Title: Relative states and the environment: Einselection, envariance, quantum Darwinism, and the existential interpretation (Part 1)

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Abstract:

Quantum Foundations

Wojciech Hubert Zurek
Theory Division, Los Alamos

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Relative States and the Environment

(Everett '57)

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"BEYOND DECOHERENCE"

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WHZ, quant-ph arXiv:0707.2832 "**Relative states & the environment:…**"
Phys. Today **44**, 36-44 (1991) (quant-ph/0306072)
Rev. Mod. Phys. **75**, 715 (2003) (quant-ph/0105127)
M. Schlosshauer, "**Decoherence**" (Springer, 2007)
…………

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Plan of the Lectures

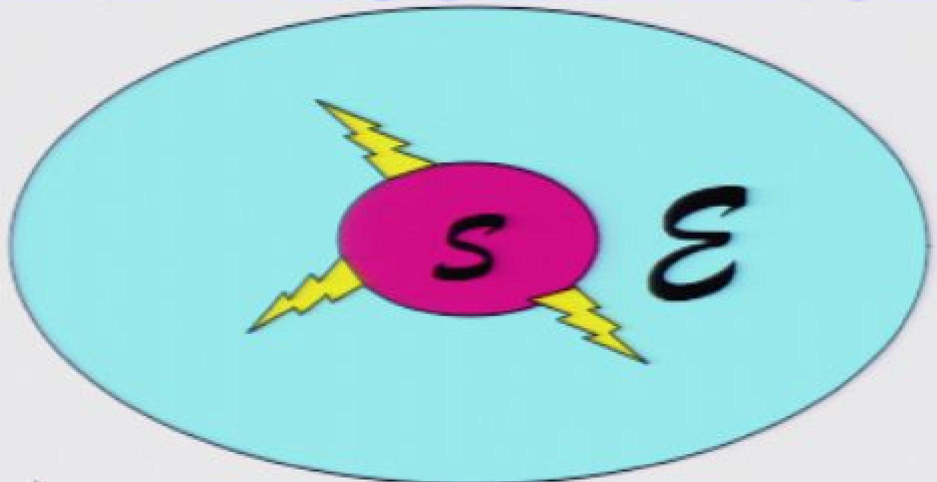
LECTURE 1

- Decoherence 101; the basic idea, and why it is not basic enough for "foundations"
- Origin of quantum jumps (orthogonality & collapse)
- Derivation of probability in quantum theory -- Born's rule $\left(p_k = |\psi_k|^2\right)$

LECTURE 2

- Decoherence & einselection (**e**nvironment - **in**duced super**selection**) of preferred "pointer states"
- Redundancy and quantum Darwinism ("environment as a witness")
- Existential interpretation (Relative states + existence)

EINSELECTION*, POINTER BASIS, AND DECOHERENCE



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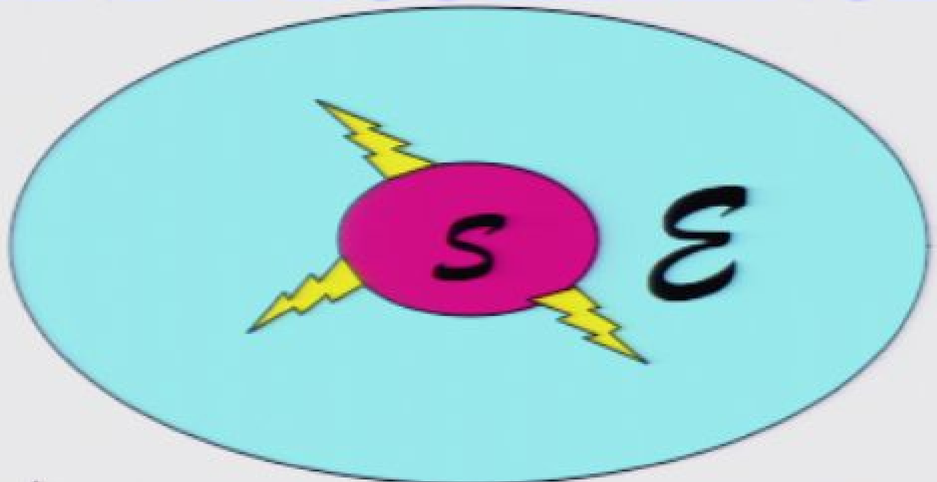

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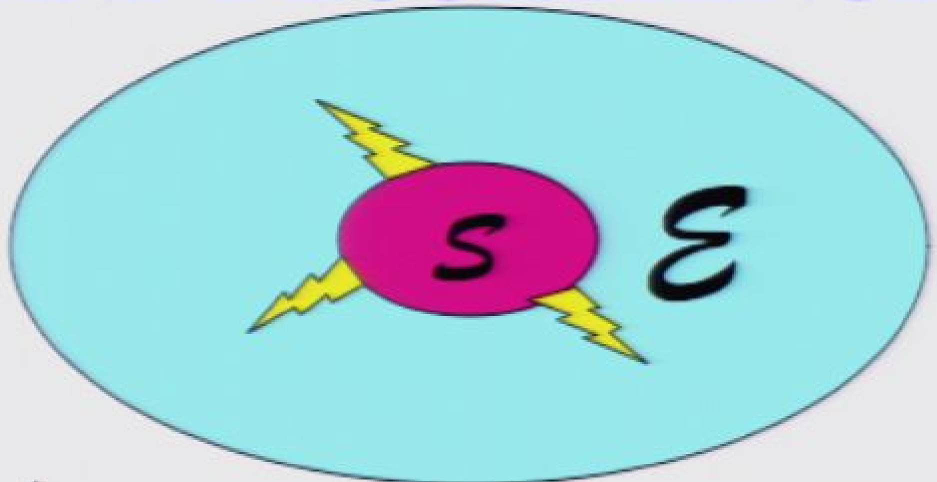

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EINSELECTION* leads to POINTER STATES

(same states appear on the diagonal of $\rho_S(t)$ for times long compared to the decoherence time; **pointer states** are effectively classical!) Pointer states **left unperturbed** by the "environmental monitoring".

***Environment INduced superSELECTION**

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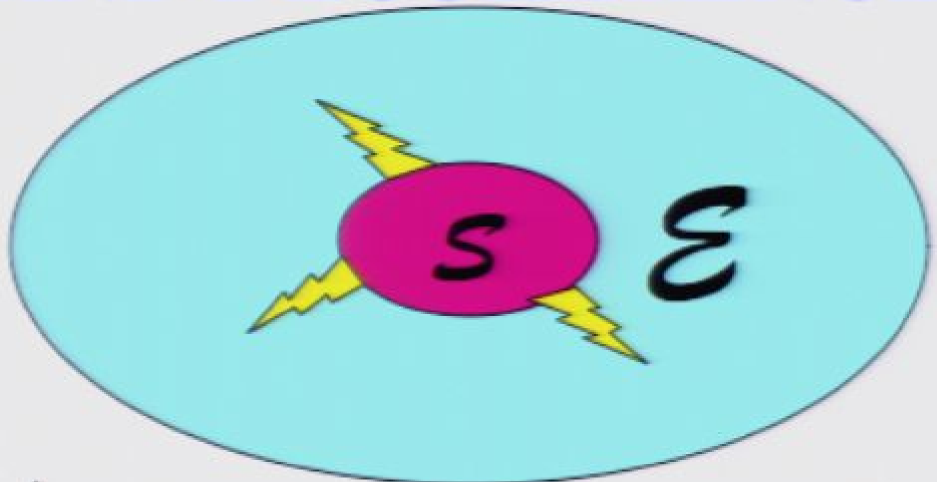

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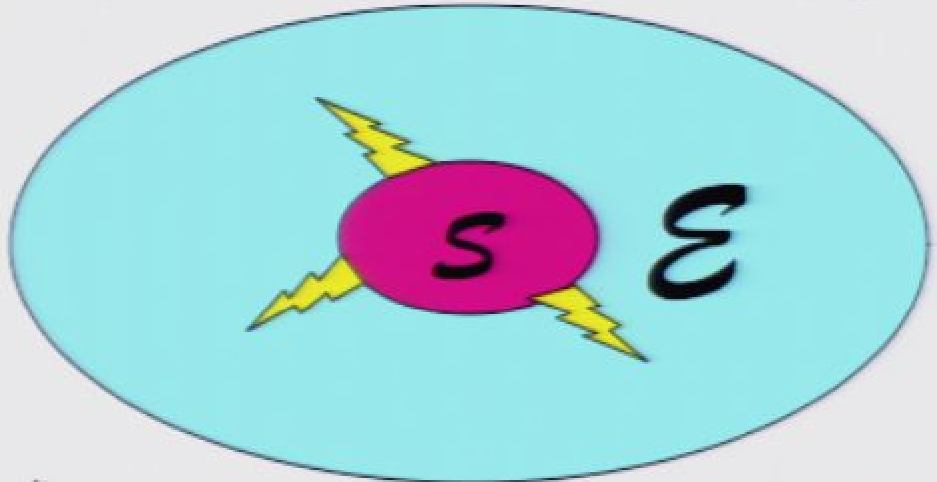

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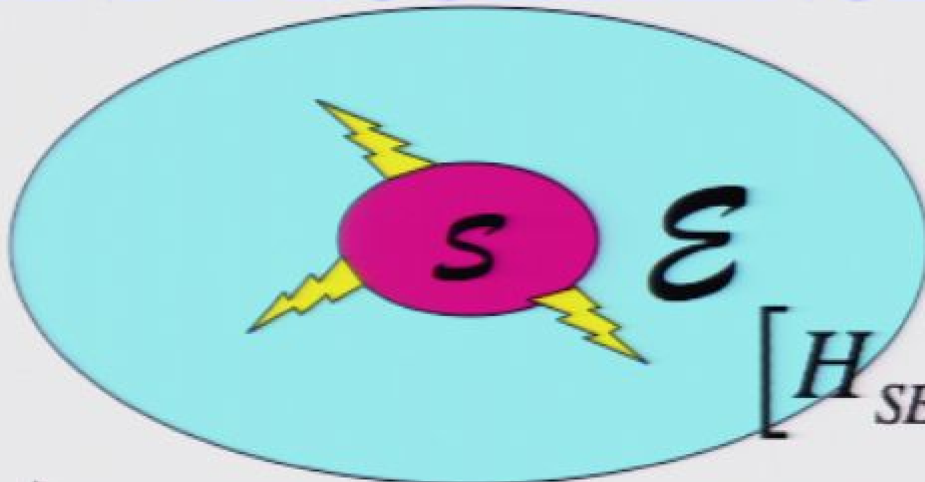

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DECOHERENCE AND EINSELECTION

'hesis: Quantum theory can explain emergence of the classical

Principle of superposition loses its validity in "open" systems, that is, systems interacting with their environments.

