

Title: Status of the Standard Model

Date: Aug 07, 2007 09:00 AM

URL: <http://pirsa.org/07080004>

Abstract:

MCTP-07-27

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Lectures at the Particle Physics, Cosmology and Strings
Summer School at Perimeter Institute (August 2007)

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Standard Model

The definition of the Standard Model

- $SU(3) \times SU(2)_L \times U(1)_Y$ gauge symmetry with appropriate gauge coupling strengths
- Three generations of quarks, leptons and neutrinos with appropriate masses and mixing angles
- Higgs boson doublet that breaks gauge symmetry to $SU(3) \times U(1)_Y$

Validity and Completeness of a theory

Particle physics aspires to a *valid* and *complete* understanding of the basic, subatomic laws of nature.

Valid: No theory implication/prediction conflicts with observations.

Complete: All observations are explained by the theory.

A theory can be *valid* without being *complete*, and a theory can aspire to *completeness* yet ultimately not be *valid*.

Nonscientists appear to allow *completeness* to trump any concerns of *validity*. **Scientists** generally value *validity* more, but are ever mindful of *completeness* concerns of a theory.

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Standard Model is not Complete

Fundamental Theorem of Particle Physics Research:
The Standard Model of particle physics is *not complete*, and it is this incompleteness that motivates almost all particle physicists to do particle physics.

How is the Standard Model incomplete? Indirectly, the Standard Model is incomplete because we do not know several important why questions that cannot be answered within the theory.

Incomplete Standard Model: Some Why Questions

- Why three generations?
- Why are there large hierarchies in the quark and lepton masses and mixings?
- Why are neutrino masses so much smaller?
- Why does electroweak symmetry break?
- Why is the strong CP violating angle so small?
- Why is the electroweak scale so much smaller than the Planck scale?
- Why is Quantum Mechanics valid?
- Why are there three spatial dimensions and one time dimension?
- Why is time so different?
- Why do the gauge couplings have their values, and appear to merge at high energy?
- . . .

Incomplete Standard Model: Some Direct Concerns

And there are more direct indications of the incompleteness of the SM:

- Dark Matter of the Universe cannot be explained by SM states or dynamics
- Baryon asymmetry of the universe cannot be explained by SM states or dynamics
- Electroweak gauge couplings diverge at high scales (triviality concerns)
- others?

The drive toward more completeness has lead to many outstanding ideas in particle physics: supersymmetry, string theory, technicolor, extra dimensions, etc.

However, we will not concern ourselves with completeness issues. Many other lectures devoted to that. Here we focus on the *validity of the Standard Model*.

Validity Tests of the Standard Model

There are three main directions to go to test the validity of the Standard Model

- Rare/forbidden events ($\mu \rightarrow e\gamma$, p decay, etc.)
- Precision tests (Γ_Z , m_W/m_Z , etc.)
- Direct tests ($d\sigma/dm_{l^+l^-}$, $e^+e^- \rightarrow HZ$, etc.)

We shall forgo the first method of rare/forbidden events, partly because of time, and partly because it is a more model-dependent exercise (how does one test for p decay without postulating how it decays?).

Focus on Precision tests and direct tests of the Standard Model.

Focus on the Higgs boson

When discussing tests of the SM there is no single issue more important than the Higgs boson. There is no direct evidence for it. The SM is a *speculative theory*, like all other theories we have that involve electroweak symmetry breaking. Our tests of the SM discussion will have particular emphasis on testing the validity of the single Higgs boson assumption.

In other words, our primary interest to focus on observables sensitivity to the existence of the SM Higgs boson.

This leaves us with two main areas of discussion: **precision electroweak analysis** and **Higgs hunting**.

But first, a quick review of the SM Higgs sector.

Standard Model Higgs Sector

The SM posits an $SU(2)_L$ doublet with hypercharge $Y = 1/2$.
The Higgs Potential is

$$V_H = m^2 |H|^2 + \lambda |H|^4$$

m^2 is assumed negative, and λ positive, such that Higgs gets a vev, breaking electroweak symmetry. Three degrees of freedom of H are absorbed as longitudinal components of W^\pm and Z , and one real physical state is left behind, h .

