Title: Quantum States as Uncertainty, pure and simple. But, Uncertainty about What? (Part 2)

Date: Aug 29, 2007 10:30 AM

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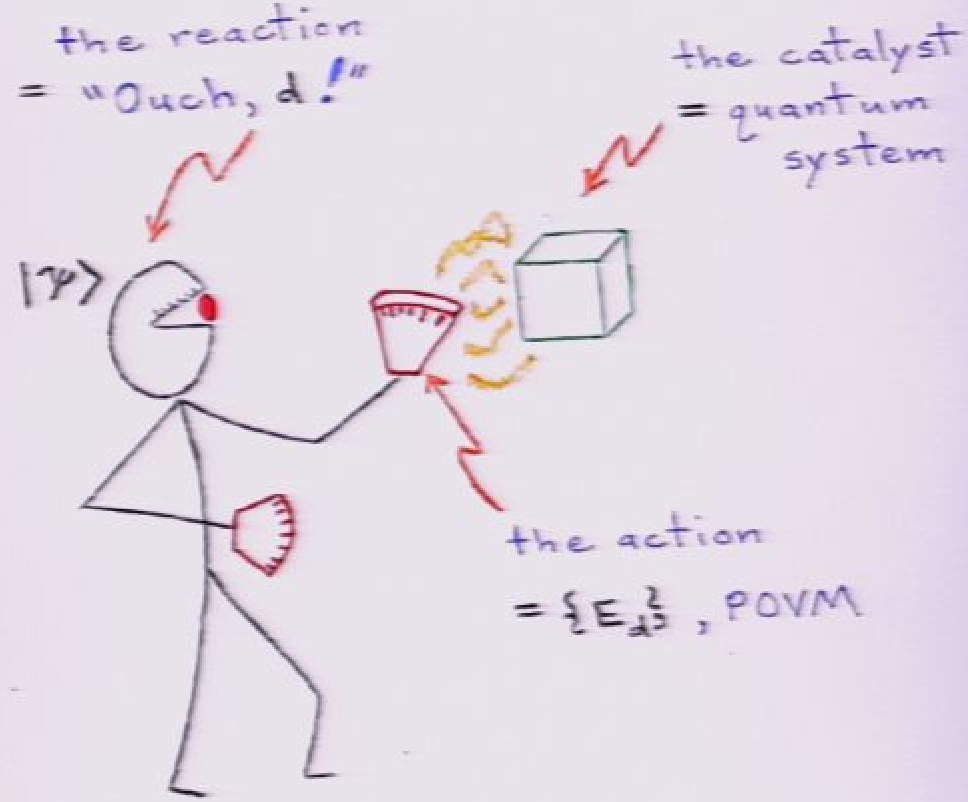
Abstract:

My thesis, paradoxically, and a little provocatively, but nonetheless genuinely, is simply this:

QUANTUM STATES DO NOT EXIST.

The abandonment of superstitious beliefs about the existence of phlogiston, the cosmic ether, absolute space and time, or fairies and witches, was an essential step along the road to scientific thinking. The quantum state, too, if regarded as something endowed with some kind of objective existence, is no less a misleading conception, an illusory attempt to exteriorize or materialize our true expectations.

— the ghost of Bruno de Finetti


the reaction
= "Ouch, d!"
the catalyst
= quantum
system
|ψ⟩
the action
= {E_d} , POVM

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A satisfactory statement about the actual (objective) characteristics of the quantum world should contain no $|\Psi\rangle$'s at all.

Really. <u>None</u>!

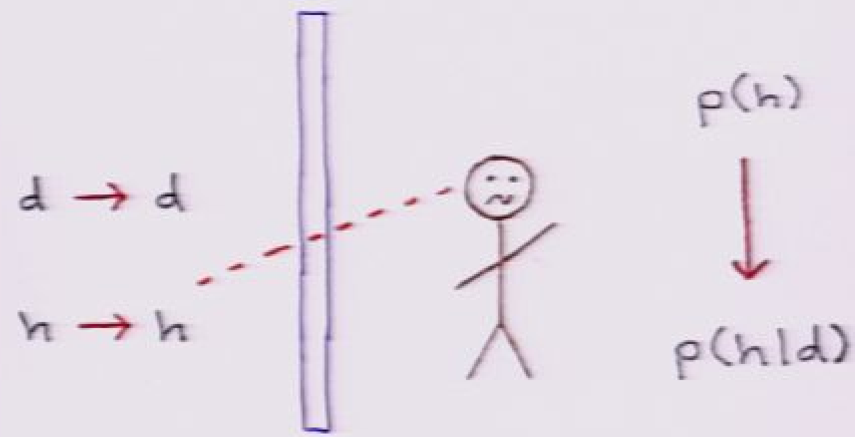
$$\rho \longleftrightarrow p(h)$$

$$\rho \longleftrightarrow p(h)$$

Jim Hartle 1968 (Section IV) Interpretation of Quantum Mechanics (suitably modified)

[redacted] tive property of the system. If the state of a system is defined as a list of [*experimental*] propositions together with [*their probabilities of occurrence*], it is not surprising that after a measurement the state must be changed to be in accord with the new information. The "reduction of the wave packet" does take place in the consciousness of the observer, not because of any unique physical process which takes place there, but only because the state is a construct of the observer and not an objective property of the physical system.

