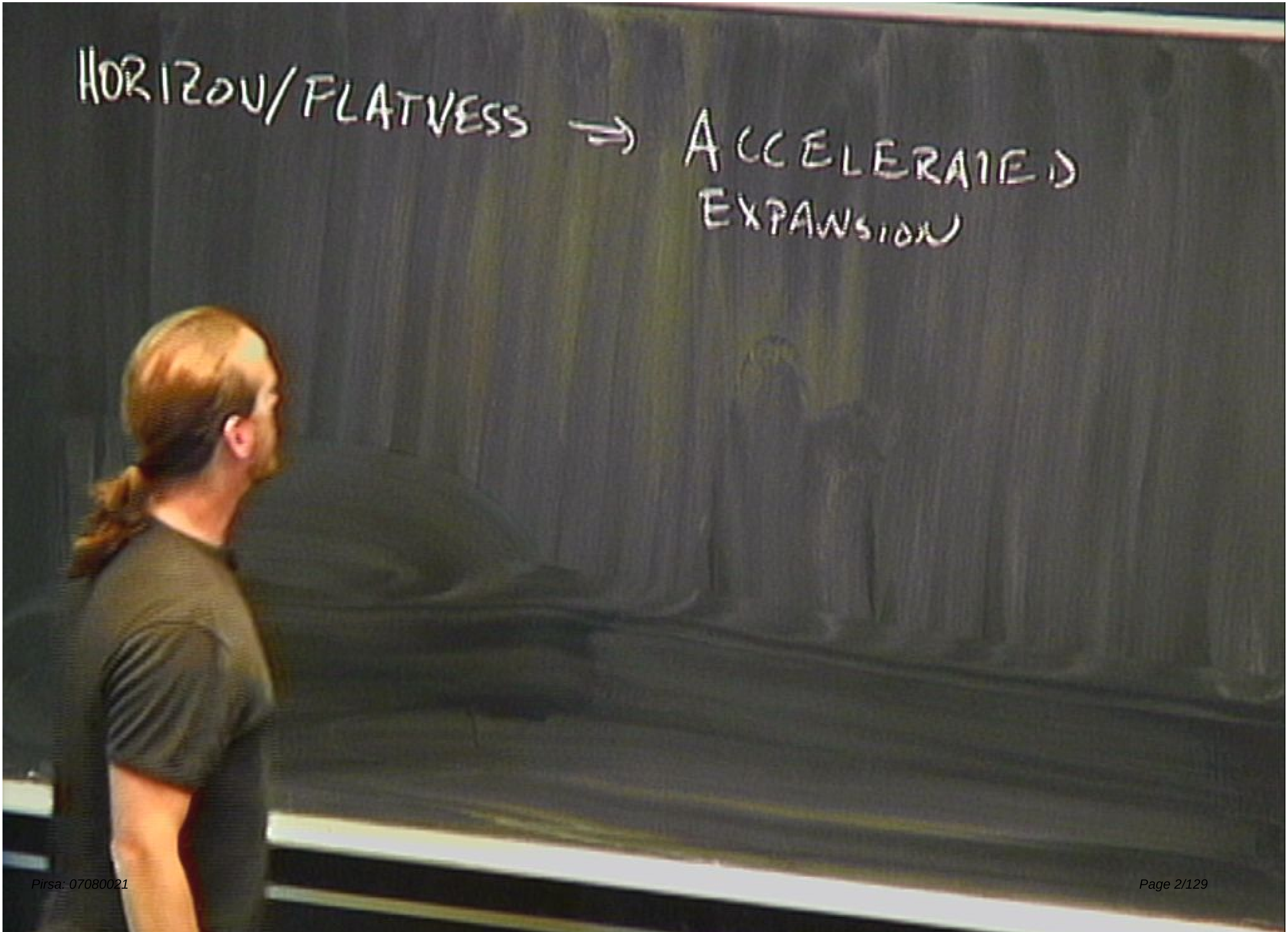
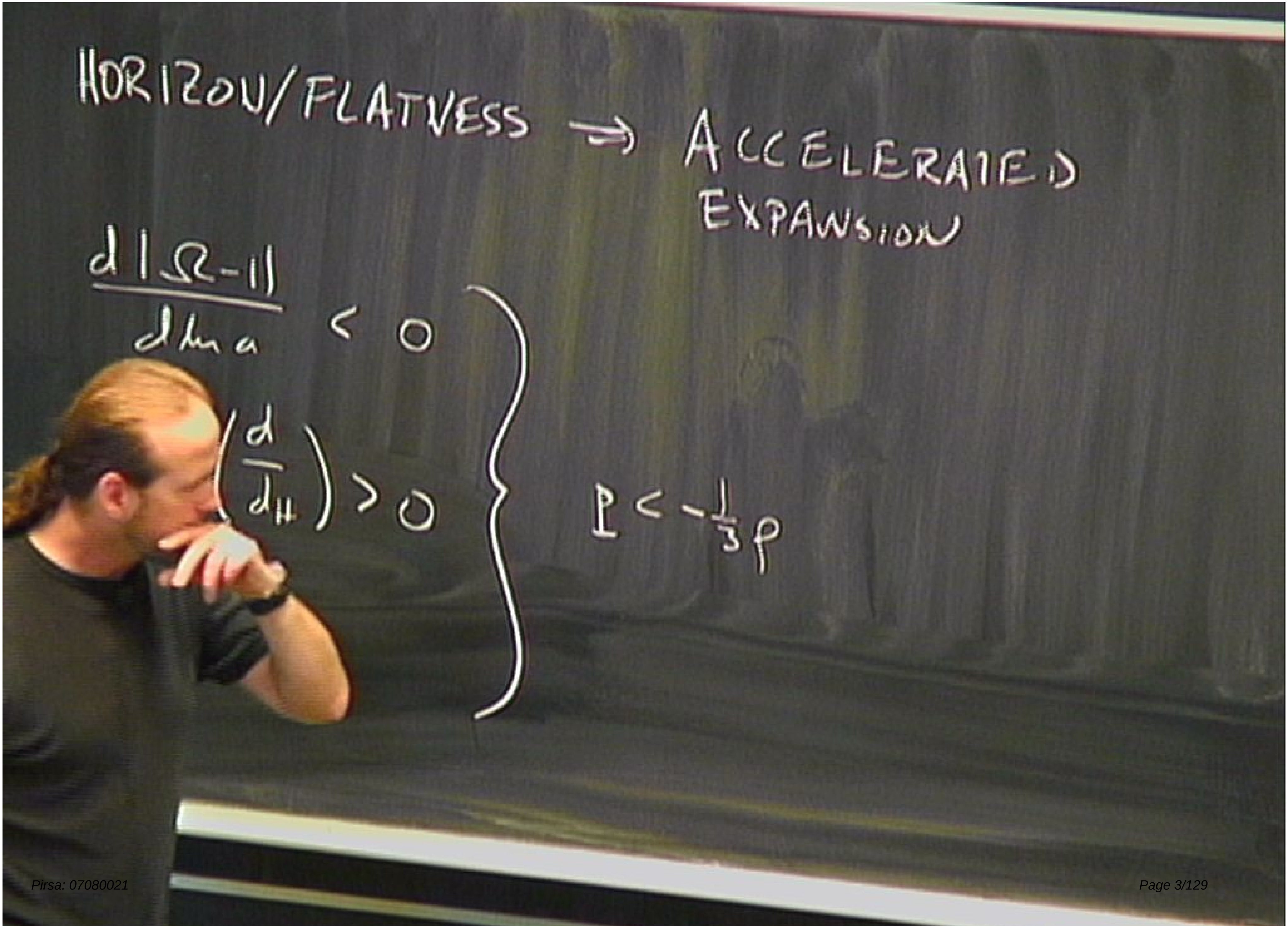
Title: Inflationary Models

Date: Aug 13, 2007 09:00 AM

URL: http://pirsa.org/07080021

Abstract:


HORIZON/FLATNESS ⇒ ACCELERATED
EXPANSION


HORIZON/FLATNESS ⇒ ACCELERATED EXPANSION
$\frac{d|\Omega-1|}{d\ln a} < 0$
$\left(\frac{d}{d_H}\right) > 0$
$P < -\frac{1}{3}\rho$

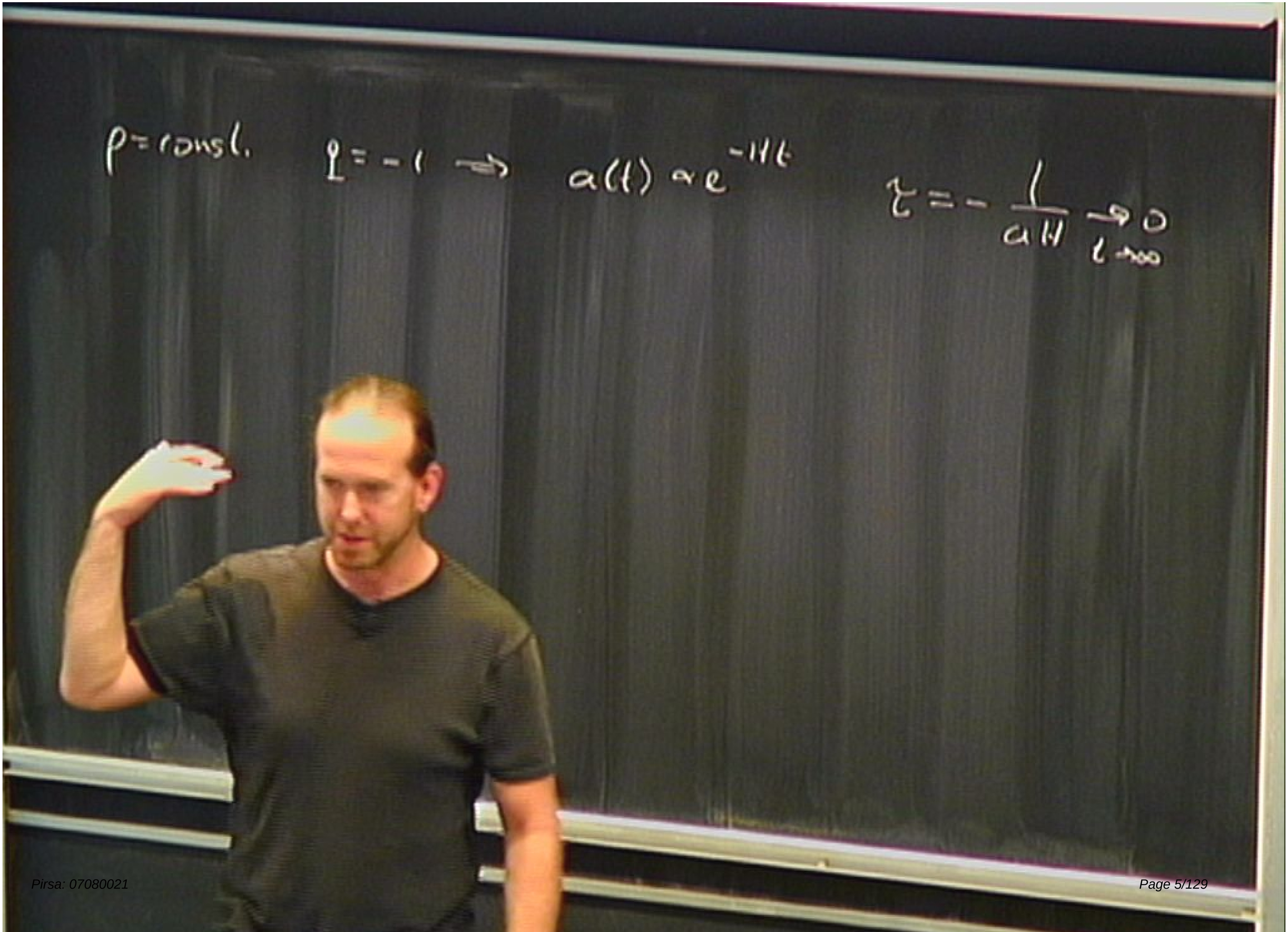
HORIZON/FLATNESS $\Rightarrow$ ACCELERATED EXPANSION

$$\frac{d|\Omega-1|}{d\ln a} < 0$$

$$\frac{d}{d\ln a}\left(\frac{d}{d_H}\right) > 0$$

$$\left(\frac{\ddot{a}}{a}\right) > 0$$

$$P < -\frac{1}{3}\rho$$


$\rho = \mathrm{const.}$
$w = -1 \Rightarrow a(t) \propto e^{-Ht}$
$\tau = -\frac{1}{aH} \xrightarrow[t \to \infty]{} 0$

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$\rho_R \propto \frac{1}{a^4}$

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$$\rho_M \propto \frac{1}{a^3} \propto e^{-3Ht}$$

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TWO <u>REQUIREMENTS</u>:

(1) END INFLATION

$\rho = \text{const.}$ $\quad q = -1 \Rightarrow a(t) \propto e^{-Ht}$ $\quad \tau = -\frac{1}{aH} \xrightarrow[t\to\infty]{} 0$

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TWO REQUIREMENTS:

(1) END INFLATION

(2) REPOPULATE MATTER / RADIATION

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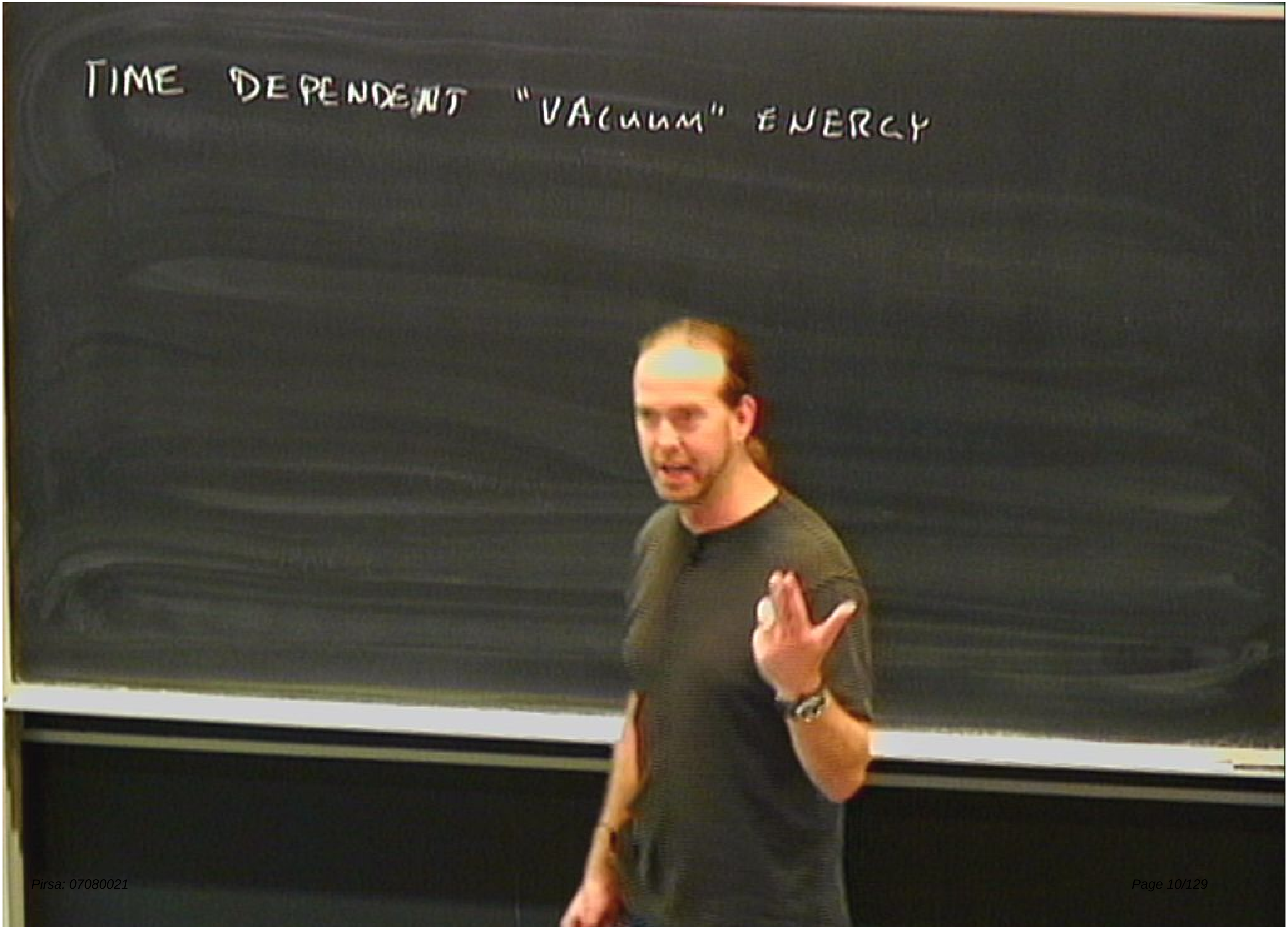
TIME DEPENDENT "VACUUM" ENERGY

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$\Rightarrow$ SCALAR ORDER PARAMETER $\phi(\vec{x})$

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LANGRANGIAN DENSITY

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi)$$

$$= \frac{1}{2}\eta^{\mu\nu}\partial_\mu \phi \partial_\nu \phi - V(\phi)$$

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$$\int d^4x \, \mathcal{L}[\Phi, \partial_\mu \Phi]$$

$\delta S = 0 \Rightarrow$ EULER LAGRANGE EQUATIONS

$$\partial_\mu \left[\frac{\delta \mathcal{L}}{\delta(\partial_\mu \Phi)} \right] - \frac{\delta \mathcal{L}}{\delta \Phi} = 0$$

ACTION $S = \int d^4x \, \mathcal{L}[\phi, \partial_\mu \phi]$

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FREE FIELD $\quad V(\phi) = 0$

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$$\phi(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[a_k \, e^{i(\vec{k}\cdot\vec{x} - \omega t)} + a_k^* \, e^{-i(\vec{k}\cdot\vec{x} - \omega t)} \right]$$

Fourier Coefficients

$$\int \frac{}{(2\pi)^{3/2}} \left[a_k\, e \quad + a_k^{*}\, e \right]$$

Fourier Coefficients

QUANTIZATION

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Canonical Momentum: $\Pi(x) \equiv \partial_0 \phi(x)$

$$\left[\phi(x), \Pi(x')\right]_{t=t'} = i\hbar\, \delta^3(\vec{x}-\vec{x}')$$

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$$\left[\hat{a}_k, \hat{a}_k^\dagger\right] = \delta^3(\vec{k}-\vec{k}')$$

SHO

$$[\hat{a}, \hat{a}^{\dagger}] = 1$$

$$H = \hbar\omega\left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right)$$

GROUND STATE $\quad \hat{a}^{\dagger}\hat{a}|0\rangle = 0$

$$\Rightarrow \quad H|0\rangle = \frac{1}{2}\hbar\omega|0\rangle$$

QFT:

$$H = \int d^3k \, \hbar\omega_k \left[\hat{a}^\dagger_k \hat{a}_k + \frac{1}{2} \right]$$

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REAL: $\Omega_\Lambda \simeq 0.7$