The lagrangian of h interacting with itself and SM gauge fields and fermions is

$$\begin{aligned} \mathcal{L} = & -\frac{m_h^2}{2} h^2 - \frac{\mu}{3!} h^3 - \frac{\eta}{4!} h^4 \\ & + \left[m_W^2 W_\mu W^\mu + \frac{m_Z^2}{2} Z_\mu Z^\mu \right] \left(1 + \frac{h}{v} \right)^2 - m_f \bar{f} f \left(1 + \frac{h}{v} \right) \end{aligned}$$

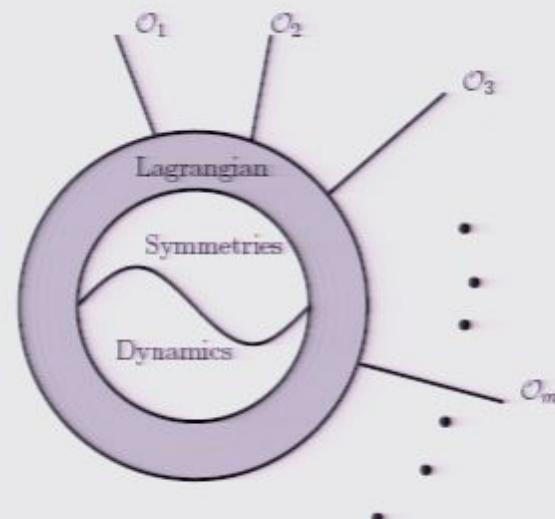
where $v = \sqrt{2}\langle H \rangle \simeq 246 \text{ GeV}$, $m_h = 2\lambda v^2$, $\mu = 3m_h^2/v$, and $\eta = 6\lambda$.

This lagrangian dictates all relevant Feynman rules we will need for the Higgs boson.

Why the SM is better than the Trivial Model

Finding a theory that matches observables is not hard at all. Give me any set of n observables $\{\mathcal{O}_i\}$ and I can give you this theory: For every \mathcal{O}_i we posit the reason \mathcal{R}_i , which simply states that \mathcal{O}_i is true.

Why is the SM better than this Trivial Model? The interrelations of symmetries and dynamics. The Lagrangian is the (usual) tool by which we can turn these into observables.



Observables in terms of Observables I

If there are n free parameters of the theory, and m computable and measurable observables, then we have $m - n$ predictions to test the theory (if observables are sufficiently “independent”):

$$\mathcal{O}_1, \mathcal{O}_2, \dots, \mathcal{O}_n$$

which can be computed from the n parameters $\{P_i\}$

$$\mathcal{O}_i^{\text{expt}} = \mathcal{O}_i^{\text{th}}(P_1, P_2, \dots, P_n)$$

These equations then can be inverted to obtain the parameters in terms of observables

$$P_i = F_i(\mathcal{O}_1^{\text{expt}}, \mathcal{O}_2^{\text{expt}}, \dots, \mathcal{O}_n^{\text{expt}}).$$

(Ignoring potential degeneracy issues.)

Now, for the remaining observables

$$\mathcal{O}_{n+1}, \mathcal{O}_{n+2}, \dots, \mathcal{O}_m$$

we have the unambiguous predictions

$$\mathcal{O}_{n+j}^{\text{th}}(P_1, P_2, \dots, P_n).$$

Observables in terms of Observables II

Actually, we have expressed observables in terms of observables:

$$\mathcal{O}_{n+j}^{\text{th}} = \mathcal{O}_{n+j}^{\text{th}}(F_1(\vec{\mathcal{O}}^{\text{expt}}), F_2(\vec{\mathcal{O}}^{\text{expt}}), \dots, F_n(\vec{\mathcal{O}}^{\text{expt}}))$$

In practice, doing this analytically can be hard, and a χ^2 analysis is conducted, where

$$\chi^2 = \sum_i \frac{(\mathcal{O}_i^{\text{expt}} - \mathcal{O}_i^{\text{th}}(\vec{P}))^2}{(\Delta \mathcal{O}_i^{\text{expt}})^2}.$$

One lets all the parameters \vec{P} vary until the best χ^2 value is obtained. If $\chi^2/\text{d.o.f.} \lesssim 1$ the theory is compatible with the experimental measurements.

Nevertheless, in these lectures I will emphasize “observables in terms of observables” because it is pedagogically important and because it is possible to do this analytically in the examples I present.