The Weatherman



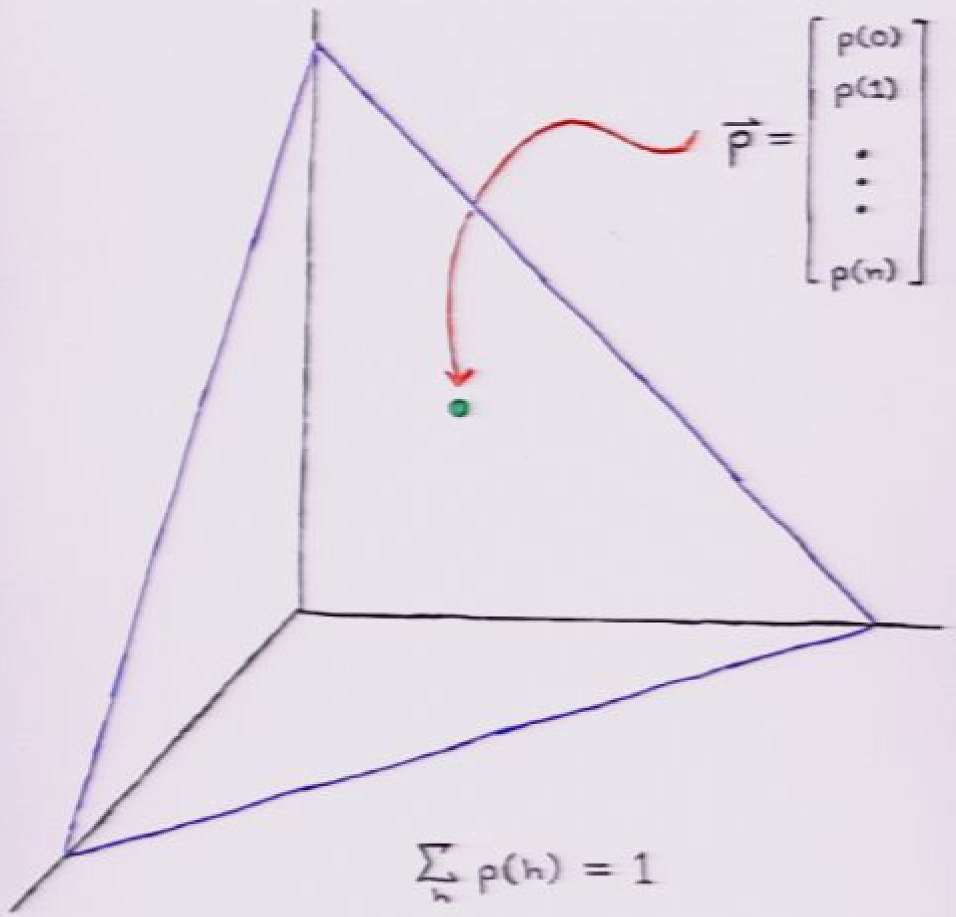
Bayesian Updating

$$p(h) = \sum_d p(h,d)$$

$$= \sum_d p(d)\, p(h|d)$$

$$p(h) \xrightarrow{d} p(h|d)$$

Probability Simplex

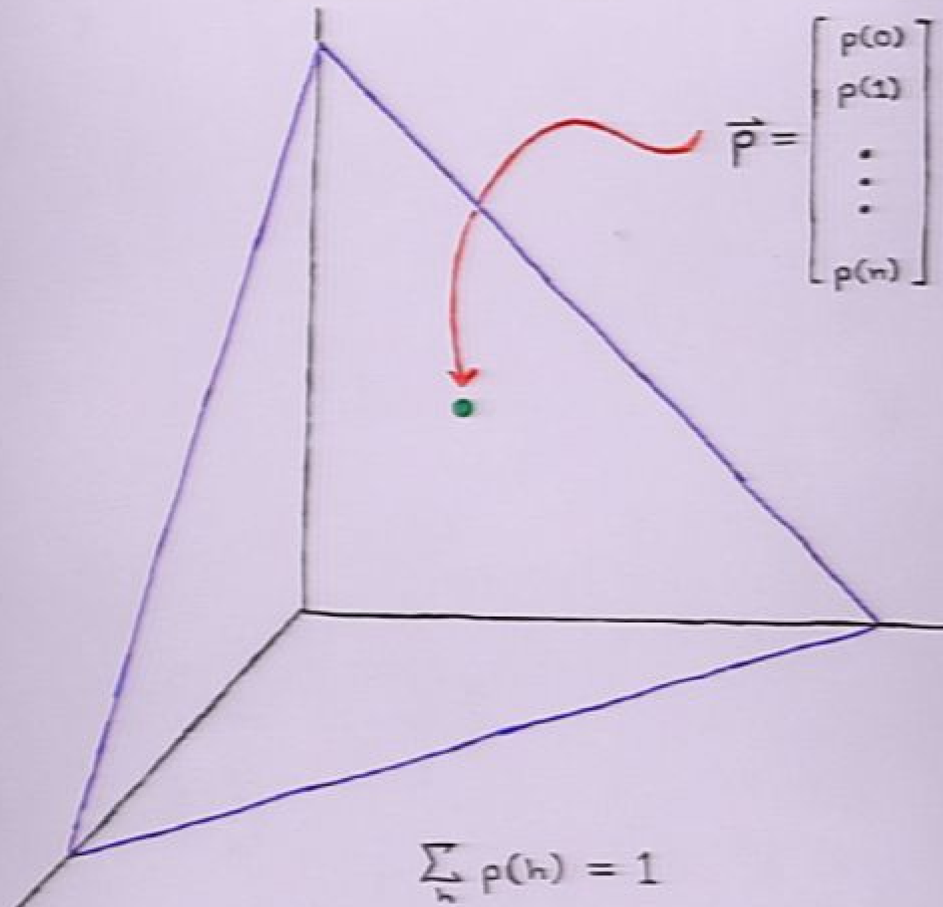


$$\vec{p} = \begin{bmatrix} p(0) \\ p(1) \\ \vdots \\ p(n) \end{bmatrix}$$

$$\sum_h p(h) = 1$$

$$p(h) \geq 0 \quad \forall h$$

Probability Simplex



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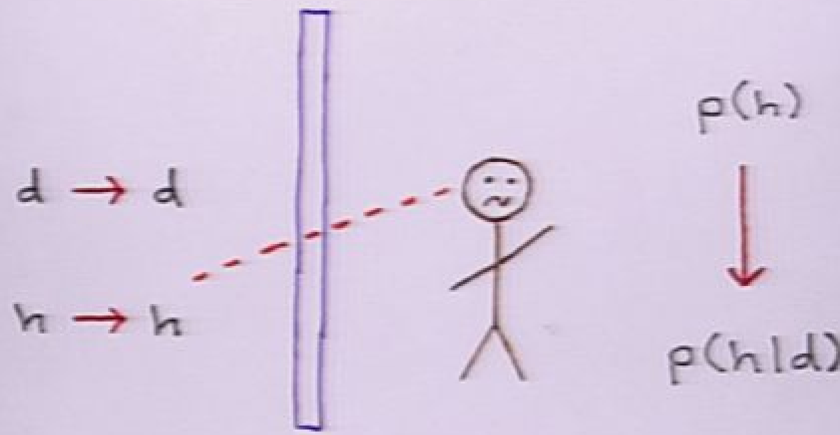

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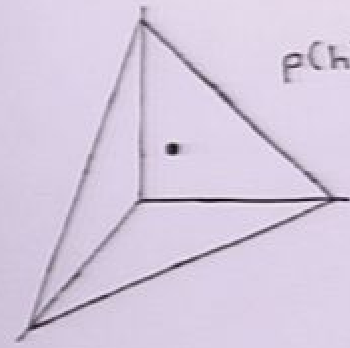
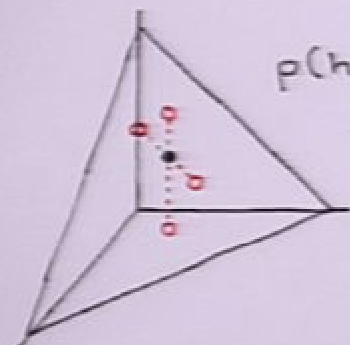


$p(h)$



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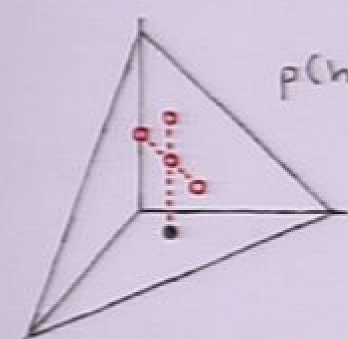

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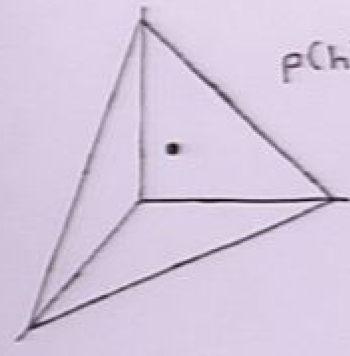

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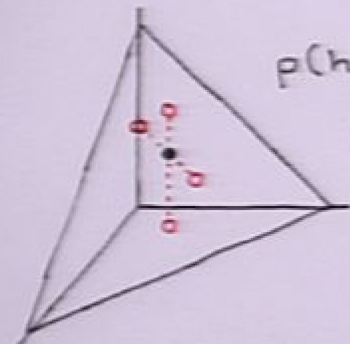

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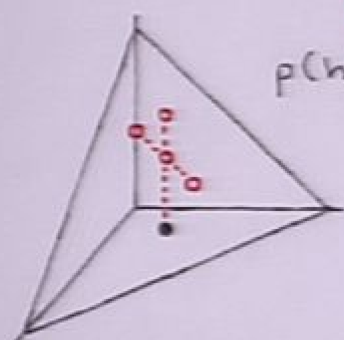
"measurement"
is any
I-know-not-what
that induces such
a transition.



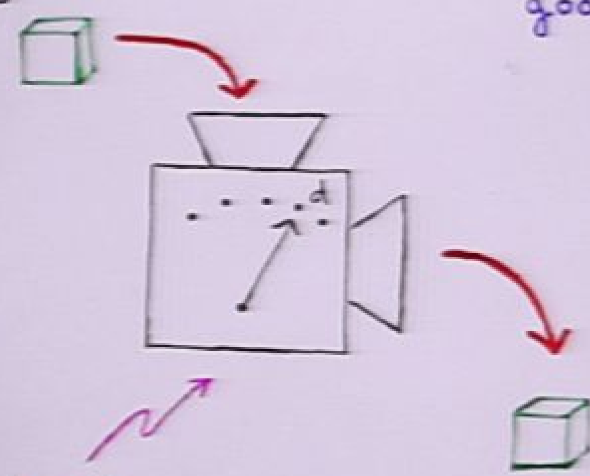
$p(h|d)$

Quantum Measurement

$\in \mathcal{L}(\mathcal{H})$

ρ

"Goodbye to the good old days."



$\{E_d\}$

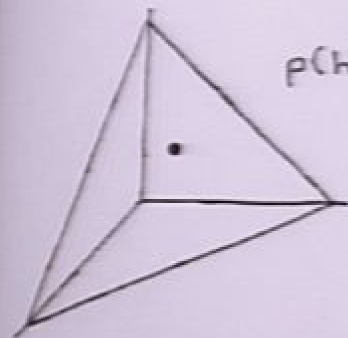
$\langle\psi|E_d|\psi\rangle \geq 0$

$\sum_d E_d = I$

ρ_d

$\rho_d = \Phi_d(\rho)$

Φ_d – renormalized trace-decreasing completely positive linear map



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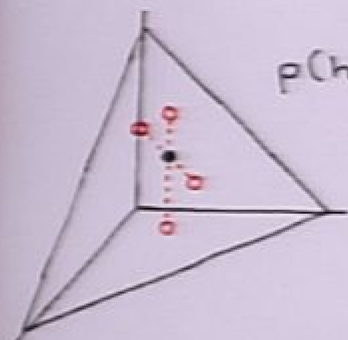

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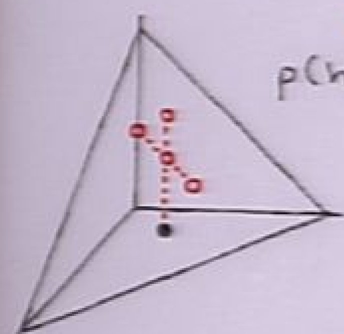

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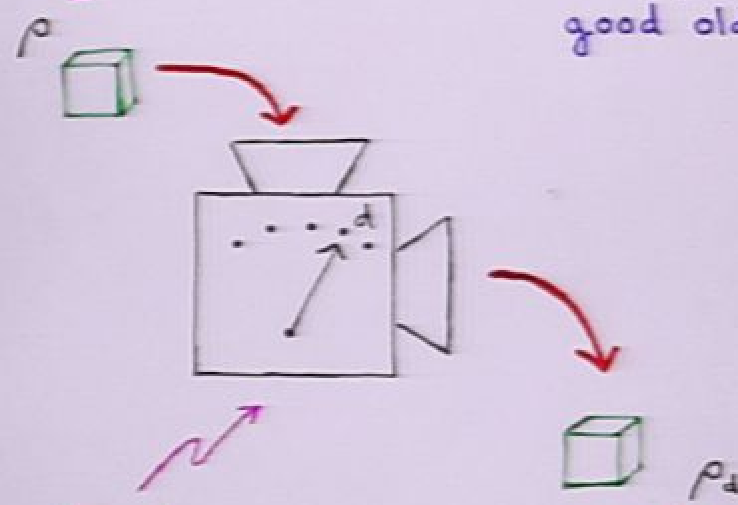

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von Neumann Measurements

"measurement"

$\Longleftrightarrow$ Hermitian operator

$$\mathcal{O} = \sum_i \alpha_i \Pi_i$$

eigenvalues

eigenprojectors

When state is ρ,

$$p(i) = \mathrm{tr}\, \rho \Pi_i \, .$$

Could say,

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Quantum Measurement

$\in \mathcal{L}(\mathcal{H})$

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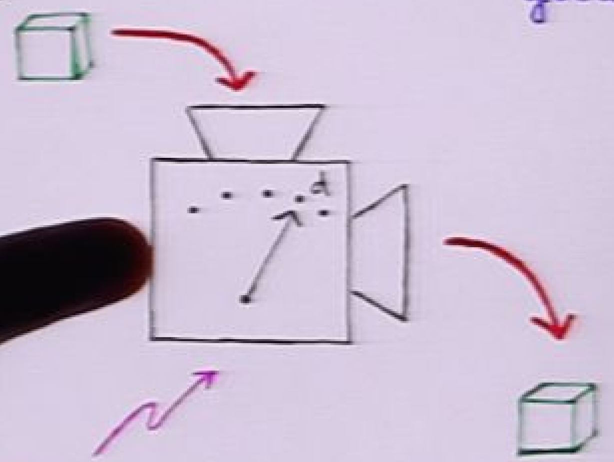
ρ