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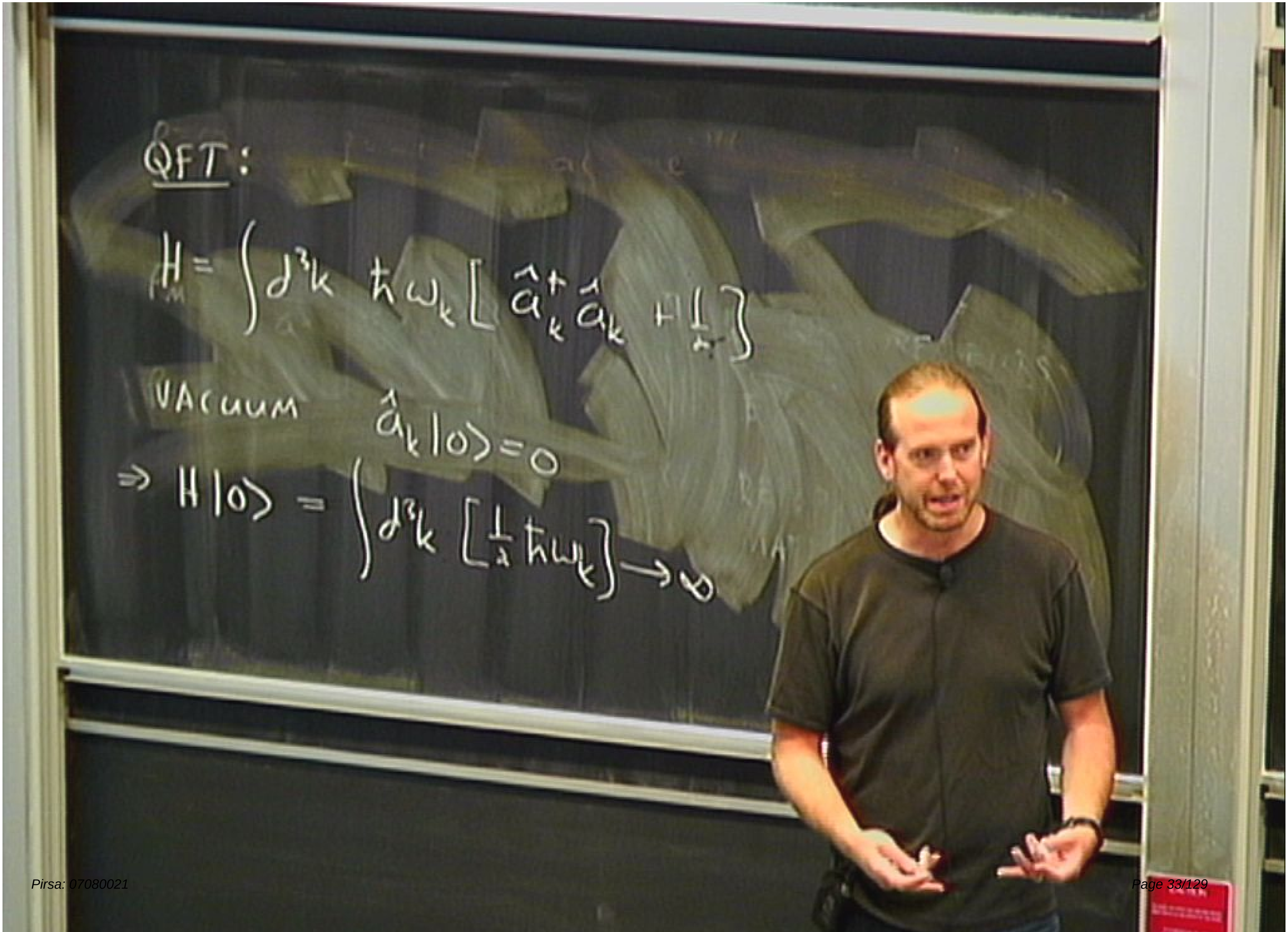
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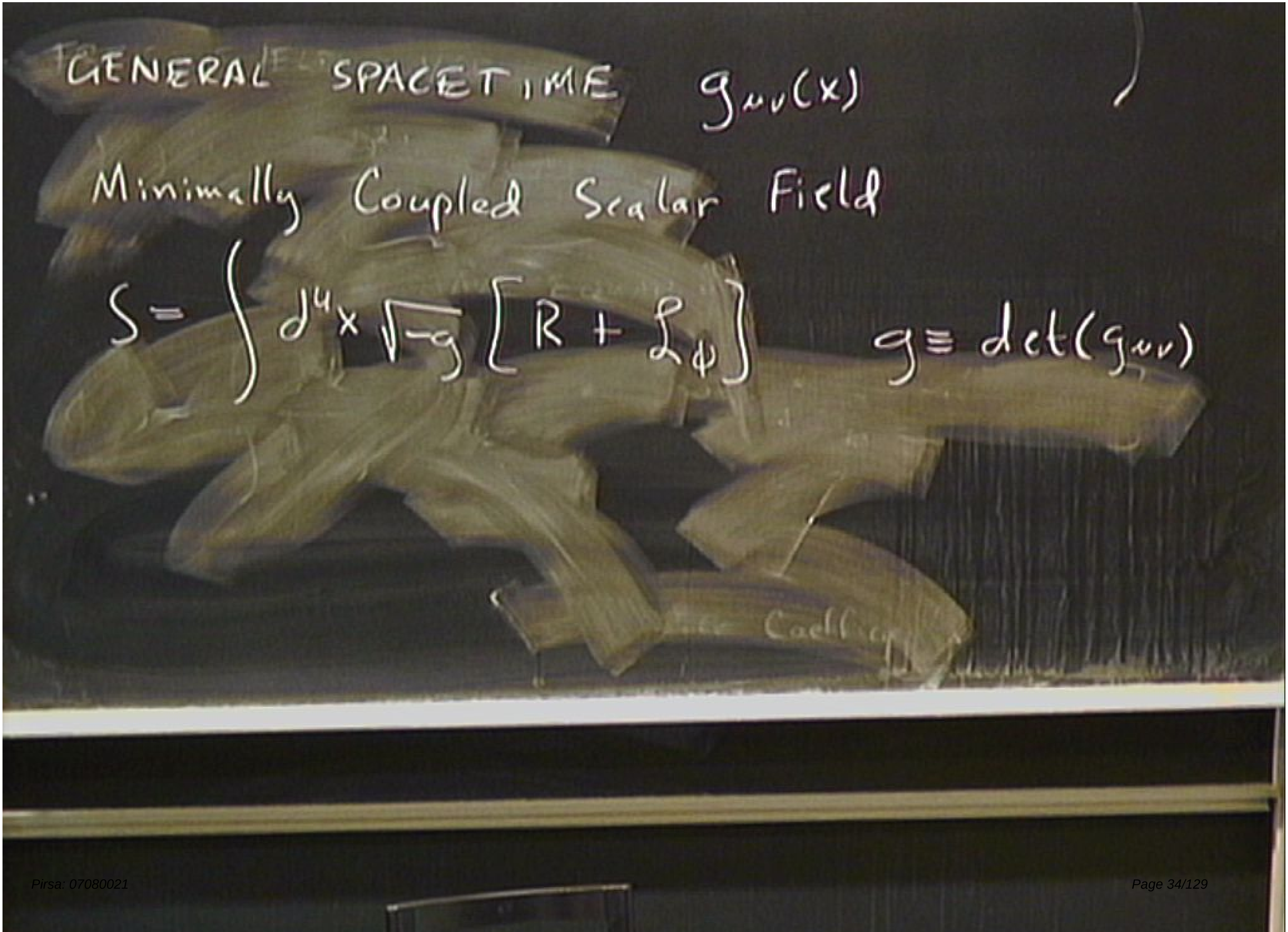
GENERAL SPACETIME $g_{\mu\nu}(x)$
Minimally Coupled Scalar Field
$S = \int d^4x \sqrt{-g}\,[R + \mathcal{L}_\phi]$
$g \equiv \det(g_{\mu\nu})$

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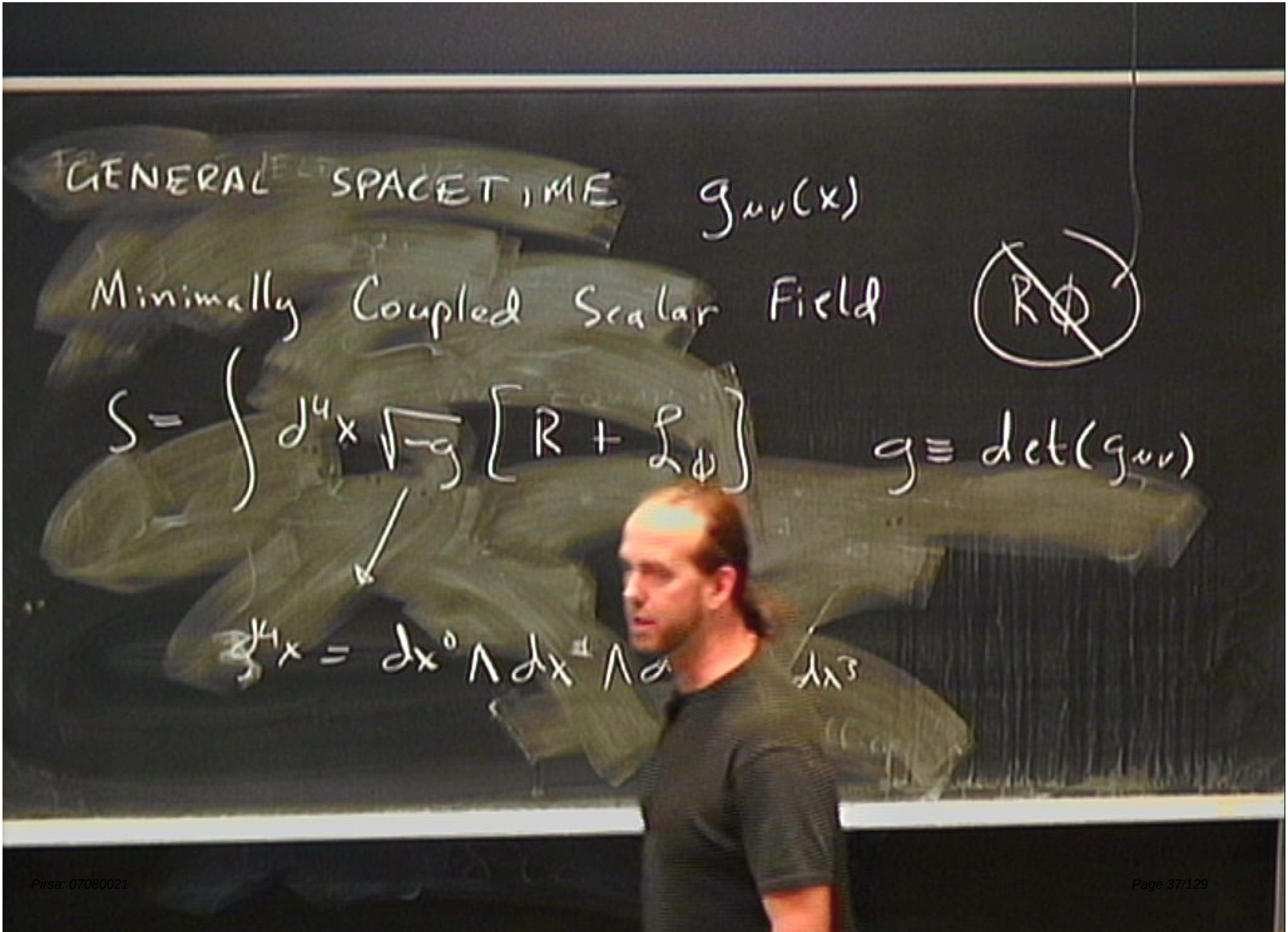
$$d^4x = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$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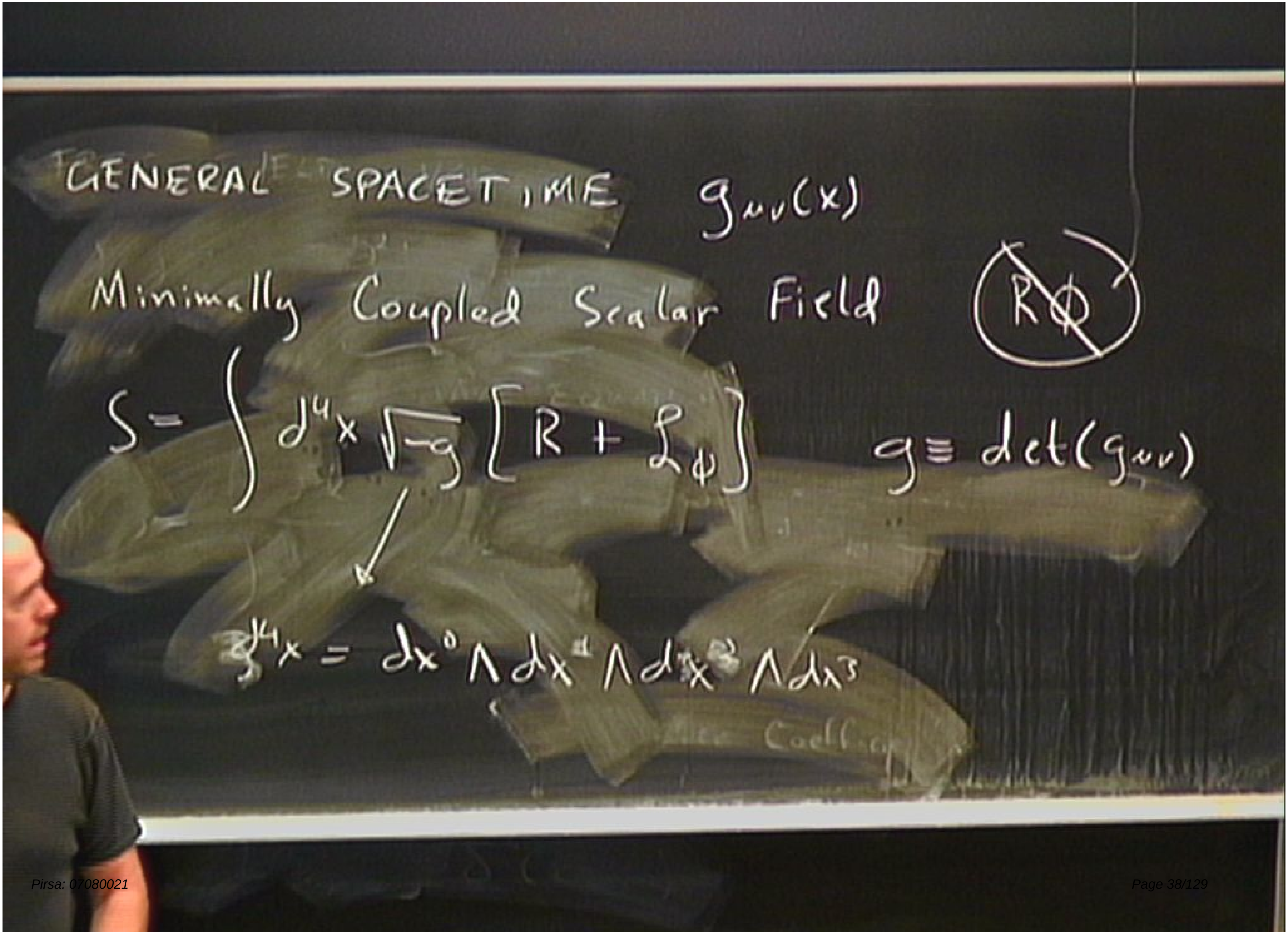
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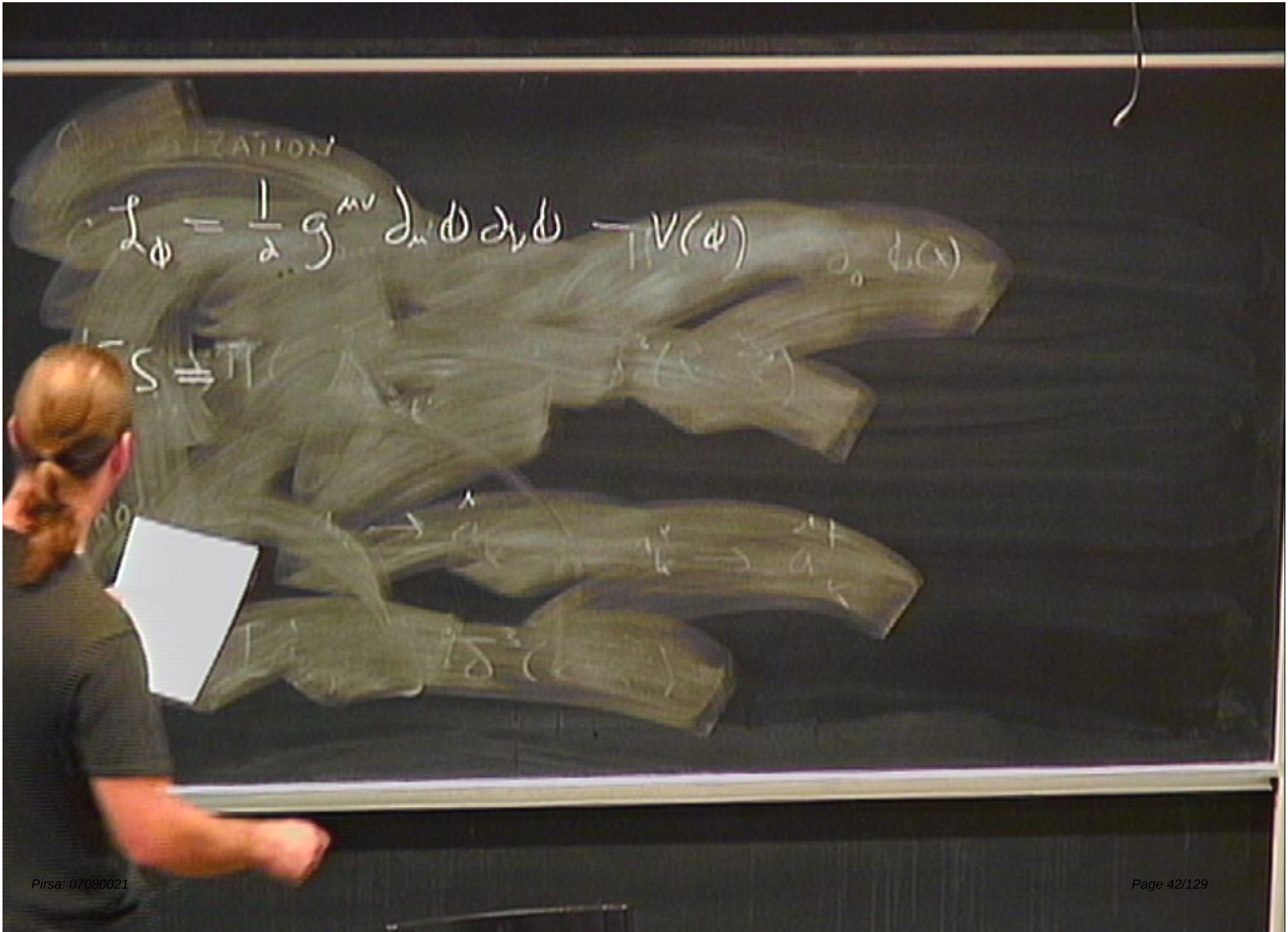
$$\mathcal{L}_\phi = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \qquad \partial_0 \phi(x)$$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$$= \int \partial_\nu\{g^{\mu\nu}\sqrt{-g}\,\partial_\mu\phi\,\delta\phi\}$$

$$\left\{\left[\partial_\nu\left(g^{\mu\nu}\sqrt{-g}\,\partial_\mu\phi\right) + \sqrt{-g}\frac{\partial V}{\delta\phi}\right]\delta\phi\right\} = 0$$

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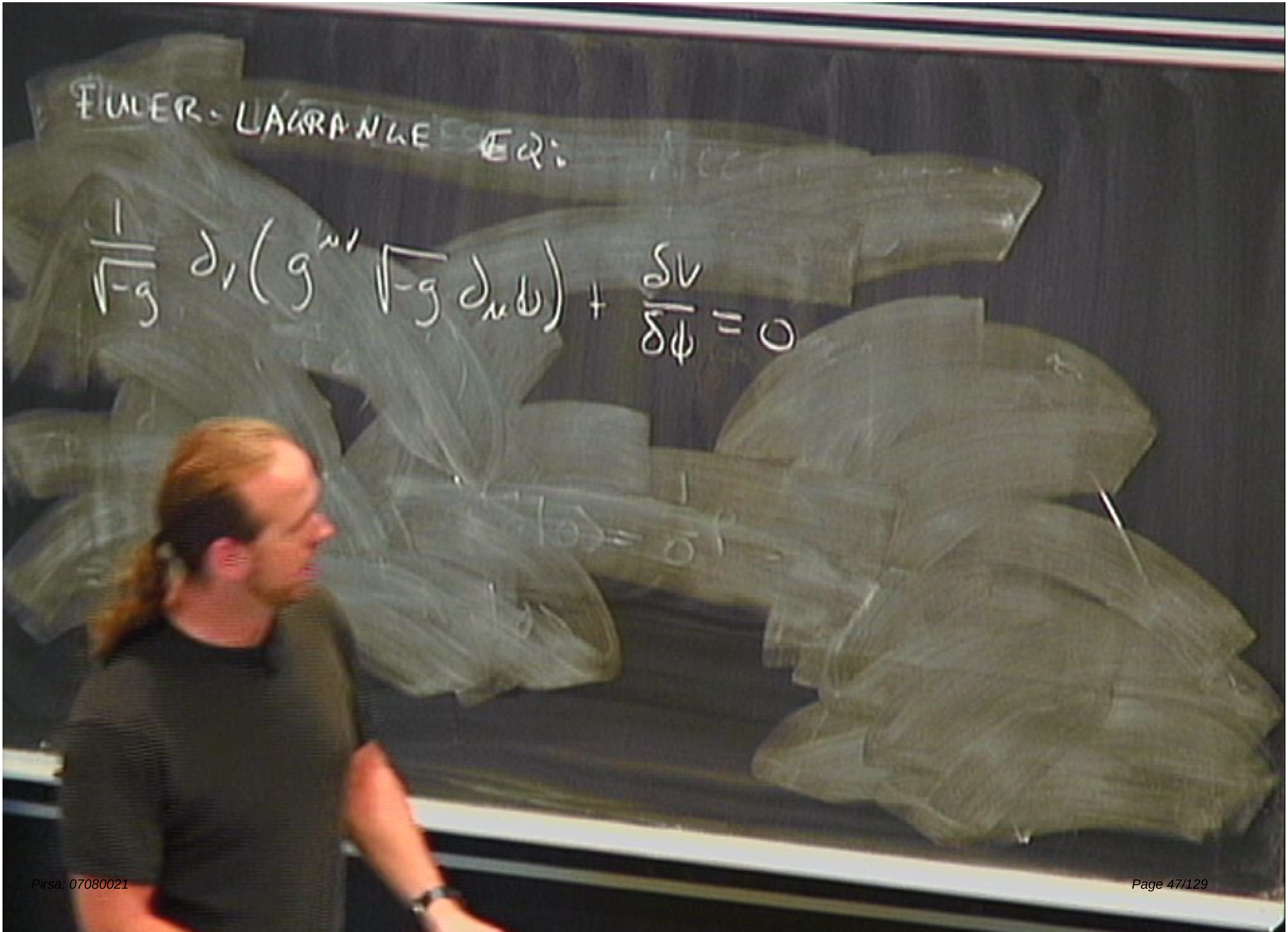
$$= \int\Big\{\overbrace{\cancel{\partial_\nu\left[g^{\mu\nu}\sqrt{-g}\,\partial_\mu\phi\,\delta\phi\right]}}^{\text{SURFACE TERM}} - \left[\partial_\nu\left(g^{\mu\nu}\sqrt{-g}\,\partial_\mu\phi\right) + \sqrt{-g}\,\frac{\partial V}{\delta\phi}\right]\delta\phi\Big\} = 0$$


EULER-LAGRANGE EQ:
$\frac{1}{\sqrt{-g}} \partial_\nu \left(g^{\mu\nu} \sqrt{-g}\, \partial_\mu \phi \right) + \frac{\delta V}{\delta \phi} = 0$

ACTION $\quad S = \int d^4x \, \mathcal{L}[\phi, \partial_\mu \phi]$

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EXERCISE

FOR

EULER-LAGRANGE EQ:

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EXERCISE

FOR $g_{\mu\nu} = a^2(t)\,\eta_{\mu\nu}$

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(a) Calculate

$g^{\mu\nu}$, $g^{\mu}{}_{\nu}$, $g \equiv \mathrm{Det}(g_{\mu\nu})$

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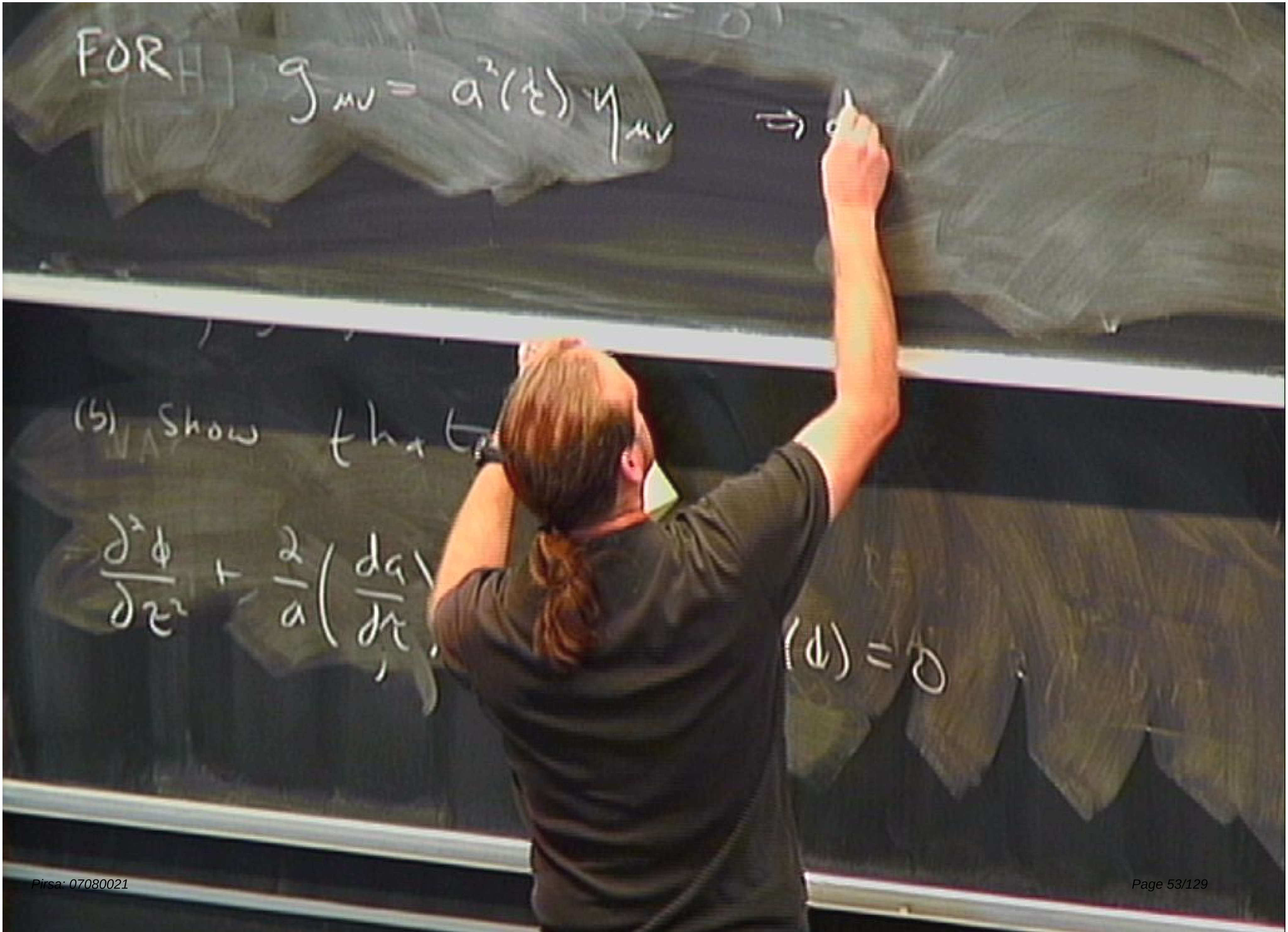
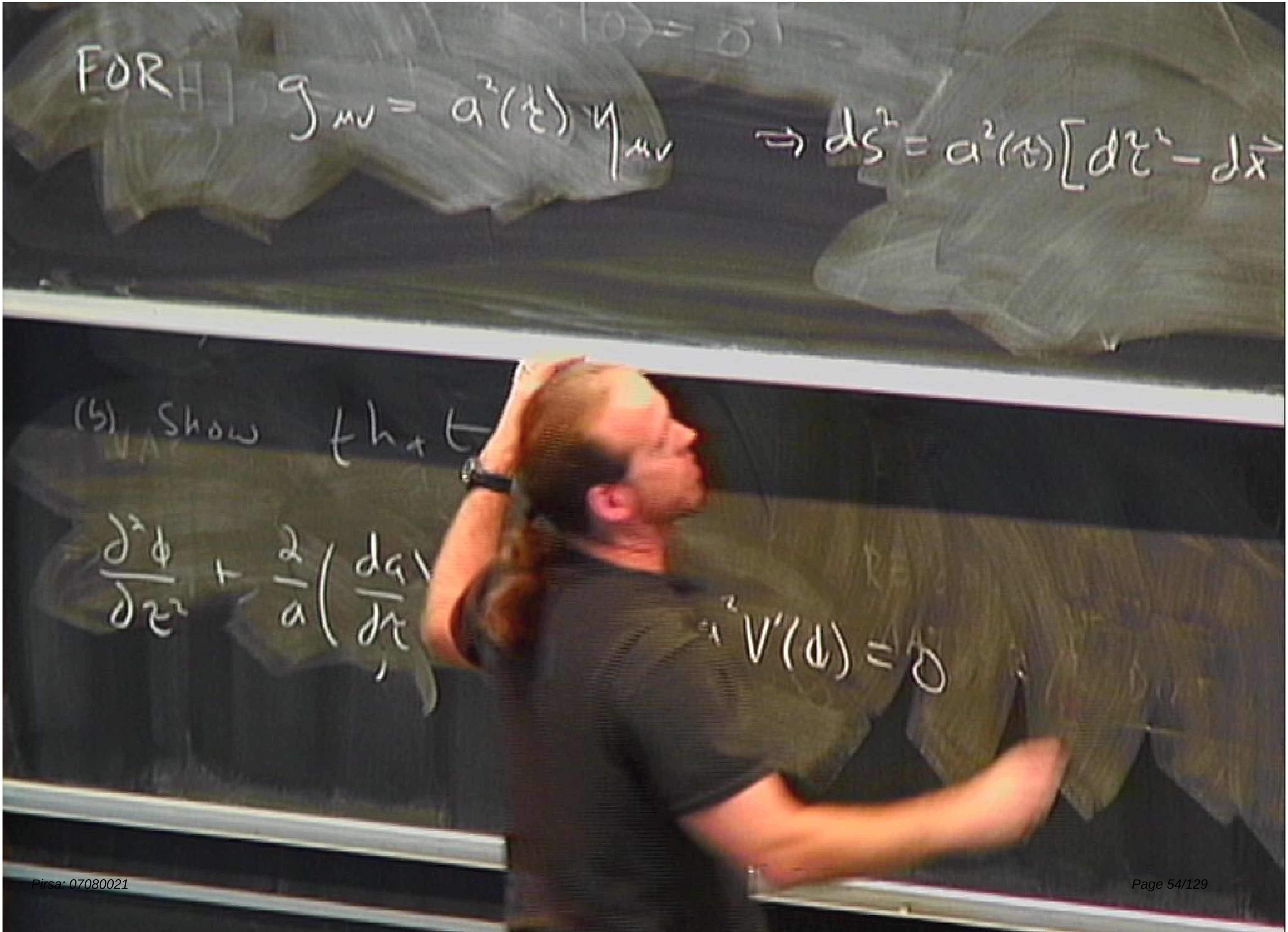
(b) Show that


FOR
$g_{\mu\nu} = a^2(t)\,\eta_{\mu\nu} \Rightarrow$
(5) Show tha
$(\phi) = 0$


FOR $g_{\mu\nu} = a^2(\tau)\,\eta_{\mu\nu}$ $\Rightarrow ds^2 = a^2(\tau)[d\tau^2 - d\vec{x}$
(5) Show that
$\frac{\partial^2\phi}{\partial\tau^2} + \frac{2}{a}\left(\frac{da}{d\tau}\right.$
$a^2 V'(\phi) = 0$

(a) Calculate

$$g^{\mu\nu}, \quad g^{\mu}{}_{\nu}, \quad g \equiv \mathrm{Det}(g_{\mu\nu})$$

(b) Show that

$$\frac{\partial^2 \phi}{\partial \tau^2} + \frac{2}{a}\left(\frac{da}{d\tau}\right) - \nabla^2 \phi + a^2 V'(\phi) = 0$$

EXERCISE

FOR [H] $g_{\mu\nu} = a^2(\tau)\,\eta_{\mu\nu} \Rightarrow ds^2 = a^2(\tau)\left[d\tau^2 - d\vec{x}^2\right]$

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FRIEDMANN EQ.:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3M_{Pl}^2}\rho$$

$$\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)(\rho + P) = 0$$

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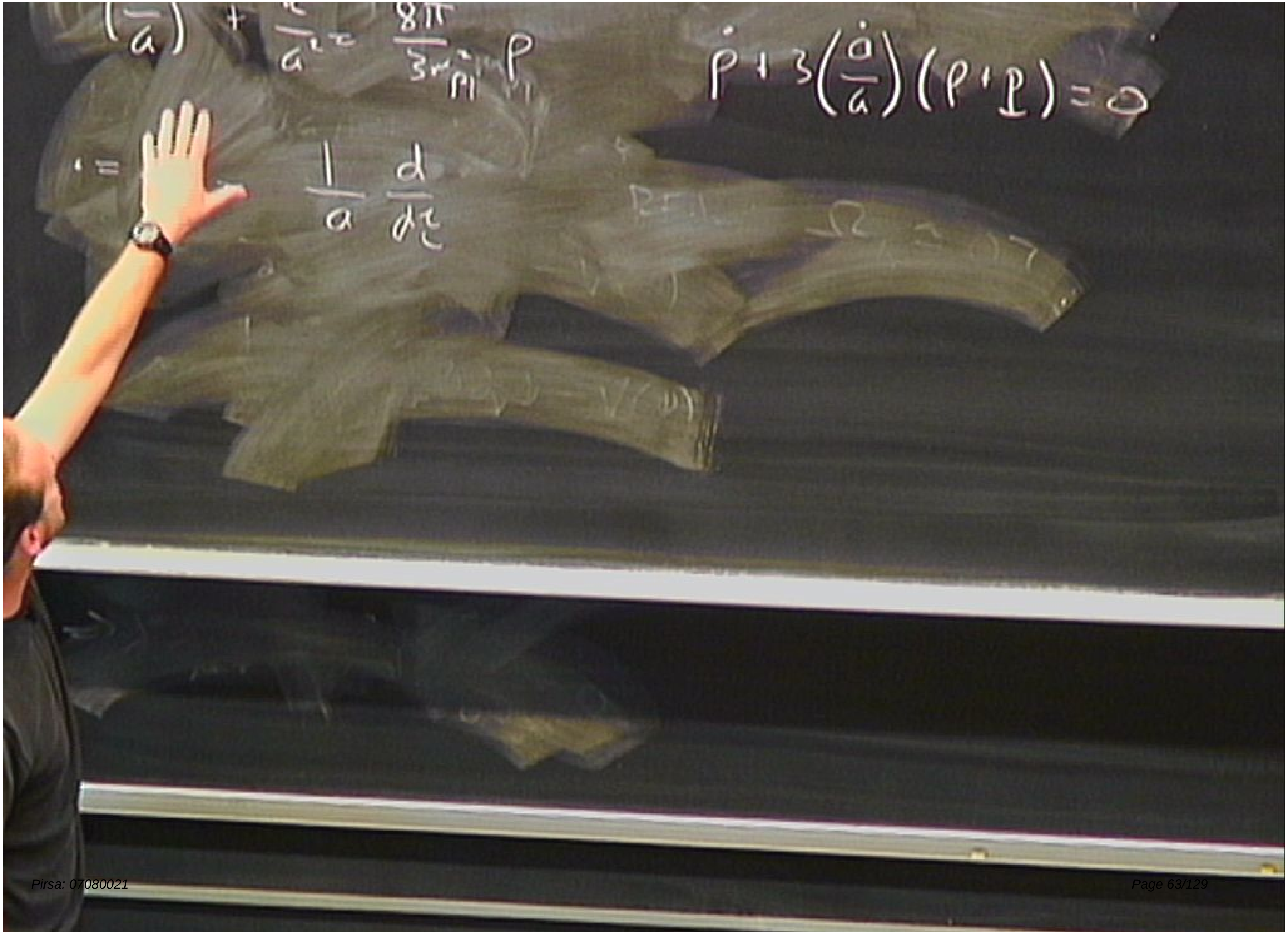
$$\dot{} = \frac{d}{dt} = \frac{1}{a}\frac{d}{d\tau}$$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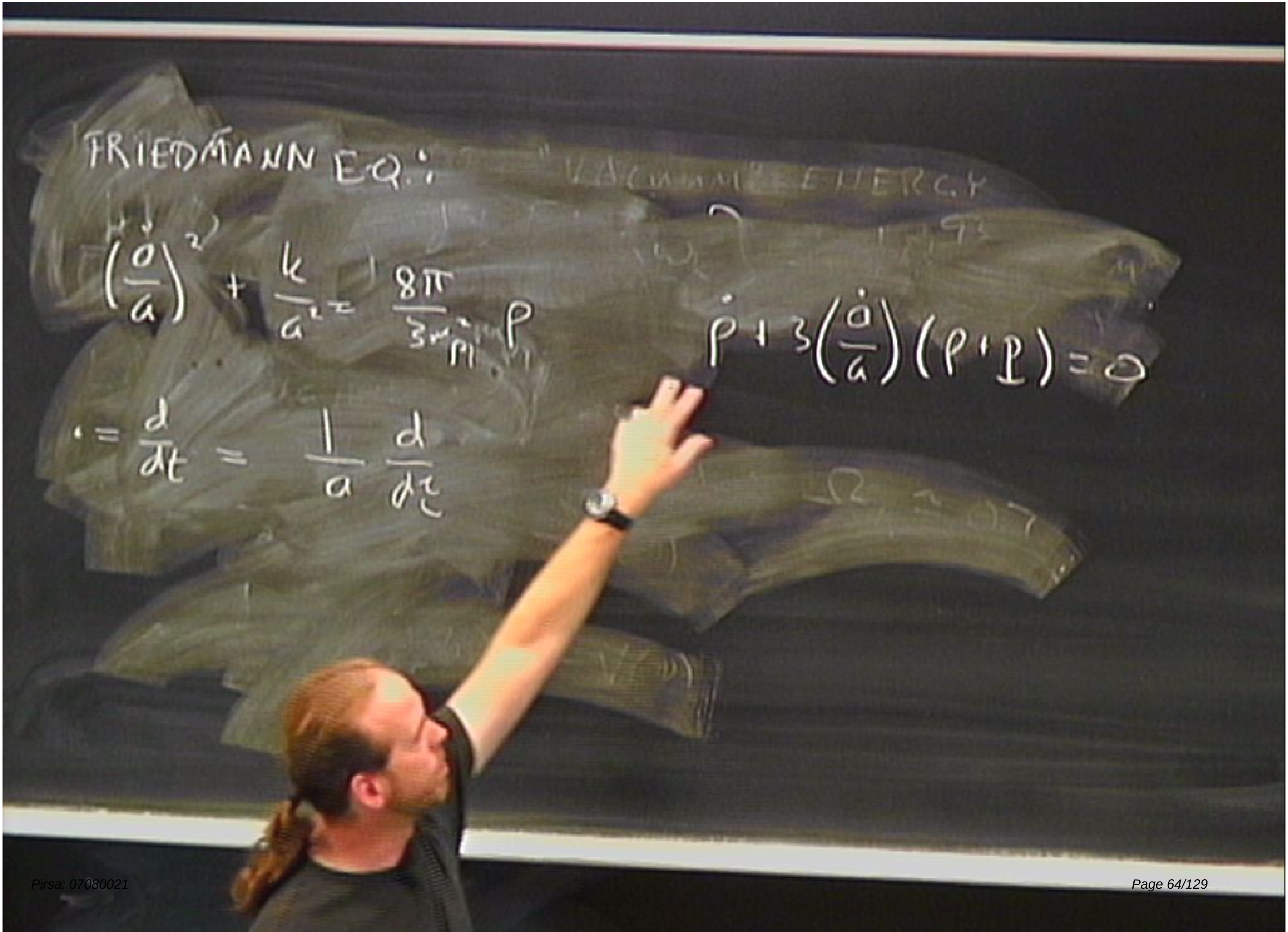
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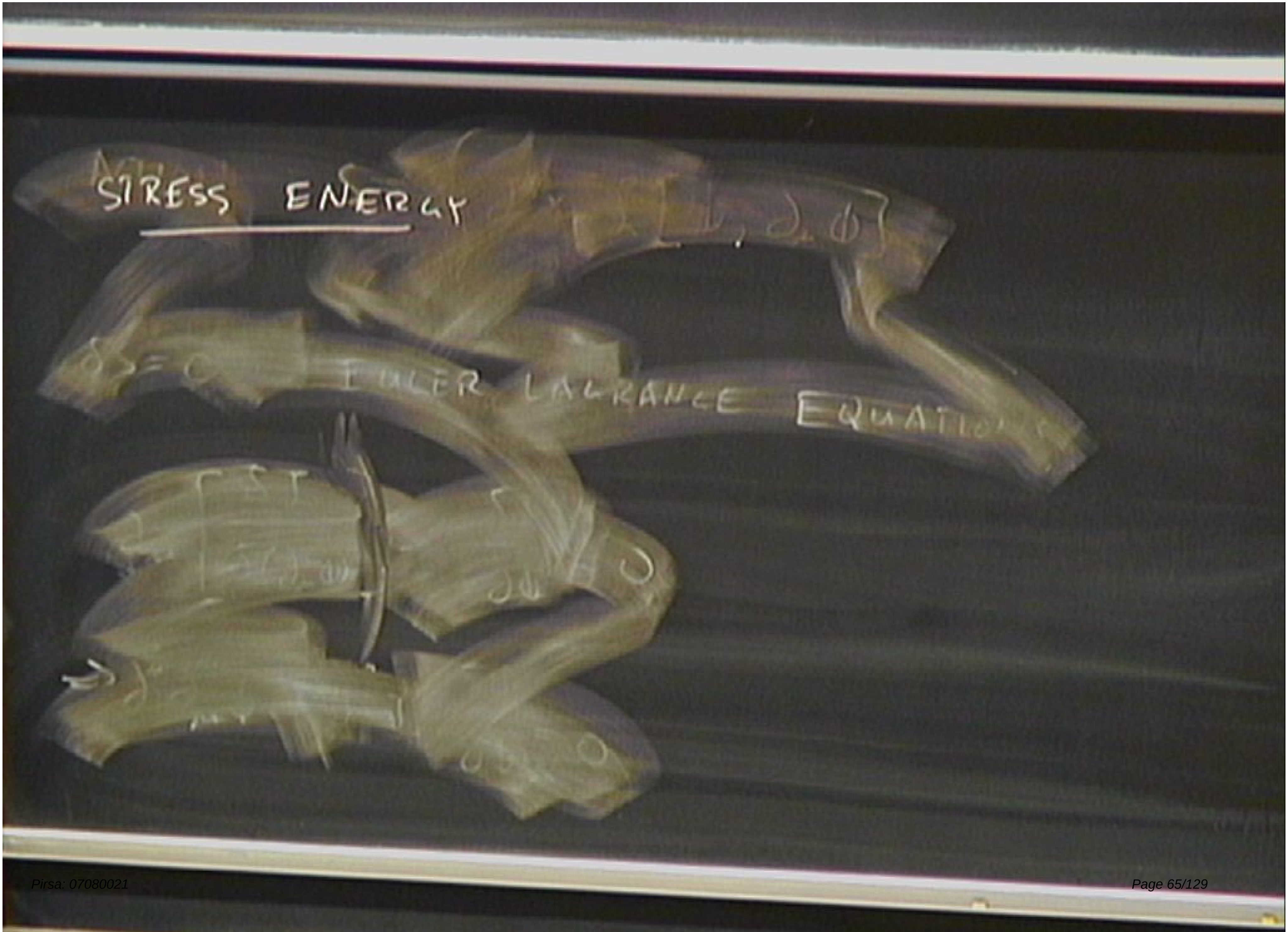
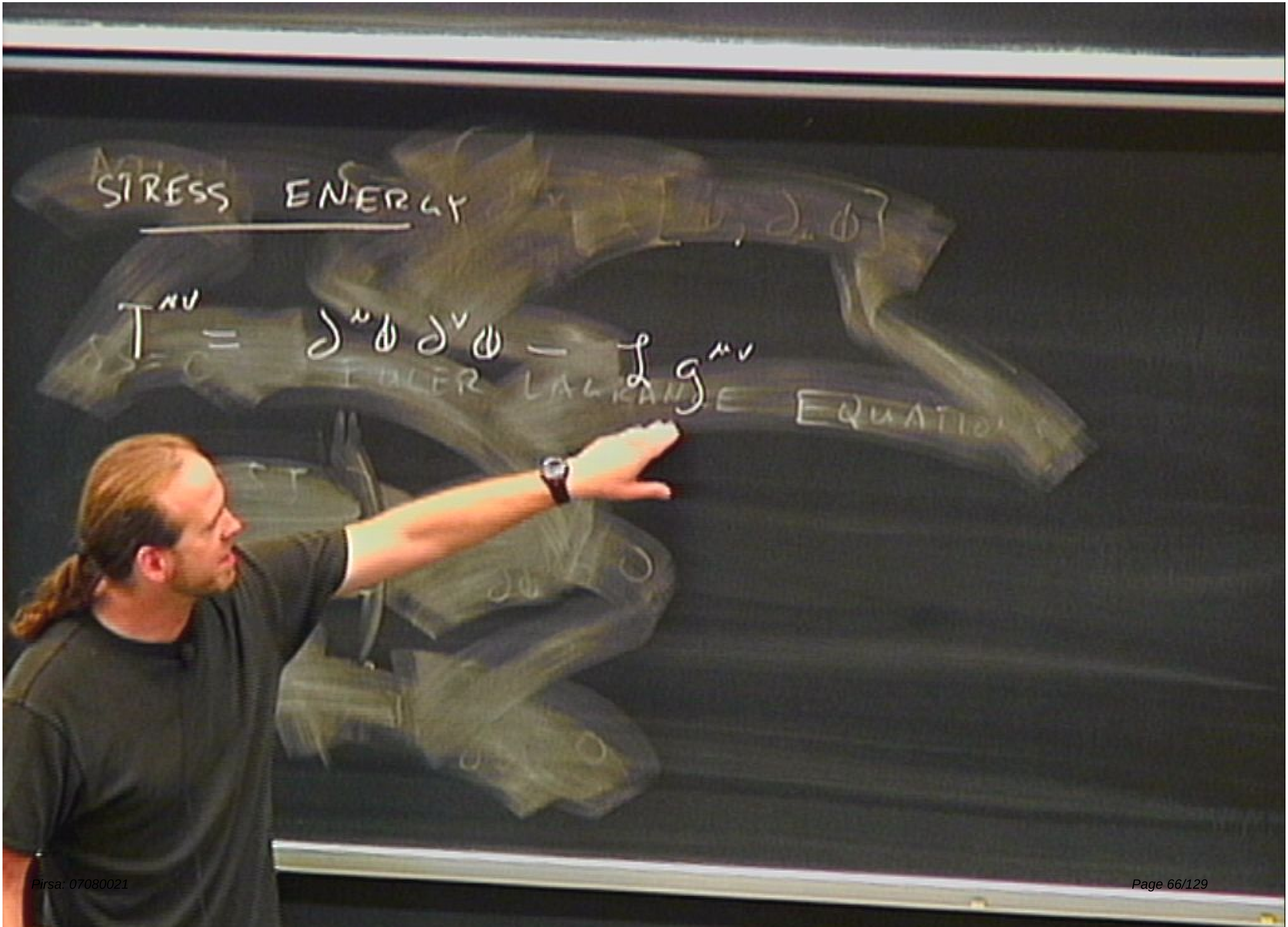
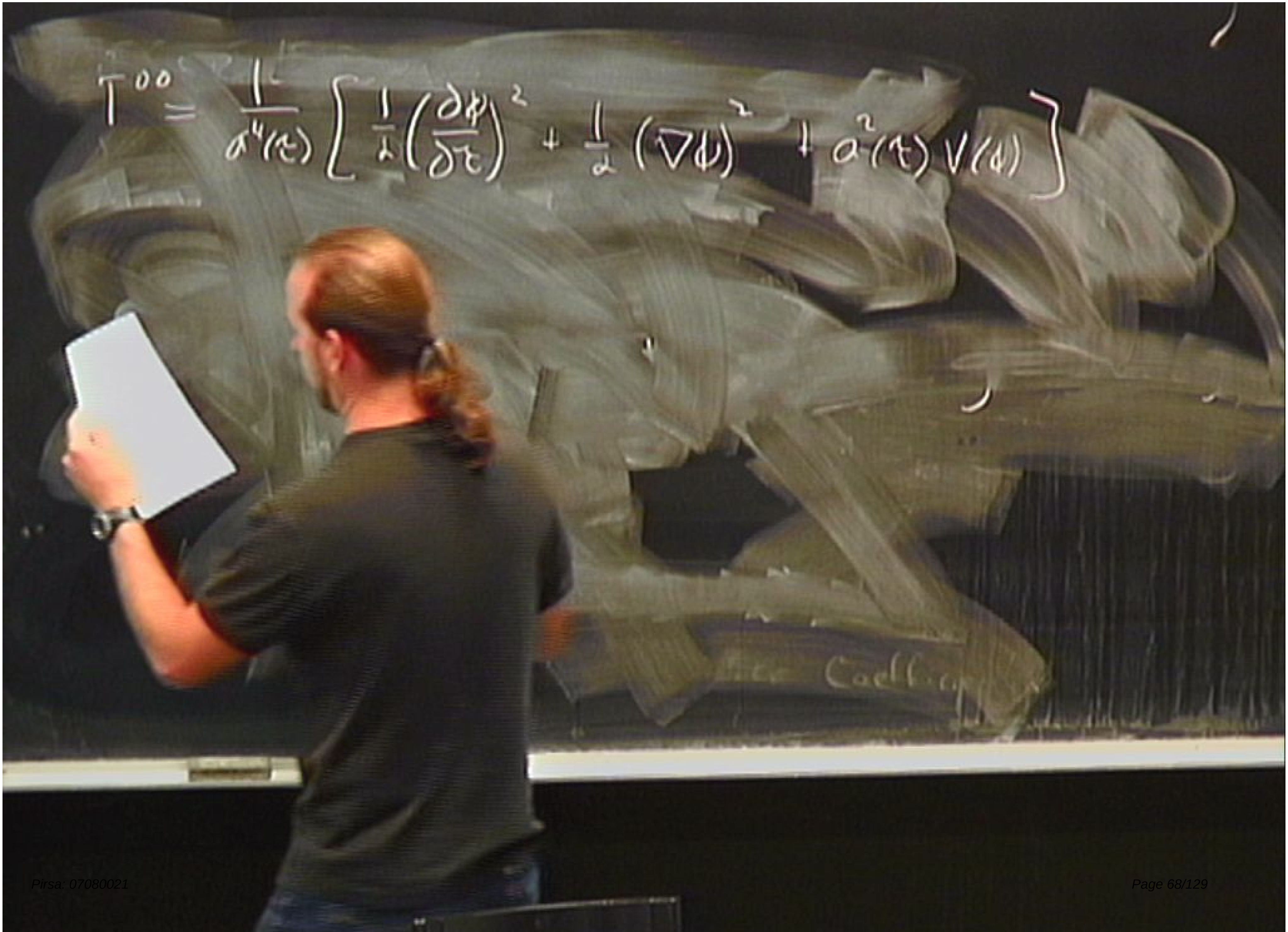