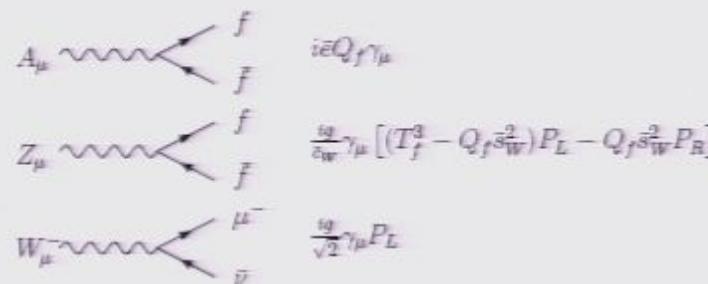
Tree-Level Analysis of SM

We analyze the EW theory at tree level, which can be described by three parameters (in addition to the fermion masses):

$$\mathcal{L} = \mathcal{L}(g, g', v)$$

where g is $SU(2)_L$ gauge coupling, g' is $U(1)_Y$ gauge coupling and v is the vev of the Higgs field.

The applicable feynman rules are



where \bar{c}_W , \bar{s}_W and \bar{e} are merely short-hand expressions for combinations of Lagrangian parameters:

$$\bar{e} \equiv \frac{gg'}{\sqrt{g^2 + g'^2}}, \quad \bar{c}_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad \bar{s}_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

Tree-Level Observables

Exchanging $\{g, g', v\}$ set for $\{e, s, v\}$ one finds

$$\begin{aligned}\hat{\alpha} &= \frac{e^2}{4\pi} \text{ Coulomb potential} \\ \hat{G}_F &= \frac{1}{\sqrt{2}v} \text{ muon decay} \\ \hat{m}_Z^2 &= \frac{e^2 v^2}{4s^2 c^2} \\ \hat{m}_W^2 &= \frac{e^2 v^2}{4s^2} \\ \hat{s}_{\text{eff}}^2 &= s^2 \\ \hat{\Gamma}_{l^+l^-} &= \frac{v}{96\pi s^3 c^3} \left[\left(-\frac{1}{2} + 2s^2 \right)^2 + \frac{1}{4} \right]\end{aligned}$$

The LHS are all observables/measurements.

The RHS are all theory predictions in terms of $\{e, s, v\}$.

s_{eff}^2 is determined from

$$\hat{A}_{LR} = \frac{\Gamma(Z \rightarrow e_L^+ e_L^-) - \Gamma(Z \rightarrow e_R^+ e_R^-)}{\Gamma(Z \rightarrow e_L^+ e_L^-) + \Gamma(Z \rightarrow e_R^+ e_R^-)} = \frac{(1/2 - \hat{s}_{\text{eff}}^2)^2 - \hat{s}_{\text{eff}}^4}{(1/2 - \hat{s}_{\text{eff}}^2)^2 + \hat{s}_{\text{eff}}^4}$$

Measurements

$$\begin{aligned}\hat{\alpha} &= 1/137.035099911(46) \text{ (PDG)} \\ \hat{G}_F &= 1.16637(1) \times 10^{-5} \text{ (PDG)} \\ \hat{m}_Z^2 &= 91.1875 \pm 0.0021 \text{ GeV (LEP)} \\ \hat{m}_W^2 &= 80.410 \pm 0.032 \text{ GeV (LEP, Tevatron)} \\ \hat{s}_{\text{eff}}^2 &= 0.23098 \pm 0.00027 \text{ (SLAC)} \\ \hat{\Gamma}_{l^+l^-} &= 83.989 \pm 0.100 \text{ MeV (LEP)}\end{aligned}$$

$\Gamma_{l^+l^-}$ obtained from Γ_Z , $R_l = \Gamma_{\text{had}}/\Gamma_l$ and σ_{had} at LEP:

$$\begin{aligned}\sigma_{\text{had}} &= \frac{12\pi\Gamma_{\text{had}}\Gamma_l}{m_Z^2 - \Gamma_Z^2} = \frac{12\pi}{m_Z^2} R_l \frac{\Gamma_l^2}{\Gamma_Z^2} \\ &\implies \Gamma_l = \hat{m}_Z \hat{\Gamma}_Z \sqrt{\frac{\hat{\sigma}_{\text{had}}}{12\pi \hat{R}_l}}\end{aligned}$$

These are six measurements to be explained by only three parameters.

Pause for a Question

Does the Standard Model predict the correct W mass?

This is mostly a vacuous question in isolation. The answer is yes.

$$\hat{m}_W^2 = \frac{1e^2v^2}{4\ s^2}$$

An infinite number of combinations of e , s and v can solve this equation.

A more meaningful question: *Can the free parameters of the Standard Model be adjusted to predict all the observables compatible with observations?*

This question is to be answered by a χ^2 analysis

$$\chi^2(e, s, v) = \sum_{i=1}^6 \frac{(\mathcal{O}_i^{\text{expt}} - \mathcal{O}_i^{\text{th}}(e, s, v))^2}{(\Delta \mathcal{O}_i^{\text{expt}})^2}$$

where the three parameters are varied to see if values can be found that match all six observables under consideration.

