

Title: Operationalism, hidden variable models, and contextuality (Part 1A)

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Abstract:

Operationalism, Hidden Variable Models, and Contextuality

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QFSS, PI

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Society

Outline

Problems with the standard postulates of quantum theory and two strategies to deal with them

Realism and the traditional notion of a noncontextual hidden variable model of quantum theory

Operational quantum mechanics and operationalism in general

An operational definition of noncontextuality

- comparison with the traditional notion
- new no-go theorems
- what contextuality is useful for

Do quantum states describe reality or our knowledge of reality?

“Orthodox” postulates of quantum theory

The rays of Hilbert space ψ correspond one-to-one with the physical states of the system.

Measurements are associated with Hermitian operators $A = \sum_k a_k P_k$. Outcomes are indeterministic; a_k occurs with probability $\langle \psi | P_k | \psi \rangle$.

The physical state of an isolated system evolves unitarily, i.e. deterministically and continuously

If a measurement associated with $A = \sum_k a_k P_k$ yields outcome a_k , the physical state of the system changes discontinuously as:
 $|\psi\rangle \rightarrow P_k |\psi\rangle$.

Inconsistencies of the orthodox interpretation

By the collapse postulate
(applied to the system)

By unitary evolution postulate
(applied to isolated system that
includes the apparatus)

Indeterministic and
discontinuous evolution

Deterministic and
continuous evolution

Determinate properties

Indeterminate properties

Operationalism vs. Realism

More generally, the term “measurement” is not defined in terms of the more primitive “physical states of systems”. Isn’t a measurement just another kind of physical interaction?

Two strategies:

- (1) Realist strategy: Eliminate measurement as a primitive concept
Elements of the formalism represent reality or our knowledge of reality
- (2) Operational strategy: Eliminate “the physical state of a system” as a primitive concept
Elements of the formalism represent lists of instructions of what to do in the lab

“It would seem that the theory is exclusively concerned about "results of measurement", and has nothing to say about anything else. What exactly qualifies some physical systems to play the role of "measurer"? ”

- John Bell

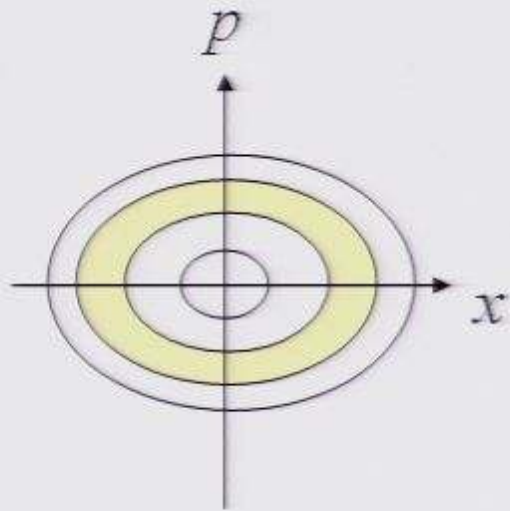
“In a strict sense, quantum theory is a set of rules allowing the computation of probabilities for the outcomes of tests which follow specified preparations.”

- Asher Peres

A realist strategy: Hidden variable models

The idea of a deterministic hidden variable model of quantum mechanics

In a classical theory, properties are associated with regions of the state space



$$E(x, p) = \frac{p^2}{2m} + \frac{kx^2}{2}$$

$$E_1 \leq E \leq E_2$$

Consider $\alpha(x, p) = a_1$ if $x < x_1$,
 $= a_2$ if $x_1 \leq x \leq x_2$,
 $= a_3$ if $x > x_2$.

Consider

$$\alpha(x, p) = a_1 \text{ if } x < x_1,$$

$$= a_2 \text{ if } x_1 \leq x \leq x_2,$$

$$= a_3 \text{ if } x > x_2.$$

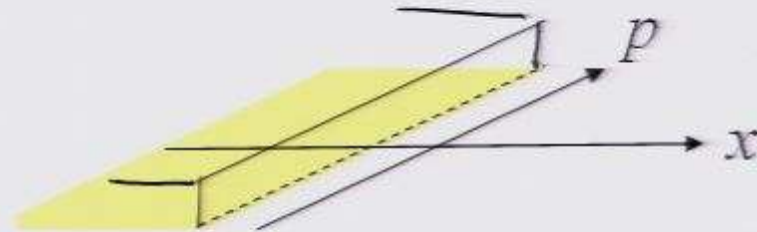
Equivalently,

$$\alpha(x, p) = \sum_k a_k \chi_k(x, p)$$

where

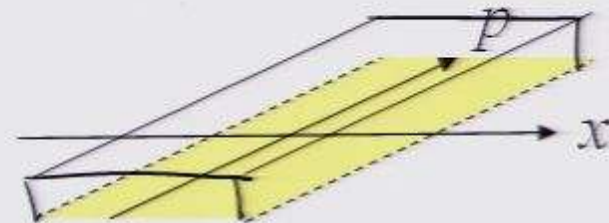
$$\chi_1(x, p) = 1 \text{ if } x < x_1$$

$$= 0 \text{ otherwise,}$$



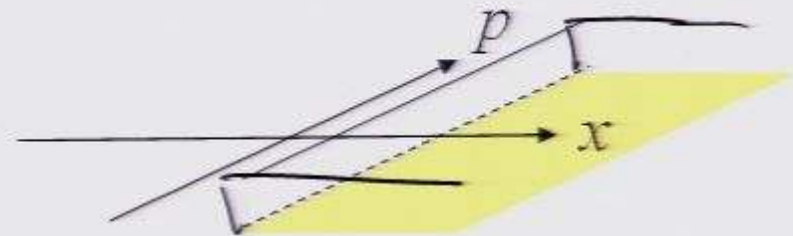
$$\chi_2(x, p) = 1 \text{ if } x_1 \leq x \leq x_2$$

$$= 0 \text{ otherwise,}$$

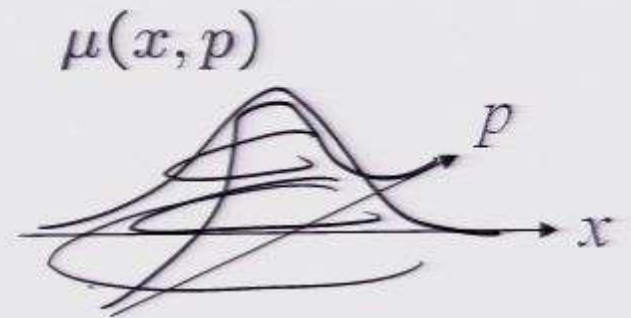


$$\chi_3(x, p) = 1 \text{ if } x > x_2$$

$$= 0 \text{ otherwise,}$$



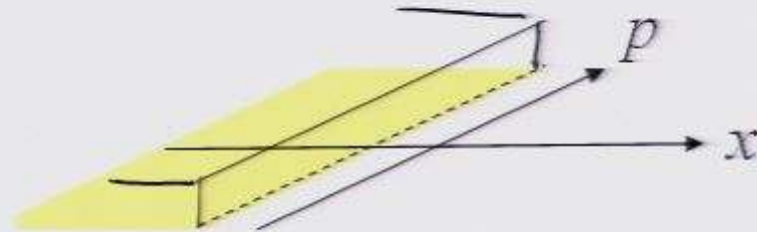
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 $= a_2$ if $x_1 \leq x \leq x_2$,
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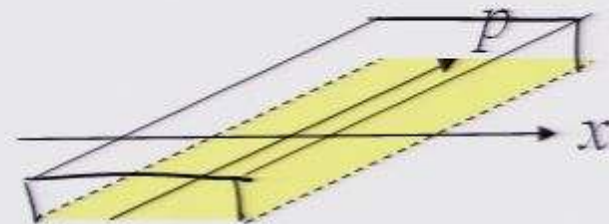
Equivalently, $\alpha(x, p) = \sum_k a_k \chi_k(x, p)$

where

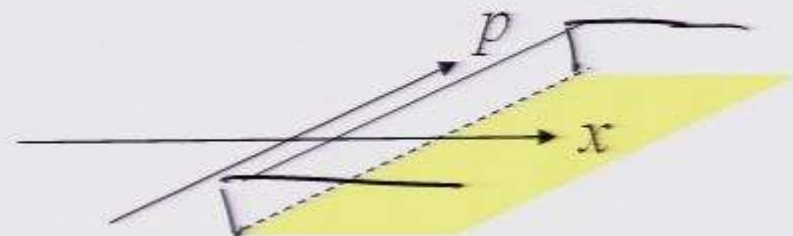
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 $= 0$ otherwise,



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 $= 0$ otherwise,



$\chi_3(x, p) = 1$ if $x > x_2$
 $= 0$ otherwise,



Can still have probabilistic outcomes if x, p is unknown

$$\text{Prob}(k) = \int dx dp \mu(x, p) \chi_k(x, p)$$

In quantum theory, we have

$$A = \sum_k a_k P_k$$

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$$p(A = a_k | |\psi\rangle) = \langle \psi | P_k | \psi \rangle$$

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The idea of a deterministic hidden variable theory is that

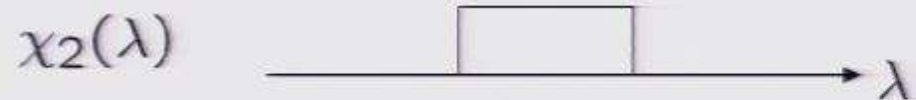
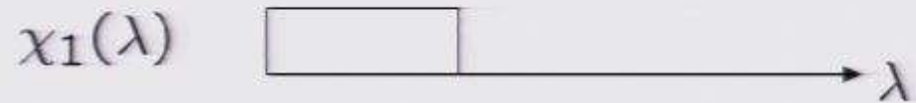
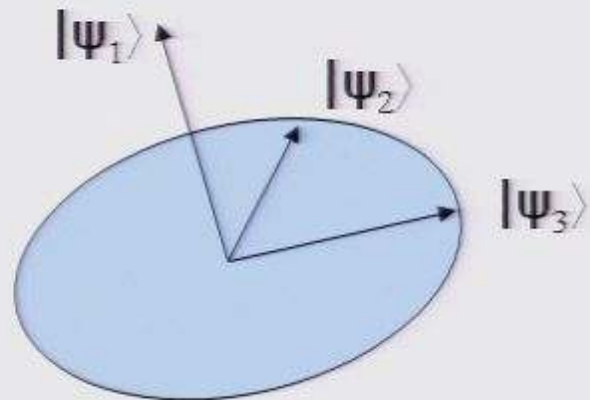
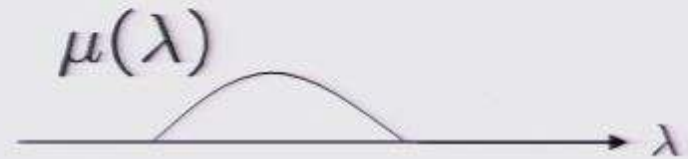
$$|\psi\rangle \leftrightarrow \mu(\lambda)$$

$$A \leftrightarrow \alpha(\lambda)$$

$$\{P_k\} \leftrightarrow \{\chi_k(\lambda)\}$$

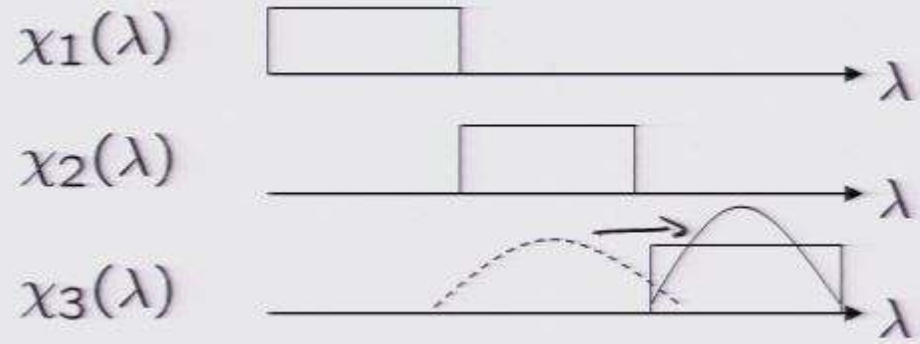
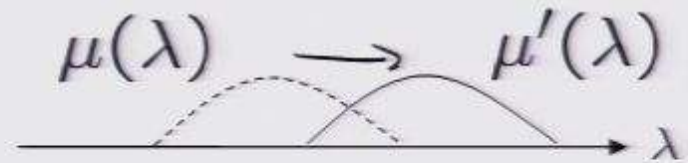
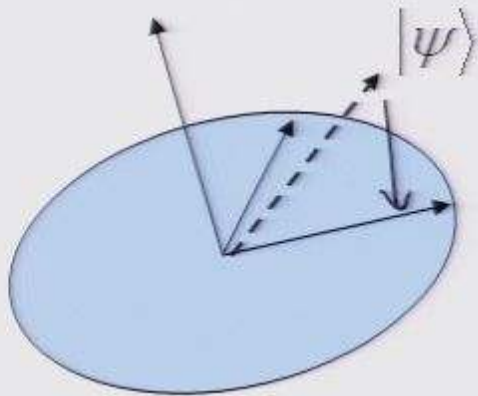
Such that

$$\langle \psi | P_k | \psi \rangle = \int d\lambda \mu(\lambda) \chi_k(\lambda)$$

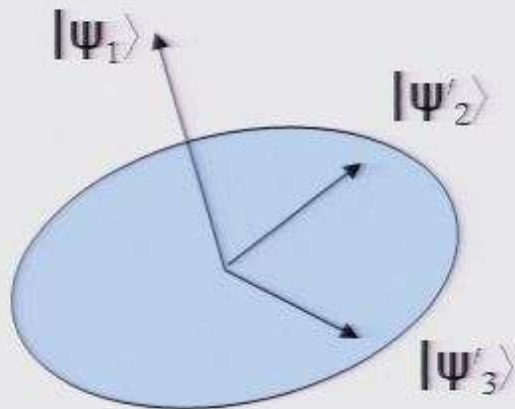
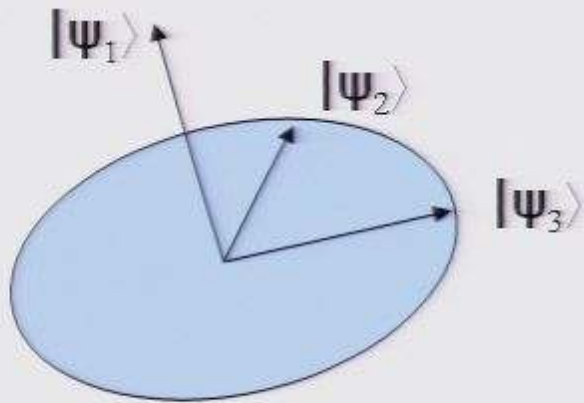


$$|\langle \psi | \chi_k \rangle|^2 = \int d\lambda \mu(\lambda) \chi_k(\lambda)$$

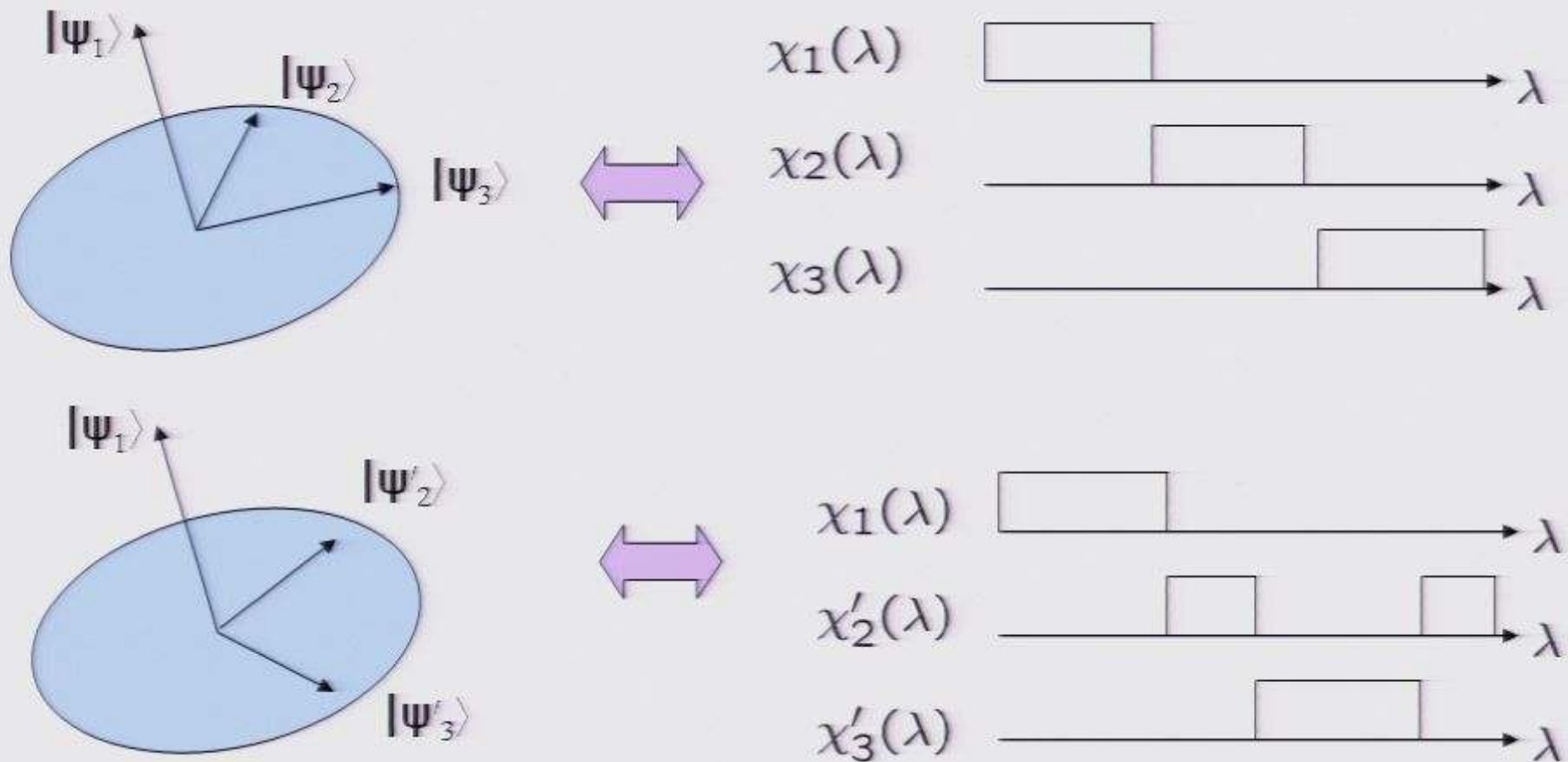
Aside:



A given projector may appear in many different measurements



A given projector may appear in many different measurements

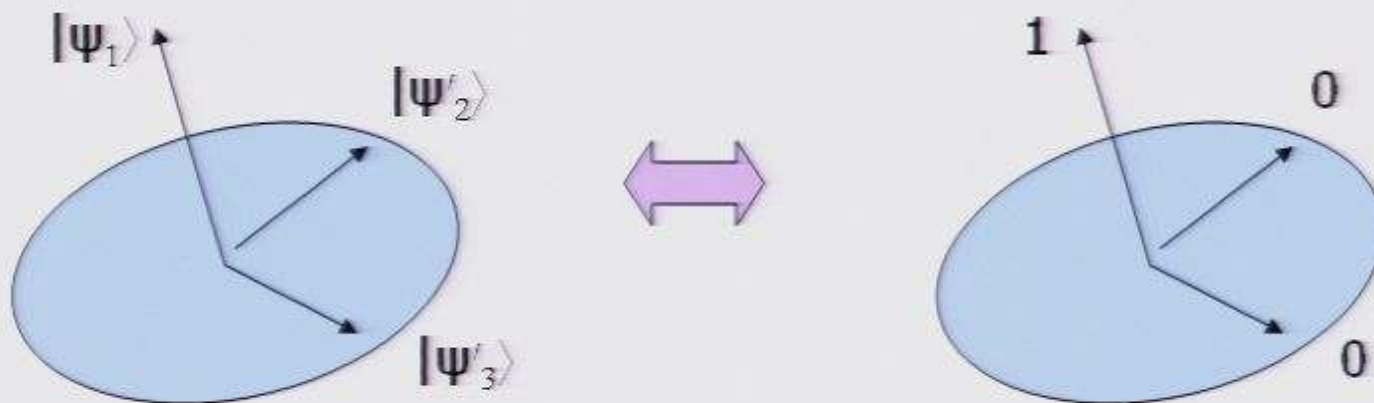
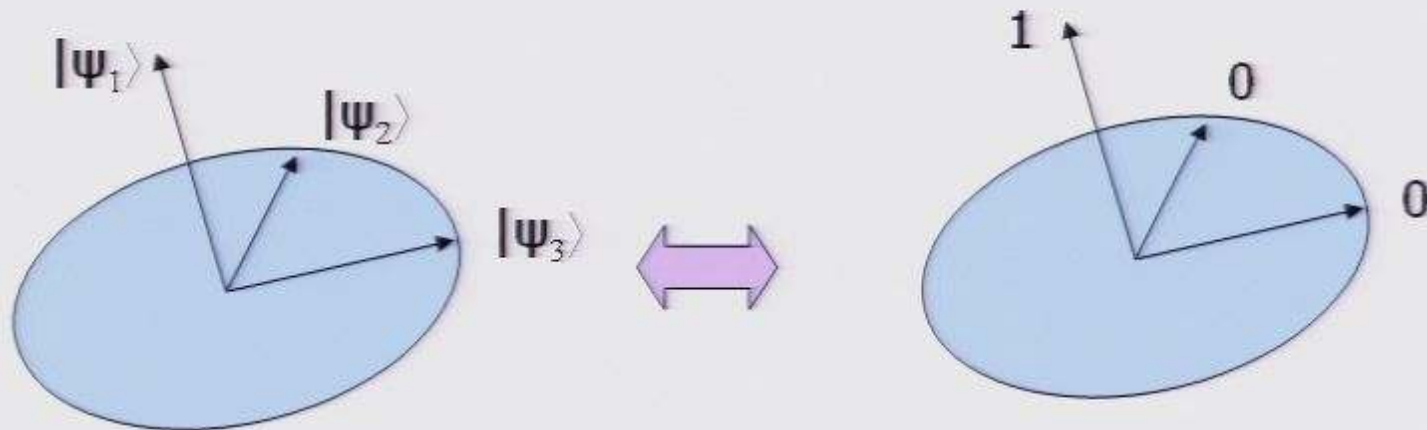


The **traditional notion of noncontextuality**:
 Every P is associated with the same $\chi(\lambda)$
 regardless of how it is measured

Alternatively, for a given λ

$$\chi_{\psi}(\lambda) = 0 \text{ or } 1$$

$$\sum_k \chi_{\psi_k}(\lambda) = 1$$



It was shown by Bell (1966) and Kochen and Specker (1967) that a noncontextual hidden variable model of quantum theory for Hilbert spaces of dimensionality 3 or greater is **impossible**. That is, **quantum theory is contextual**

This is the **Bell-Kochen-Specker theorem**

Example (Cabello's algebraic 18 ray proof in 4d):

Each of the 18 rays appears twice in the following list

0,0,0,1	0,0,0,1	1,-1,1,-1	1,-1,1,-1	0,0,1,0	1,-1,-1,1	1,1,-1,1	1,1,-1,1	1,1,1,-1
0,0,1,0	0,1,0,0	1,-1,-1,1	1,1,1,1	0,1,0,0	1,1,1,1	1,1,1,-1	-1,1,1,1	-1,1,1,1
1,1,0,0	1,0,1,0	1,1,0,0	1,0,-1,0	1,0,0,1	1,0,0,-1	1,-1,0,0	1,0,1,0	1,0,0,1
1,-1,0,0	1,0,-1,0	0,0,1,1	0,1,0,-1	1,0,0,-1	0,1,-1,0	0,0,1,1	0,1,0,-1	0,1,-1,0

Example (Cabello's algebraic 18 ray proof in 4d):

Each of the 18 rays appears twice in the following list

0,0,0,1	0,0,0,1	1,-1,1,-1	1,-1,1,-1	0,0,1,0	1,-1,-1,1	1,1,-1,1	1,1,-1,1	1,1,1,-1
0,0,1,0	0,1,0,0	1,-1,-1,1	1,1,1,1	0,1,0,0	1,1,1,1	1,1,1,-1	-1,1,1,1	-1,1,1,1
1,1,0,0	1,0,1,0	1,1,0,0	1,0,-1,0	1,0,0,1	1,0,0,-1	1,-1,0,0	1,0,1,0	1,0,0,1
1,-1,0,0	1,0,-1,0	0,0,1,1	0,1,0,-1	1,0,0,-1	0,1,-1,0	0,0,1,1	0,1,0,-1	0,1,-1,0

In each of the 9 columns, one ray is assigned 1, the other three 0
Therefore, 9 rays must be assigned 1

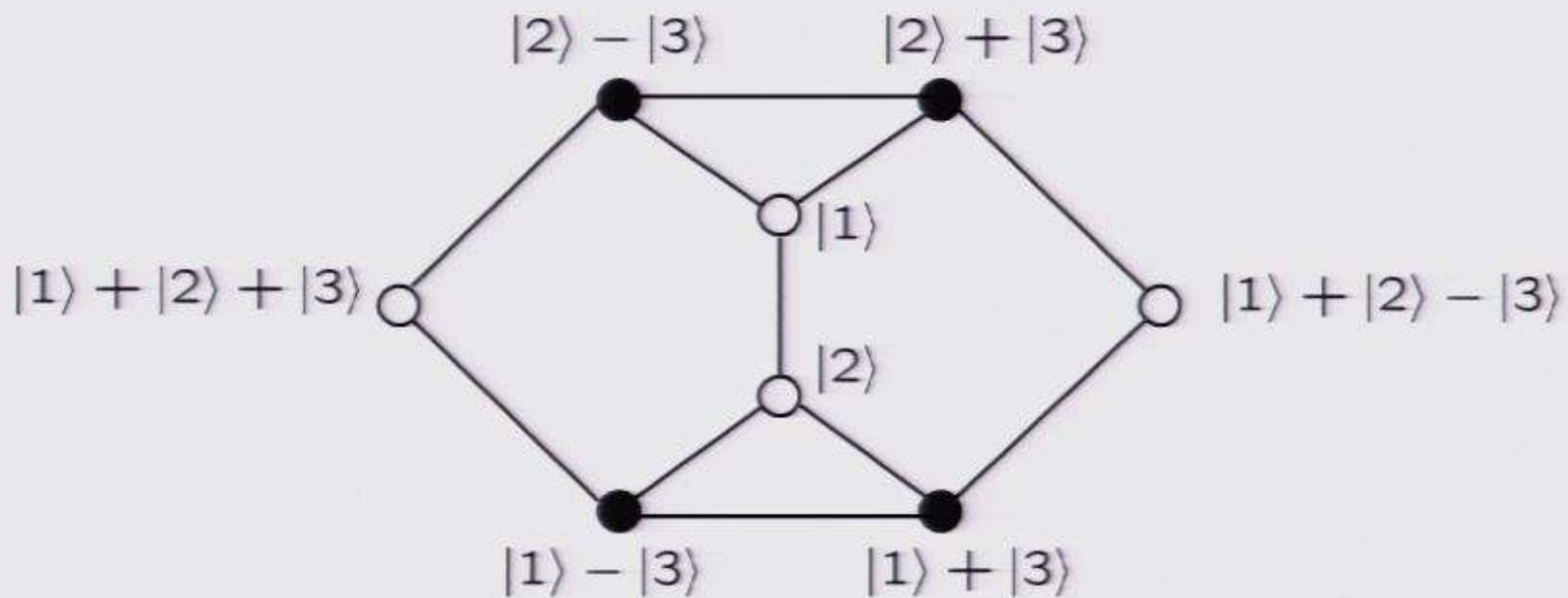
But each ray appears twice and so there must be an even number
of rays assigned 1

CONTRADICTION!

Example (Clifton's statistical 8 ray proof in 3d):

$$|\psi\rangle \circ \rightarrow \chi_{|\psi\rangle}(\lambda) = 1$$

$$|\psi\rangle \bullet \rightarrow \chi_{|\psi\rangle}(\lambda) = 0$$



CONTRADICTION!

Problems with the traditional definition of noncontextuality:

- applies only to deterministic hidden variable models
- applies only to quantum theory

Can we define it in such a way that we can judge

- whether any given theory is contextual or not
- whether any given data requires contextuality for its explanation

Yes, by being operational.

A purely operational formulation of quantum theory

Operational Quantum Mechanics



Preparation

Transformation

Measurement

P

T

M

Vector

Unitary map

Projector-valued
measure (PVM)

$|\psi\rangle$

U

$\{P_k\}$

$$Pr(k|P, T, M) = \langle \psi | U^\dagger P_k U | \psi \rangle$$

More general preparations

Probability p , prepare $|\psi\rangle$

Probability q , prepare $|\chi\rangle$

Measure $\{P_k\}$

$$\begin{aligned}\text{Prob}(k) &= p\langle\psi|P_k|\psi\rangle + q\langle\chi|P_k|\chi\rangle \\ &= p\text{Tr}(|\psi\rangle\langle\psi|P_k) + q\text{Tr}(|\chi\rangle\langle\chi|P_k) \\ &= \text{Tr}(\rho P_k) \\ \rho &= p|\psi\rangle\langle\psi| + q|\chi\rangle\langle\chi|\end{aligned}$$

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$$\rho = p|\psi\rangle\langle\psi| + q|\chi\rangle\langle\chi|$$

A density operator

$$\rho \in \mathcal{L}(\mathbb{C}_d)$$

$$\langle\psi|\rho|\psi\rangle \geq 0, \forall\psi$$

$$\text{Tr}(\rho) = 1$$

$$\rho = |\psi\rangle\langle\psi| \quad \Leftrightarrow \text{Pure preparation}$$

$$\rho \neq |\psi\rangle\langle\psi| \quad \Leftrightarrow \text{Mixed preparation}$$

More general measurements

Prepare ρ

Probability p , measure the PVM $\{P_k\}$

Probability q , measure the PVM $\{Q_k\}$

$$\begin{aligned}\text{Prob}(k) &= p \text{Tr}(\rho P_k) + q \text{Tr}(\rho Q_k) \\ &= \text{Tr}(\rho E_k)\end{aligned}$$

$$E_k = pP_k + qQ_k$$

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$$E_k = pP_k + qQ_k$$

A Positive operator valued measure (POVM)

$$E_k \in \mathcal{L}(\mathbb{C}_d)$$

$$\langle \psi | E_k | \psi \rangle \geq 0, \forall \psi$$

$$\sum_{k=1}^d E_k = I$$

$\{E_k\} = \{P_k\} \Leftrightarrow$ Sharp measurement

$\{E_k\} \neq \{P_k\} \Leftrightarrow$ Unsharp measurement

More general transformations

Prepare ρ

Probability p , transform with U

Probability q , transform with V

measure $\{E_k\}$

$$\begin{aligned}\text{Prob}(k) &= p \text{Tr}(U \rho U^\dagger E_k) + q \text{Tr}(V \rho V^\dagger E_k) \\ &= \text{Tr}(\mathcal{T}(\rho) E_k)\end{aligned}$$

$$\mathcal{T}(\cdot) = pU(\cdot)U^\dagger + qV(\cdot)V^\dagger$$

More general transformations

Prepare ρ

Probability p , transform with U

Probability q , transform with V

measure $\{E_k\}$

$$\begin{aligned}\text{Prob}(k) &= p \text{Tr}(U \rho U^\dagger E_k) + q \text{Tr}(V \rho V^\dagger E_k) \\ &= \text{Tr}(\mathcal{T}(\rho) E_k)\end{aligned}$$

$$\mathcal{T}(\cdot) = pU(\cdot)U^\dagger + qV(\cdot)V^\dagger$$

A completely positive map (CP map)

$$\mathcal{T} : \mathcal{L}(\mathbb{C}_d) \rightarrow \mathcal{L}(\mathbb{C}_d)$$

$$\mathcal{T}(\rho) = \sum_{\mu} A_{\mu} \rho A_{\mu}^{\dagger}$$

$$\sum_{\mu} A_{\mu}^{\dagger} A_{\mu} = I$$

$$\mathcal{T}(\rho) = U \rho U^{\dagger} \Leftrightarrow \text{Reversible transformation}$$

$$\mathcal{T}(\rho) \neq U \rho U^{\dagger} \Leftrightarrow \text{Irreversible transformation}$$

Operational Quantum Mechanics



Preparation

Transformation

Measurement

P

T

M

Density operator

Trace-preserving
completely positive
linear map (CP map)

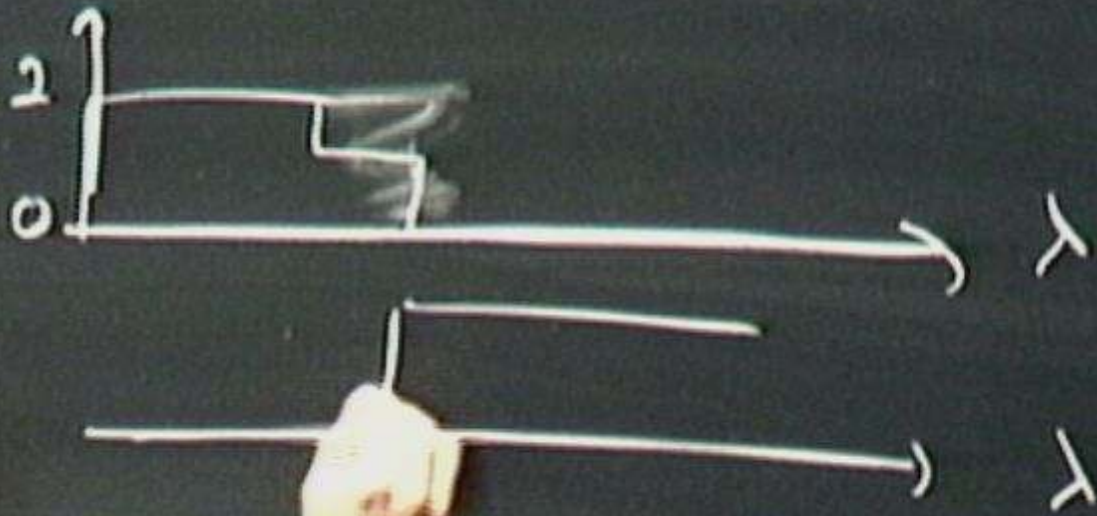
Positive operator-valued
measure (POVM)

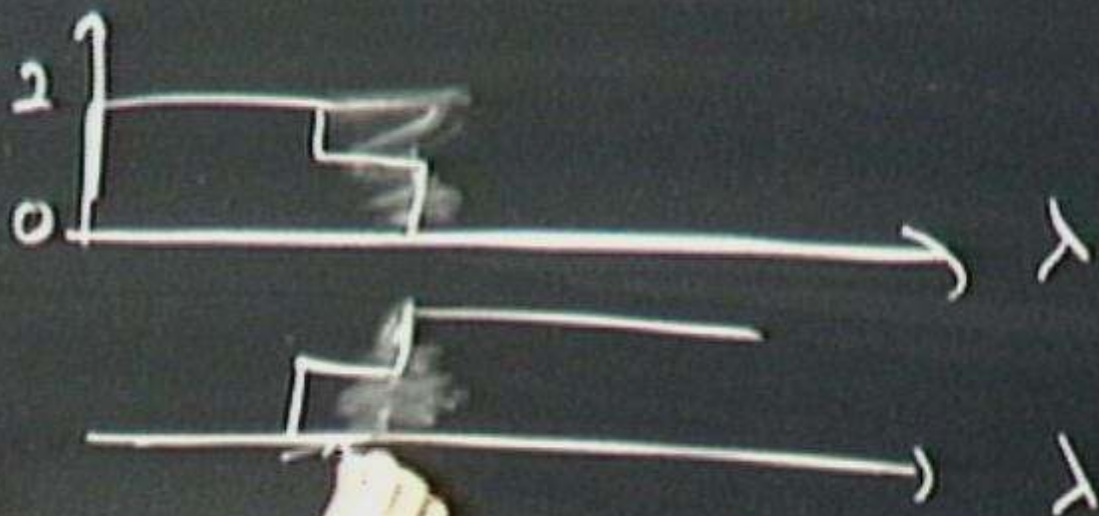
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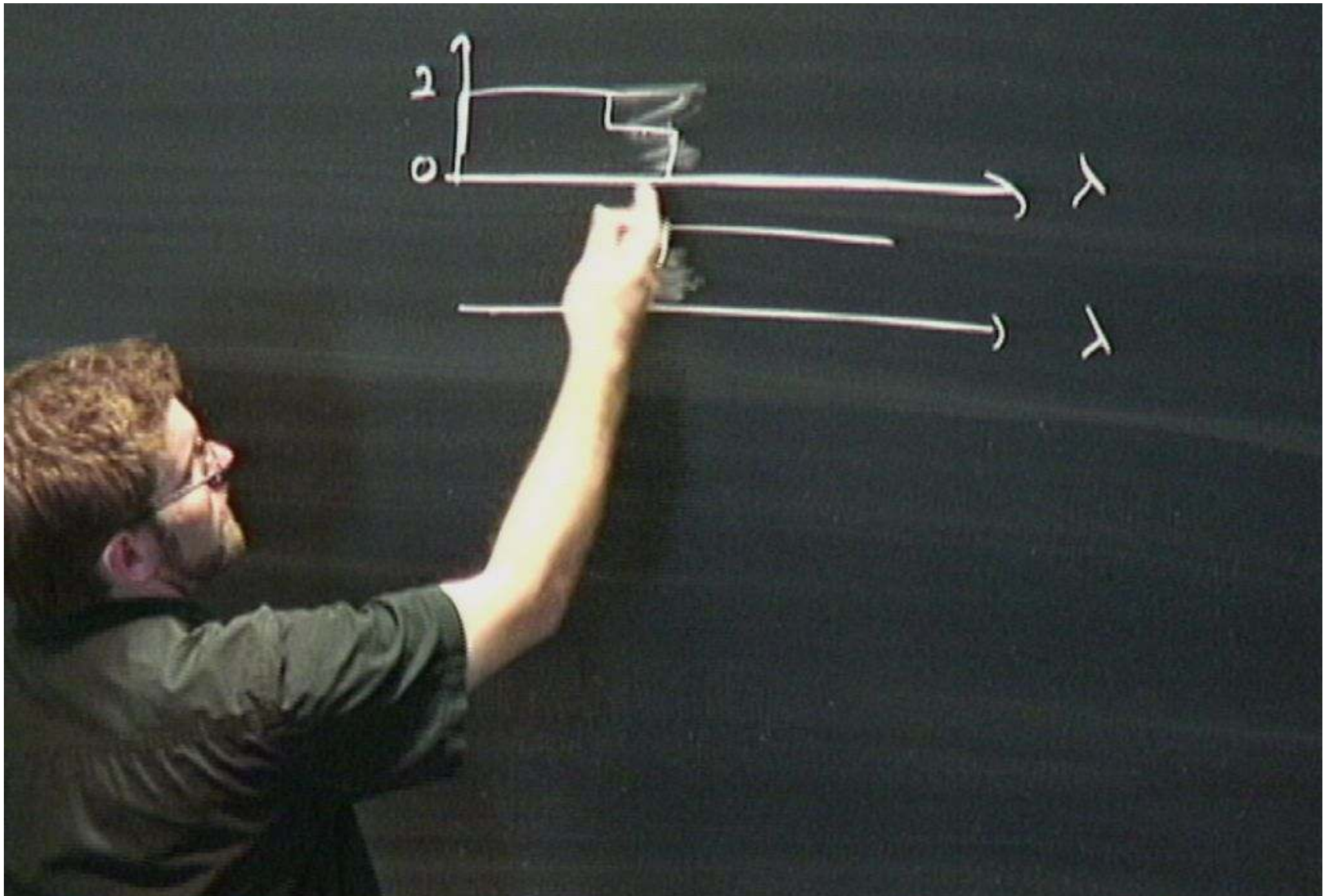
\mathcal{T}