STRESS ENERGY
EULER LAGRANGE EQUATIONS


STRESS ENERGY
$T^{\mu\nu} = \partial^\mu \phi \, \partial^\nu \phi - \mathcal{L} g^{\mu\nu}$

STRESS ENERGY

$$T^{\mu\nu} = \partial^\mu\phi\,\partial^\nu\phi - \mathcal{L}\, g^{\mu\nu}$$

$$= \frac{1}{a^4(t)}\left[\left(\eta^{\mu\alpha}\eta^{\nu\beta} - \frac{1}{2}\eta^{\mu\nu}\eta^{\alpha\beta}\right)\partial_\alpha\phi\,\partial_\beta\phi - \eta^{\mu\nu} a^2(t)\, V(\phi)\right]$$


$T^{00} = \frac{1}{a^4(t)} \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 + \frac{1}{2}(\nabla\phi)^2 + a^2(t)V(\phi) \right]$

$$T^{00} = \frac{1}{a^4(\tau)}\left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 + \frac{1}{2}(\nabla\phi)^2 + a^2(\tau)V(\phi)\right]$$

$$T^{0i} = -\frac{1}{a^4(\tau)}\left(\frac{\partial\phi}{\partial\tau}\right)(\nabla\phi)_i$$

$$T^{ij} = \frac{1}{a^4(\tau)}\left\{\left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 - \frac{1}{2}(\nabla\phi)^2 - a^2V(\phi)\right]\delta^{ij} + (\nabla\phi)_i(\nabla\phi)_j\right\}$$

R-W SPACE : $\nabla\phi \to 0$

$$T^{00} = \frac{1}{a^4(\tau)}\left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 + \frac{1}{2}(\nabla\phi)^2 + a^2(\tau)V(\phi)\right]$$

$$T^{0i} = -\frac{1}{a^4(\tau)}\left(\frac{\partial\phi}{\partial\tau}\right)(\nabla\phi)_i$$

$$T^{ij} = \frac{1}{a^4(\tau)}\left\{\left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 - \frac{1}{2}(\nabla\phi)^2 - a^2V(\phi)\right]\delta^{ij} + (\nabla\phi)_i(\nabla\phi)_j\right\}$$

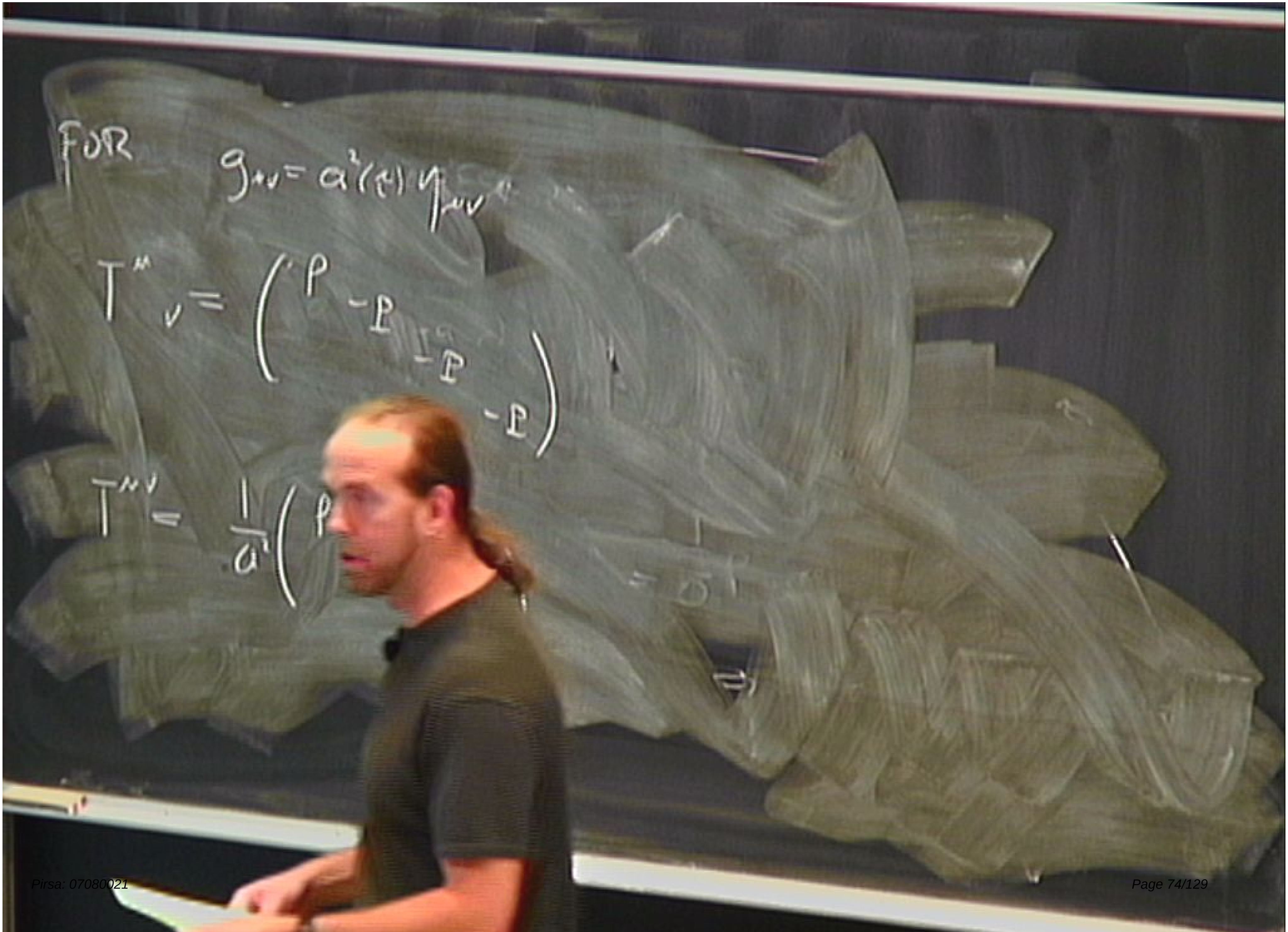
R-W SPACE : $\nabla\phi \to 0$

OFF-DIAGONAL TERMS $\to 0$

$$\boxed{\begin{aligned} T^{00} &= \frac{1}{a^4(\tau)}\left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 + a^2(\tau)V(\phi)\right] \\ T^{ii} &= \frac{1}{a^4(\tau)}\left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 - a^2(\tau)V(\phi)\right] \end{aligned}}$$

FOR $g_{\mu\nu} = a^2(t)\,\eta_{\mu\nu}$

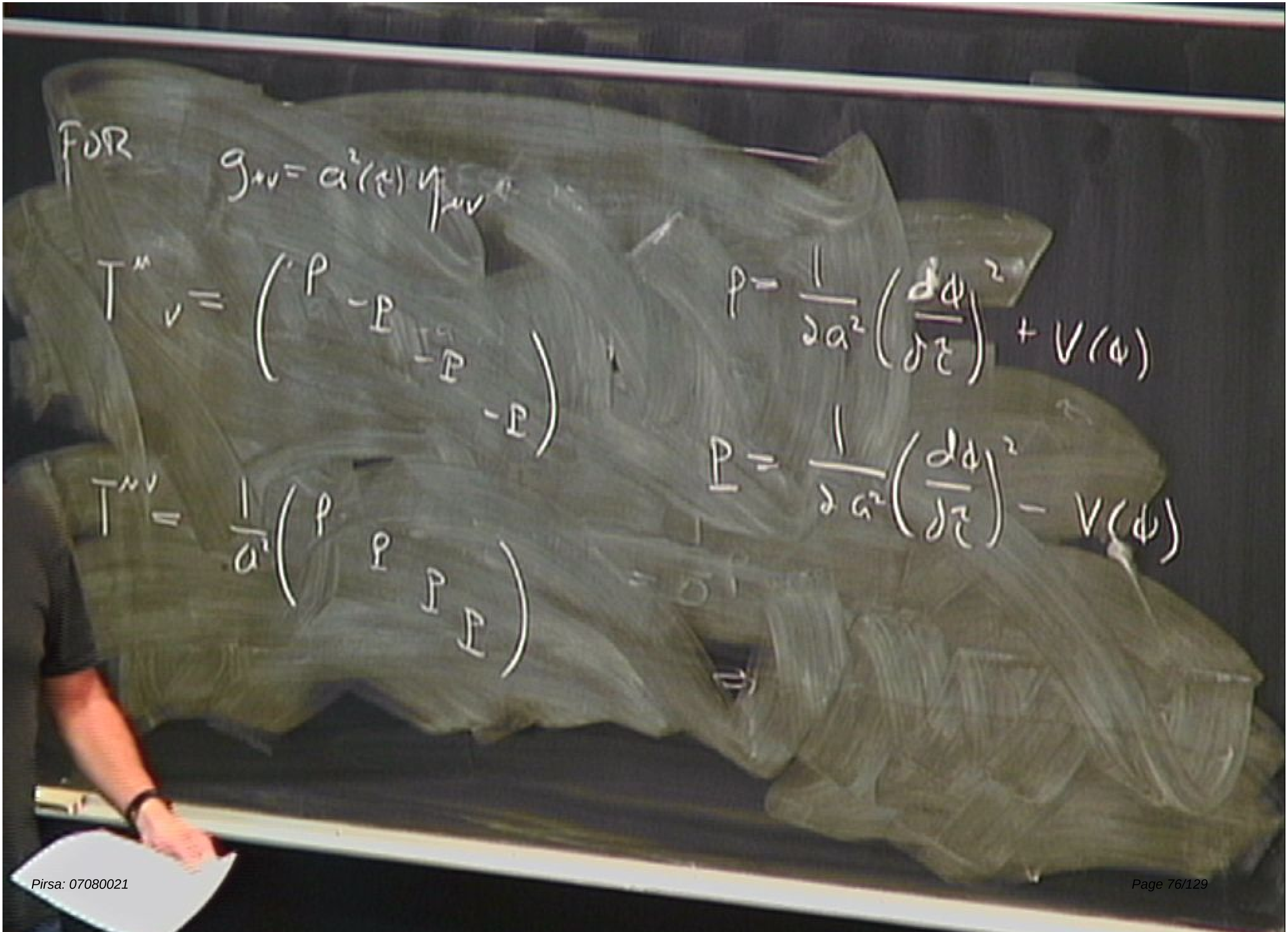
$$T^{\mu}{}_{\nu} = \begin{pmatrix} \rho & & & \\ & -p & & \\ & & -p & \\ & & & -p \end{pmatrix}$$


FOR
$g_{\mu\nu} = a^2(t)\,\eta_{\mu\nu}$
$T^{\mu}{}_{\nu} = \begin{pmatrix} \rho & & & \\ & -p & & \\ & & -p & \\ & & & -p \end{pmatrix}$
$T^{\mu\nu} = \frac{1}{a^2}\Big(\rho$

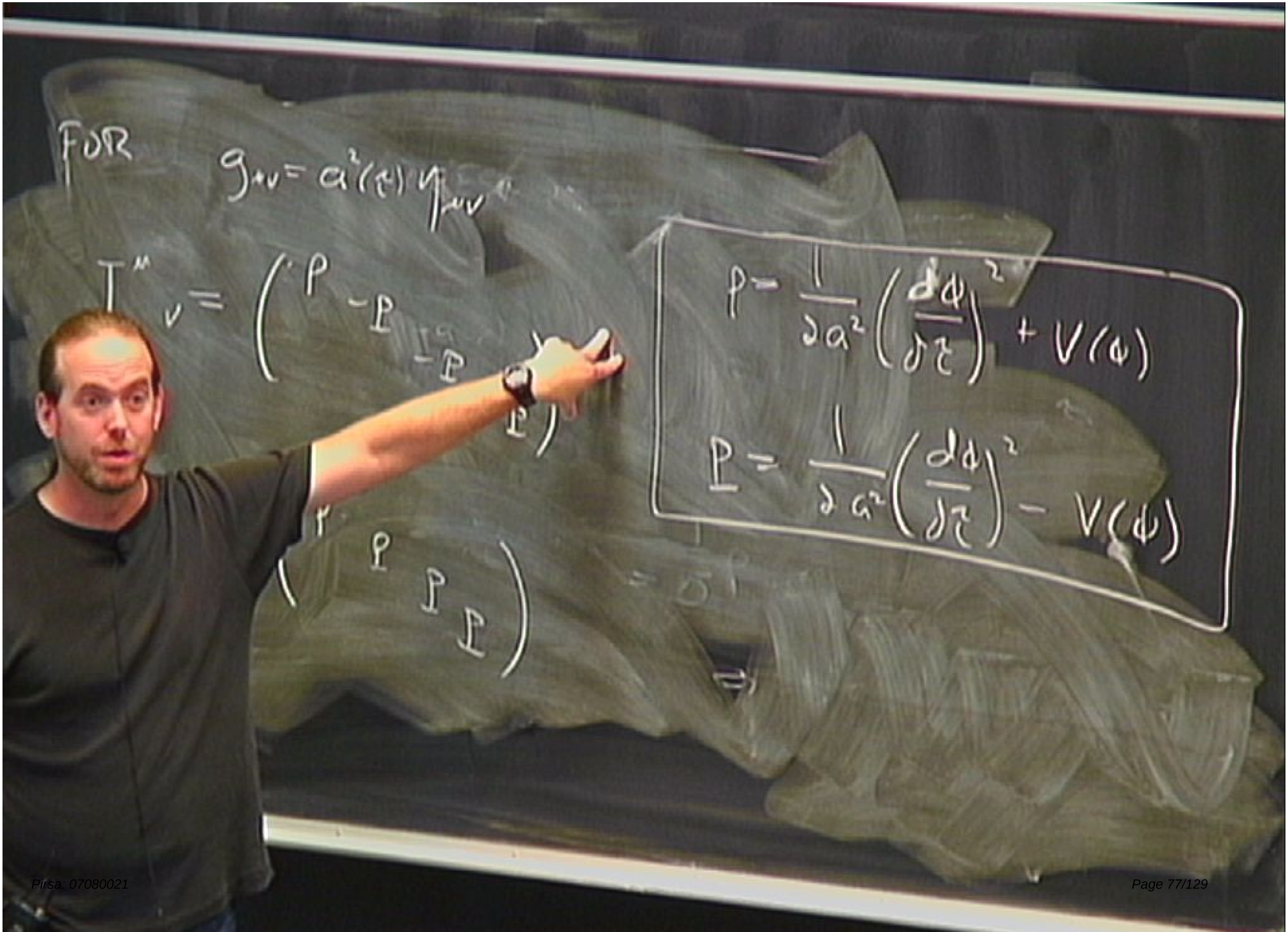
FOR $g_{\mu\nu} = a^2(t)\,\eta_{\mu\nu}$

$$T^{\mu}{}_{\nu} = \begin{pmatrix} \rho & & & \\ & -p & & \\ & & -p & \\ & & & -p \end{pmatrix}$$

$$T^{\mu\nu} = \frac{1}{a^2}\begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$$



FOR $g_{\mu\nu} = a^2(\tau)\,\eta_{\mu\nu}$
$T^{\mu}{}_{\nu} = \begin{pmatrix} \rho & & & \\ & -P & & \\ & & -P & \\ & & & -P \end{pmatrix}$
$\rho = \frac{1}{2a^2}\left(\frac{d\phi}{d\tau}\right)^2 + V(\phi)$
$P = \frac{1}{2a^2}\left(\frac{d\phi}{d\tau}\right)^2 - V(\phi)$
$T^{\mu\nu} = \frac{1}{a^2}\begin{pmatrix} \rho & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}$


FOR
$g_{\mu\nu} = a^2(\tau)\,\eta_{\mu\nu}$
$\rho = \frac{1}{2a^2}\left(\frac{d\phi}{d\tau}\right)^2 + V(\phi)$
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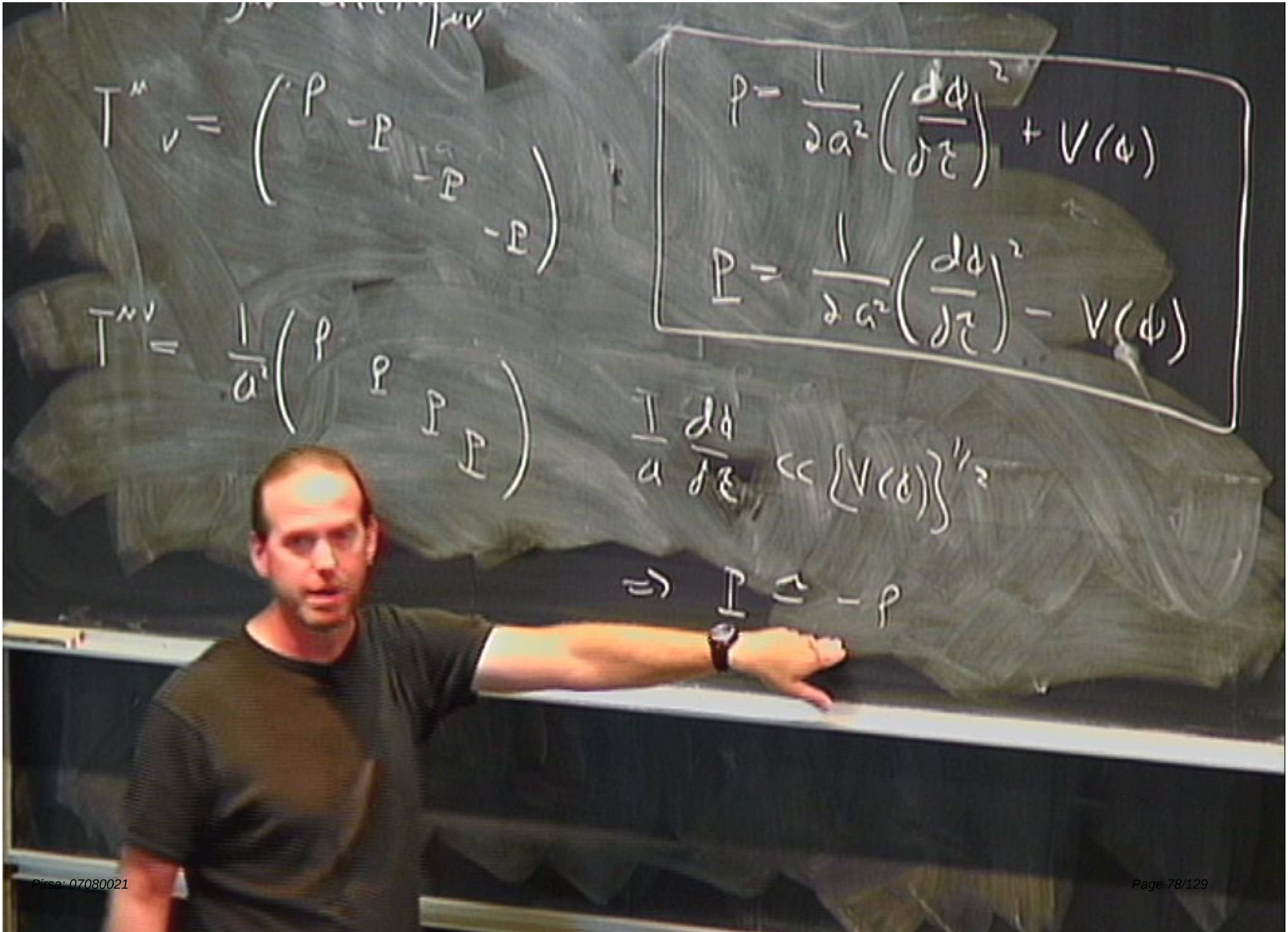

