Title: Introduction to Flux Compactifications

Date: Aug 07, 2007 11:00 AM

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Abstract:

INTRODUCTION TO FLUX COMPACTIFICATIONS

① Moduli stabilization & flux vacua : general idea & challenges

② Explicit models in IIB (F-theory)

③ Living with the landscape

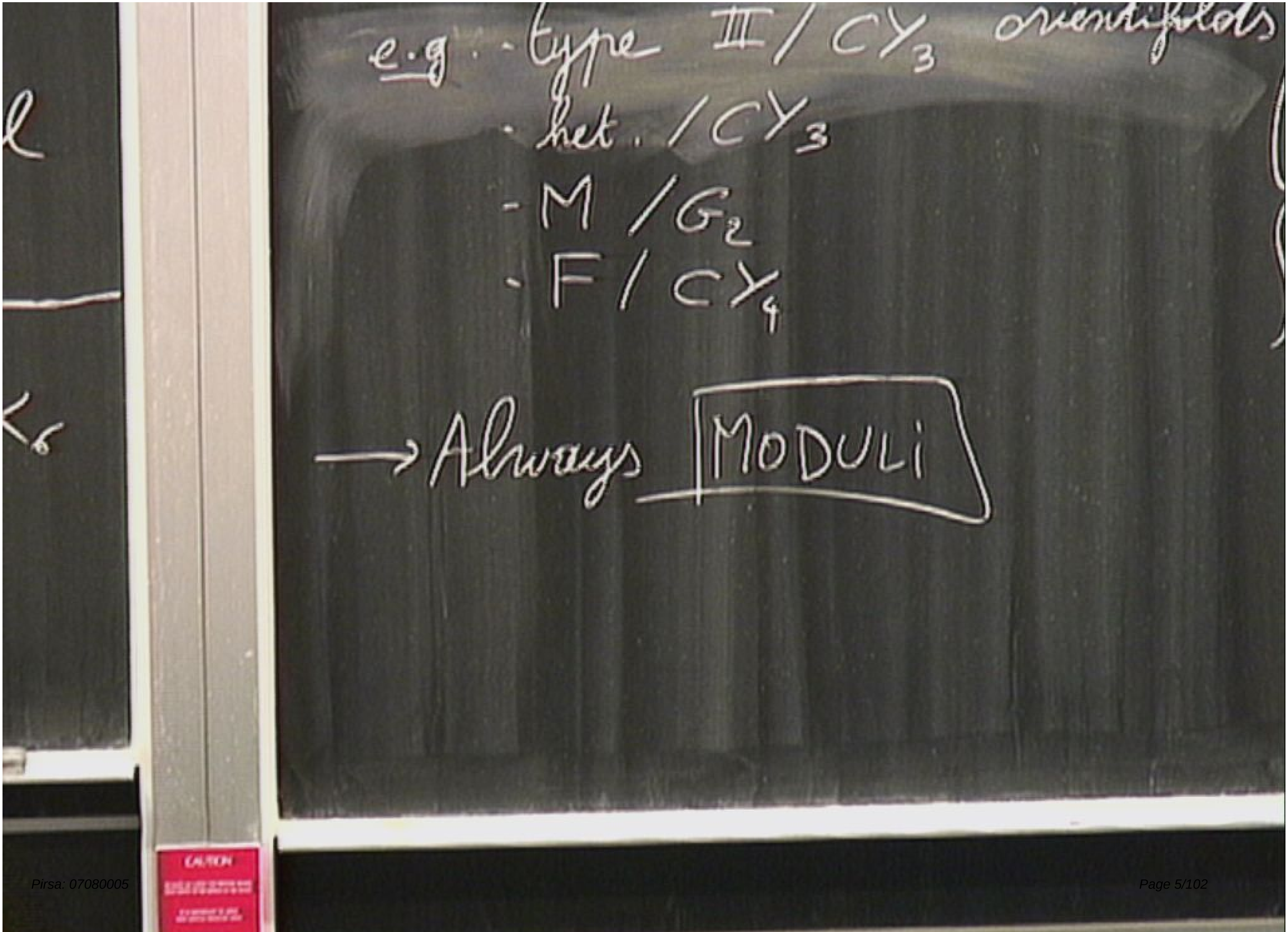
INTRODUCTION TO FLUX COMPACTIFICATIONS

① Moduli stabilization & flux vacua : general idea & challenges

Ⓐ Start with susy ($\mathcal{N}=1$) string compact. on X_6 $\times$ flat $\mathbb{R}^{1,3}$

e.g. - type II / CY_3 orientifolds
- het. / CY_3
- M / G_2
- F / CY_4

$\rightarrow \mathcal{N}=1$ in 4d


e.g. - type II / CY_3 orientifolds
- het. / CY_3
- M / G_2
- F / CY_4
→ Always MODULI

$\} \rightarrow \mathcal{N} = 1$ in 4d

G_2

CY_4

$\boxed{\text{MODULI}}$ = def. of compactification at no cost in energy

$\rightarrow$

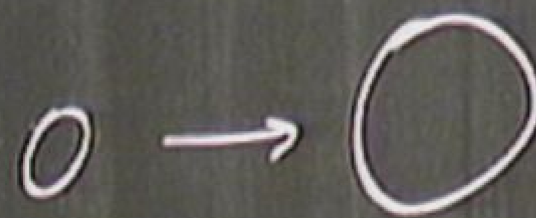
G_2
CY_4

$\boxed{\text{MODULI}}$ = def. of compactification at no cost in energy

$\longrightarrow$ massless scalars in 4d

Examples

- dilaton $e^{\phi} = g_s$
- Kähler (size) moduli



e.g. volume

$\cdot\ F/CY_4$)

→ Always $\boxed{\text{MODULI}}$ = def. of compactification at cost in energy

[CLASSICALLY] → massless scalars in 4d

Examples

- dilaton $e^{\phi} = g_s$
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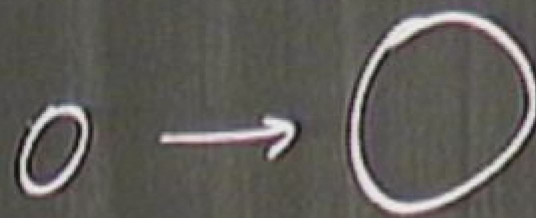
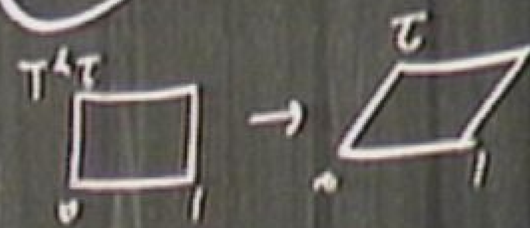


e.g. volume

- Complex structure moduli (shape)



Examples

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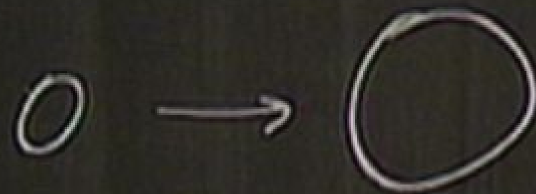
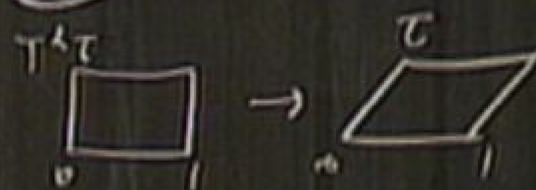
- Kähler (size) moduli  e.g. volume
- Complex structure moduli (shape) 

- D-brane / bundle moduli 

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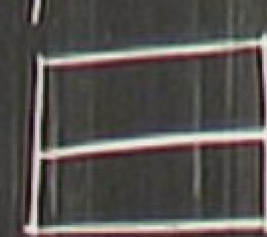
- Axions