)ecoherence restricts stable states (states that can persist, and, therefore, "exist") to the exceptional…

'ointer states that exist or evolve predictably in spite of the immersion of the system in the environment.

'redictability sieve can be used to 'sift' through the Hilbert space of any open quantum system in search of these pointer states.

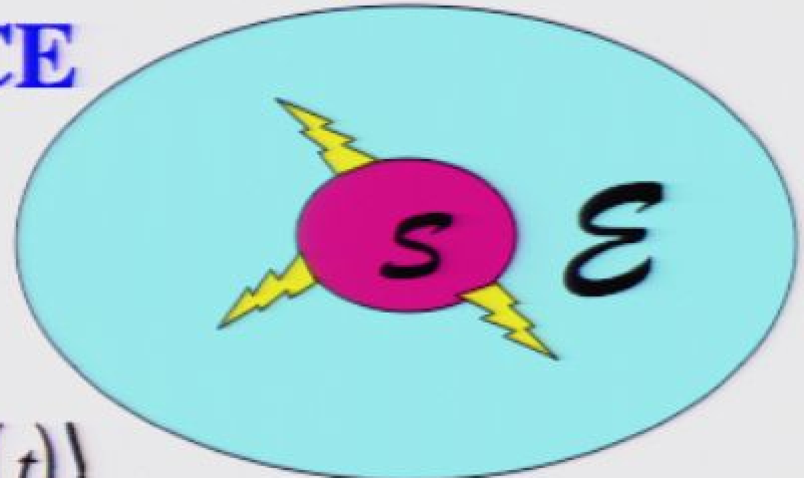
:INSELECTION (or Environment INduced superSELECTION) is the process of selection of these preferred pointer states.

or macroscopic systems, decoherence and einselection can be very effective, enforcing ban on Schroedinger cats.

inselection enforces an effective border that divides quantum from classical, making a point of view similar to Bohr's Copenhagen Interpretation possible, although starting from a rather different standpoint (i. e., no *ab initio* classical domain of the universe).

'eh, Joos, Paz, Caldeira, Leggett, Kiefer, Gell-Mann, Hartle, Omnes, Dalvit, Dziarmaga, Cucchietti …

EINSELECTION, POINTER BASIS, AND DECOHERENCE



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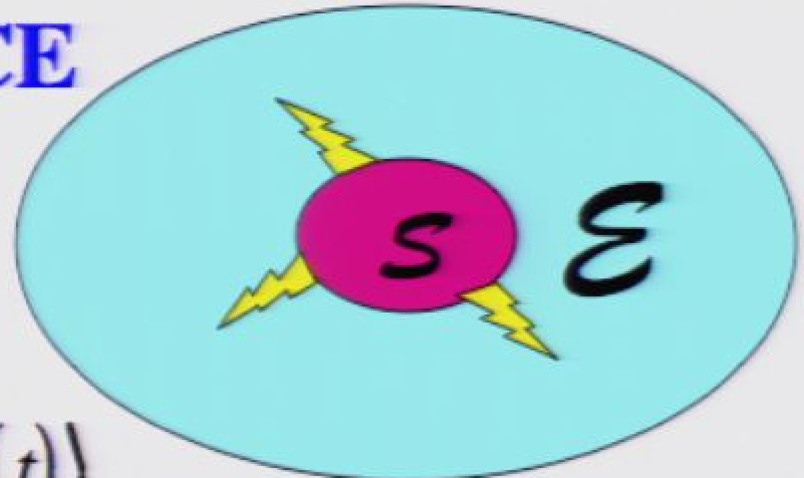

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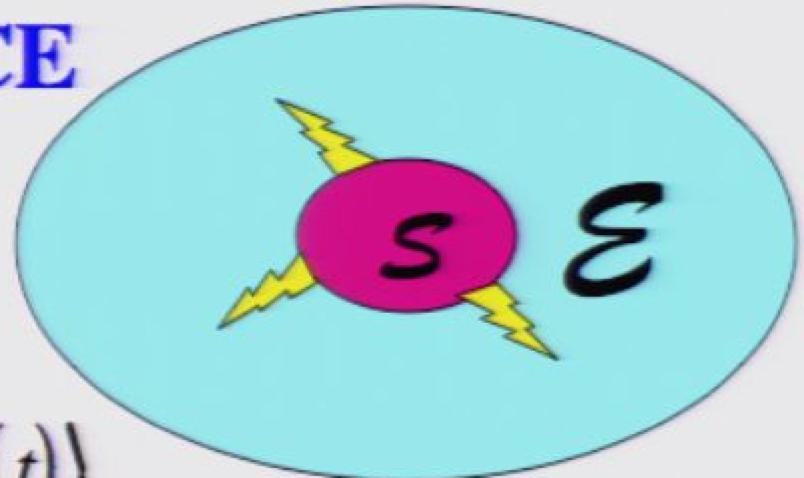

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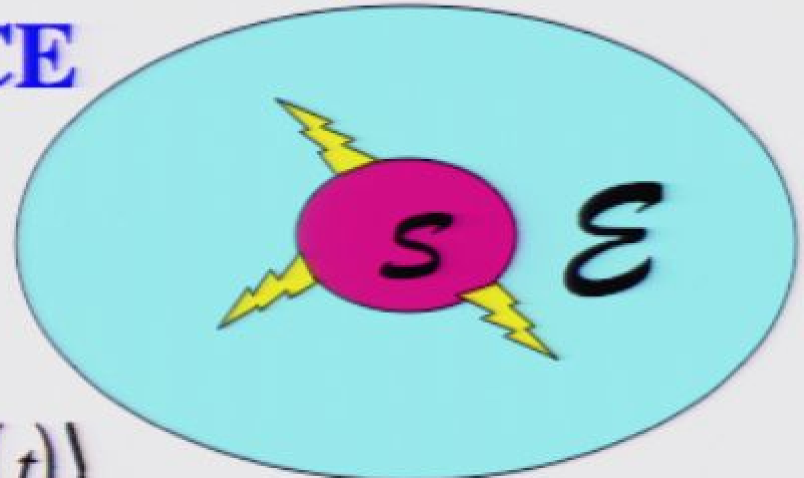

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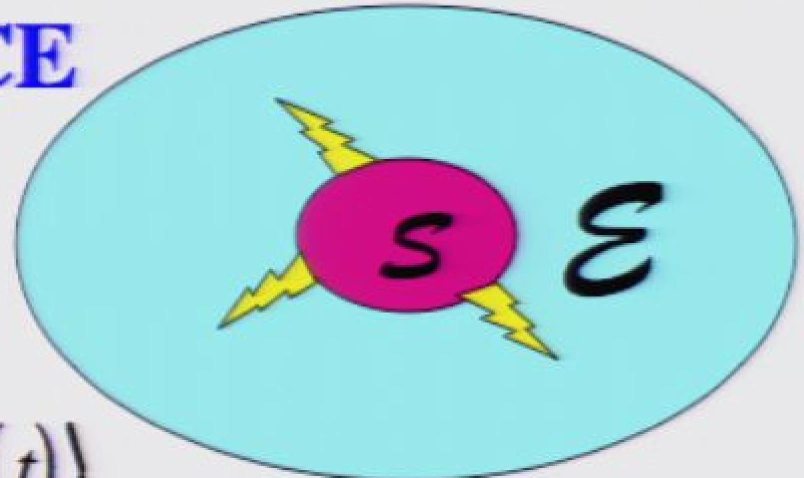

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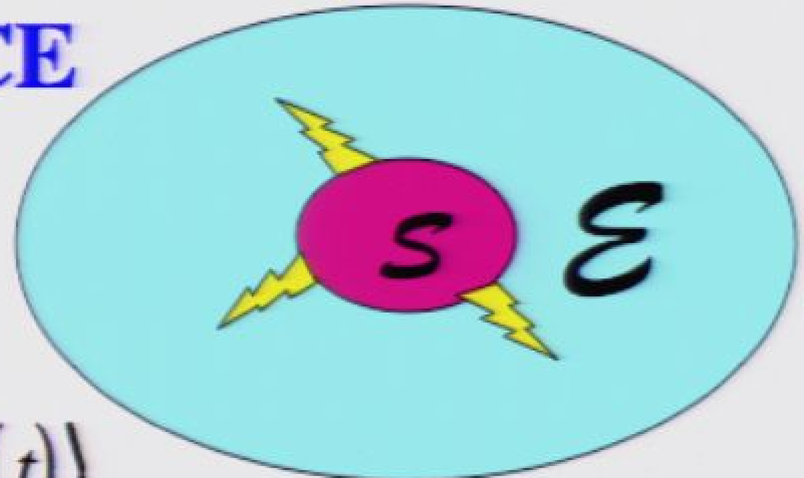

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Justify axioms 4&5 using the noncontroversial 0-3.

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- **Why are the measurement outcomes limited to an orthogonal subset of all the possible states in the Hilbert states? (as in "Collapse")**

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- How can "objective classical reality" -- states we can find out -- arise from the fragile quantum states that are perturbed by measurements? ("Quantum Darwinism")

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States that can survive "being found out" intact must be orthogonal.