$T^{\mu}{}_{\nu} = \begin{pmatrix} \rho & & & \\ & -P & & \\ & & -P & \\ & & & -P \end{pmatrix}$
$\rho = \frac{1}{2a^2}\left(\frac{d\phi}{d\tau}\right)^2 + V(\phi)$
$P = \frac{1}{2a^2}\left(\frac{d\phi}{d\tau}\right)^2 - V(\phi)$
$T^{\mu\nu} = \frac{1}{a^2}\begin{pmatrix} \rho & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}$
$\frac{1}{a}\frac{d\phi}{d\tau} \ll \{V(\phi)\}^{1/2}$
$\Rightarrow P \simeq -\rho$

FOR $g_{\mu\nu} = a^2(\tau)\,\eta_{\mu\nu}$

$$T^{\mu}{}_{\nu} = \begin{pmatrix} \rho & & & \\ & -P & & \\ & & -P & \\ & & & -P \end{pmatrix}$$

$$= \frac{1}{a^2}\begin{pmatrix} \rho & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}$$

$$\boxed{\begin{aligned} \rho &= \frac{1}{2a^2}\left(\frac{d\phi}{d\tau}\right)^2 + V(\phi) \\ P &= \frac{1}{2a^2}\left(\frac{d\phi}{d\tau}\right)^2 - V(\phi) \end{aligned}}$$

$$\frac{1}{a}\frac{d\phi}{d\tau} \ll [V(\phi)]^{1/2}$$

$$\Rightarrow P \simeq -\rho$$

FOR $g_{\mu\nu} = a^2(\tau)\,\eta_{\mu\nu}$ ← $k=0$ / $\Omega=1$

$$T^{\mu}{}_{\nu} = \begin{pmatrix} \rho & & & \\ & -P & & \\ & & -P & \\ & & & -P \end{pmatrix}$$

$$T^{\mu\nu} = \frac{1}{a^2}\begin{pmatrix} \rho & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}$$

$$\boxed{\begin{aligned} \rho &= \frac{1}{2a^2}\left(\frac{d\phi}{d\tau}\right)^2 + V(\phi) \\ P &= \frac{1}{2a^2}\left(\frac{d\phi}{d\tau}\right)^2 - V(\phi) \end{aligned}}$$

$$\frac{1}{a}\frac{d\phi}{d\tau} \ll \{V(\phi)\}^{1/2}$$

$$\Rightarrow P \simeq -\rho$$

$k=0$ (FLAT SPACETIME)

FRW

$$\frac{1}{a^2}\left(\frac{da}{d\tau}\right)^2 = \frac{8\pi}{3m_{Pl}^2}\left[a^2V(\phi) + \frac{1}{2}\left(\frac{d\phi}{d\tau}\right)^2\right]$$

$$\frac{1}{a}\left(\frac{d^2a}{d\tau^2}\right) - \frac{1}{a^2}\left(\frac{da}{d\tau}\right)^2 = \frac{8\pi}{3m_{Pl}^2}\left[a^2V(\phi) - \left(\frac{d\phi}{d\tau}\right)^2\right]$$

$$\frac{d^2\phi}{d\tau^2} + 2aH\frac{d\phi}{d\tau} + a^2V'(\phi) = 0$$

$$H = \left(\frac{\dot a}{a}\right) = \frac{1}{a^2}\left(\frac{da}{d\tau}\right)$$

$k=0$ (FLAT SPACETIME)

FRW

$$\frac{1}{a^2}\left(\frac{da}{d\tau}\right)^2 = \frac{8\pi}{3m_{Pl}^2}\left[a^2 V(\phi) + \frac{1}{2}\left(\frac{d\phi}{d\tau}\right)^2\right]$$

$$\frac{1}{a}\left(\frac{d^2a}{d\tau^2}\right) - \frac{1}{a^2}\left(\frac{da}{d\tau}\right)^2 = \frac{8\pi}{3m_{Pl}^2}\left[a^2 V(\phi) - \left(\frac{d\phi}{d\tau}\right)^2\right]$$

$$\frac{d^2\phi}{d\tau^2} + 2aH\frac{d\phi}{d\tau} + a^2 V'(\phi) = 0$$

$$H = \left(\frac{\dot{a}}{a}\right) = \frac{1}{a^2}\left(\frac{da}{d\tau}\right)$$

FOR COORDINATE TIME: $dt = a\,d\tau$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3m_{Pl}^2}\left[\frac{1}{2}\dot\phi^2 + V(\phi)\right]$$

$$\rho = \frac{1}{2}\dot\phi^2 + V(\phi)$$

$$p = \frac{1}{2}\dot\phi^2 - V(\phi)$$

$$\ddot\phi + 3H\dot\phi + V'(\phi) = 0$$

FOR COORDINATE TIME: $dt = a\,d\tau$

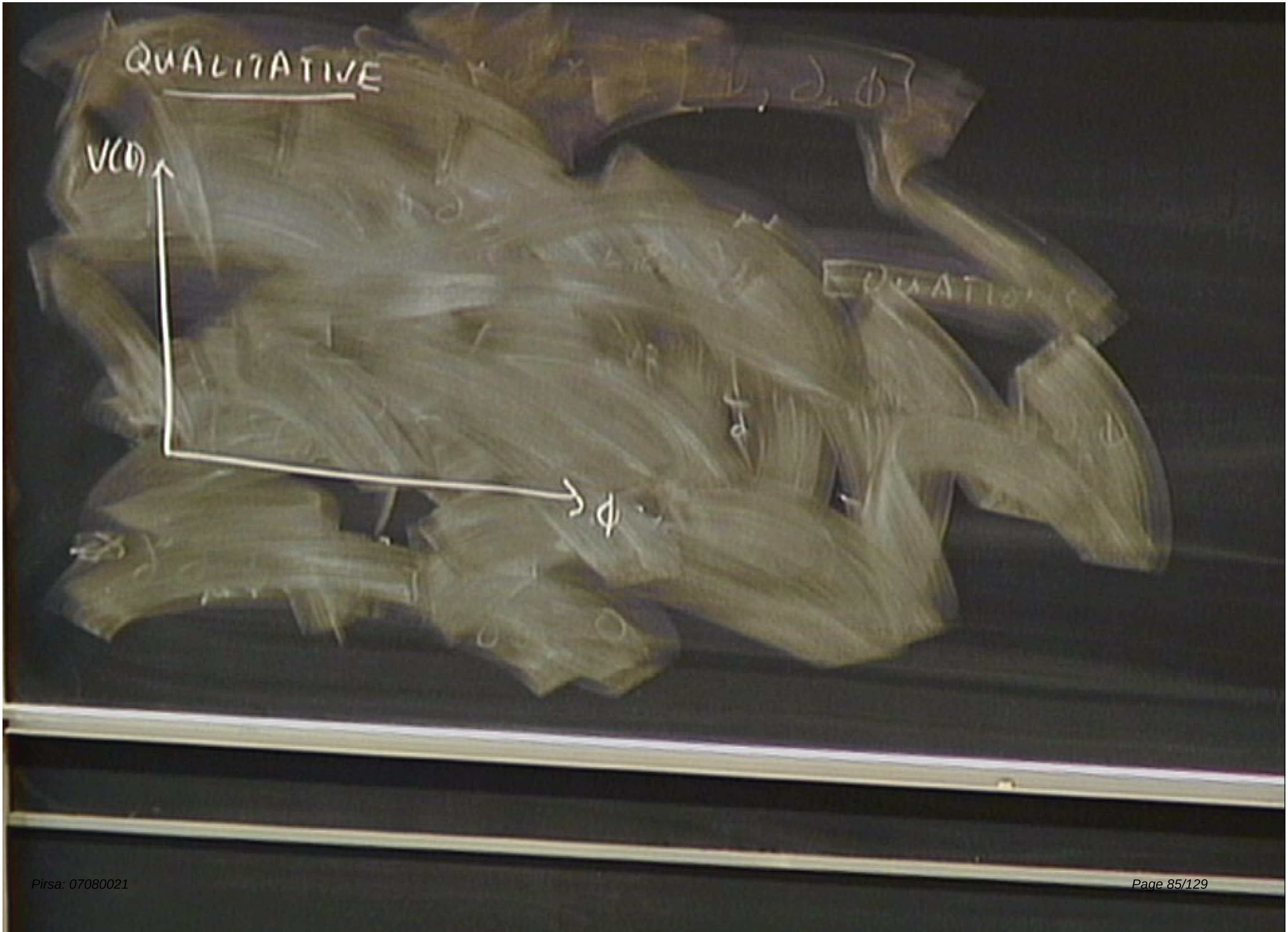
(1) $$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3m_{Pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

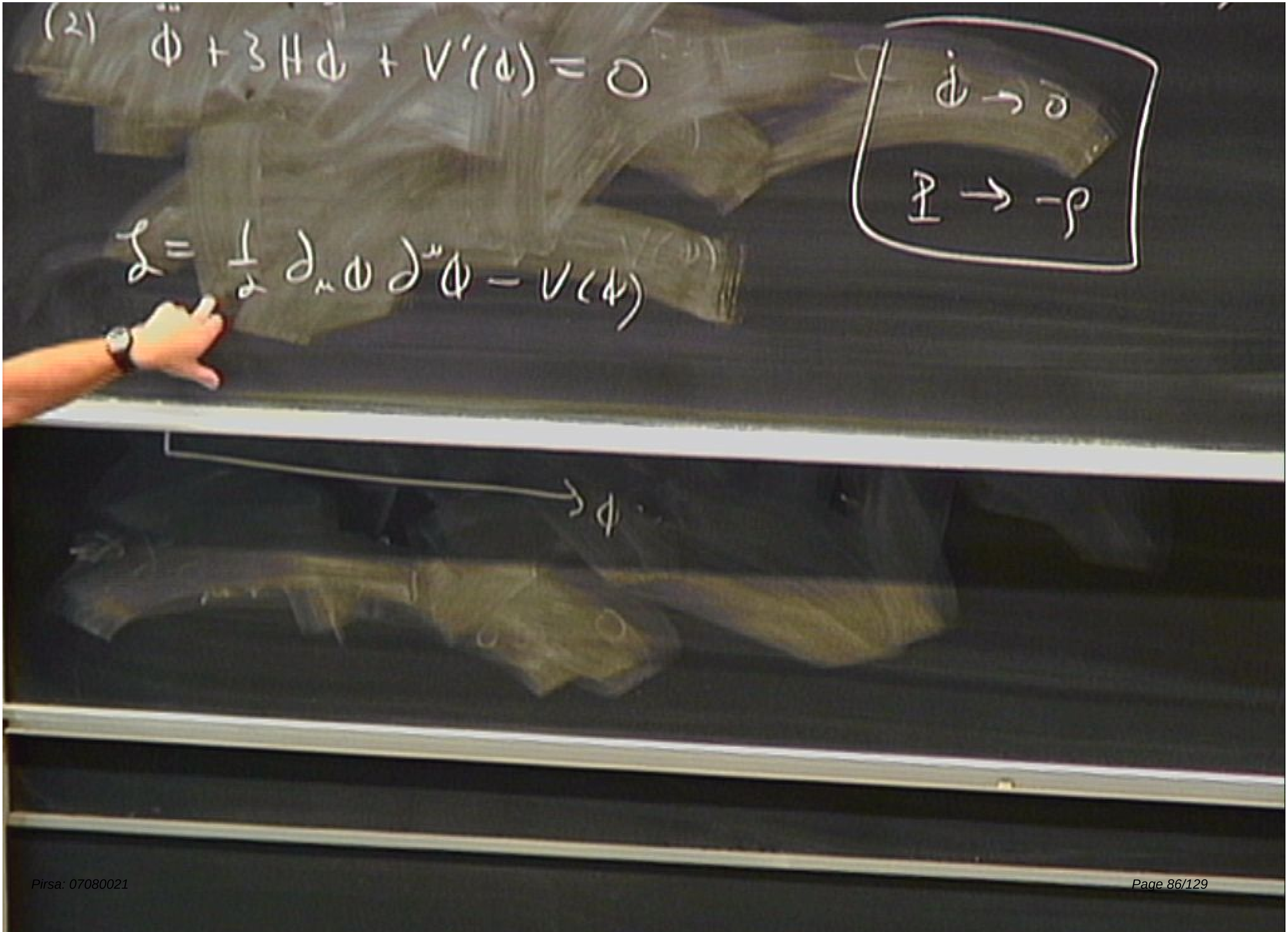
$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

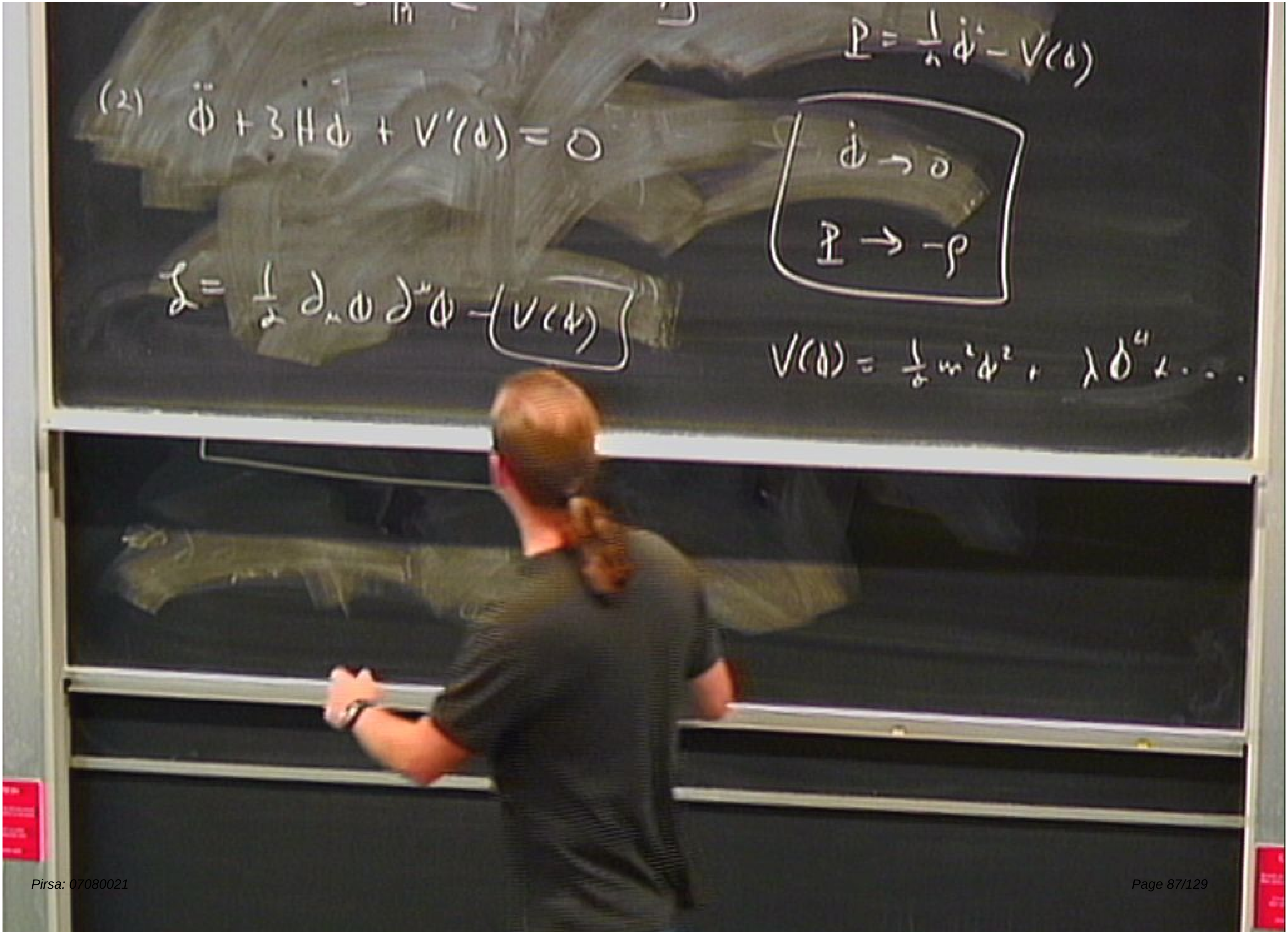
$$P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

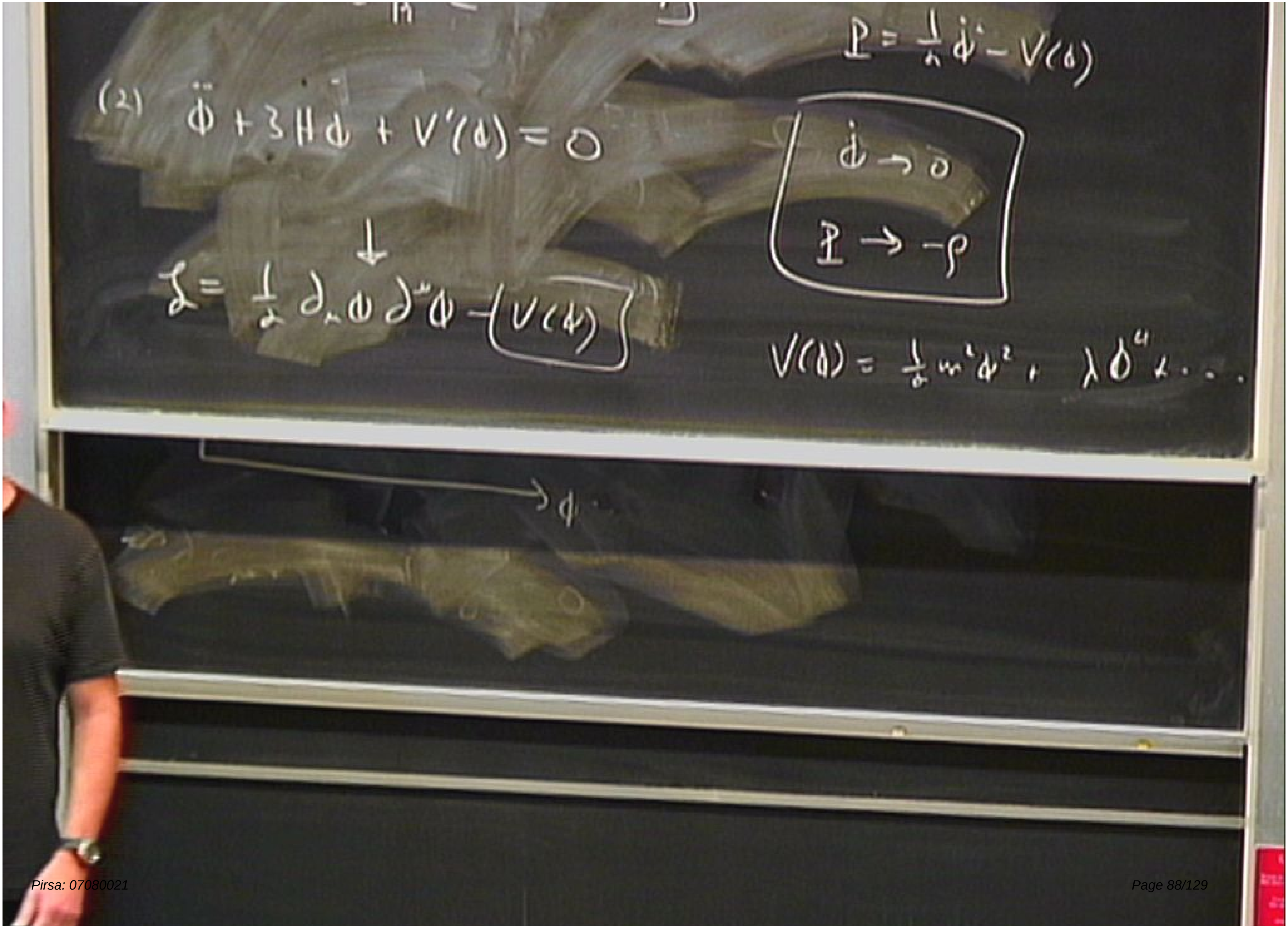
(2) $$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$\boxed{\begin{array}{c}\dot{\phi} \to 0 \\ P \to -\rho\end{array}}$$


QUALITATIVE
V(ϕ)
ϕ


(2) $\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0$
$\dot{\phi} \to \partial$
$P \to -\rho$
$\mathcal{L} = \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - V(\Phi)$
ϕ


$P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$
(2) $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$
$\dot{\phi} \to 0$
$P \to -\rho$
$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)$
$V(\phi) = \frac{1}{2}m^2\phi^2 + \lambda\phi^4 + \dots$