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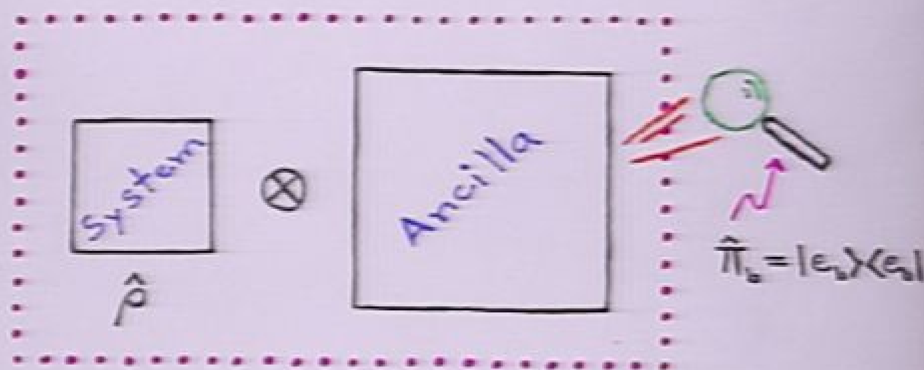
Standard Measurements	Generalized Measurements
$\{\Pi_i\}$	$\{E_b\}$
$\langle\psi\|\Pi_i\|\psi\rangle \geq 0,\ \forall\|\psi\rangle$	$\langle\psi\|E_b\|\psi\rangle \geq 0,\ \forall\|\psi\rangle$
$\sum_i \Pi_i = I$	$\sum_b E_b = I$
$p(i) = \mathrm{tr}\,\rho\Pi_i$	$p(b) = \mathrm{tr}\,\rho E_b$
$\Pi_i\Pi_j = \delta_{ij}\Pi_i$	—

Generalized Measurements

or **POVMs** — positive operator-valued measures



1) Couple system to be "measured" to some "ancilla".
2) Let the two interact unitarily.
3) Perform standard measurement on ancilla.

$p(b) = \mathrm{tr}\, \hat{\rho} \hat{E}_b$

$\hat{E}_b$ arbitrary otherwise

with $\langle\psi|\hat{E}_b|\psi\rangle \geq 0 \quad \forall\, |\psi\rangle$

and $\sum_b \hat{E}_b = \hat{I}$

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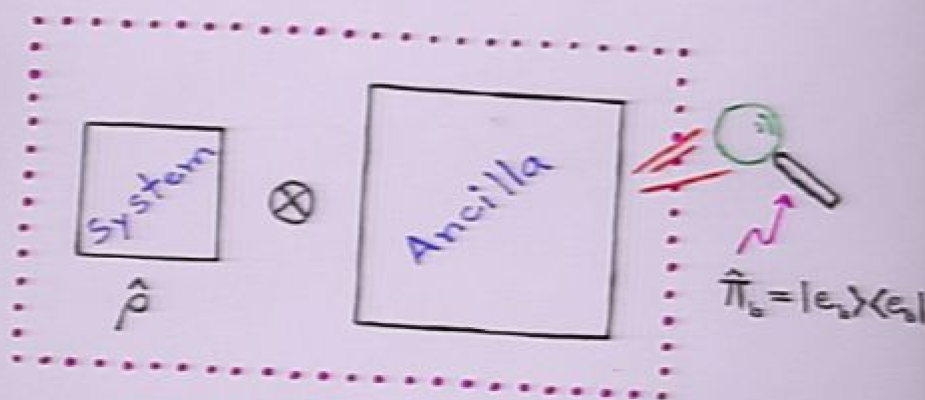

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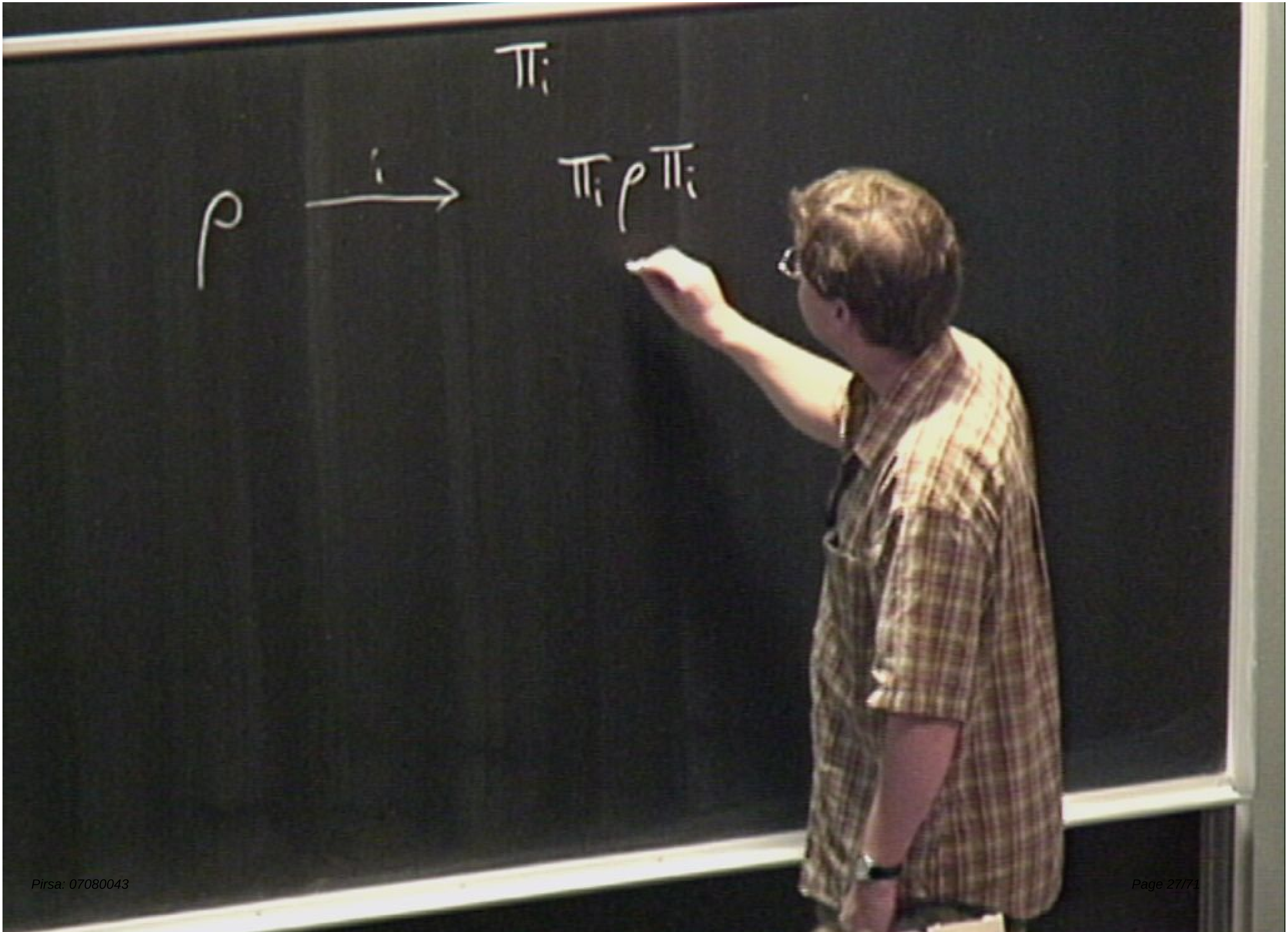
Π_i
$\rho \xrightarrow{i} \Pi_i \rho \Pi_i$

$$\Pi_i$$

$$\rho \xrightarrow{i} \rho_i = \frac{\Pi_i \rho \Pi_i}{\mathrm{tr}\,\rho \Pi_i}$$

State Change

When measure POVM $\{E_b\}$ efficiently

$$\rho \xrightarrow{b} \tilde{\rho}_b = \frac{1}{p_b} A_b \rho A_b^\dagger$$

where

$$p_b = \operatorname{tr} \rho E_b \quad \text{and} \quad A_b^\dagger A_b = E_b \,.$$

By polar decomposition theorem

$$\tilde{\rho}_b = \frac{1}{p_b} \underbrace{U_b}_{\text{"feedback"}} \underbrace{E_b^{1/2} \rho E_b^{1/2}}_{\text{"collapse"}} \underbrace{U_b^\dagger}_{\text{"feedback"}}$$

U_b depends upon detailed form of measurement interaction

$$\Pi_i$$

$$\rho \xrightarrow{\;i\;} \rho_i = \frac{\Pi_i \rho \Pi_i}{\mathrm{tr}\,\rho \Pi_i}$$

$$A = PU$$

Π_i

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"feedback"

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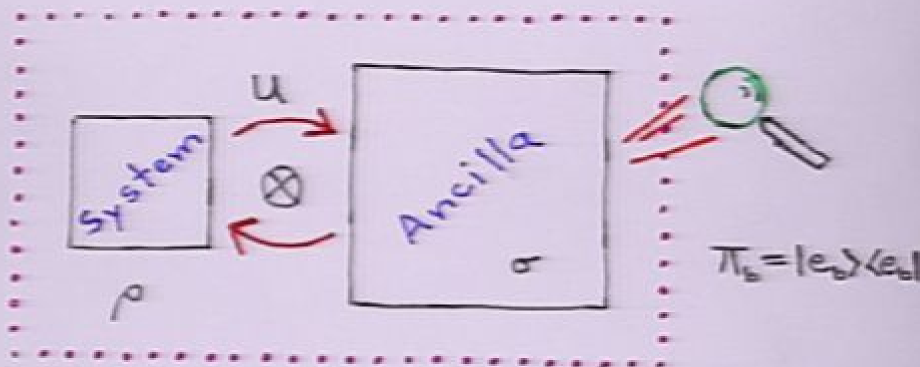
Usual Justification



1) $\rho\otimes\sigma \xrightarrow{\text{couple}} U(\rho\otimes\sigma)U^{+}$

2) $\xrightarrow{\text{collapse}} (I\otimes\Pi_b)\,U(\rho\otimes\sigma)U^{+}(I\otimes\Pi_b)$

3) $\xrightarrow{\text{discard}} \mathrm{tr}_A[\cdots] \xrightarrow{\text{renormalize}}$

Upshot

$$\rho \xrightarrow{b} \tilde{\rho}_b = \frac{1}{p_b} A_b \rho A_b^{+}$$

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Note

Unlike with Bayes rule

$$\rho \neq \sum_b p_b \tilde{\rho}_b$$

pre-measurement state

post-mmt. states

Moreover, not even if we delete the "feedback"

