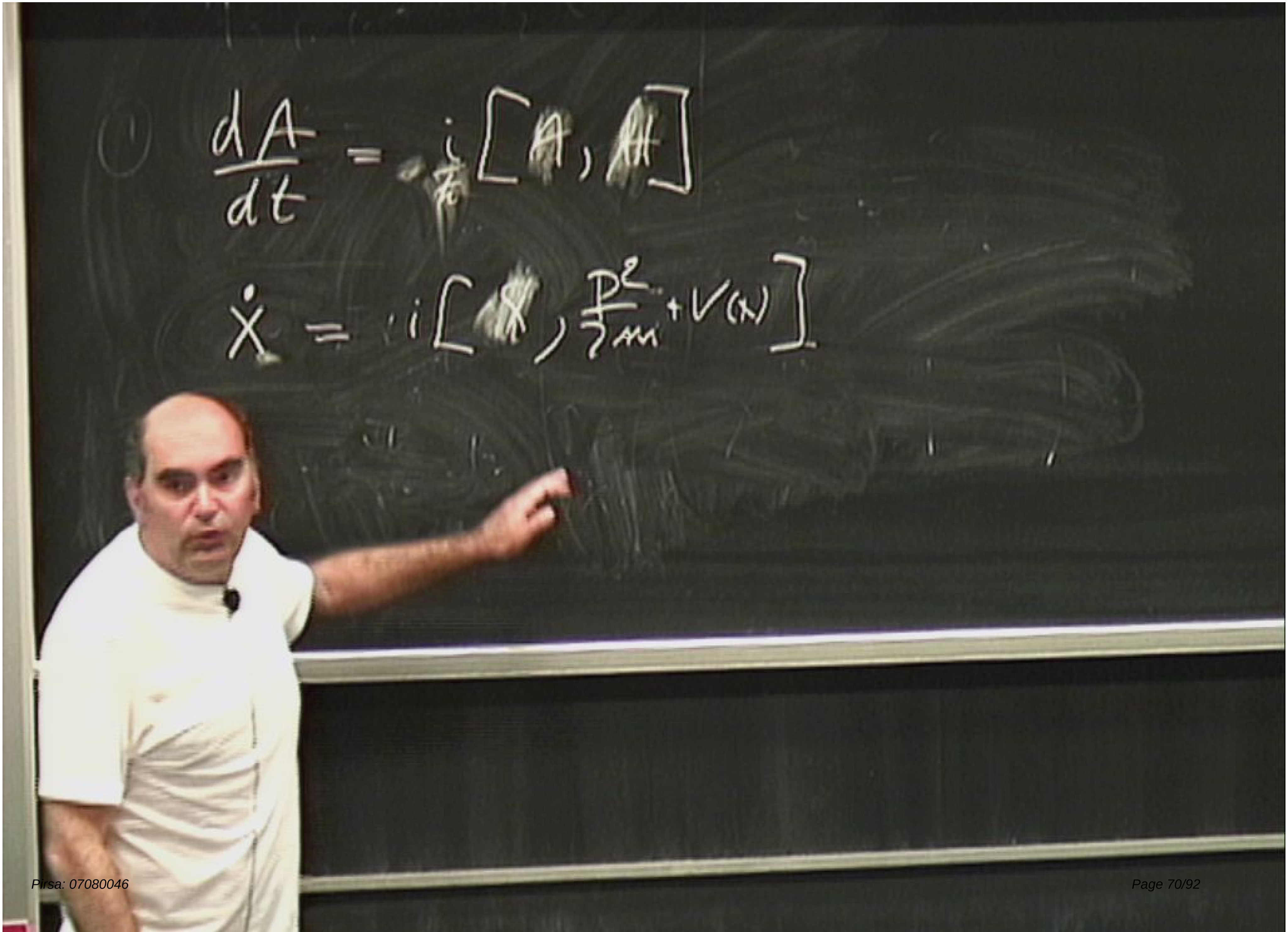

Prob (p)
3
P(X)
p


Prob (p)
1
ψ_α
Prob(p)
p
3
ψ1 + ψ2
ψ1 − ψ2
P(X)

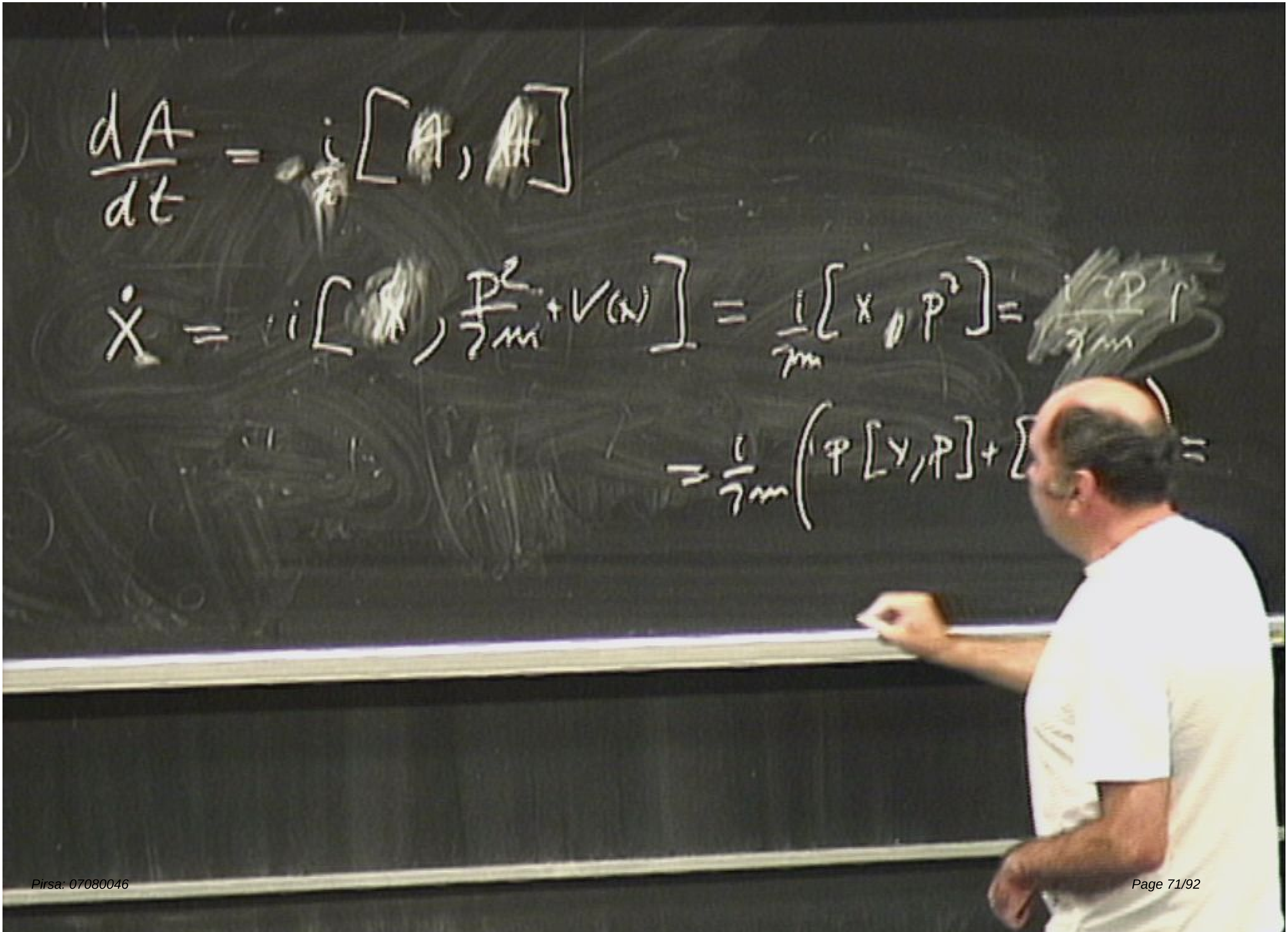

H= P^2/2m + V(x)