$P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$
(2) $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$
$\dot{\phi} \to 0$
$P \to -\rho$
$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \boxed{V(\phi)}$
$V(\phi) = \frac{1}{2}m^2\phi^2 + \lambda\phi^4 + \dots$
$\to \phi$

$$P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

(2) $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

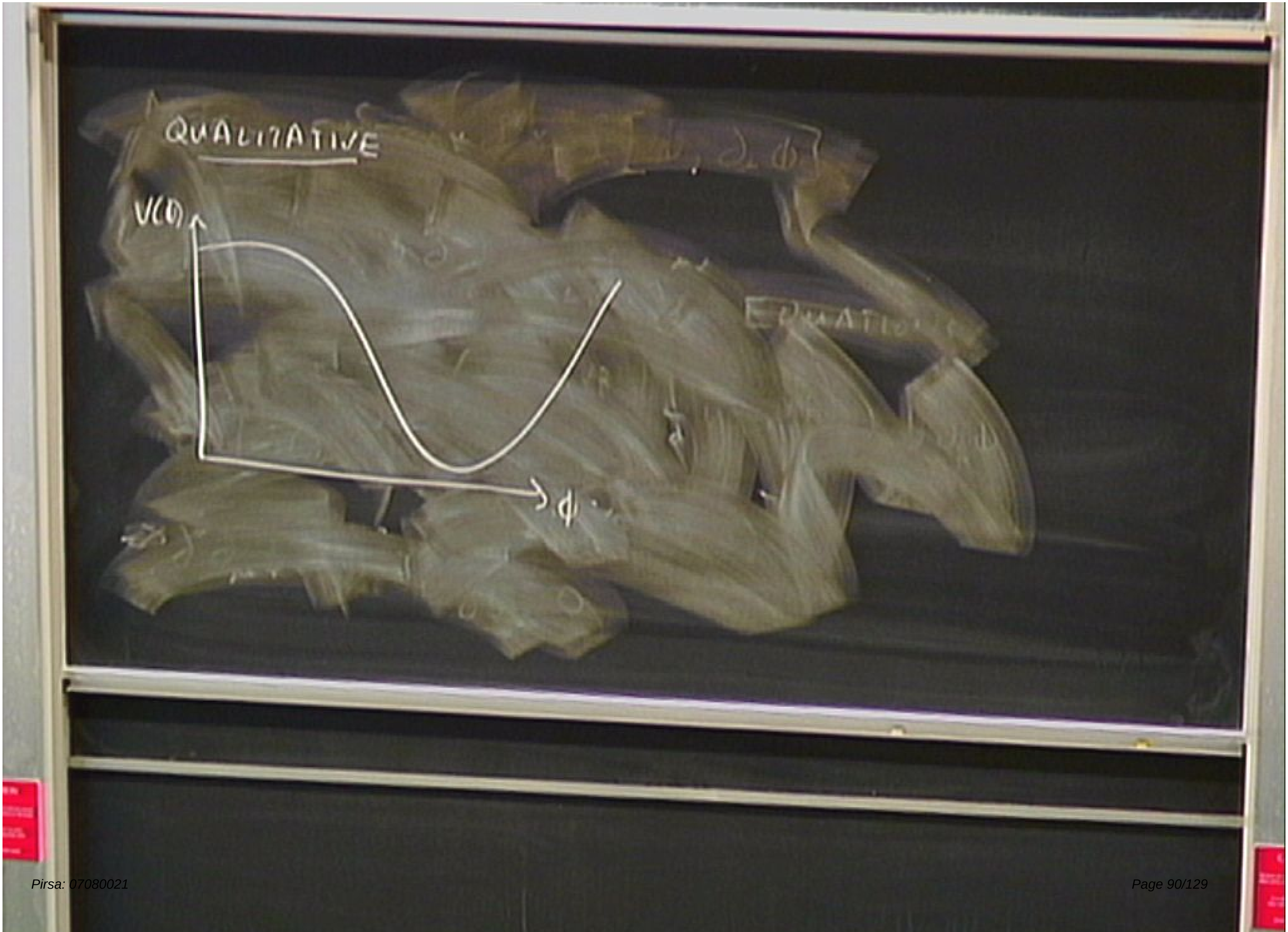
$$\boxed{\begin{array}{c} \dot{\phi} \to 0 \\ P \to -\rho \end{array}}$$

$f(\phi,\dot{\phi}) \downarrow$ DBI

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \boxed{V(\phi)}$$

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \lambda\phi^4 + \ldots$$

$\to \phi \ldots$


QUALITATIVE
V(ϕ)
ϕ

(1) $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3m_{Pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$

$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$

$P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

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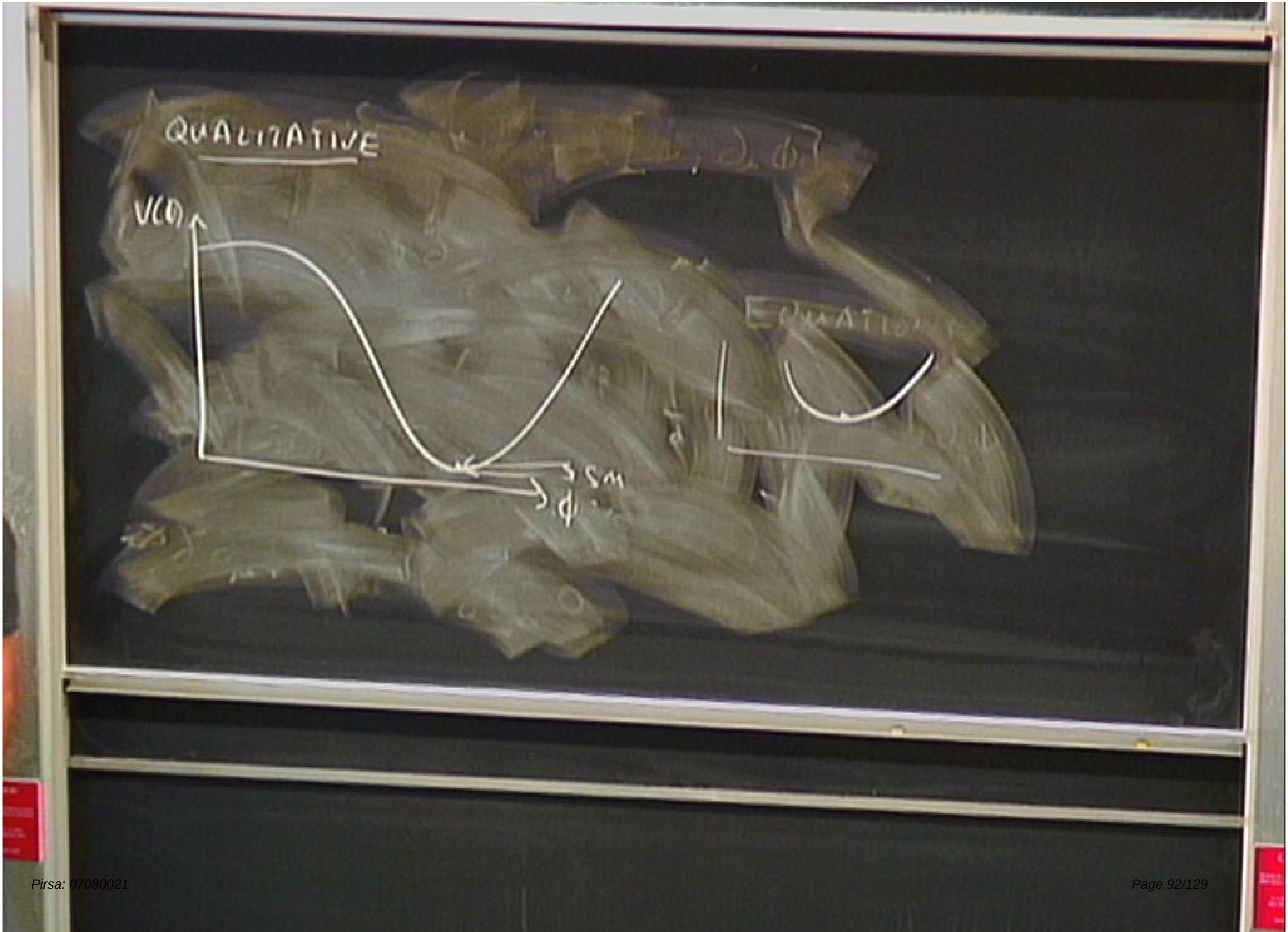
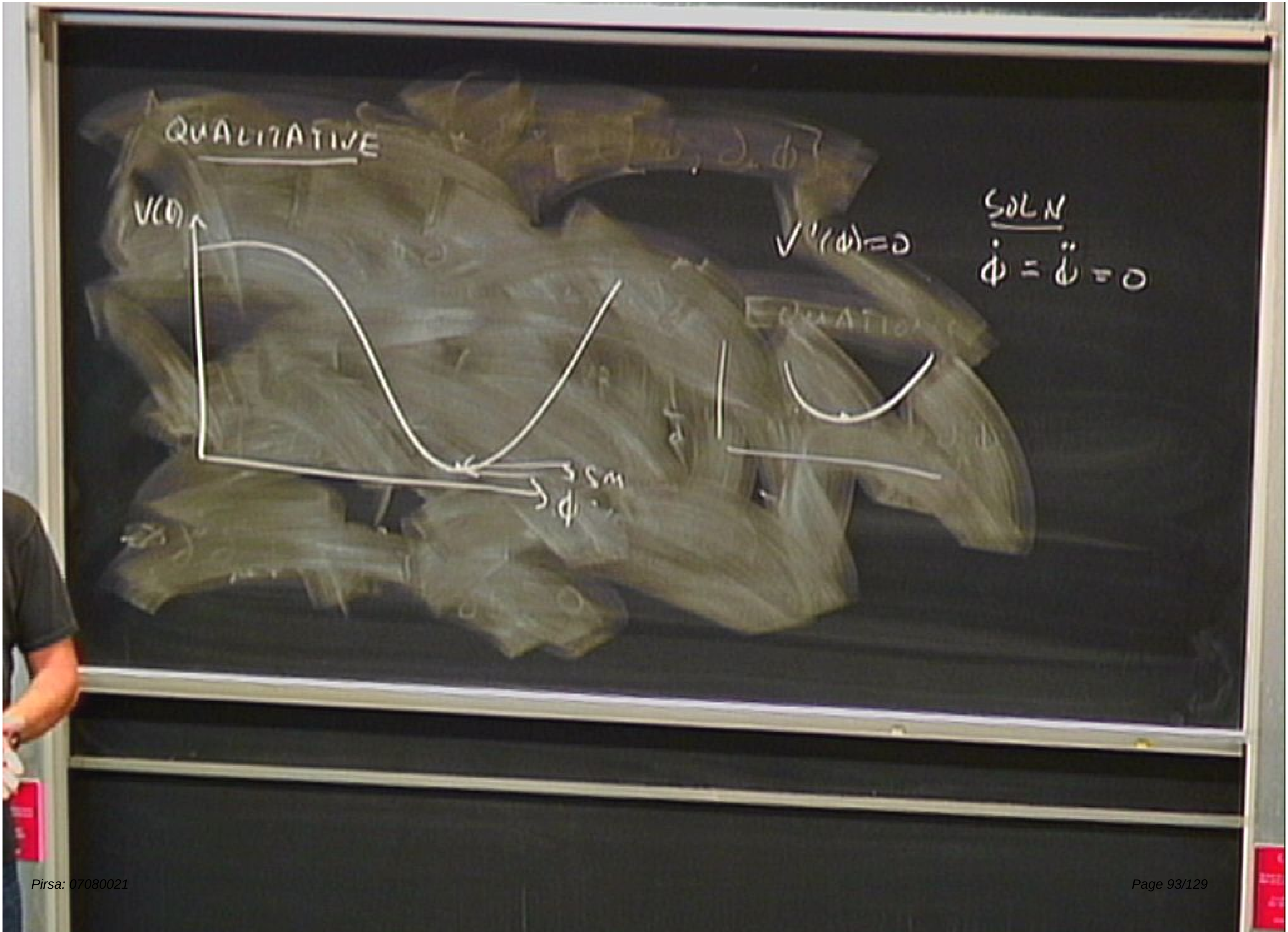
$$\boxed{\begin{array}{c} \dot{\phi} \to 0 \\ P \to -\rho \end{array}}$$

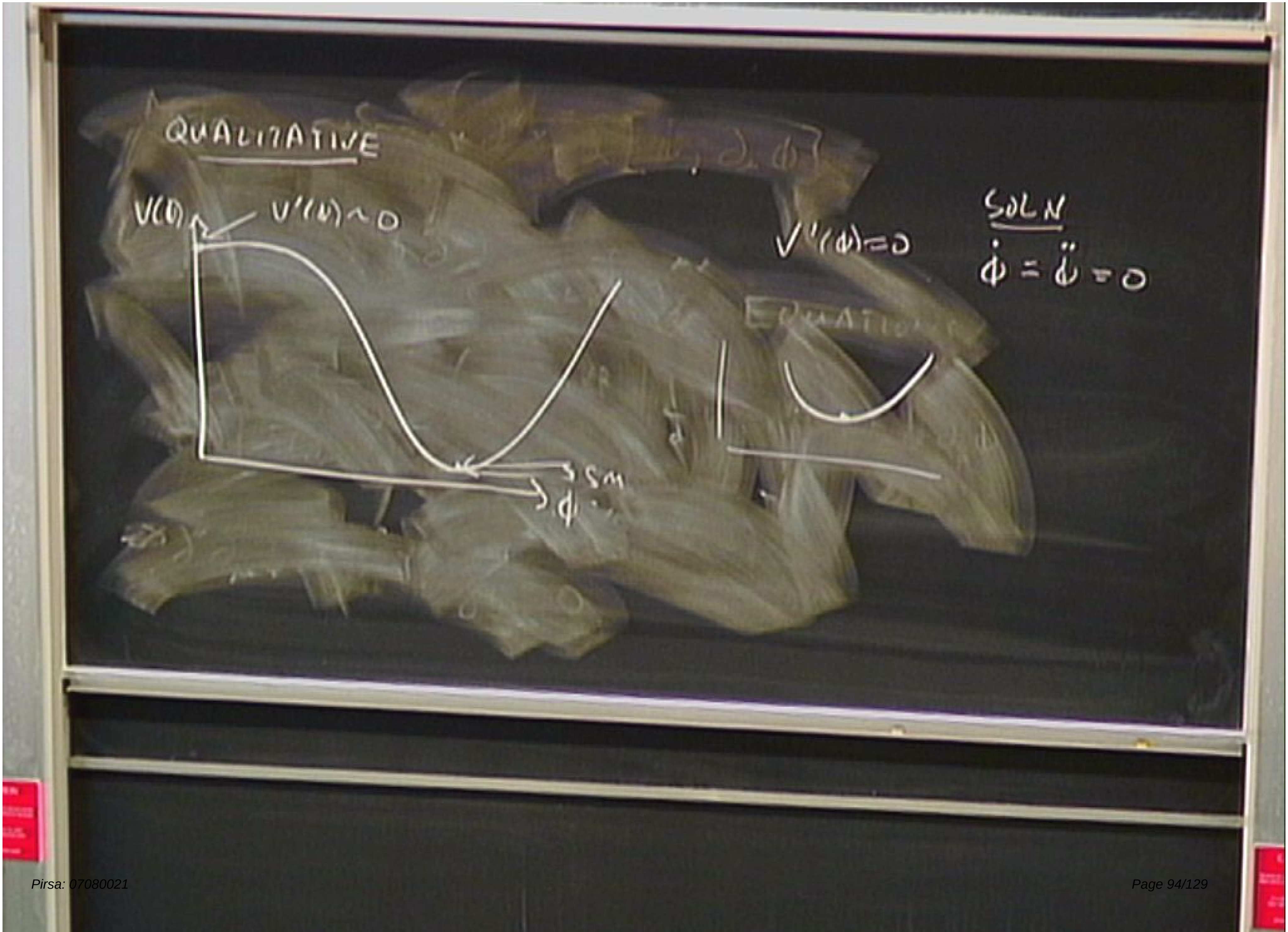
$f(\phi, \dot{\phi}) \downarrow$ DBI

$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \, \partial^\mu \phi - \boxed{V(\phi)}$

$V(\phi) = \frac{1}{2}m^2\phi^2 + \lambda\phi^4 + \ldots$


QUALITATIVE
V(ϕ)
ϕ SM
ϕ


QUALITATIVE
V(φ)
SM
φ
V'(φ)=0
SOLN
φ̇ = φ̈ = 0


QUALITATIVE
V(φ)
V'(φ)~0
SM
φ
V'(φ)=0
SOLN
φ̇ = φ̈ = 0

SLOW ROLL APPROXIMATION ($V'(\phi)$ small)

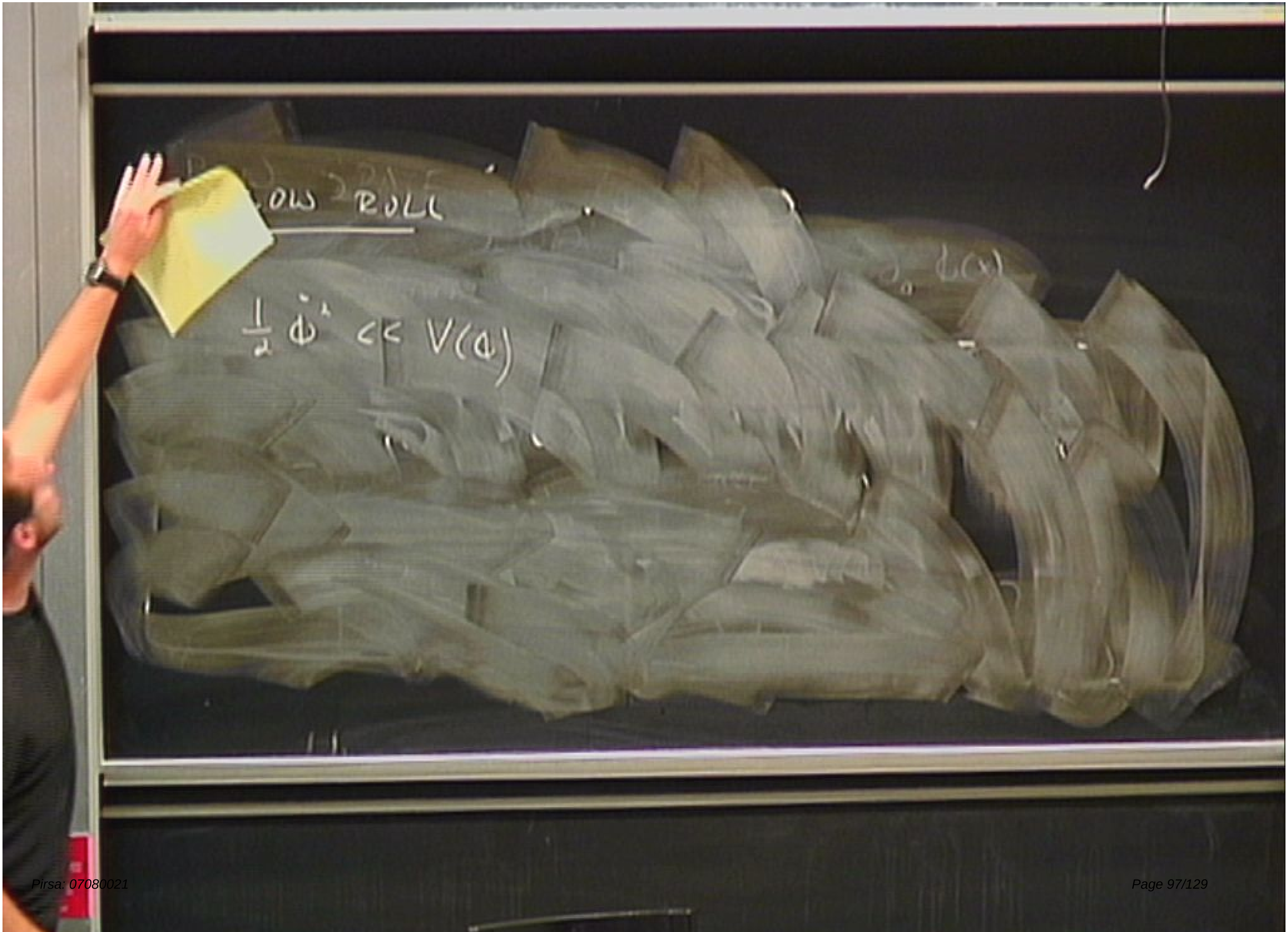
$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3 m_{Pl}^2}\left[\cancel{\tfrac{1}{2}\dot\phi^2}^{\,\to 0} + V(\phi)\right]$$

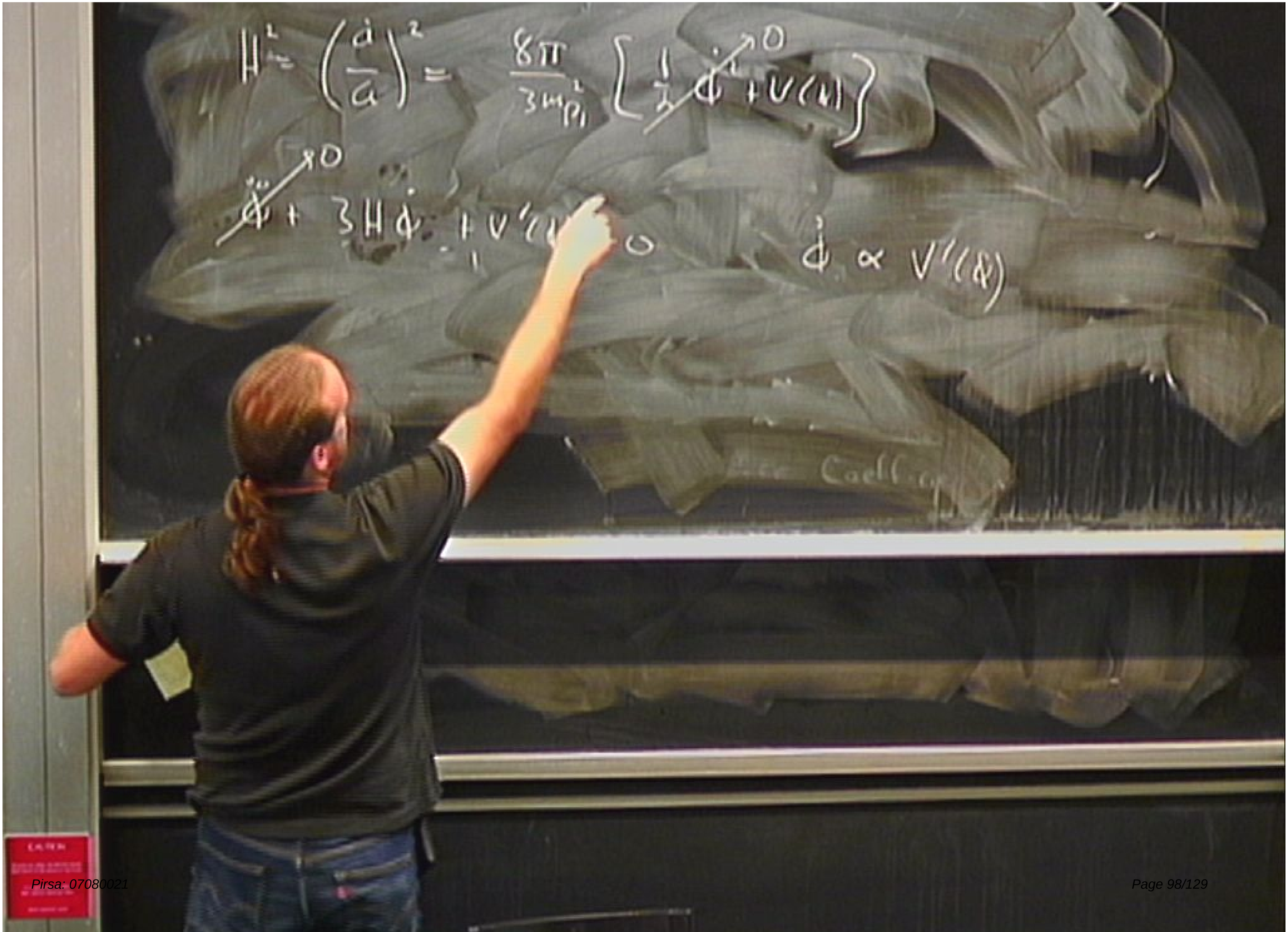
$$\cancel{\ddot\phi}^{\,\to 0} + 3H\dot\phi + V'(\phi) = 0$$