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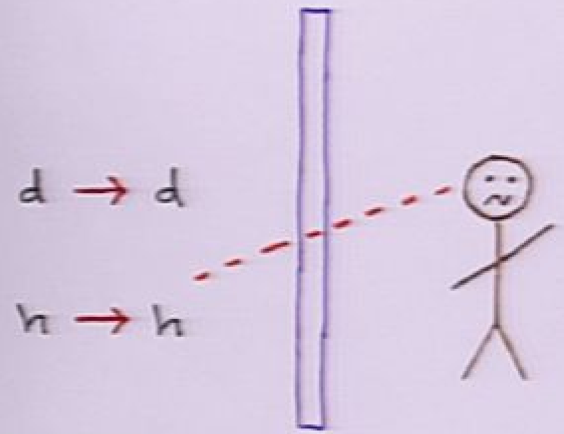

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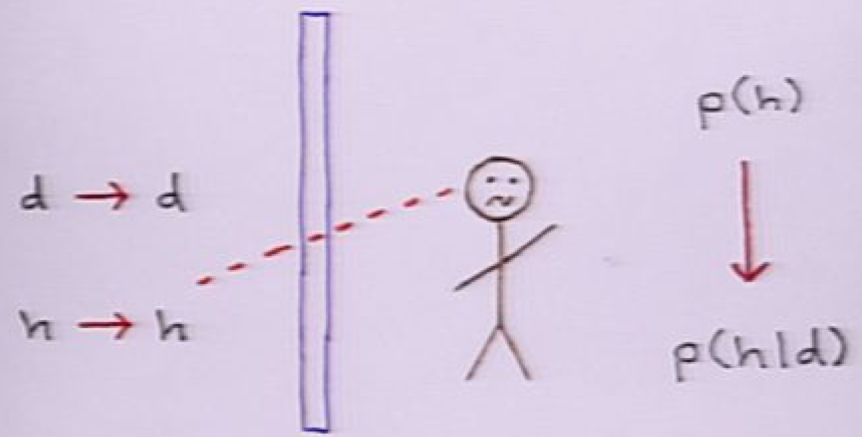

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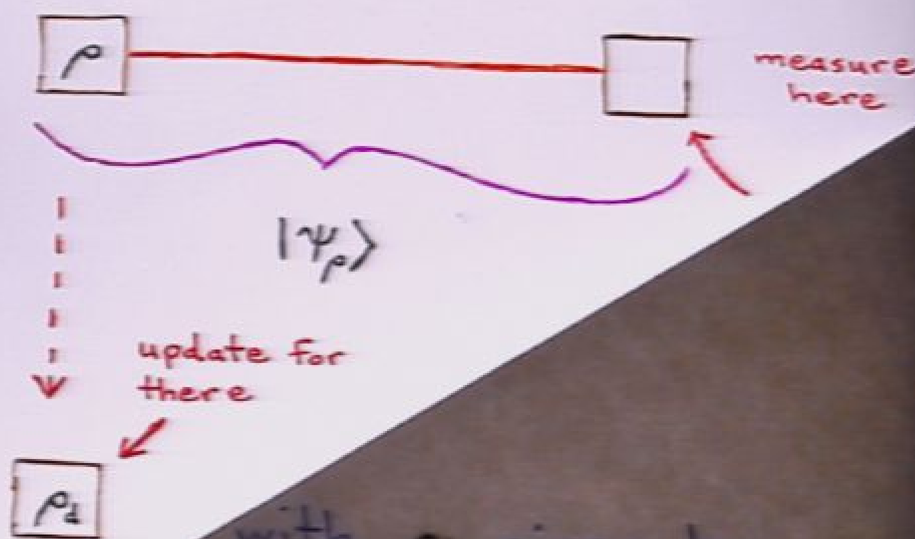
Particularly Important Case



with ρ_A given by

pure conditionalization!

(no extra unitary)

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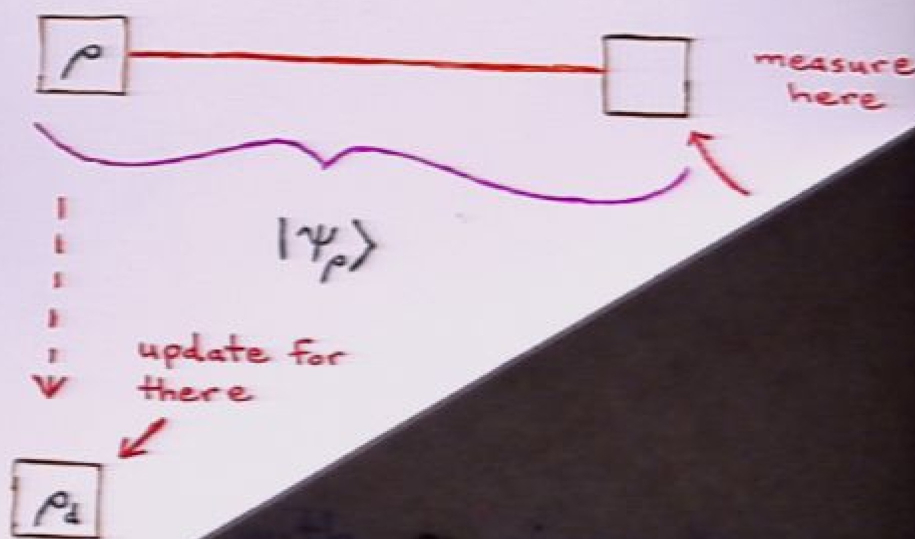

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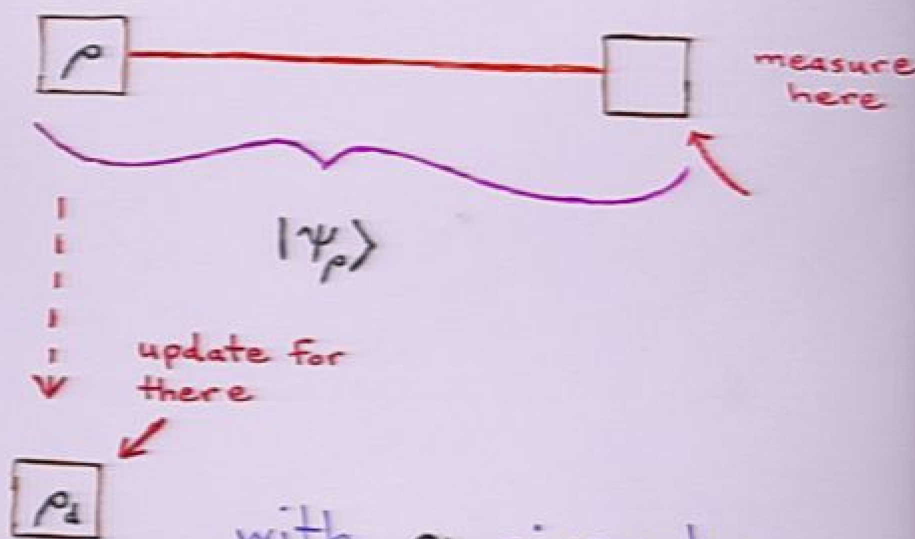

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State Change at Distance

$$|\psi_\rho\rangle = \sum_i \sqrt{\lambda_i}\, |i\rangle|i\rangle$$

Measurement causes update:

$$|\psi_\rho\rangle\langle\psi_\rho| \longrightarrow (A_d \otimes I)|\psi_\rho\rangle\langle\psi_\rho|(A_d^\dagger \otimes I)$$

Partial trace:

$$tr_A(\,\cdot\,) =$$

$$= \sum_{ijk} \sqrt{\lambda_j}\sqrt{\lambda_k}\,\langle i|A_d \otimes I|j\rangle|j\rangle\langle k|\langle k|A_d^\dagger \otimes I|i\rangle$$

$$= \sum_{ijk} \sqrt{\lambda_j}\sqrt{\lambda_k}\,\langle k|A_d^\dagger|i\rangle\langle i|A_d|j\rangle\; |j\rangle\langle k|$$

$$= \sum_{jk} \sqrt{\lambda_j}\sqrt{\lambda_k}\,\langle j|(A_d^\dagger A_d)^T|k\rangle\; |j\rangle\langle k|$$

$$= \Big(\sum_j \sqrt{\lambda_j}\,|j\rangle\langle j|\Big)(A_d^\dagger A_d)^T\Big(\sum_k \sqrt{\lambda_k}\,|k\rangle\langle k|\Big)$$

$$= \rho^{1/2} E_d^T \rho^{1/2}$$

$$\rho_A = \frac{\rho^{1/2} E_A^T \rho^{1/2}}{\operatorname{tr} \rho E_A^T}$$

$$\rho_\alpha = \frac{\rho^{1/2} E_\alpha^T \rho^{1/2}}{\mathrm{tr}\,\rho E_\alpha^T}$$

$$\rho = \sum \rho^{1/2} E_\alpha^T \rho^{1/2} = \rho^{1/2}\Big(\sum_\alpha E_\alpha^T\Big)\rho^{1/2}$$

$$=$$

$$= \rho$$

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$$= \rho^{1/2} E_d^T \rho^{1/2}$$

