Title: Operationalism, hidden variable models, and contextuality (Part 1B)

Date: Aug 27, 2007 04:30 PM

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Abstract:

Proof of preparation contextuality

(a preparation noncontextual hidden variable model is impossible)

Important features of hidden variable models

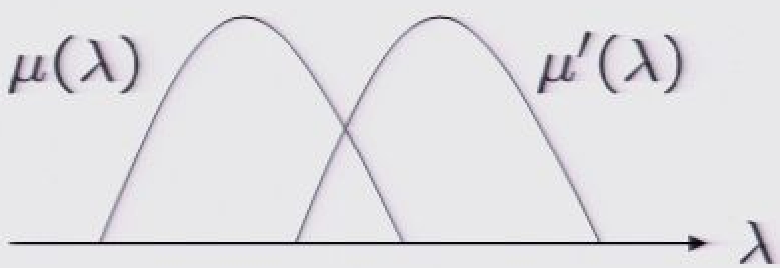
Let $\mathsf{P} \leftrightarrow \mu(\lambda)$

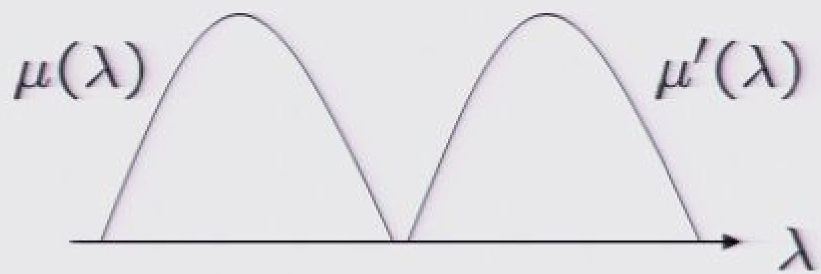
$\mathsf{P}' \leftrightarrow \mu'(\lambda)$

Representing one-shot distinguishability:

If P and P′ are distinguishable with certainty

then $\mu(\lambda)\,\mu'(\lambda) = 0$





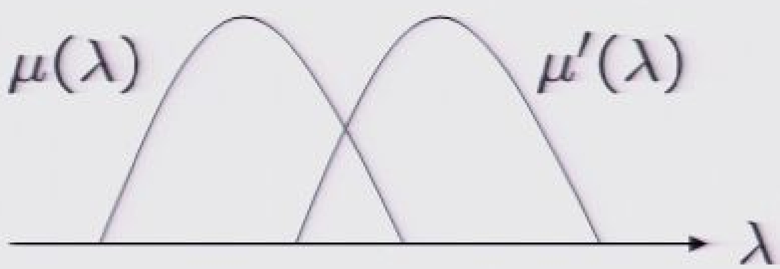
Important features of hidden variable models

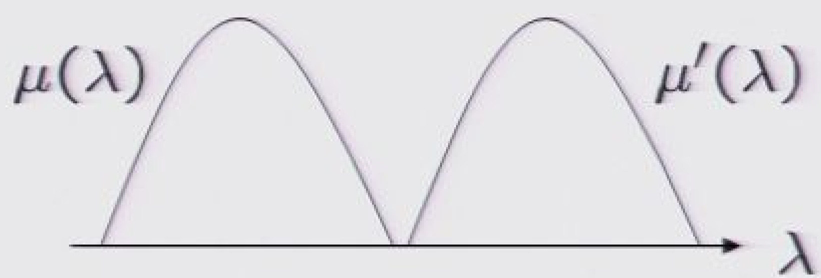
$$\text{Let } \mathsf{P} \leftrightarrow \mu(\lambda)$$
$$\mathsf{P}' \leftrightarrow \mu'(\lambda)$$

Representing one-shot distinguishability:

If P and P$'$ are distinguishable with certainty

$$\text{then } \mu(\lambda)\, \mu'(\lambda) = 0$$





Representing convex combination:

If P$'' =$ P with prob. p and P$'$ with prob. $1-p$

Then $\mu''(\lambda) = p\, \mu(\lambda) + (1-p)\, \mu'(\lambda)$

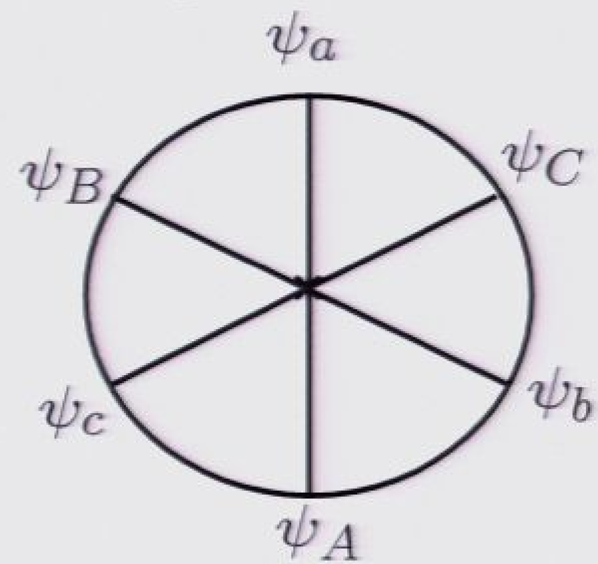
Proof based on finite construction in 2d

$$
\begin{aligned}
\mathsf{P}_a &\leftrightarrow \psi_a = (1,0)\\
\mathsf{P}_A &\leftrightarrow \psi_A = (0,1)\\
\mathsf{P}_b &\leftrightarrow \psi_b = (1/2, \sqrt{3}/2)\\
\mathsf{P}_B &\leftrightarrow \psi_B = (\sqrt{3}/2, -1/2)\\
\mathsf{P}_c &\leftrightarrow \psi_c = (1/2, -\sqrt{3}/2)\\
\mathsf{P}_C &\leftrightarrow \psi_C = (\sqrt{3}/2, 1/2)
\end{aligned}
$$

$$\psi_a \perp \psi_A$$

$$\psi_b \perp \psi_B$$

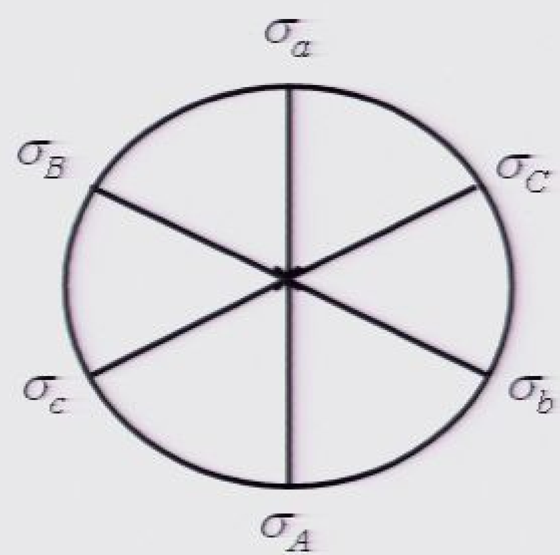
$$\psi_c \perp \psi_C$$



Proof based on finite construction in 2d

$$
\begin{aligned}
P_a &\leftrightarrow & \sigma_a &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\
P_A &\leftrightarrow & \sigma_A &= \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\
P_b &\leftrightarrow & \sigma_b &= \begin{pmatrix} \frac{1}{4} & \frac{1}{4}\sqrt{3} \\ \frac{1}{4}\sqrt{3} & \frac{3}{4} \end{pmatrix} \\
P_B &\leftrightarrow & \sigma_B &= \begin{pmatrix} \frac{3}{4} & -\frac{1}{4}\sqrt{3} \\ -\frac{1}{4}\sqrt{3} & \frac{1}{4} \end{pmatrix} \\
P_c &\leftrightarrow & \sigma_c &= \begin{pmatrix} \frac{1}{4} & -\frac{1}{4}\sqrt{3} \\ -\frac{1}{4}\sqrt{3} & \frac{3}{4} \end{pmatrix} \\
P_C &\leftrightarrow & \sigma_C &= \begin{pmatrix} \frac{3}{4} & \frac{1}{4}\sqrt{3} \\ \frac{1}{4}\sqrt{3} & \frac{1}{4} \end{pmatrix}
\end{aligned}
$$

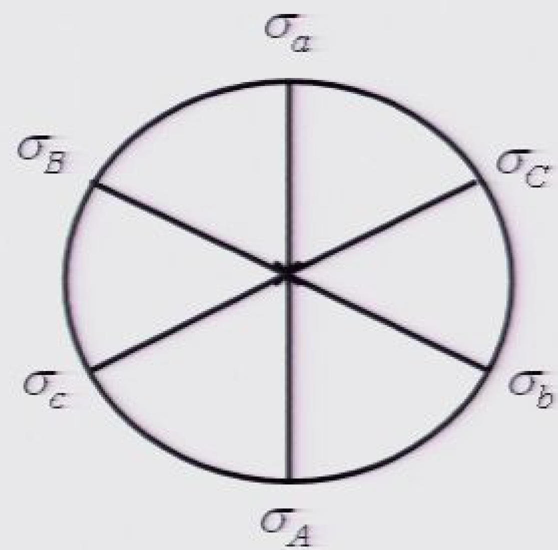
$$
\begin{aligned}
\sigma_a \sigma_A &= 0 \\
\sigma_b \sigma_B &= 0 \\
\sigma_c \sigma_C &= 0
\end{aligned}
$$