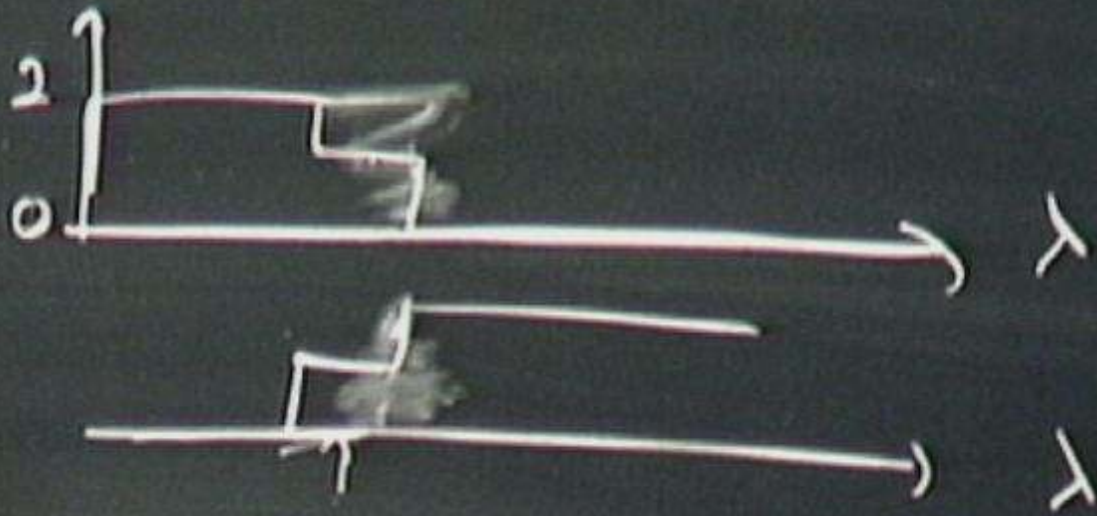
$\{E_k\}$

$$Pr(k|P, T, M) = \text{Tr}[E_k \mathcal{T}(\rho)]$$









More general preparations

Probability p , prepare $|\psi\rangle$

Probability q , prepare $|\chi\rangle$

Measure $\{P_k\}$

$$\begin{aligned}\text{Prob}(k) &= p\langle\psi|P_k|\psi\rangle + q\langle\chi|P_k|\chi\rangle \\ &= p\text{Tr}(|\psi\rangle\langle\psi|P_k) + q\text{Tr}(|\chi\rangle\langle\chi|P_k) \\ &= \text{Tr}(\rho P_k)\end{aligned}$$

$$\rho = p|\psi\rangle\langle\psi| + q|\chi\rangle\langle\chi|$$

A density operator

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$$\rho = |\psi\rangle\langle\psi| \quad \Leftrightarrow \text{Pure preparation}$$

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Operational Quantum Mechanics



Preparation

Transformation

Measurement

P

T

M

Density operator

Trace-preserving
completely positive
linear map (CP map)

Positive operator-valued
measure (POVM)

ρ

\mathcal{T}

$\{E_k\}$

$$Pr(k|P, T, M) = \text{Tr}[E_k \mathcal{T}(\rho)]$$

Operational postulates of quantum theory

Every preparation P is associated with a density operator ρ

Every measurement M is associated with a positive operator-valued measure $\{E_k\}$. The probability of M yielding outcome k given a preparation P is $p_k = \text{Tr}(E_k\rho)$.

Every transformation is associated with a trace-preserving completely-positive linear map $\rho \rightarrow \rho' = T(\rho)$,

Every measurement outcome k is associated with a trace-nonincreasing completely-positive linear map $\rho \rightarrow \rho' = T_k(\rho)$.

The formalism of density operators, POVMs, and CP maps is critical in quantum information theory and arguably quantum foundations as well.

Operational alternatives to quantum theory

Axiomatization

L. Hardy, "Quantum theory from five reasonable axioms", quant-ph/0101012

Foil theories

J. Barrett, "Information Processing in Generalized Probabilistic Theories", quant-ph/0508211

RS, "In defense of the epistemic view of quantum states: a toy theory" quant-ph/0401052

For more references, see:

<http://qubit.damtp.cam.ac.uk/users/rob/foilswebpage.htm>

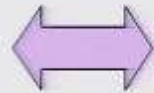
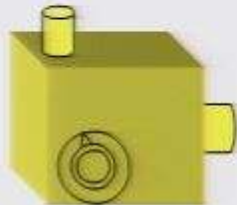
Operationalism as a tool for the realist:

Devising a theory-independent
definition of contextuality

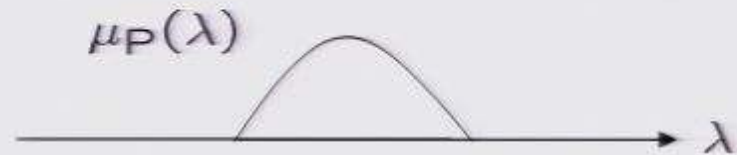
A hidden variable model of an operational theory assumes primitives of systems and properties

Preparation

P



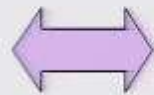
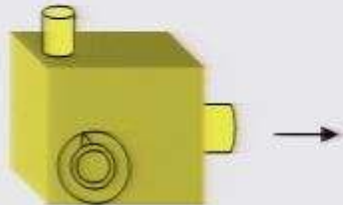
$$\int \mu_P(\lambda) d\lambda = 1$$



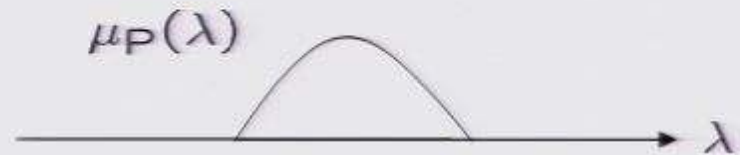
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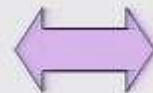
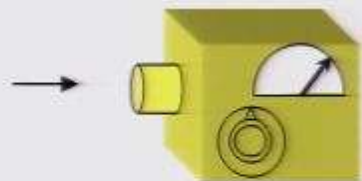


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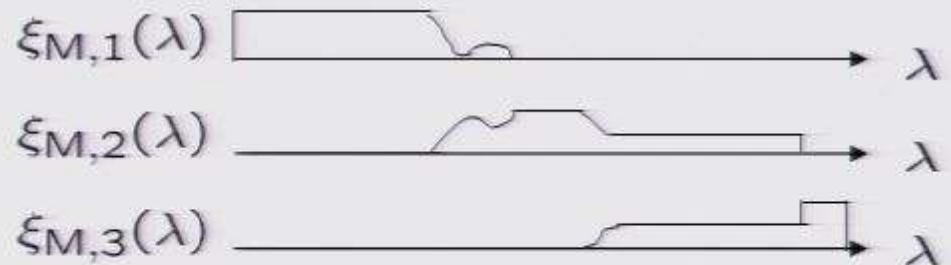
Measurement

M



$$0 \leq \xi_{M,k} \leq 1$$

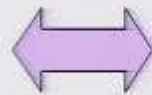
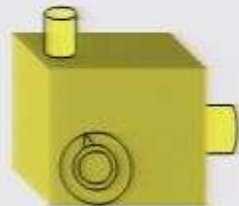
$$\sum_k \xi_{M,k}(\lambda) = 1 \text{ for all } \lambda$$



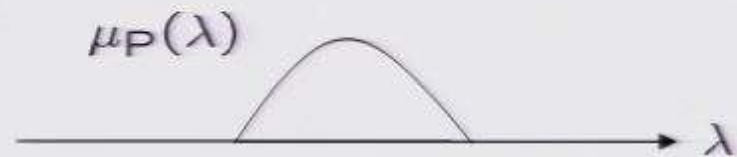
A hidden variable model of an operational theory assumes primitives of systems and properties

Preparation

P

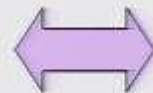
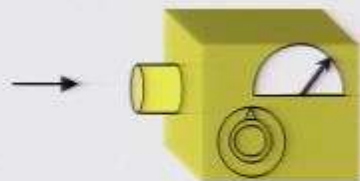


$$\int \mu_P(\lambda) d\lambda = 1$$



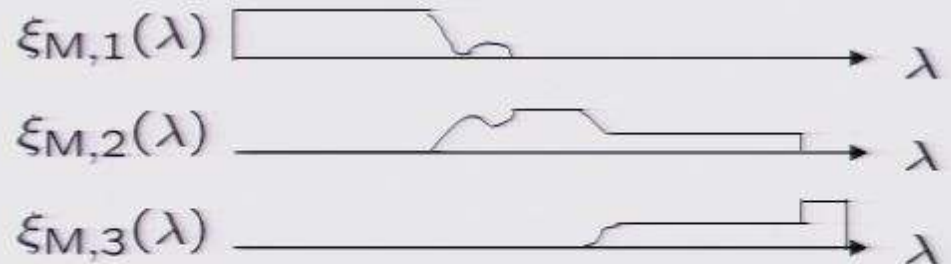
Measurement

M



$$0 \leq \xi_{M,k} \leq 1$$

$$\sum_k \xi_{M,k}(\lambda) = 1 \text{ for all } \lambda$$



$$p(k|P, M) = \int d\lambda \xi_{M,k}(\lambda) \mu_P(\lambda)$$

Proposed new definition of noncontextuality:

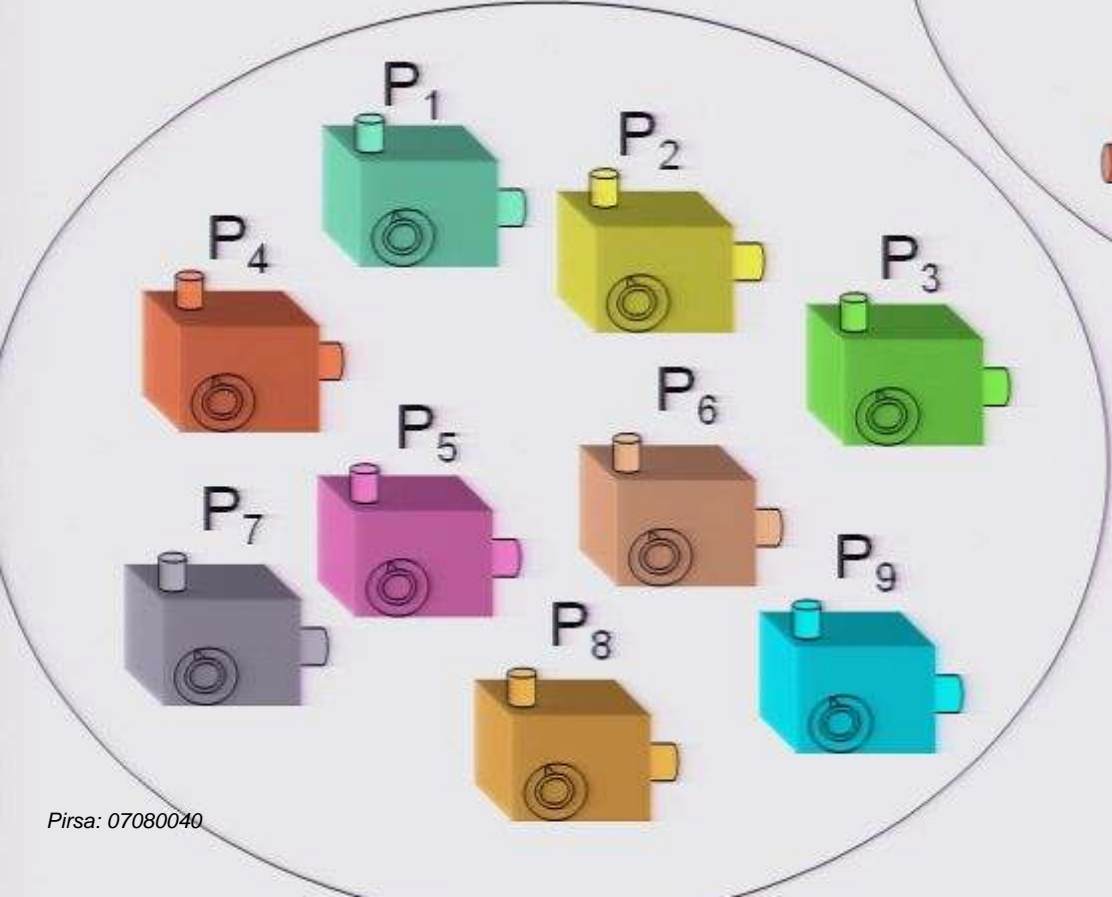
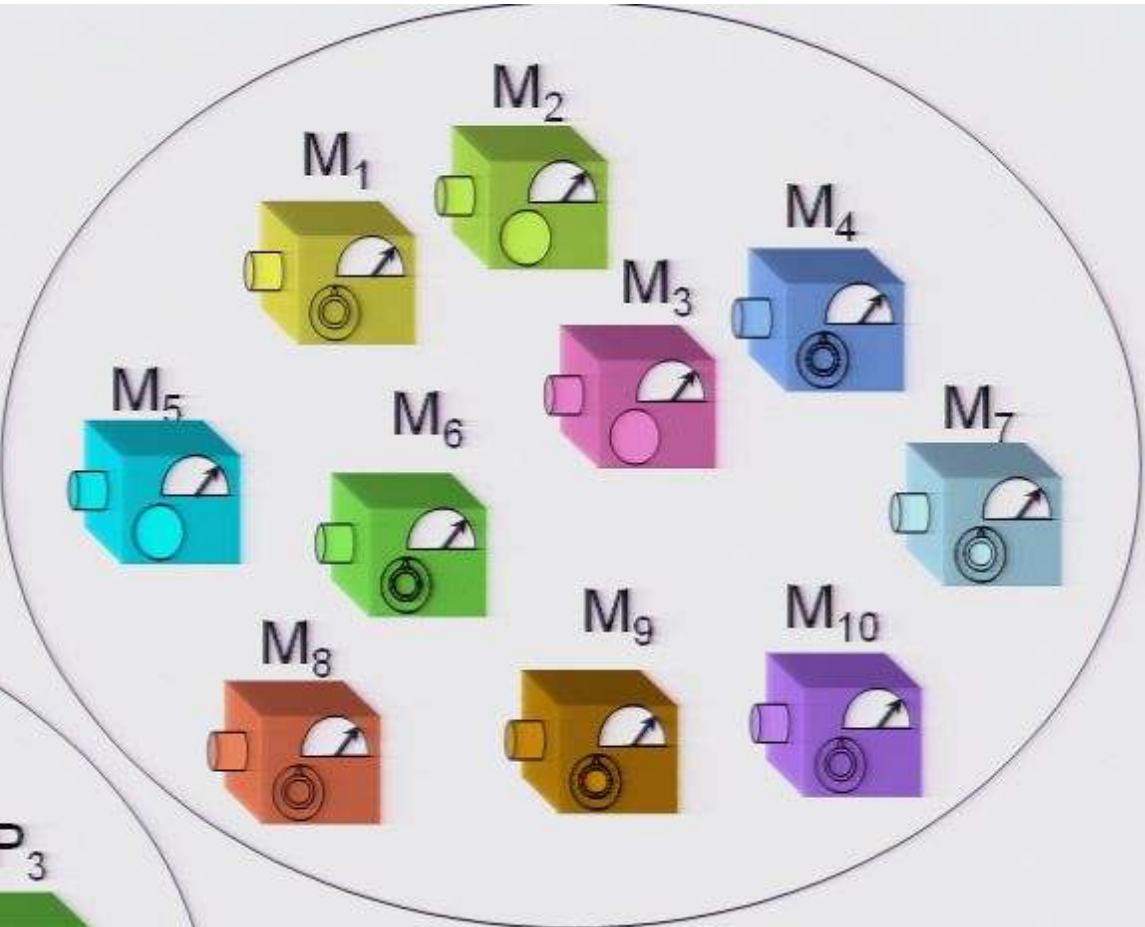
A HV model of an operational theory is **noncontextual** if

Operational equivalence
of two experimental
procedures



Equivalent
representations
in the HV model

Operational Equivalence

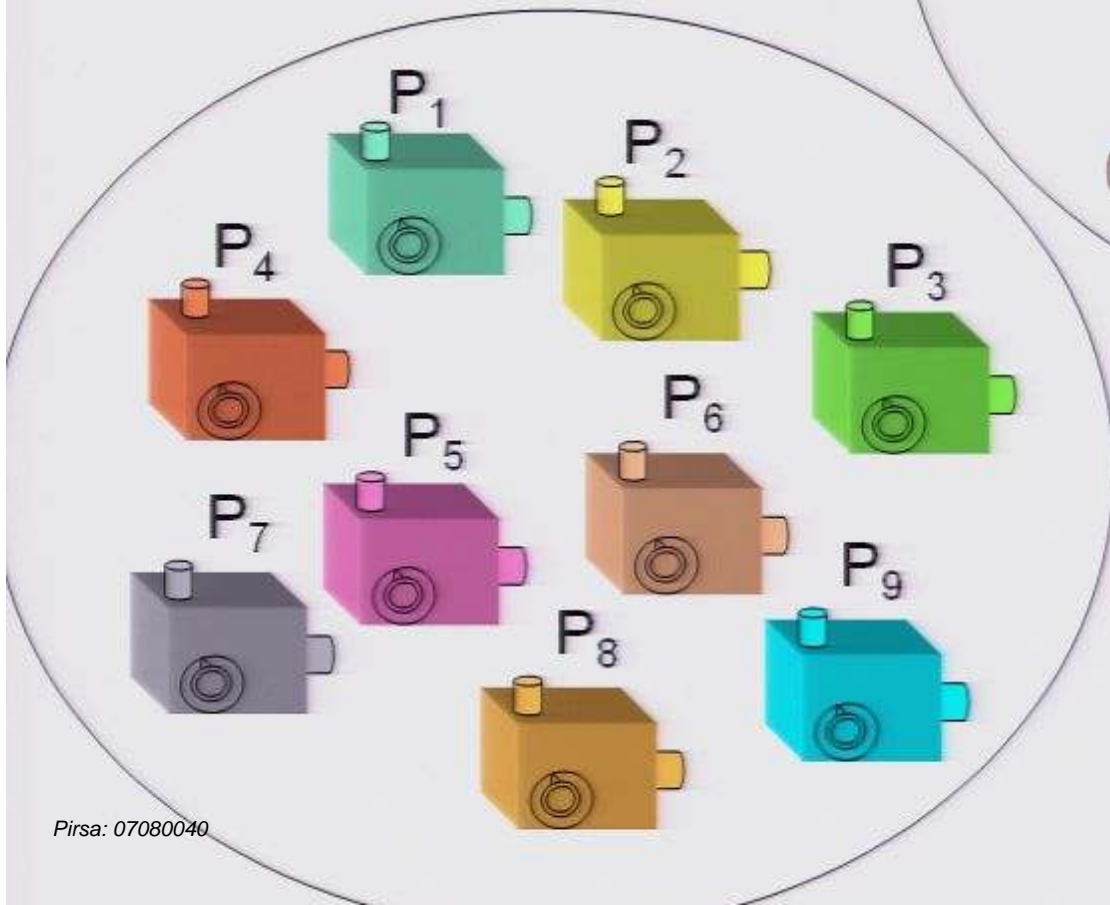
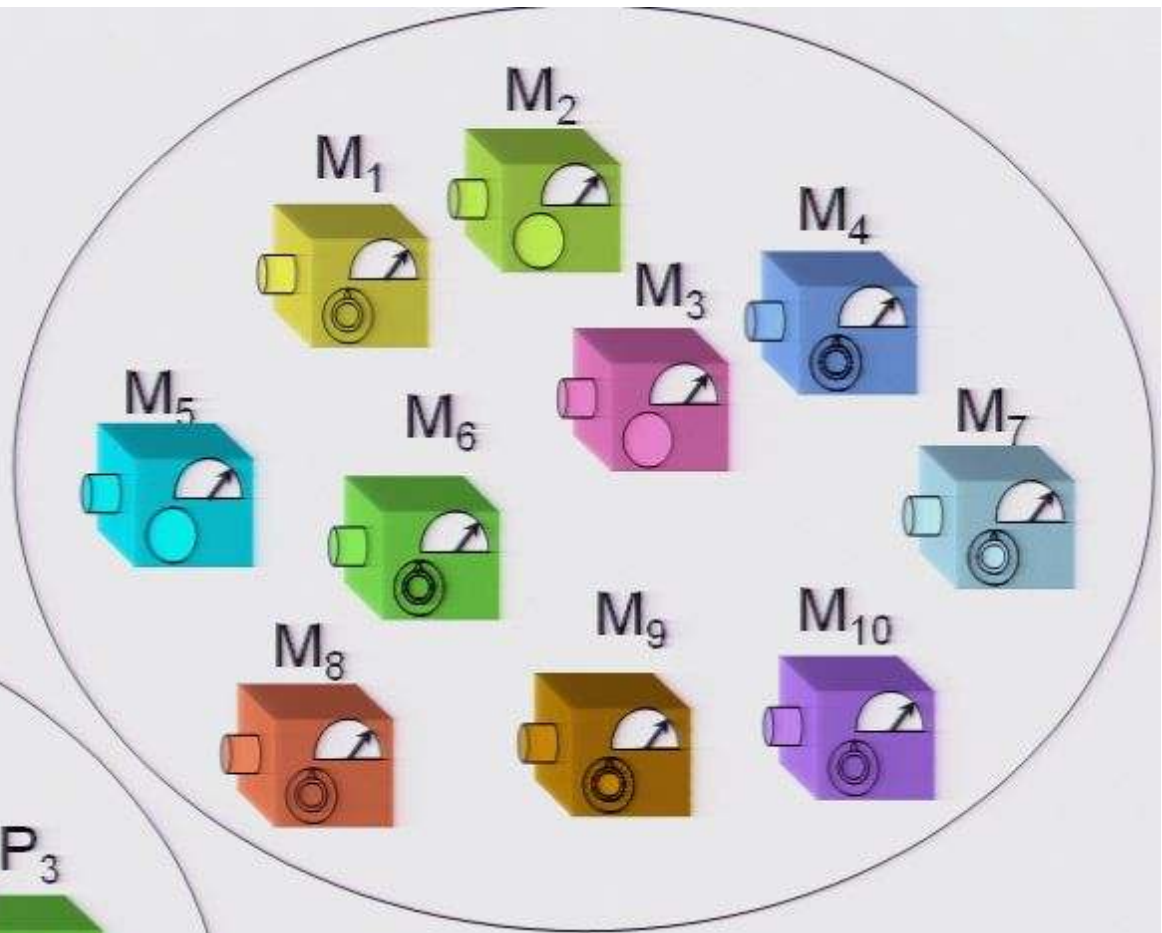


Operational Equivalence

$$P \simeq P'$$

$$\equiv p(k|P, M) = p(k|P', M)$$

for all M .