Emphasis

Classical

$$p(H) = \sum_D p(D)\, p(H|D)$$

$$p(H) \xrightarrow{D} p(H|D)$$

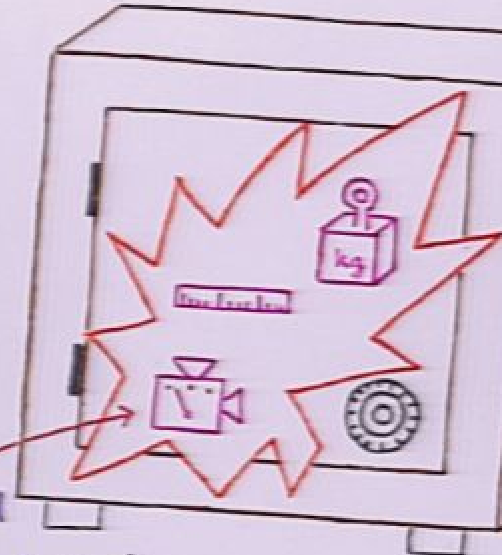
Quantum

$$\rho = \sum_b p_b \rho_b$$

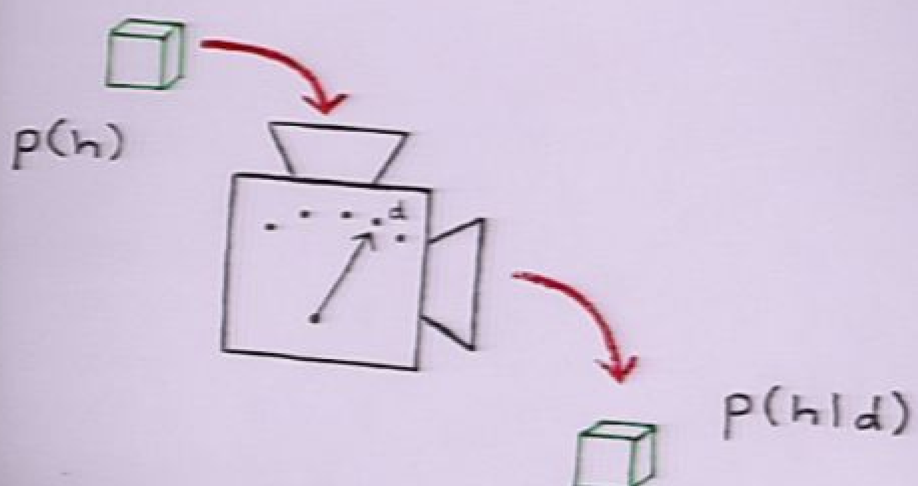
$$\rho \xrightarrow{b} \rho_b$$

modulo a further
unitary
readjustment

Bureau of Standards



international standard quantum measurement



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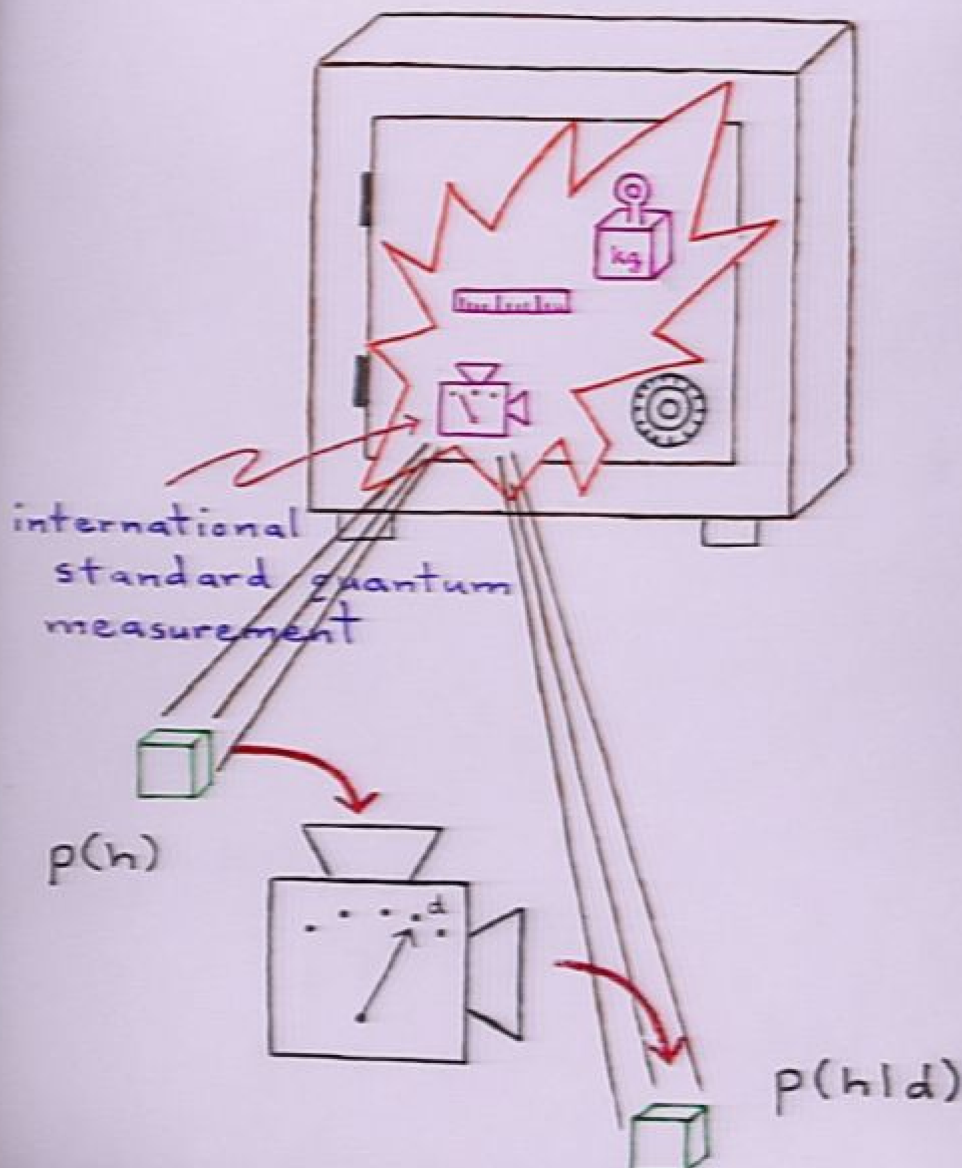


Standard measurements not good enough for the bureau.

$$H = \sum_i \alpha_i \Pi_i \quad , \quad \Pi_i = |i\rangle\langle i|$$

$$p(i) = \mathrm{tr}\,\rho\,\Pi_i = \langle i|\rho|i\rangle$$

$$\Rightarrow \begin{pmatrix} \rho_{11} & & \\ & \rho_{22} & \\ & & \ddots \end{pmatrix}$$

Informational Completeness

quantum states

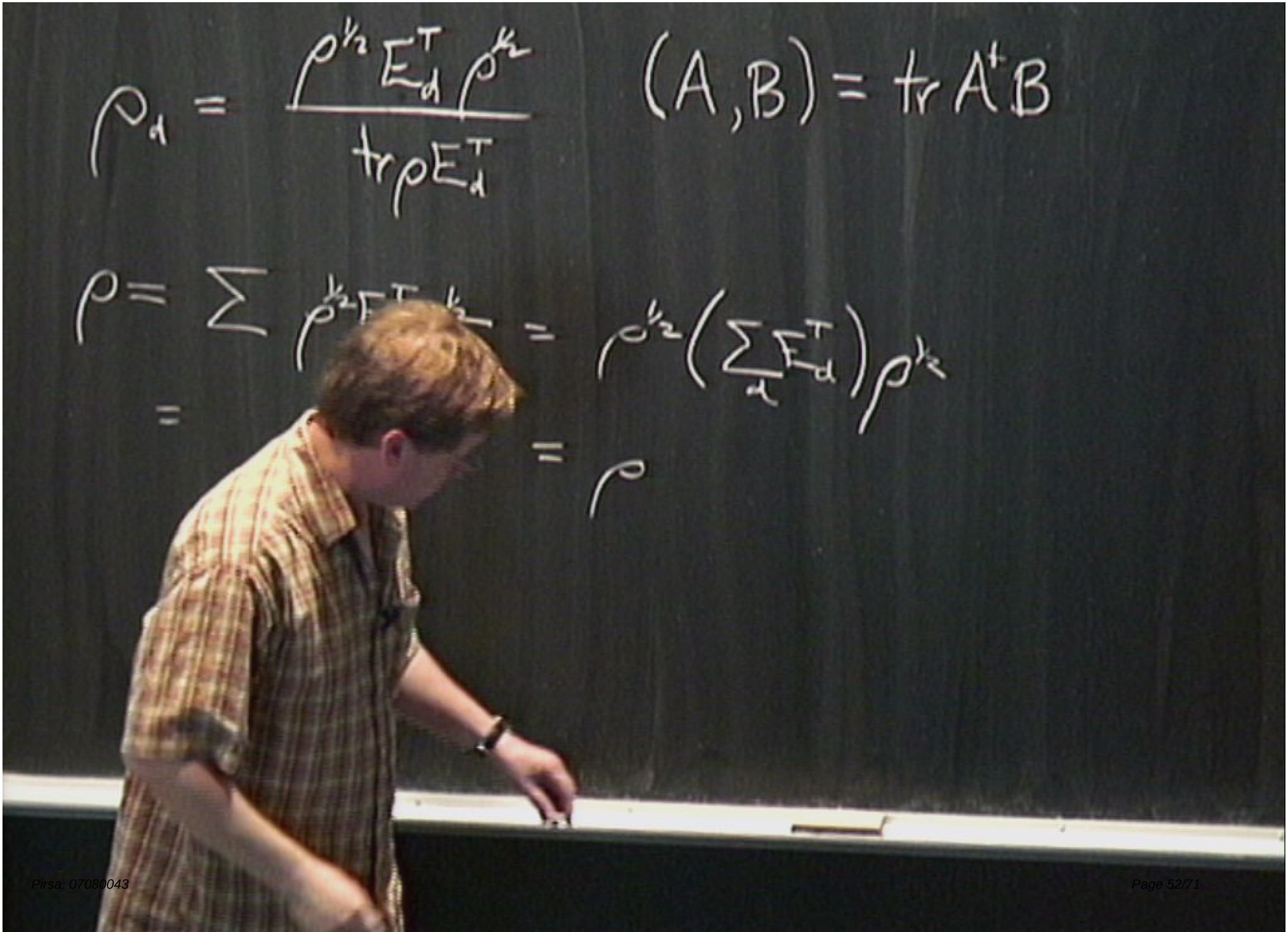
$\rho \in \mathcal{L}(\mathcal{H}_D)$ — D^2-dimensional vector space

Choose POVM $\{E_h\}$, $h = 1, \ldots, D^2$, with E_h all linearly independent.
(Can be done.)

D^2 numbers $p(h) = \mathrm{tr}\,\rho E_h$ determine ρ.

projection of ρ onto E_h

Any such $\{E_h\}$ can be the standard quantum measurement.