A Simplified Analysis I

Of the six observables under consideration, historically three were measured extraordinarily well (α , G_F and m_Z) and served as inputs to the predictions of other observables. Let's do that here.

Inverting the lagrangian parameters and three well-measured observables, one finds

$$\begin{aligned} e^2 &= 4\pi\hat{\alpha} \\ v^2 &= \frac{\hat{G}_F^{-1}}{\sqrt{2}} \\ s^2 &= \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4\hat{x}^2} \quad \text{where } \hat{x} = \frac{\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F m_Z^2} \end{aligned}$$

A Simplified Analysis II

Now, use these to rewrite the remaining observables in terms of these first three:

$$\begin{aligned}\hat{m}_W^2 &= \pi\sqrt{2}\hat{G}_F^{-1}\hat{\alpha} \left(1 - \sqrt{1 - \frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}\right)^{-1} \\ \hat{s}_{\text{eff}}^2 &= \frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}} \\ \hat{\Gamma}_{l^+l^-} &= \frac{\sqrt{2}\hat{G}_F\hat{m}_Z^3}{12\pi} \left\{ \left(\frac{1}{2} - \sqrt{1 - \frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}\right)^2 + \frac{1}{4} \right\}\end{aligned}$$

Substitute in experimental values to make predictions:

	Prediction	Deviation
\hat{m}_W	$80.939 \pm 0.003 \text{ GeV}$	$\sim 17\sigma$
\hat{s}_{eff}^2	0.21215 ± 0.00003	$\sim 70\sigma$
$\hat{\Gamma}_{l^+l^-}$	$84.834 \pm 0.012 \text{ MeV}$	$\sim 8\sigma$

Is Standard Model ruled out?

Of course not. But, we do not know that for sure without doing the next order computation. We can suspect that there are large corrections (compared to what? to experimental uncertainty) to the previous analysis.

$$\frac{\Delta \mathcal{O}}{\mathcal{O}} \sim \frac{g^2}{4\pi^2} \sim 1.2\%$$

and

$$\frac{m_W^{\text{tree}} - \hat{m}_W}{\hat{m}_W} \sim 0.7\%$$

$$\frac{\hat{s}_{\text{eff}}^2 - s_{\text{eff,tree}}^2}{\hat{s}_{\text{eff}}^2} \sim 8.2\%$$

$$\frac{\Gamma_{l^+l^-}^{\text{tree}} - \hat{\Gamma}_{l^+l^-}}{\hat{\Gamma}_{l^+l^-}} \sim 1.0\%$$

Why is s_{eff}^2 deviation so large? Very sensitive to α corrections

$$\frac{\Delta \alpha}{\alpha} \simeq \frac{1/129 - 1/137}{1/137} = 6.2\%$$

Methods of establishing Standard Model

In this lecture we focus on the class of corrections that arise solely from the self-energy corrections of the γ , W^\pm , and Z vector bosons. Restricting our analysis to this class of corrections enables us to do something complete and meaningful in the short time we have together.

A full-scale renormalization of the SM with all corrections explicitly calculated is a significantly more time-consuming project without significantly enhancing the conceptual learning.

Vector-Boson Self-Energies

By convention the one-loop corrections to the vector boson self-energies



is of the form

$$i[\Pi_{VV'}(q^2)g^{\mu\nu} - \Delta_{VV'}(q^2)q^\mu q^\nu].$$

Only the $\Pi_{VV'}$ piece of the self-energies matters for our analysis since the q^μ part of the second term is dotted into a light-fermion current and is zero by the Dirac equation, since the corresponding fermion masses is well-approximated to be zero:

$$q^\mu J_\mu^{\text{light fermion}} \rightarrow \bar{f} \gamma^\mu q_\mu f \rightarrow \bar{f} m f \rightarrow 0.$$

The way the self-energies are defined, they add to the vector boson masses by convention:

$$m_V^2 \rightarrow m_V^2 + \Pi_{VV}(q^2 = m_V^2)$$

Photon Self-Energies

Because the photon is massless we know that $\Pi_{\gamma\gamma}(0) = 0$ and $\Pi_{\gamma Z}(0) = 0$, and so we do not have to compute them.