$H = \frac{p^2}{2m} + V(x)$
1 $\frac{dA}{dt} =$

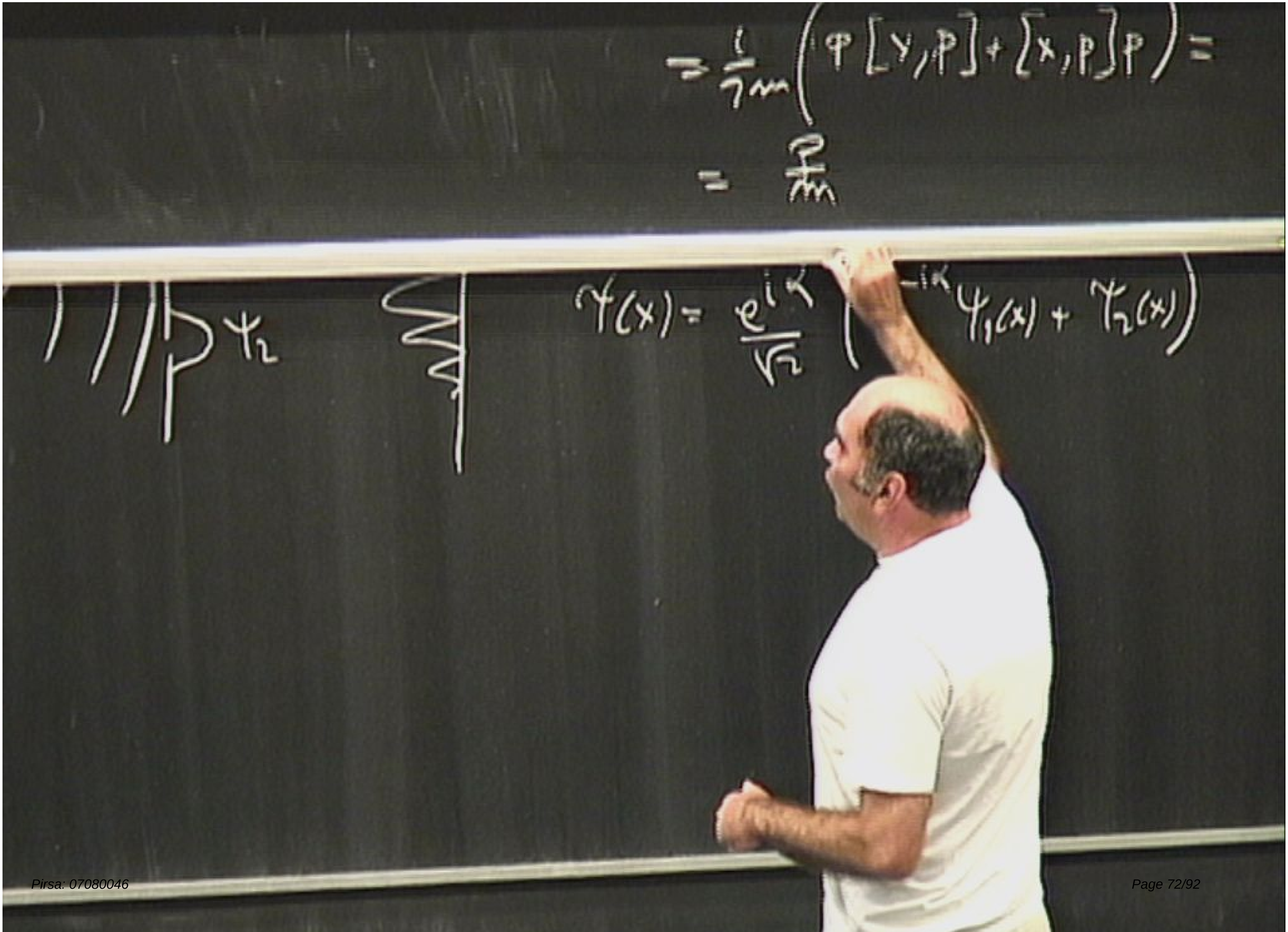

$H = \frac{p^2}{2m} + V(x)$
$(1) \quad \frac{dA}{dt} = \frac{i}{\hbar}[H, A]$



$\frac{dA}{dt} = \frac{i}{\hbar}[A, H]$
$\dot{X} = i[X, \frac{p^2}{2m} + V(x)]$



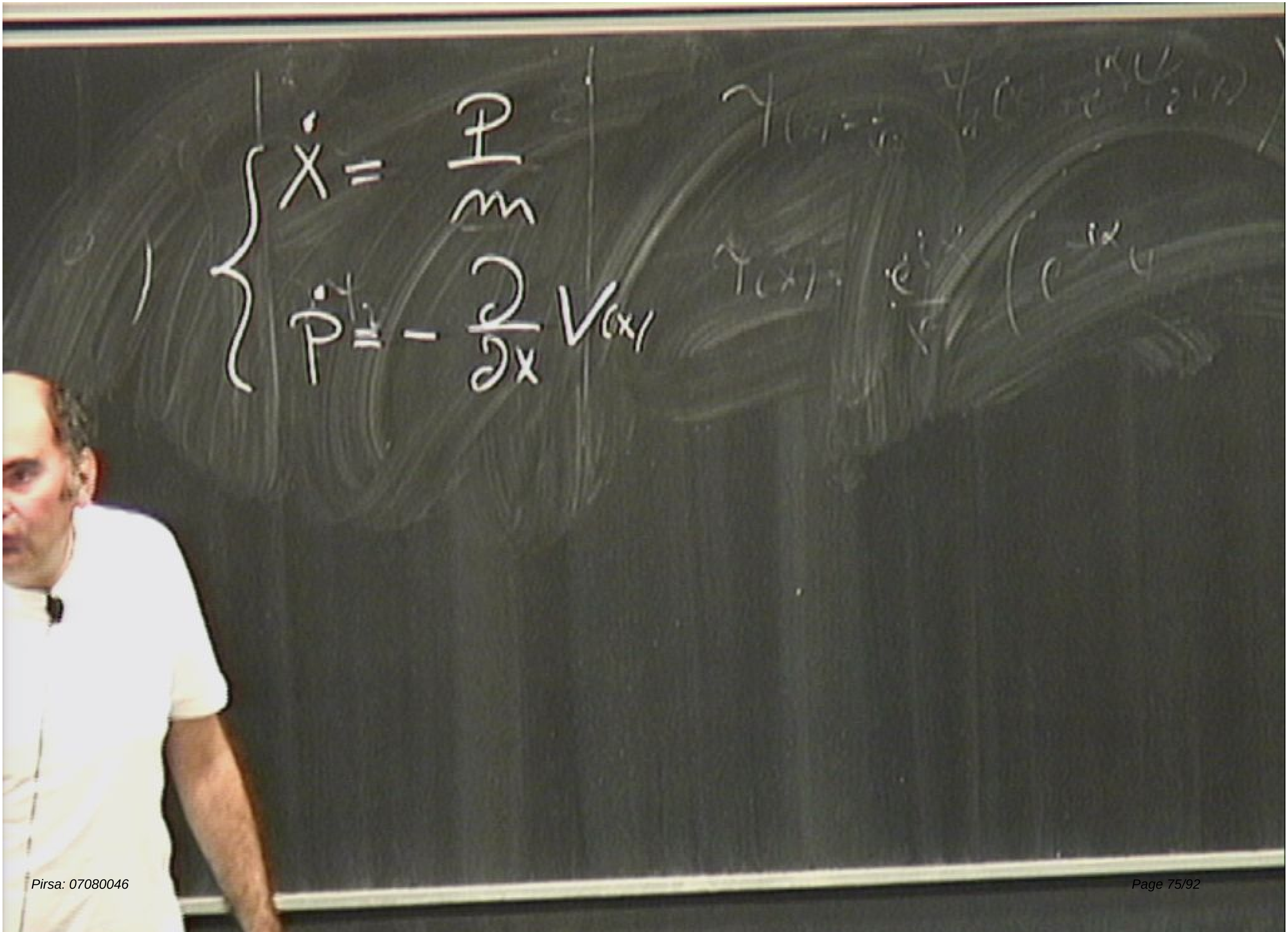
$\frac{dA}{dt} = \frac{i}{\hbar}[A, H]$
$\dot{x} = i\left[x, \frac{p^2}{2m} + V(x)\right] = \frac{i}{2m}[x, p^2] =$
$= \frac{i}{2m}\left(p[x,p] + [\right.$ $=$