$\rho_d = \frac{\rho^{1/2} E_d^T \rho^{1/2}}{\mathrm{tr}\,\rho E_d^T}$
$(A,B) = \mathrm{tr}\,A^\dagger B$
$\rho = \sum \rho^{1/2} E^T$
$= \rho^{1/2}\left(\sum_d E_d^T\right)\rho^{1/2}$
$=$
$= \rho$

Informational Completeness

quantum states

$\rho \in \mathcal{L}(\mathcal{H}_D)$ — D^2-dimensional vector space

Choose POVM $\{E_h\}$, $h=1,\ldots,D^2$, with E_h all linearly independent.
(Can be done.)

D^2 numbers $p(h) = \mathrm{tr}\,\rho E_h$ determine ρ.

projection of ρ onto E_h

Any such $\{E_h\}$ can be the standard quantum measurement.

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A Very Fundamental Mmt ?

Caves, 1999

Suppose d^2 projectors $\Pi_i = |\psi_i\rangle\langle\psi_i|$ satisfying

$$\mathrm{tr}\,\Pi_i \Pi_j = \frac{1}{d+1} \quad , \quad i \neq j$$

exist.

Can prove:

1) the Π_i linearly independent

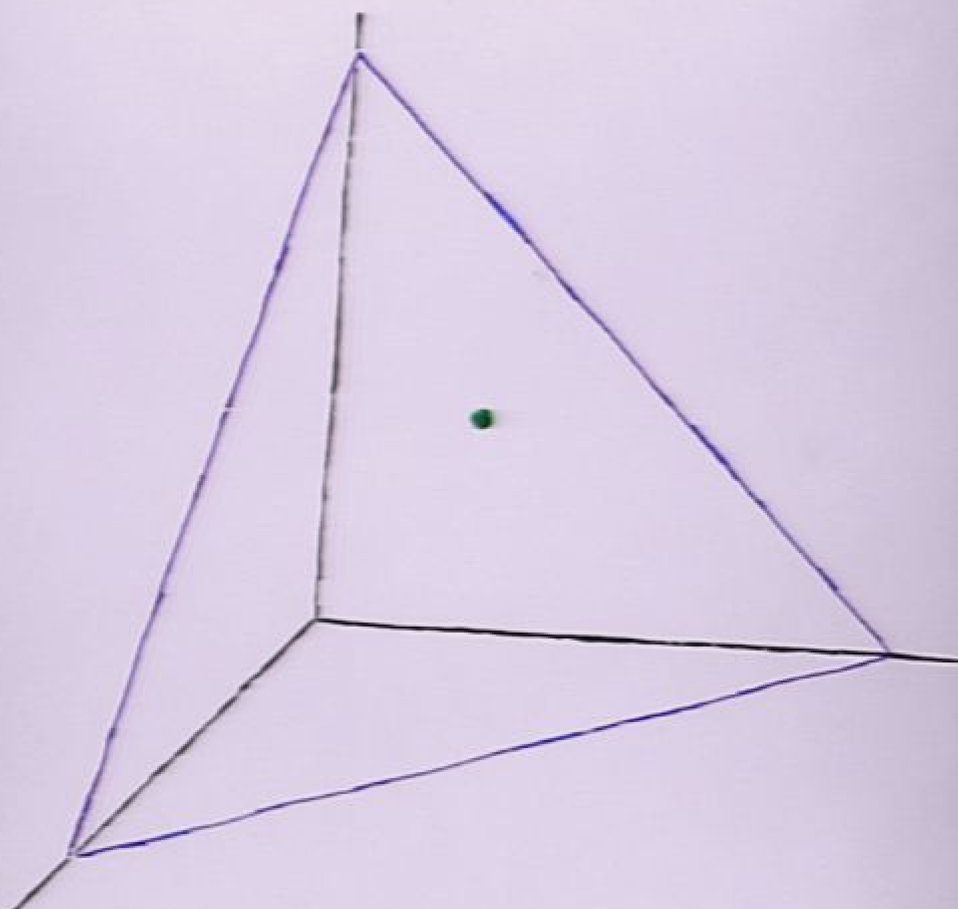
2) $\sum_i \frac{1}{d} \Pi_i = I$

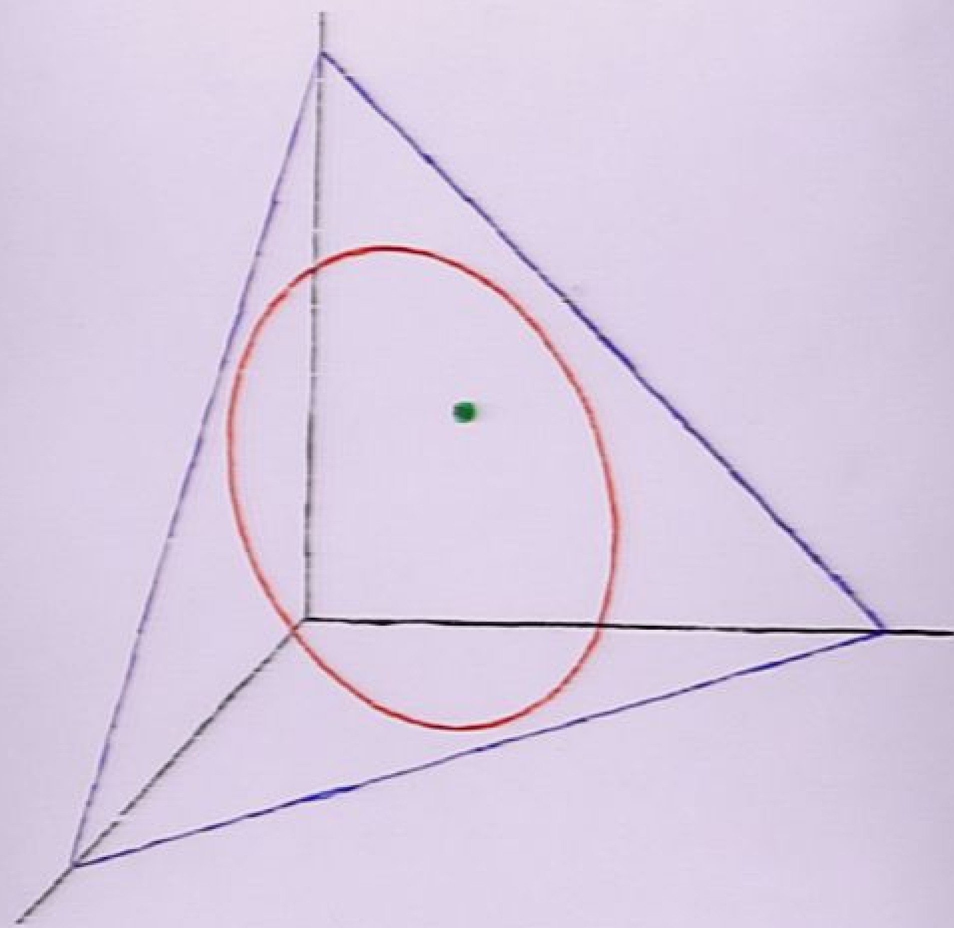
So good for Bureau of Standards.

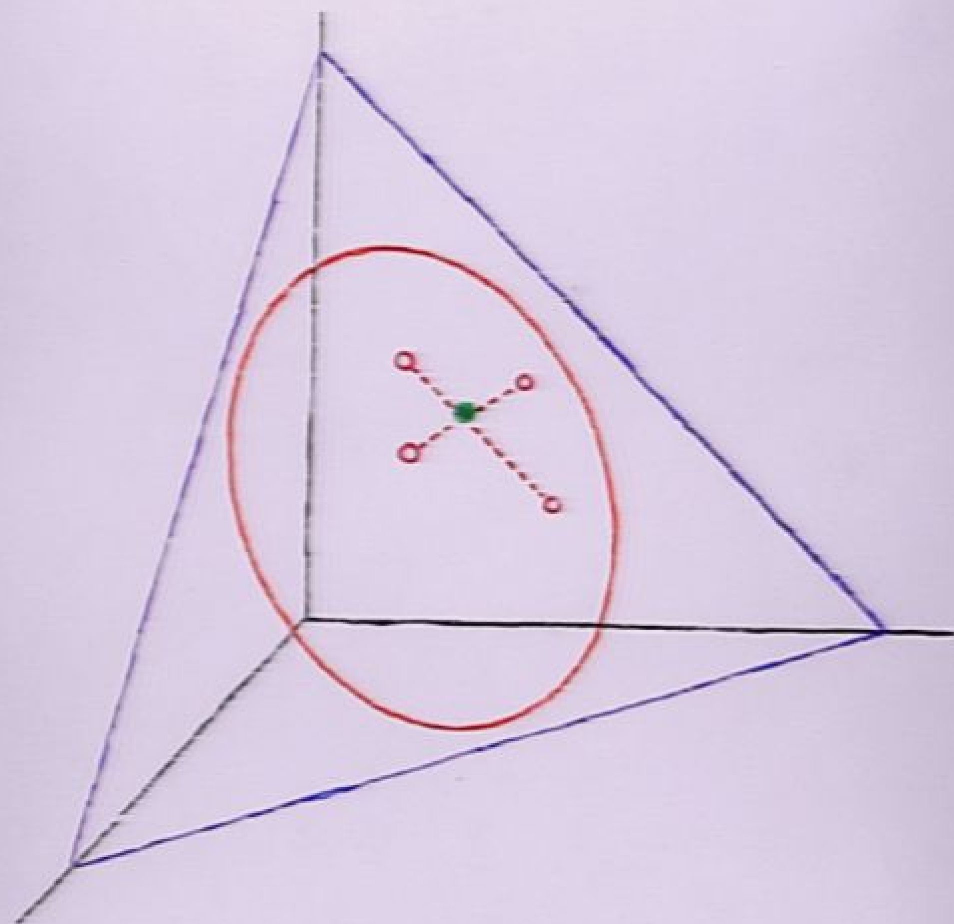
Also

$$p(i) = \frac{1}{d}\,\mathrm{tr}\,\rho \Pi_i$$

$$\boxed{\rho = \sum_i \left[(d+1)p(i) - \frac{1}{d}\right] \Pi_i}$$







Measurement

Given state ρ and mmt $\{F_d\}$.