Caveat: There is one subtlety to keep in mind. $\Pi_{\gamma Z}(0)$ is not zero when the W^\pm bosons are included in the loop. This is special to the W^\pm bosons (gauge degree of freedom partners of the W^3). In new physics scenarios (e.g., supersymmetry) there are no additional one-loop contributions to $\Pi_{\gamma Z}(0)$, and it is usually appropriate in analyses of beyond-the-SM contributions to precision EW observables to ignore it.

Our results when we are done will be of direct use for oblique analysis of physics beyond the SM, but as for SM a more complete analysis is needed, including taking into account vertex corrections.

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Z and W Masses

The computation of the Z and W masses is straightforward. The resulting theoretical prediction of m_Z and m_W in terms of the lagrangian parameters and the one-loop self-energy corrections is

$$(\hat{m}_Z)^{\text{th}} = \frac{e^2 v^2}{4 s^2 c^2} + \Pi_{ZZ}(m_Z^2)$$

$$(\hat{m}_W)^{\text{th}} = \frac{e^2 v^2}{4 s^2} + \Pi_{WW}(m_W^2)$$

Computing α

We next compute the theory prediction for α . It sounds odd to use the words “theory prediction of α ” since we often are sloppy in our wording (or thinking) and view α as just a coupling. In reality, it is an observable defined in the Thomson limit of Compton scattering and probes the Coulomb potential at $q^2 \rightarrow 0$:



which is proportional to

$$-i \frac{4\pi\hat{\alpha}}{q^2} \Big|_{q^2=0} = \frac{-ie^2}{q^2} \left[1 + \frac{\Pi_{\gamma\gamma}(q^2)}{q^2} \right]_{q^2=0}$$

If we define

$$\Pi'_{\gamma\gamma}(0) \equiv \lim_{q^2 \rightarrow 0} \frac{\Pi_{\gamma\gamma}(q^2)}{q^2}$$

then we can write the theory prediction for α as

$$(\hat{\alpha})^{\text{th}} = \frac{e^2}{4\pi} (1 + \Pi'_{\gamma\gamma}(0))$$

Muon Decay

The muon decay observable \hat{G}_F is computed from the lifetime of the muon



which is proportional to $\hat{G}_F/\sqrt{2}$. This amplitude is then used to compute the muon lifetime

$$\tau_\mu^{-1} = \frac{\hat{G}_F^2 m_\mu^5}{192\pi^3} K(\alpha, m_e, m_\mu, m_W)$$

where the function K is mainly a kinematics function and can be obtained from the electroweak chapter in the PDG. The theory prediction for \hat{G}_F is

$$\begin{aligned} \frac{(\hat{G}_F)^{\text{th}}}{\sqrt{2}} &= \frac{g^2}{8m_W^2} \left[1 + i\Pi_{WW}(q^2) \left(\frac{-i}{q^2 - m_W^2} \right) \right]_{q \rightarrow 0} \\ &= \frac{1}{2v^2} \left[1 - \frac{\Pi_{WW}(0)}{m_W^2} \right]. \end{aligned}$$

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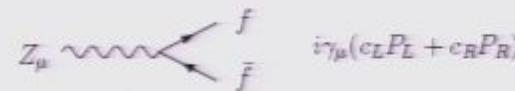
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Computation of \hat{s}_{eff}^2 (I)

The observable associated with \hat{s}_{eff}^2 is a little trickier than the other ones. For one, there are many different types of \hat{s}_{eff}^2 observables, depending on the final state fermion. We have defined \hat{s}_{eff}^2 to be the observable associated with the left-right asymmetry of Z decays to leptons. We assume universality of the leptons.

$$A_{LR}^l = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \equiv \frac{c_L^2 - c_R^2}{c_L^2 + c_R^2}$$

where at tree-level the c_L and c_R couplings are defined by



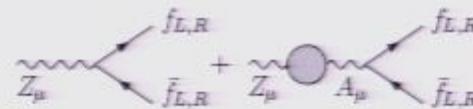
and

$$c_L = \frac{e}{sc}(T^3 - Qs^2) \quad \text{and} \quad c_R = -\frac{-eQs^2}{sc}$$

Computation of \hat{s}_{eff}^2 (II)

The definition of \hat{s}_{eff}^2 is chosen such that observable \hat{A}_{LR}^l is written in terms of \hat{s}_{eff}^2 using the tree-level expression above with $s^2 \rightarrow \hat{s}_{\text{eff}}^2$.