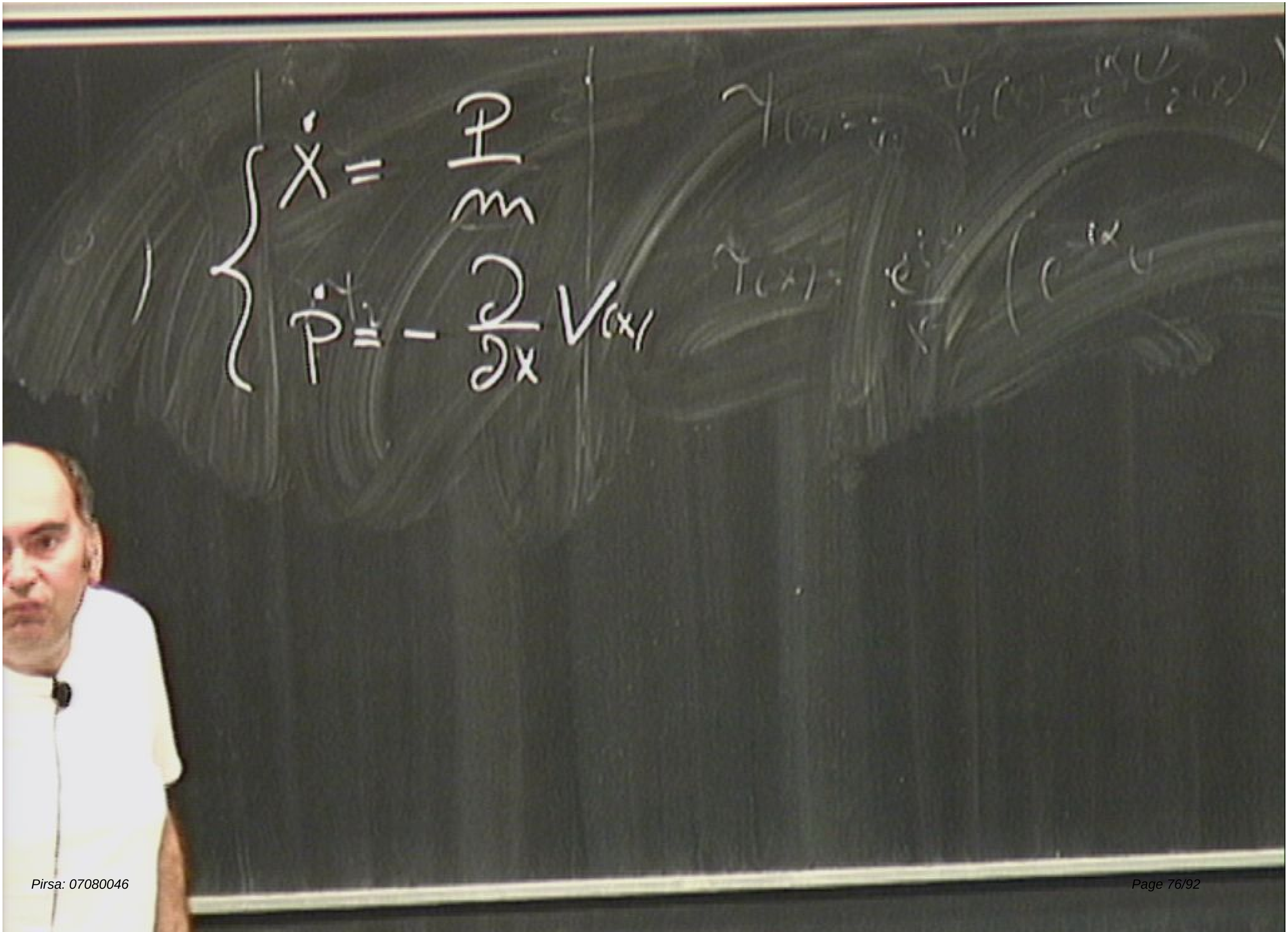

$= \frac{i}{\hbar m}\left(\varphi\,[y,p] + [x,p]\,p\right) =$
$= \frac{\hbar}{m}$
ψ_2
$\psi(x) = \frac{e^{i\alpha}}{\sqrt{2}}\left(\psi_1(x) + \psi_2(x)\right)$