$\oint_{\gamma} A$

- complex ...
- D-brane / bundle moduli

T^2

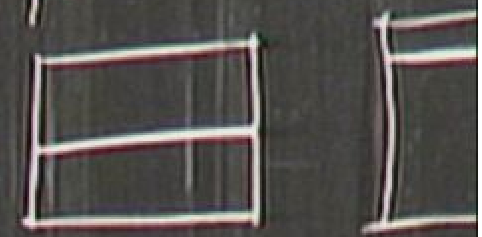


- Axions

$\oint_\gamma A \qquad \int B_2$

- complex
- D-brane / bundle moduli

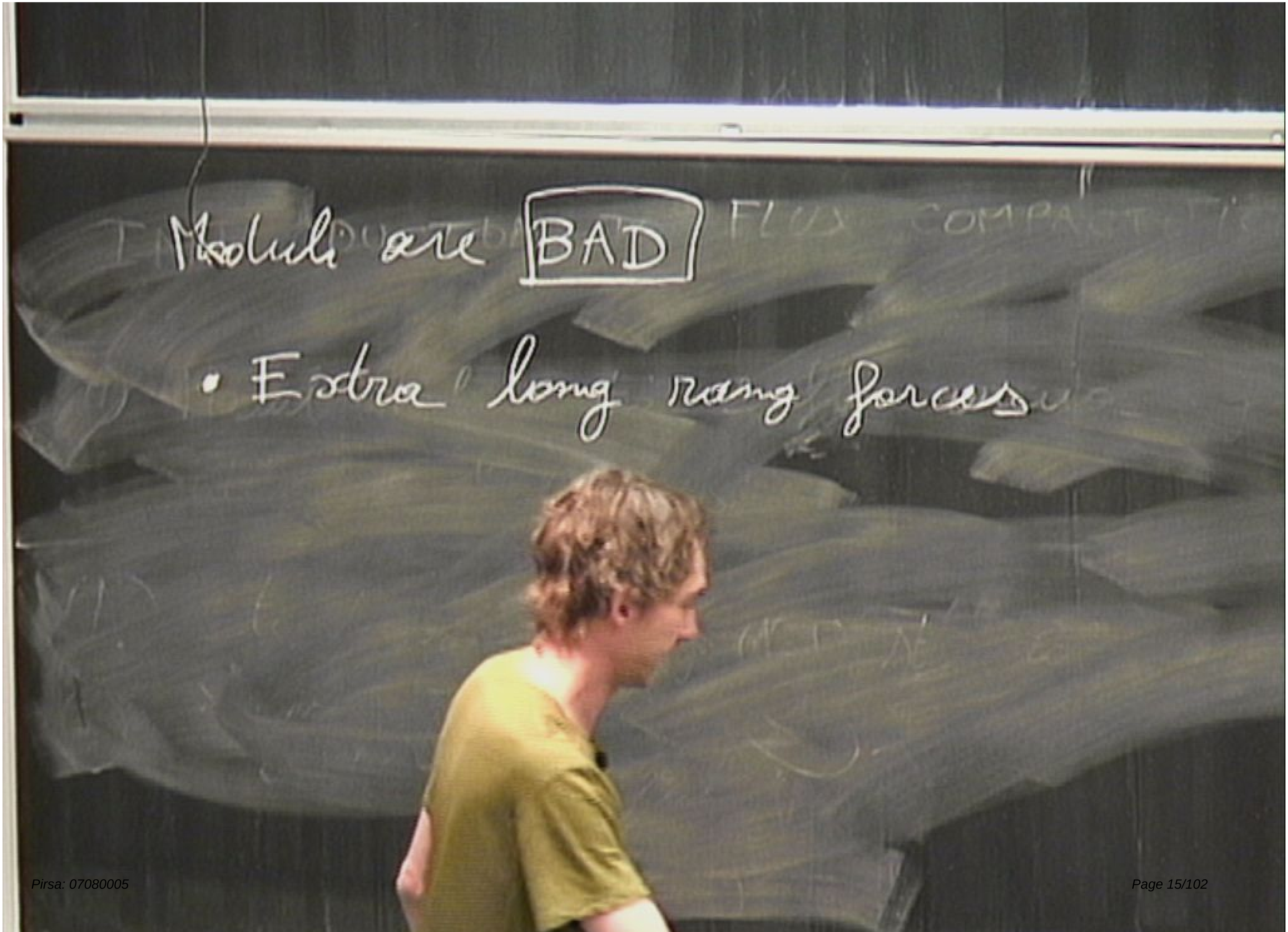
T^2

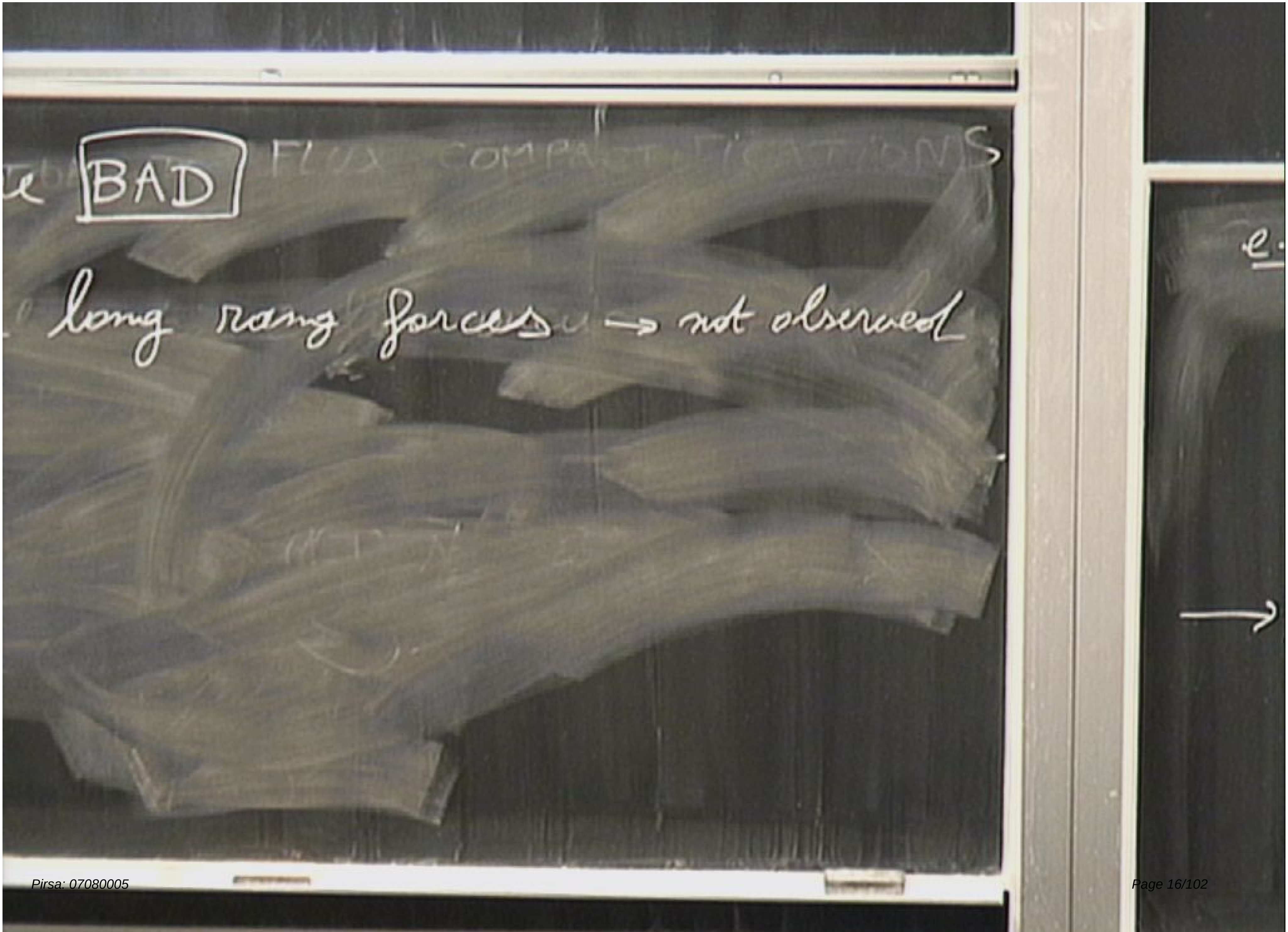


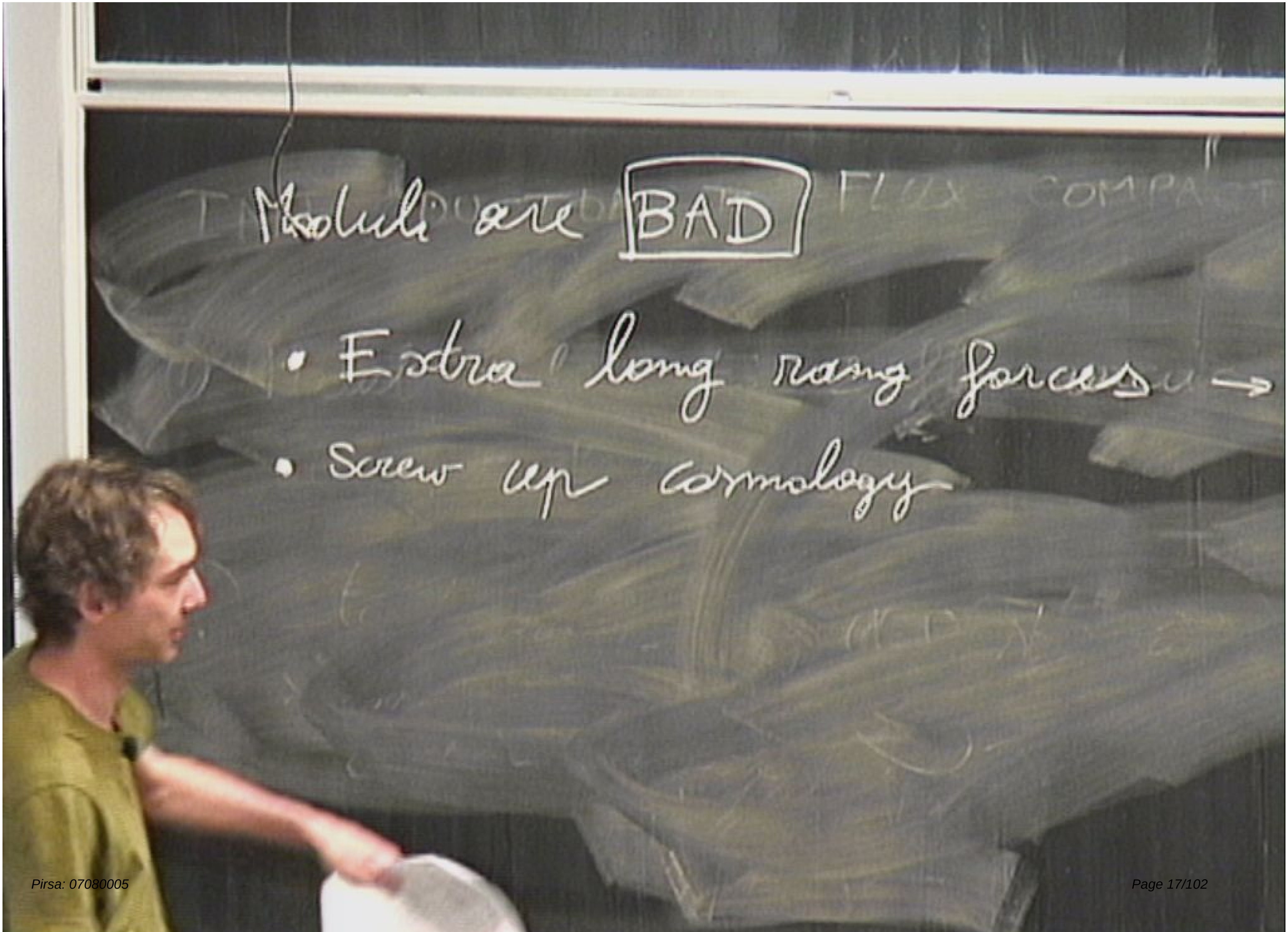
- Axions

γ

$\oint_{\gamma} A \qquad \int_{\Sigma_2} B_2 \qquad \int_{\Sigma_p} C_p$

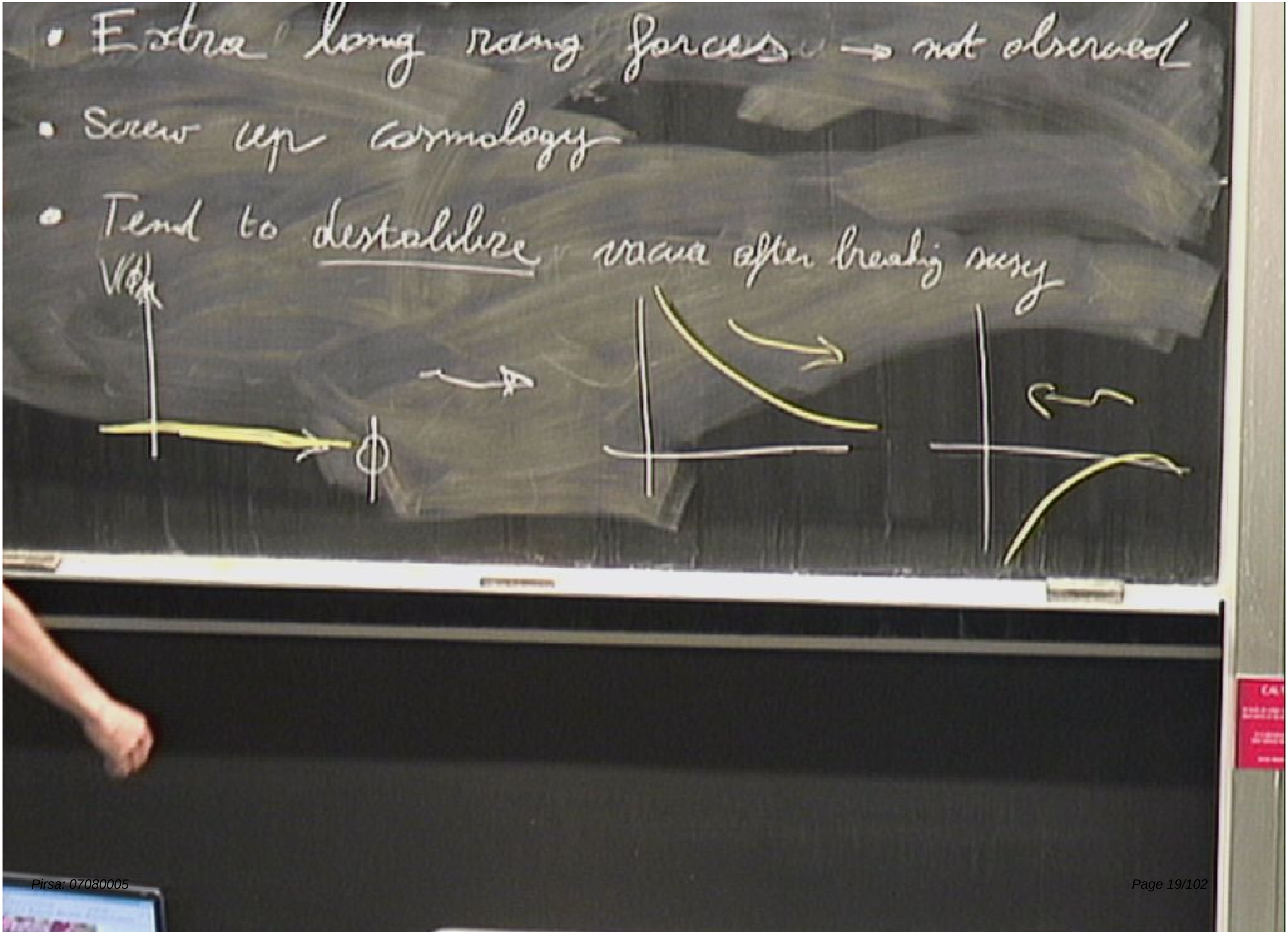

Moduli are BAD
• Extra long rang forces


BAD
long rang forces → not observed


Moduli are BAD
• Extra long rang forces →
• Screw up cosmology

Moduli are $\boxed{\text{BAD}}$

- Extra long rang forces → not observe
- Screw up cosmology
- Tend to destabilize vacua after breaking susy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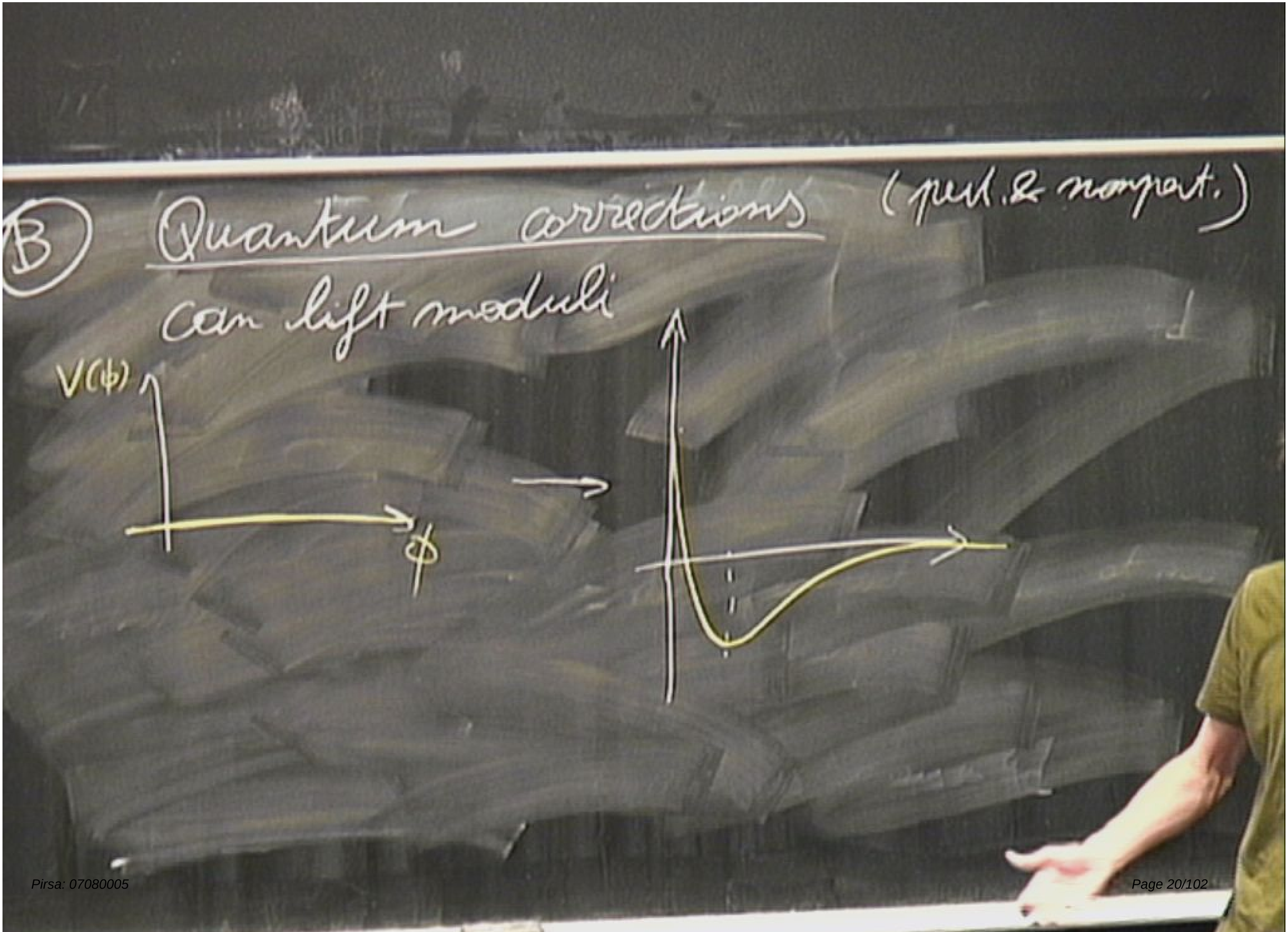
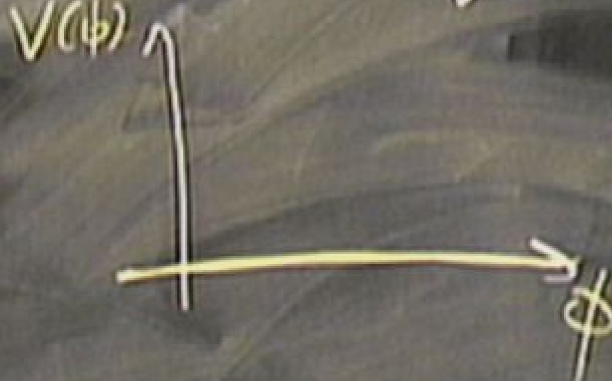
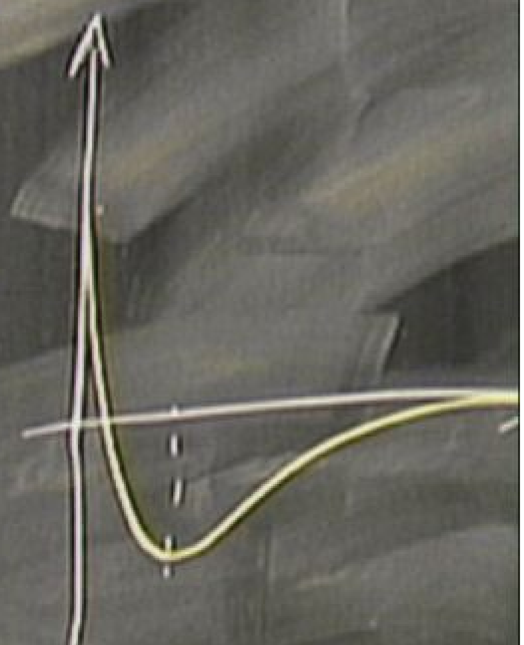
V(ϕ)
ϕ


B
Quantum corrections
(pert. & nonpert.)
can lift moduli
V(ϕ)
ϕ

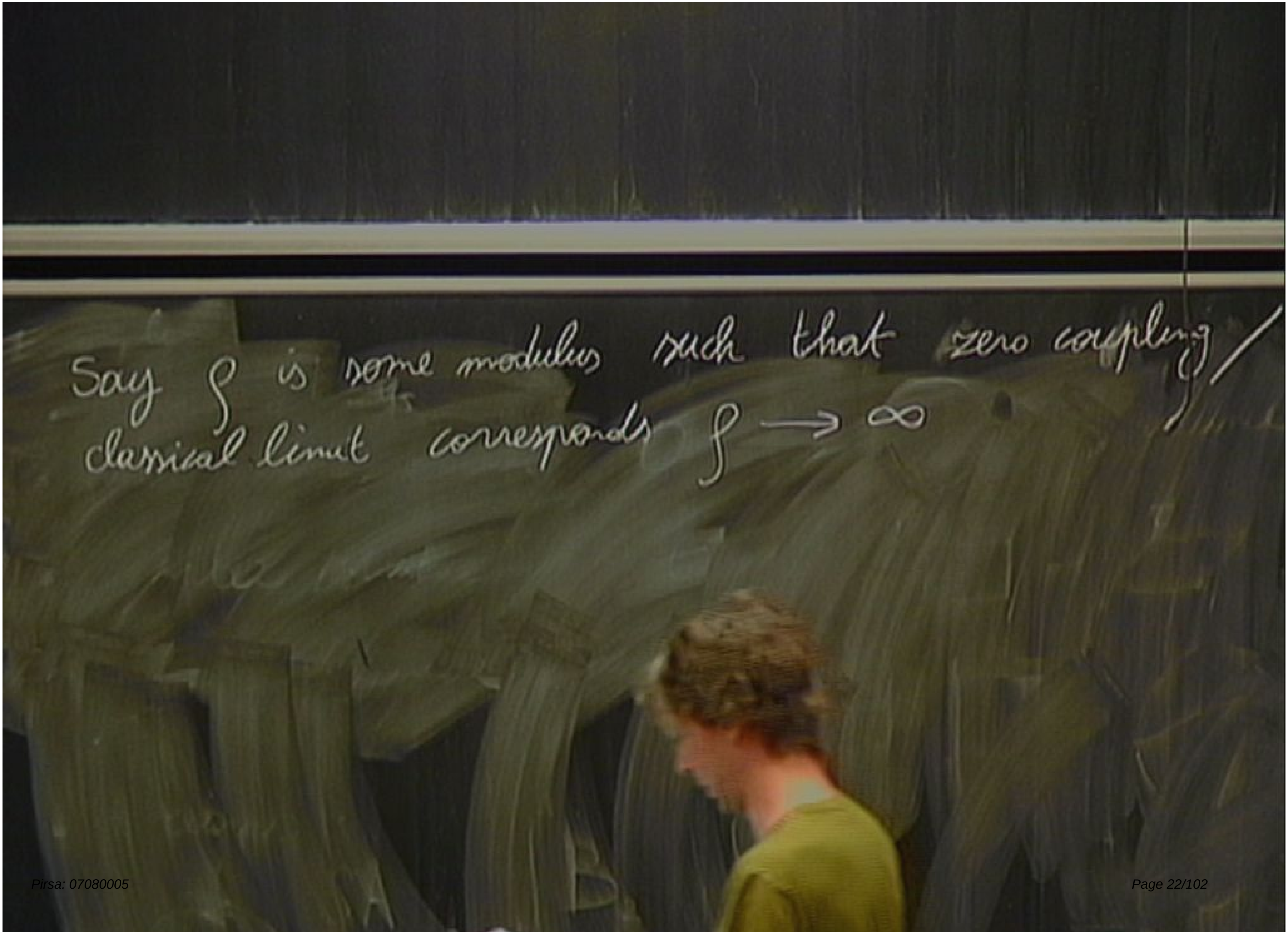
(B) Quantum corrections

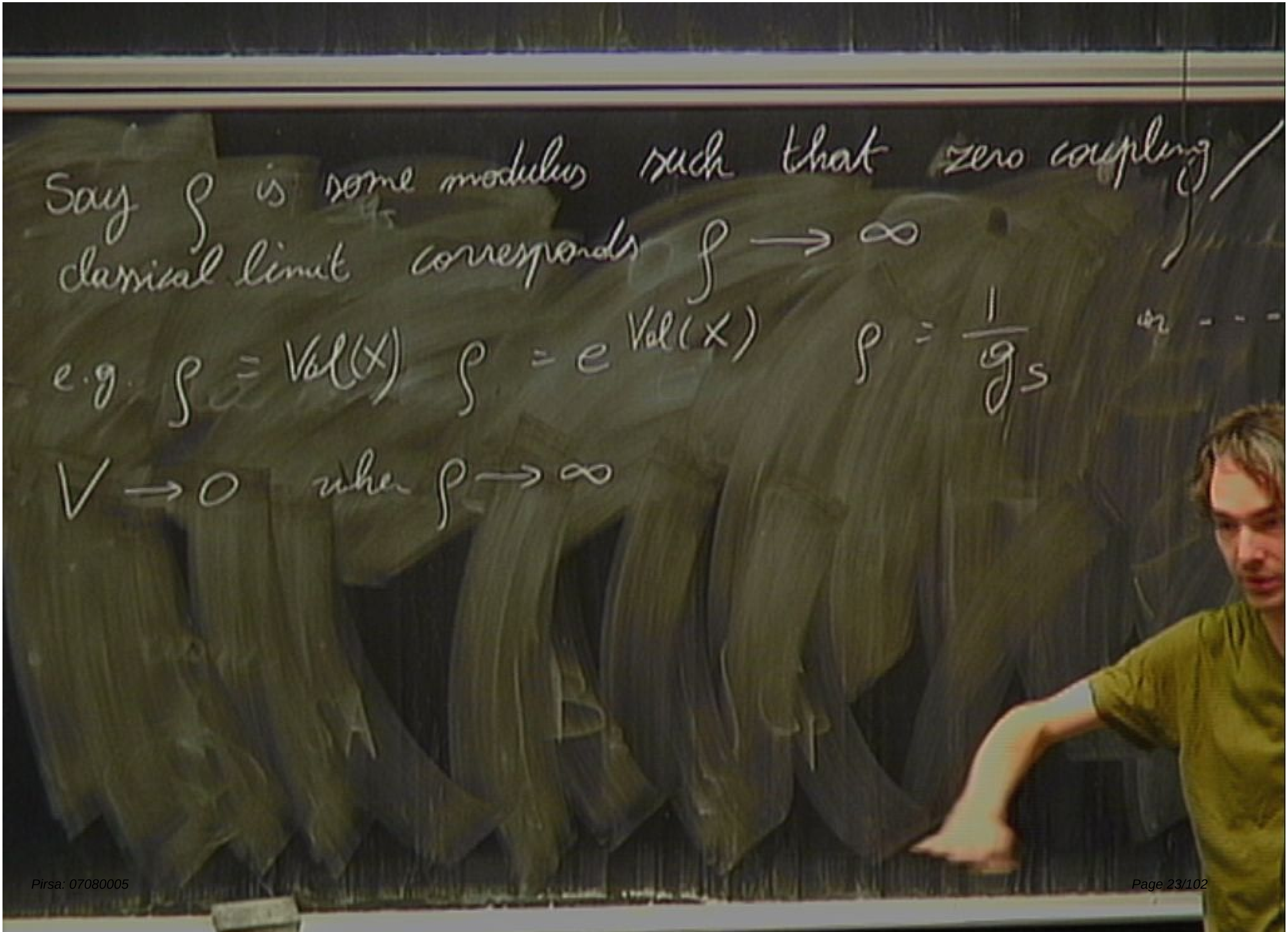
can lift moduli



→ 

→ $\boxed{\text{CONTROL}}$ PROBLEM


Say ρ is some modulus such that zero coupling/
classical limit corresponds $\rho \longrightarrow \infty$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e.g. $\rho = \mathrm{Vol}(X)$ $\rho = e^{\mathrm{Vol}(X)}$ $\rho = \frac{1}{g_s}$ or ...
$V \to 0$ when $\rho \to \infty$

