Title: Quantum Information, Entanglement and Nonlocality - EinsteinPlus Keynote Session

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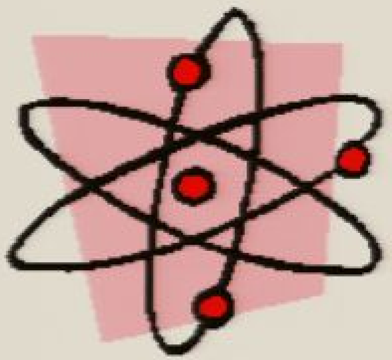
Abstract: One simple way to think about physics is in terms of information. We gain information about physical systems by observing them, and with luck this data allows us to predict what they will do next. Quantum mechanics doesn\'t just change the rules about how physical objects behave - it changes the rules about how information behaves. In this talk we explore what quantum information is, and how strangely it differs from our intuitions. In particular we see how information about quantum particles can become entangled, leading to seemingly impossibly coordinated behaviour for separate objects.

Quantum Information, Entanglement and Nonlocality



PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Physical Information

Quantum theory is *not* a fully formed model of the physical world.

It is a framework – a mathematical structure we can colour in with whatever physics we need at for any situation.

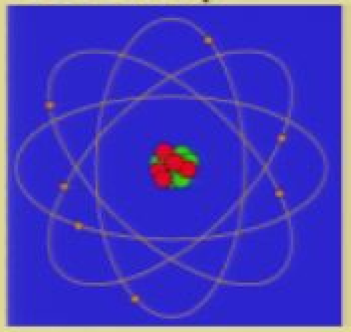
It tells us something remarkable:

Every physical system follows identical rules of behaviour.

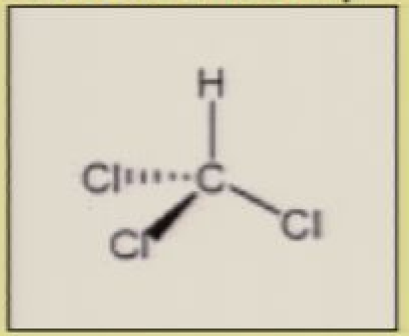
Photons,

Electrons,

Atoms,



Chloroform,



Buckyballs.

Physical Information

Every physical system follows the same, counterintuitive rules.

Quantum information theory is the study of these rules.

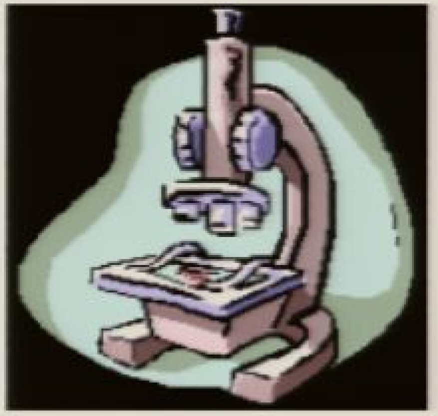
What do these rules let us do?

- Quantum Cryptography
- Quantum Computing

What do these systems know that we don't?

Why Information?

As physicists, we're interested in getting information about the world around us – we want to *observe* the world:



Where? When? How heavy? How hot? How charged? How fast?...

Every measurement we make, every observation we take, is an interaction with a physical system to extract information from it.

Why Information?

Classically, we think of properties like ‘position’ and ‘momentum’ as the simple, fundamental building blocks of the world.

Information-wise, a position measurement is a huge undertaking.

With better and better equipment, position measurements yield *arbitrarily large* amounts of information.

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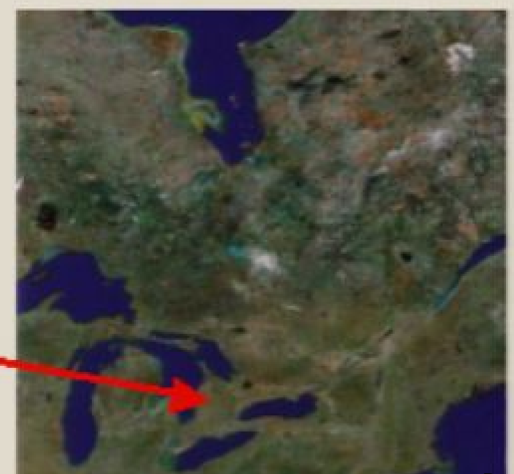
80 W



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Bits and Qubits

What's the simplest bit of physical information we can talk about?

The answer to a binary question:

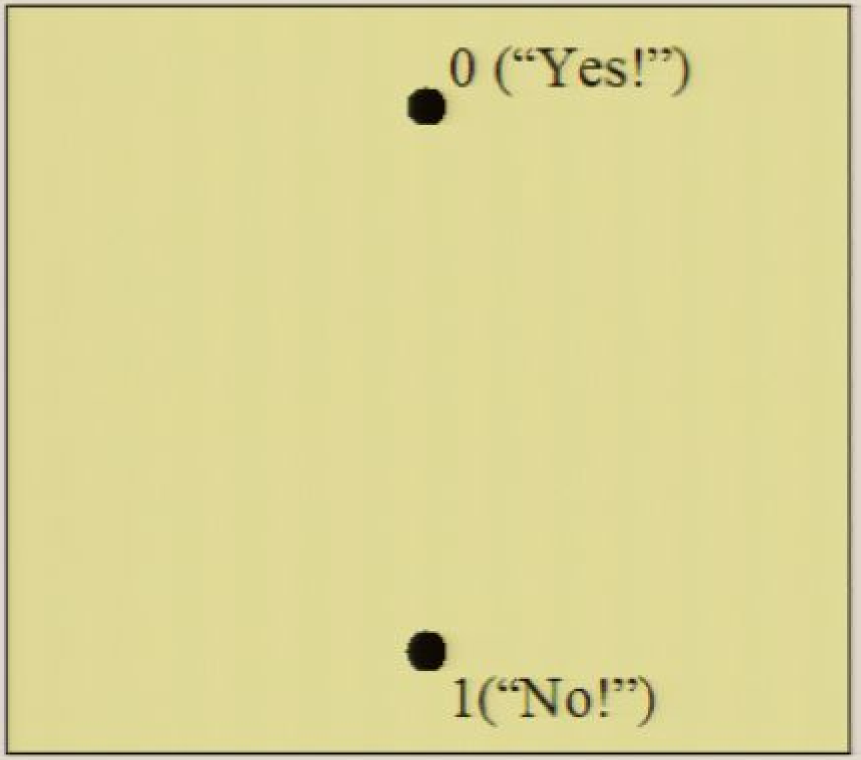
> "Is the atom in its ground state, or not?"
>
> "Is there a photon there, or not?"
>
> "Is the cat dead, or not?"

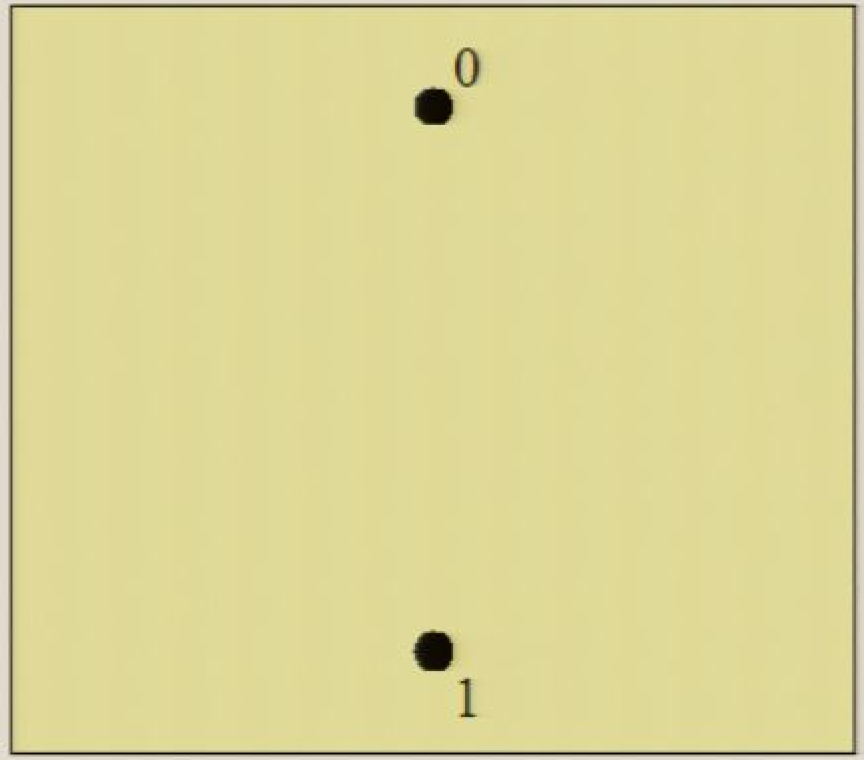
There are only two possible answers to these questions:

> **Yes** or **No.**

This is the key kind of question to ask. Every other question we can ask of the physical world can be built up out of these.