$$\dot{x} = \frac{P}{m}$$

$$\dot{P} = i\,[H, P] = i\left[\frac{P^2}{2m} + V(x), P\right] =$$

$$= i\,[V(x), P] = -\frac{\partial V}{\partial x}$$


$\dot{X} = \frac{P}{m}$
$\dot{P} = -\frac{\partial}{\partial x} V(x)$

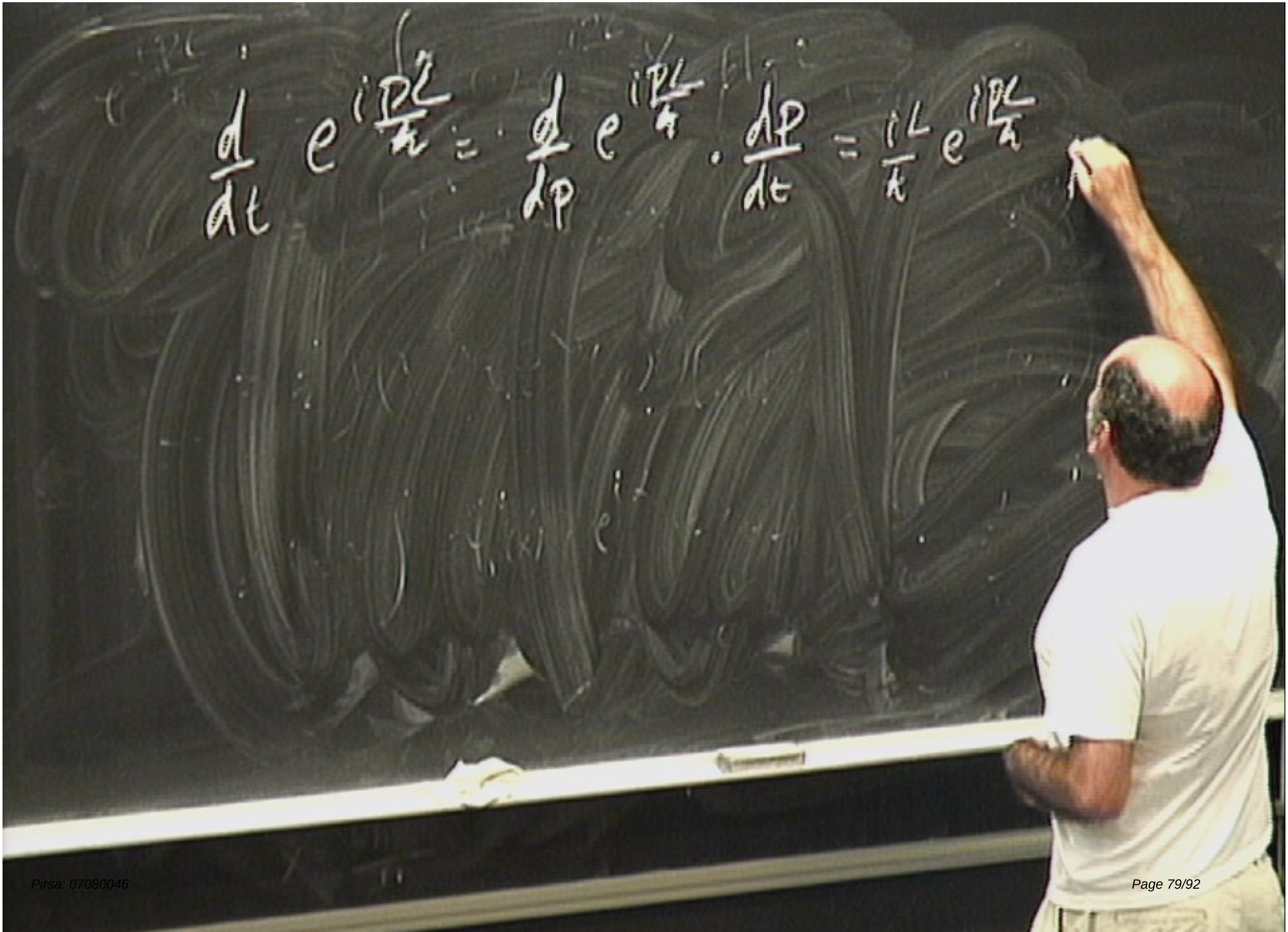

$\dot{X} = \frac{P}{m}$
$\dot{P} = -\frac{\partial}{\partial x} V(x)$

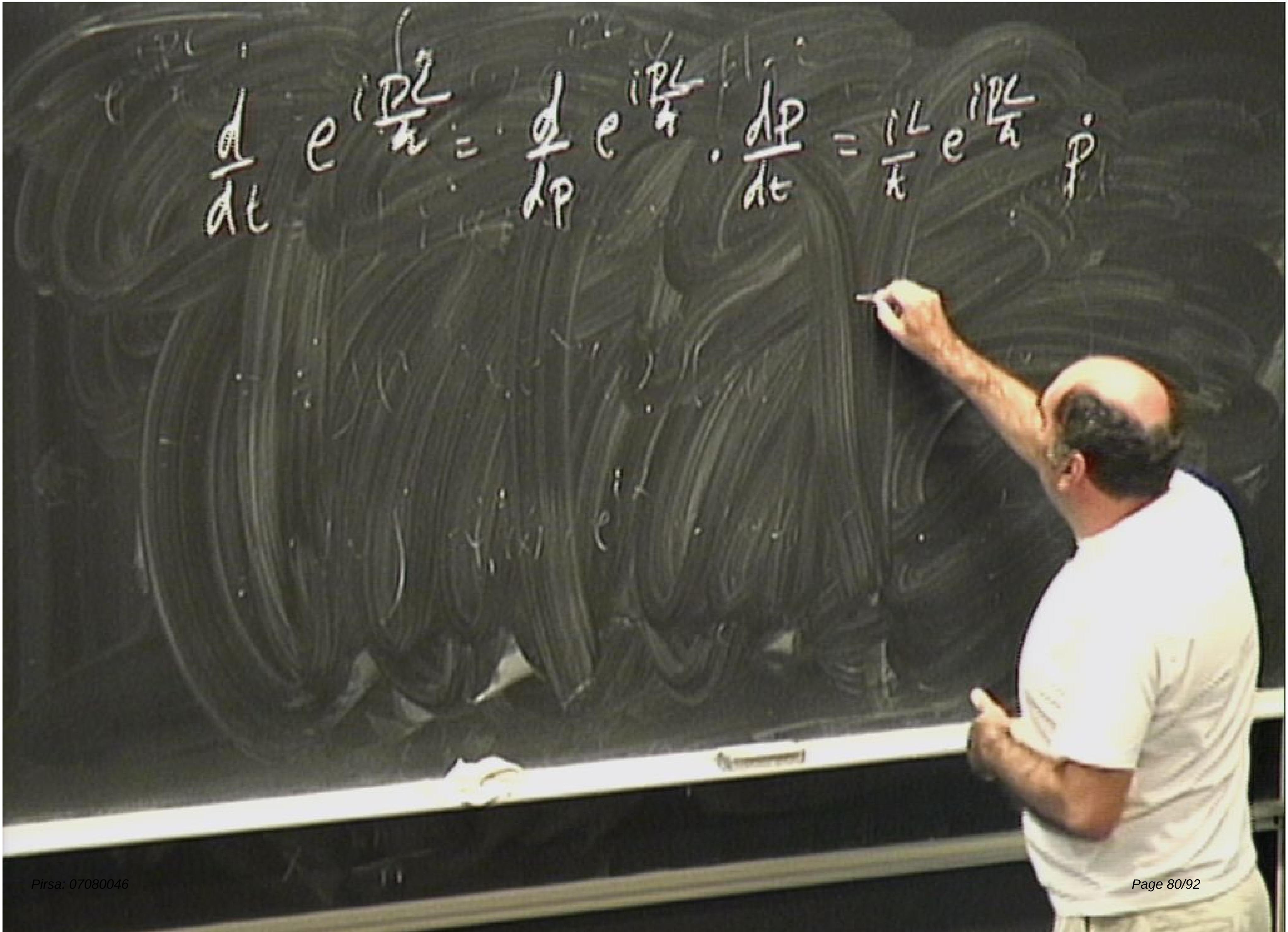
$$\begin{cases} \dot{x} = \dfrac{p}{m} \\ \dot{p} = -\dfrac{\partial}{\partial x} V(x) \end{cases}$$