Proof based on finite construction in 2d

$$\mathsf{P}_a \leftrightarrow \sigma_a = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$
$$\mathsf{P}_A \leftrightarrow \sigma_A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$
$$\mathsf{P}_b \leftrightarrow \sigma_b = \begin{pmatrix} \frac{1}{4} & \frac{1}{4}\sqrt{3} \\ \frac{1}{4}\sqrt{3} & \frac{3}{4} \end{pmatrix}$$
$$\mathsf{P}_B \leftrightarrow \sigma_B = \begin{pmatrix} \frac{3}{4} & -\frac{1}{4}\sqrt{3} \\ -\frac{1}{4}\sqrt{3} & \frac{1}{4} \end{pmatrix}$$
$$\mathsf{P}_c \leftrightarrow \sigma_c = \begin{pmatrix} \frac{1}{4} & -\frac{1}{4}\sqrt{3} \\ -\frac{1}{4}\sqrt{3} & \frac{3}{4} \end{pmatrix}$$
$$\mathsf{P}_C \leftrightarrow \sigma_C = \begin{pmatrix} \frac{3}{4} & \frac{1}{4}\sqrt{3} \\ \frac{1}{4}\sqrt{3} & \frac{1}{4} \end{pmatrix}$$

$$\sigma_a \sigma_A = 0$$
$$\sigma_b \sigma_B = 0$$
$$\sigma_c \sigma_C = 0$$



P_a and P_A are distinguishable with certainty
P_b and P_B are distinguishable with certainty
P_c and P_C are distinguishable with certainty

$$\mu_a(\lambda)\, \mu_A(\lambda) = 0$$
$$\longrightarrow \quad \mu_b(\lambda)\, \mu_B(\lambda) = 0$$
$$\mu_c(\lambda)\, \mu_C(\lambda) = 0$$

$$
\begin{aligned}
\mathsf{P}_{aA} &\equiv \mathsf{P}_a \text{ and } \mathsf{P}_A \text{ with prob. } 1/2 \text{ each} \\
\mathsf{P}_{bB} &\equiv \mathsf{P}_b \text{ and } \mathsf{P}_B \text{ with prob. } 1/2 \text{ each} \\
\mathsf{P}_{cC} &\equiv \mathsf{P}_c \text{ and } \mathsf{P}_C \text{ with prob. } 1/2 \text{ each} \\
\mathsf{P}_{abc} &\equiv \mathsf{P}_a, \mathsf{P}_b \text{ and } \mathsf{P}_c \text{ with prob. } 1/3 \text{ each} \\
\mathsf{P}_{ABC} &\equiv \mathsf{P}_A, \mathsf{P}_B \text{ and } \mathsf{P}_C \text{ with prob. } 1/3 \text{ each}
\end{aligned}
$$

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\mathsf{P}_{aA} &\equiv \mathsf{P}_a \text{ and } \mathsf{P}_A \text{ with prob. } 1/2 \text{ each} \\
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\mathsf{P}_{abc} &\equiv \mathsf{P}_a, \mathsf{P}_b \text{ and } \mathsf{P}_c \text{ with prob. } 1/3 \text{ each} \\
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\end{aligned}
$$

$$
\longrightarrow \quad
\begin{aligned}
\mu_{aA}(\lambda) &= \frac{1}{2}\mu_a(\lambda) + \frac{1}{2}\mu_A(\lambda) \\
\mu_{bB}(\lambda) &= \frac{1}{2}\mu_b(\lambda) + \frac{1}{2}\mu_B(\lambda) \\
\mu_{cC}(\lambda) &= \frac{1}{2}\mu_c(\lambda) + \frac{1}{2}\mu_C(\lambda) \\
\mu_{abc}(\lambda) &= \frac{1}{3}\mu_a(\lambda) + \frac{1}{3}\mu_b(\lambda) + \frac{1}{3}\mu_c(\lambda) \\
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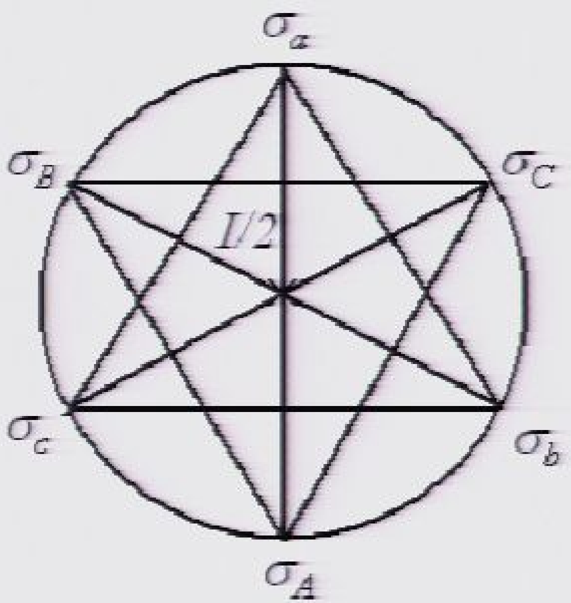
\end{aligned}
$$



$$
\begin{aligned}
I/2 &= \frac{1}{2}\sigma_a + \frac{1}{2}\sigma_A \\
&= \frac{1}{2}\sigma_b + \frac{1}{2}\sigma_B \\
&= \frac{1}{2}\sigma_c + \frac{1}{2}\sigma_C \\
&= \frac{1}{3}\sigma_a + \frac{1}{3}\sigma_b + \frac{1}{3}\sigma_c \\
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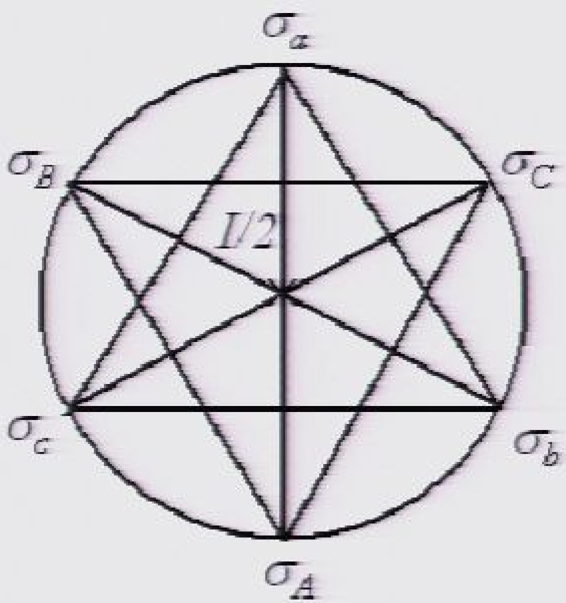
\end{aligned}
$$



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&= \frac{1}{2}\sigma_c + \frac{1}{2}\sigma_C \\
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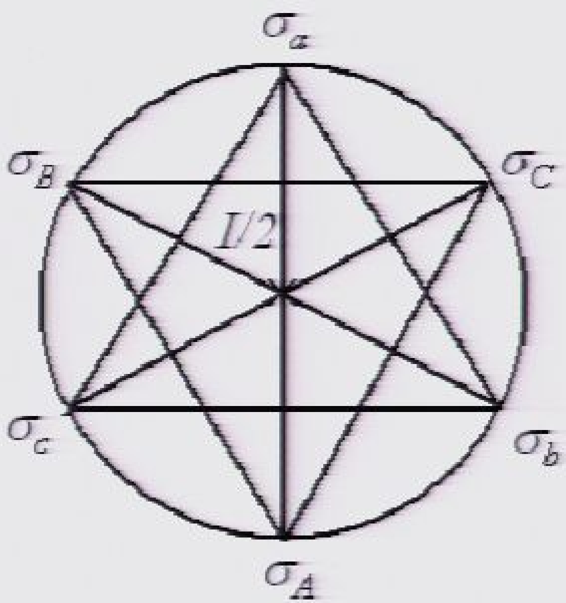
&= \frac{1}{3}\sigma_A + \frac{1}{3}\sigma_B + \frac{1}{3}\sigma_C.
\end{aligned}
$$

$$
\mathsf{P}_{aA} \simeq \mathsf{P}_{bB} \simeq \mathsf{P}_{cC}
$$
$$
\simeq \mathsf{P}_{abc} \simeq \mathsf{P}_{ABC}
$$



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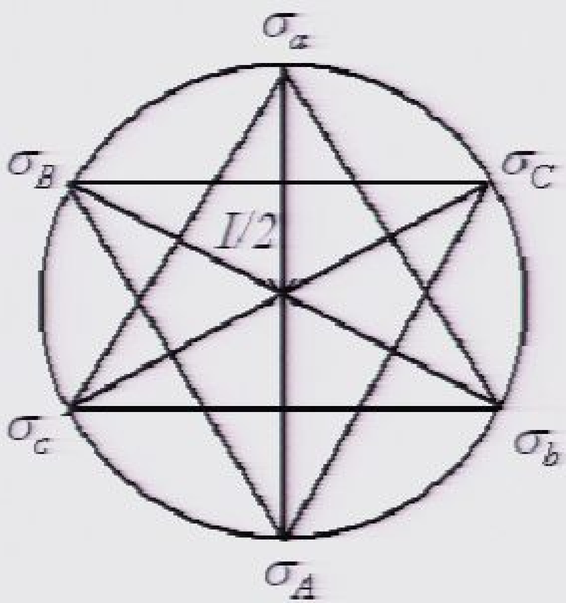
&\mathsf{P}_{aA} \simeq \mathsf{P}_{bB} \simeq \mathsf{P}_{cC} \\
&\simeq \mathsf{P}_{abc} \simeq \mathsf{P}_{ABC}
\end{aligned}
$$

By preparation noncontextuality

$$
\begin{aligned}
&\mu_{aA}(\lambda) = \mu_{bB}(\lambda) = \mu_{cC}(\lambda) \\
&= \mu_{abc}(\lambda) = \mu_{ABC}(\lambda) \\
&\equiv \nu(\lambda)
\end{aligned}
$$



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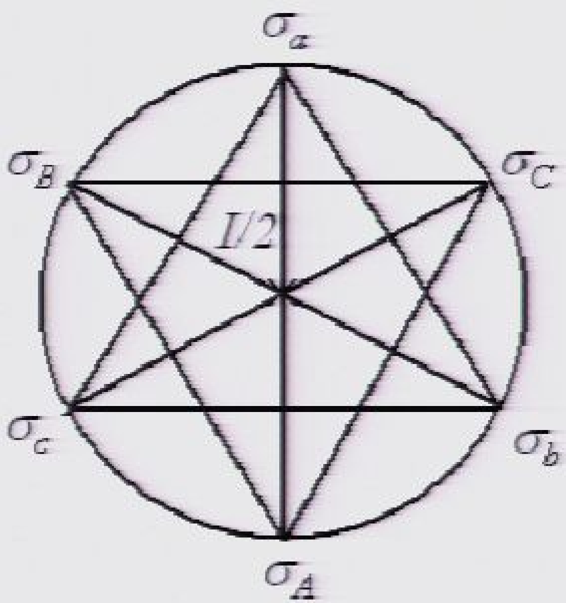
\end{aligned}
$$