$$I = \sum_d F_d$$

$$\Rightarrow \quad \rho = \sum_d \rho^{1/2} F_d \rho^{1/2}$$

$$\Rightarrow \quad \rho = \sum_d p(d)\, \rho_d$$

where $p(d) = \mathrm{tr}\, \rho F_d$

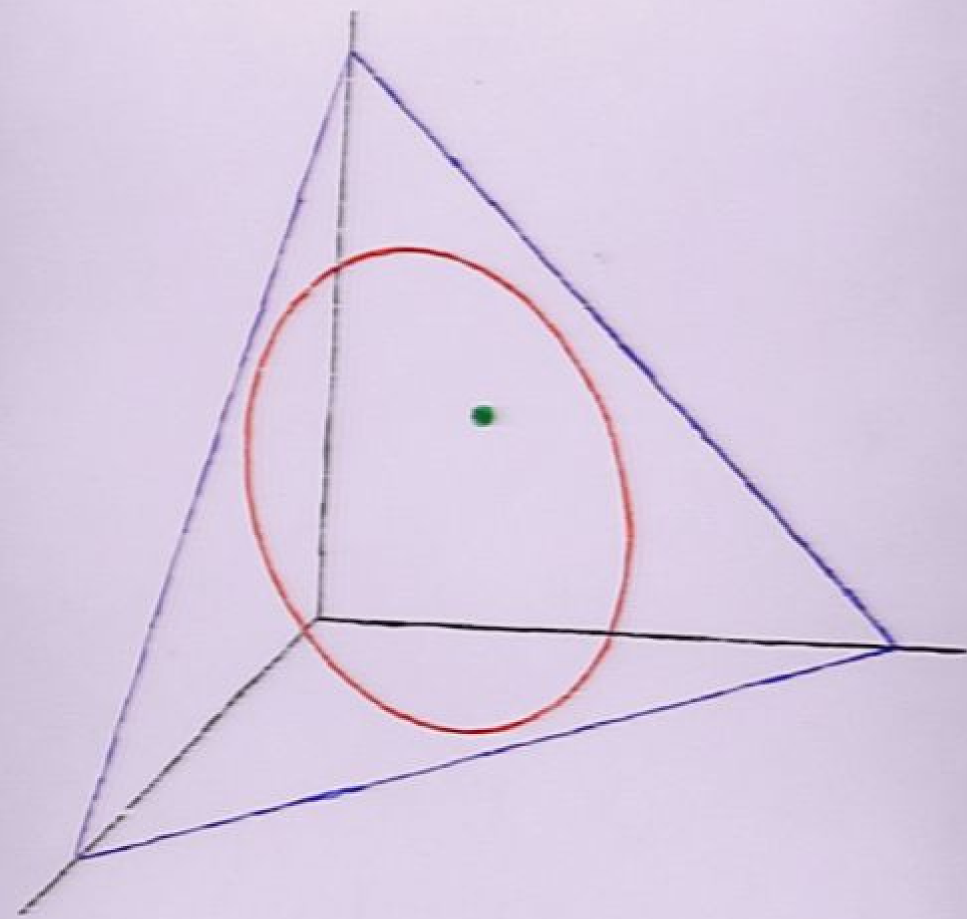
$$\rho_d = \frac{1}{p(d)} \rho^{1/2} F_d \rho^{1/2}$$