Consider two states that can be "found out":

$$|u\rangle|A_0\rangle \Rightarrow |u\rangle|A_u\rangle$$

$$|v\rangle|A_0\rangle \Rightarrow |v\rangle|A_v\rangle$$

Consider an initial **superposition** of these two states:

$$\left(\alpha|u\rangle + \beta|v\rangle\right)|A_0\rangle \Rightarrow \alpha|u\rangle|A_u\rangle + \beta|v\rangle|A_v\rangle$$

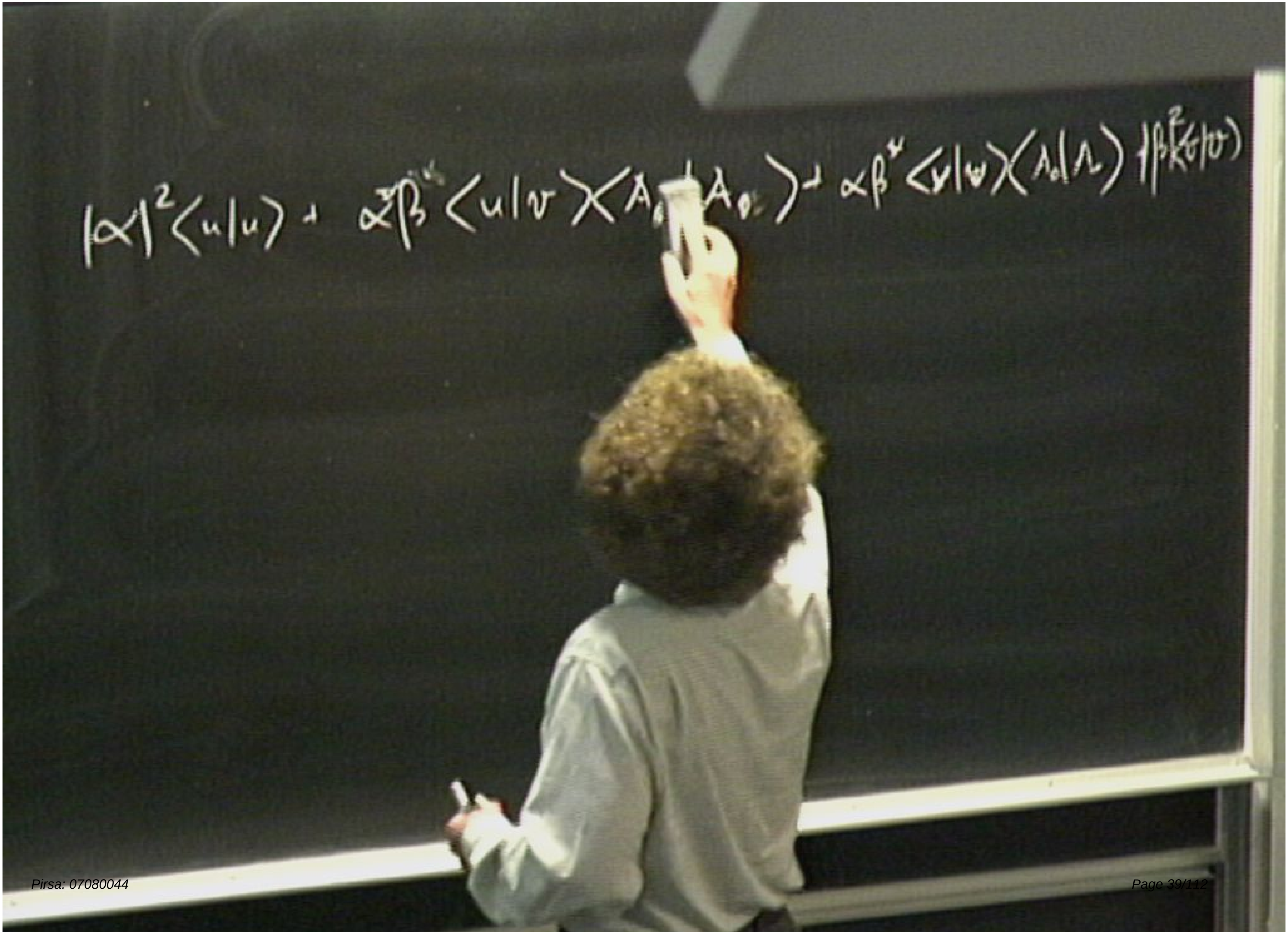
Norm must be preserved. Hence: $\mathrm{Re}(\alpha^*\beta\langle u|v\rangle) = \mathrm{Re}(\alpha^*\beta\langle u|v\rangle\langle A_u|A_v\rangle)$

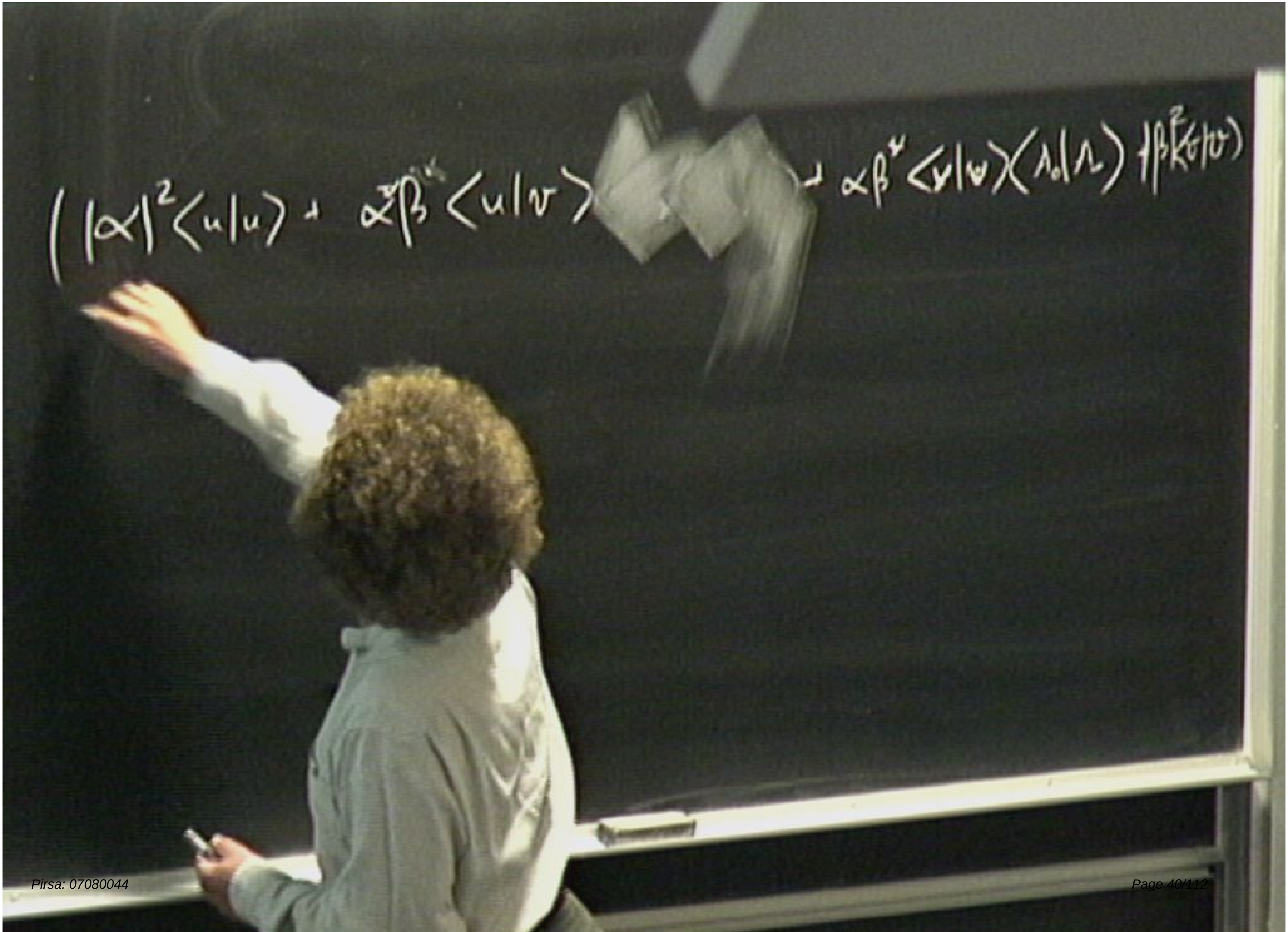
Phases of the coefficients can be adjusted at will. So:

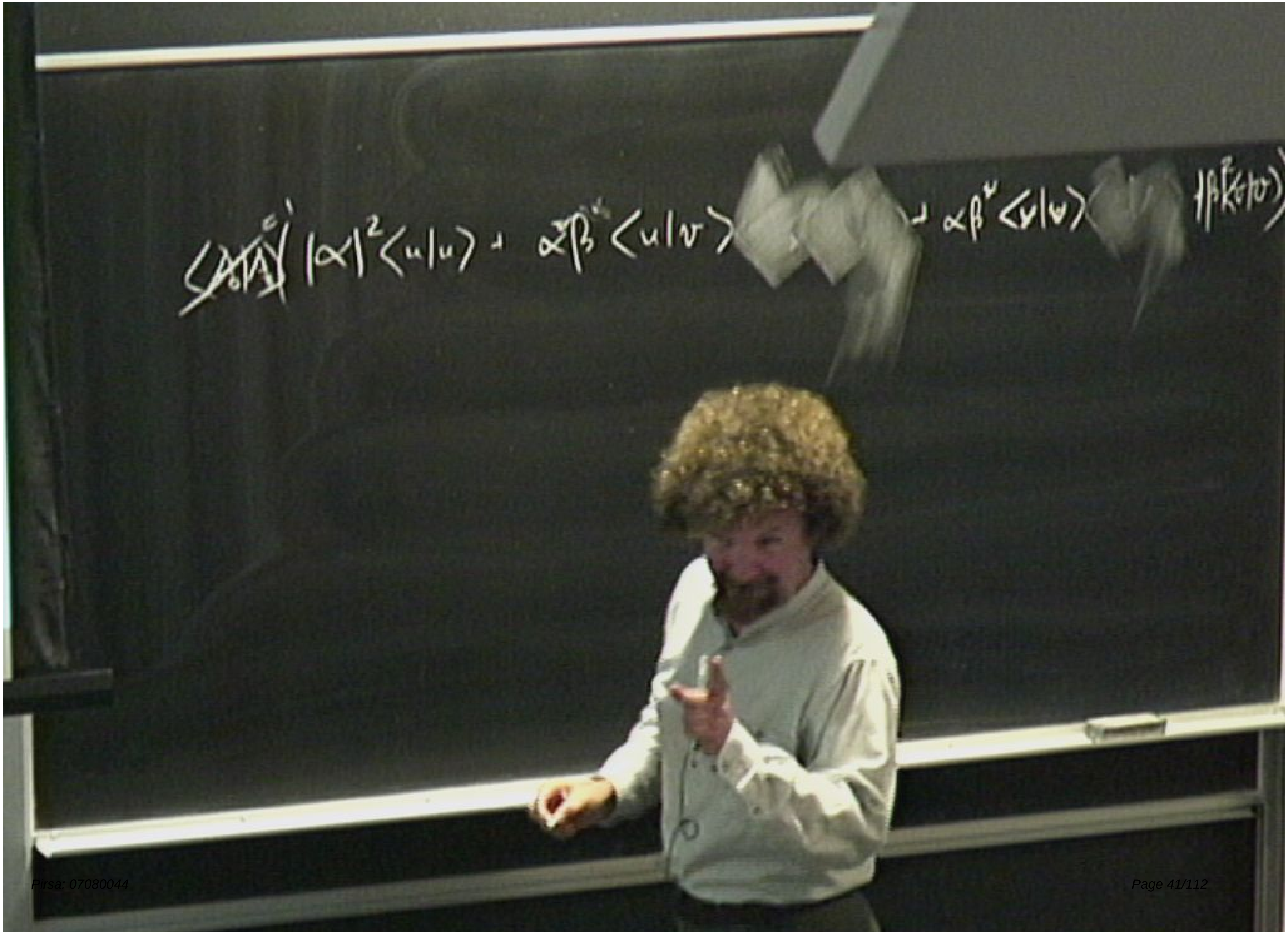
$$\langle u|v\rangle = \langle u|v\rangle\langle A_u|A_v\rangle$$