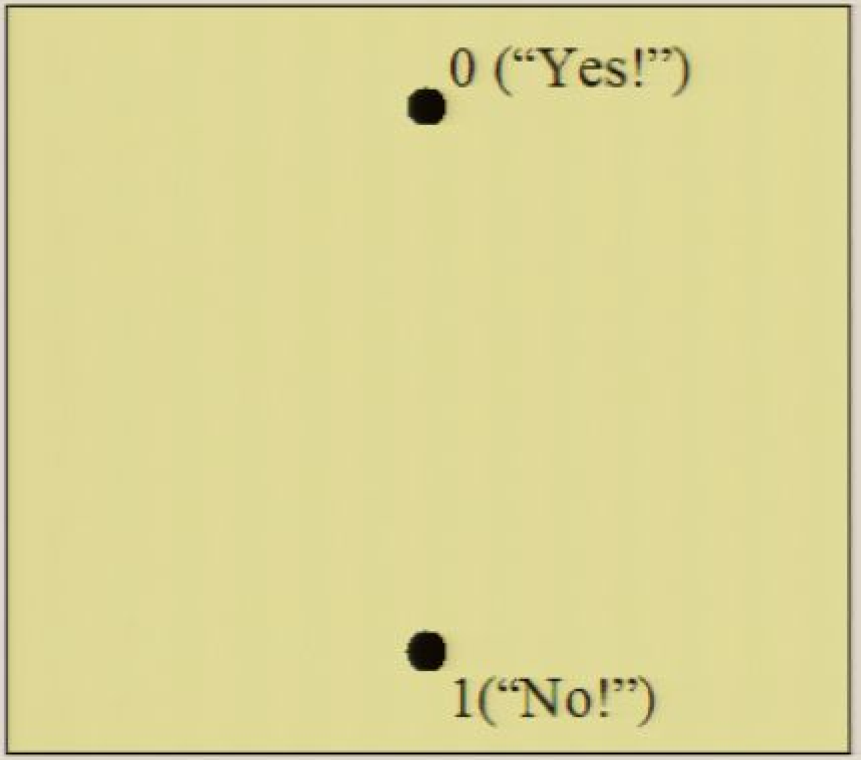
Bits and Qubits

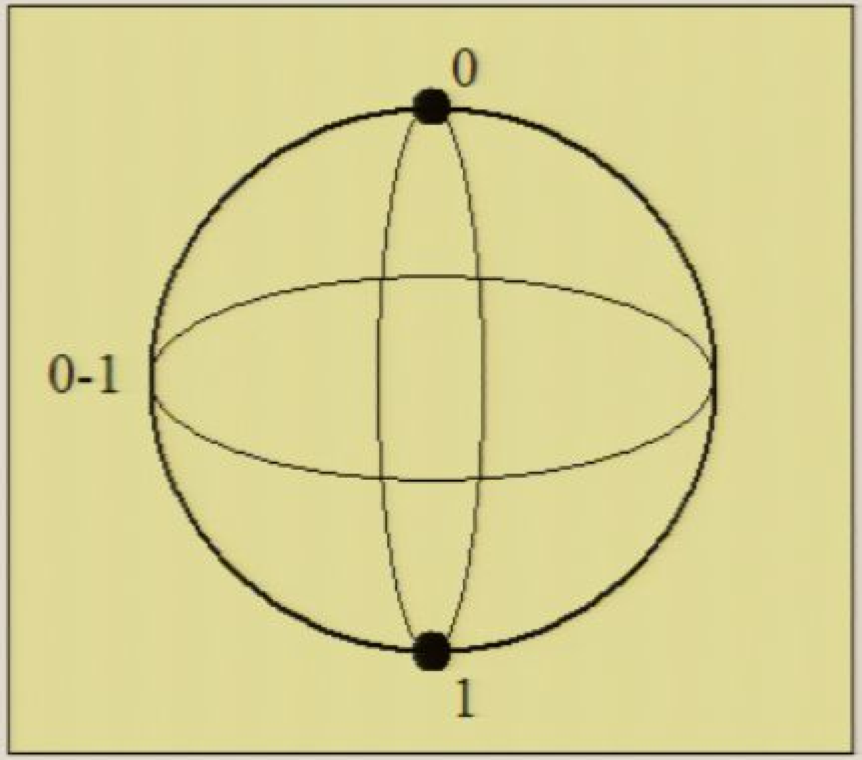




In a classical worldview, we think of the world as being in one of two possible states, each corresponding to the possible answers "Yes" and "No".

Bits and Qubits

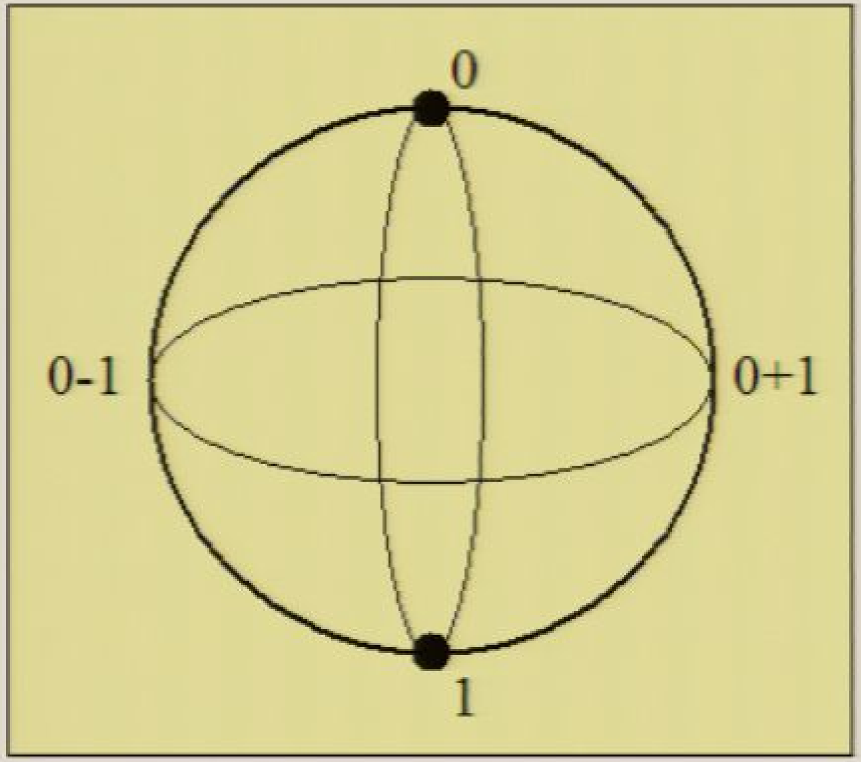




In a classical worldview, we think of the world as being in one of two possible states, each corresponding to the possible answers “Yes” and “No”.

A quantum bit, or *qubit*, has an infinite set possible pure states, the surface of the Bloch sphere. The states **0** and **1** are just two opposing points.

Ket Notation and the Superposition Principle



We write different possible states of physical systems inside 'ket's.

So if **"0"** is a possible state of the system, then we write that state as

$$|0\rangle$$

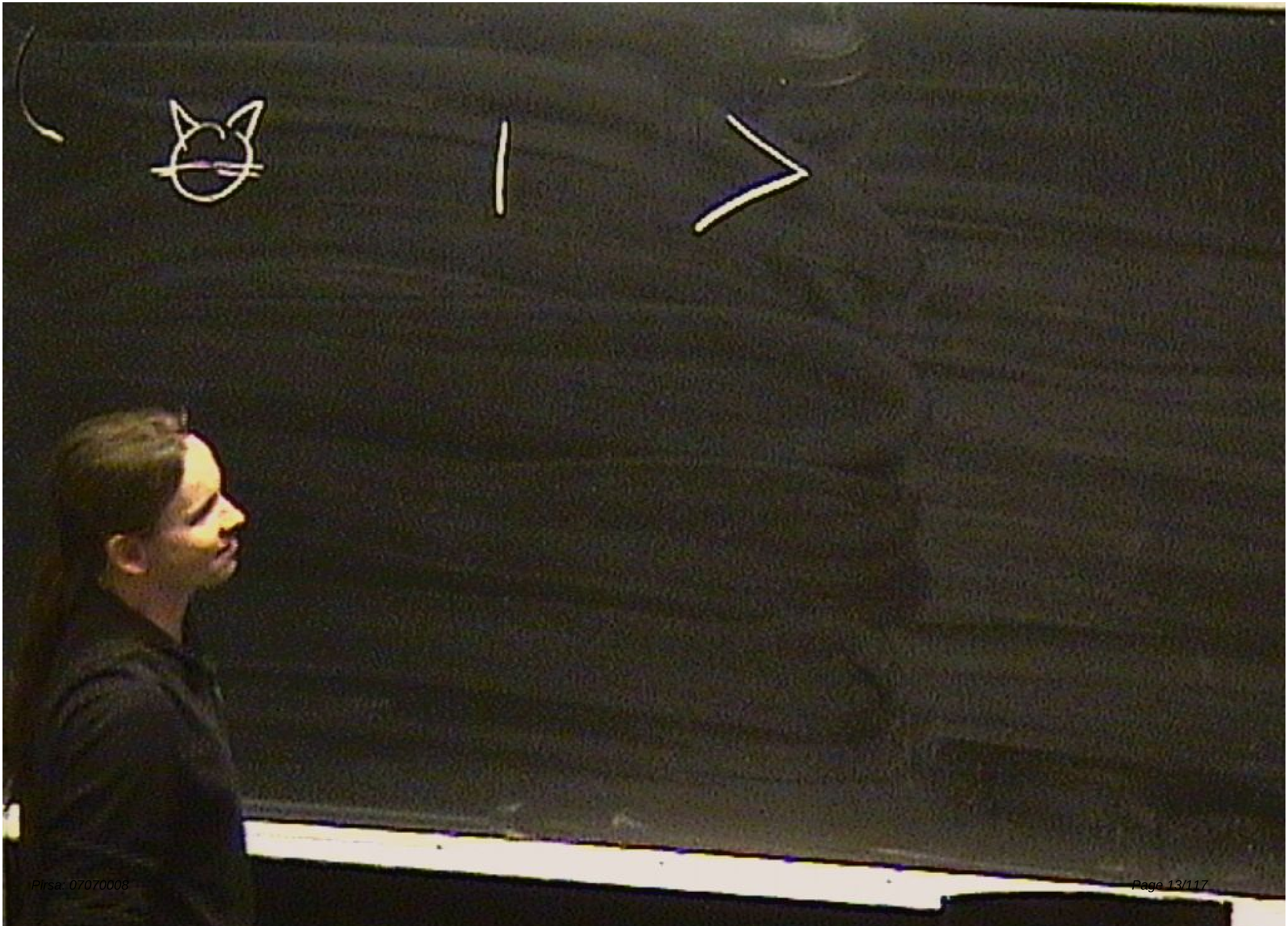
If **"1"** is a different possible state of the system, then just write that state as