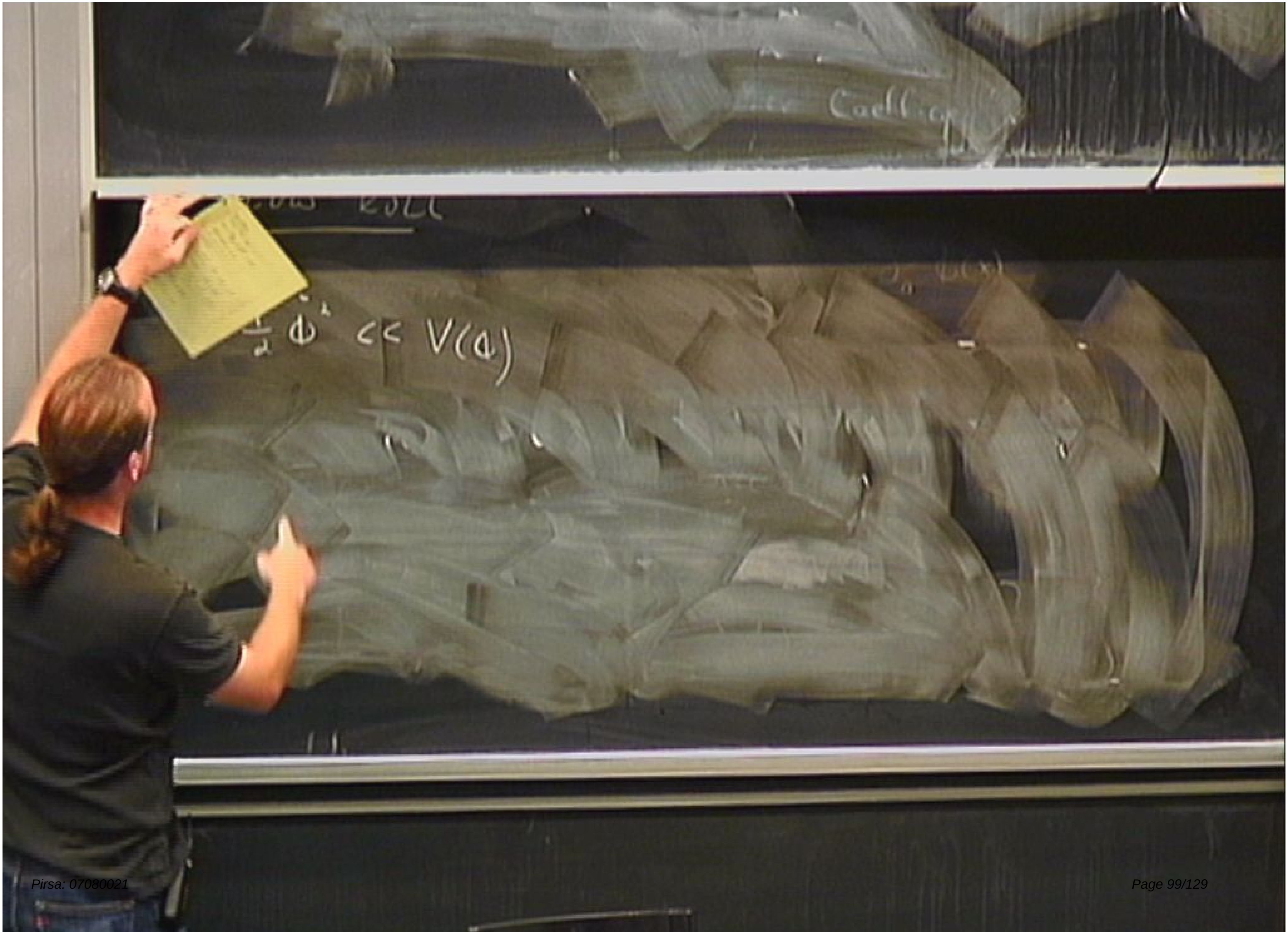
SLOW ROLL APPROXIMATION ($V'(\phi)$ small)

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3m_{Pl}^2}\left[\cancelto{0}{\frac{1}{2}\dot\phi^2} + V(\phi)\right]$$

$$\cancelto{0}{\ddot\phi} + 3H\dot\phi + V'(\phi) = 0 \qquad \dot\phi \propto V'(\phi)$$


SLOW ROLL
$\frac{1}{2}\dot{\phi}^2 << V(\phi)$
$\phi(x)$


$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3m_{Pl}^2}\left[\cancel{\tfrac{1}{2}\dot\phi^2} + V(\phi)\right]$
$\cancel{\ddot\phi} + 3H\dot\phi + V'(\phi) = 0$
$\dot\phi \propto V'(\phi)$



<< V(Φ)

SLOW ROLL

$$\frac{1}{2}\dot{\phi}^2 << V(\phi) \qquad \ddot{\phi} << 3H\dot{\phi}$$

CAN SHOW

$$\left|\frac{m_{Pl}\, V'(\phi)}{V(\phi)}\right| << \sqrt{48\pi} \qquad \left|\frac{m_{Pl}^2\, V''(\phi)}{V(\phi)}\right| << 24\pi$$

$$H^2 = \frac{8\pi}{3 m_{Pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$H^2 = \frac{8\pi}{3m_{Pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$\phi(t)$ MONOTONIC

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$H^2 = \frac{8\pi}{3m_{Pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$\phi(t)$ MONOTONIC

$t \rightarrow \phi$

$$H^2 = \frac{8\pi}{3 m_{Pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Ex Show that

$\phi(t)$ MONOTONIC

$t \rightarrow \phi$

$H(t) \rightarrow H(\phi)$

$$H^2 = \frac{8\pi}{3 m_{Pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$\phi(t)$ MONOTONIC

$t \rightarrow \phi$

$H(t) \rightarrow H(\phi)$

Ex Show that

$$H'(\phi) = -\frac{4\pi}{m_{Pl}^2}\dot{\phi}$$

$$H^2(\phi)\left[1 - \frac{1}{3}\epsilon(\phi)\right] = \frac{8\pi}{3 m_{Pl}^2} V(\phi) \qquad \epsilon \equiv \frac{m_{Pl}^2}{4\pi}\left(\frac{H'(\phi)}{H(\phi)}\right)^2$$

$$H^2 = \frac{8\pi}{3m_{Pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$\phi(t)$ MONOTONIC

$t \to \phi$

$H(t) \to H(\phi)$

$$\frac{d}{dt} = \frac{1}{\phi}\frac{d}{d\phi}$$

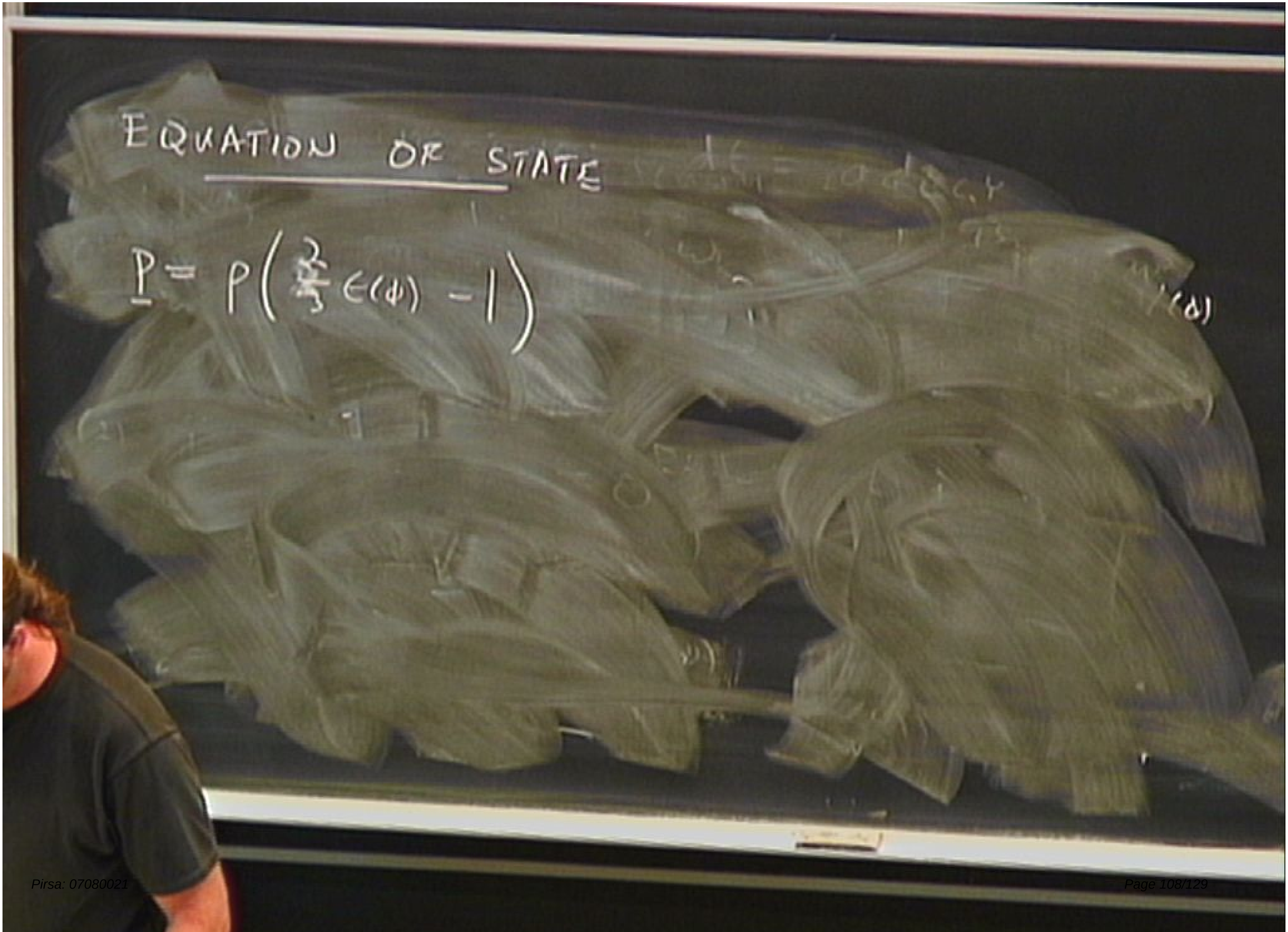
Ex Show that

$$H'(\phi) = -\frac{4\pi}{m_{Pl}^2}\dot{\phi}$$

$$H^2(\phi)\left[1 - \frac{1}{3}\epsilon(\phi)\right] = \frac{8\pi}{3m_{Pl}^2}V(\phi)$$

$$\epsilon = \frac{m_{Pl}^2}{4\pi}\left(\frac{H'(\phi)}{H(\phi)}\right)^2$$

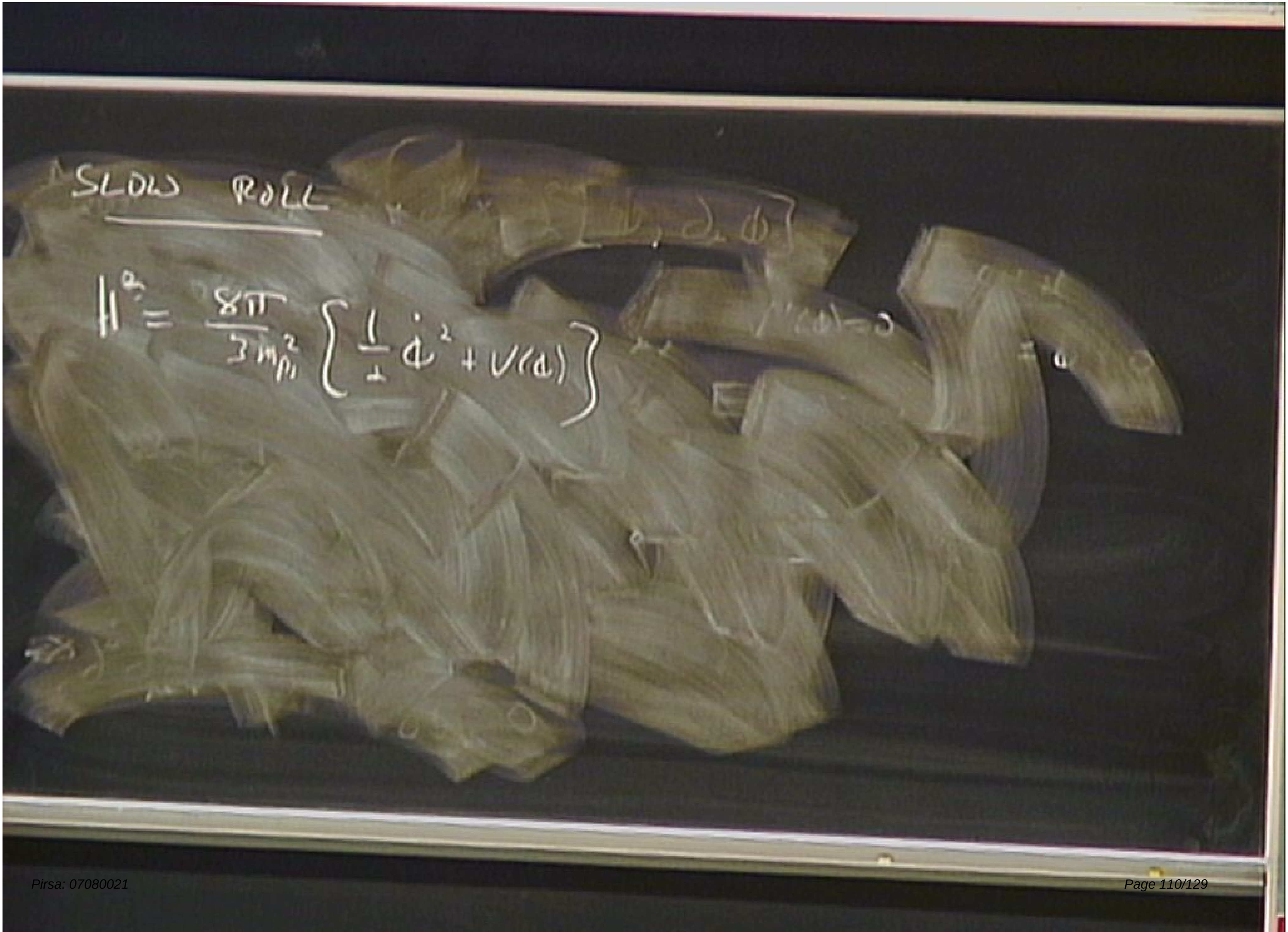
EQUATION OF STATE


EQUATION OF STATE
$\underline{P} = \rho\left(\frac{2}{3}\epsilon(\phi) - 1\right)$

EQUATION OF STATE

$$P = \rho\left(\frac{2}{3}\epsilon(\phi) - 1\right) \Rightarrow \left(\frac{\ddot{a}}{a}\right) = H^2(\phi)\left[1 - \epsilon(\phi)\right]$$

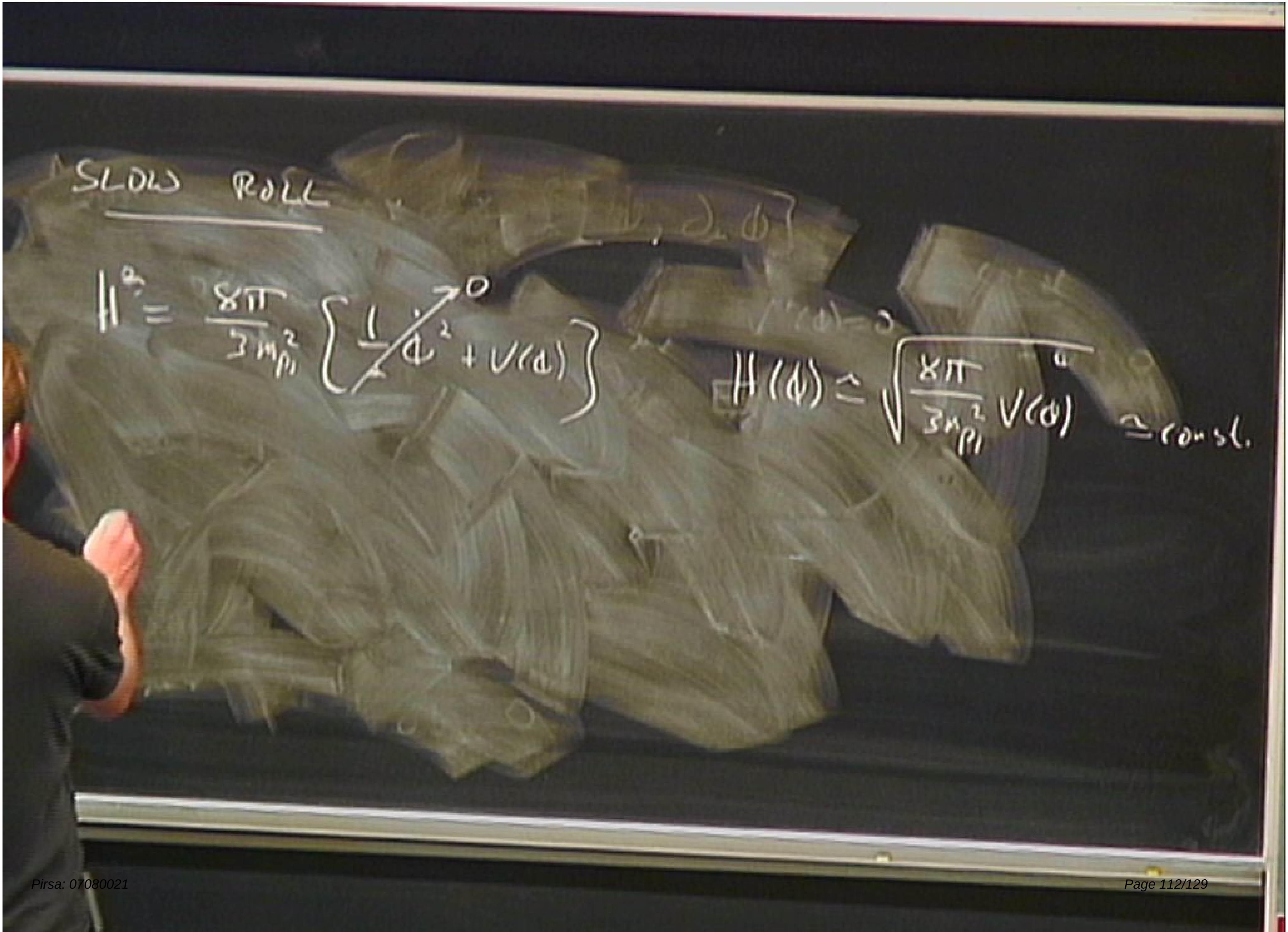
$$\left(\frac{\ddot{a}}{a}\right) > 0 \qquad P < -\frac{1}{3}\rho \Longleftrightarrow \boxed{\epsilon(\phi) < 1}$$


SLOW ROLL
$H^2 = \frac{8\pi}{3m_{Pl}^2}\left\{\frac{1}{2}\dot{\phi}^2 + V(\phi)\right\}$

SLOW ROLL

$$H^2 = \frac{8\pi}{3 m_{Pl}^2}\left\{\cancelto{0}{\frac{1}{2}\dot{\phi}^2} + V(\phi)\right\}$$

$$H(\phi) \simeq \sqrt{\frac{8\pi}{3 m_{Pl}^2} V(\phi)}$$


SLOW ROLL
$H^2 = \frac{8\pi}{3 m_{pl}^2} \left\{ \cancel{\frac{1}{2}\dot{\phi}^2} + V(\phi) \right\}$
$H(\phi) \simeq \sqrt{\frac{8\pi}{3 m_{pl}^2} V(\phi)} \simeq \text{const.}$

SLOW ROLL

$$\frac{1}{2}\dot{\phi}^2 \ll V(\phi) \qquad \ddot{\phi} \ll 3H\dot{\phi}$$

CAN SHOW

$$\left|\frac{m_{Pl}\, V'(\phi)}{V(\phi)}\right| \ll \sqrt{48\pi} \qquad \left|\frac{m_{Pl}^2\, V''(\phi)}{V(\phi)}\right| \ll 24\pi$$

SLOW ROLL

$$H^2 = \frac{8\pi}{3 m_{Pl}^2}\left\{ \cancelto{0}{\frac{1}{2}\dot{\phi}^2} + V(\phi) \right\}$$

$$H(\phi) \simeq \sqrt{\frac{8\pi}{3 m_{Pl}^2} V(\phi)} \simeq \text{const.}$$

$$\epsilon \simeq \frac{m_{Pl}^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1$$