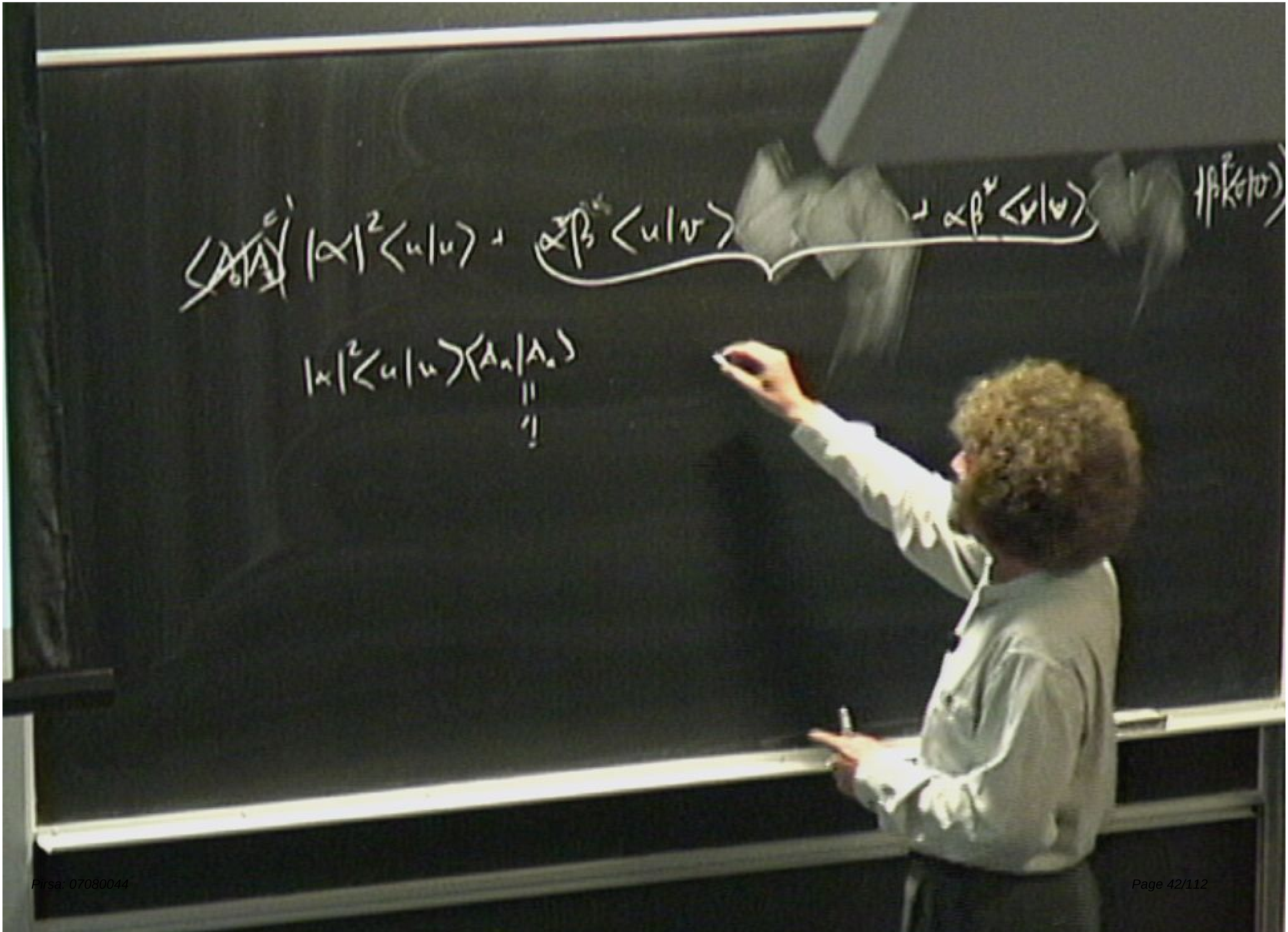
So either $\langle A_u|A_v\rangle = 1$ (measurement was not successful)

or $\langle u|v\rangle = 0$ **QED!!!!**


$|\alpha|^2 \langle u|u\rangle + \alpha^*\beta \langle u|v\rangle\langle A_\alpha|A_\beta\rangle + \alpha\beta^* \langle v|u\rangle\langle A_\beta|A_\alpha\rangle + |\beta|^2\langle v|v\rangle$


$\left(|\alpha|^2 \langle u|u\rangle + \alpha^*\beta \langle u|v\rangle\right.$
$\left. + \alpha\beta^* \langle v|u\rangle\langle \lambda|\lambda\rangle + |\beta|^2\langle v|v\rangle \right)$


|α|² ⟨u|u⟩ + α*β ⟨u|v⟩


|α|²⟨u|u⟩⟨A_n|A_n⟩
1

$$\cancel{\langle\psi_A|\psi_A\rangle}\; |\alpha|^2\langle u|u\rangle + \underbrace{\alpha^*\beta\langle u|v\rangle + \alpha\beta^*\langle v|u\rangle} + |\beta|^2\langle v|v\rangle$$

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Consider two states that can be "found out":

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Information transfer need not be due to a deliberate measurement: any information transfer that does not perturb outcome states will have to abide by this rule: Pointer states, predictability sieve, and DECOHERENCE.

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Theorem: Outcomes of a measurement that satisfy postulates 1-3 must be orthogonal.

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"information gain implies disturbance"

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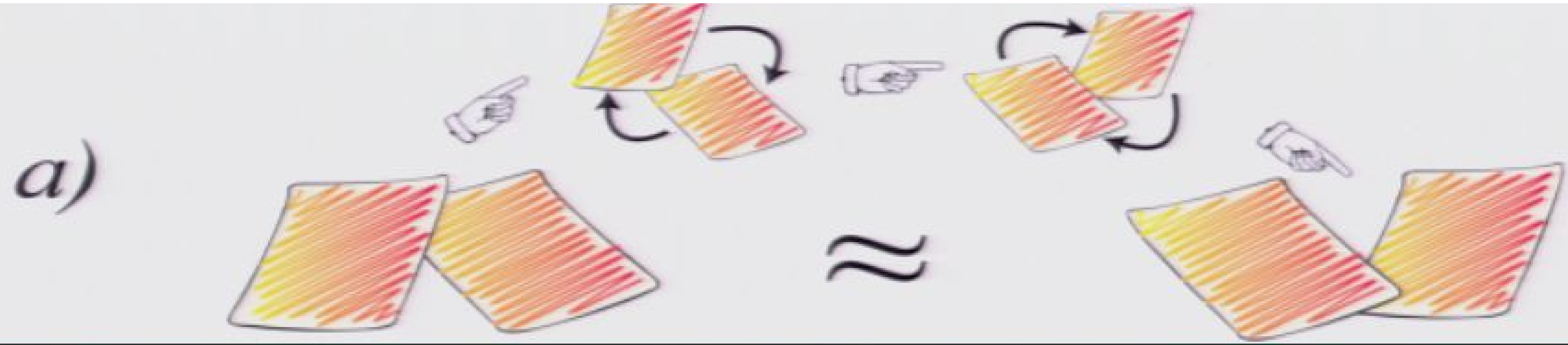