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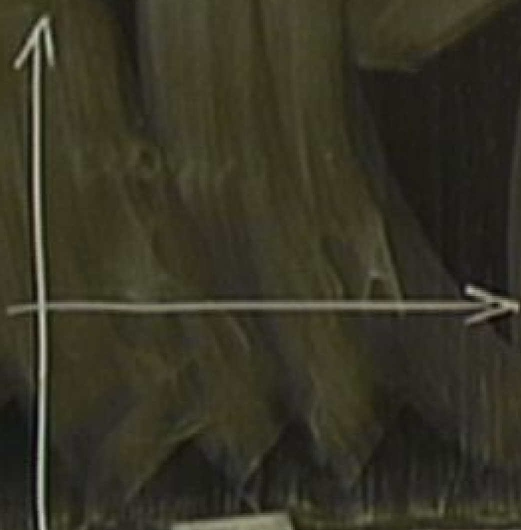
$V \rightarrow 0$ when $\rho \longrightarrow \infty$ $\Rightarrow$ first correction $V(\rho) \sim \pm \frac{1}{\rho}$



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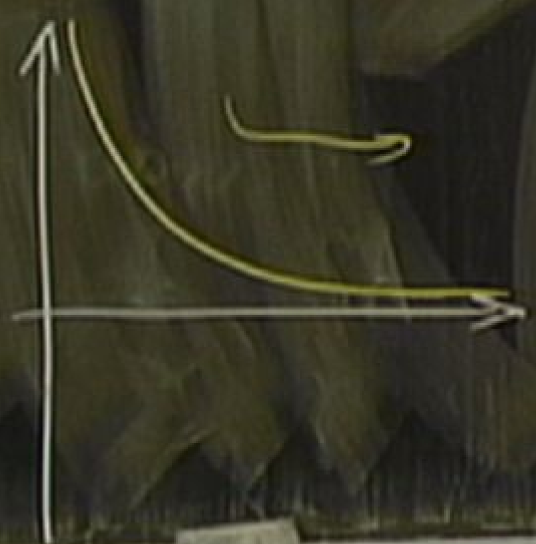

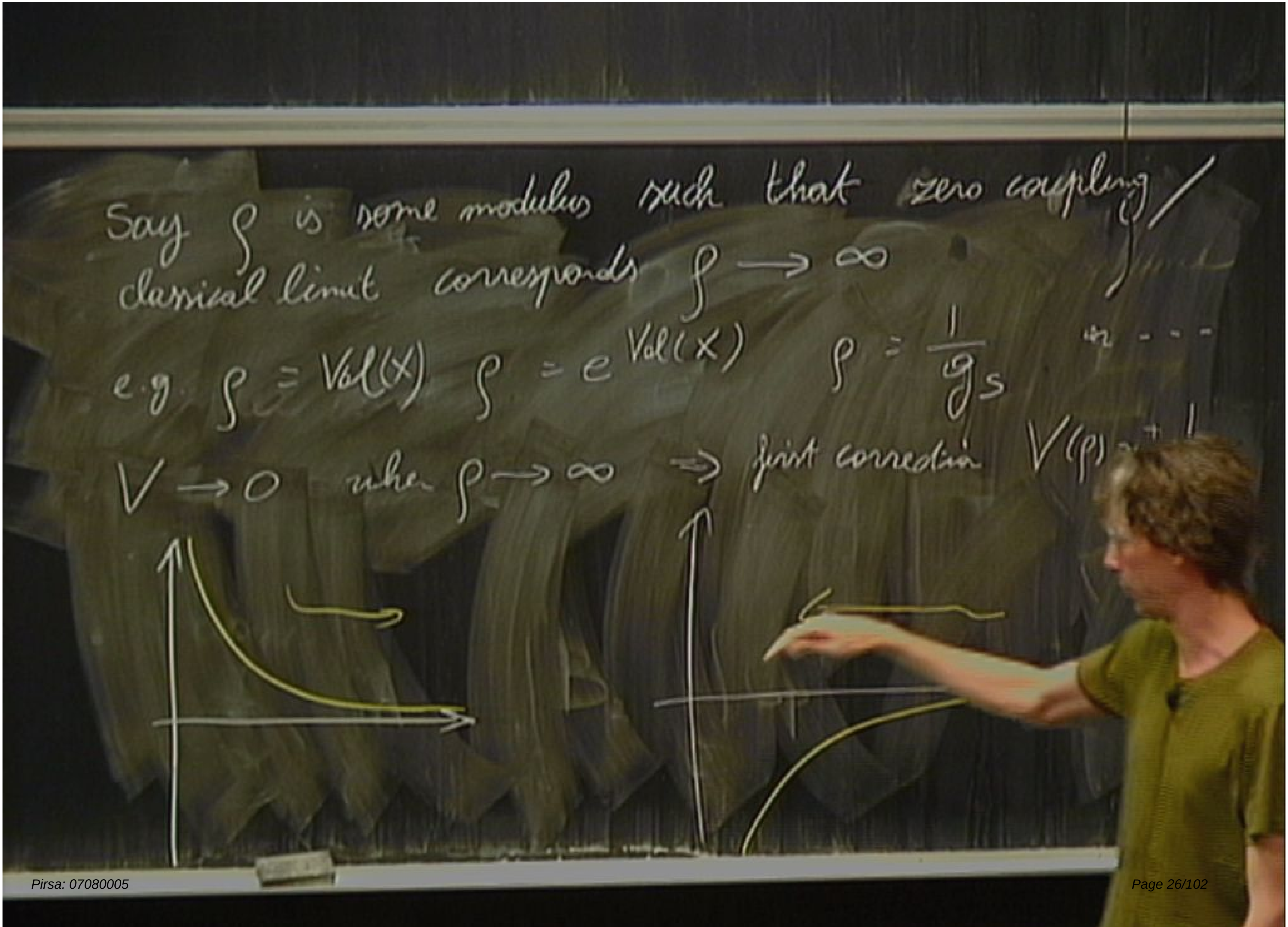
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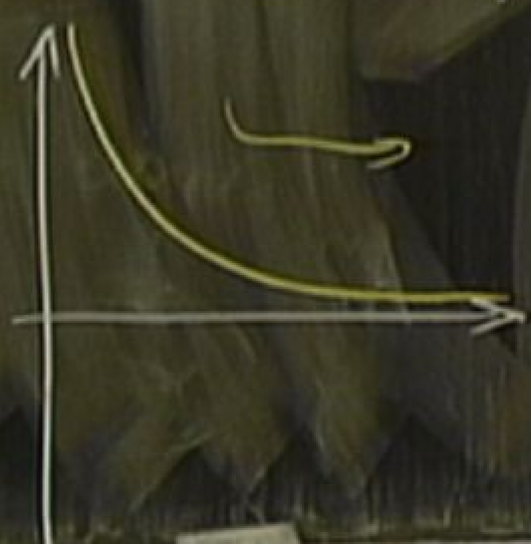
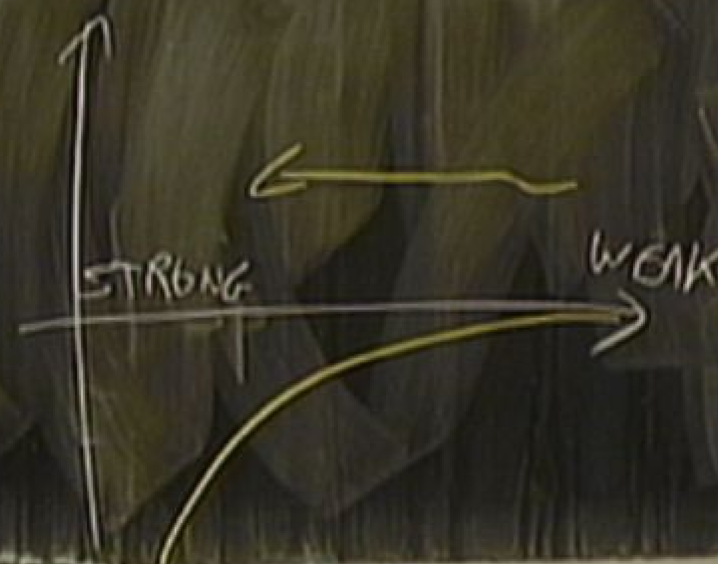
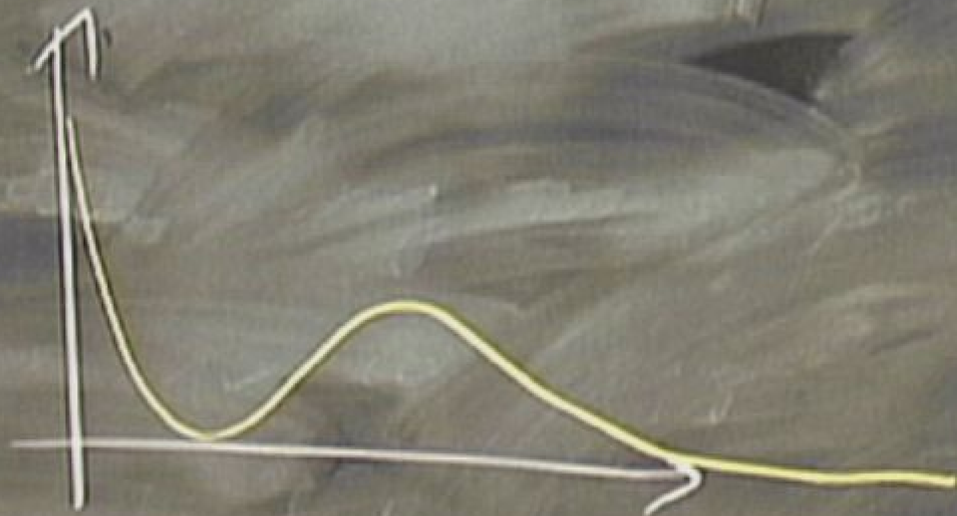
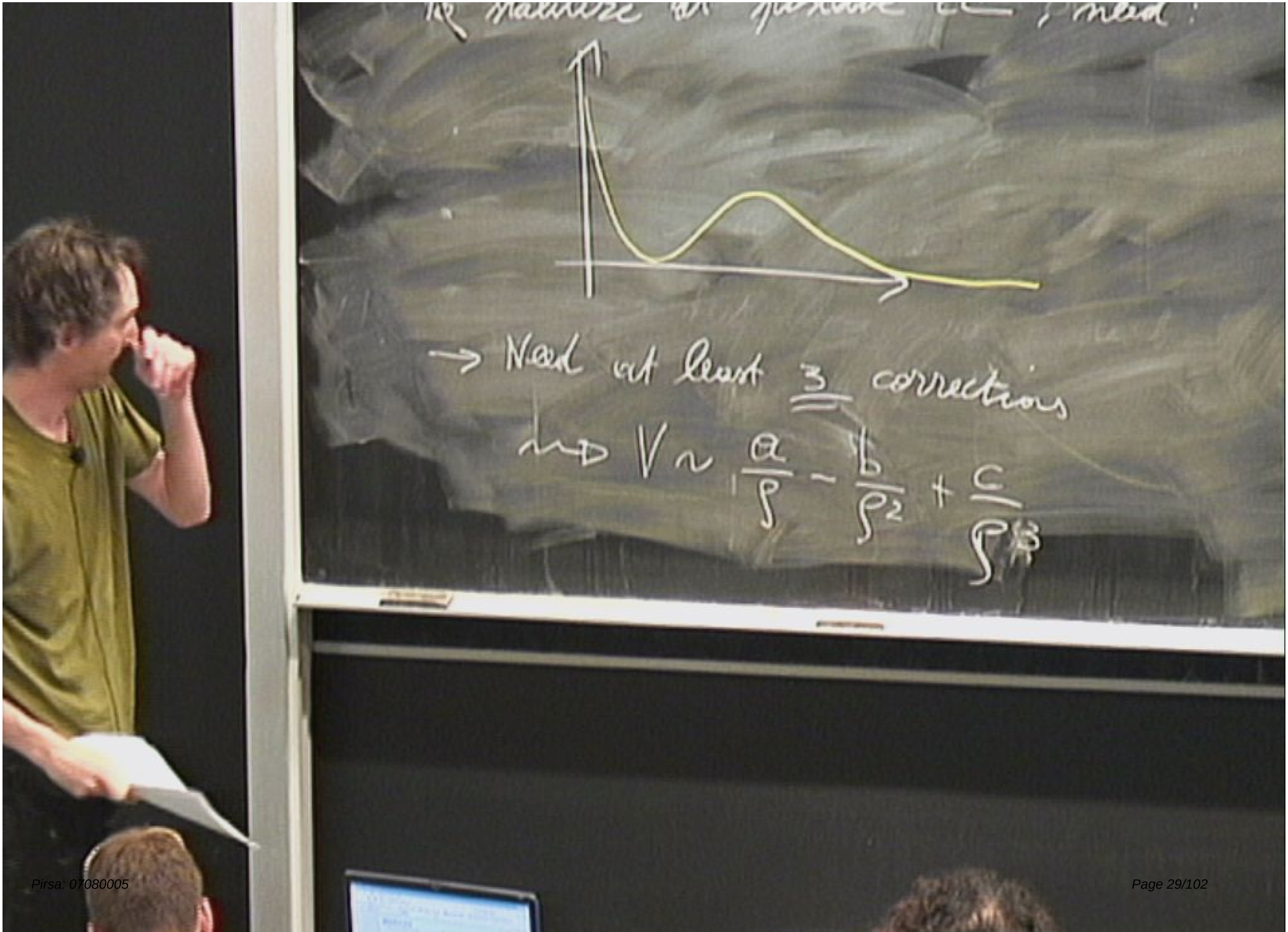




To stabilize at positive cc, need:




→ Need at least 3 corrections
V ~ a/ρ − b/ρ2 + c/ρ3

To stabilize at positive CC, need

→ Need at least $\underline{\underline{3}}$ corrections

$$\Longrightarrow V \sim \frac{a}{\rho} - \frac{b}{\rho^2} + \frac{c}{\rho^3}$$

- Typically $a, b, c \sim \mathcal{O}(1) \Rightarrow \rho_* \approx \mathcal{O}(1)$