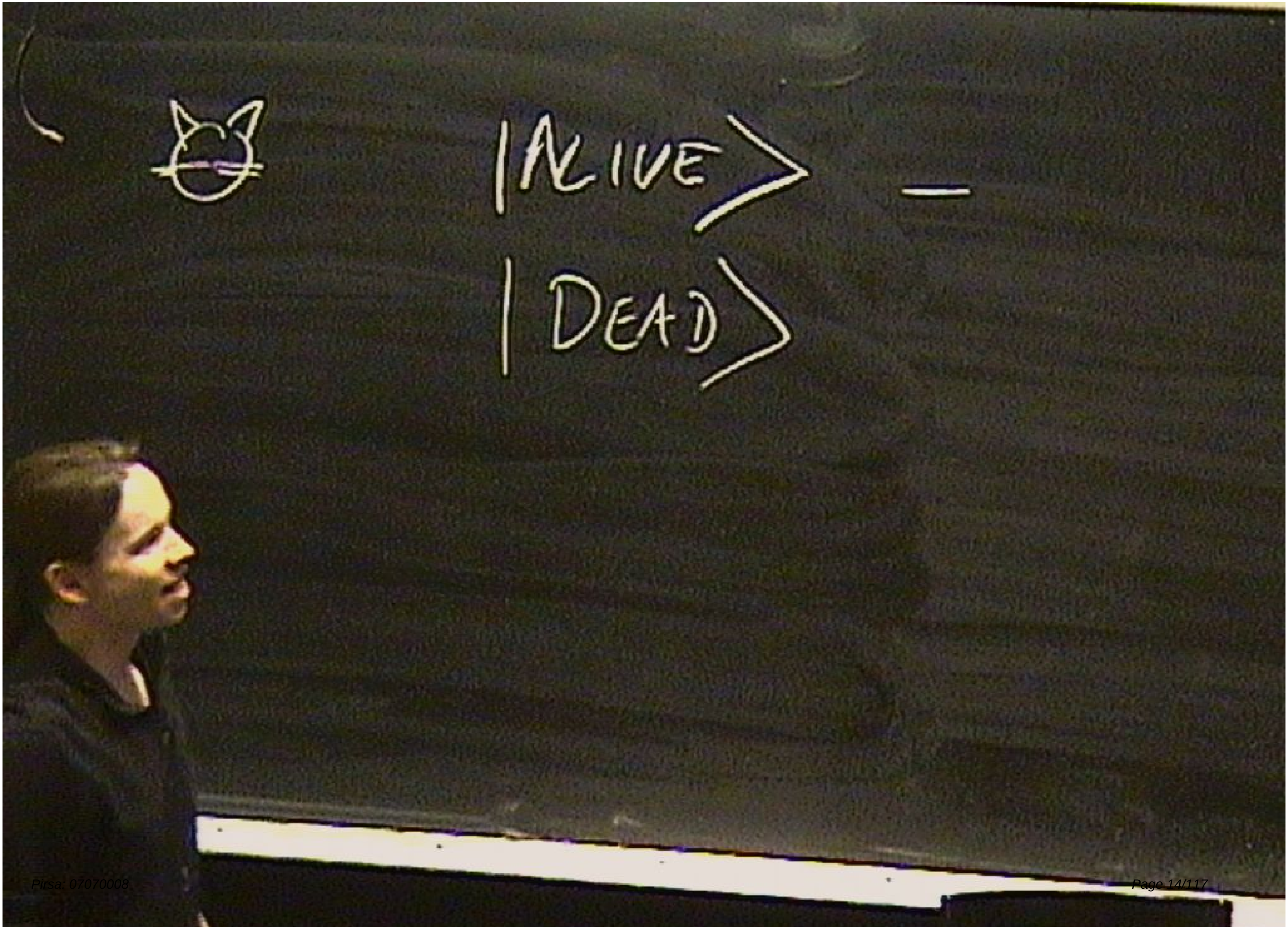
$$|1\rangle$$

If $|0\rangle$ and $|1\rangle$ are different possible states of a system, then **any** complex linear combination of them is also a possible state.

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle,$$

We call $|0\rangle$ and $|1\rangle$ *basis vectors* for the system.

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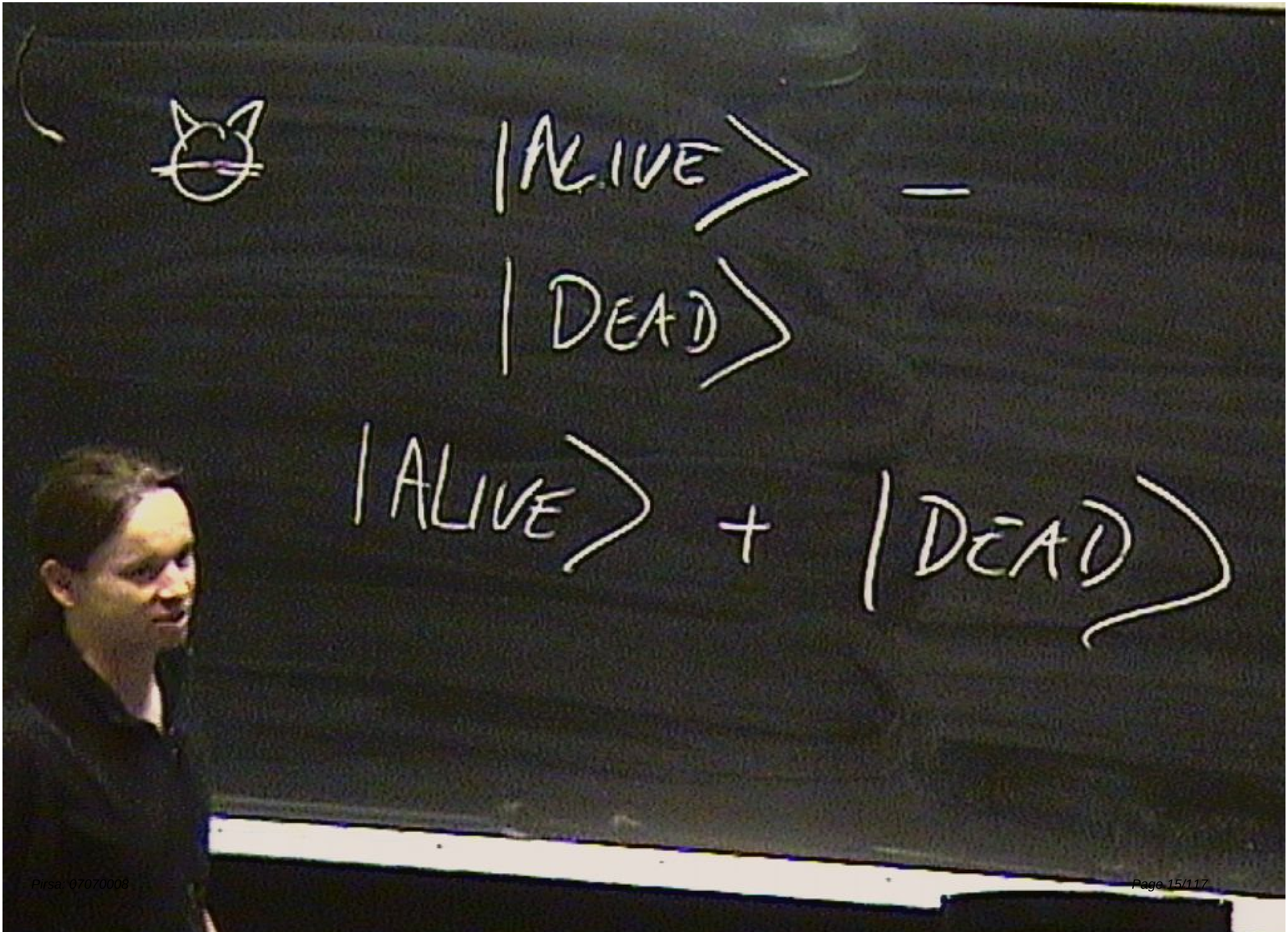
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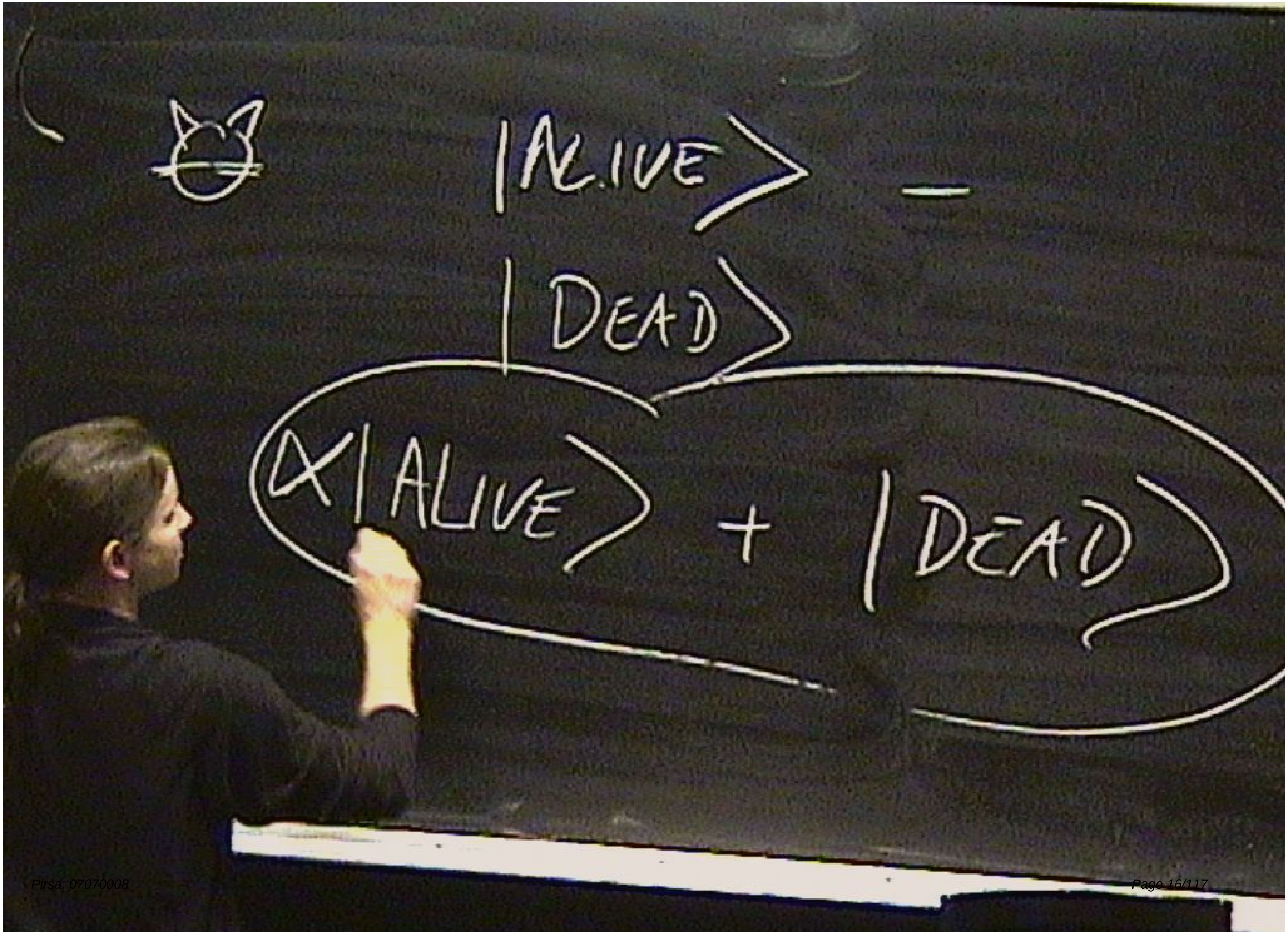
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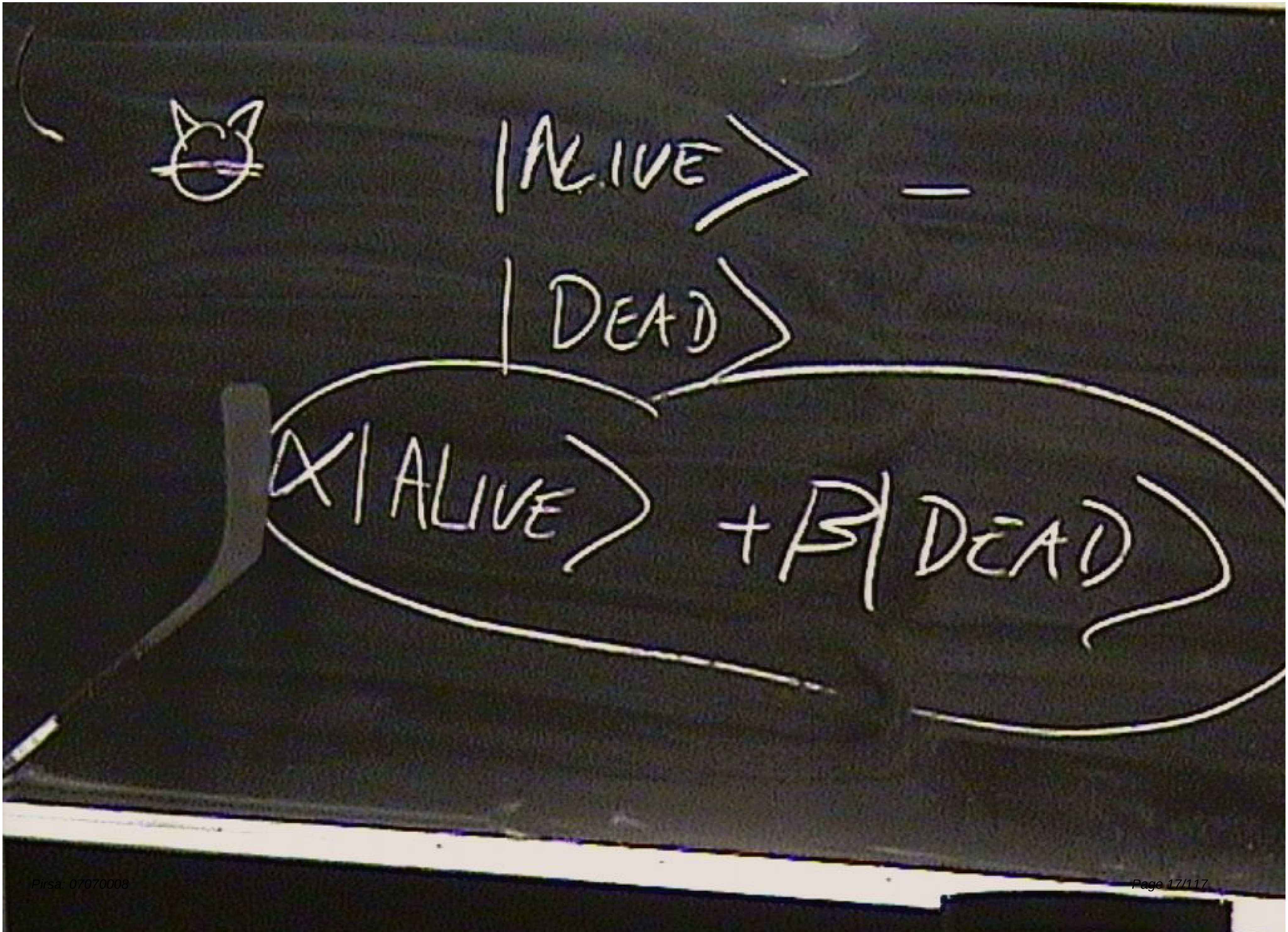
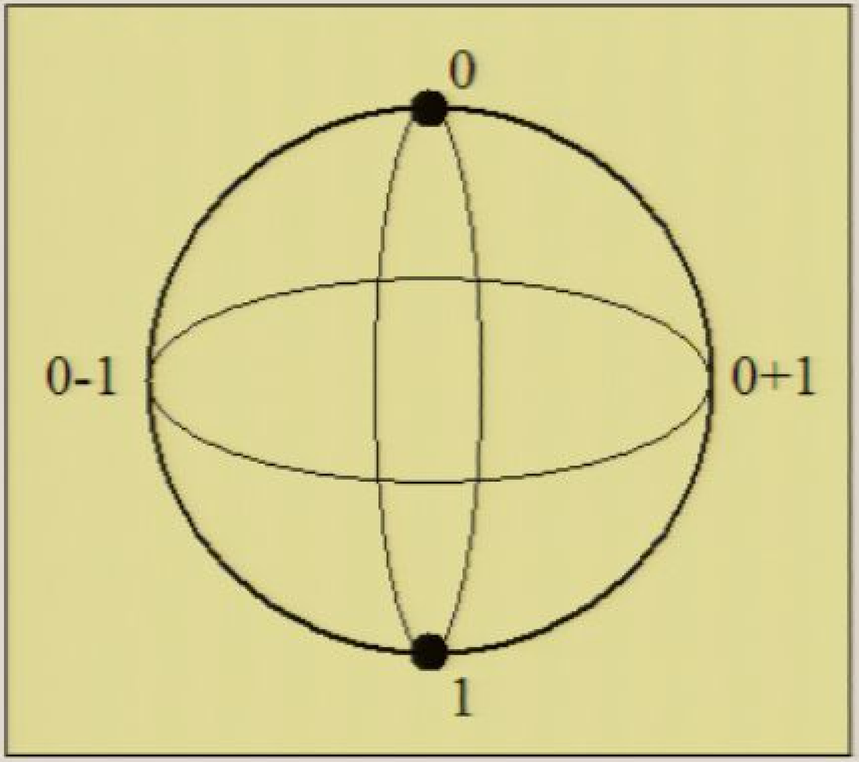