Compute the one-loop shifts in c_L and c_R . Neglect all Π_{ZZ} contributions since they will only affect the overall factor of c_L and c_R which cancels. On the other hand, the $Z - A$ mixing self-energy does contribute to the c_L and c_R couplings:



where

$$\begin{aligned} c_L &= \frac{e}{sc} (T^3 - Qs^2) + i\Pi_{\gamma Z}(m_Z^2) \left(\frac{-i}{m_Z^2} \right) (eQ) \\ &= \frac{e}{sc} \left[T^3 - Q \left(s^2 - sc \frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2} \right) \right] \end{aligned}$$

$$\begin{aligned} c_R &= \frac{-eQs^2}{sc} + i\Pi_{\gamma Z}(m_Z^2) \left(\frac{-i}{m_Z^2} \right) (eQ) \\ &= -\frac{eQ}{sc} \left[s^2 - sc \frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2} \right]. \end{aligned}$$

Computation of \hat{s}_{eff}^2 (III)

The above c_L and c_R expressions are exactly the same as the tree-level expressions except $s^2 \rightarrow s^2 - sc\Pi_{\gamma Z}(m_Z^2)/m_Z^2$ in the numerator. Thus, at the Z -pole

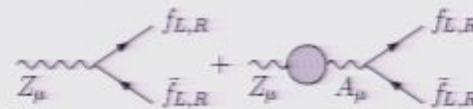
$$(\hat{s}_{\text{eff}}^2)^{\text{th}} = s^2 - sc \frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2}$$

$$\text{where } \hat{A}_{LR} = \frac{(1/2 - \hat{s}_{\text{eff}}^2)^2 - (\hat{s}_{\text{eff}}^2)^2}{(1/2 - \hat{s}_{\text{eff}}^2)^2 + (\hat{s}_{\text{eff}}^2)^2}.$$

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$$\begin{aligned} c_R &= \frac{-eQs^2}{sc} + i\Pi_{\gamma Z}(m_Z^2) \left(\frac{-i}{m_Z^2} \right) (eQ) \\ &= -\frac{eQ}{sc} \left[s^2 - sc \frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2} \right]. \end{aligned}$$

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$$(\hat{s}_{\text{eff}}^2)^{\text{th}} = s^2 - sc \frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2}$$

$$\text{where } \hat{A}_{LR} = \frac{(1/2 - \hat{s}_{\text{eff}}^2)^2 - (\hat{s}_{\text{eff}}^2)^2}{(1/2 - \hat{s}_{\text{eff}}^2)^2 + (\hat{s}_{\text{eff}}^2)^2}.$$

Computation of $\hat{\Gamma}_{l^+l^-}$ (I)

Now we compute $\hat{\Gamma}_{l^+l^-}$ from



The theoretical prediction for this observable in terms of independent lagrangian parameters and one-loop self-energies is

$$(\hat{\Gamma}_{l^+l^-})^{\text{th}} = \frac{Z_Z}{48\pi s^2 c^2} \hat{m}_Z \left[\left(-\frac{1}{2} + 2(\hat{s}_{\text{eff}}^2)^{\text{th}} \right)^2 + \frac{1}{4} \right]$$

Recall that $\Pi_{\gamma Z}$ had the effect of just putting $s^2 \rightarrow (\hat{s}_{\text{eff}}^2)^{\text{th}}$ into the numerator of the c_L and c_R expressions. The \hat{m}_Z comes as a kinematical phase space mass of the Z decay.

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Computation of $\hat{\Gamma}_{l+l^-}$ (II)

Z_Z is a wavefunction residue piece. To compute it, start with $\Pi_{ZZ}(q^2)$, a self-energy that when resummed affects the Z boson propagator in a simple way

$$\text{Resummed Propagator} \longrightarrow P_Z^{\mu\nu}(q^2) = \frac{-ig^{\mu\nu}}{q^2 - m_Z^2 - \Pi_{ZZ}(q^2)}.$$

But,

$$\Pi_{ZZ}(q^2) = \Pi_{ZZ}(m_{\text{phys}}^2) + \Pi'_{ZZ}(m_{\text{phys}}^2)(q^2 - m_{\text{phys}}^2) + \dots$$

The mass of the Z is defined to be the position of the real part of the pole of the propagator. In the neighborhood of $q^2 = m_{\text{phys}}^2$

$$\begin{aligned} q^2 - m_Z^2 - \Pi_{ZZ}(q^2) &= q^2 - m_Z^2 - \Pi_{ZZ}(m_{\text{phys}}^2) - \Pi'_{ZZ}(m_{\text{phys}}^2)(q^2 - m_{\text{phys}}^2) + \dots \\ &= (q^2 - m_{\text{phys}}^2)(1 - \Pi'_{ZZ}(m_{\text{phys}}^2)) + \dots \end{aligned}$$