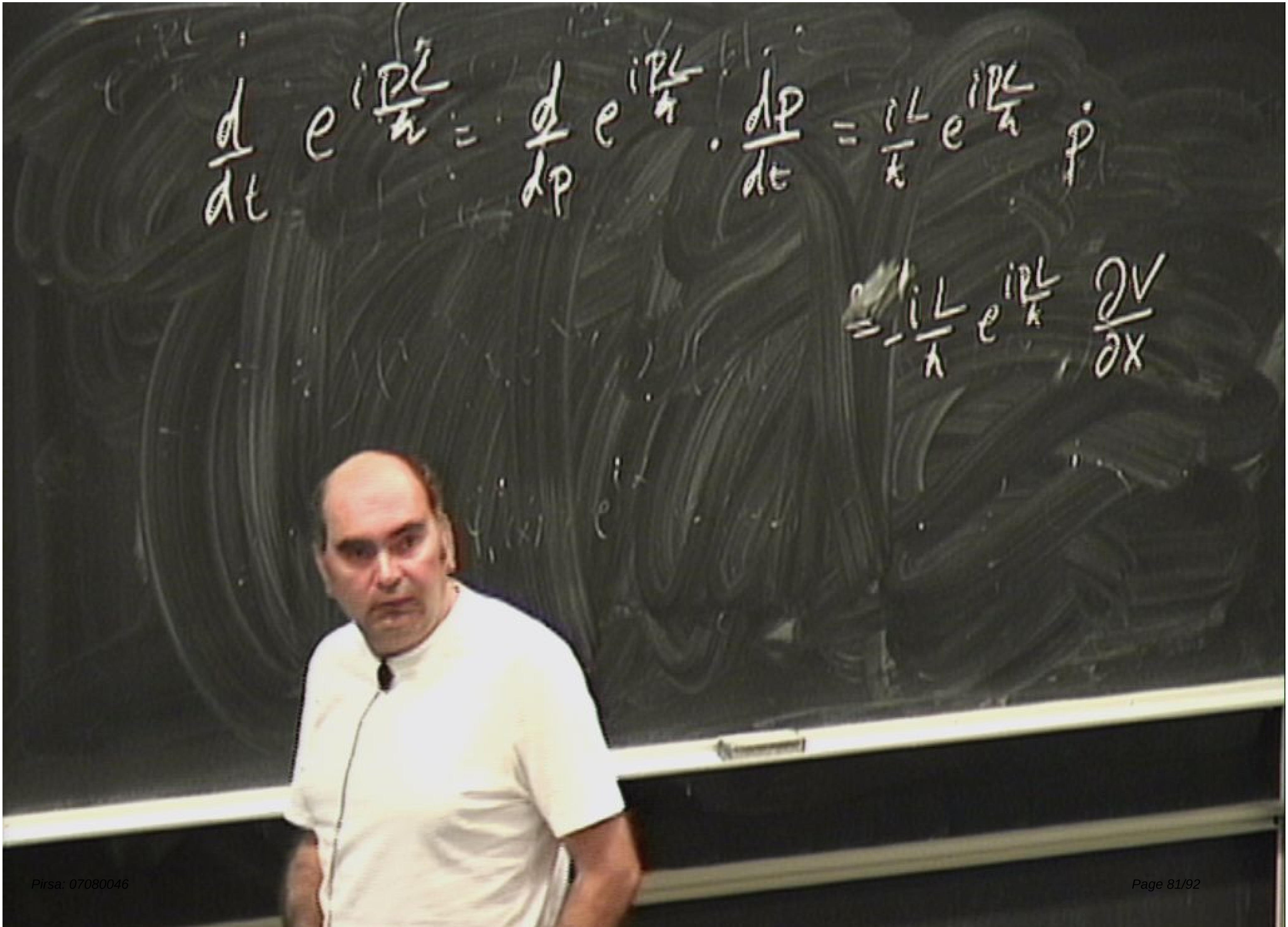
$$\frac{d}{dt} e^{\frac{ipL}{\hbar}}$$

$$\begin{cases} \dot{x} = \dfrac{p}{m} \\ \dot{p} = -\dfrac{\partial}{\partial x} V(x) \end{cases}$$