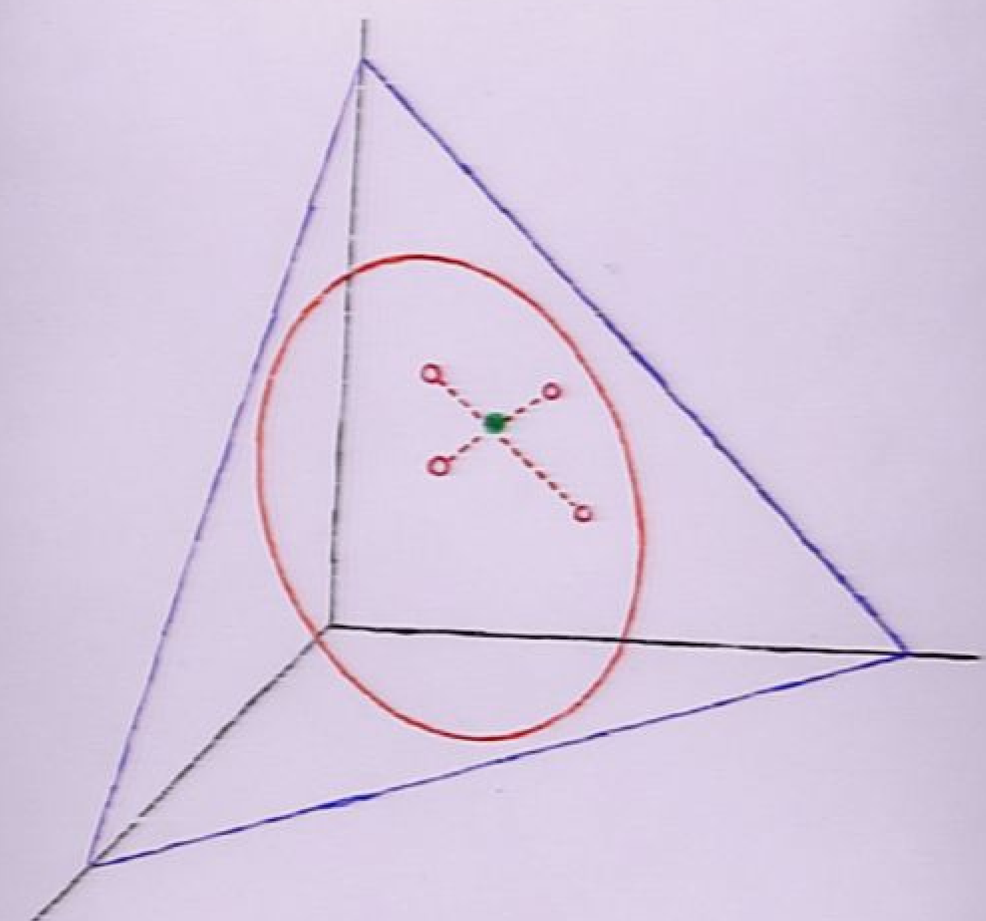
Introducing SQMD $\{E_h\}$

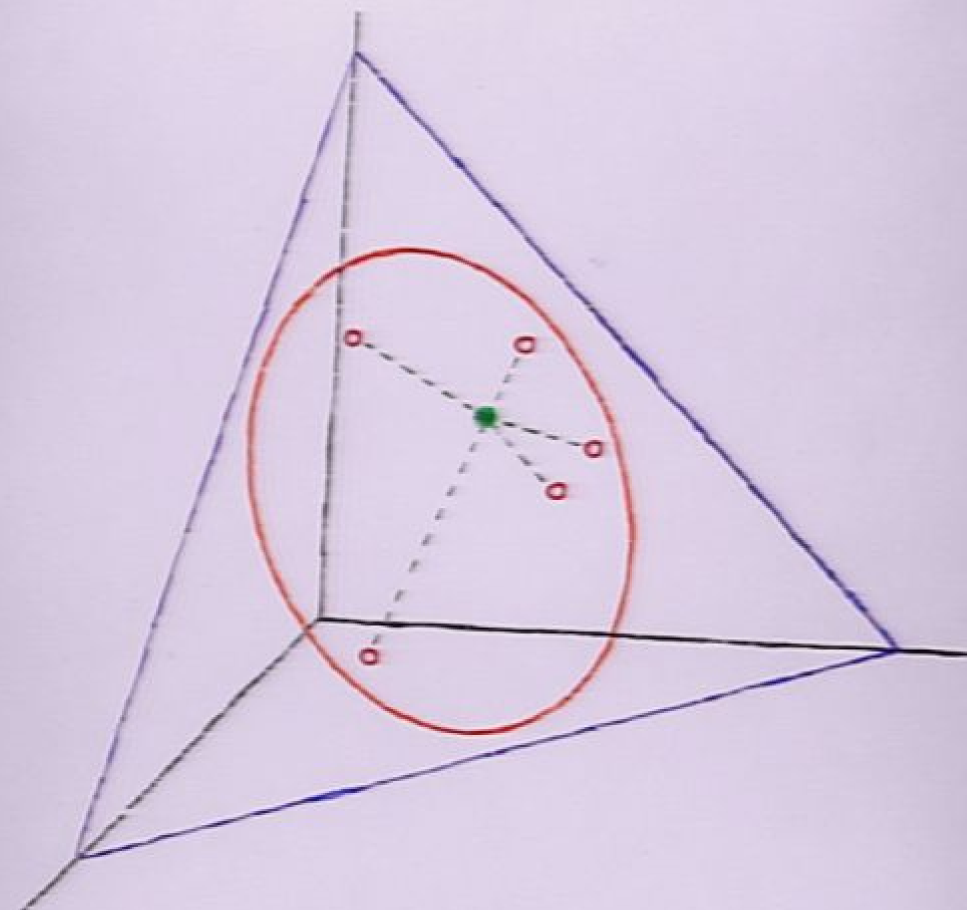
$$p(h) = \mathrm{tr}\, \rho E_h$$

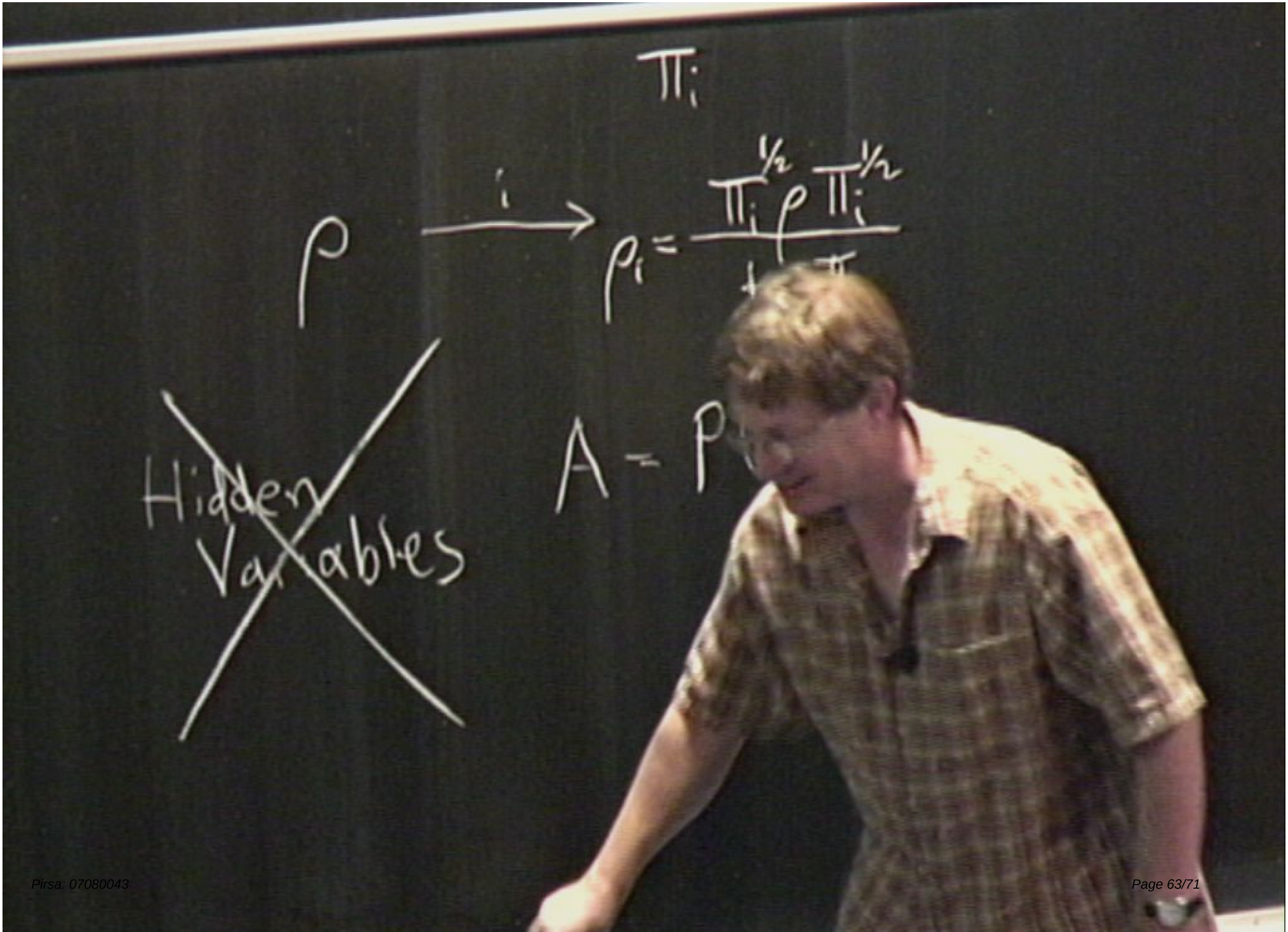
$$p(h|d) = \mathrm{tr}\, \rho_d E_h$$

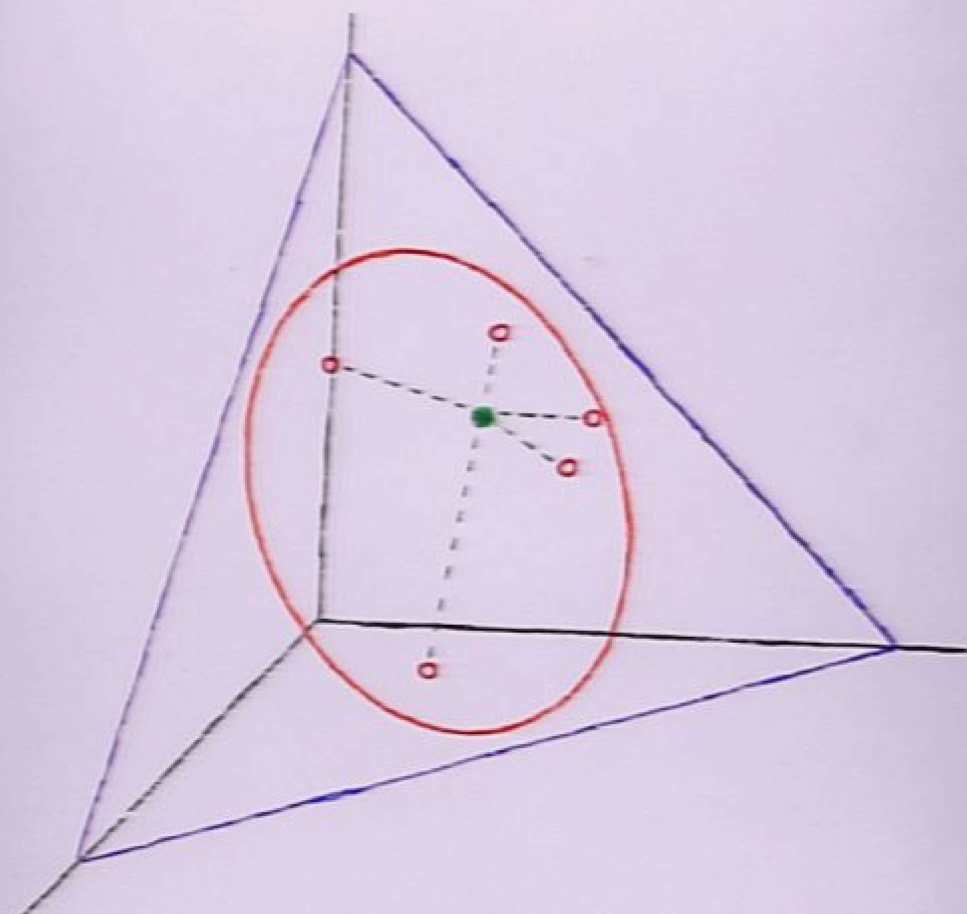
$$p(h) = \sum_d p(d)\, p(h|d)$$








Π_i
$\rho \xrightarrow{i} \rho_i = \frac{\Pi_i^{1/2} \rho \Pi_i^{1/2}}{}$
Hidden Variables
$A = P$



"Measurement"

Does it reveal a pre-existing,
but unknown, value?

or

Does it in some sense go toward
creating the very value?

EPR Criterion of Reality

"If, without in any way disturbing a system [one can gather the information required to] predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

$P(h)$

$P(h)$

~~states of pre-existent reality~~

consequences of measurement interactions

The Pauli'an Idea, I

[Einstein and I] often discussed these questions, and I invariably profited very greatly even when I could not agree with Einstein's views. "Physics is after all the description of reality," he said to me, continuing, with a sarcastic glance in my direction, "or should I perhaps say physics is the description of what one merely imagines?" This question clearly shows Einstein's concern that the objective character of physics might be lost through a theory of the type of quantum mechanics, in that as a consequence of its wider conception of objectivity of an explanation of nature the difference between physical reality and dream or hallucination become blurred.

The objectivity of physics is however fully ensured in quantum mechanics in the following sense. Although in principle, according to the theory, it is in general only the statistics of series of experiments that is determined by laws, the observer is unable, even in the unpredictable single case, to influence the result of his observation—as for example the response of a counter at a particular instant of time. ~~Further, personal qualities of the observer do not come into the theory in any way—the observation can be made by objective registering apparatus, the results of which are objectively available for anyone's inspection.~~ Just as in the theory of relativity a group of mathematical transformations connects all possible coordinate systems, so in quantum mechanics a group of mathematical transformations connects the possible experimental arrangements.

Einstein however advocated a narrower form of the reality concept . . .

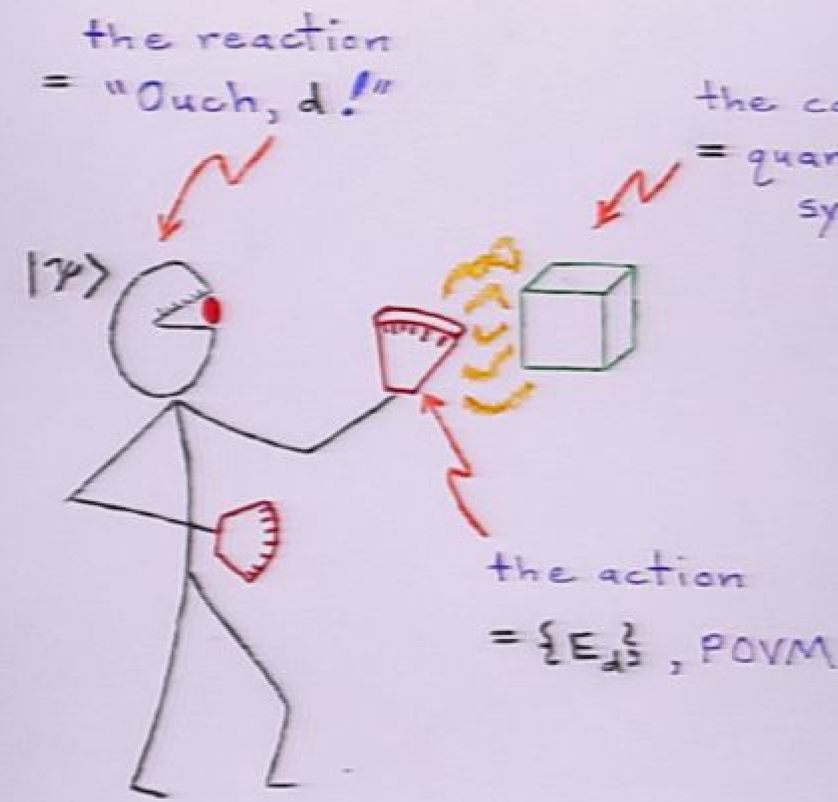
— Wolfgang Pauli
"Albert Einstein and the Development of Physics," 1958

as there is no sharp distinction possible between these and macroscopic objects. Observation cannot *create* an element of reality like a position, there must be something contained in the complete description of physical reality which corresponds to the *possibility* of observing a position, already before the observation has been actually made." It is this kind of postulate which I call the ideal of the detached observer.

In quantum mechanics, on the contrary, an observation changes in general the "state" of the observed system in a way not contained in the mathematically formulated *laws*, which only apply to the time dependence of the state of a *closed* system. I think here on the passage to a new phenomenon by observation which is taken into account by the so called "reduction of the wave packets." As it is allowed to consider the instruments of observation as a kind of prolongation of the sense organs of the observer, I consider the impredictable change of the state by a single observation to be *an abandonment of the idea of the isolation (detachment) of the observer from the course of physical events outside himself.*

— Wolfgang Pauli

Letter to Bohr, 1955


the reaction
= "Ouch, d!"
the catalyst
= quantum system
|ψ⟩
the action
= {E_d}, POVM