$|\text{ALIVE}\rangle$ —
$|\text{DEAD}\rangle$
$\alpha|\text{ALIVE}\rangle + \beta|\text{DEAD}\rangle$

Ket Notation and the Superposition Principle



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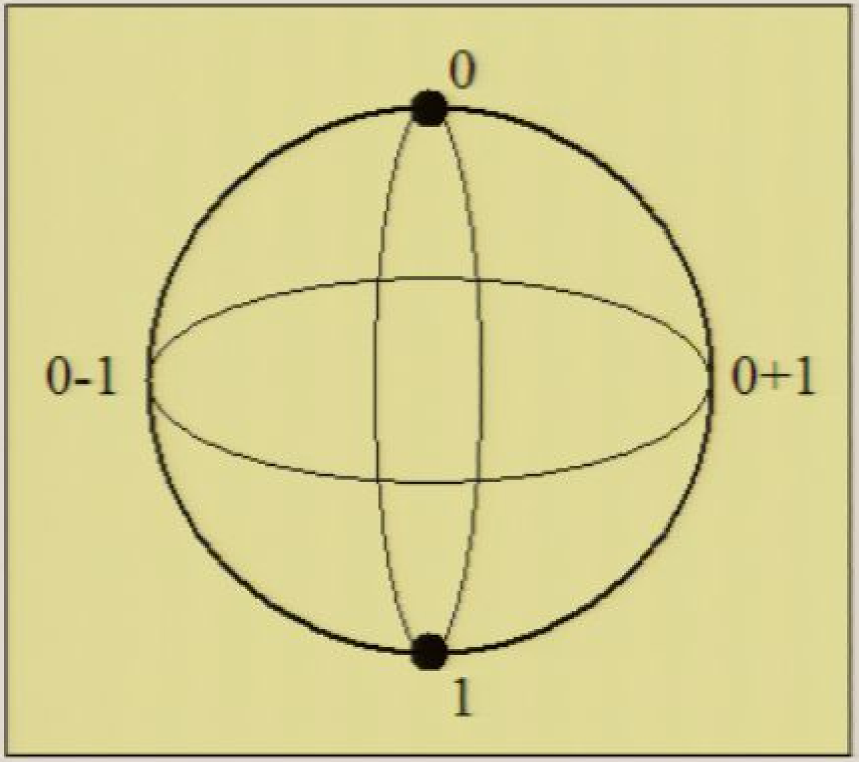
The Superposition Principle



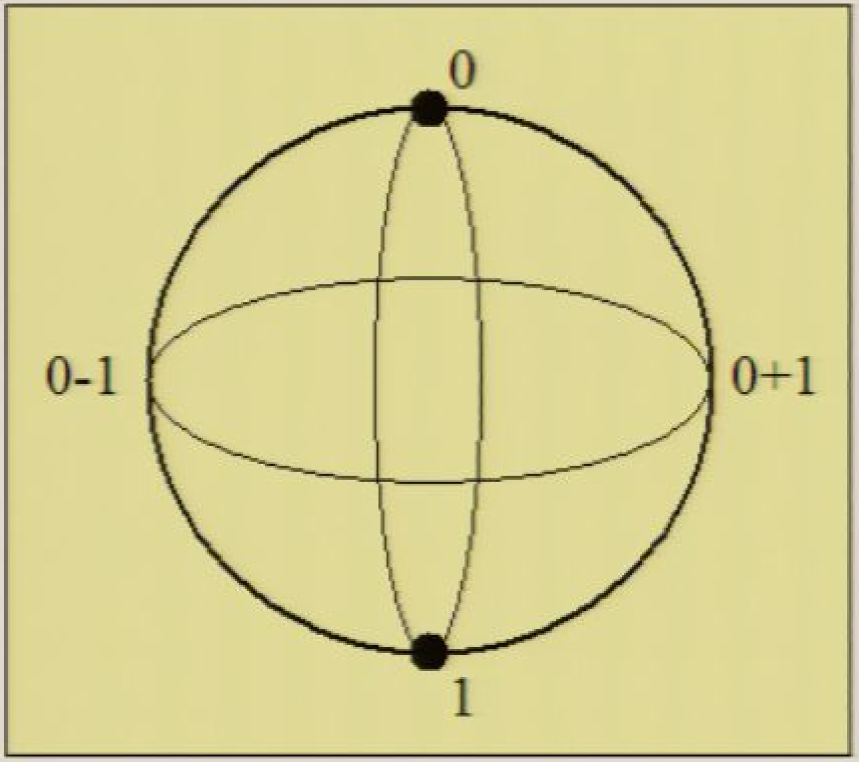
$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle,$$

$|\alpha|^2$ and $|\beta|^2$ are the probabilities of getting the answers "**0**" and "**1**" respectively.

There is nothing special about the points $|0\rangle$ and $|1\rangle$.

Different measurements are possible as well.