$$\frac{d}{dt} e^{\frac{ipL}{\hbar}}$$

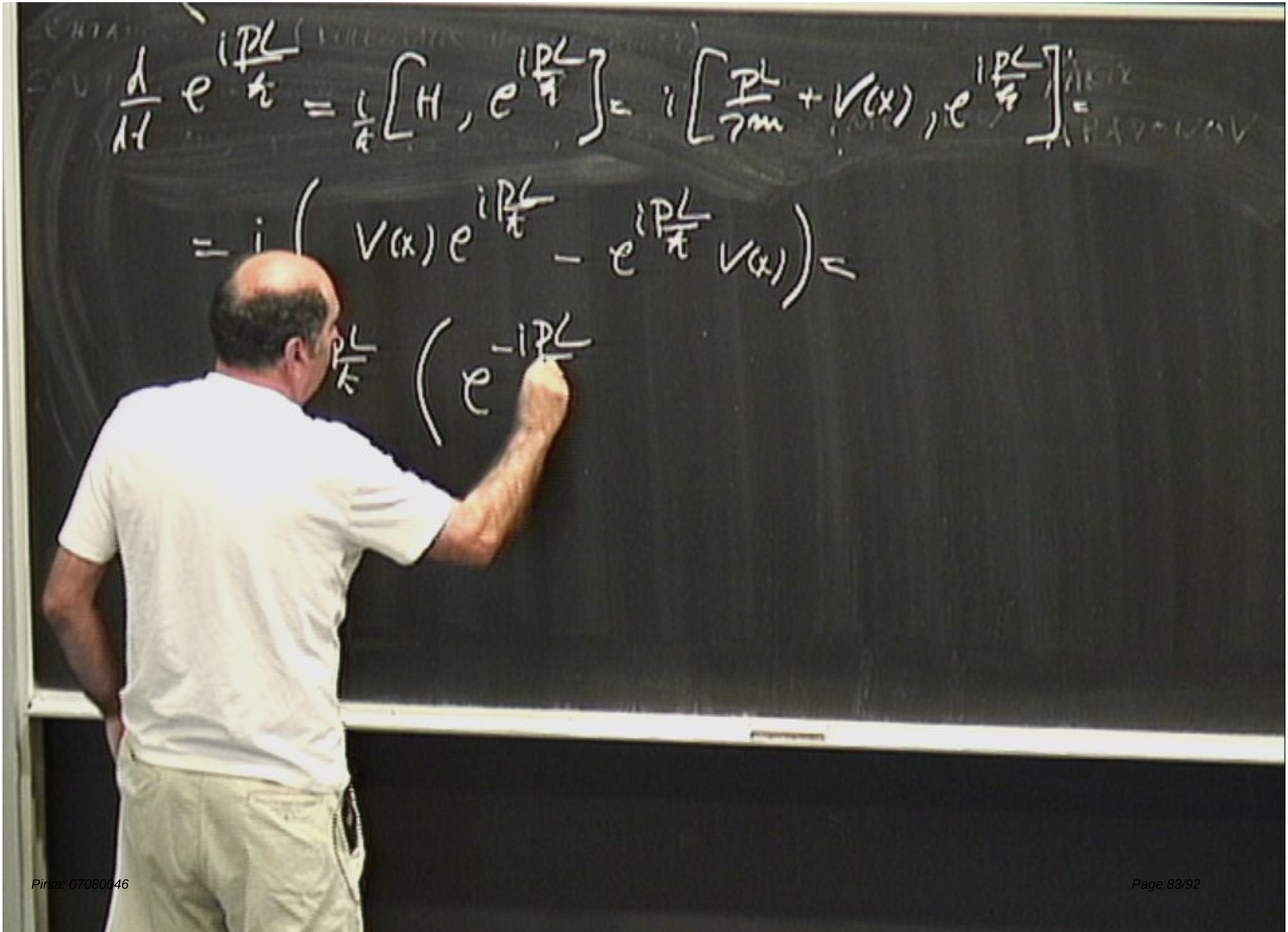

$\frac{d}{dt} e^{\frac{ipL}{\hbar}} = \frac{d}{dp} e^{\frac{ipL}{\hbar}} \cdot \frac{dp}{dt} = \frac{iL}{\hbar} e^{\frac{ipL}{\hbar}}$

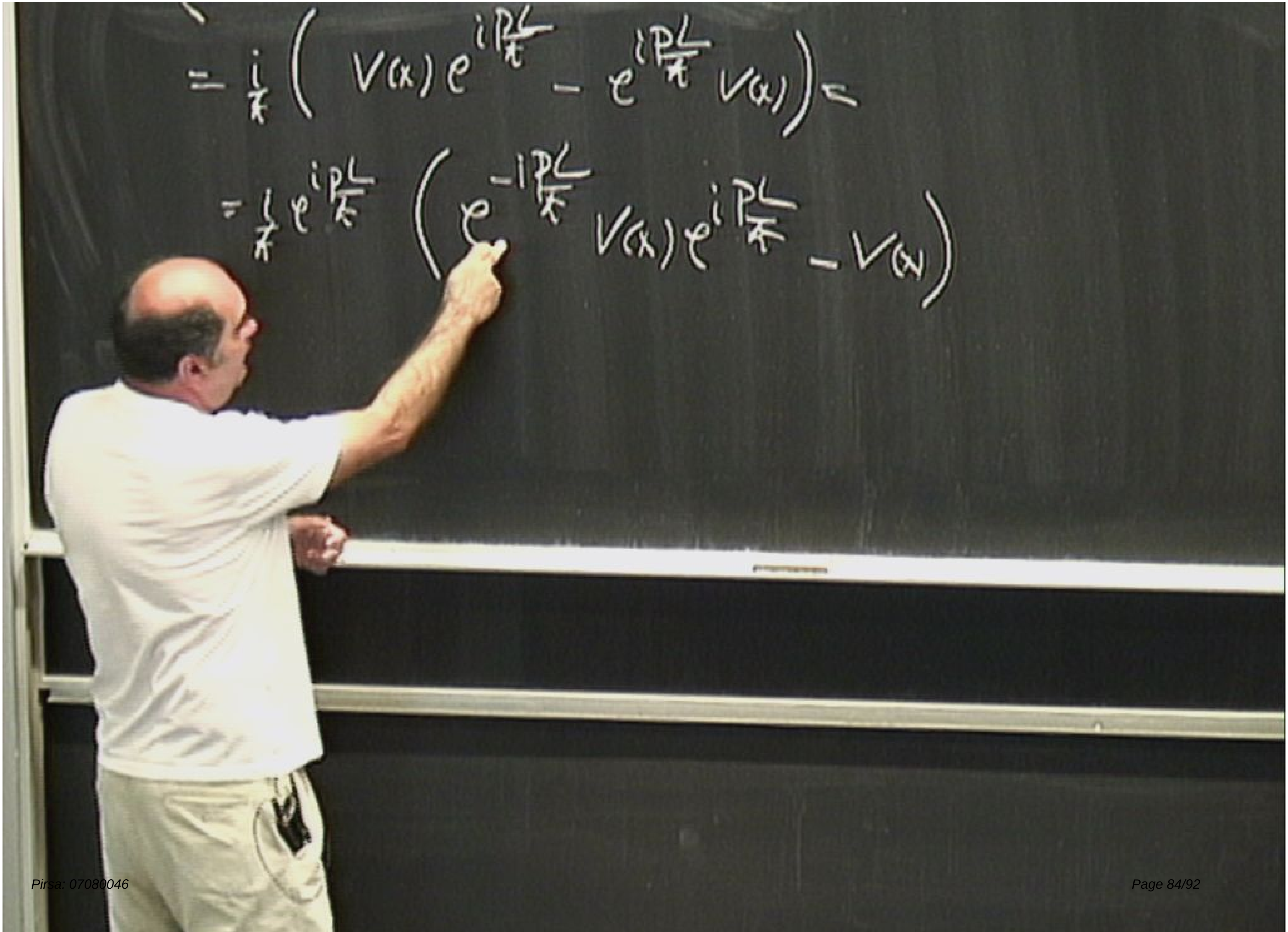

$\frac{d}{dt} e^{\frac{ipL}{\hbar}} = \frac{d}{dp} e^{\frac{ipL}{\hbar}} \cdot \frac{dp}{dt} = \frac{iL}{\hbar} e^{\frac{ipL}{\hbar}} \dot{p}$