Therefore, in the neighborhood of $q^2 = m_{\text{phys}}^2$ the Z propagator can be written as

$$\frac{-ig^{\mu\nu}}{(q^2 - m_{\text{phys}}^2)(1 - \Pi'_{ZZ}(m_{\text{phys}}^2))} = \frac{-iZ_Z g^{\mu\nu}}{(q^2 - m_{\text{phys}}^2)}$$

where

$$Z_Z = 1 + \Pi'_{ZZ}(\hat{m}_Z) + \text{higher order terms}$$

Computation of $\hat{\Gamma}_{l^+l^-}$ (III)

At this point, we have all factors needed to compute $\hat{\Gamma}_{l^+l^-}$.

Keep in mind a standard approximation for $\Pi'_{ZZ}(m_Z^2)$:

$$\Pi'_{ZZ}(m_Z^2) = \frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2}$$

although it is not needed given the many good numerical tools available now. Nevertheless, I will use it at times.

Sometimes I will also utilize the variable δ_Z which is defined as $Z_Z = 1 + \delta_Z$, where

$$\delta_Z = \Pi'_{ZZ}(m_Z^2) \simeq \frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2} = \frac{\Pi_{ZZ}(m_Z^2)}{m_Z^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2}$$

Reflection Pause

Where we are at: We now have written all of our observables in terms of lagrangian parameters and Π functions (one-loop corrections).

Usual next step: Construct χ^2 function of all relevant observables in terms of the input parameters, and fit.

Our next step: Let's now do analytic inversions write parameters in terms of observables, and ultimately observables in terms of observables. Often this step is not possible in practice to do analytically. Works here due to relative noncomplexity of one-loop self-energy corrections.

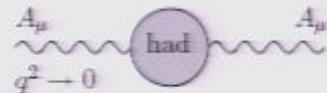
Why this analytic step? We will see infinities cancel automatically when we write observables in terms of observables, and perhaps provide a different perspective about the infinities that supposedly afflict our theories.

The troublesome case of $\hat{\alpha}$ (I)

Before we do those calculations, we need to say a few more things about the $\hat{\alpha}$ observable. It is an unusual observable among our list, because it is obviously incalculable. Recall from before that we found

$$e^2 = \frac{4\pi\hat{\alpha}}{1 + \Pi'_{\gamma\gamma}(0)}$$

The problem is with $\Pi'_{\gamma\gamma}(0)$, which requires us to know the result of the photon self energy as $q^2 \rightarrow 0$:



Of course we know from the beginning of this section that

$$\Pi_{\gamma\gamma}(q^2) \rightarrow q^2 B \text{ as } q^2 \rightarrow 0,$$

where B is some constant. There is no reason for B to be zero, and so there is no reason for the derivative of the self-energy $\Pi'_{\gamma\gamma}(0) \rightarrow B$ to be zero. Unfortunately, however, it is not calculable.

The troublesome case of $\hat{\alpha}$ (II)

The incalculability of $\Pi'_{\gamma\gamma}(0)$ threatens to derail our precision electroweak analysis. However, it has been known for some time now that we can get at this value by using a combination of theory tricks and experimental data. The first thing we do is to rewrite $\Pi'_{\gamma\gamma}(0)$ by adding and subtracting the self-energy at the higher scale $q^2 = m_Z^2$:

$$\Pi'_{\gamma\gamma}(0) = \text{Re} \frac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2} - \left[\frac{\text{Re} \Pi_{\gamma\gamma}}{m_Z^2} - \Pi'_{\gamma\gamma}(0) \right]$$

The first term is calculable as computations are done at the scale $q^2 = m_Z^2$ where all interactions are perturbative in the SM. The two terms in the bracket are not calculable, but we will give it a name $\Delta\alpha(m_Z)$. There are three main contributions to $\Delta\alpha(m_Z)$:

$$\Delta\alpha(m_Z) = \Delta\alpha_l(m_Z) + \Delta\alpha_{\text{top}}(m_Z) + \Delta\alpha_{\text{had}}^{(5)}(m_Z)$$

where

$$\Delta\alpha_l(m_Z) = 0.03150 \text{ with essentially no error}$$

$$\Delta\alpha_{\text{top}}(m_Z) = -0.0007(1) \text{ } m_t \text{ dependent but negligible}$$

$$\Delta\alpha_{\text{had}}^{(5)} = \text{incalculable light hadrons contributions}$$