The Superposition Principle



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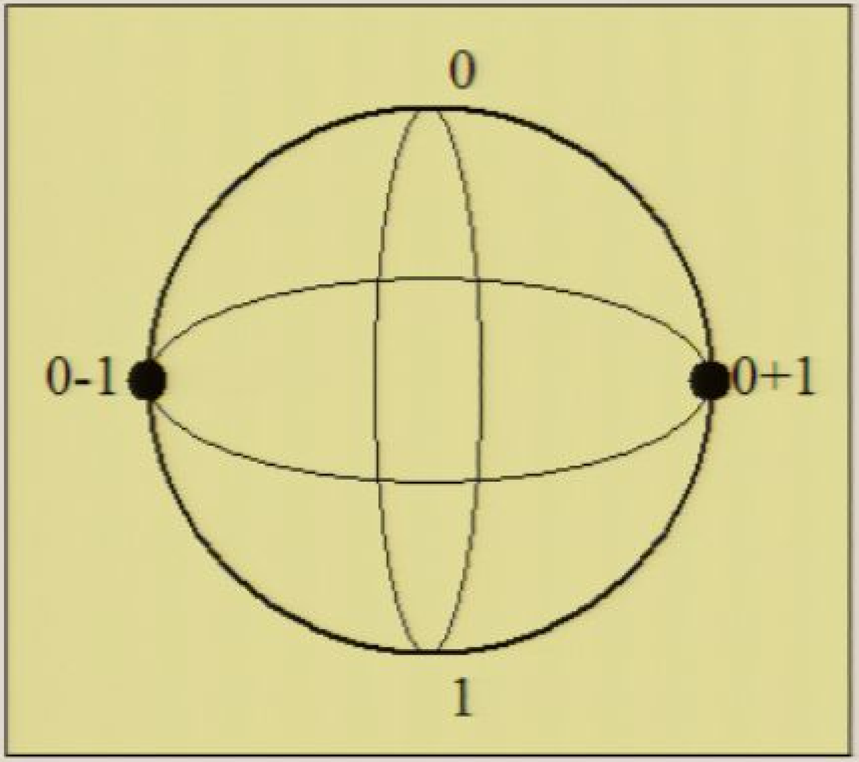

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But we have to ask a binary question!

We can't learn α and β directly.

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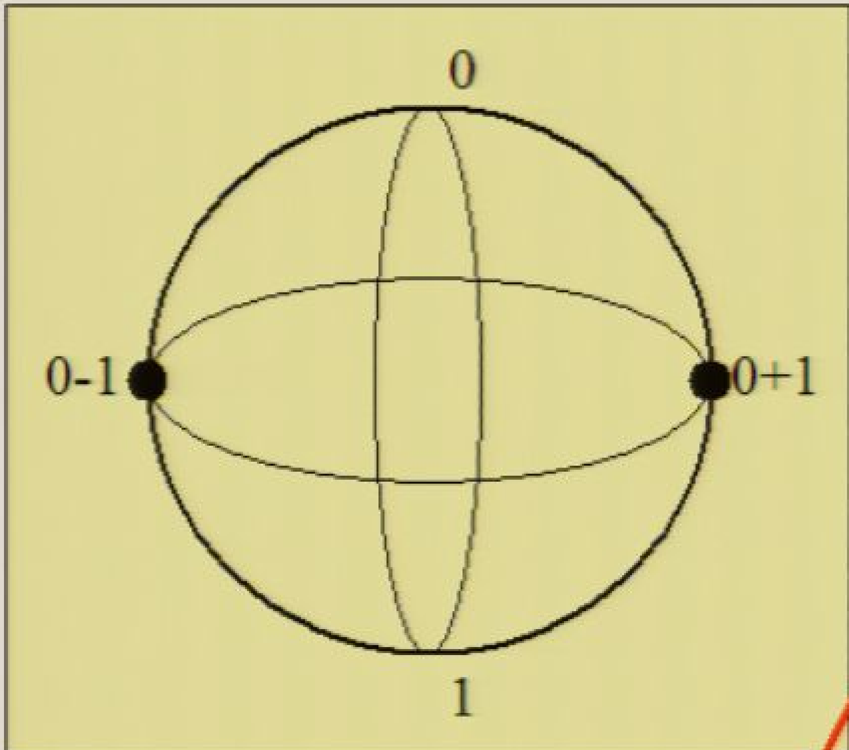

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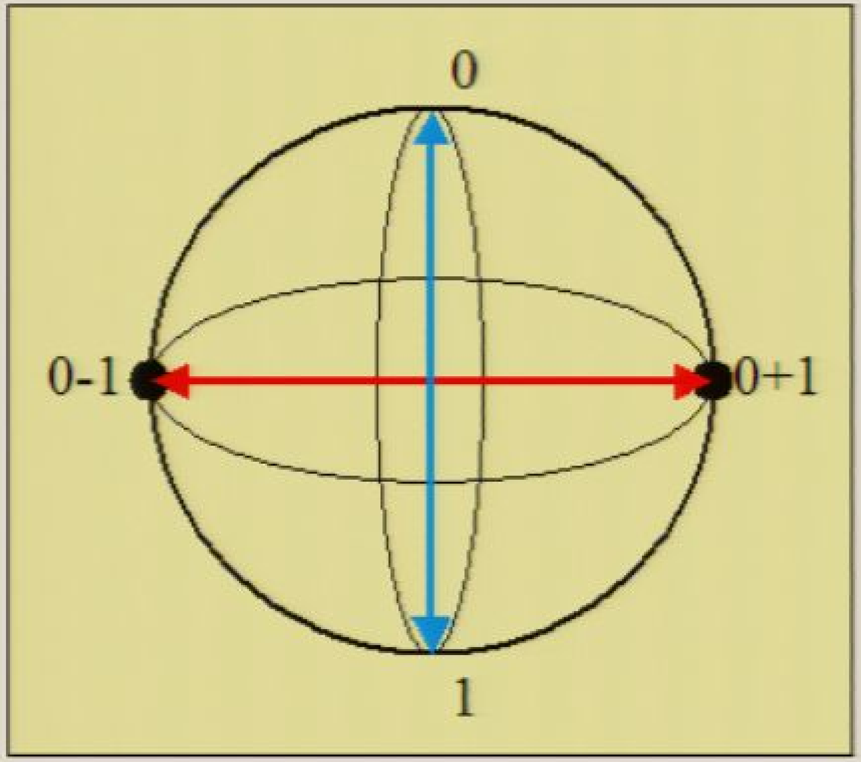
$$\frac{\alpha+\beta}{\sqrt{2}}|0+1\rangle + \frac{\alpha-\beta}{\sqrt{2}}|0-1\rangle$$

Uncertainty and nonorthogonality



All measurements in physics behave this way, from the simplest on up.

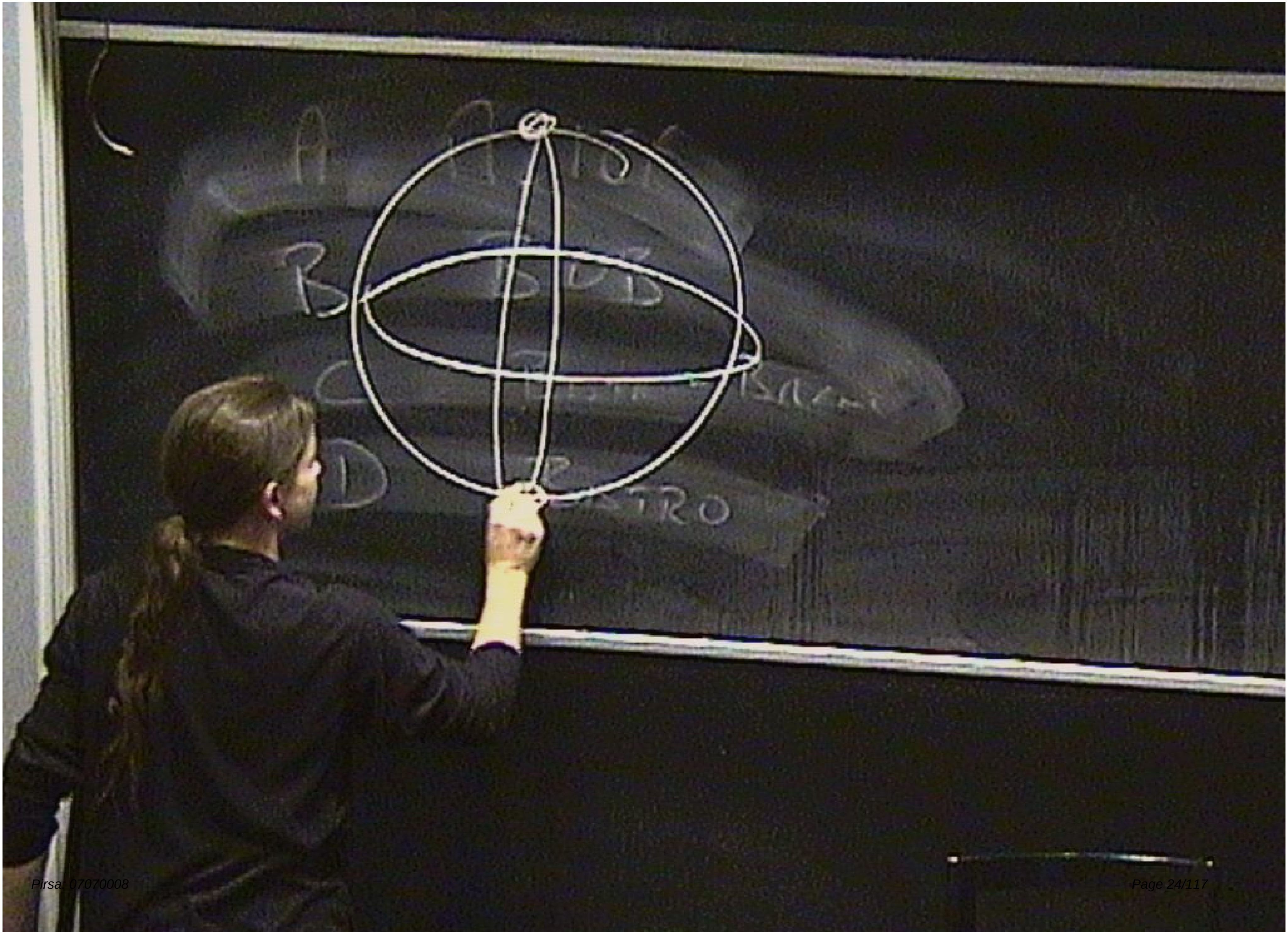
If "the atom is in its ground state" is a valid result, and "the atom is in its excited state" is a valid result, then any complex superposition of the two is:

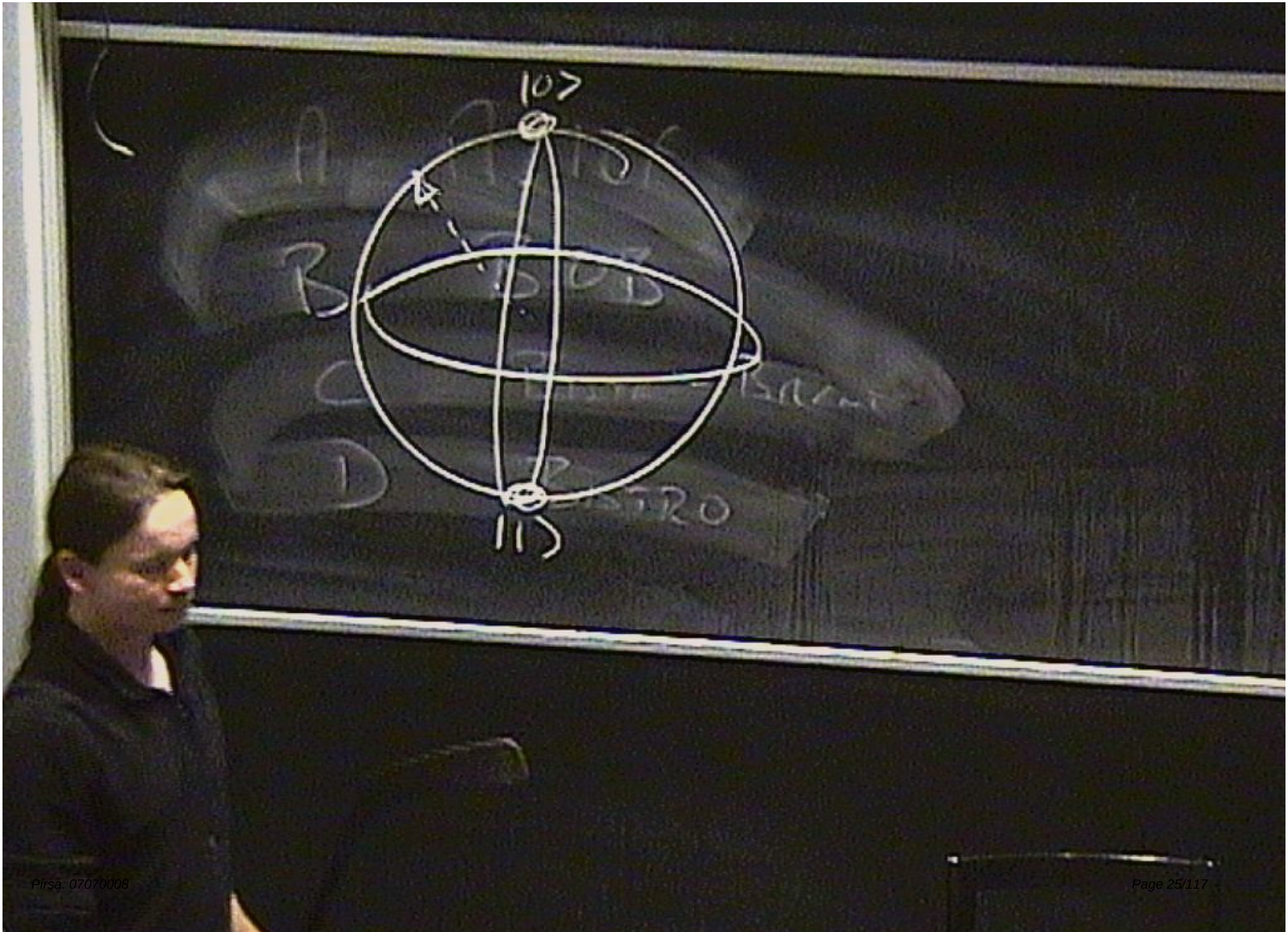
- A possible state of the system.
- A possible result of a different measurement.

This is one way to approach Heisenberg's uncertainty principle.

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \qquad\qquad |0\rangle$$

Uncertainty is at the heart of all "yes/no" questions in physics.

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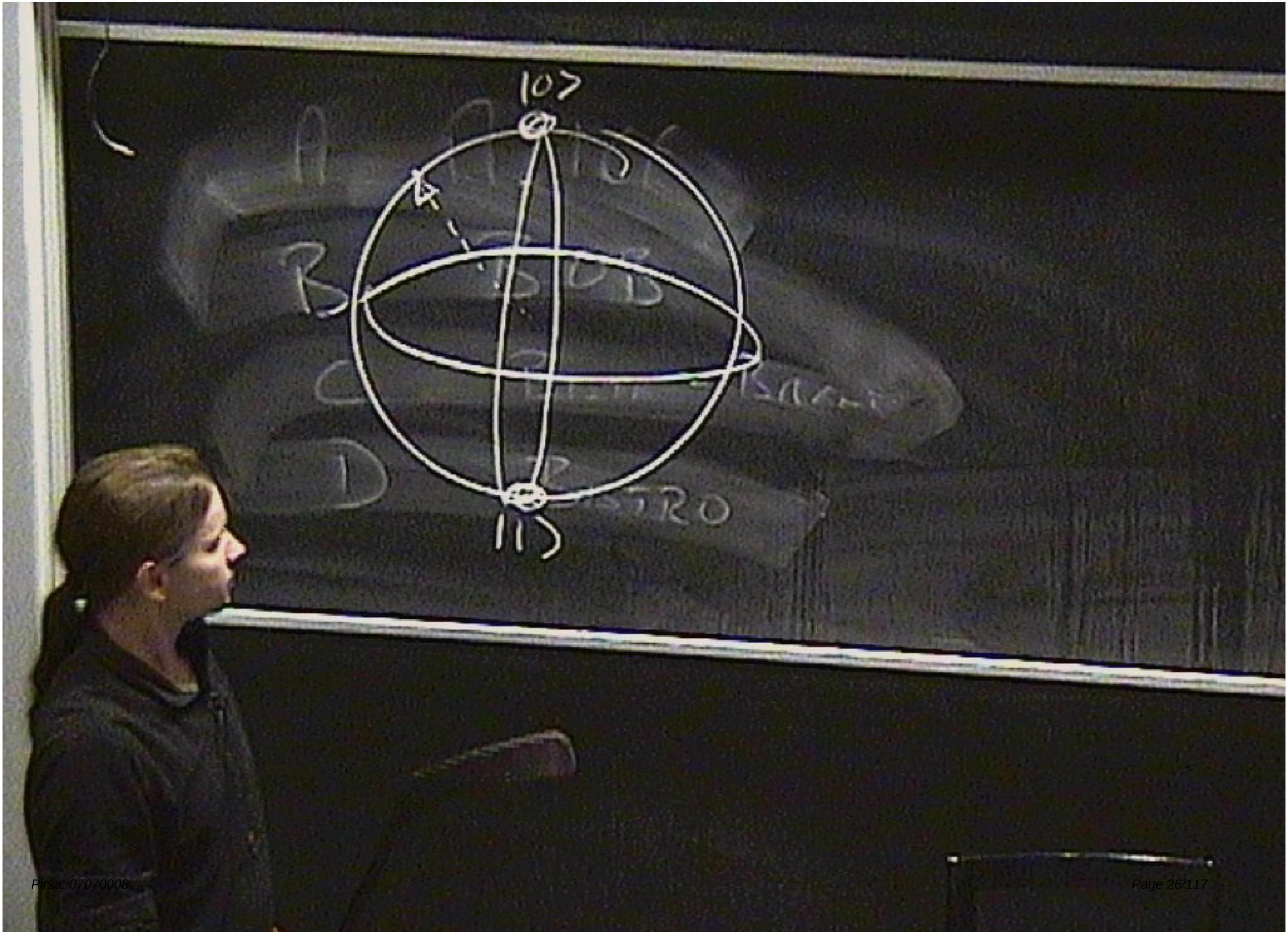
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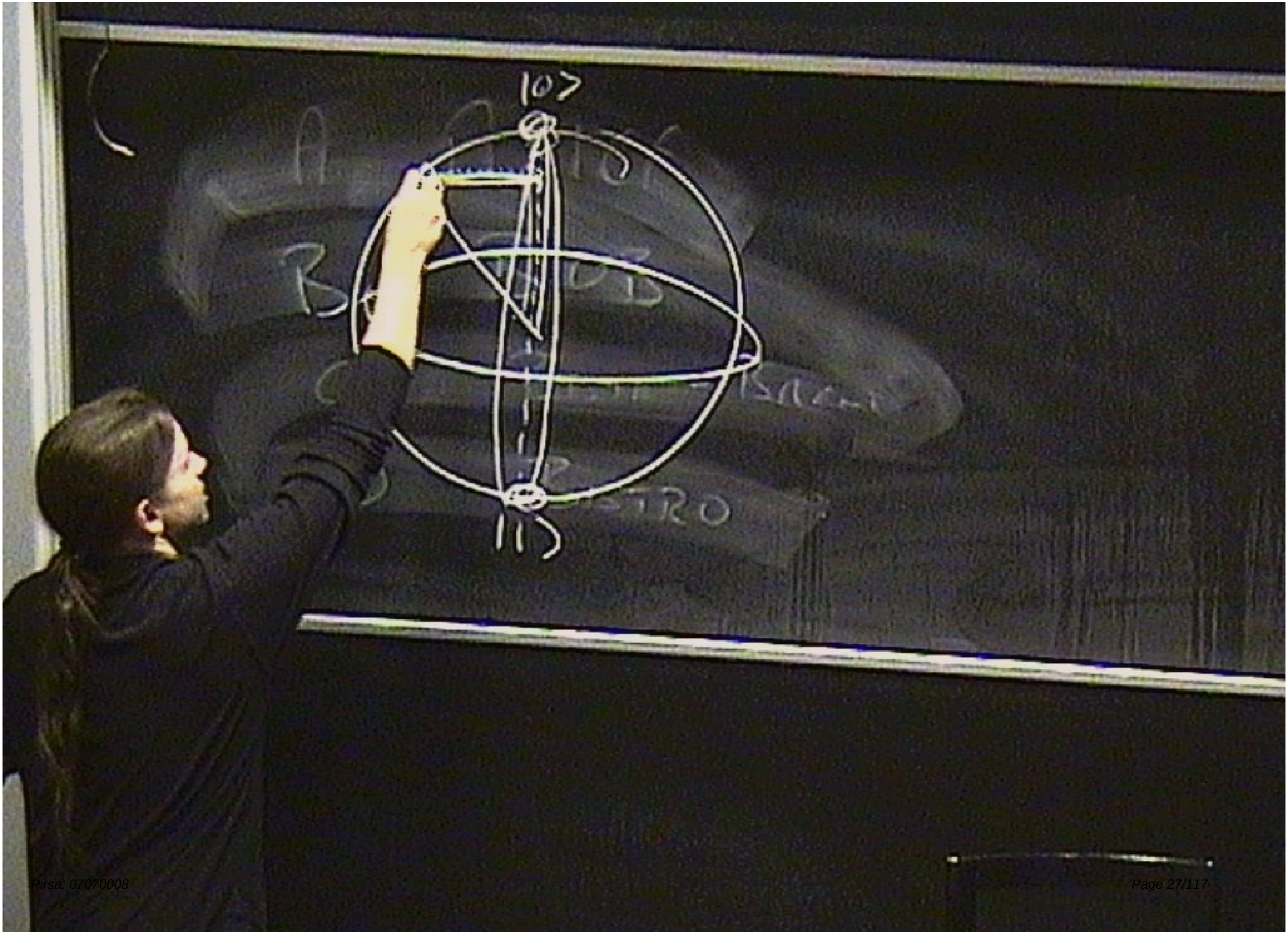