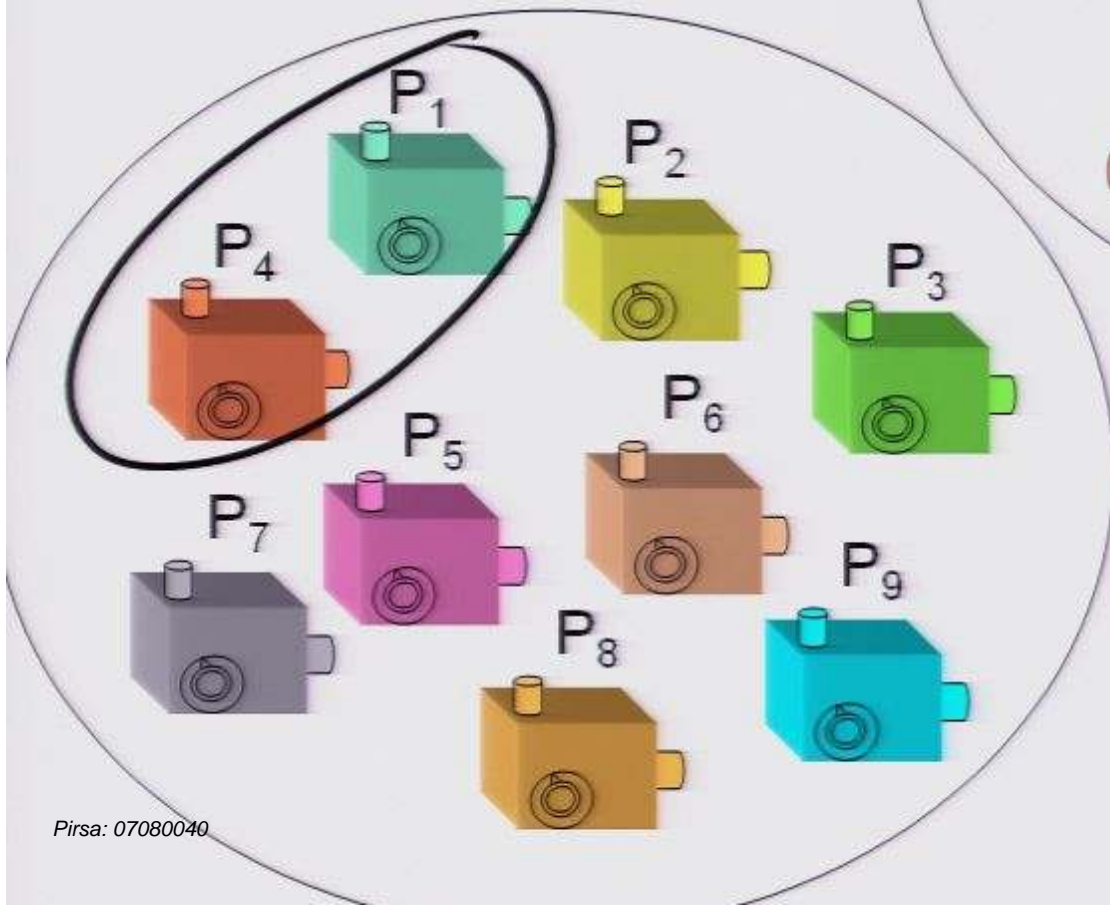
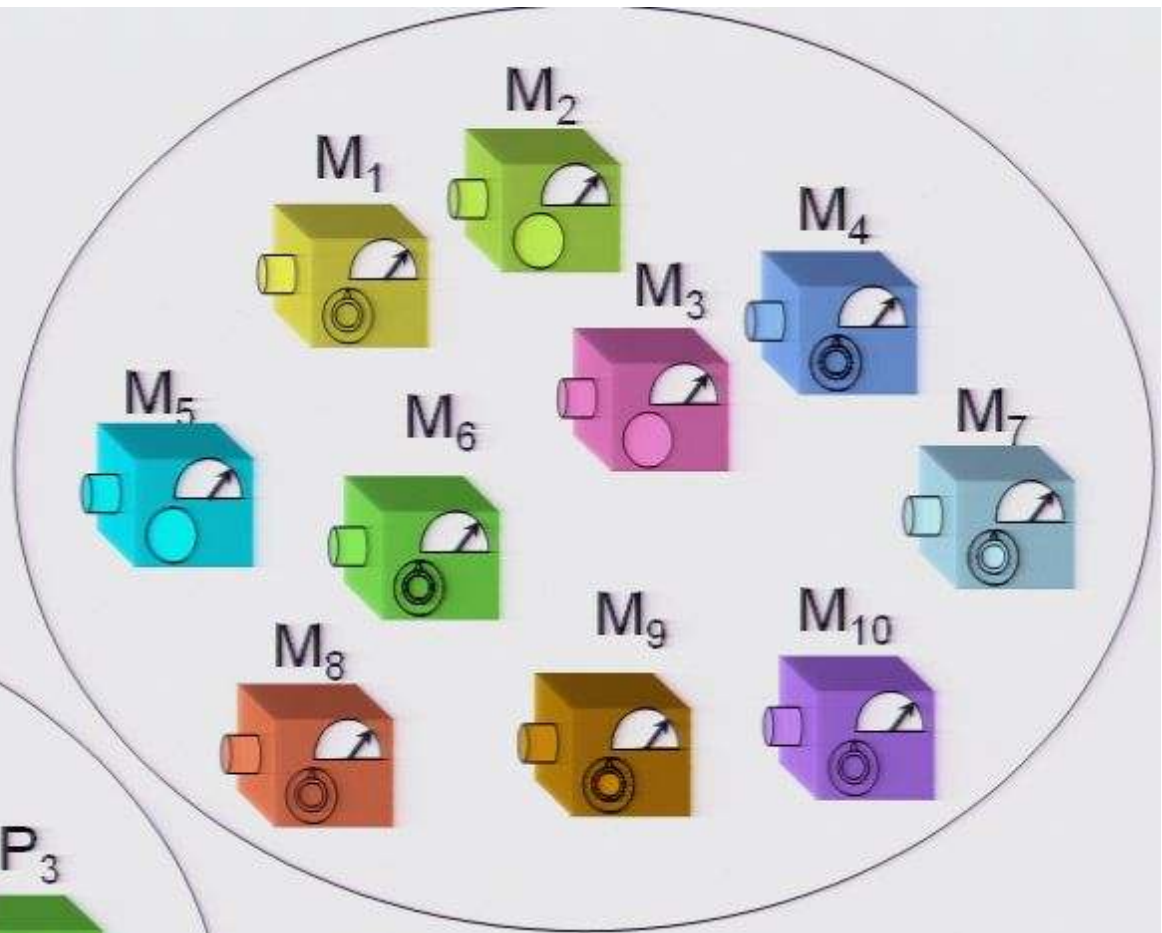


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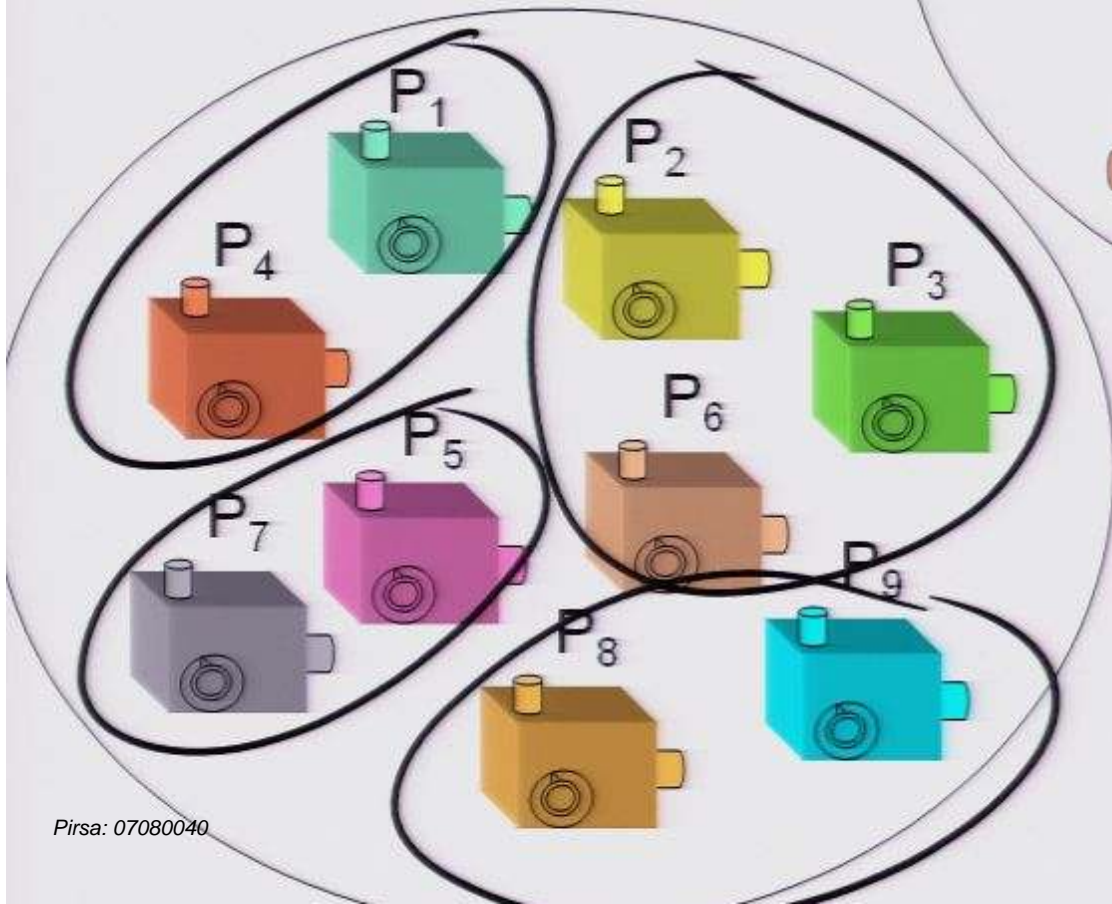
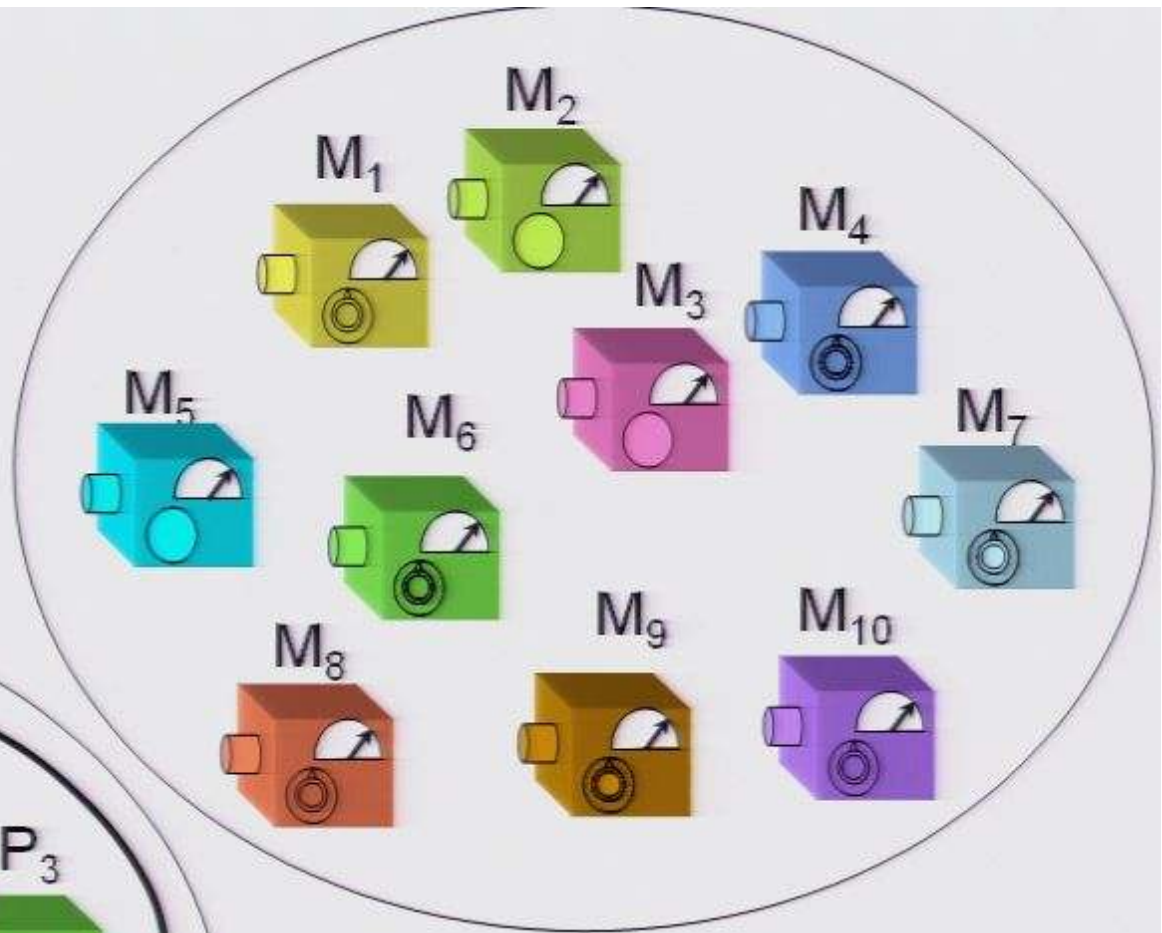


Operational Equivalence

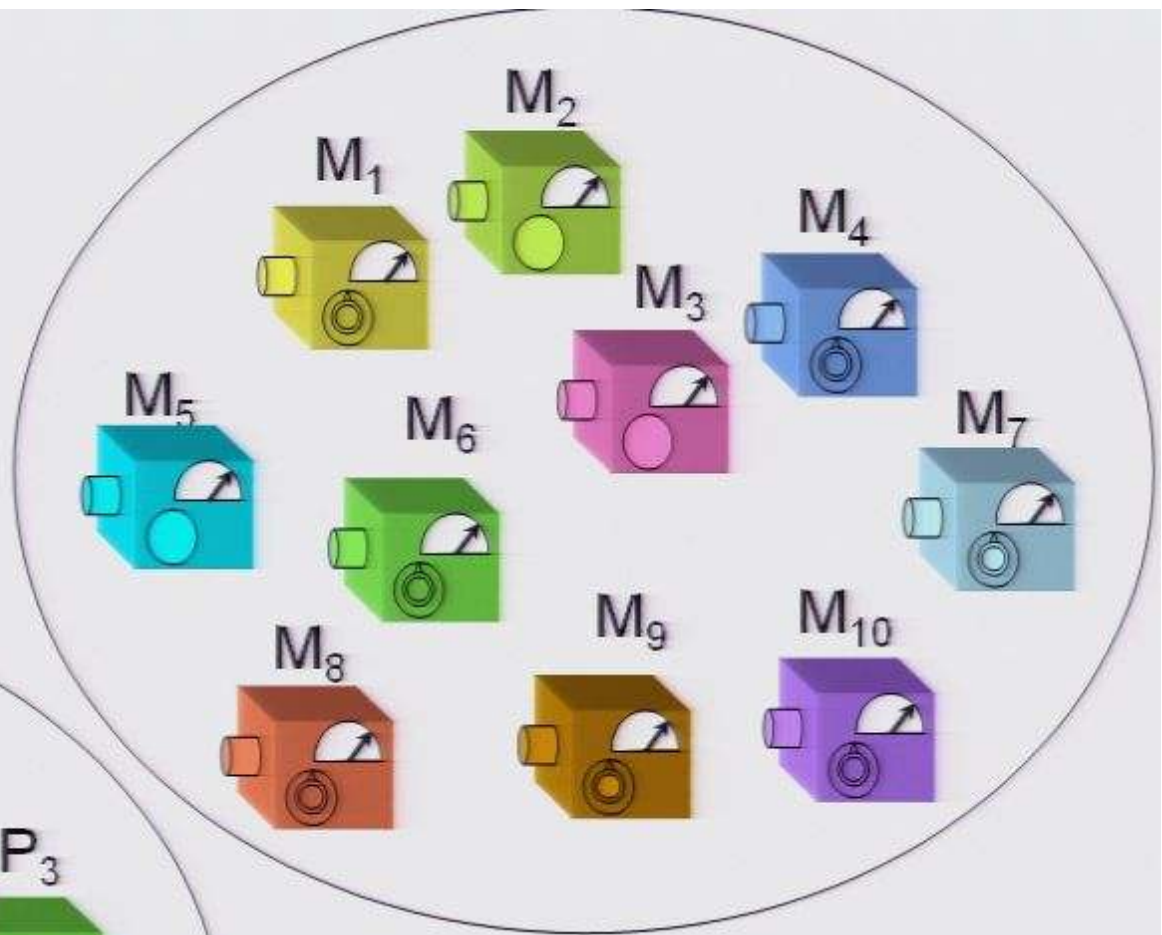
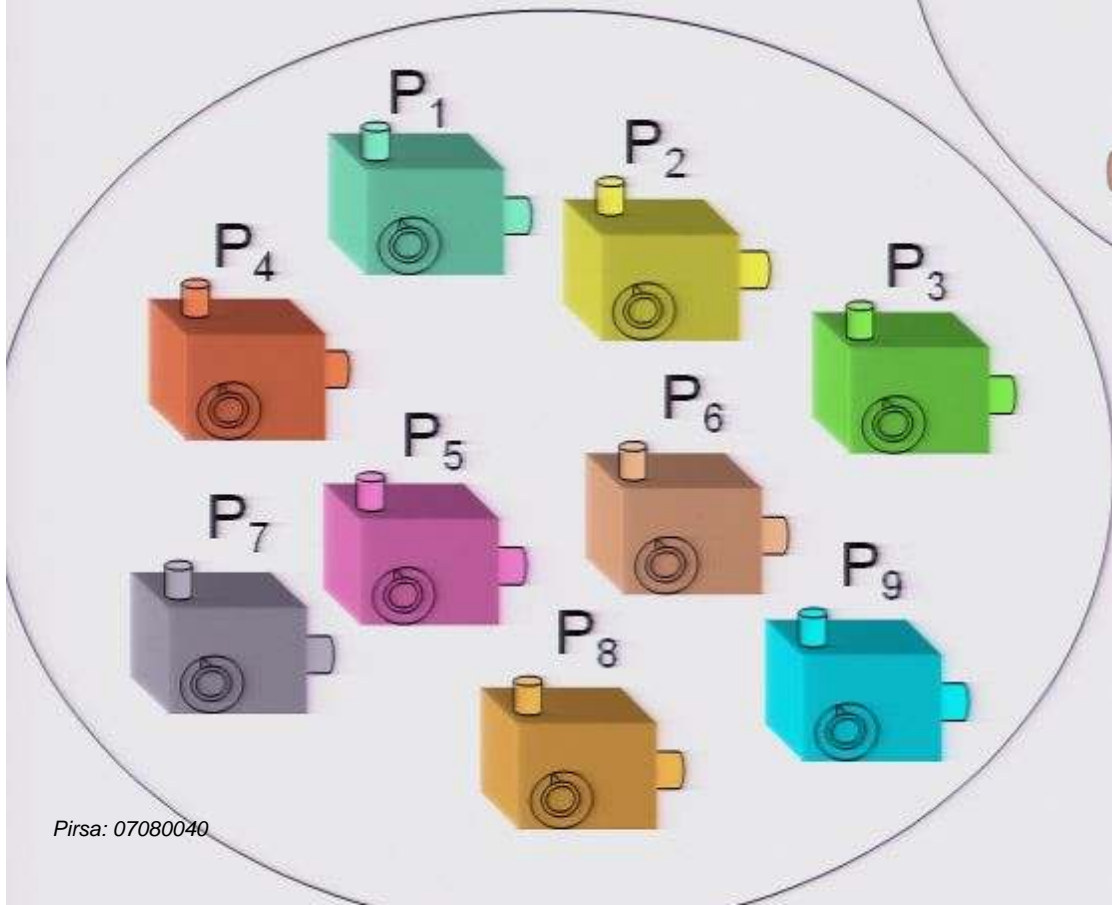
$$P \simeq P'$$