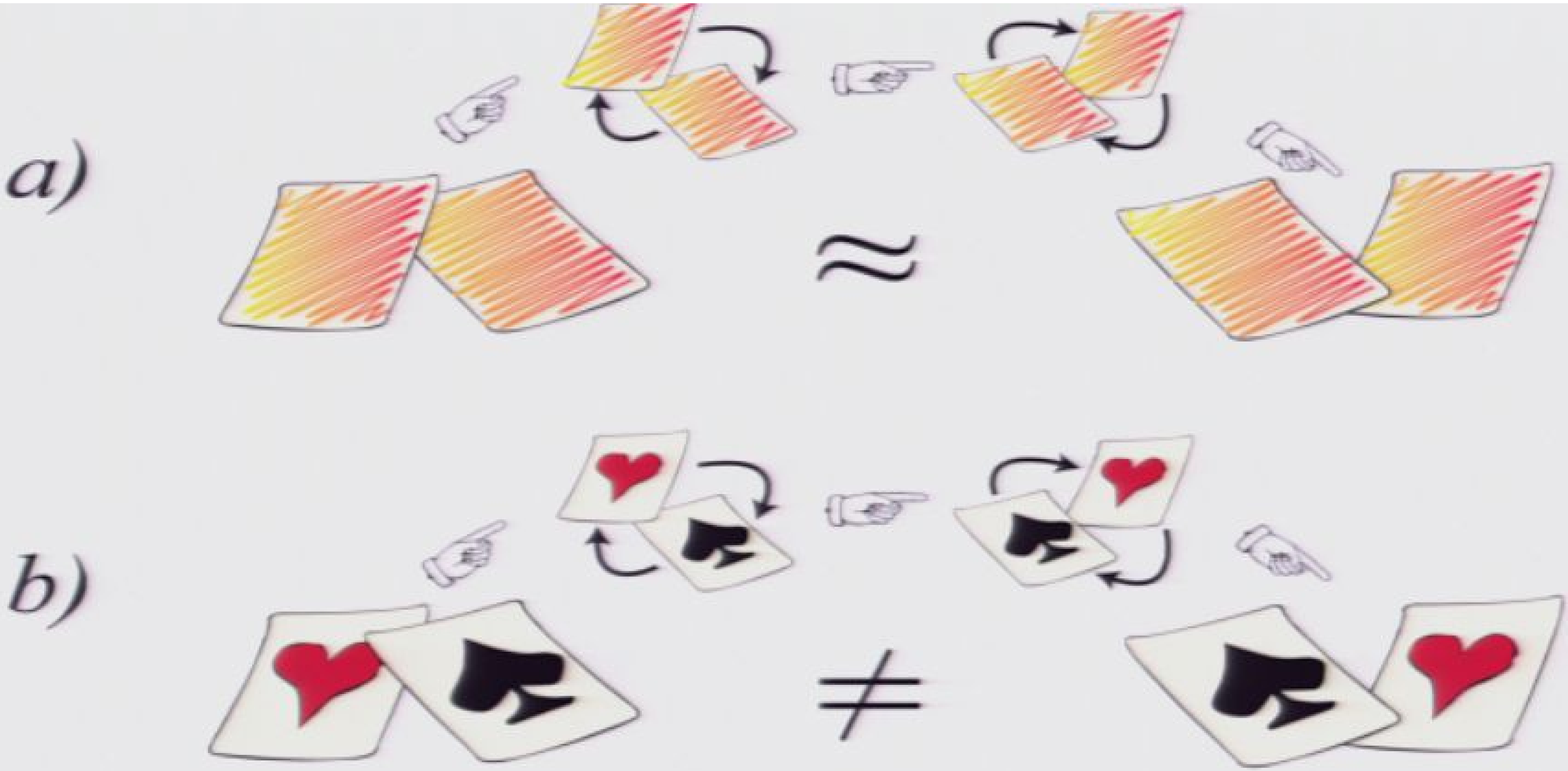
a)
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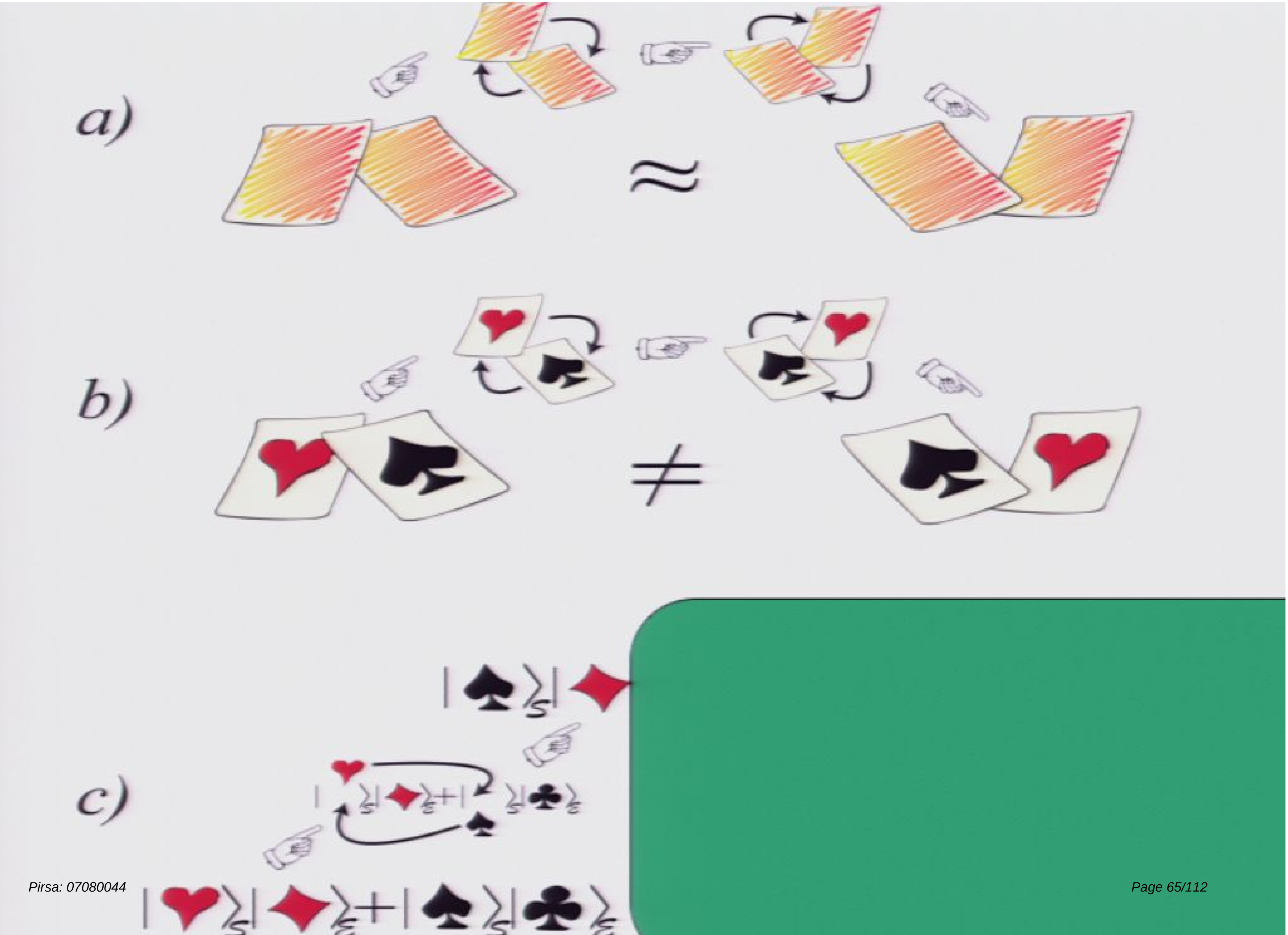
a)
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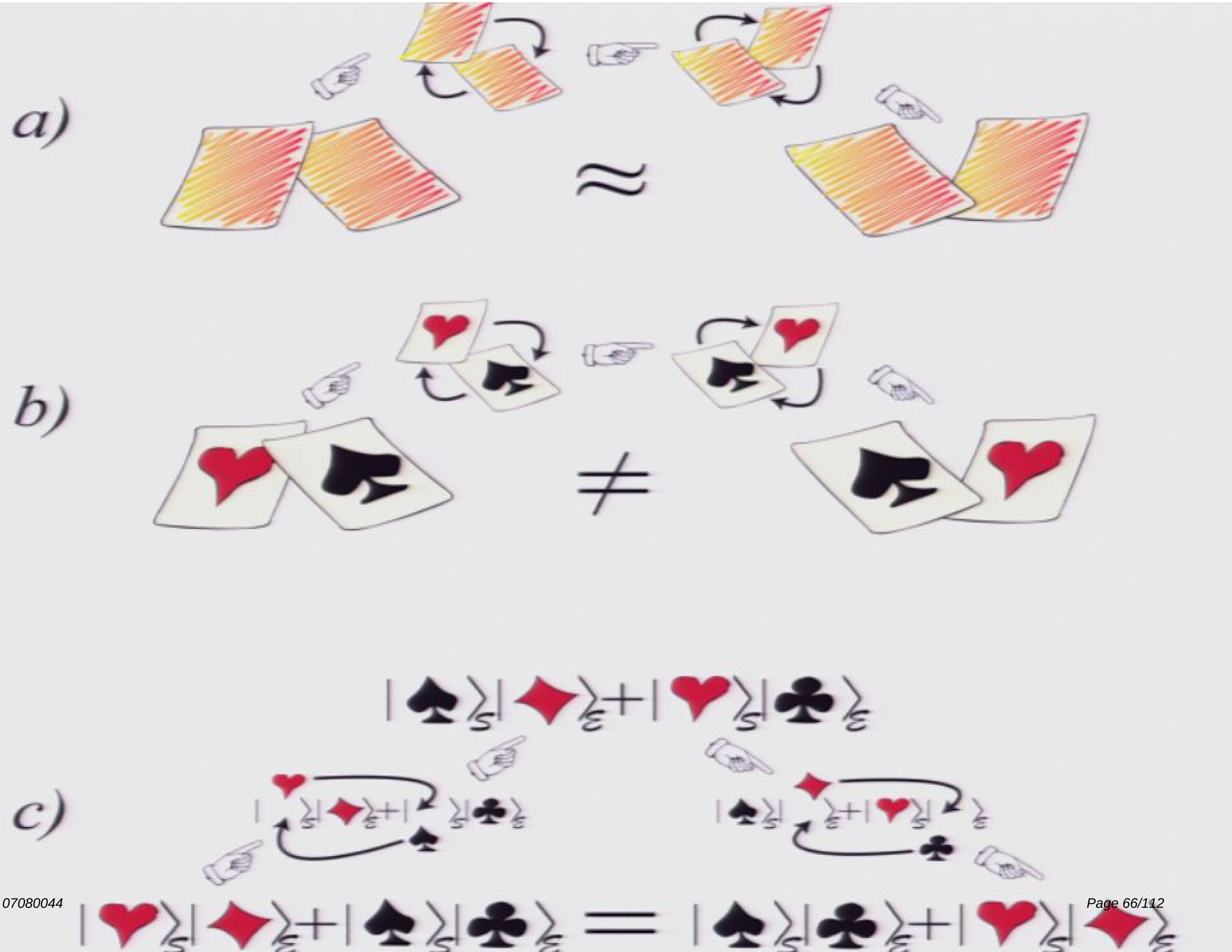
c)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ENVARIANCE

(Entanglement-Assisted Invariance)

DEFINITION:

Consider a composite quantum object consisting of system $\mathcal{S}$ and environment $\mathcal{E}$. **When the combined state $\psi_{\mathcal{SE}}$ is transformed by:**

$$U_S = u_S \otimes \mathbf{1}_E$$

but can be "untransformed" by acting solely on $\mathcal{E}$, that is, if there exists:

$$U_{\mathcal{E}} = \mathbf{1}_{\mathcal{S}} \otimes u_{\mathcal{E}}$$

then $\psi_{\mathcal{SE}}$ is ENVARIANT with respect to $u_{\mathcal{S}}$.

$$U_{\mathcal{E}}(U_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) = U_{\mathcal{E}}|\varphi_{\mathcal{SE}}\rangle = |\psi_{\mathcal{SE}}\rangle$$

Envariance is a property of U_S and the joint state $\psi_{\mathcal{SE}}$ of two systems, $\mathcal{S}$ & $\mathcal{E}$.