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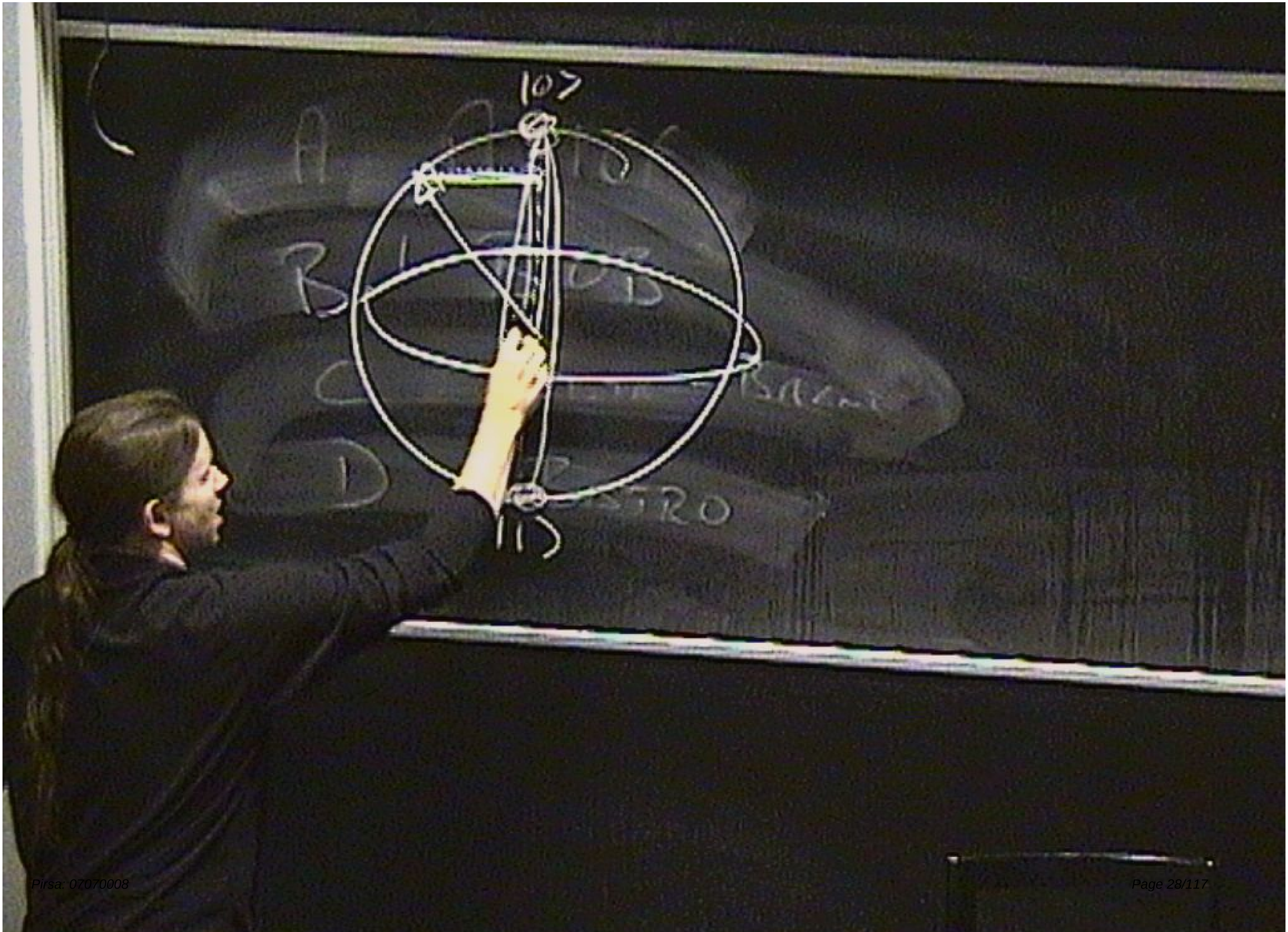
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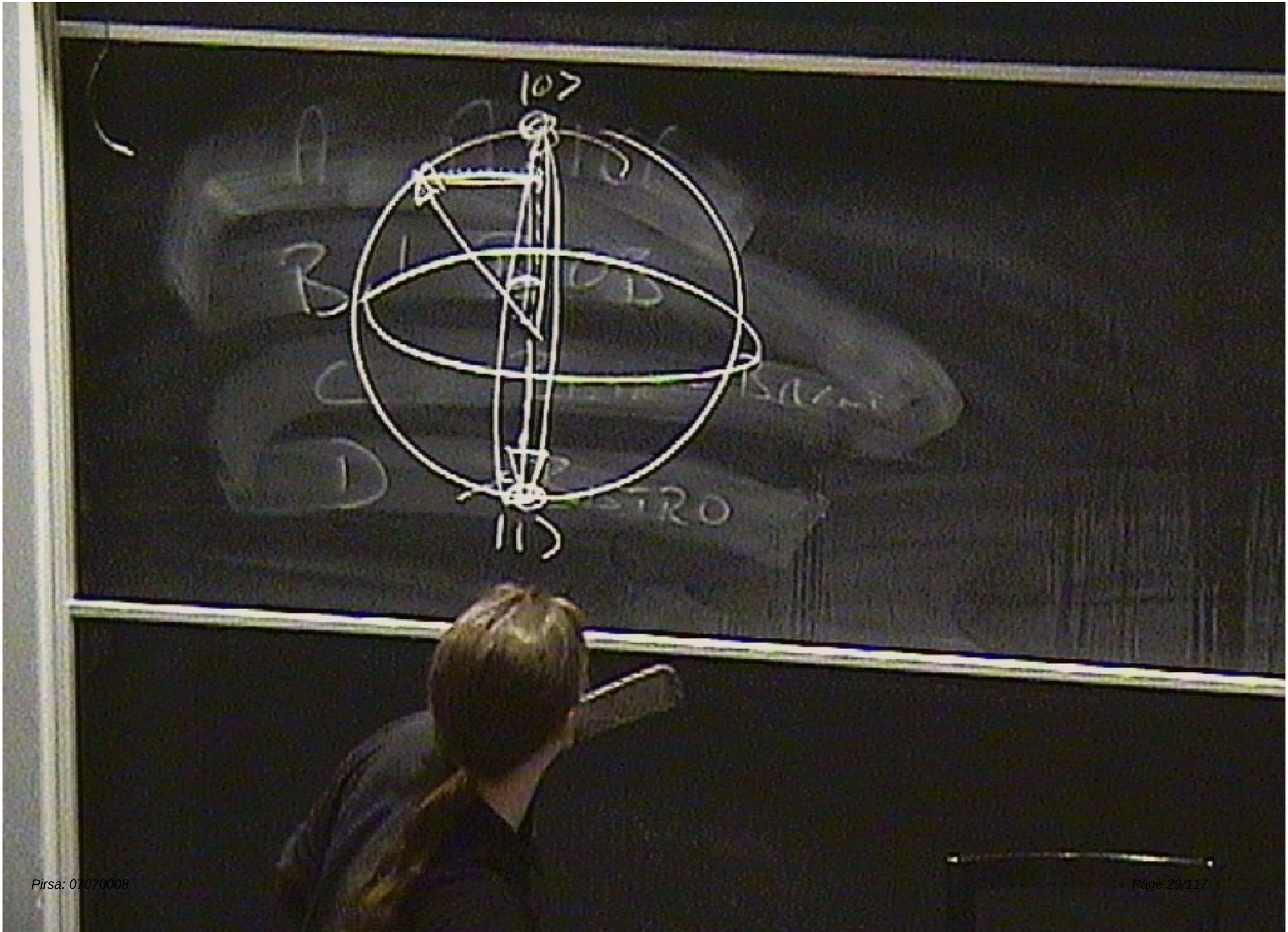
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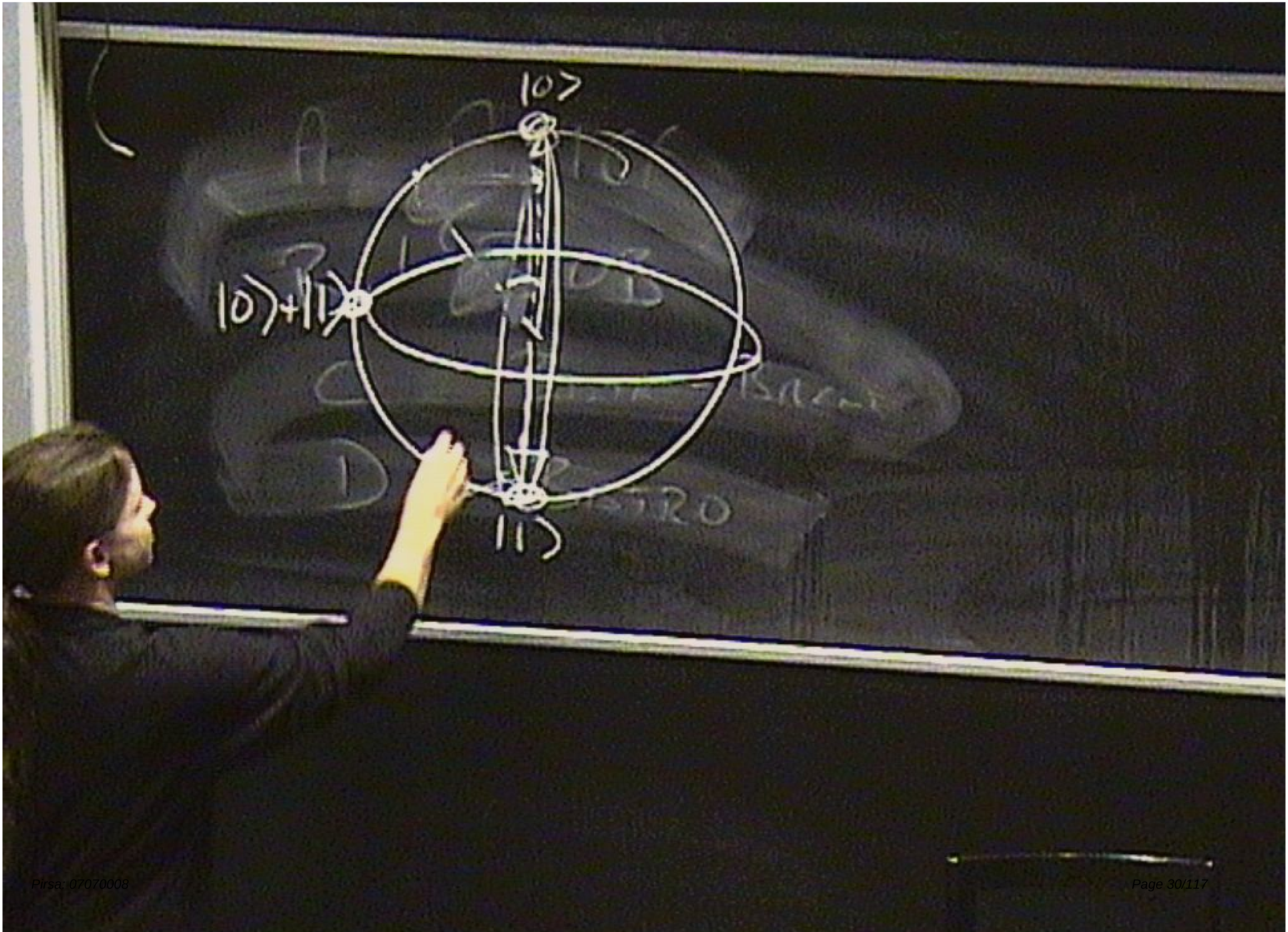
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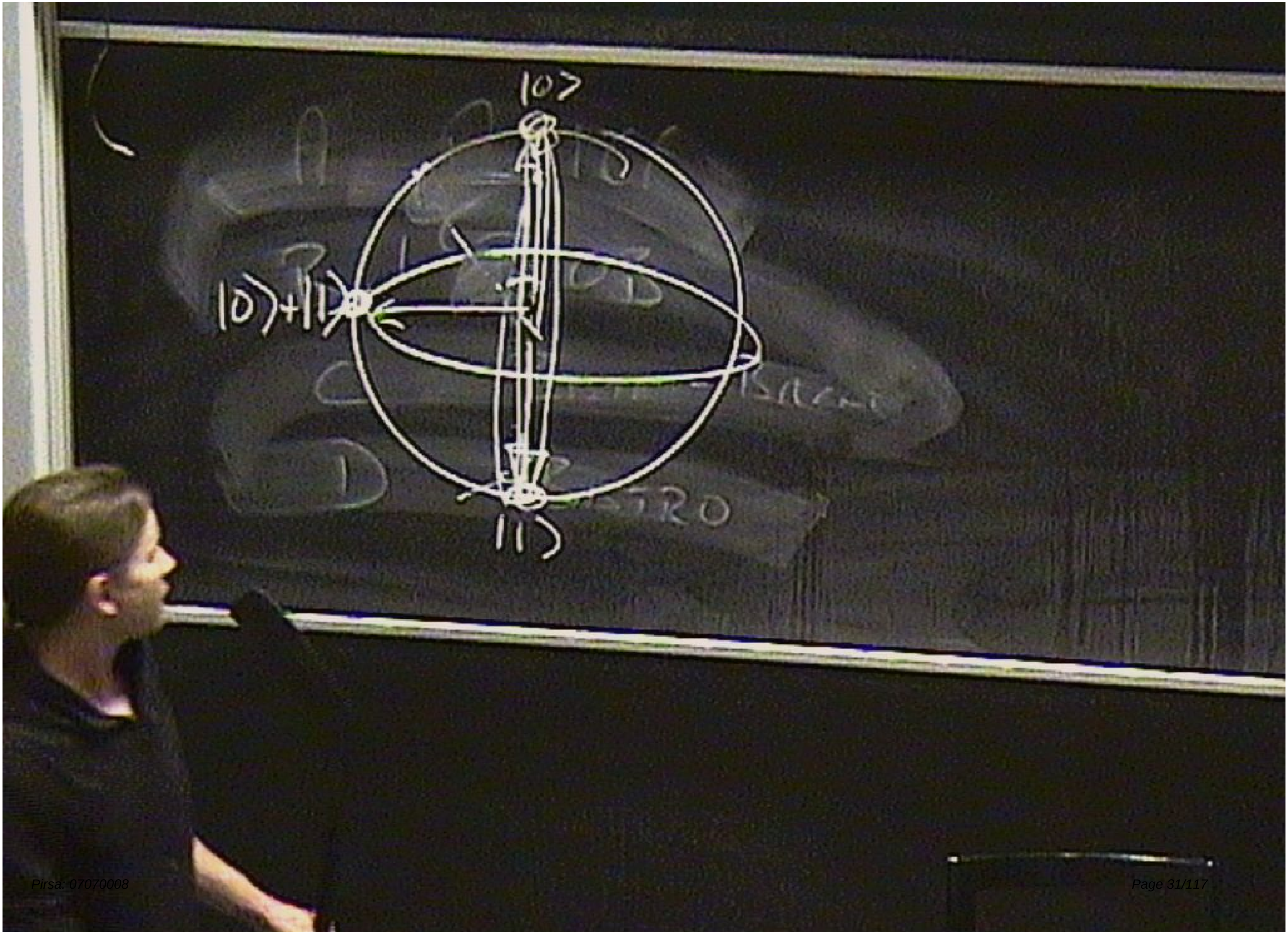
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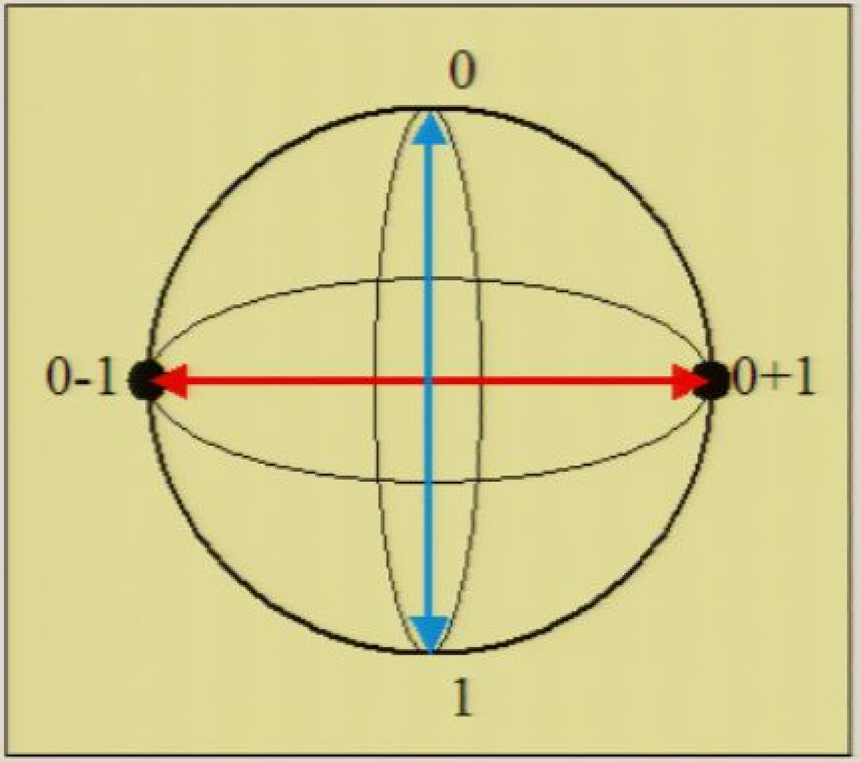