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&= \frac{1}{3}\sigma_a + \frac{1}{3}\sigma_b + \frac{1}{3}\sigma_c \\
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$$
\mathsf{P}_{aA} \simeq \mathsf{P}_{bB} \simeq \mathsf{P}_{cC} \\
\simeq \mathsf{P}_{abc} \simeq \mathsf{P}_{ABC}
$$

By preparation noncontextuality

$$
\mu_{aA}(\lambda) = \mu_{bB}(\lambda) = \mu_{cC}(\lambda) \\
= \mu_{abc}(\lambda) = \mu_{ABC}(\lambda) \\
\equiv \nu(\lambda)
$$

$$
\begin{aligned}
\nu(\lambda) &= \frac{1}{2}\mu_a(\lambda) + \frac{1}{2}\mu_A(\lambda) \\
&= \frac{1}{2}\mu_b(\lambda) + \frac{1}{2}\mu_B(\lambda) \\
&= \frac{1}{2}\mu_c(\lambda) + \frac{1}{2}\mu_C(\lambda) \\
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\end{aligned}
$$

Our task is to find $\mu_a(\lambda)$, $\mu_A(\lambda)$, $\mu_b(\lambda)$, $\mu_B(\lambda)$, $\mu_c(\lambda)$, $\mu_C(\lambda)$, and $\nu(\lambda)$ such that

$$\begin{aligned}
\mu_a(\lambda)\,\mu_A(\lambda) &= 0 \\
\mu_b(\lambda)\,\mu_B(\lambda) &= 0 \\
\mu_c(\lambda)\,\mu_C(\lambda) &= 0
\end{aligned}$$

$$\begin{aligned}
\nu(\lambda) &= \frac{1}{2}\mu_a(\lambda) + \frac{1}{2}\mu_A(\lambda) \\
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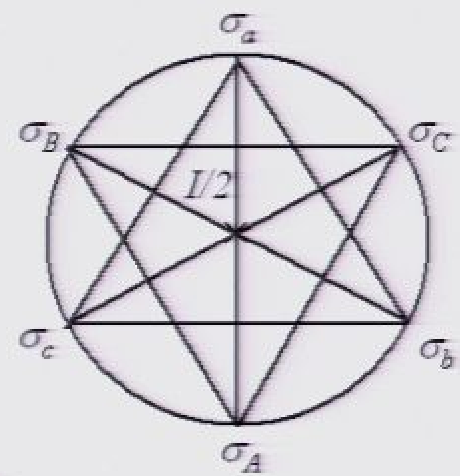
$$\begin{aligned} \mu_a(\lambda)\, \mu_A(\lambda) &= 0 \\ \mu_b(\lambda)\, \mu_B(\lambda) &= 0 \\ \mu_c(\lambda)\, \mu_C(\lambda) &= 0 \end{aligned}$$

$$\begin{aligned} \nu(\lambda) &= \frac{1}{2}\mu_a(\lambda) + \frac{1}{2}\mu_A(\lambda) \\ &= \frac{1}{2}\mu_b(\lambda) + \frac{1}{2}\mu_B(\lambda) \\ &= \frac{1}{2}\mu_c(\lambda) + \frac{1}{2}\mu_C(\lambda) \\ &= \frac{1}{3}\mu_a(\lambda) + \frac{1}{3}\mu_b(\lambda) + \frac{1}{3}\mu_c(\lambda) \\ &= \frac{1}{3}\mu_A(\lambda) + \frac{1}{3}\mu_B(\lambda) + \frac{1}{3}\mu_C(\lambda). \end{aligned}$$

i.e., paralleling the quantum structure:



$$\begin{aligned} \sigma_a\sigma_A &= 0 \\ \sigma_b\sigma_B &= 0 \\ \sigma_c\sigma_C &= 0 \end{aligned}$$

$$\begin{aligned} I/2 &= \frac{1}{2}\sigma_a + \frac{1}{2}\sigma_A \\ &= \frac{1}{2}\sigma_b + \frac{1}{2}\sigma_B \\ &= \frac{1}{2}\sigma_c + \frac{1}{2}\sigma_C \\ &= \frac{1}{3}\sigma_a + \frac{1}{3}\sigma_b + \frac{1}{3}\sigma_c \\ &= \frac{1}{3}\sigma_A + \frac{1}{3}\sigma_B + \frac{1}{3}\sigma_C. \end{aligned}$$

Our task is to find
$\mu_a(\lambda)$, $\mu_A(\lambda)$, $\mu_b(\lambda)$,
$\mu_B(\lambda)$, $\mu_c(\lambda)$, $\mu_C(\lambda)$,
and $\nu(\lambda)$ such that

$$\begin{aligned}
\mu_a(\lambda)\,\mu_A(\lambda) &= 0 \\
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\mu_c(\lambda)\,\mu_C(\lambda) &= 0
\end{aligned}$$

$$\begin{aligned}
\nu(\lambda) &= \frac{1}{2}\mu_a(\lambda) + \frac{1}{2}\mu_A(\lambda) \\
&= \frac{1}{2}\mu_b(\lambda) + \frac{1}{2}\mu_B(\lambda) \\
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\end{aligned}$$

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&= \frac{1}{3}\mu_a(\lambda) + \frac{1}{3}\mu_b(\lambda) + \frac{1}{3}\mu_c(\lambda)\\
&= \frac{1}{3}\mu_A(\lambda) + \frac{1}{3}\mu_B(\lambda) + \frac{1}{3}\mu_C(\lambda)
\end{aligned}$$

From decompositions (1)-(3), for $\lambda = \lambda'$

$$\begin{aligned}
\mu_a(\lambda') &= 0 \text{ or } 2\nu(\lambda')\\
\mu_b(\lambda') &= 0 \text{ or } 2\nu(\lambda')\\
\mu_c(\lambda') &= 0 \text{ or } 2\nu(\lambda')
\end{aligned}$$

Our task is to find $\mu_a(\lambda)$, $\mu_A(\lambda)$, $\mu_b(\lambda)$, $\mu_B(\lambda)$, $\mu_c(\lambda)$, $\mu_C(\lambda)$, and $\nu(\lambda)$ such that

$$
\begin{aligned}
\mu_a(\lambda)\,\mu_A(\lambda) &= 0 \\
\mu_b(\lambda)\,\mu_B(\lambda) &= 0 \\
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\end{aligned}
$$

$$
\begin{aligned}
\nu(\lambda) &= \frac{1}{2}\mu_a(\lambda) + \frac{1}{2}\mu_A(\lambda) \\
&= \frac{1}{2}\mu_b(\lambda) + \frac{1}{2}\mu_B(\lambda) \\
&= \frac{1}{2}\mu_c(\lambda) + \frac{1}{2}\mu_C(\lambda) \\
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\end{aligned}
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From decompositions (1)-(3), for $\lambda = \lambda'$

$$
\begin{aligned}
\mu_a(\lambda') &= 0 \text{ or } 2\nu(\lambda') \\
\mu_b(\lambda') &= 0 \text{ or } 2\nu(\lambda') \\
\mu_c(\lambda') &= 0 \text{ or } 2\nu(\lambda')
\end{aligned}
$$

But then the RHS of decomposition (4) is

$$
0, \tfrac{2}{3}\nu(\lambda'), \tfrac{4}{3}\nu(\lambda'), 2\nu(\lambda') \\
\neq \nu(\lambda')
$$

for λ' such that $\nu(\lambda') \neq 0$

CONTRADICTION

Measurement noncontextuality

new definition versus old

Another feature of a hidden variable model

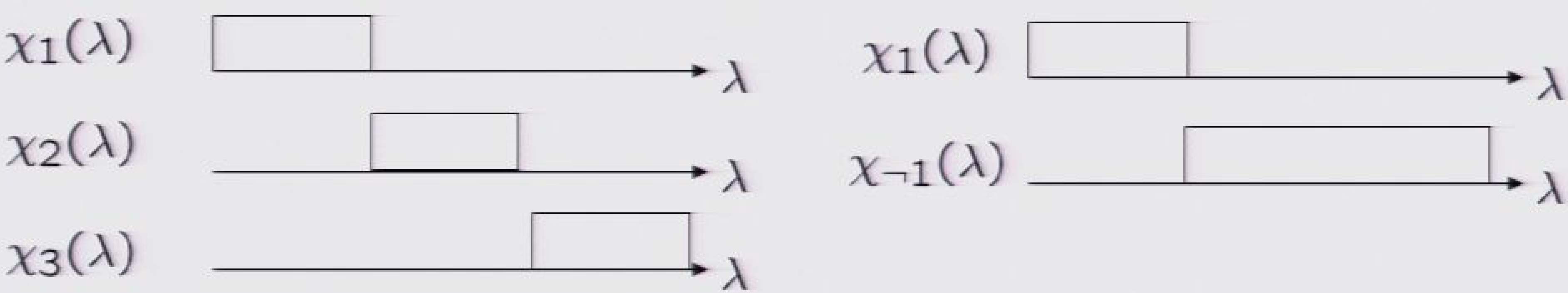
Let M $\leftrightarrow \{\chi_k(\lambda)\}$