$$\equiv p(k|P, M) = p(k|P', M)$$

for all M .



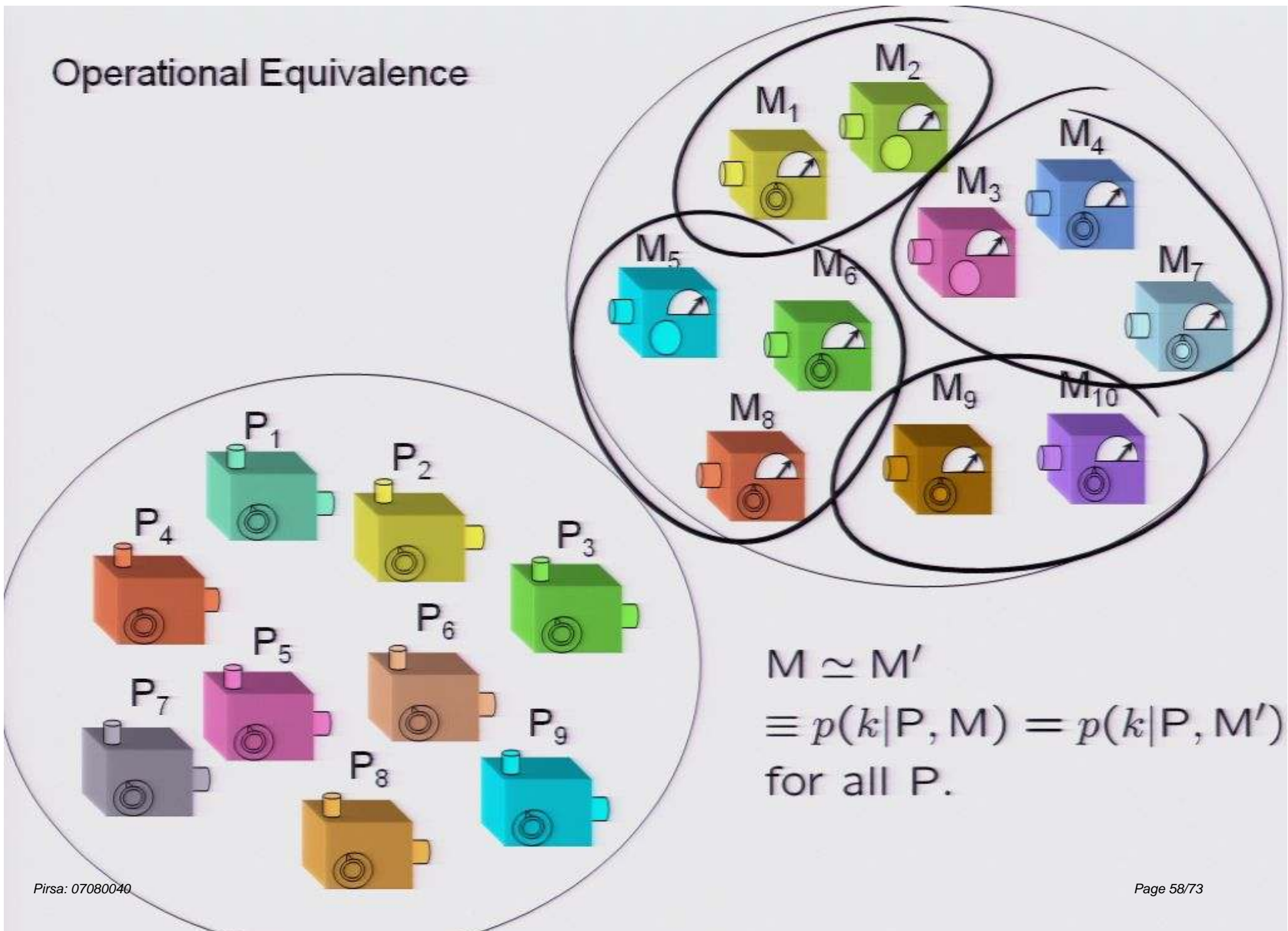
Operational Equivalence



$$M \simeq M'$$
$$\equiv p(k|P, M) = p(k|P, M')$$

for all P.

Operational Equivalence



$$M \simeq M'$$
$$\equiv p(k|P, M) = p(k|P, M')$$

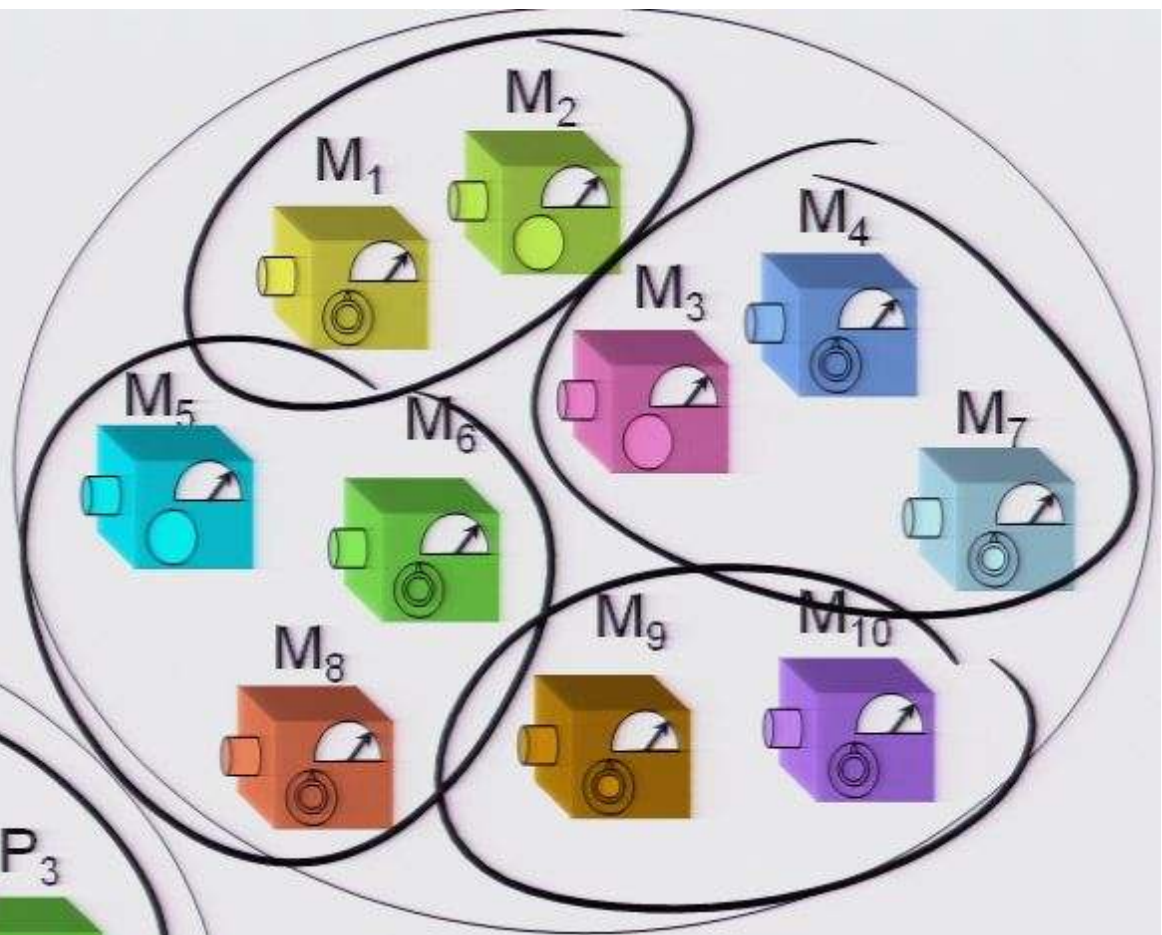
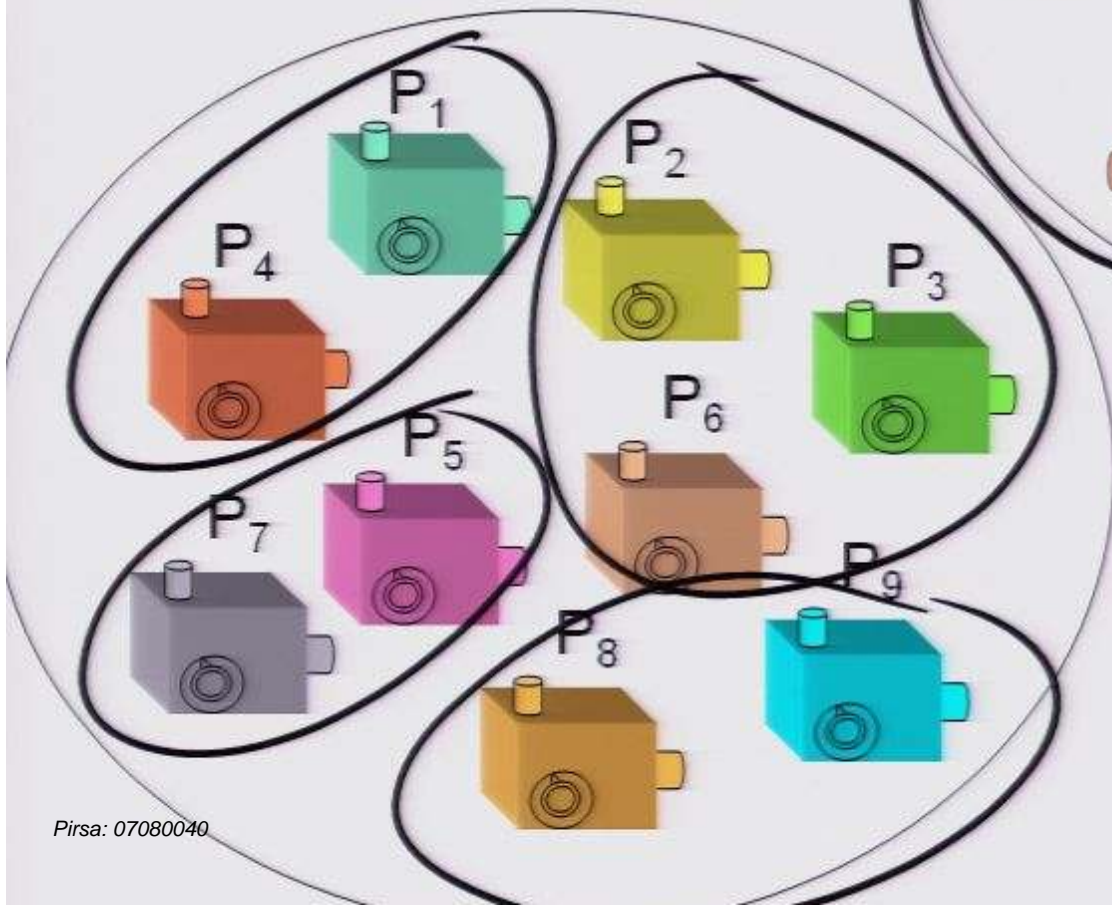
for all P.

Operational Equivalence

$$P \simeq P'$$

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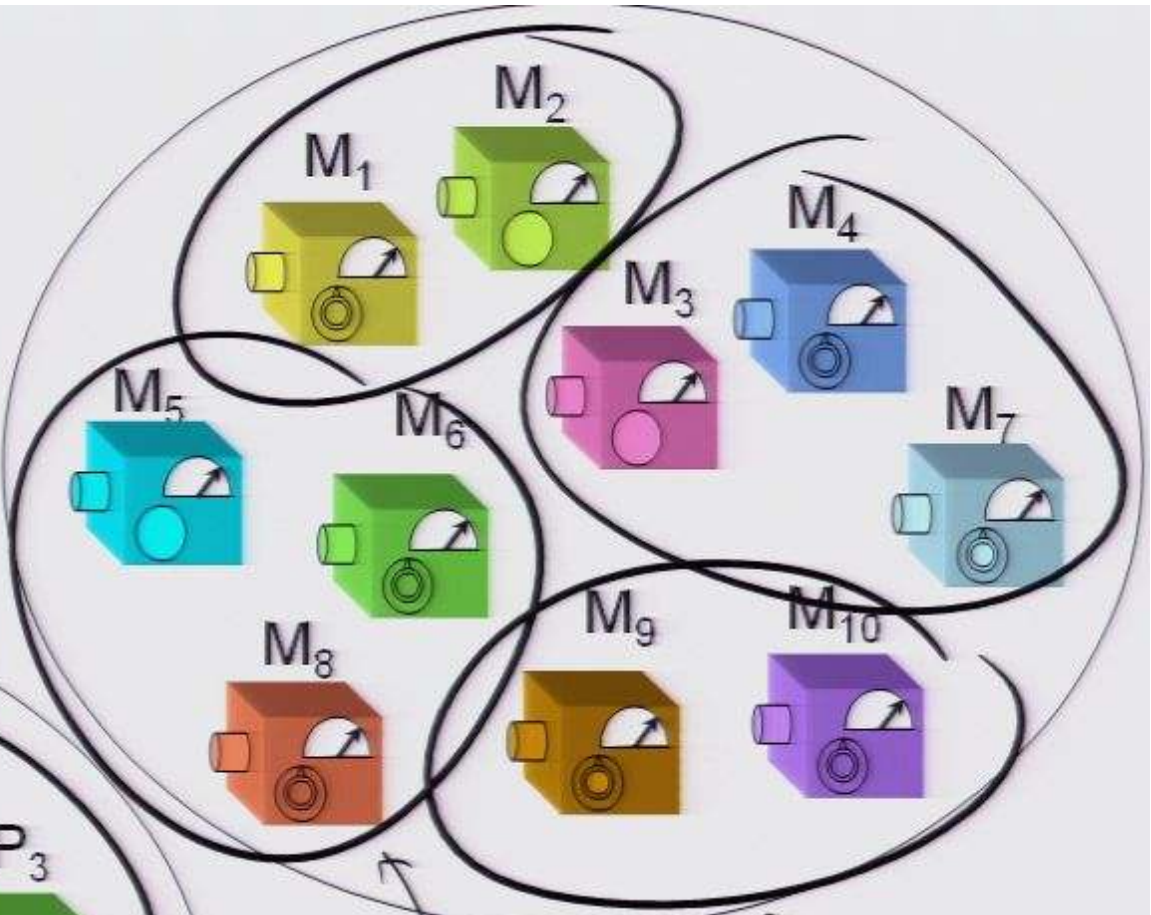
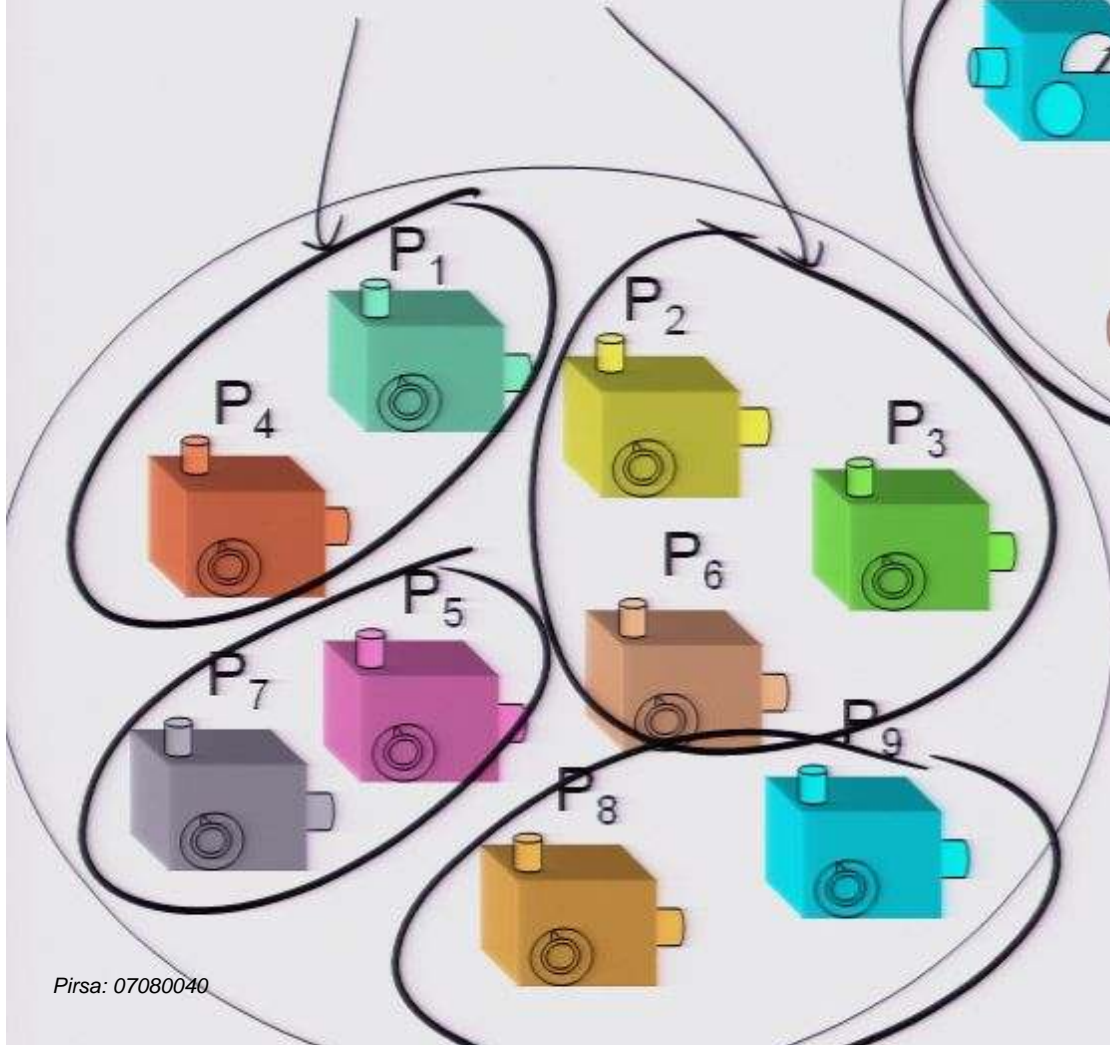