ENTANGLED STATE AS AN EXAMPLE OF ENVARIANCE:

Schmidt decomposition:

$$|\psi_{S\mathcal{E}}\rangle = \sum_{k=1}^{N} \alpha_k |s_k\rangle |\varepsilon_k\rangle$$

Above Schmidt states $|s_k\rangle$, $|\varepsilon_k\rangle$ are orthonormal and α_k complex.

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$$u_S(s_k) = \sum_{k=1} \exp(i\phi_k) |s_k\rangle\langle s_k|$$

leave $\psi_{S\mathcal{E}}$ envariant.

'roof: $u_S(s_k)|\psi_{S\mathcal{E}}\rangle = \sum_{k=1} \alpha_k \exp(i\phi_k)|s_k\rangle|\varepsilon_k\rangle$. $u_{\mathcal{E}}(\varepsilon_k) = \sum_{k=1} \exp\{i(-\phi_k + 2\pi\, l_k)\}|\varepsilon_k\rangle\langle\varepsilon_k|$

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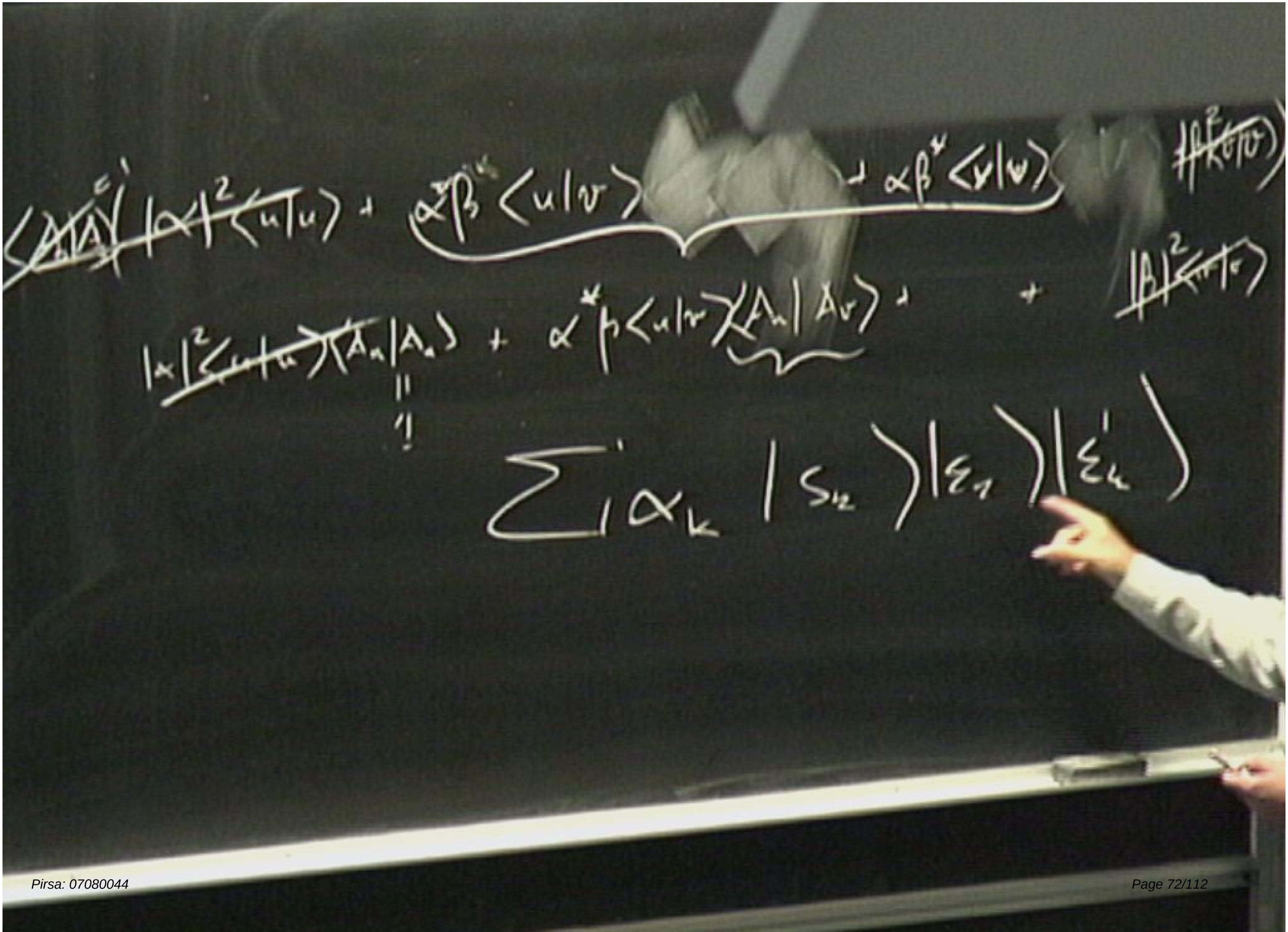
LOCALLY, SCHMIDT PHASES DO NOT MATTER: DECOHERENCE!!!

ENVARIANCE -- SOME PROPERTIES

$$U_{\mathcal{E}}(U_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) = U_{\mathcal{E}}|\varphi_{\mathcal{SE}}\rangle = \boxed{\exp(i\phi)}|\psi_{\mathcal{SE}}\rangle$$

- Envariant $\psi_{\mathcal{SE}}$ is an eigenstate of two unitary transformations with a unit (or unimodular) eigenvalue.
- Envariance can be defined for density matrices of $\mathcal{SE}$, but this will not be necessary, as one can instead purify the state of $\mathcal{SE}$ in the usual way, by introducing $\mathcal{E}'$, so the density matrix of $\mathcal{SE}$ is given by: $\rho_{\mathcal{SE}} = Tr_{\mathcal{E}'}|\Psi_{\mathcal{SEE}'}\rangle\langle\Psi_{\mathcal{SEE}'}|$
- A product of envariant transformations of $\psi_{\mathcal{SE}}$ is an envariant transformation of $\psi_{\mathcal{SE}}$
- All envariant transformations have Schmidt eigenstates.
- There may be many environments that undo an effect of the same unitary transformation on the system

For additional discussion, see WHZ, quant-ph/0211037, PRL, 90, 120404 (2003); also *Decoherence, einselection, and the quantum origin of the classical* RMP, 75, 715 (2003); and especially *Probabilities from entanglement,* ... quant-ph/0405161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$= 1$
$\sum \alpha_k |s_k\rangle|\varepsilon_1\rangle|\varepsilon'_k\rangle$

PHASE ENVARIANCE THEOREM

Fact 1: Unitary transformations must act on the system to alter its state (if they act only somewhere else, system is not effected).

Fact 2: The state of the system is all that is necessary/available to predict measurement outcomes (including their probabilities).

Fact 3: A state of the composite system is all that is needed/available to determine the state of the system.

Moreover, "entanglement happens":

$$|\psi_{\mathcal{SE}}\rangle \propto \sum_{k=1}^{N} \alpha_k |s_k\rangle |\varepsilon_k\rangle$$

HEOREM 1: State (and probabilities) of $\mathcal{S}$ alone can depend only on he absolute values of Schmidt coefficients $|\alpha_k|$, and **not on their phases.**

Proof: Phases of α_k can be changed by acting on $\mathcal{S}$ alone. But the state of the whole can be restored by acting only on $\mathcal{E}$. So change of phases of Schmidt coefficients could not have affected $\mathcal{S}$! QED.

$\therefore$ By phase envariance, $\{|\alpha_k|, |s_k\rangle\}$ must provide a complete local description of the system alone.

Same info as reduced density matrix!!!

Envariance of entangled states: the case of **equal coefficients**

$$|\psi_{\mathcal{SE}}\rangle \propto \sum_{k=1}^{N} \exp(i\phi_k)|s_k\rangle|\varepsilon_k\rangle$$

In this case ANY orthonormal basis is Schmidt. In particular, in the Hilbert subspace spanned by any two $\{|s_k\rangle, |s_l\rangle\}$ one can define a Hadamard basis;

$$|\pm\rangle = (|s_k\rangle \pm |s_l\rangle)/\sqrt{2}$$

This can be used to generate 'new kind' of envariant transformations:

A SWAP: $u_{\mathcal{S}}(k \leftrightarrow l) = \exp(i\varphi_{kl})|s_k\rangle\langle s_l| + h.c.$

Can be 'undone' by the COUNTERSWAP:

$$u_{\mathcal{E}}(k \leftrightarrow l) = \exp\{i(-\varphi_{kl} - \varphi_k + \varphi_l)\}|\varepsilon_l\rangle\langle\varepsilon_k| + h.c.$$

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Probabilities of Schmidt partners are the same (detecting 0 in S implies 0 in E, etc.).

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Probabilities can “stay the same” and also “get exchanged” only when they are equal!!! (p(0)=p(1))

(Schlosshauer & Fine, Barnum, WHZ)

Probability of envariantly swappable states

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By the Phase Envariance Theorem the set of pairs $|\alpha_k|$, $|s_k\rangle$ provides a complete description of $\mathcal{S}$. But all $|\alpha_k|$ are equal.