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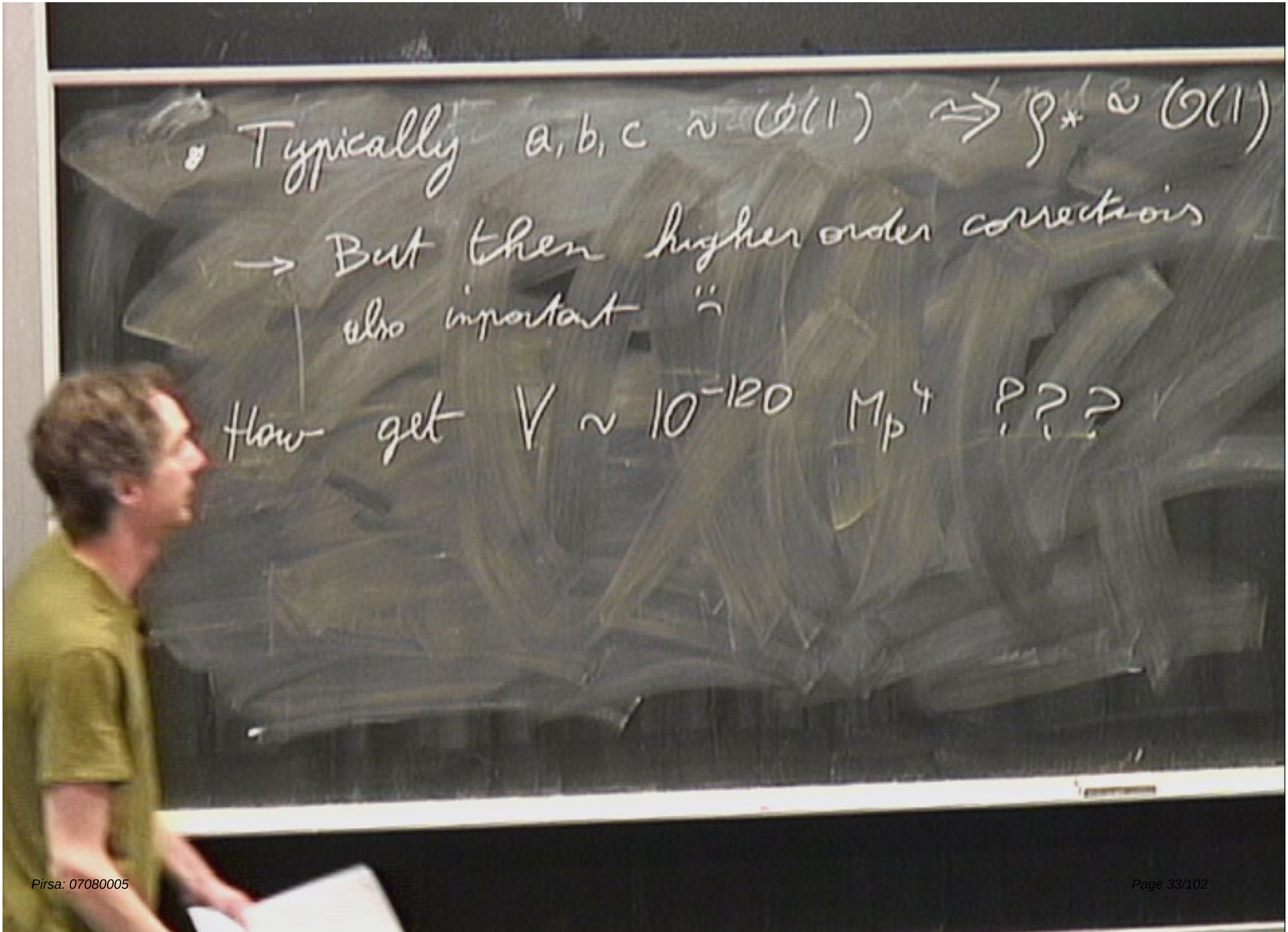
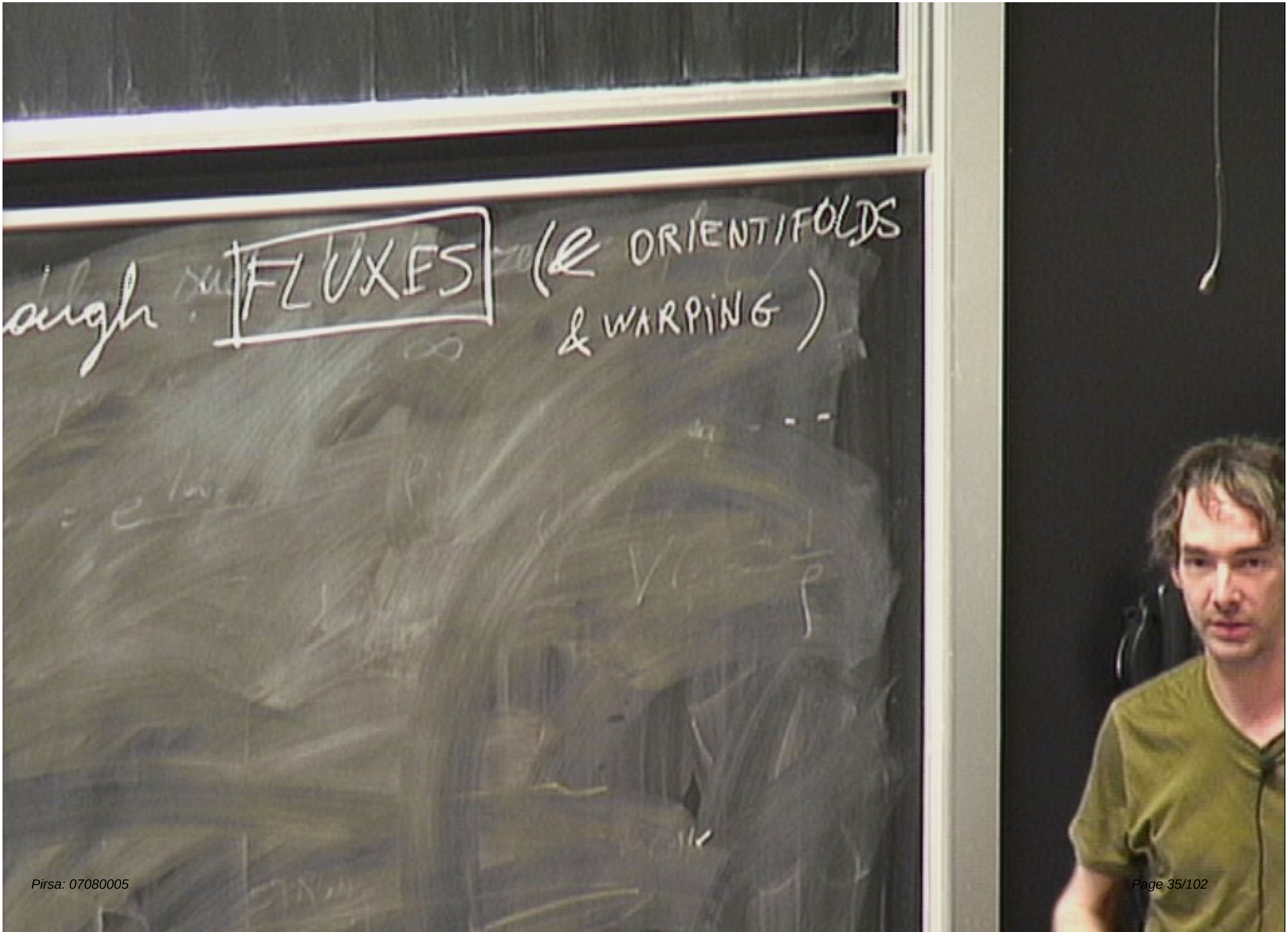


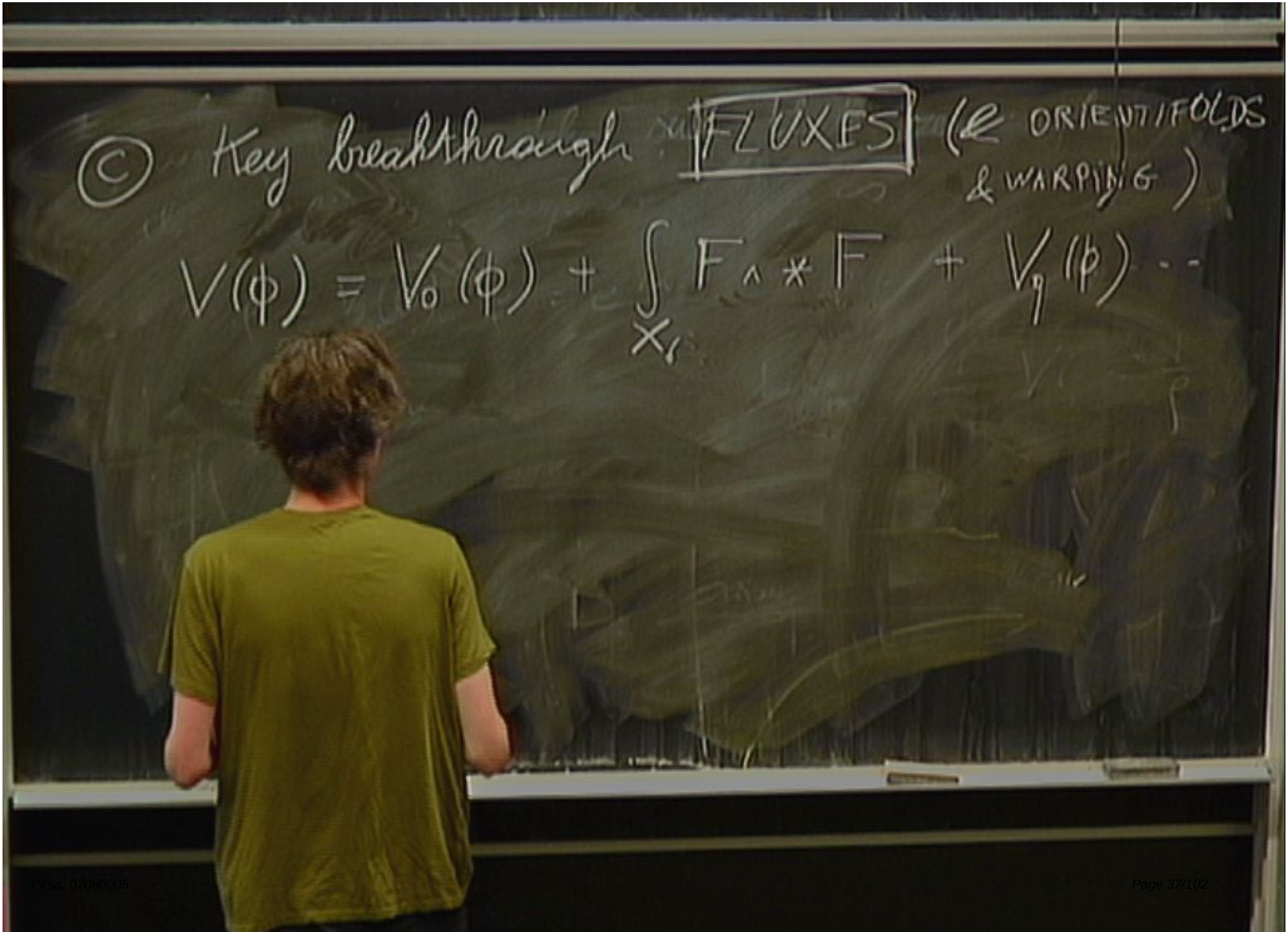
• Typically a, b, c ~ O(1) ⇒ ρ* ~ O(1)
→ But then higher order corrections
also important ¨
How get V ~ 10^-120 Mp^4 ???

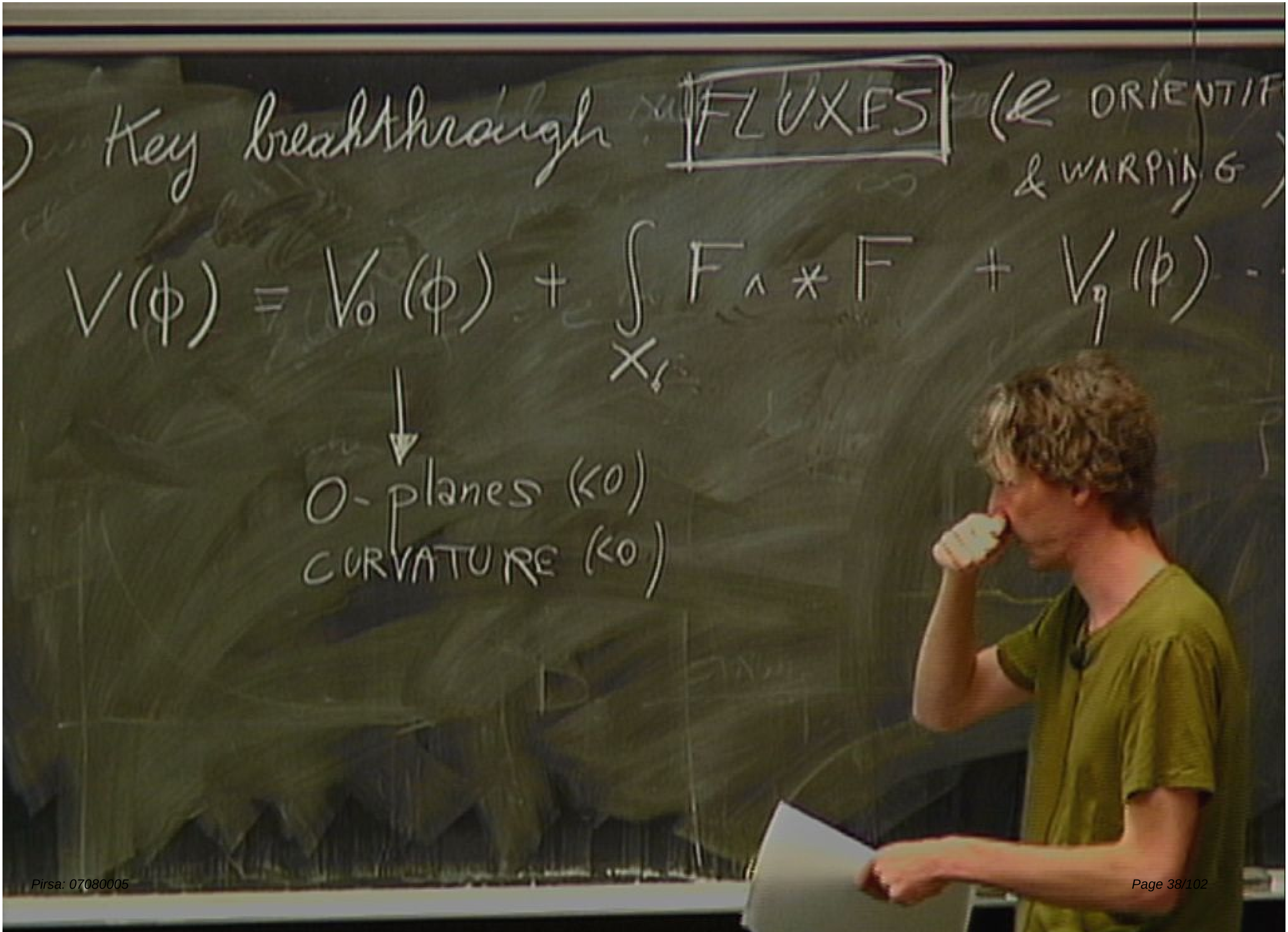
(c) Key breakthrough : $\boxed{\text{FLUXES}}$


ough : FLUXES (& ORIENTIFOLDS
& WARPING)

(c) Key breakthrough: $\boxed{\text{FLUXES}}$ (& ORIENTIF
& WARPING

$$V(\phi) = V_0(\phi) + \int F \wedge * F$$


(c) Key breakthrough: FLUXES (& ORIENTIFOLDS & WARPING)
$V(\phi) = V_0(\phi) + \int_{X_6} F \wedge * F + V_q(\phi) \ldots$


Key breakthrough: FLUXES (& ORIENTIF
& WARPING)
V(ϕ) = V0(ϕ) + ∫X6 F ∧ *F + Vq(ϕ)
O-planes (<0)
CURVATURE (<0)

$$V(\phi) = V_0(\phi) + \int_{X_6} F \wedge * F + V_q$$

O-planes (<0)

CURVATURE (<0)

D-BRANES (>0)

& WARPING)

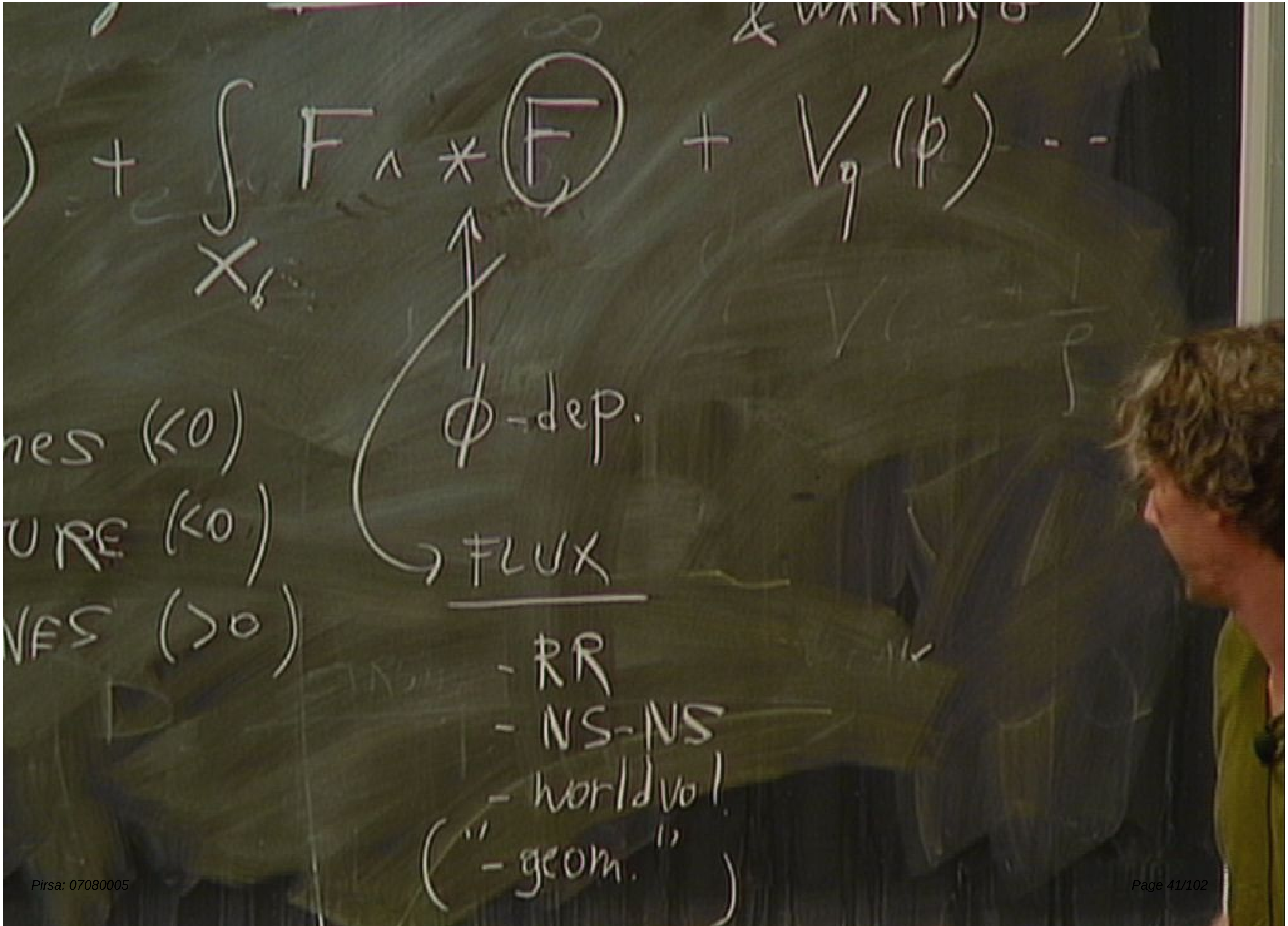
$$V_0(\phi) + \int_{X_6} F \wedge * F + V_q(\phi) \cdots$$