M' $\leftrightarrow \{\chi'_j(\lambda)\}$

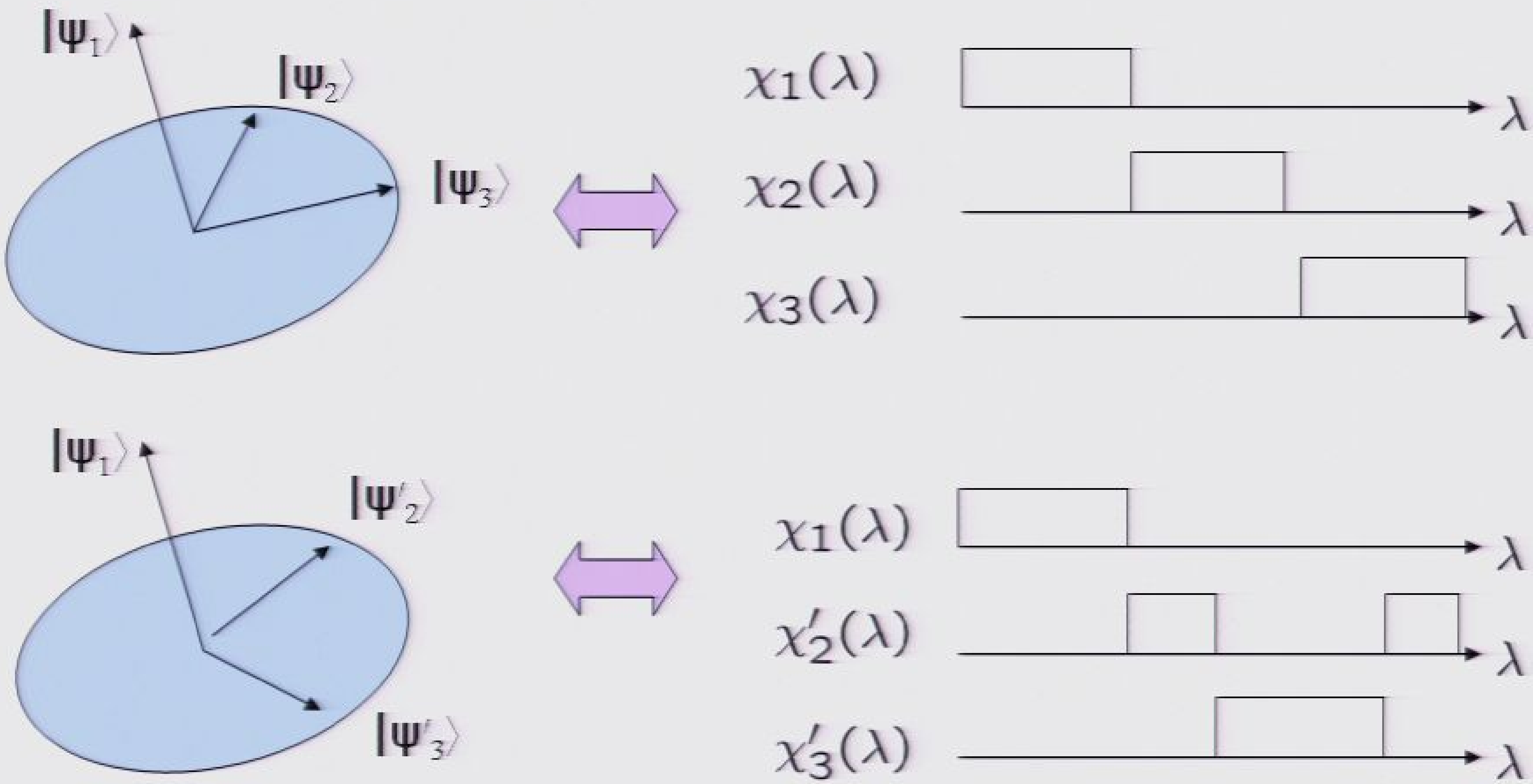
Representing coarse-graining of measurement outcomes:

Suppose the outcomes k of M are sorted into subsets S_j. Suppose M$'$ $\equiv$ implement M and upon obtaining outcome k, record the j such that $k \in S_j$.

Then $\chi'_j(\lambda) = \sum_{k \in S_j} \chi_k(\lambda)$

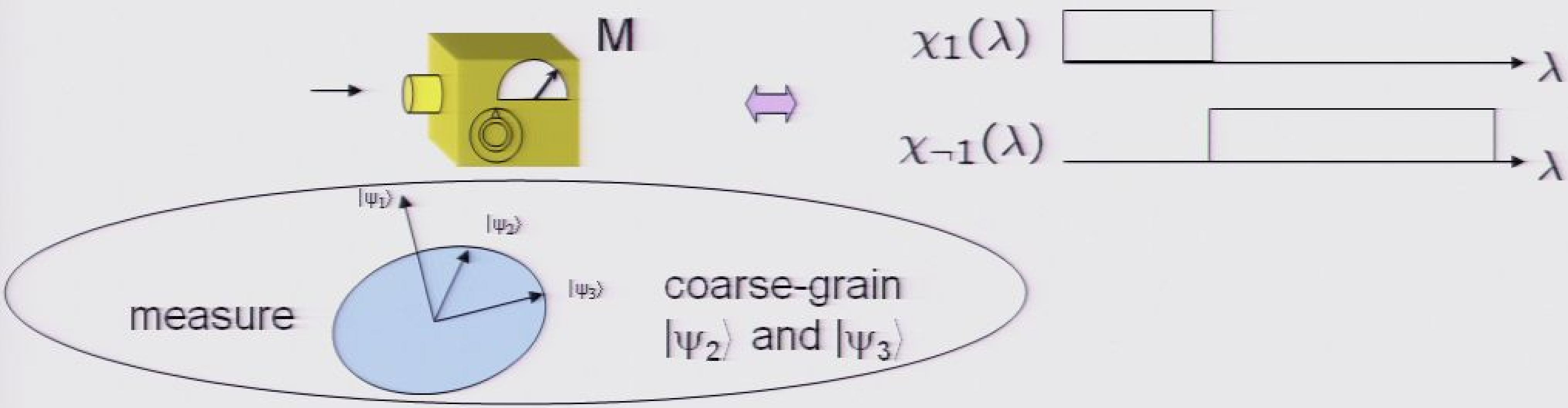


Recall the traditional notion of noncontextuality:



$\chi_1(\lambda)$ is the same in the two cases

This is equivalent to assuming:



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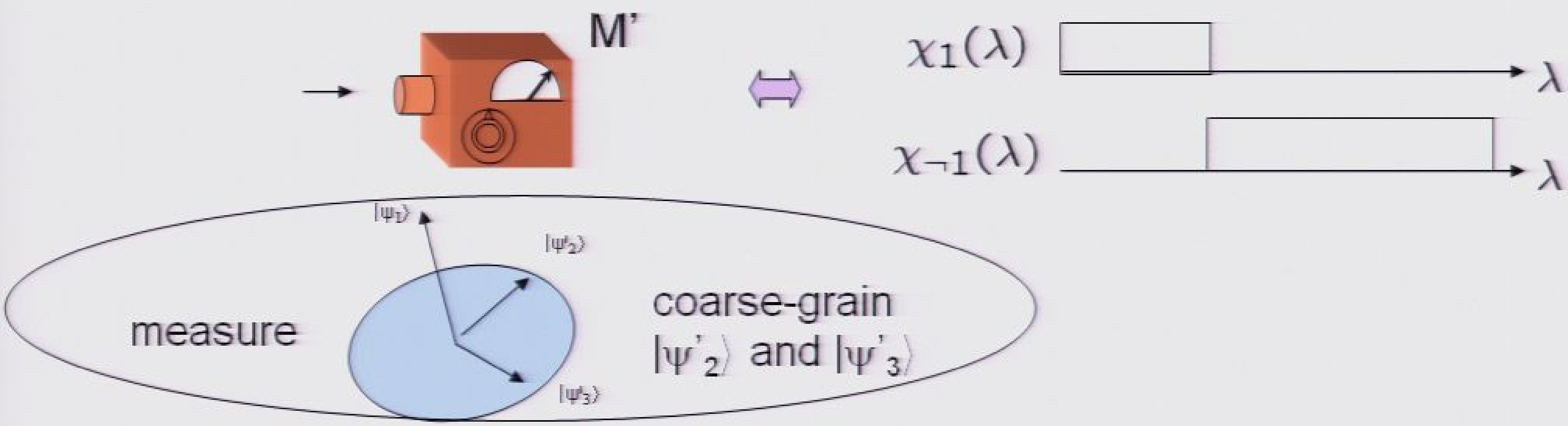