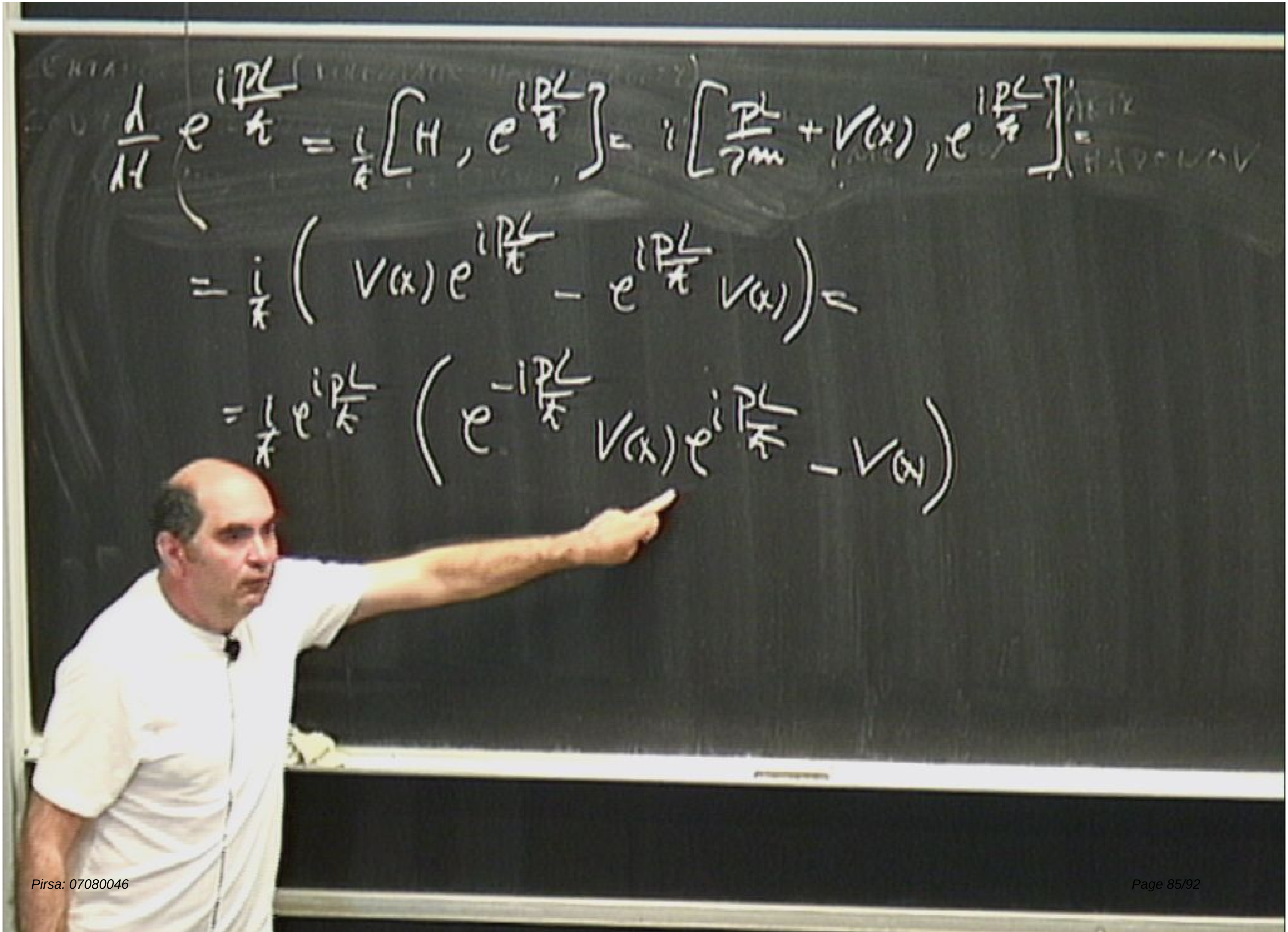

$\frac{d}{dt} e^{\frac{ipL}{\hbar}} = \frac{d}{dp} e^{\frac{ipL}{\hbar}} \cdot \frac{dp}{dt} = \frac{iL}{\hbar} e^{\frac{ipL}{\hbar}} \dot{p}$
$= -\frac{iL}{\hbar} e^{\frac{ipL}{\hbar}} \frac{\partial V}{\partial x}$

$$\frac{d}{dt} e^{\frac{ipL}{\hbar}} = \frac{d}{dp} e^{\frac{ipL}{\hbar}} \cdot \frac{dp}{dt} = \frac{iL}{\hbar} e^{\frac{ipL}{\hbar}} \dot{p}$$

$$= -\frac{iL}{\hbar} e^{\frac{ipL}{\hbar}} \frac{\partial V}{\partial x}$$


$\frac{d}{dt} e^{\frac{ip^2}{\hbar}} = \frac{i}{\hbar}\left[H, e^{\frac{ip^2}{\hbar}}\right] = i\left[\frac{p^2}{2m} + V(x), e^{\frac{ip^2}{\hbar}}\right] =$
$= i\left(V(x) e^{\frac{ip^2}{\hbar}} - e^{\frac{ip^2}{\hbar}} V(x)\right) =$
$e^{\frac{ip^2}{\hbar}} \left(e^{-\frac{ip^2}{}}\right.$


$= \frac{i}{\hbar}\left(V(x)\, e^{iPL/\hbar} - e^{iPL/\hbar}\, V(x) \right) =$
$= \frac{i}{\hbar} e^{iPL/\hbar} \left(e^{-iPL/\hbar}\, V(x)\, e^{iPL/\hbar} - V(x) \right)$


$\frac{d}{dt} e^{\frac{ipL}{\hbar}} = \frac{i}{\hbar}\left[H, e^{\frac{ipL}{\hbar}}\right] = \frac{i}{\hbar}\left[\frac{p^2}{2m} + V(x), e^{\frac{ipL}{\hbar}}\right] =$
$= \frac{i}{\hbar}\left(V(x) e^{\frac{ipL}{\hbar}} - e^{\frac{ipL}{\hbar}} V(x) \right) =$
$= \frac{i}{\hbar} e^{\frac{ipL}{\hbar}} \left(e^{-\frac{ipL}{\hbar}} V(x) e^{\frac{ipL}{\hbar}} - V(x) \right)$

$$\frac{d}{dt} e^{\frac{iPL}{\hbar}} = \frac{i}{\hbar}\left[H, e^{\frac{iPL}{\hbar}}\right] = i\left[\frac{P^2}{2m} + V(x), e^{\frac{iPL}{\hbar}}\right] =$$

$$= \frac{i}{\hbar}\left(V(x) e^{\frac{iPL}{\hbar}} - e^{\frac{iPL}{\hbar}} V(x)\right) =$$

$$= \frac{i}{\hbar} e^{\frac{iPL}{\hbar}} \left(e^{-\frac{iPL}{\hbar}} V(x) e^{\frac{iPL}{\hbar}} - V(x)\right)$$