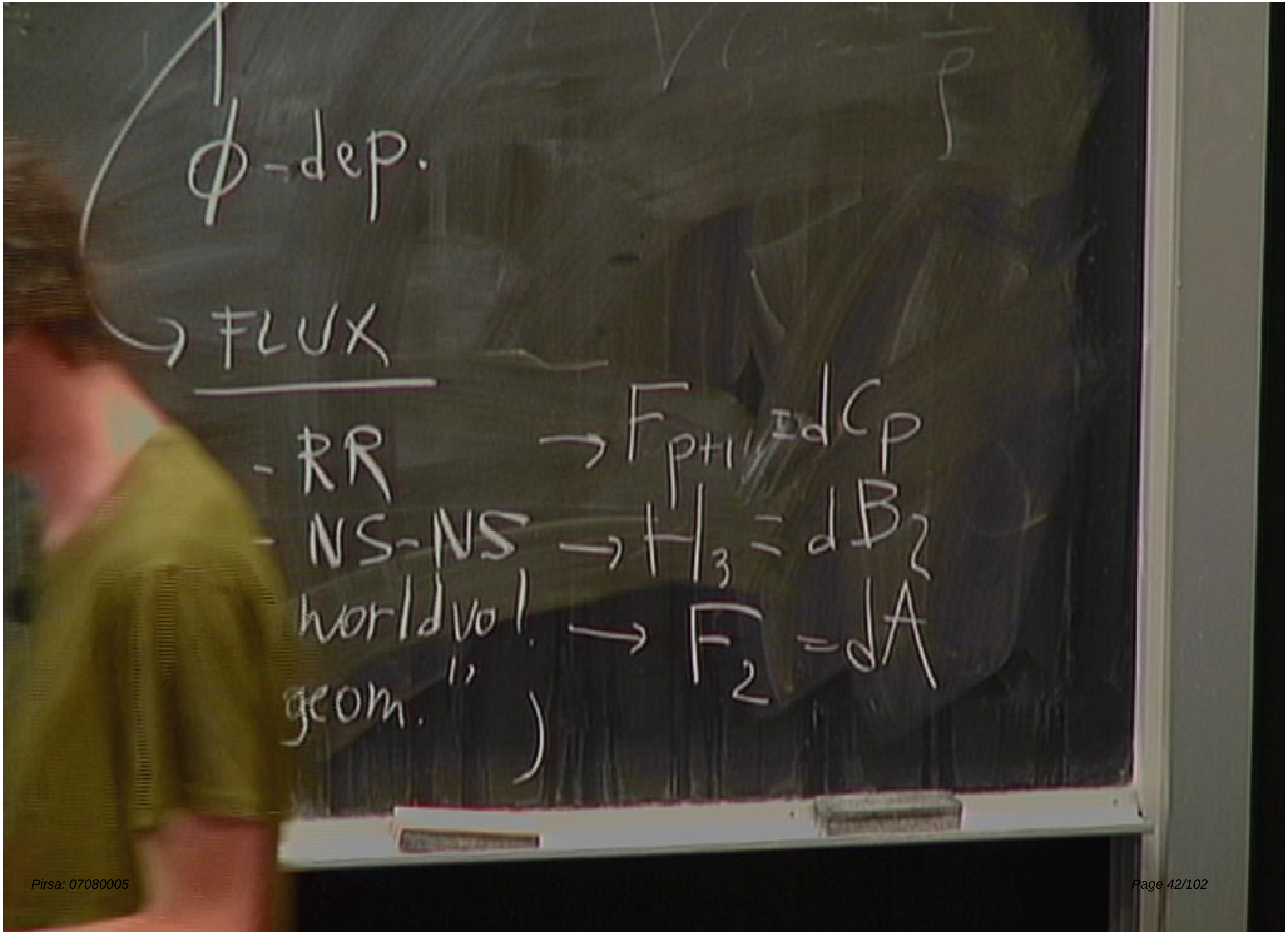
ϕ-dep.

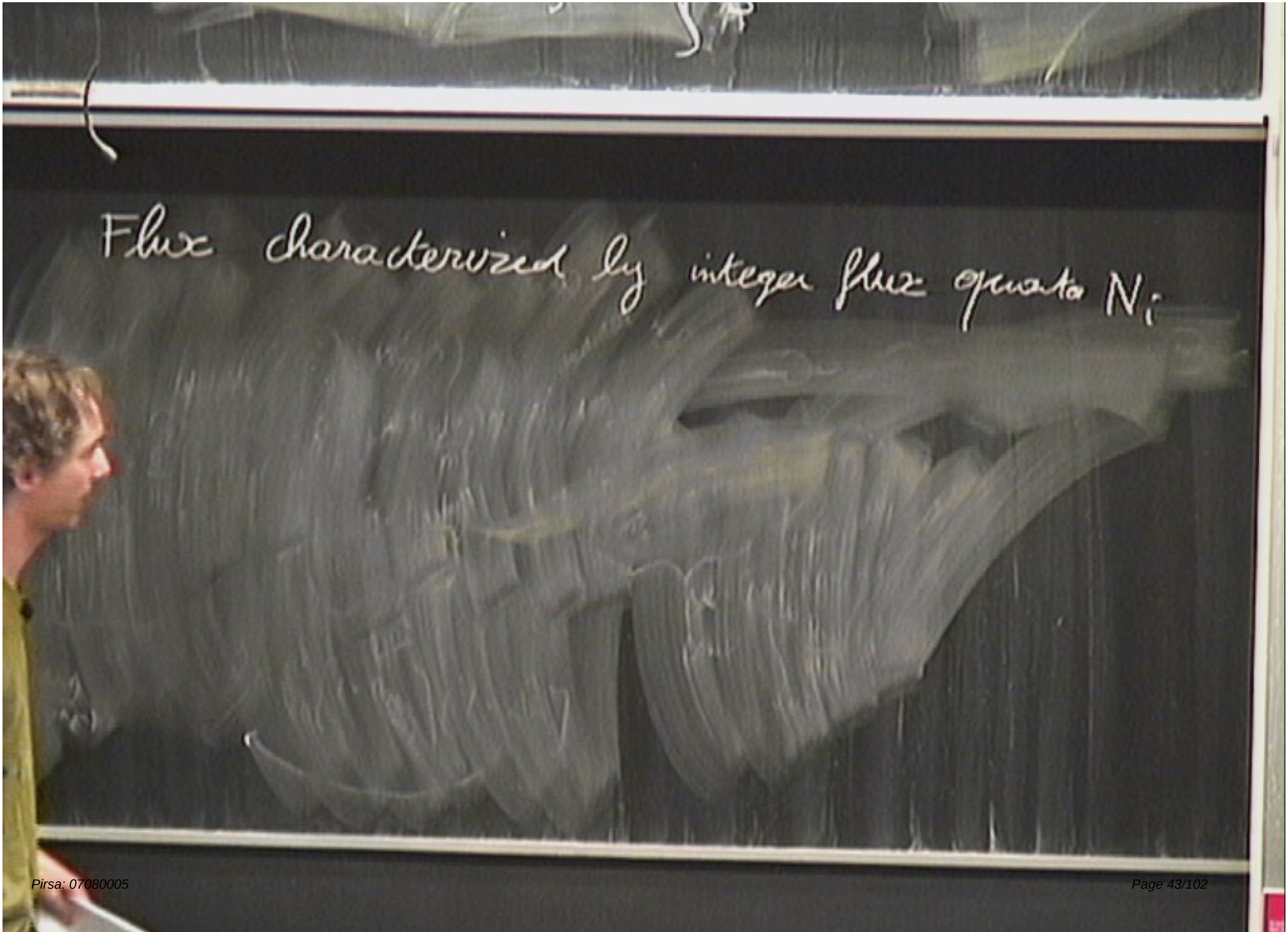
O-planes (<0)

CURVATURE (<0)

D-BRANES (>0)


& warping)
) + $\int_{X_6} F \wedge * F$ + $V_q(\phi)$ ...
nes (<0)
URE (<0)
NES (>0)
ϕ-dep.
FLUX
- RR
- NS-NS
- worldvol
("- geom.")


ϕ-dep.
FLUX
- RR $\rightarrow F_{p+1} = dC_p$
- NS-NS $\rightarrow H_3 = dB_2$
- worldvol. $\rightarrow F_2 = dA$
- geom. "


Flux characterized by integer flux quanta N_i

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$$N_i = \int_{\Sigma_i} F$$

Flux characterized by integer flux quanta N_i

$$N_i = \int_{\Sigma_i} F \qquad i = 1, \dots, \#\text{indep. cycles in } X \equiv d$$

$$V(\rho) = \frac{1}{\rho^3}\left(a_i N^i \rho - 1\right)^2$$

$$V(\rho) = \frac{1}{\rho^3}\left(\sum_i a_i N^i \rho - 1\right)^2 \qquad a_i \in \mathbb{R}$$

Minima $\rho_* = \dfrac{1}{\sum_i a_i N^i}$

$$V(\rho) = \frac{1}{\rho^3}\left(\sum_i a_i N^i \rho - 1\right)^2 \qquad a_i \in \mathbb{R}$$

$$\vec{a} \in \mathbb{R}^d$$

$$\text{Minima} \quad \rho_* = \frac{1}{\sum_i a_i N^i} = \frac{1}{\vec{a}\cdot\vec{N}} \qquad \vec{N} \in \mathbb{Z}^d$$

Minima $\rho_* = \dfrac{1}{\sum_i a_i N^i} =$

$\sum_i N_i^2$

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Minima $\rho_* = \frac{1}{\sum_i a_i N^i} = \frac{1}{\vec{a}\cdot\vec{N}}$

$\vec{a}$

$\vec{N}$

$$\sum_i N_i^2 \leq L$$

$$V(\rho) = \frac{1}{\rho^3}\left(\sum_i a_i N^i \rho - 1\right)^2 \qquad a_i \in$$

$$\vec{a} \in \mathbb{R}^d$$

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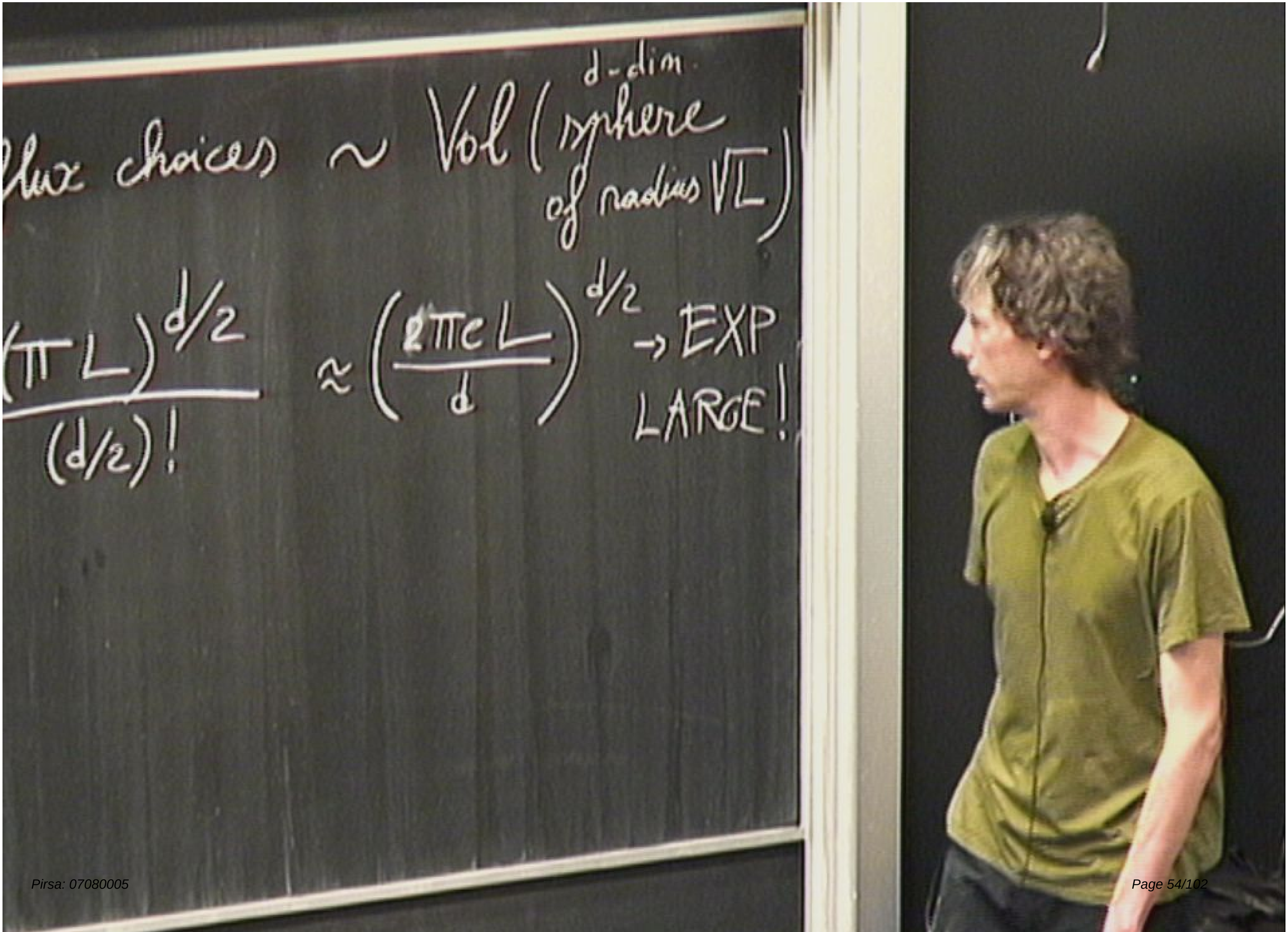
$$\sum_i N_i^2 \leq L \qquad L \in \mathbb{R}^+$$

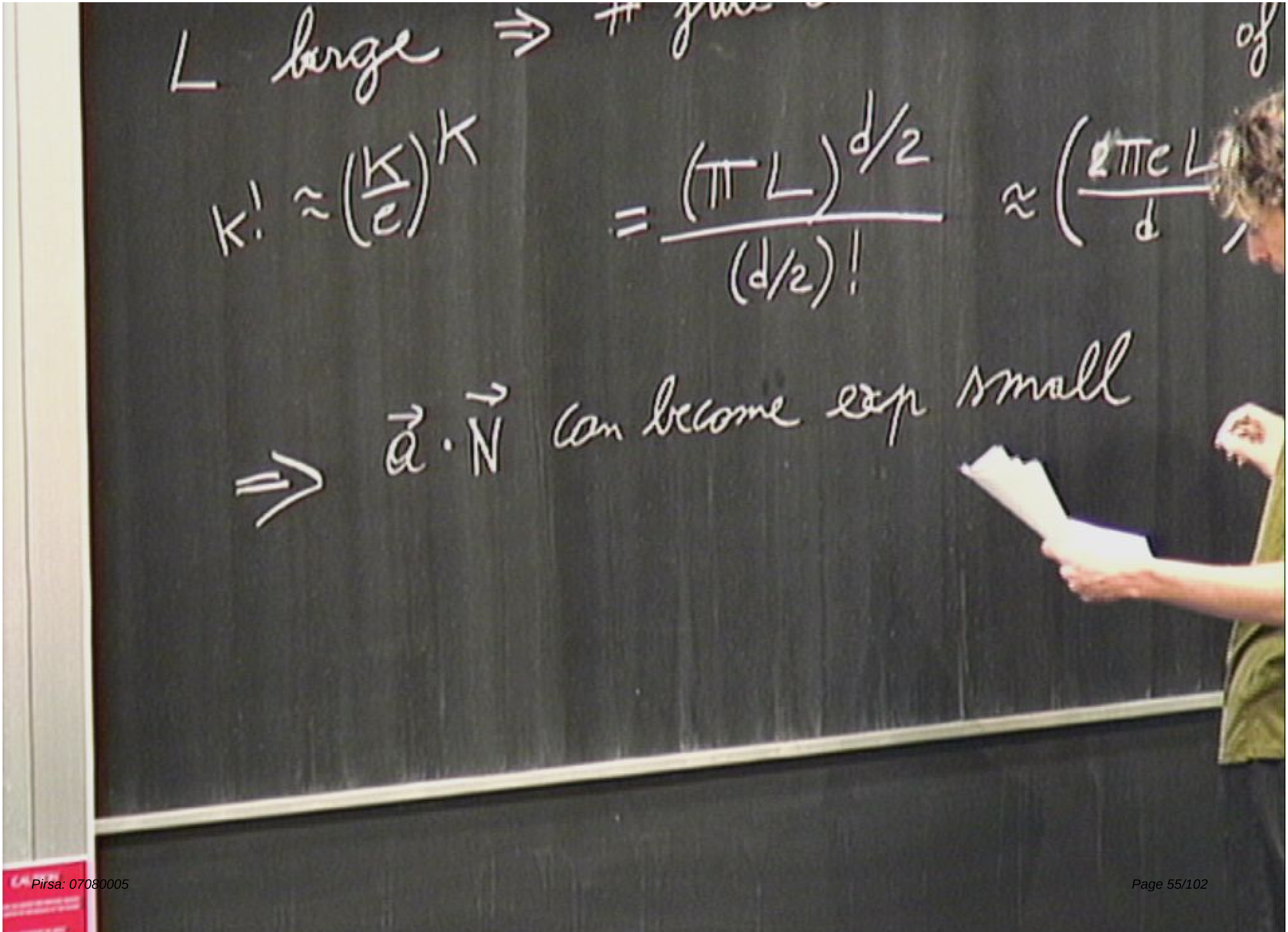
("-geom.")