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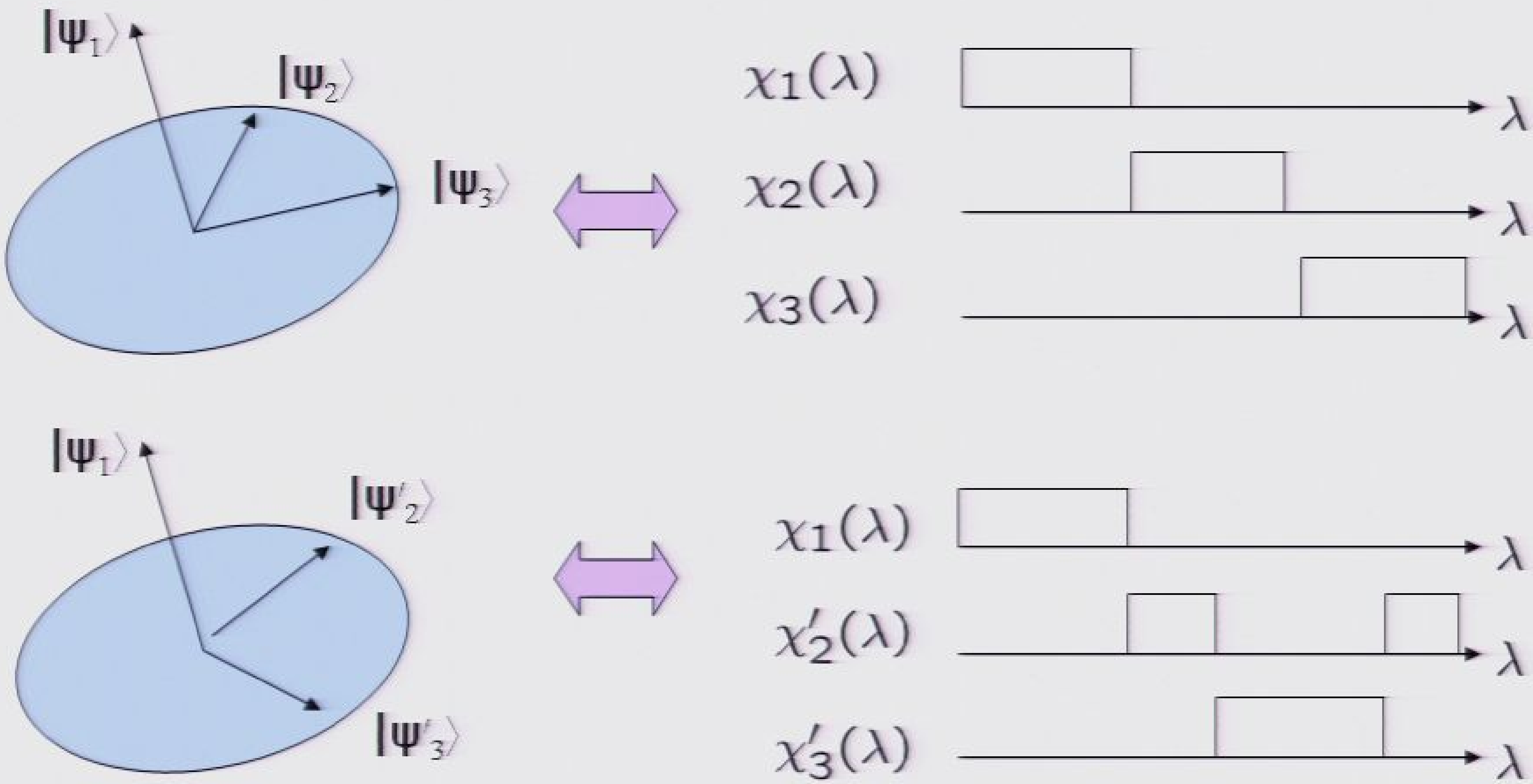

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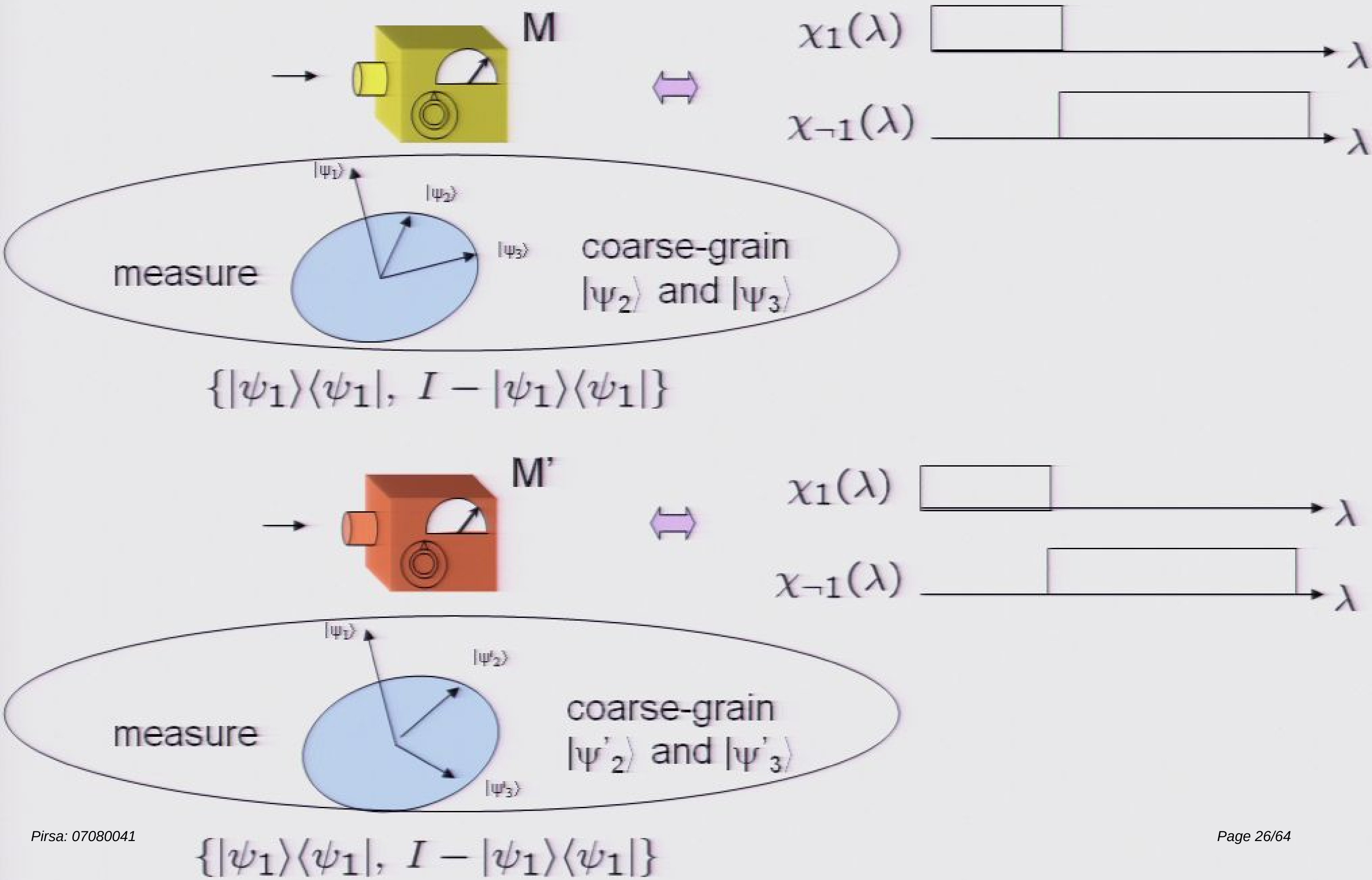

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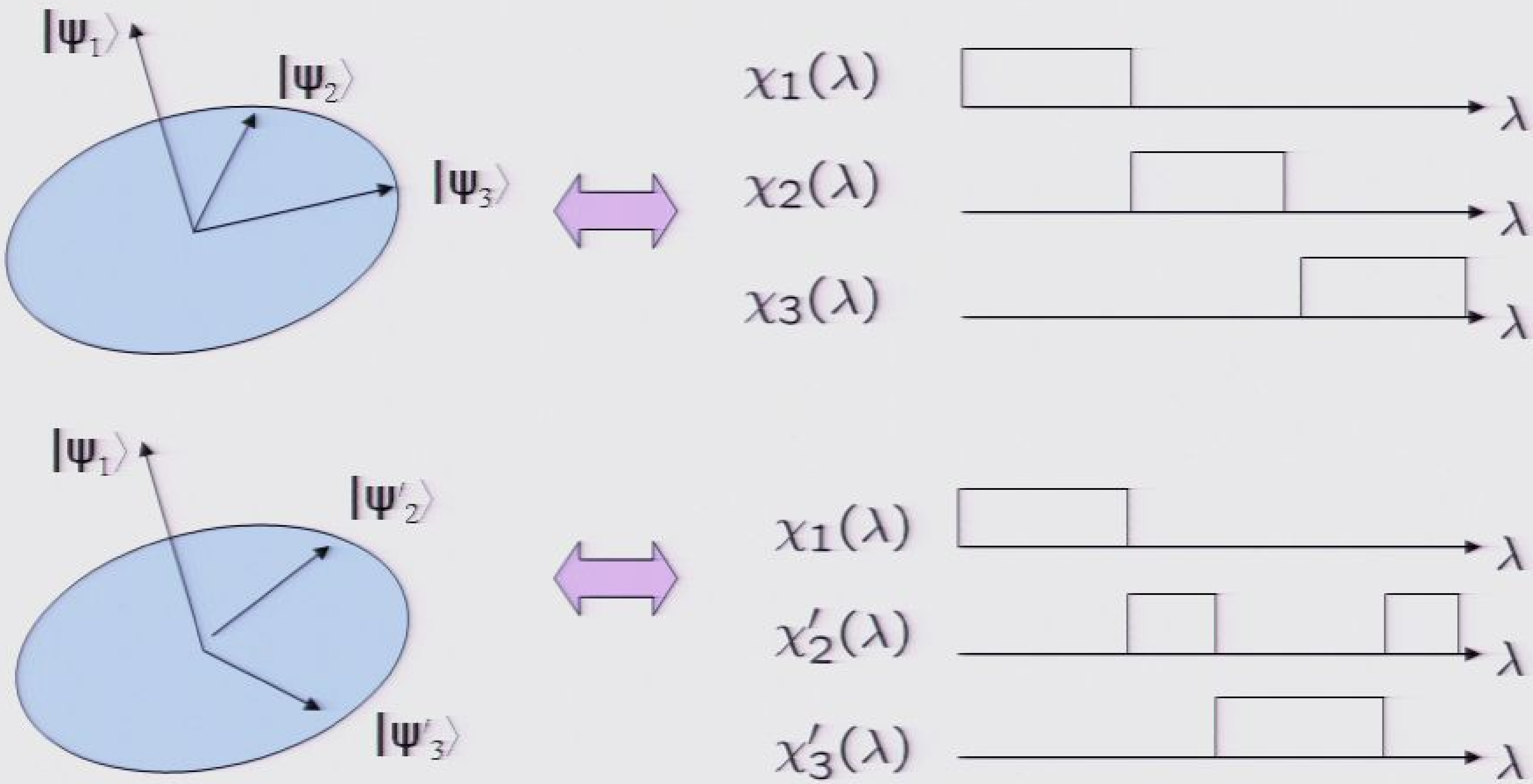