$$M \simeq M'$$

$$\equiv p(k|P, M) = p(k|P, M')$$

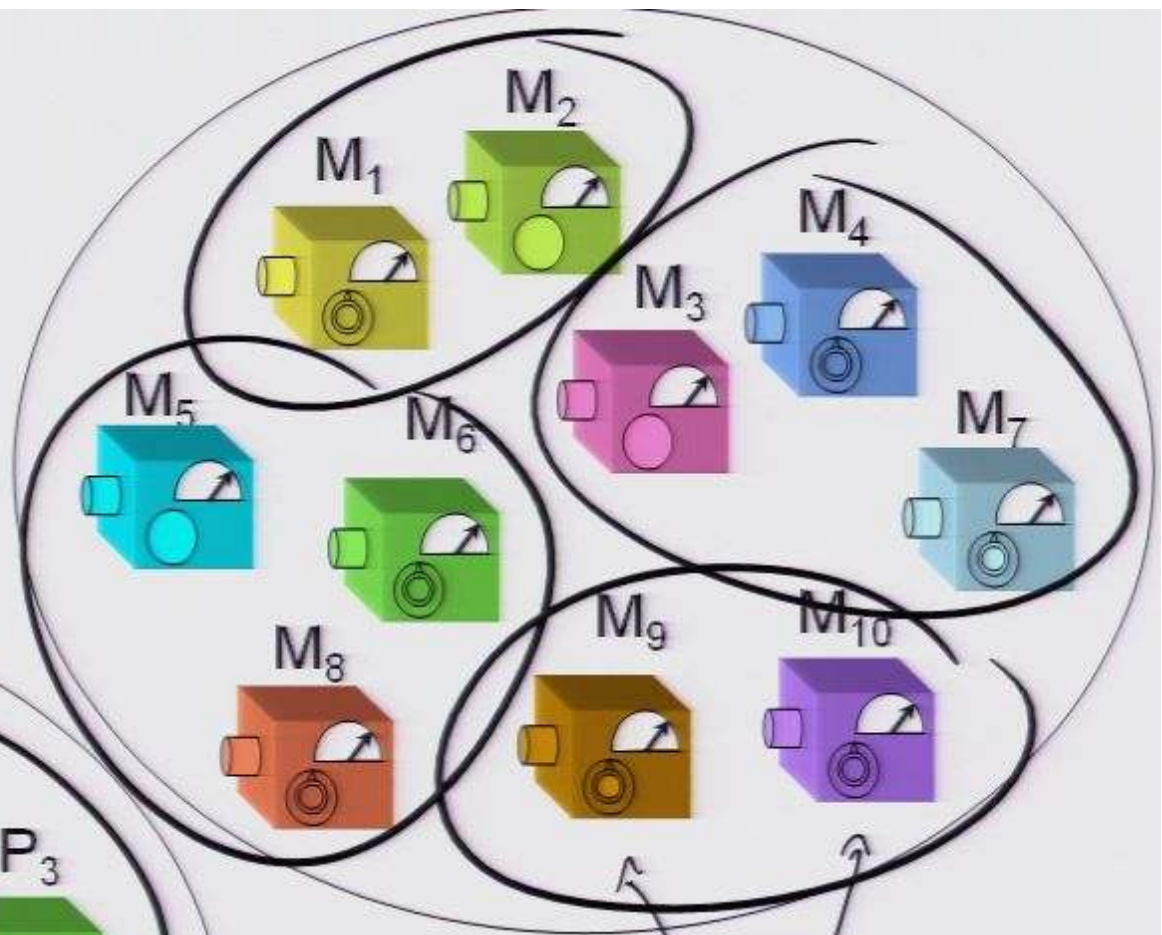
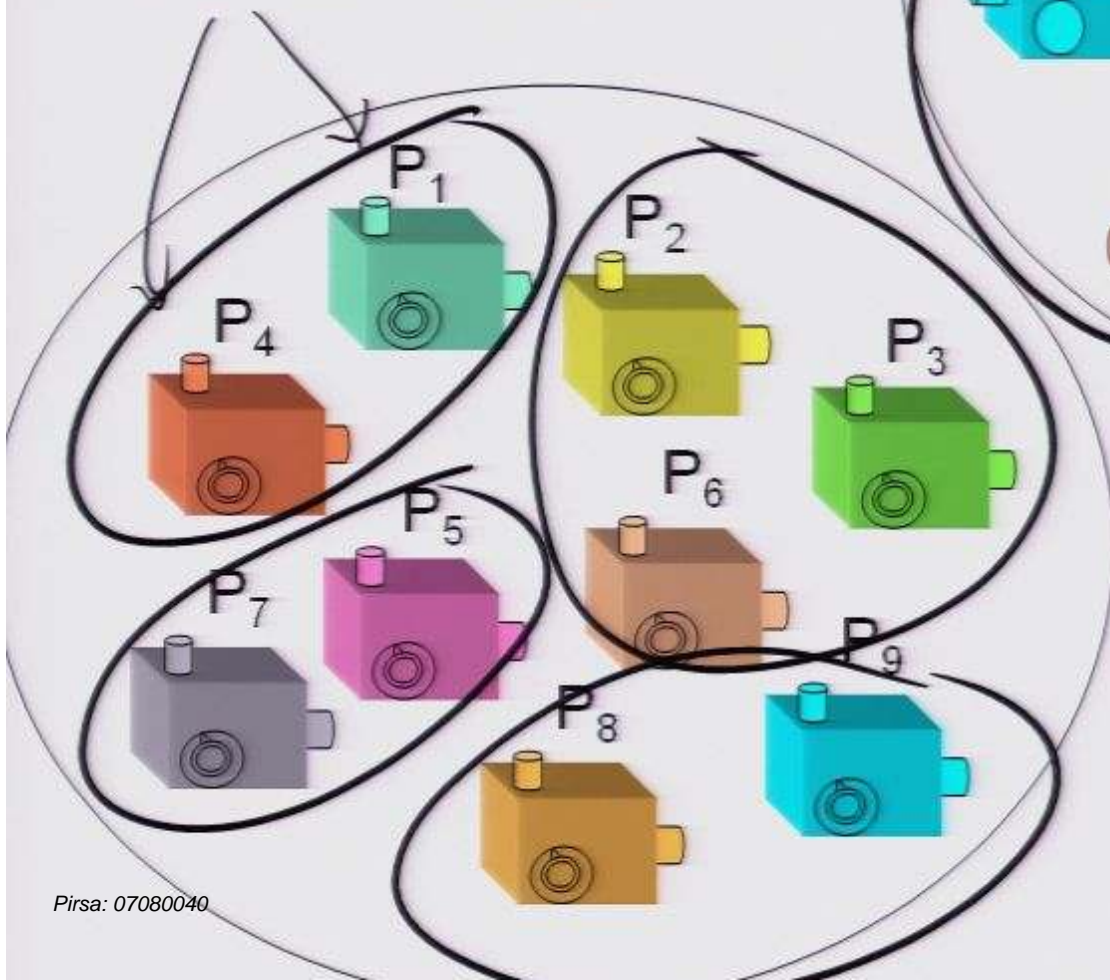
for all P .

Difference of Equivalence class



Difference of Equivalence class

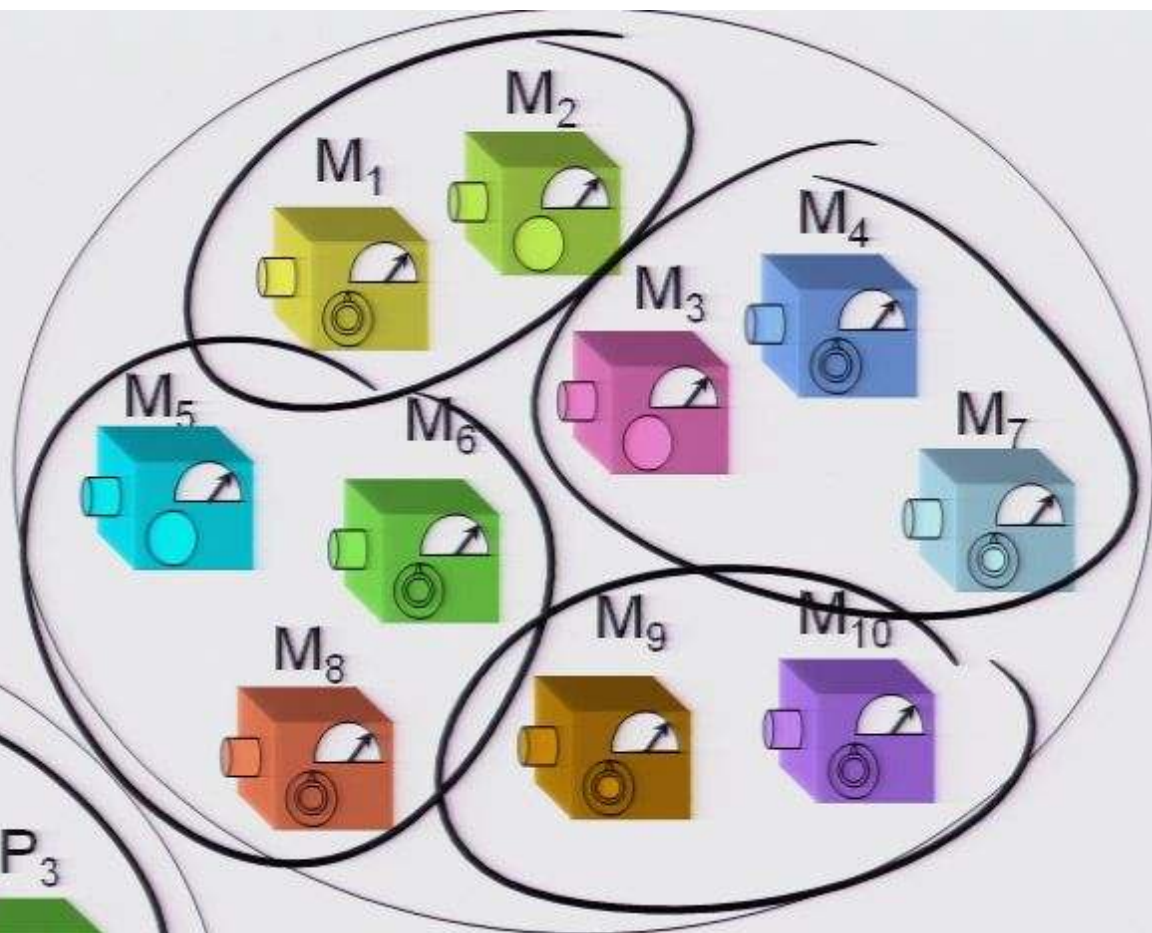
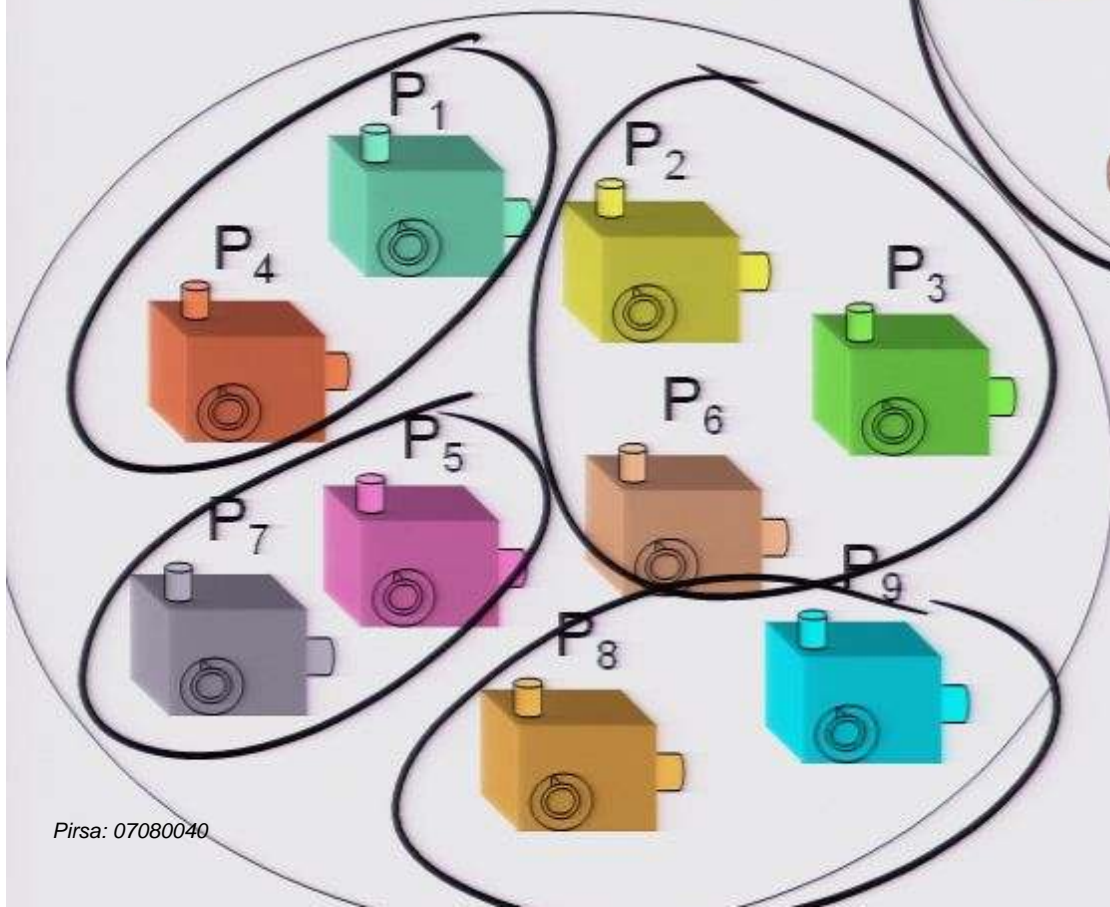
Difference of context



Difference of context

Preparation Noncontextuality

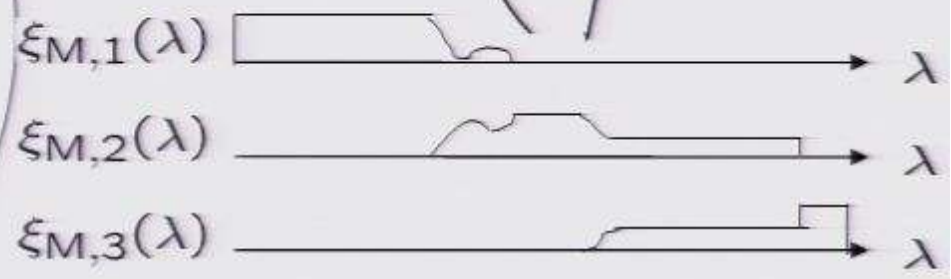
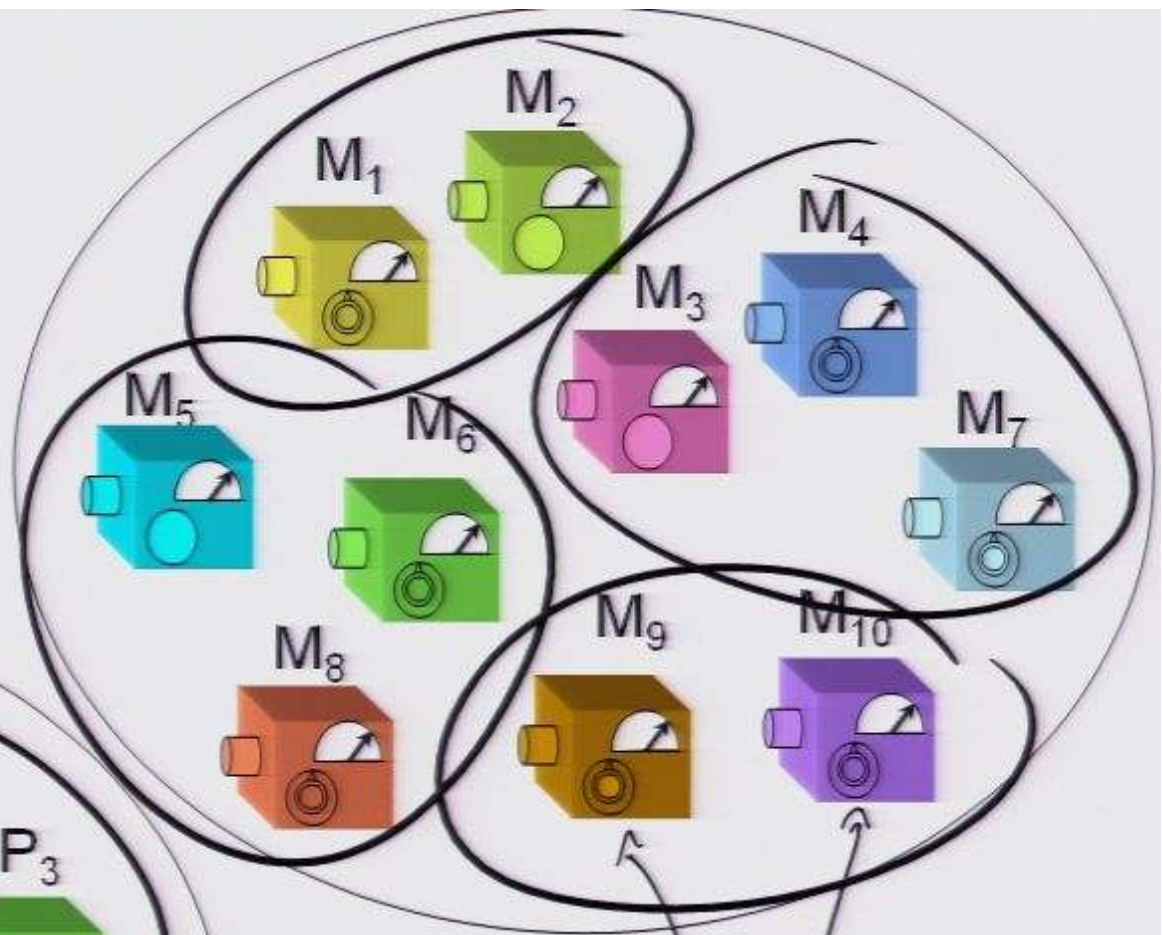
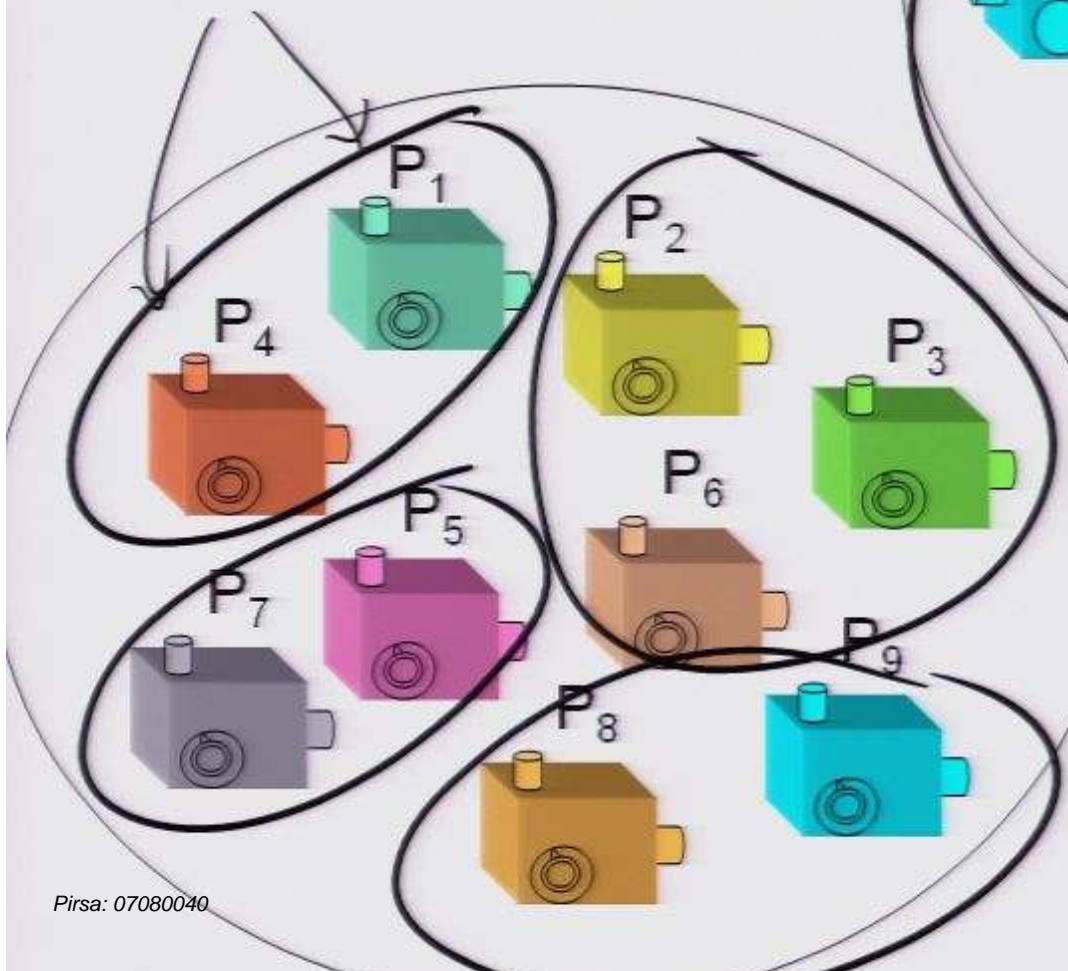
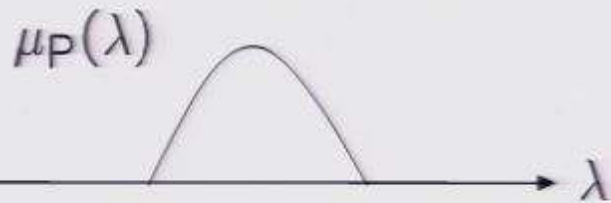
if $P \simeq P'$ then
 $\mu_P(\lambda) = \mu_{P'}(\lambda)$