L large $\Rightarrow$ # flux choices $\sim$ Vol(d-dim sphere of radius $\sqrt{L}$)

L large $\Rightarrow$ # flux choices $\sim$ Vol (d-dim sphere of radius

$$k! \approx \left(\frac{k}{e}\right)^k \qquad = \frac{(\pi L)^{d/2}}{(d/2)!} \approx \left(\frac{2\pi e L}{d}\right)^{d/2}$$


flux choices ~ Vol (d-dim sphere of radius √L)
(πL)^(d/2) / (d/2)! ≈ (2πeL / d)^(d/2) → EXP LARGE!


L large $\Rightarrow$
of
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$\approx \left(\frac{2\pi e L}{d}\right.$
$\Rightarrow \vec{a} \cdot \vec{N}$ can become exp small

L large

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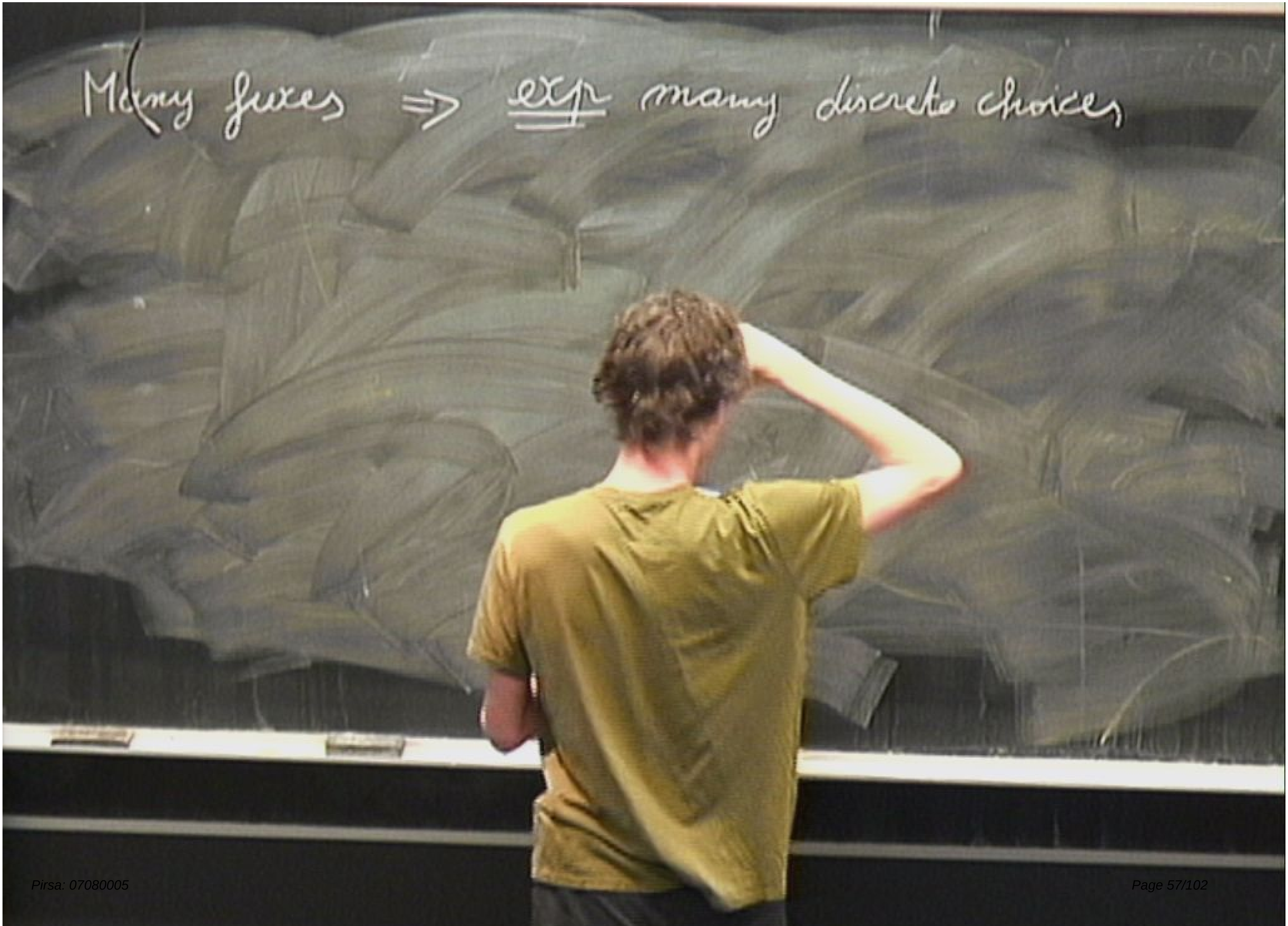
$\Rightarrow \vec{a}\cdot\vec{N}$ can become exp small

$\Rightarrow \rho_*$ can become exp LARGE


Many fluxes ⇒ exp many discrete choices

Many fluxes $\Rightarrow$ <u>exp</u> many discrete choices

$\Rightarrow$ "discretuum"

uxes $\Rightarrow$ <u>exp</u> many discrete choices ("vacua")

$\Rightarrow$ "discretuum"

$\Rightarrow$ tunability of low-energy parameters

but without massless scalars

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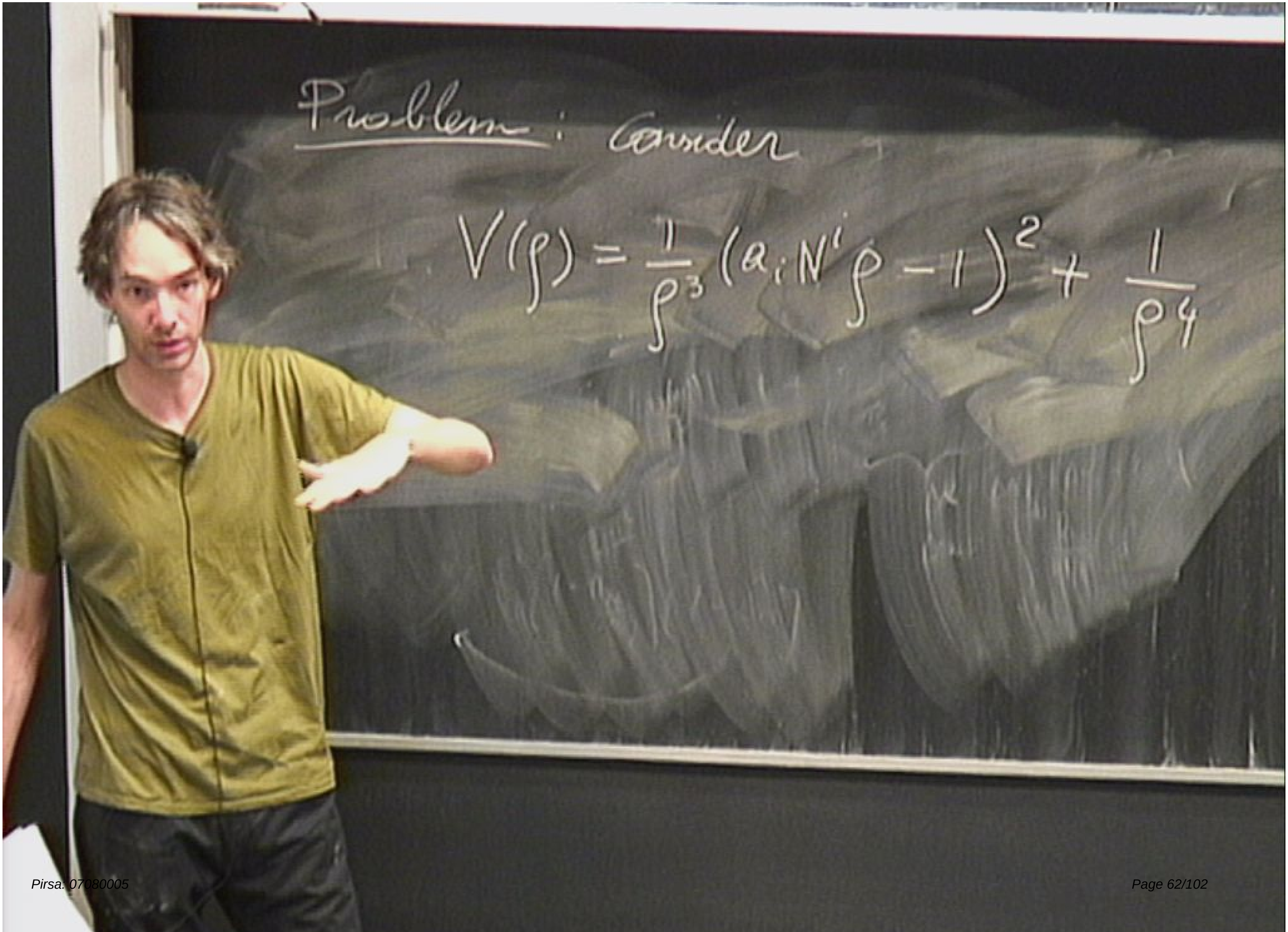
[Quantum effect : Dirac quant.]

but without massless scalars

[Quantum effect : Dirac quant.]

* Bousso Polchinski * → even $\Lambda \sim 10^{-120}\ M_p^4$ can be achieved with few hundred flux quanta


Problem: Consider
$V(\rho) = \frac{1}{\rho^3}(a_i N^i \rho - 1)^2 + \frac{1}{\rho^4}$

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For random $\vec{a}$, $|\vec{a}| = 1$

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$$\sum_i N_i^2 \leq L$$

For random $\vec{a}$, $|\vec{a}| = 1$, how many vacua do you expect with c.c. V : $0 < V < \varepsilon$

consider

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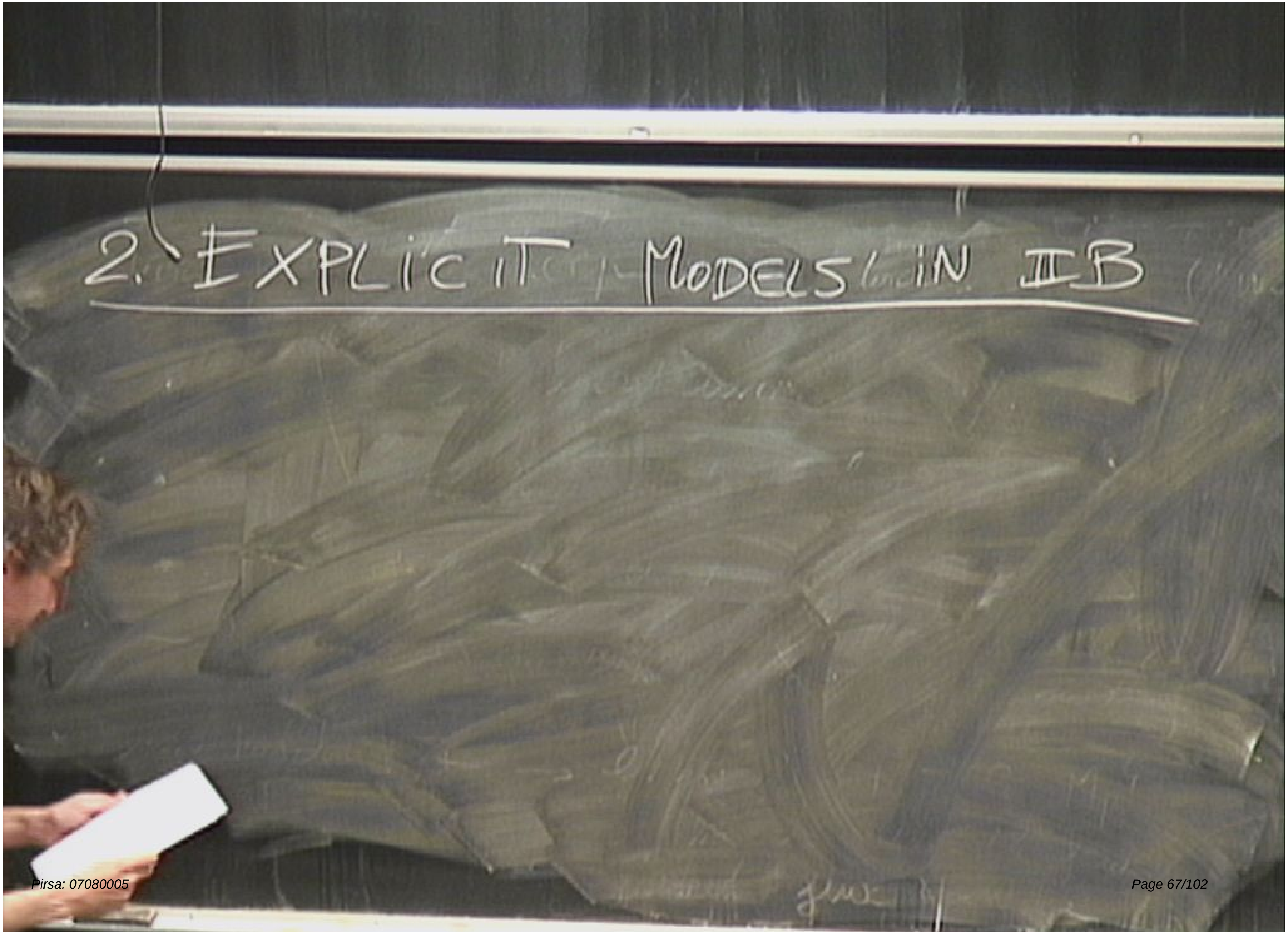
$$\varepsilon \lll 1$$


2. EXPLICIT MODELS IN IIB

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A. CALABI-YAU'S

B. ORIENTIFOLDS (& F-THEORY LIFTS)

C. FLUX & MODULI STAB.

D. KÄHLER (SIZE) STAB. SCENARIO'S : · KKLT

· BBCQ

(A) CONSTRUCTING CY'S

Quintic in $\mathbb{CP}^4$:

$$X_1^5 + X_2^5 + X_3^5 + X_4^5 + X_5^5 \; (+ \cdots) = 0$$

$$(X_1, \ldots, X_5) \simeq$$

(A) CONSTRUCTING CY'S

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Ⓐ CONSTRUCTING CY'S

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$$(X_1, \ldots, X_5) \neq (0, \ldots, 0)$$

- other popular examples : toroidal orbifolds

$T^6 / \mathbb{Z}_2 \times \mathbb{Z}_2$

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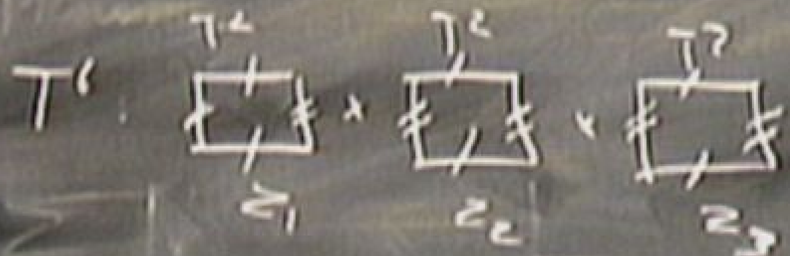
e.g. $T^6/\mathbb{Z}_2\times\mathbb{Z}_2$