Vith additional assumption about probabilities, can prove

HEOREM 2: Probabilities of envariantly swappable states are equal.

a) "Pedantic assumption"; when states get swapped, so do probabilitites;

b) When the state of the system does not change under any unitary in part of its Hilbert space, probabilities of any set of basis states are equal

c) Because there is one-to-one correlation between $|s_k\rangle, |\varepsilon_k\rangle$

Therefore, by normalization: $$p_k = \frac{1}{N} \quad \forall_k$$

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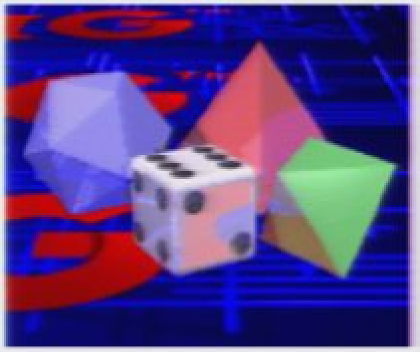
Therefore, by normalization: $p_k = \dfrac{1}{N} \quad \forall_k$



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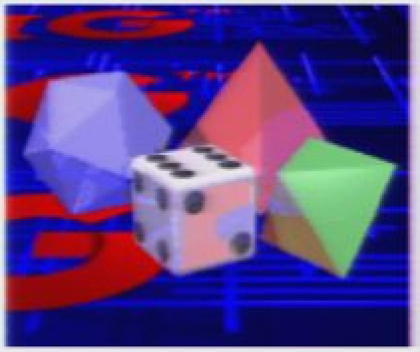

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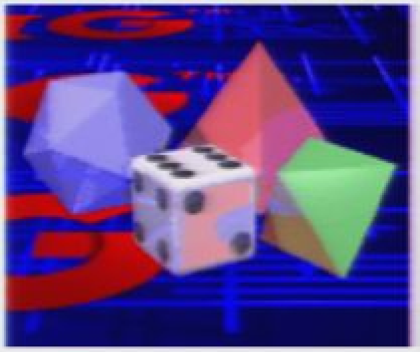

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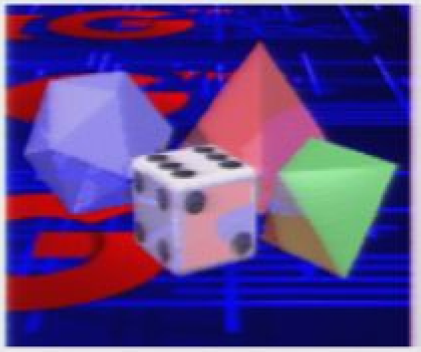

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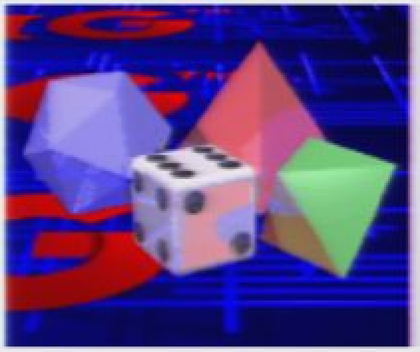
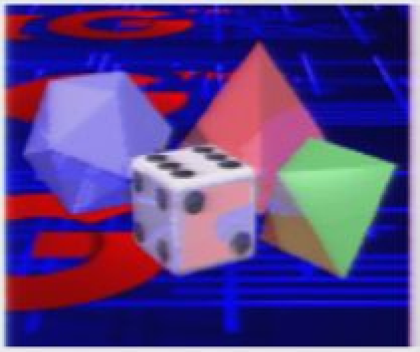


Symmetries can reflect ignorance

Probabilities from envariance



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(Environment-assisted iNVARIANCE)

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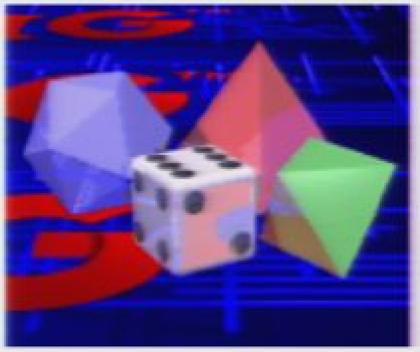

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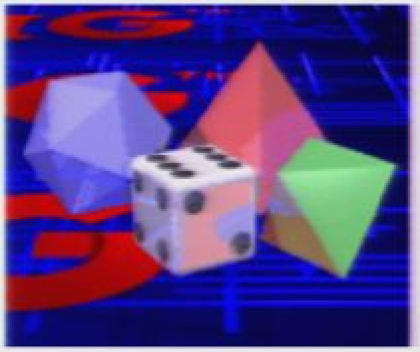

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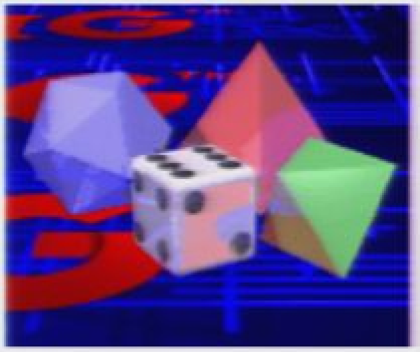

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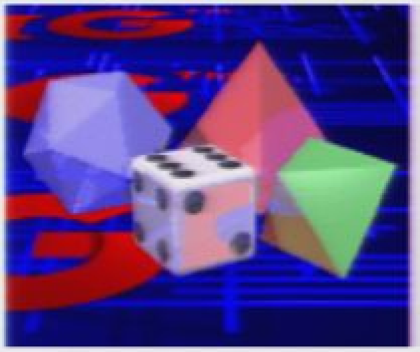

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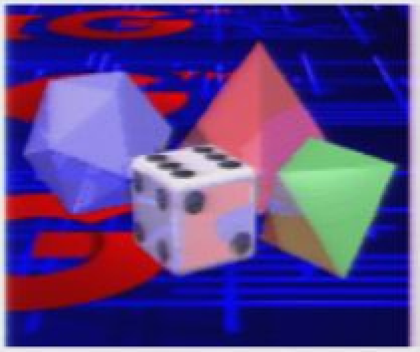

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Special case with unequal coefficients

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he environment $\mathcal{E}$ has three states $\{|0\rangle, |1\rangle, |2\rangle\}$ and $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$

$$\left|\psi_{\mathcal{SE}}\right\rangle = \sqrt{\frac{2}{3}}\left|0\right\rangle\left|+\right\rangle + \sqrt{\frac{1}{3}}\left|2\right\rangle\left|2\right\rangle$$

An auxilliary environment $\mathcal{E}'$ interacts with $\mathcal{E}$ so that:

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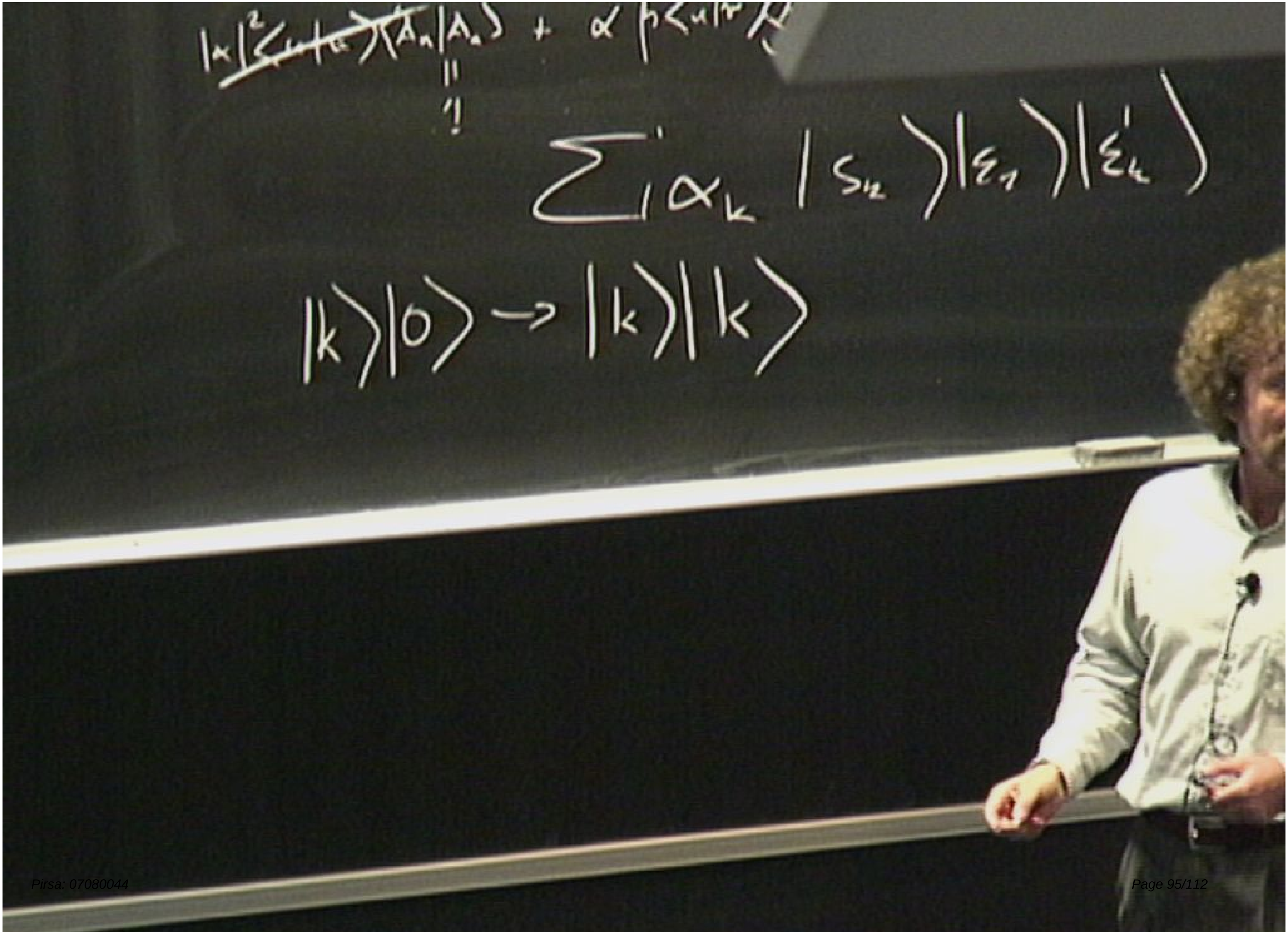
States $|0\rangle|0\rangle$, $|0\rangle|1\rangle$, $|2\rangle|2\rangle$ have equal coefficients. Therefore, Each of them has probability of 1/3. Consequently:

p(0) = p(0,0)+p(0,1) = 2/3, and p(2) = 1/3.

...... BORN's RULE!!!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no need to assume additivity! (p(0)=1-p(2))!