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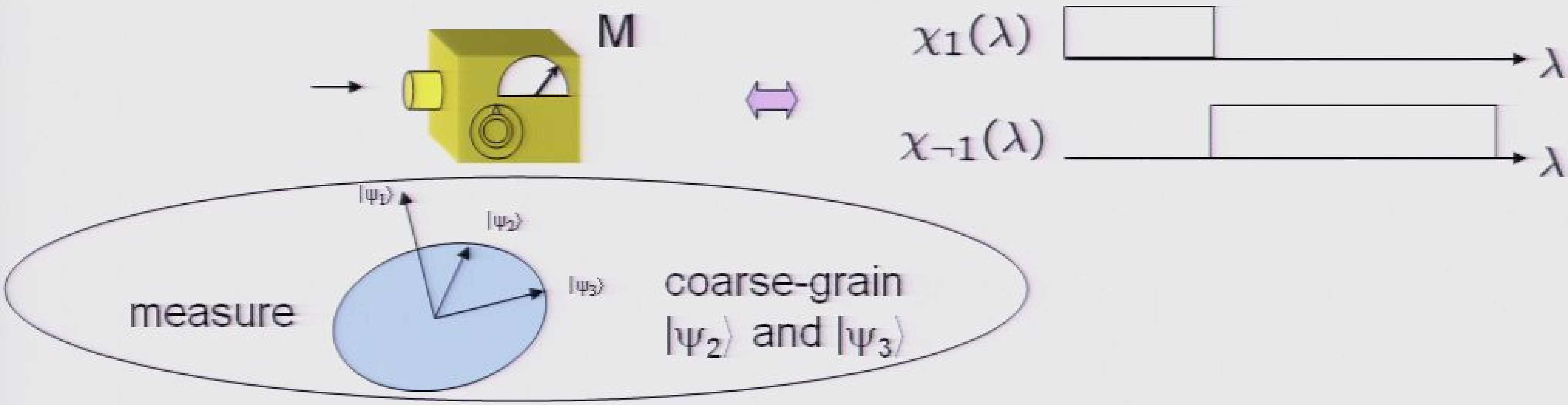

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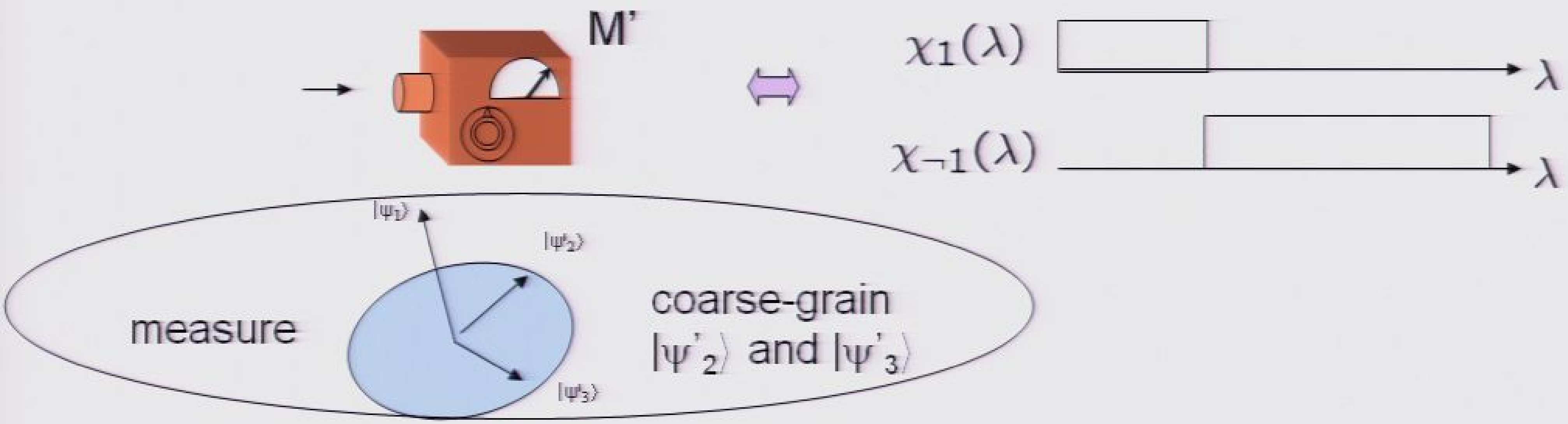

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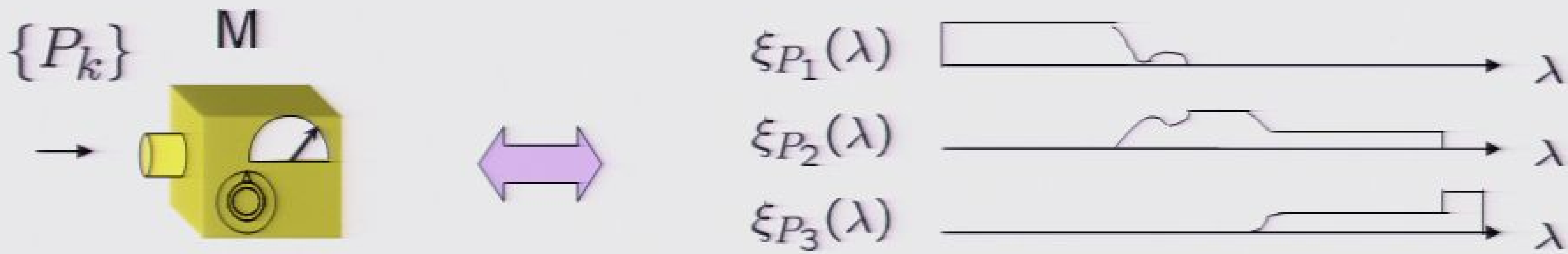

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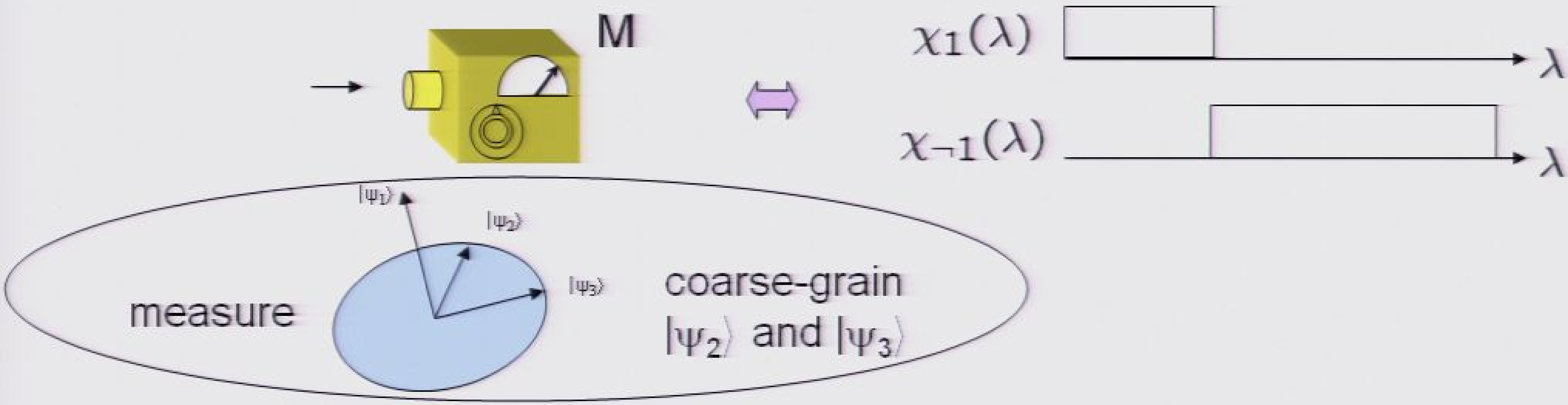

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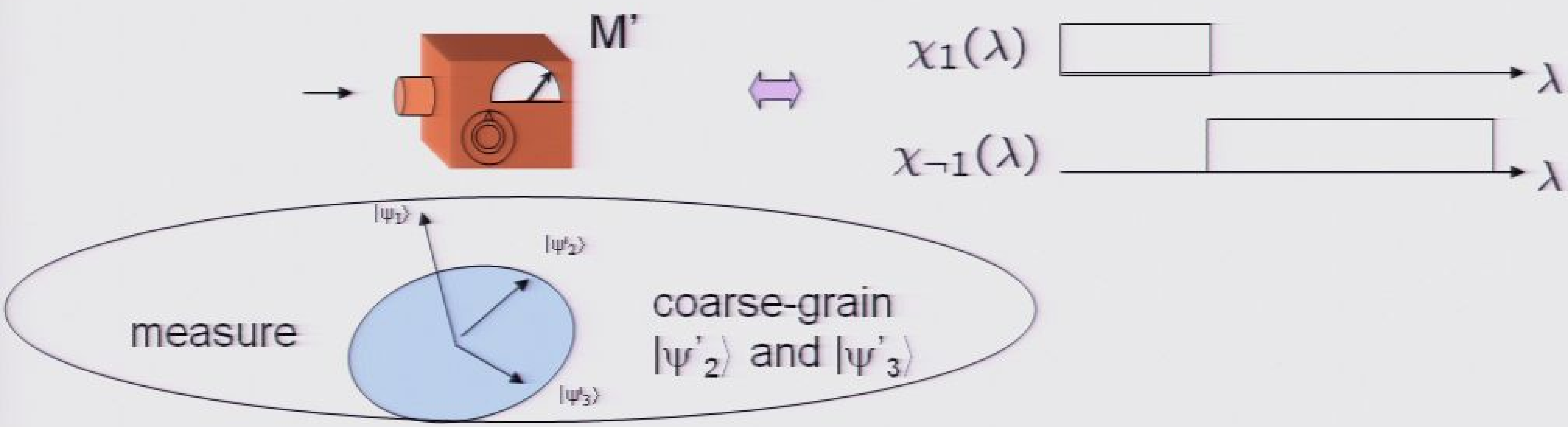