$$= \frac{i}{\hbar} e^{\frac{iPL}{\hbar}} \left(V(x+L) - V(x)\right)$$

$= \hbar m$

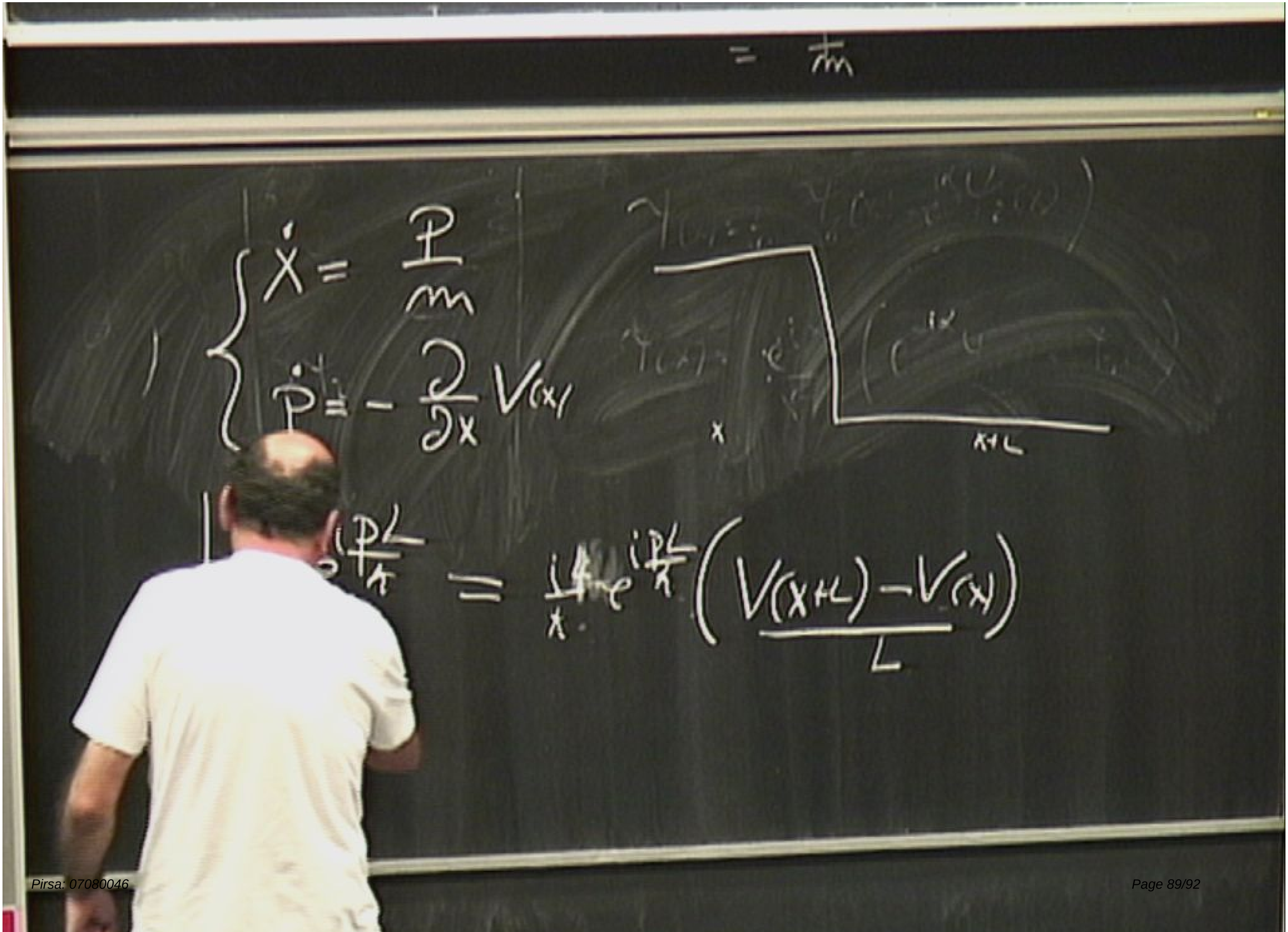
$$\begin{cases} \dot{x} = \dfrac{p}{m} \\ \dot{p} = -\dfrac{\partial}{\partial x} V(x) \end{cases}$$

$$\frac{d}{dt} e^{ipL/\hbar} = \frac{i}{\hbar} e^{ipL/\hbar} \left(\frac{V(x+L) - V(x)}{L} \right)$$

$= \frac{\hbar}{m}$

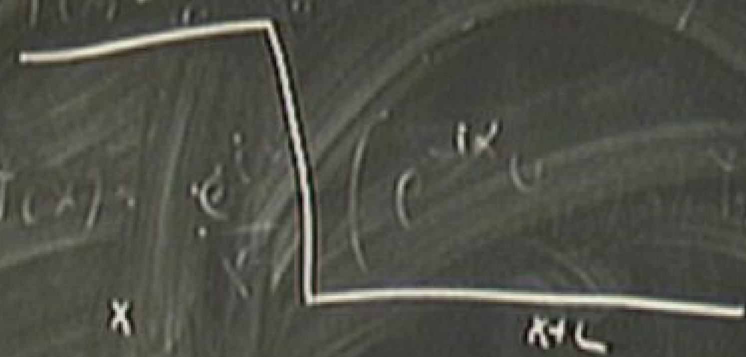
$$\begin{cases} \dot{x} = \frac{p}{m} \\ \dot{p} = -\frac{\partial}{\partial x} V(x) \end{cases}$$

$$\frac{d}{dt} e^{\frac{ipL}{\hbar}} = \frac{i}{\hbar} e^{\frac{ipL}{\hbar}} \left(\frac{V(x+L) - V(x)}{L} \right)$$


$\dot{x} = \frac{p}{m}$
$\dot{p} = -\frac{\partial}{\partial x} V(x)$
x
x+L
$= \frac{i}{\hbar} e^{i\frac{pL}{\hbar}} \left(\frac{V(x+L) - V(x)}{L} \right)$

$= \frac{\hbar}{m}$

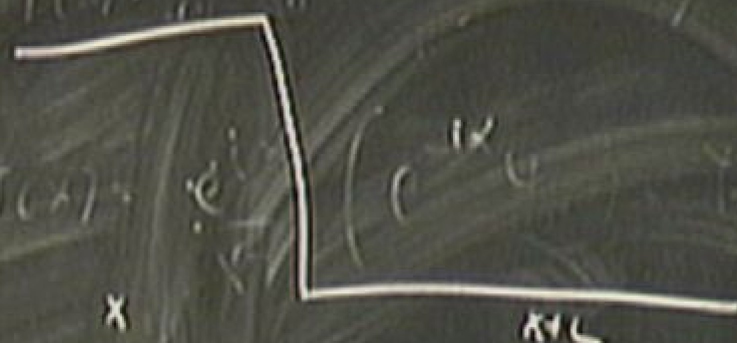
$$\begin{cases} \dot{x} = \dfrac{p}{m} \\ \dot{p} = -\dfrac{\partial}{\partial x} V(x) \end{cases}$$



$$\boxed{\frac{d}{dt} e^{ipL/\hbar} = \frac{i}{\hbar} e^{ipL/\hbar}\left(\frac{V(x+L)-V(x)}{L}\right)}$$

$= \frac{\hbar}{m}$

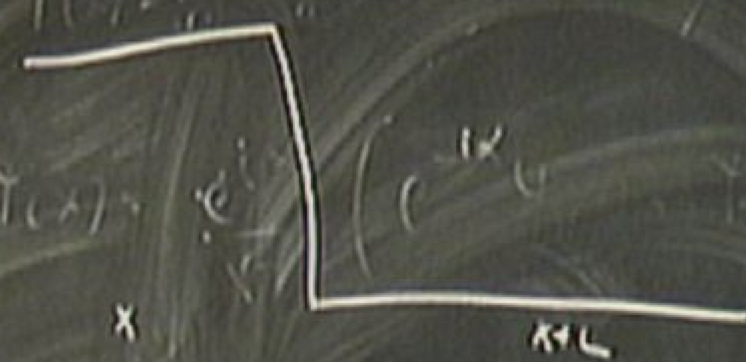
$$\begin{cases} \dot{x} = \frac{p}{m} \\ \dot{p} = -\frac{\partial}{\partial x} V(x) \end{cases}$$



$$\boxed{\frac{d}{dt} e^{\frac{ipL}{\hbar}} = \frac{i}{\hbar} e^{\frac{ipL}{\hbar}} \left(\frac{V(x+L) - V(x)}{L} \right)}$$

$= \frac{\hbar}{m}$

$$\begin{cases} \dot{x} = \frac{p}{m} \\ \dot{p} = -\frac{\partial}{\partial x} V(x) \end{cases}$$



$$\boxed{\frac{d}{dt} e^{ipL/\hbar} = \frac{i}{\hbar} e^{ipL/\hbar}\left(\frac{V(x+L)-V(x)}{L}\right)}$$