Probabilities from Envariance

The case of commensurate probabilities: $\left|\psi_{\mathcal{SE}}\right\rangle = \sum_{k=1}^{N} \underbrace{\sqrt{m_k / M}}_{\alpha_k} \left|s_k\right\rangle \left|\varepsilon_k\right\rangle$

Attach the auxiliary "counter" environment $\mathcal{C}$:

$$\left|\psi_{\mathrm{SE}}\right\rangle \left|e'_0\right\rangle = \left(\sum_{k=1}^{N} \underbrace{\sqrt{m_k / M}}_{\alpha_k} \left|s_k\right\rangle \left(\underbrace{\sum_{j_k=1}^{m_k} \frac{1}{\sqrt{m_k}} \left|e_{j_k}\right\rangle}_{\left|\varepsilon_k\right\rangle} \right) \right) \left|c_0\right\rangle \Rightarrow$$

$$\Rightarrow \frac{1}{\sqrt{M}} \sum_{j=1}^{M} \boxed{\left|s_{k(j)}\right\rangle \left|e_j\right\rangle \left|c_j\right\rangle}$$

THEOREM 3: The case with commensurate probabilities can be reduced to the case with equal probabilities. **BORN's RULE follows:**

$$p_j = \frac{1}{M}, \qquad \boxed{p_k =} \sum_{j_k=1}^{m_k} p_{j_k} = \frac{m_k}{M} = \boxed{\left|\alpha_k\right|^2}$$

General case -- by continuity. QED

Why the proof works

- Need to know how to relate quantum states and "events". ("Symmetry breaking" induced by information transfer.)
- Need to prove that phases of the coefficients do not matter (otherwise swapping alters state even when absolute values of coeff's equal). ("Decoherence without decoherence")

ENVARIANCE* -- SUMMARY

1. New symmetry - ENVARIANCE - of joint states of quantum systems. It is related to causality.

2. In quantum physics perfect knowledge of the whole may imply complete ignorance of a part.

3. **BORN's RULE** follows as a consequence of envariance.

4. Relative frequency interpretation of probabilities naturally follows.

5. Envariance supplies a new foundation for environment - induced superselection, decoherence, quantum statistical physics, etc., by justifying the form and interpretation of reduced density matrices.

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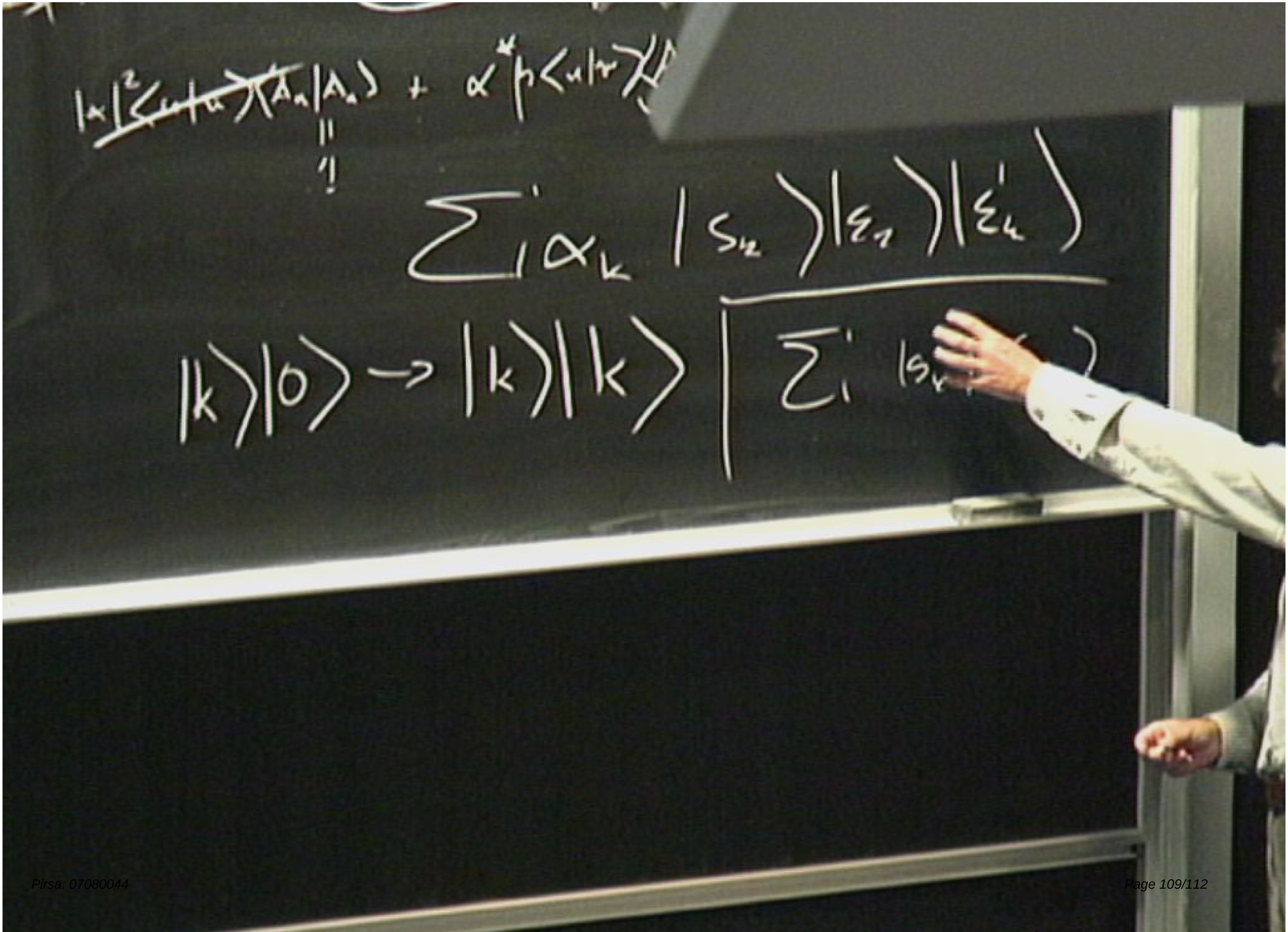
$|\alpha|^2 \langle u|u\rangle \langle A_n|A_n\rangle + \alpha^* \beta \langle u|v\rangle$
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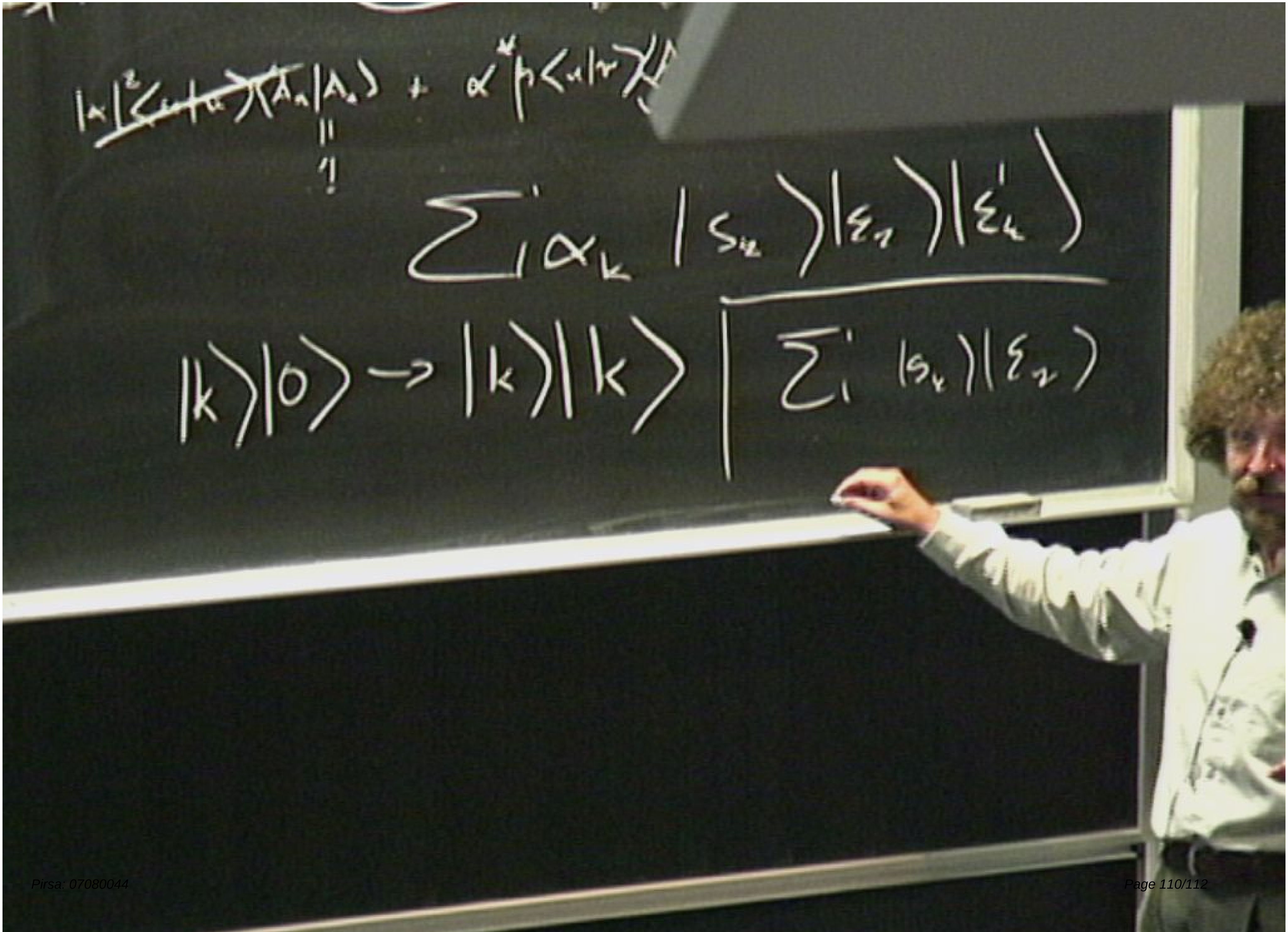
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PHASE ENVARIANCE THEOREM

Fact 1: Unitary transformations must act on the system to alter its state (if they act only somewhere else, system is not effected).

Fact 2: The state of the system is all that is necessary/available to predict measurement outcomes (including their probabilities).

Fact 3: A state of the composite system is all that is needed/available to determine the state of the system.

Moreover, "entanglement happens":

$$|\psi_{\mathcal{SE}}\rangle \propto \sum_{k=1}^{N} \alpha_k |s_k\rangle|\varepsilon_k\rangle$$

HEOREM 1: State (and probabilities) of $\mathcal{S}$ alone can depend only on ne absolute values of Schmidt coefficients $|\alpha_k|$, and **not on their phases.**

Proof: Phases of α_k can be changed by acting on $\mathcal{S}$ alone. But the state of the whole can be restored by acting only on $\mathcal{E}$. So change of phases of Schmidt coefficients could not have affected $\mathcal{S}$! QED.

$\therefore$ By phase envariance, $\{|\alpha_k|, |s_k\rangle\}$ must provide a complete local description of the system alone.

Same info as reduced density matrix!!!