The troublesome case of $\hat{\alpha}$ (III)

Fortunately, there is a way to measure $\Delta\alpha_{\text{had}}^{(5)}$. From the optical theorem and the methods of analytic continuation, one finds that

$$\Delta\alpha_{\text{had}}^{(5)} = -\frac{\alpha m_Z^2}{3\pi} \int_{4m_\pi^2}^\infty \frac{R_{\text{had}}(q^2) dq^2}{q^2(q^2 - m_Z^2)} \quad \text{where}$$

$$R_{\text{had}}(q^2) = \frac{\sigma_{\text{had}}(q^2)}{\sigma_{l+l-}(q^2)}.$$

Therefore, to get a numerical value for $\Delta\alpha_{\text{had}}^{(5)}$ one must integrate over the experimental hadronic cross-section over a wide energy range.

As soon as q^2 is significantly above Λ_{QCD} the theoretical cross-section can be used without concern. However, for lower q^2 (lower than about 5 GeV in practice), only the experimental data can be used.

There are numerous experiments that contribute data for this integral in differing energy bins, and it is a challenge to understand all the systematics and statistical errors that go into the final number for $\Delta\alpha_{\text{had}}^{(5)}$.

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Experimental Determination of $\Delta\alpha_{\text{had}}^{(5)}$

Many groups have gone through this difficult exercise and there are many different values obtained. The one the LEP Electroweak Working Group has been using is by Burkhardt and Pietrzyk, who conclude that

$$\Delta\alpha_{\text{had}}^{(5)} = 0.02761 \pm 0.0036.$$

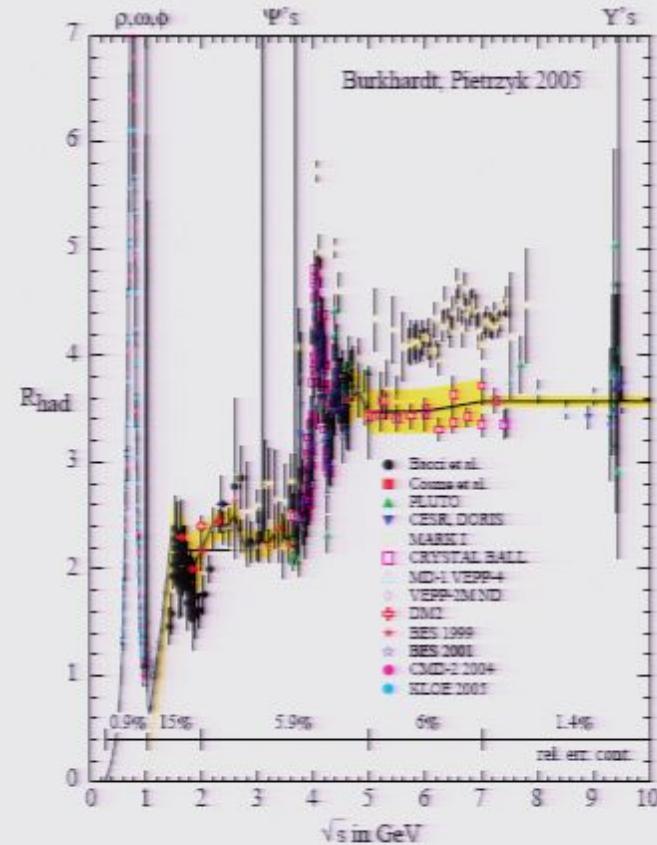
We will now trade in the incalculable $\hat{\alpha}$ for the calculable/measured $\hat{\alpha}(m_Z)$, which is related to the lagrangian parameters and Π 's by

$$\hat{\alpha}(m_Z) = \frac{\hat{\alpha}}{1 - \Delta\alpha(m_Z)} = \frac{e^2}{4\pi} \left[1 + \frac{\Pi_{\gamma\gamma}(m_Z)}{m_Z^2} \right]$$

Always remember, $\hat{\alpha}(m_Z)$ is an observable, which is a meaningful combination of many different experiments (Thomson scattering cross-section plus integration over $R_{\text{had}}(q^2)$), and its experimental value is

$$\frac{1}{\hat{\alpha}(m_Z)} = 128.936 \pm 0.046.$$

Update of $\Delta\alpha$ data scan



Burkhardt, Pietrzyk, hep-ph/0506323