$(z_1, z_2, z_3) \simeq (-z_1, -z_2, z_3)$

$\simeq (z_1, -z_2, -z_3)$

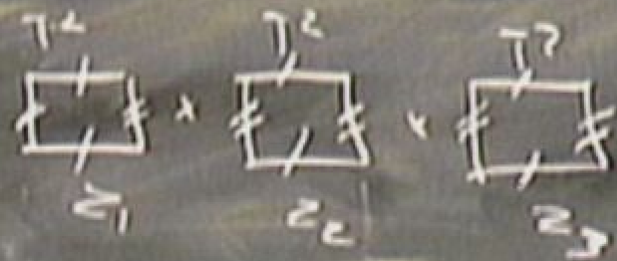


- Other popular examples : toroidal orbifolds

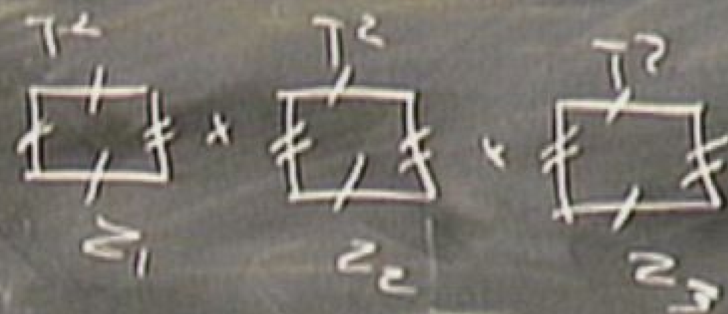
e.g. $T^6 / \mathbb{Z}_2 \times \mathbb{Z}_2$

$$(z_1, z_2, z_3) \simeq (-z_1, -z_2, z_3) \simeq (z_1, -z_2, -z_3)$$

T^6 :



e.g. $T^6 / \mathbb{Z}_2 \times \mathbb{Z}_2$ $(z_1, z$

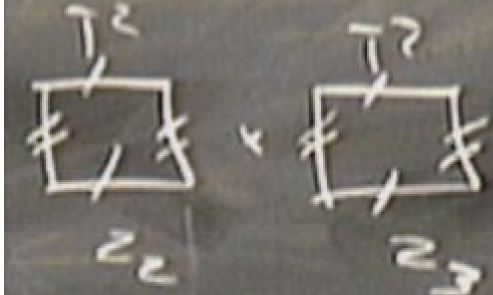
T^6 : 

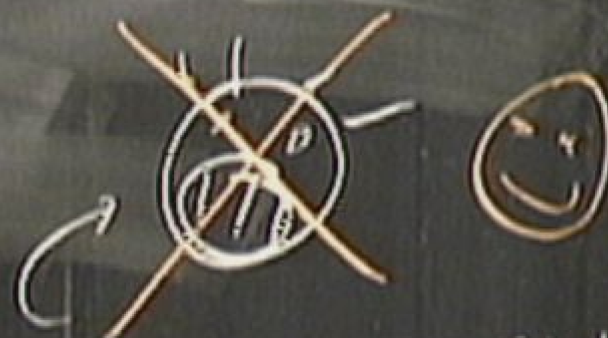
$\longrightarrow$ Want more general construction popular examples : toroidal orbifolds

$$(z_1, z_2, z_3) \simeq (-z_1, -z_2, z_3)$$

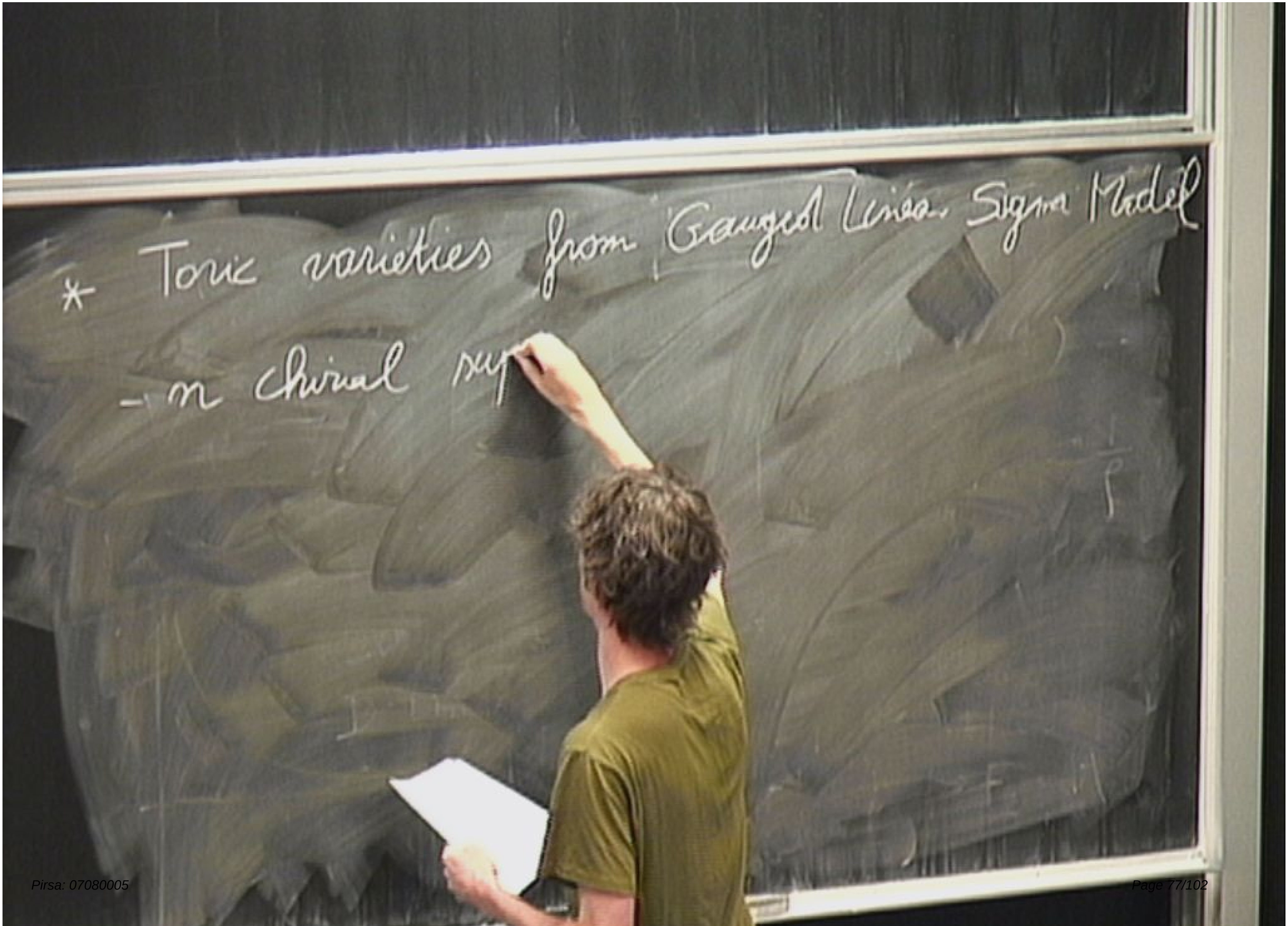
$$\simeq (z_1, -z_2, -z_3)$$

$\mathbb{Z}_2 \times \mathbb{Z}_2$



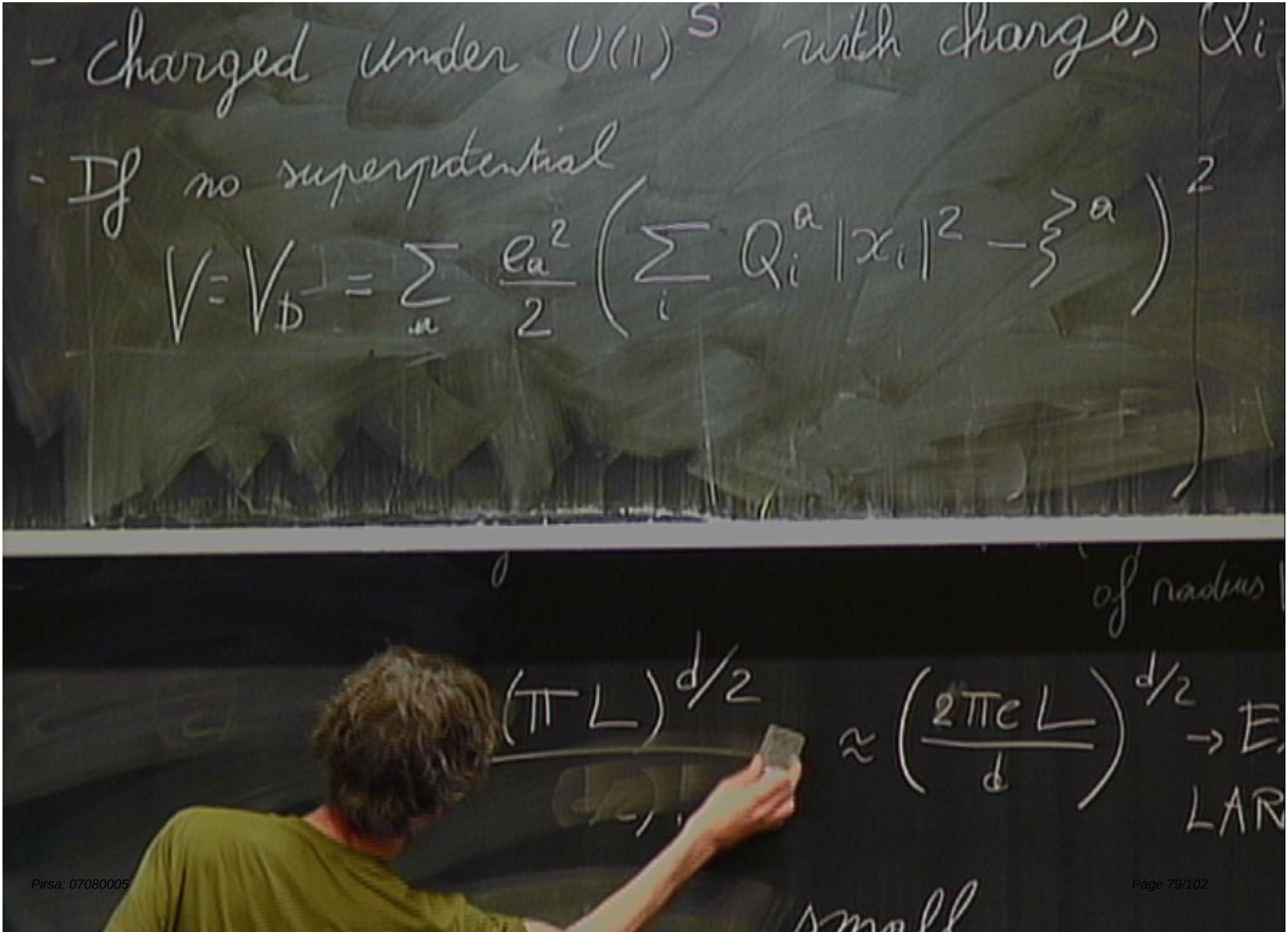


ore general contruction → TORIC GEOMETRY


* Toric varieties from Gauged Linear Sigma Model
– n chiral sup

Toric varieties from Gauged Linear Sigma Model

- n chiral superfields X_i $\quad i=1,\dots,n$
  with complex scalar components x_i $\quad a=1,\dots,S$
- charged under $U(1)^S$ with charges Q_i^a


- charged under U(1)^S with charges Q_i
- If no superpotential
$V = V_D = \sum_a \frac{e_a^2}{2} \left(\sum_i Q_i^a |x_i|^2 - \xi^a \right)^2$
of radius
$(\pi L)^{d/2} \approx \left(\frac{2\pi c L}{d} \right)^{d/2}$ → E
LAR
small

Susy moduli space : $V=0$ mod $U(1)^s$

$$\mathcal{M} = \left\{ x \in \mathbb{C}^n \,\middle|\, \sum_i Q_i^a |x_i|^2 = \xi^a \right\} / U(1)^s$$

$$U(1)^s : x_i \to e^{i Q_i^a \varphi_a} x_i$$

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$$U(1)^S : x_i \to e^{i Q_i^a \varphi_a} x_i$$

If ξ^a such that $\dim_{\mathbb{C}} \mathcal{M} = n - S$

$Q_i^a \varphi_a \quad x_i$

$n - s \longrightarrow M$ toric variety

- **EXAMPLE 1**

$\mathcal{M} = \mathbb{CP}^4$ $\qquad n = 5 \quad , \quad s = 1$

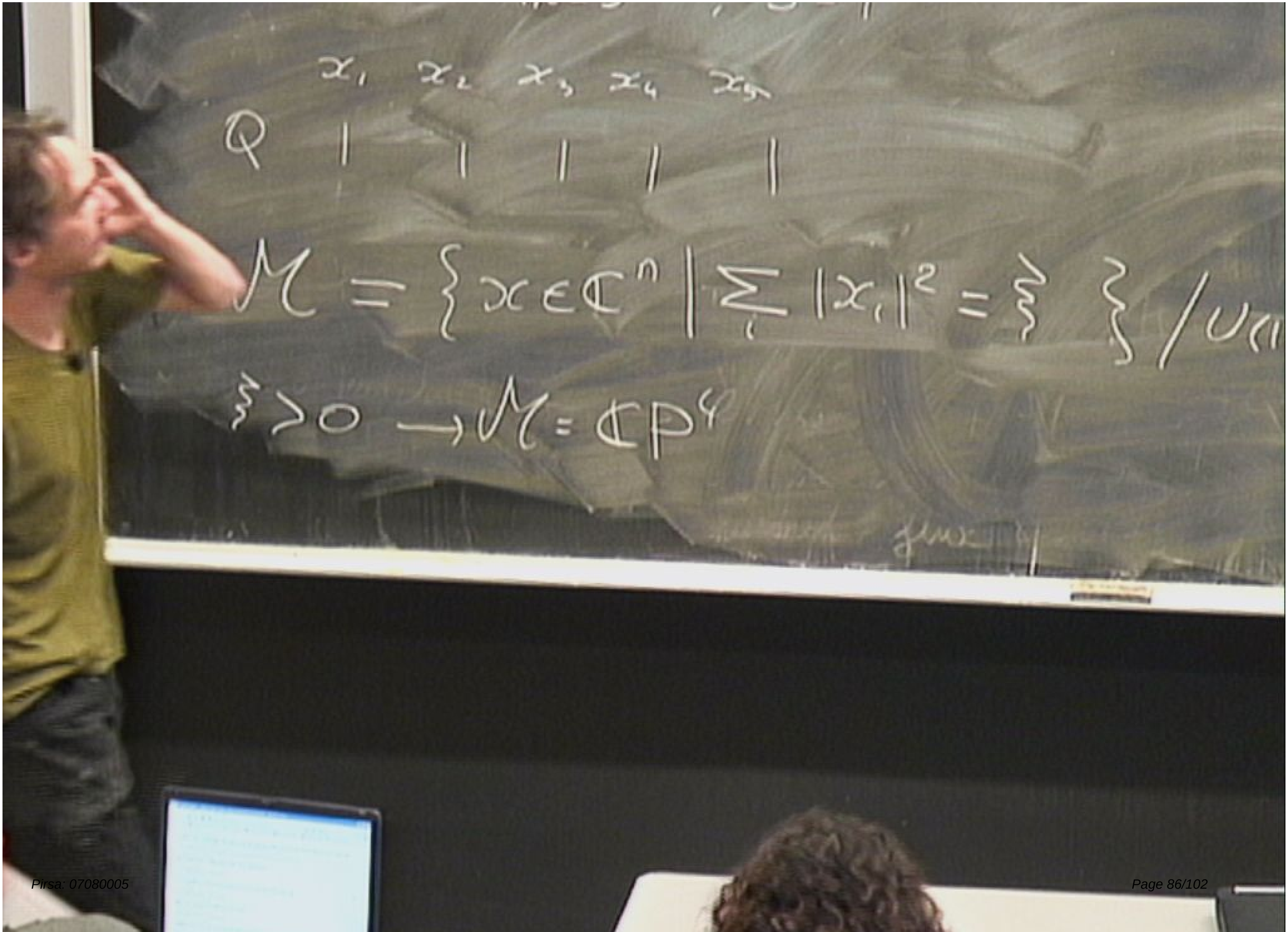
	x_1	x_2	x_3	x_4	x_5
Q	1	1	1	1	1

• **EXAMPLE 1**

$\mathcal{M} = \mathbb{C}P^4$ $\quad n = 5 \;,\; s = 1$

	x_1	x_2	x_3	x_4	x_5
Q	1	1	1	1	1

$$\rightarrow \mathcal{M} = \left\{ x \in \mathbb{C}^n \,\middle|\, \sum_i |x_i|^2 = \xi \right\} / U(1)$$


$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$
$Q \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$
$\mathcal{M} = \{ x \in \mathbb{C}^n \mid \sum_i |x_i|^2 = \xi \} / U(1$
$\xi > 0 \rightarrow \mathcal{M} = \mathbb{CP}^4$