Measurement Noncontextuality

if $M \simeq M'$ then
 $\xi_{M,k}(\lambda) = \xi_{M',k}(\lambda)$

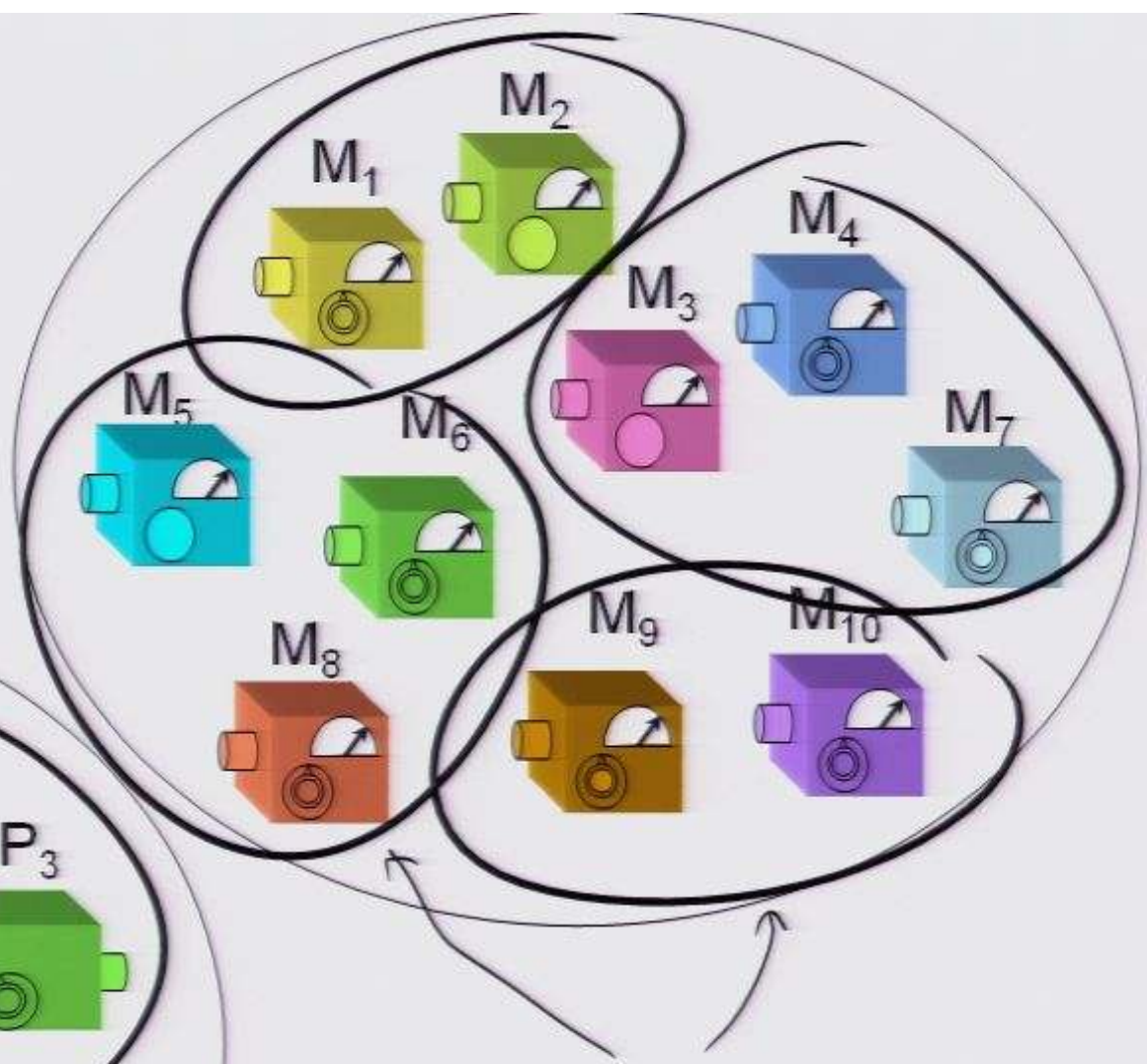
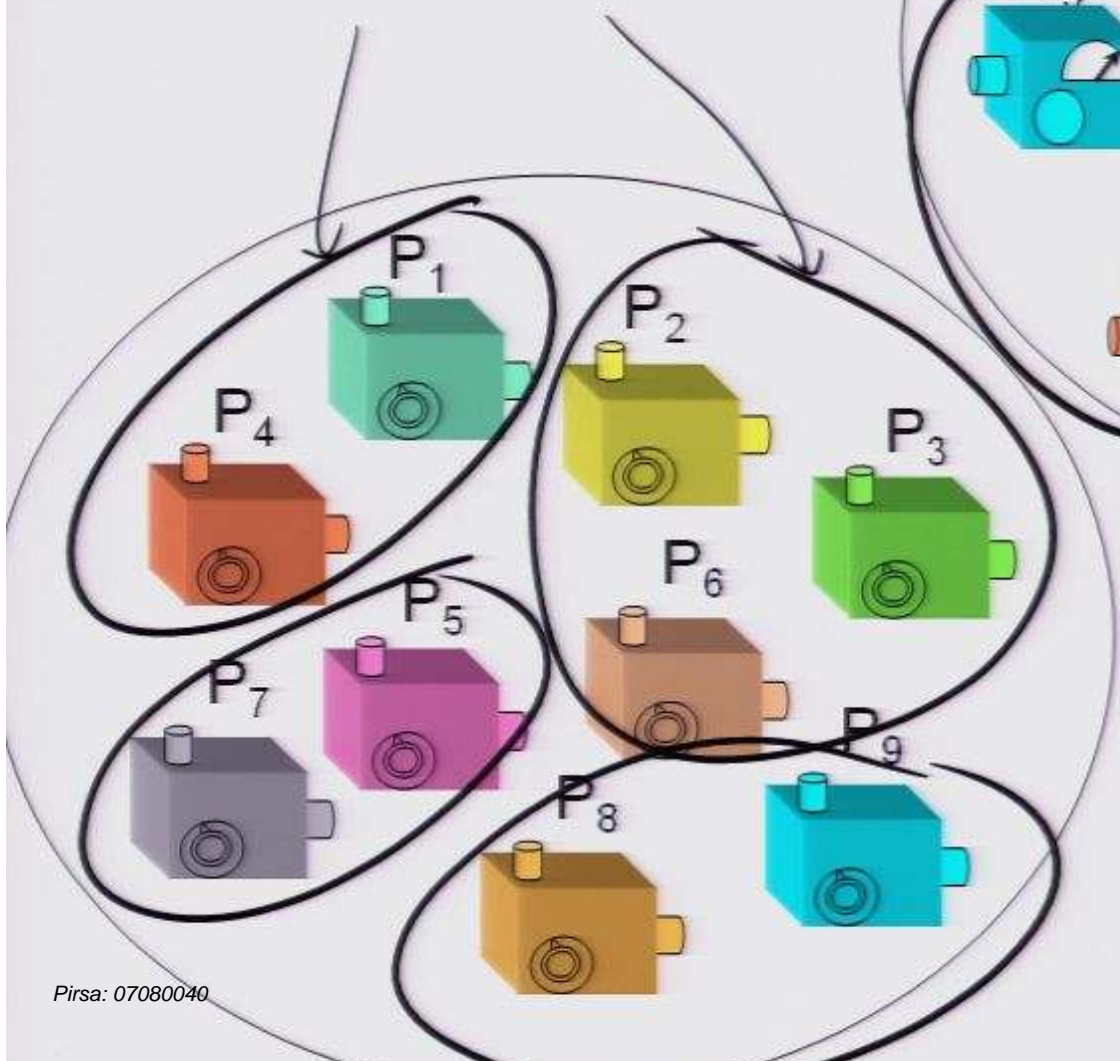
Preparation Noncontextuality



Measurement Noncontextuality

Contextuality in quantum theory revisited

Different density operators



Different observables

Ex: different **convex decompositions** of ρ

Many $\{p_j, |\psi_j\rangle\}$ such that

$$\rho = \sum_j p_j |\psi_j\rangle \langle \psi_j|$$

$$\frac{1}{2}I = \frac{1}{2}|0\rangle \langle 0| + \frac{1}{2}|1\rangle \langle 1|$$

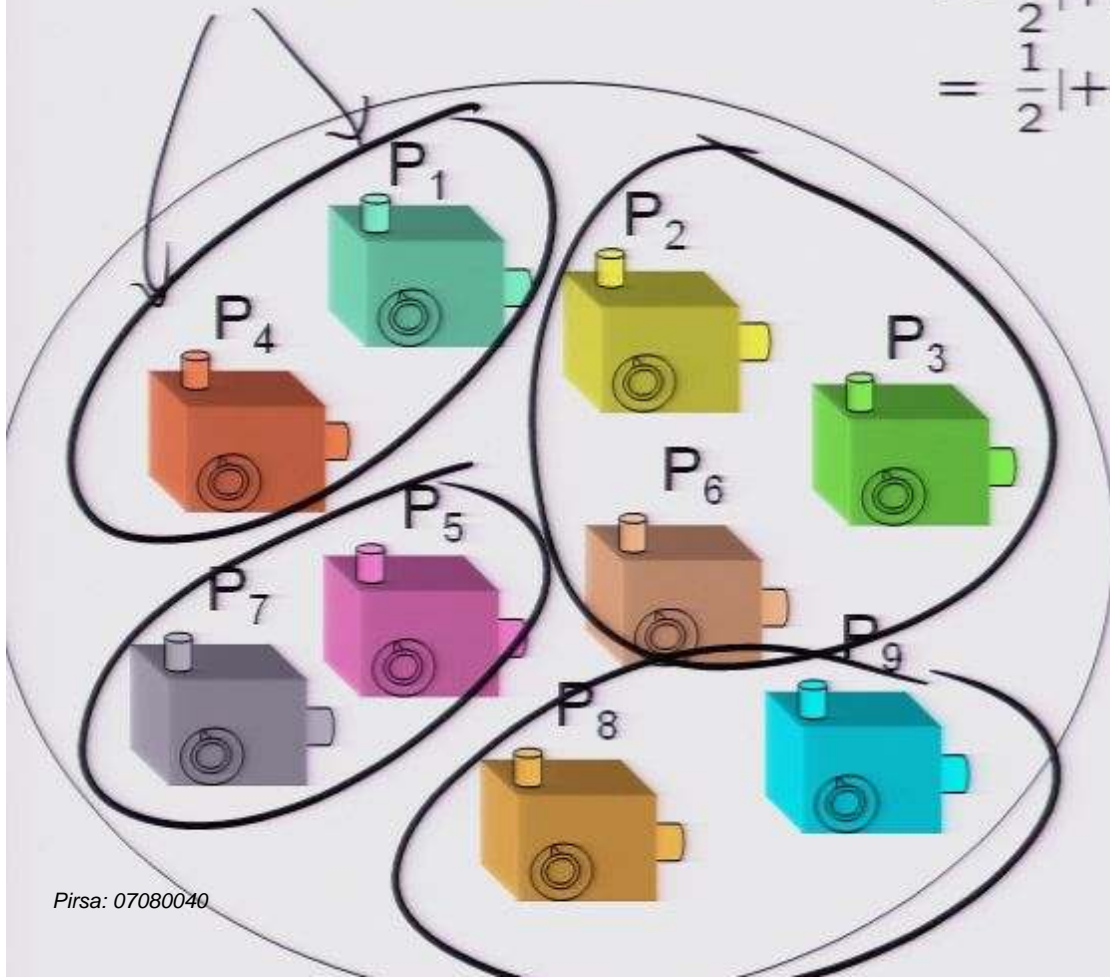
$$= \frac{1}{2}|+\rangle \langle +| + \frac{1}{2}|-\rangle \langle -|$$

$$= \frac{1}{2}|+i\rangle \langle +i| + \frac{1}{2}|-i\rangle \langle -i|$$

$$|\pm\rangle = |0\rangle \pm |1\rangle$$

$$|\pm i\rangle = |0\rangle \pm i|1\rangle$$

Difference of context



Ex: different **convex decompositions** of ρ

Many $\{p_j, |\psi_j\rangle\}$ such that

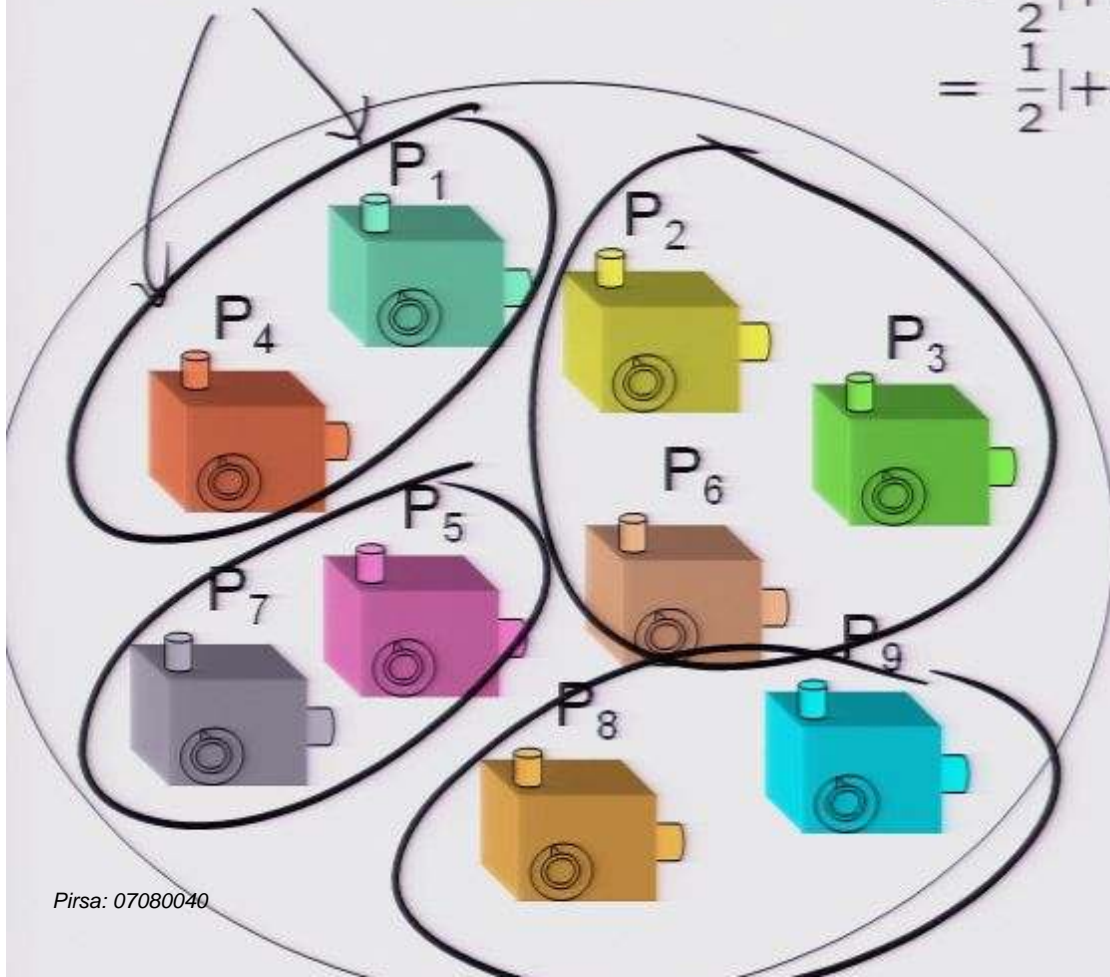
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$$= \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -| \quad |\pm\rangle = |0\rangle \pm |1\rangle$$

$$= \frac{1}{2}|+i\rangle\langle +i| + \frac{1}{2}|-i\rangle\langle -i| \quad |\pm i\rangle = |0\rangle \pm i|1\rangle$$