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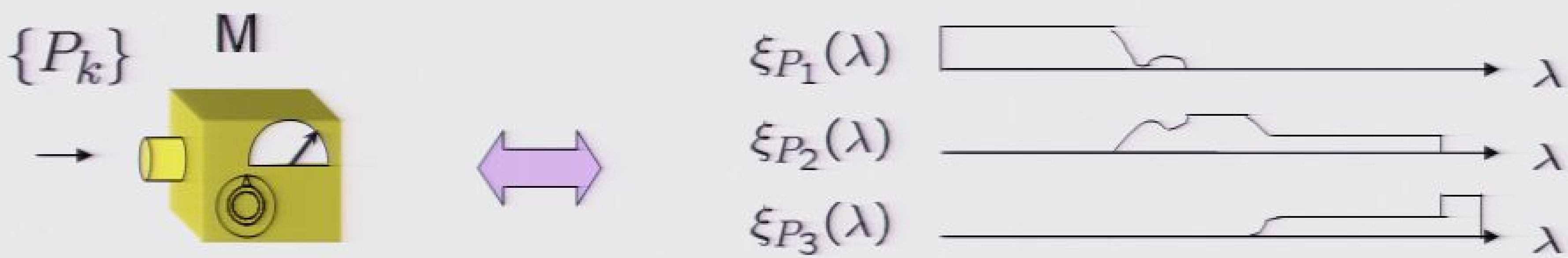

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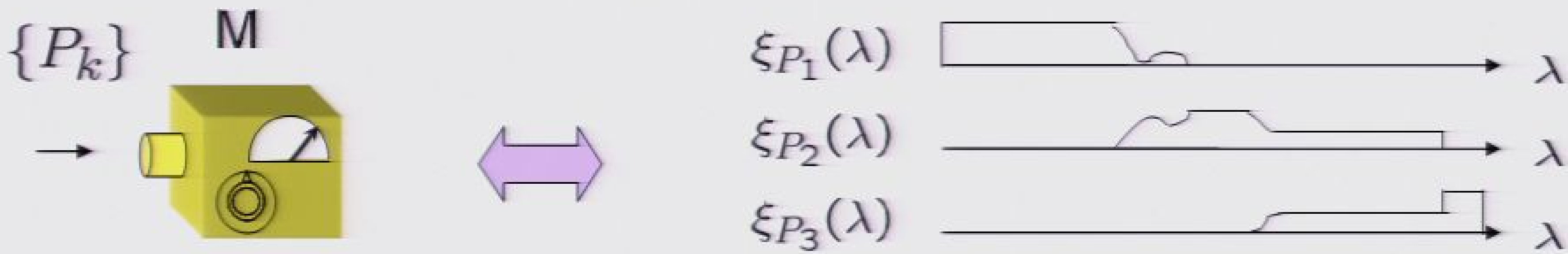

Therefore:

traditional notion of noncontextuality = revised notion of noncontextuality for sharp measurements

and

outcome determinism for sharp measurements

So, the proposed definition of noncontextuality is not simply a generalization of the traditional notion

For sharp measurements, it is a revision of the traditional notion

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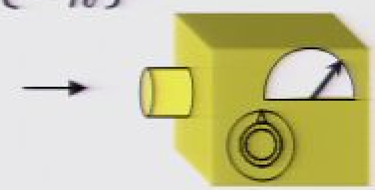
Noncontextuality and determinism are separate issues!

But recall that the most general representation was

$\{P_k\}$ M

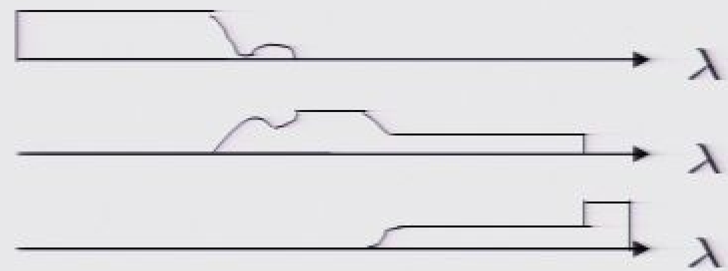


$\Leftrightarrow$

$\xi_{P_1}(\lambda)$

$\xi_{P_2}(\lambda)$

$\xi_{P_3}(\lambda)$



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No-go theorems for previous notion are not necessarily no-go theorems for the new notion!

In face of contradiction, could give up ODSM

But

preparation noncontextuality → outcome determinism for sharp measurements

But

preparation noncontextuality → outcome determinism for sharp measurements

Therefore:

measurement noncontextuality and preparation noncontextuality → noncontextuality for sharp measurements and outcome determinism for sharp measurements

But

preparation noncontextuality → outcome determinism for sharp measurements

Therefore:

measurement noncontextuality
and
preparation noncontextuality → Traditional notion of noncontextuality

But

preparation noncontextuality → outcome determinism for sharp measurements

Therefore:

measurement noncontextuality and preparation noncontextuality → Traditional notion of noncontextuality

no-go theorems for the traditional notion of noncontextuality can be salvaged as no-go theorems for the generalized notion

… and there are many new proofs, even in 2d

Is contextuality mysterious?

Is contextuality mysterious?

I would say YES.

Is contextuality mysterious?

I would say YES.

- There is a tension between
  the dependence of representation on certain details of the experimental procedure
  and
  the independence of outcome statistics on those details of the experimental procedure

Phenomena that are a form of generalized contextuality

- all variants of the Bell-Kochen-Specker theorem (algebraic, state-specific, statistical, continuous, discrete)

- all variants of Bell's theorem

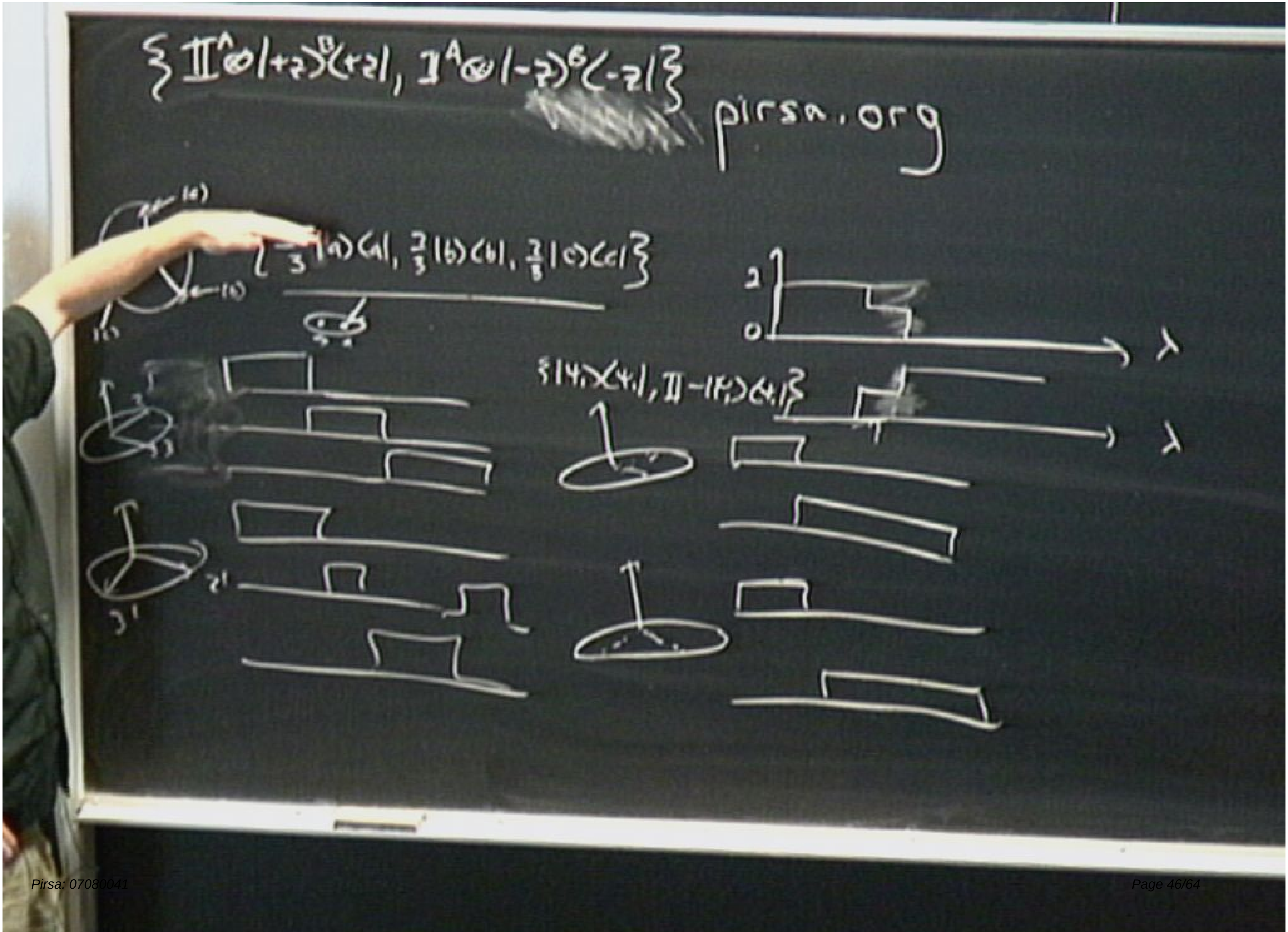
- all the novel no-go theorems, including the 2d ones (see RS, PRA **71**, 052108)

- Aspects of pre- and post-selected "paradoxes" (joint work with M. Leifer, PRL **95,** 200405)

-The necessity of having negativity in quasi-probability representations of quantum theory