• EXAMPLE ②

WCP^4_{11114}

	x_1	x_2	x_3	x_4	x_5
Q	1	1	1	1	4

$$M : \sum_{i=1}^{4} |x_i|^2 + 4|x_5|^2 = \xi > 0.$$

• EXAMPLE 3 : $\mathbb{P}I_1$

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6$

Q^1

Q^2

• EXAMPLE 3 : $\mathbb{P}I_1$

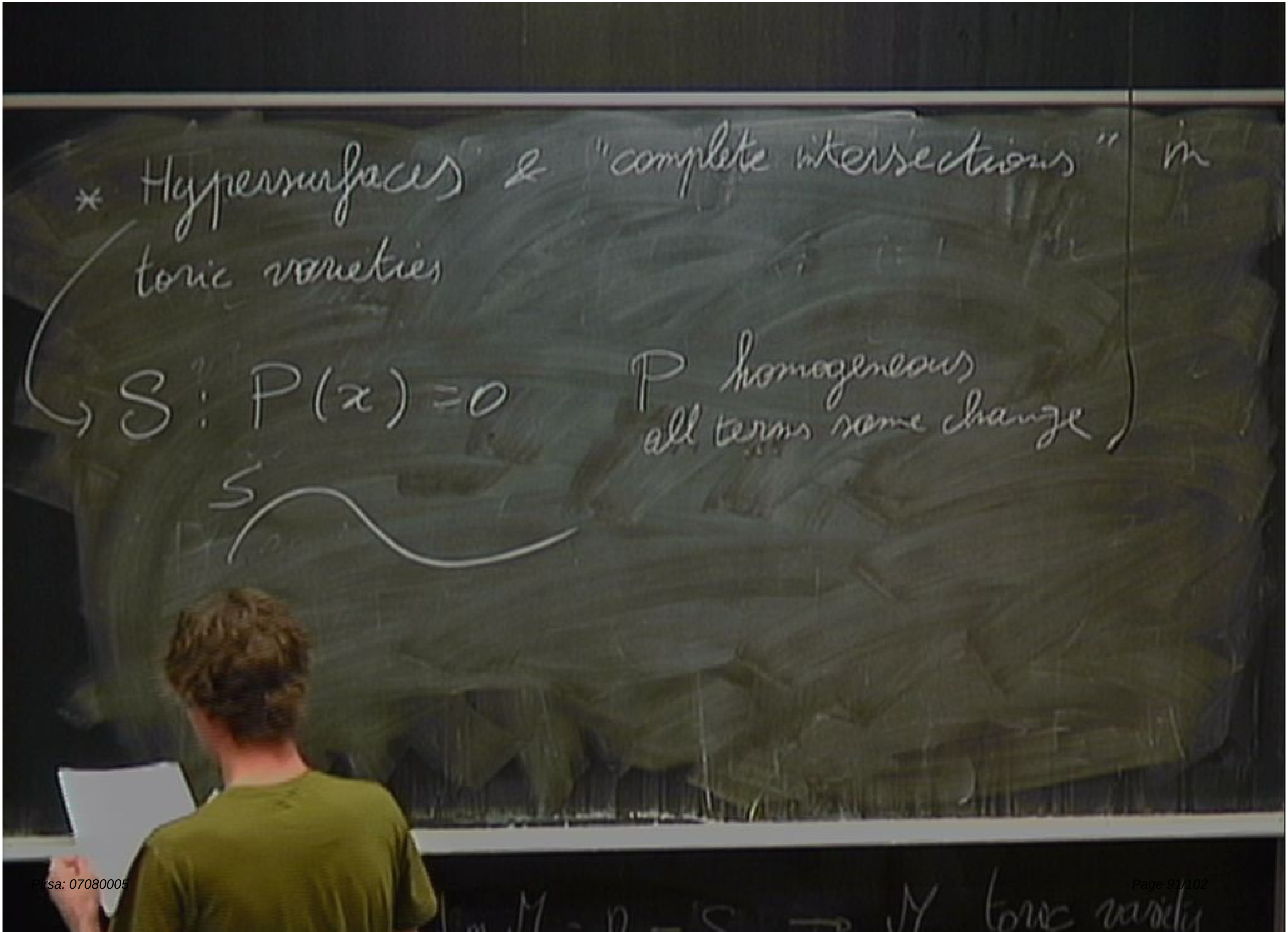
$n=6$
$s=2$

	x_1	x_2	x_3	x_4	x_5	x_6
Q^1	1	1	1	-1	0	2
Q^2	0	0	0	1	1	2

$$
\begin{array}{c|cccccc}
 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\
Q^1 & 1 & 1 & 1 & -1 & 0 & 2 \\
Q^2 & 0 & 0 & 0 & 1 & 1 & 2
\end{array}
$$

M is CY iff $\sum_i Q_i^a = 0 \quad \forall a$

e.g. $Q = (1\ 1\ 1\ \ -3)$


* Hypersurfaces & "complete intersections" in
toric varieties
S : P(x) = 0
P homogeneous
all terms same charge
S

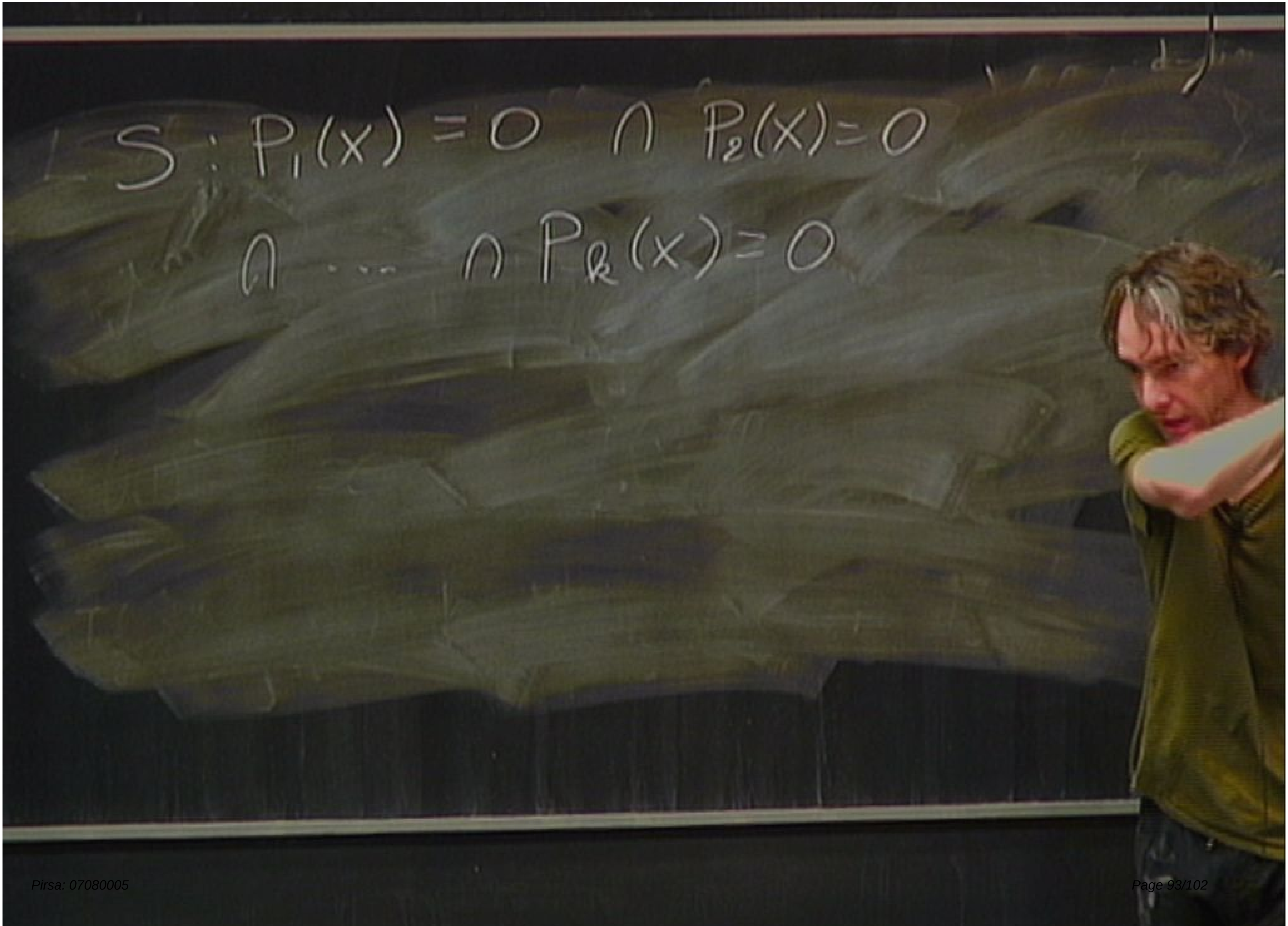
toric varieties

$S : P(x) = 0$ $\quad$ P homogeneous
all terms same charge

S

$$S \text{ C.Y.} \Leftrightarrow Q^a(P(x)) = \sum_i Q_i^a$$

If ξ^a such that $\dim_{\mathbb{C}} M = n - s \rightarrow M$ toric variet

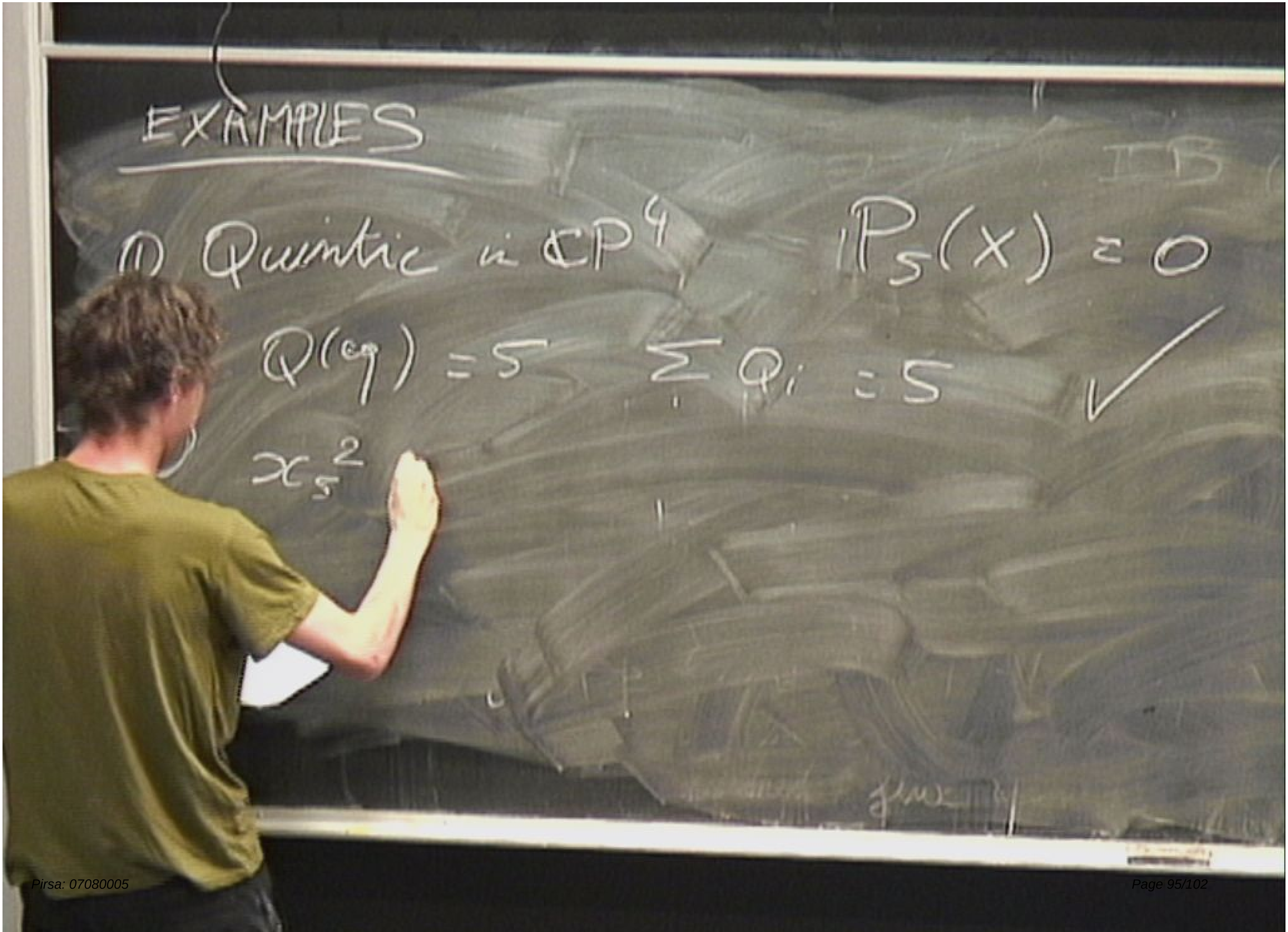

S : P₁(x) = 0 ∩ P₂(x) = 0
∩ … ∩ P_k(x) = 0

$$S: P_1(x) = 0 \cap P_2(x) = 0$$

$$\cap \dots \cap P_k(x) = 0$$

is CY $\iff \sum_\alpha Q(P_\alpha(x)) = \sum_i Q_i$

$$\iff \boxed{\sum Q(\text{eqs}) = \sum Q(\text{fields})}$$


EXAMPLES
① Quintic in $\mathbb{CP}^4$
$P_5(x) = 0$
$Q(\varphi) = 5$
$\sum Q_i = 5$
②
x_5^2

EXAMPLES

① Quintic in $\mathbb{CP}^4$ $\qquad \mathbb{P}_5(x) = 0$

$$Q(eq) = 5 \qquad \sum_i Q_i = 5 \quad \checkmark$$

② $$x_5^{\,2} = h_8(x_1, \ldots, x_4) \qquad \left(\text{e.g.} = \sum_{i=1}^{4} x_i^8 + x_1^2 x_2^2 x_3^2 x_4^2 + x_1^7 x_2 + \ldots \right)$$

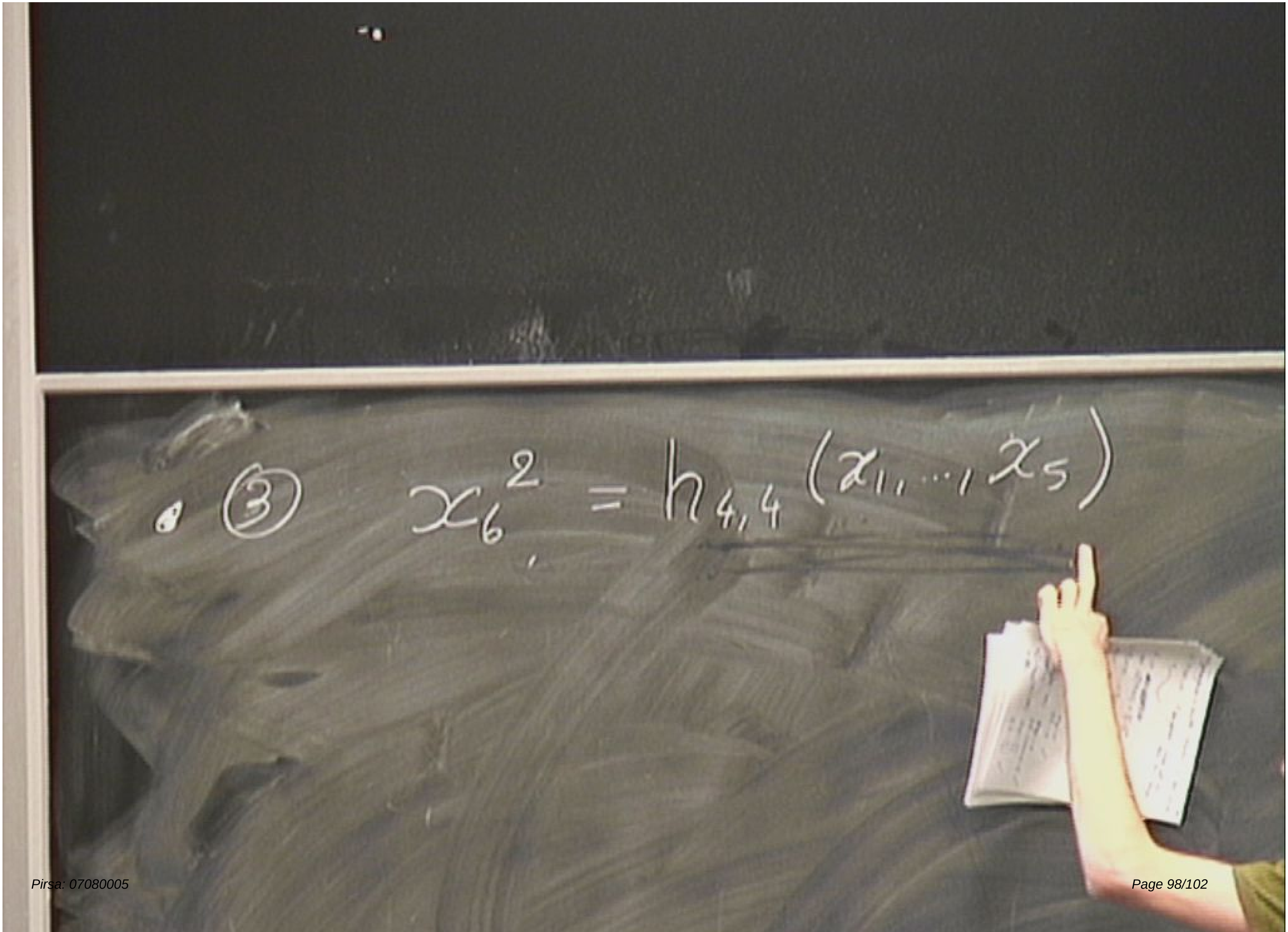
① Quintic in $\mathbb{CP}^4$ $P_5(X) =$

$Q(eq) = 5$ $\quad \sum_i Q_i = 5$ ✓

② $x_5^2 = h_8(x_1, \dots, x_4)$ $\quad$ (e.g. $= \sum_{i=1}^{4} x$

$Q(eq) = 8 = Q(\text{fields})$

C.Y. ✓



(3) $x_6^2 = h_{4,4}(x_1, \dots, x_5)$

③ $x_6^2 = h_{4,4}(x_1, \ldots, x_5)$ in $\mathbb{P}\Delta_1$

$Q(eq) = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ $\sum_i Q_i = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

$\Rightarrow$ CY ✓ CY $\mathbb{P}\Delta_1$

$\Rightarrow$ CY ✓ CY $\not{P}\,\not{\Sigma}$ |

PROB. M built out of x_i $i = 1, \ldots, n$
w. charges Q_i^{a}
$\rightarrow$ find charge assignment for x_{n+1}
(extra field) such that eq.

$\rightarrow$ CY ✓ CY ~~B~~ ~~Σ~~

PROB: M built out of x_i $i = 1, \ldots, n$

w. charges Q_i^a

$\rightarrow$ find charge assignment for x_{n+1} (extra field) such that eq. of form

$$x_{n+1}^2 = h(x_1, \ldots, x_n)$$

$\Rightarrow$ CY ✓ CY ~~PS~~

PROB. $\mathcal{M}$ built out of x_i $i = 1, \ldots, n$

w. charges Q_i^a

$\rightarrow$ find charge assignment for x_{n+1} (extra field) such that eq. of form

$$x_{n+1}^2 = h(x_1, \ldots, x_n)$$

defines a C.Y.