← CONDITION FOR SLOW ROLL INFLATION

SLOW ROLL

$$H^2 = \frac{8\pi}{3m_{Pl}^2}\left\{\cancel{\frac{1}{2}\dot{\phi}^2}^{\,0} + V(\phi)\right\}$$

$$H(\phi) \simeq \sqrt{\frac{8\pi}{3m_{Pl}^2}V(\phi)} \simeq \text{const.}$$

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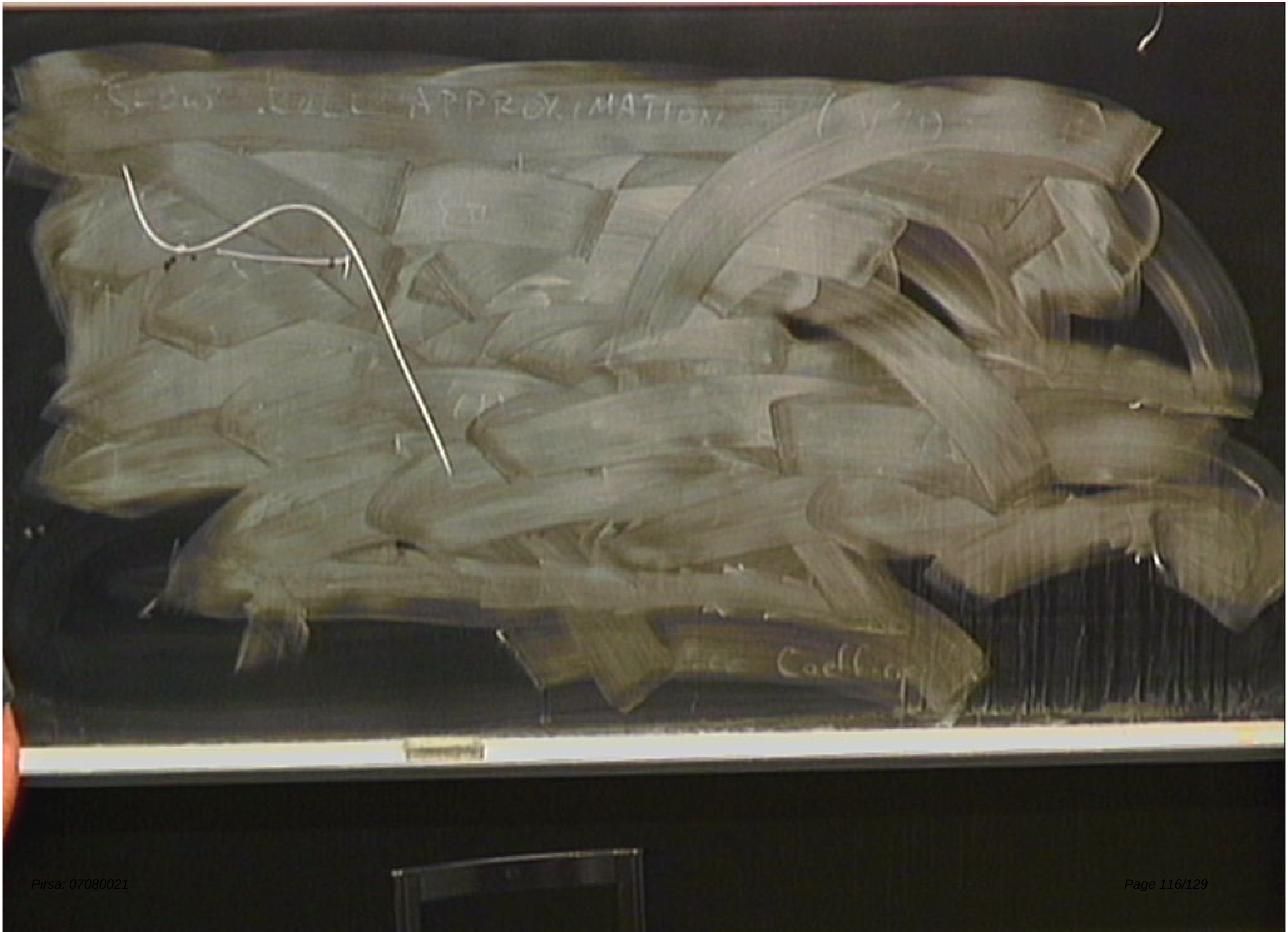
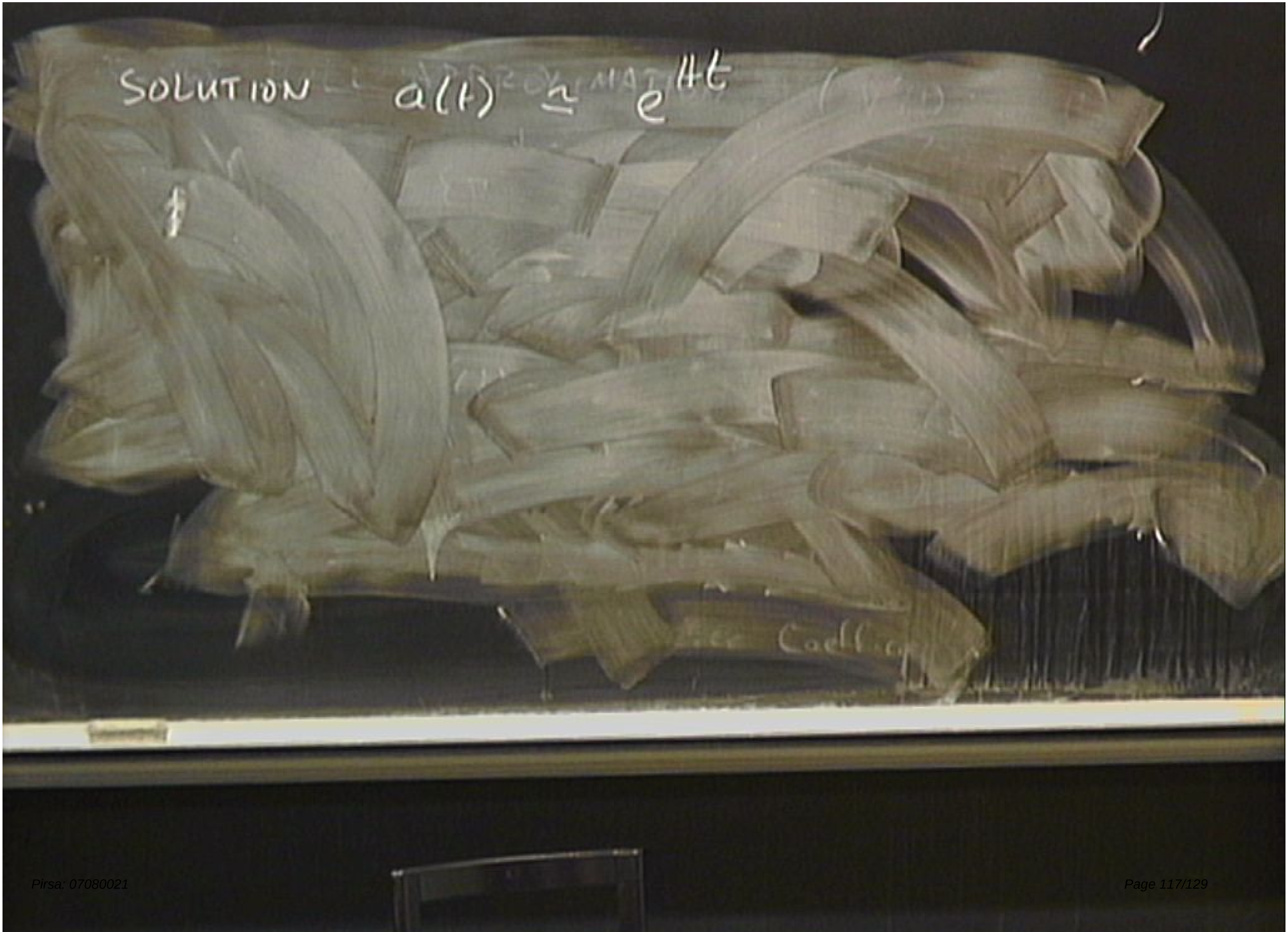
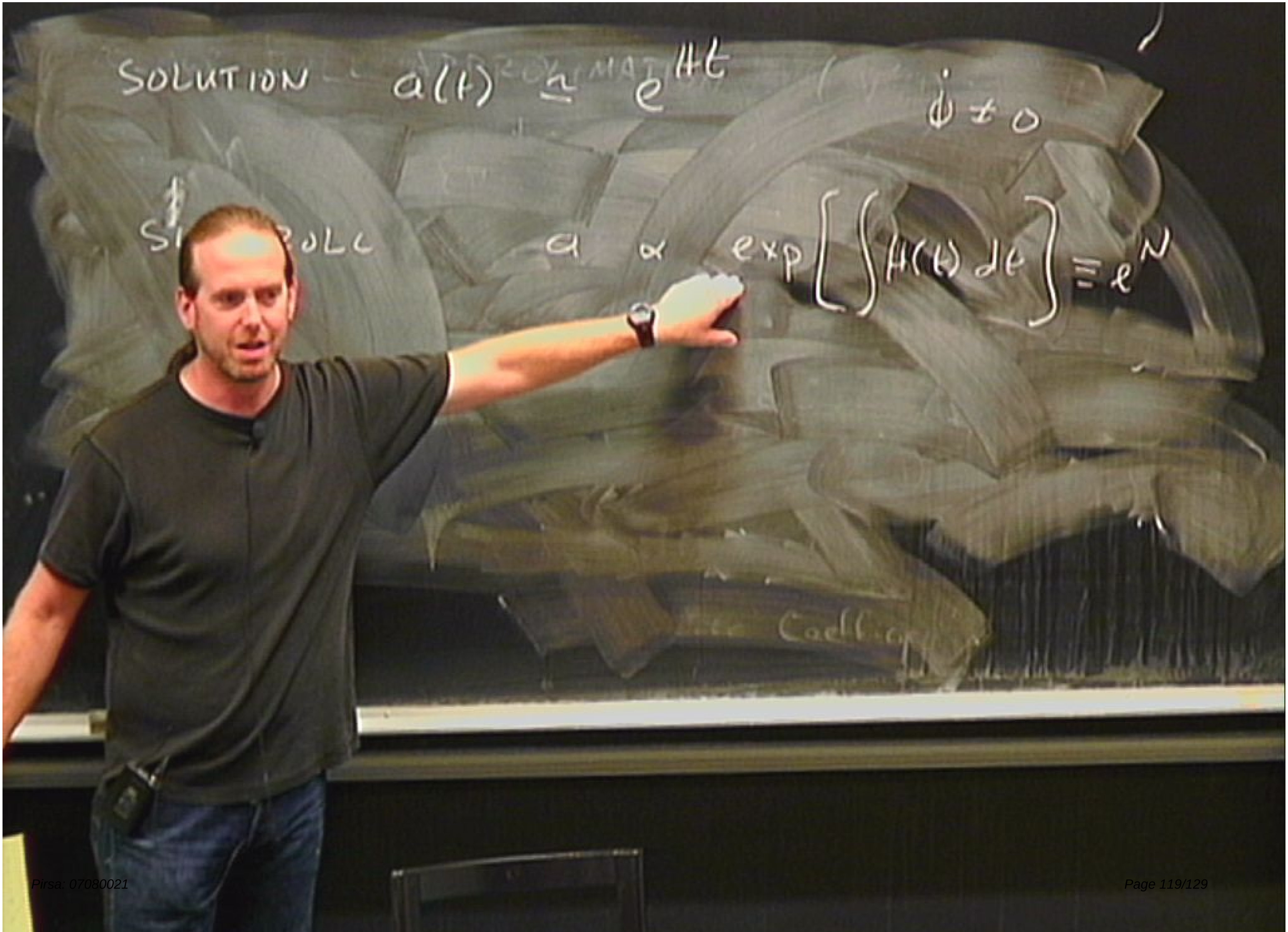
CONDITION FOR SLOW ROLL INFLATION


APPROXIMATION


SOLUTION $a(t) \simeq e^{Ht}$

SOLUTION $a(t) \simeq e^{Ht}$ $\dot{\phi} \neq 0$


SOLUTION a(t) ≃ e^Ht
φ̇ ≠ 0
a ∝ exp[∫H(t)dt] ≡ e^N

SOLUTION $a(t) \simeq e^{Ht}$

$\dot{\phi} \neq 0$

SLOW ROLL $a \propto \exp\left[\int H(t)\,dt\right] \equiv e^{N}$

$H = \frac{\dot{a}}{a}$

$$N = \int H\,dt - \int \frac{H(\phi)}{\dot{\phi}}\,d\phi$$

$$N = \int H\,dt = \int \frac{H(\phi)}{\dot{\phi}}\,d\phi$$

$$= \frac{2\pi}{m_{pl}} \int \frac{d\phi}{\sqrt{\epsilon(\phi)}}$$

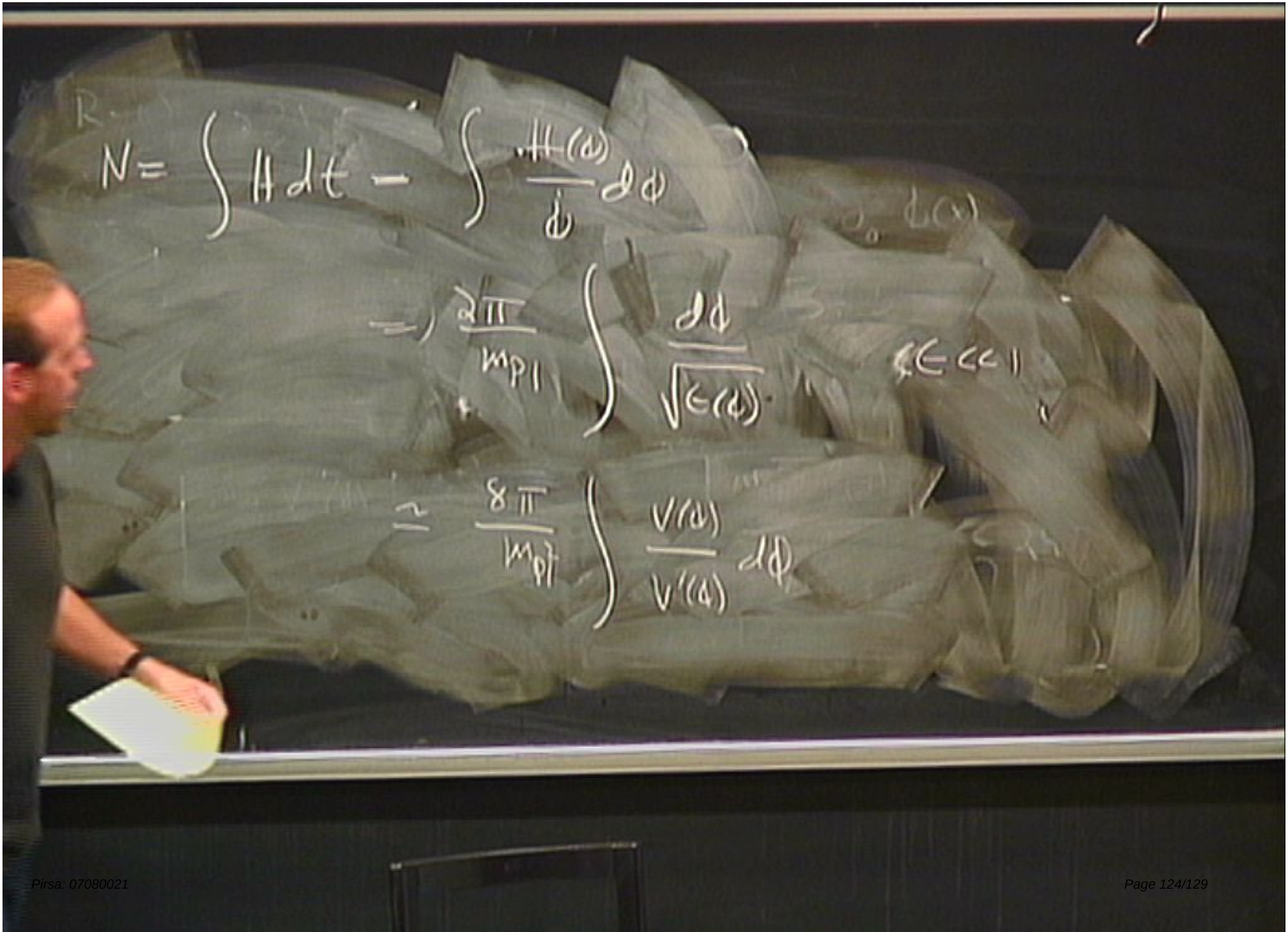
SLOW ROLL

$$H^2 = \frac{8\pi}{3 m_{Pl}^2}\left\{\cancel{\frac{1}{2}\dot{\phi}^2}^{\,0} + V(\phi)\right\}$$

$$H(\phi) \simeq \sqrt{\frac{8\pi}{3 m_{Pl}^2} V(\phi)} \simeq \text{cons}$$

$$\epsilon \simeq \frac{m_{Pl}^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1$$

← CONDITION FOR SLOW ROLL INFLATION


$N = \int H\,dt = \int \frac{H(\phi)}{\dot\phi}\,d\phi$
$= \frac{2\pi}{|m_{pl}|}\int \frac{d\phi}{\sqrt{\epsilon(\phi)}}$
$\epsilon << 1$
$\simeq \frac{8\pi}{m_{pl}^2}\int \frac{V(\phi)}{V'(\phi)}\,d\phi$

$$N = \int H\,dt = \int \frac{H(\phi)}{\dot\phi}\,d\phi$$

$$= \frac{2\pi}{m_{Pl}} \int \frac{d\phi}{\sqrt{\epsilon(\phi)}} \qquad \epsilon \ll 1 \Rightarrow N \gg 1$$

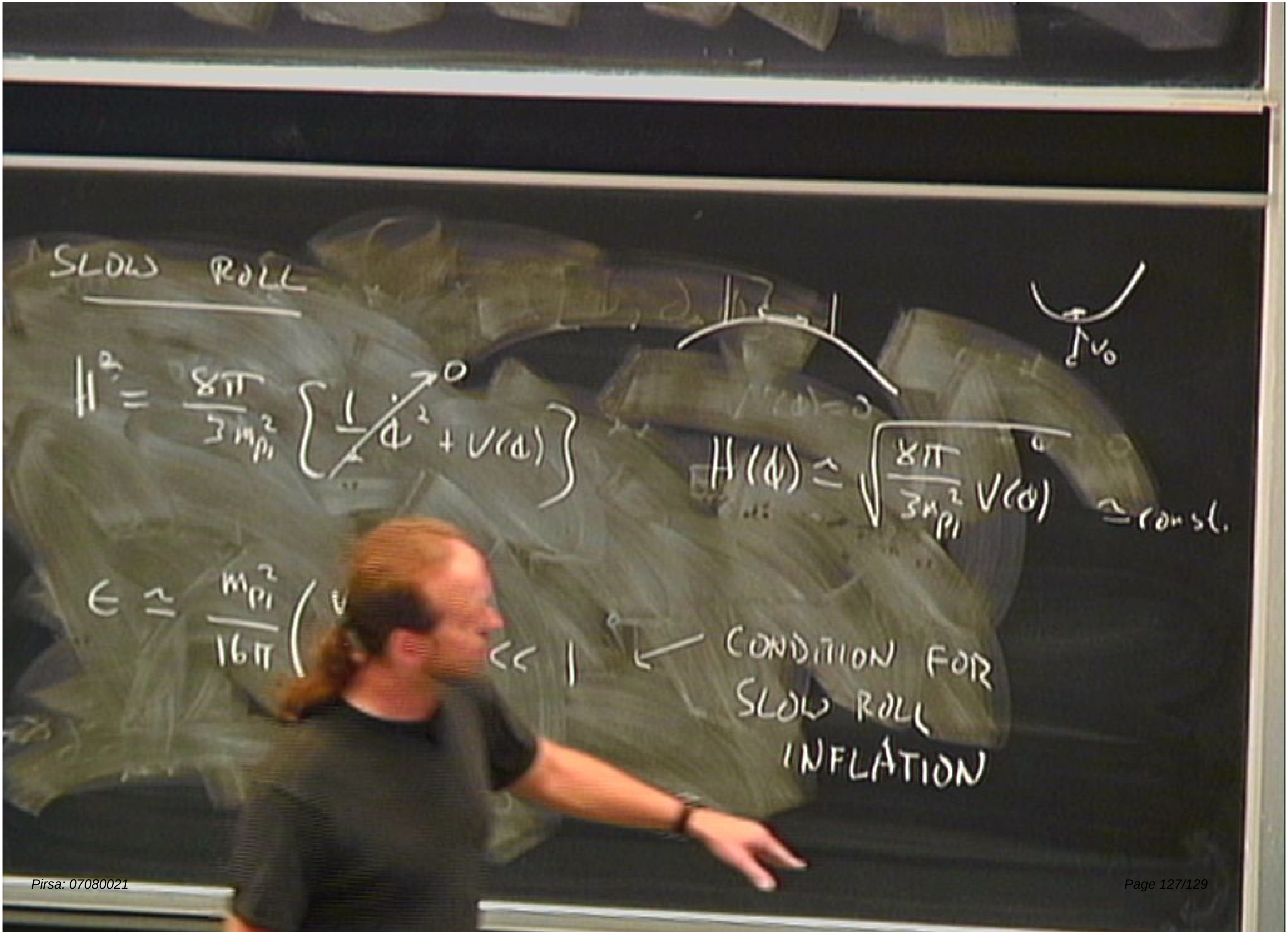
$$= \frac{8\pi}{m_{Pl}^2} \int \frac{V(\phi)}{V'(\phi)}\,d\phi$$

EQUATION OF STATE

$$P = \rho\left(\frac{2}{3}\epsilon(\phi) - 1\right) \Longrightarrow \left(\frac{\ddot{a}}{a}\right) = \mu^2(\phi)\left[1 - \epsilon(\right.$$

$$\left(\frac{\ddot{a}}{a}\right) > 0 \qquad P < -\frac{1}{3}\rho \iff \boxed{\epsilon(\phi) < 1}$$

SLOW ROLL


SLOW ROLL
$H^2 = \frac{8\pi}{3m_{pl}^2}\left\{\frac{1}{2}\dot{\phi}^2 + V(\phi)\right\}$
$H(\phi) \simeq \sqrt{\frac{8\pi}{3m_{pl}^2}V(\phi)} \simeq$ const.
V_0
$\epsilon \simeq \frac{m_{pl}^2}{16\pi}$
$\ll 1$
CONDITION FOR
SLOW ROLL
INFLATION

LOW ROLL

$$H^2 = \frac{8\pi}{3 m_{Pl}^2}\left\{ \cancel{\frac{1}{2}\dot{\phi}^2}^{\,0} + V(\phi) \right\}$$

$$H(\phi) \simeq \sqrt{\frac{8\pi}{3 m_{Pl}^2} V(\phi)} \simeq \text{const.}$$

$$\epsilon \simeq \frac{m_{Pl}^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1$$

CONDITION FOR SLOW ROLL INFLATION

$$N = \int H\,dt = \int \frac{H(\phi)}{\dot\phi}\,d\phi$$

$$\Rightarrow \frac{2\pi}{m_{pl}} \int \frac{d\phi}{\sqrt{\epsilon(\phi)}}$$

$\epsilon \ll 1 \Rightarrow N \gg 1$

$$\simeq \frac{8\pi}{m_{pl}} \int \frac{V(\phi)}{V'(\phi)}\,d\phi$$