- all variants of von Neumann's no-go theorem

- Quantum improvements in certain IP tasks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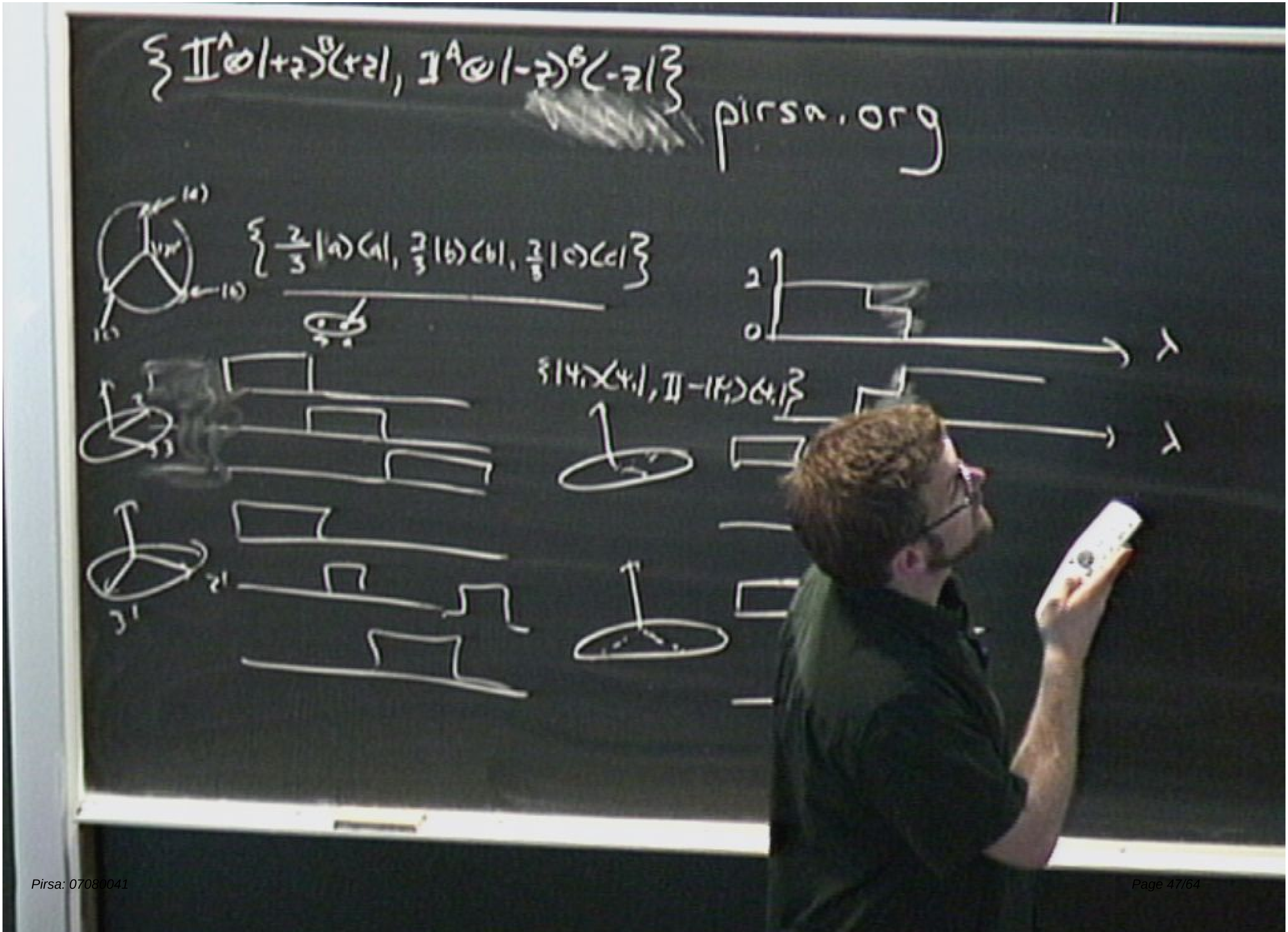
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Conclusions about contextuality

The notion of contextuality can and should be separated from that of outcome indeterminism

It can be extended to preparations and unsharp measurements.

It can be made operational and thus subject to experimental test

It powers better-than-classical performance of certain information-processing tasks

The generalized notion is seen to be an umbrella for many notions of nonclassicality

Open questions

What other notions of nonclassicality might be instances of contextuality? Fermionic statistics?

What other information-processing tasks might be powered by contextuality? Quantum computation?

Can we quantify contextuality as a resource?

Why isn't the world more contextual? For instance, why can't we implement **perfect** parity-oblivious 2-to-1 random access code?

What physical principle relieves the tension between the context-dependence at the hidden variable level and the lack of context-dependence at the operational level?

See: RS, Phys. Rev. A **71**, 052108 (2005); quant-ph/0406166

Do quantum states describe reality or our knowledge of reality?

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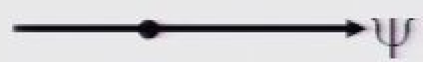
“But our present QM formalism is not purely epistemological; it is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature --- all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble. Yet we think that the unscrambling is a prerequisite for any further advance in basic physical theory. For, if we cannot separate the subjective and objective aspects of the formalism, we cannot know what we are talking about; it is just that simple.”

--E.T. Jaynes

ψ-complete vs. ψ-incomplete
ψ-ontic vs. ψ-epistemic

ψ-complete



Complete state is ψ

ψ-complete vs. ψ-incomplete
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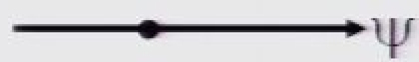

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Complete state is (ψ, λ)

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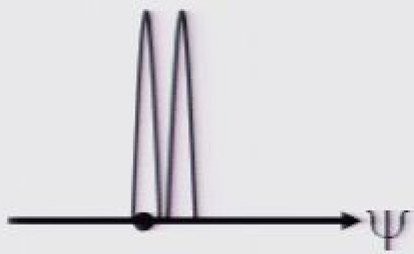

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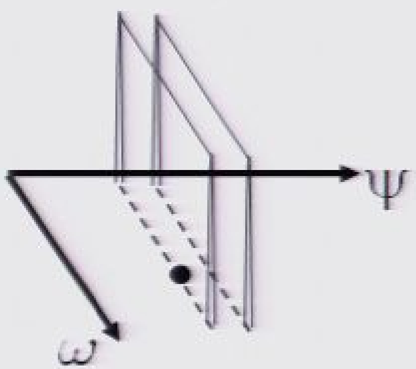

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ψ-complete model:

Space of physical states = space of rays in Hilbert space

$$\lambda = \psi$$

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For preparation procedures $P_{|\psi_1\rangle}, P_{|\psi_2\rangle}$ with $|\psi_1\rangle \neq |\psi_2\rangle$

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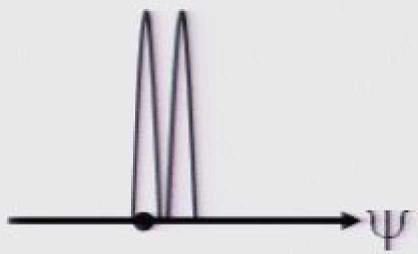

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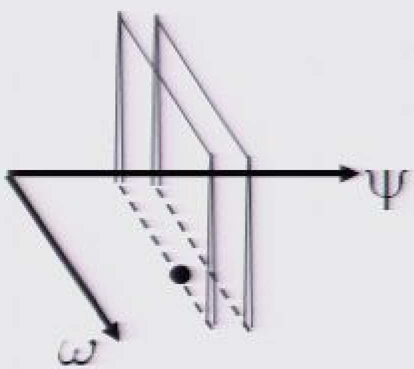

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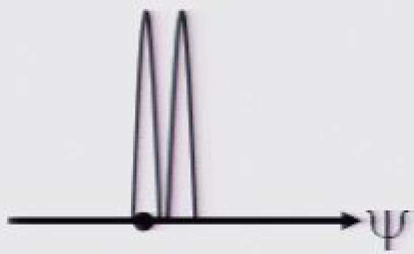

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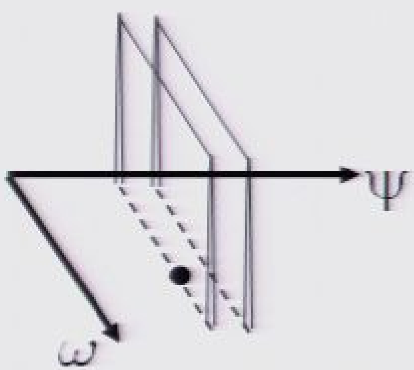

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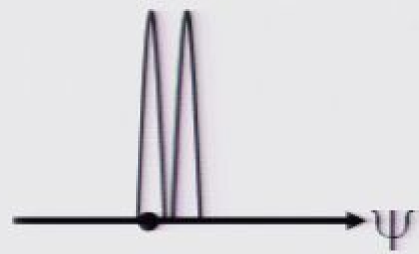

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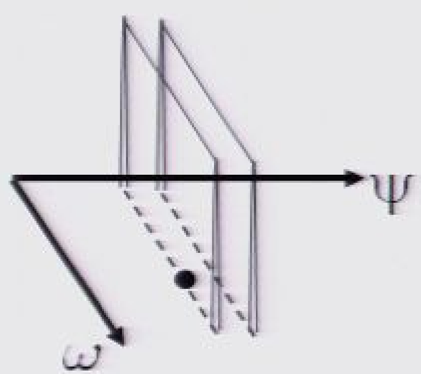

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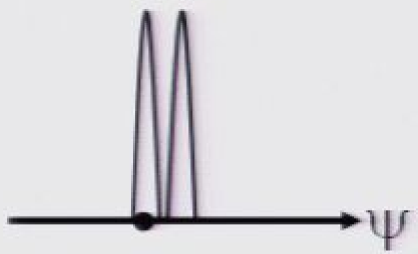

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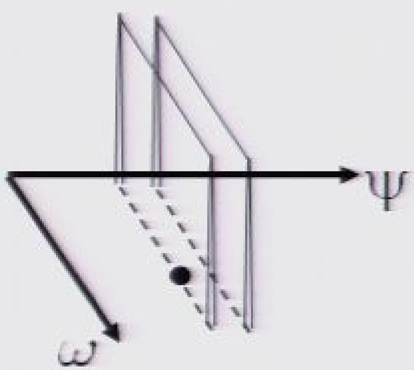

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