Difference of context



Ex: Different **purifications** of ρ

Many $|\Psi\rangle_{AB}$ such that

$$\rho = \text{Tr}_B(|\Psi\rangle_{AB}\langle\Psi|)$$

Ex: different **convex decompositions** of ρ

Many $\{p_j, |\psi_j\rangle\}$ such that

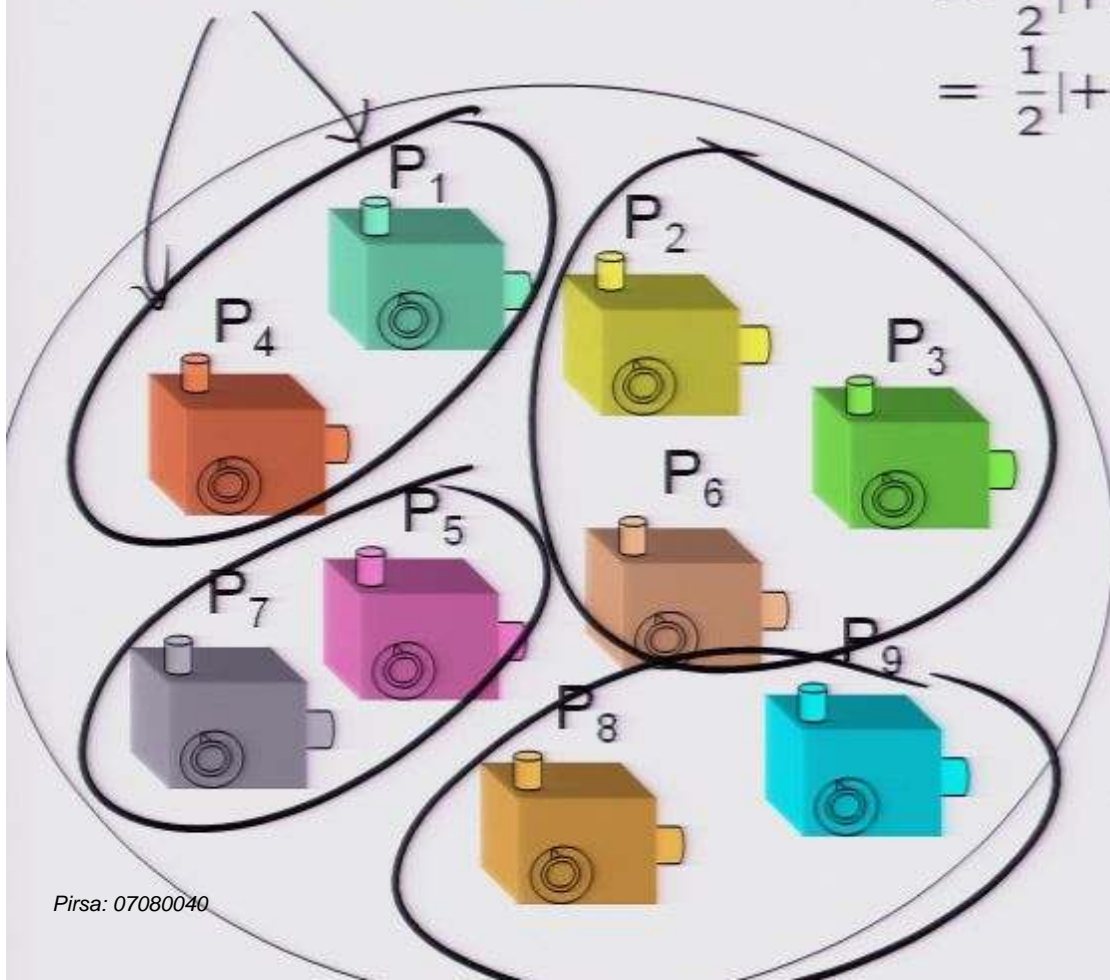
$$\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$$

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Difference of context



Ex: Different **purifications** of ρ

Many $|\Psi\rangle_{AB}$ such that

$$\rho = \text{Tr}_B(|\Psi\rangle_{AB}\langle\Psi|)$$

Preparation

Noncontextuality

$$\mu_P(\lambda) = \mu_\rho(\lambda)$$

Different **fine-grainings** of $\{E_k\}$

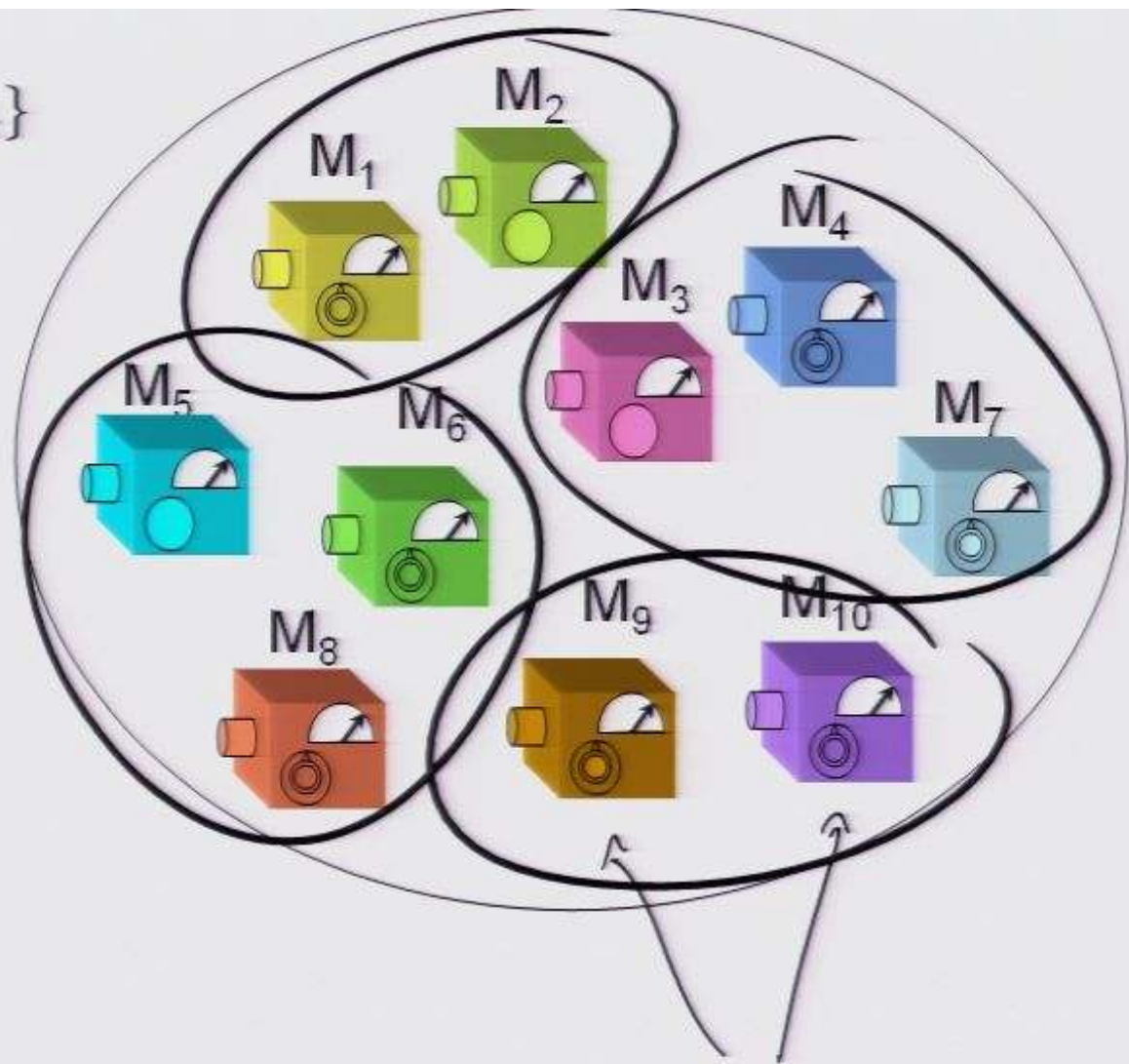
Many $\{E_{k,s}\}$ such that

$$E_k = \sum_s E_{k,s}$$

$$I - |\psi_1\rangle\langle\psi_1|$$

$$= |\psi_2\rangle\langle\psi_2| + |\psi_3\rangle\langle\psi_3|$$

$$= |\psi'_2\rangle\langle\psi'_2| + |\psi'_3\rangle\langle\psi'_3|$$



Difference of context

Different **fine-grainings** of $\{E_k\}$

Many $\{E_{k,s}\}$ such that

$$E_k = \sum_s E_{k,s}$$

$$I - |\psi_1\rangle\langle\psi_1|$$

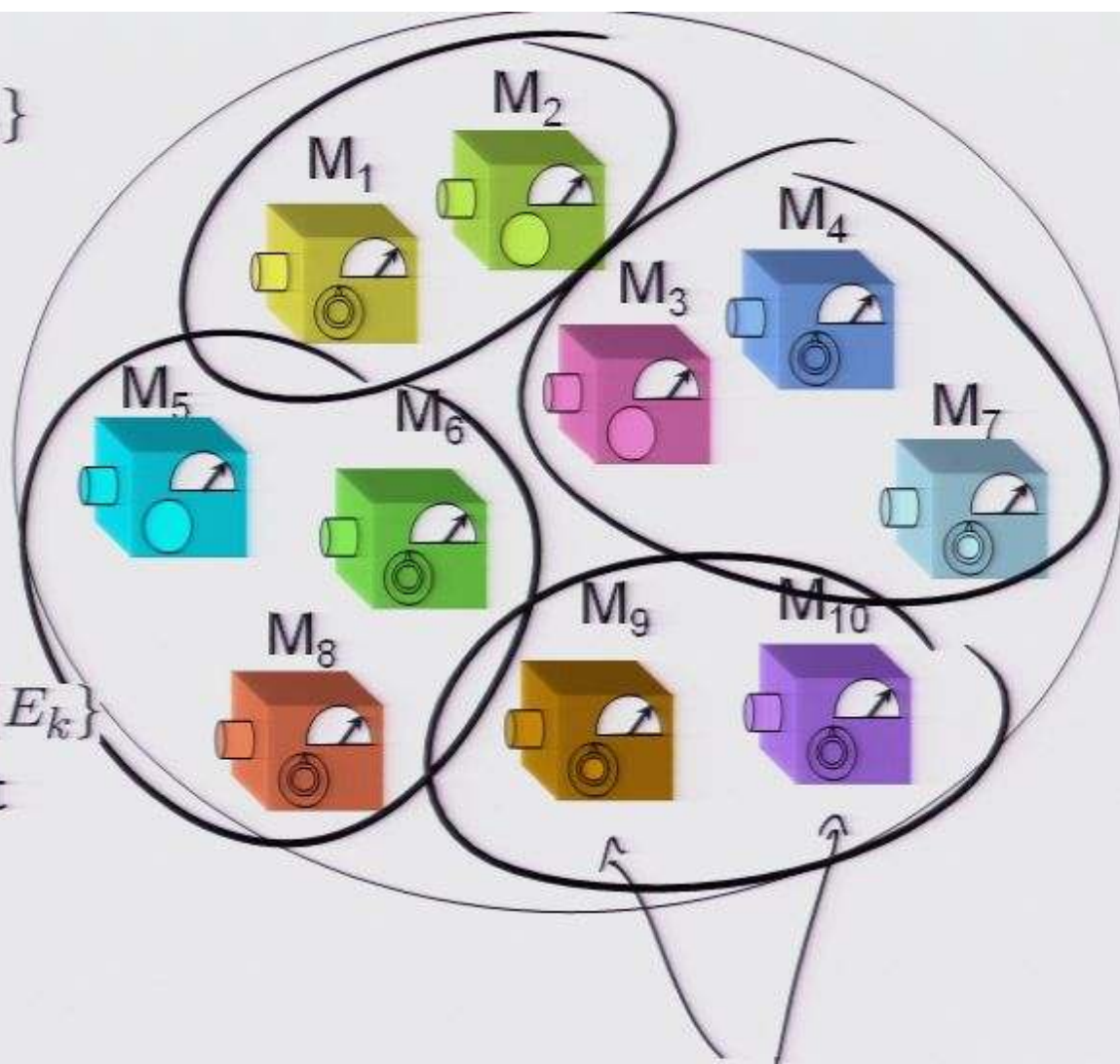
$$= |\psi_2\rangle\langle\psi_2| + |\psi_3\rangle\langle\psi_3|$$

$$= |\psi'_2\rangle\langle\psi'_2| + |\psi'_3\rangle\langle\psi'_3|$$

Ex: different **convex decomps** of $\{E_k\}$

Many $\{p_j, \{E_k^j\}\}$ such that

$$E_k = \sum_j p_j E_k^j$$



Difference of context

Different **fine-grainings** of $\{E_k\}$

Many $\{E_{k,s}\}$ such that

$$E_k = \sum_s E_{k,s}$$

$$I - |\psi_1\rangle\langle\psi_1|$$

$$= |\psi_2\rangle\langle\psi_2| + |\psi_3\rangle\langle\psi_3|$$

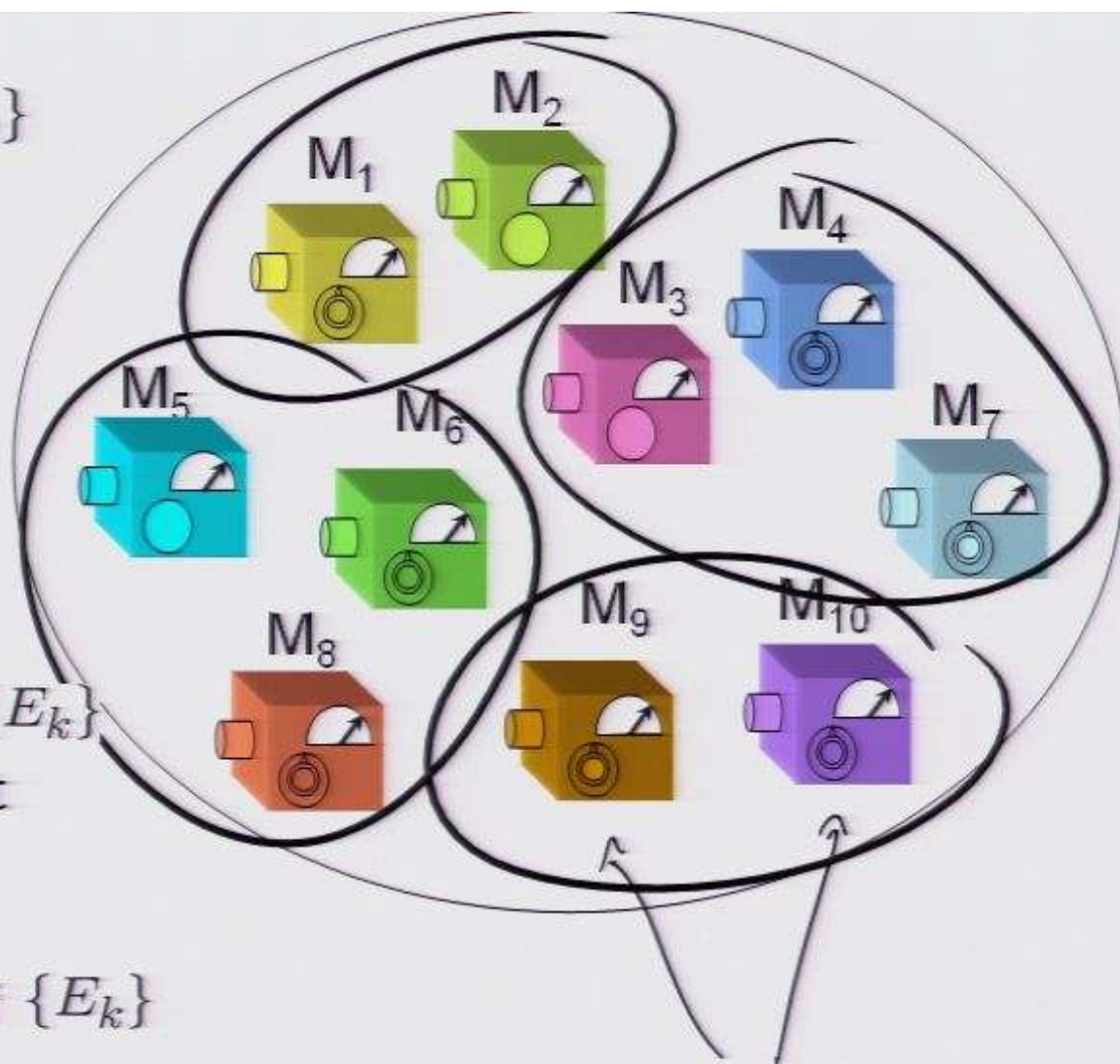
$$= |\psi'_2\rangle\langle\psi'_2| + |\psi'_3\rangle\langle\psi'_3|$$

Ex: different **convex decomps** of $\{E_k\}$

Many $\{p_j, \{E_k^j\}\}$ such that

$$E_k = \sum_j p_j E_k^j$$

Different **Neumark extensions** of $\{E_k\}$



Difference of context

Different **fine-grainings** of $\{E_k\}$

Many $\{E_{k,s}\}$ such that

$$E_k = \sum_s E_{k,s}$$

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Ex: different **convex decomps** of $\{E_k\}$

Many $\{p_j, \{E_k^j\}\}$ such that

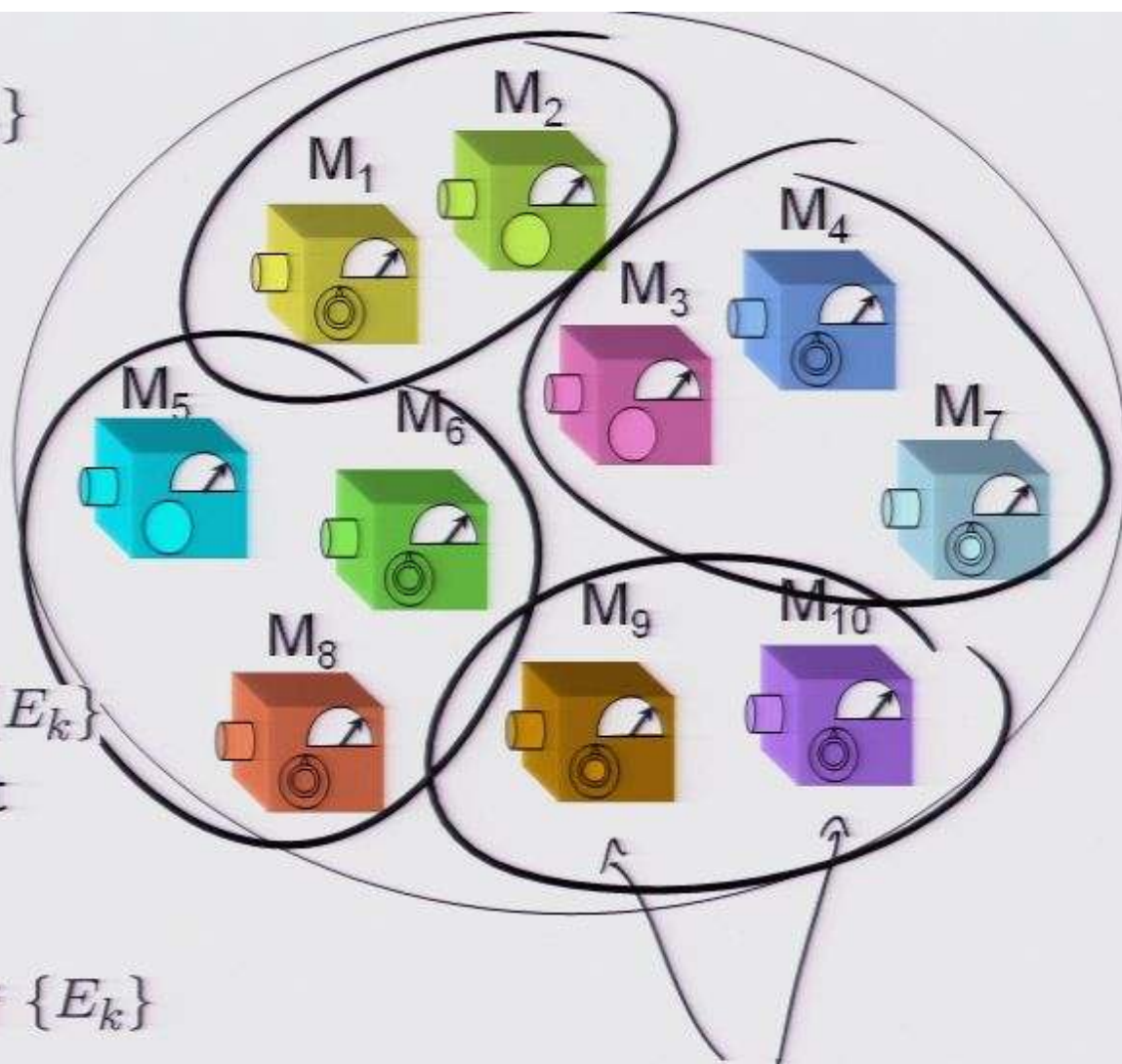
$$E_k = \sum_j p_j E_k^j$$

Different **Neumark extensions** of $\{E_k\}$

Measurement

Noncontextuality

$$\xi_{M,j}(\lambda) = \xi_{\{E_k\},j}(\lambda)$$



Difference of context

Proof of preparation contextuality

(a preparation noncontextual hidden variable model is impossible)