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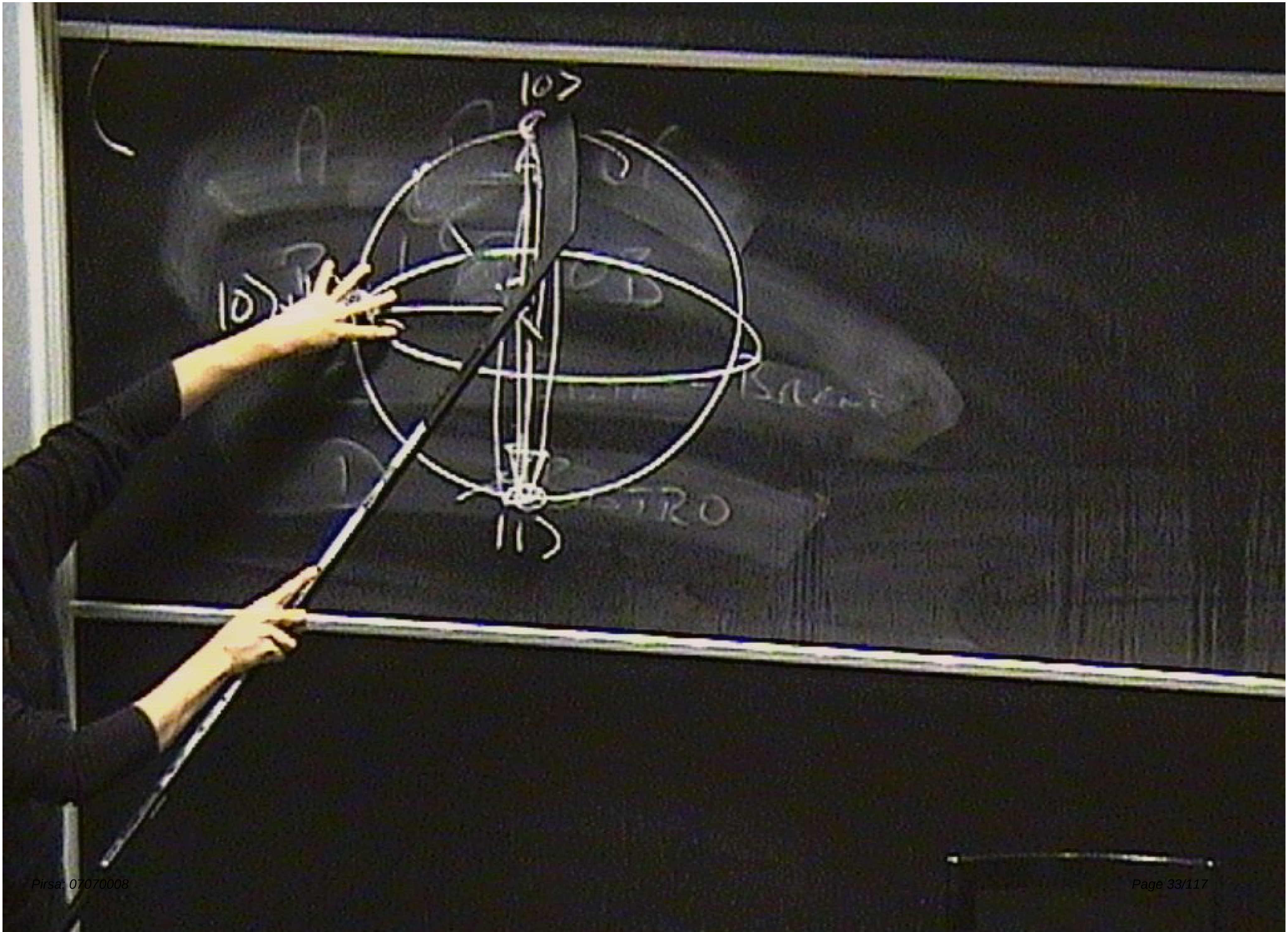
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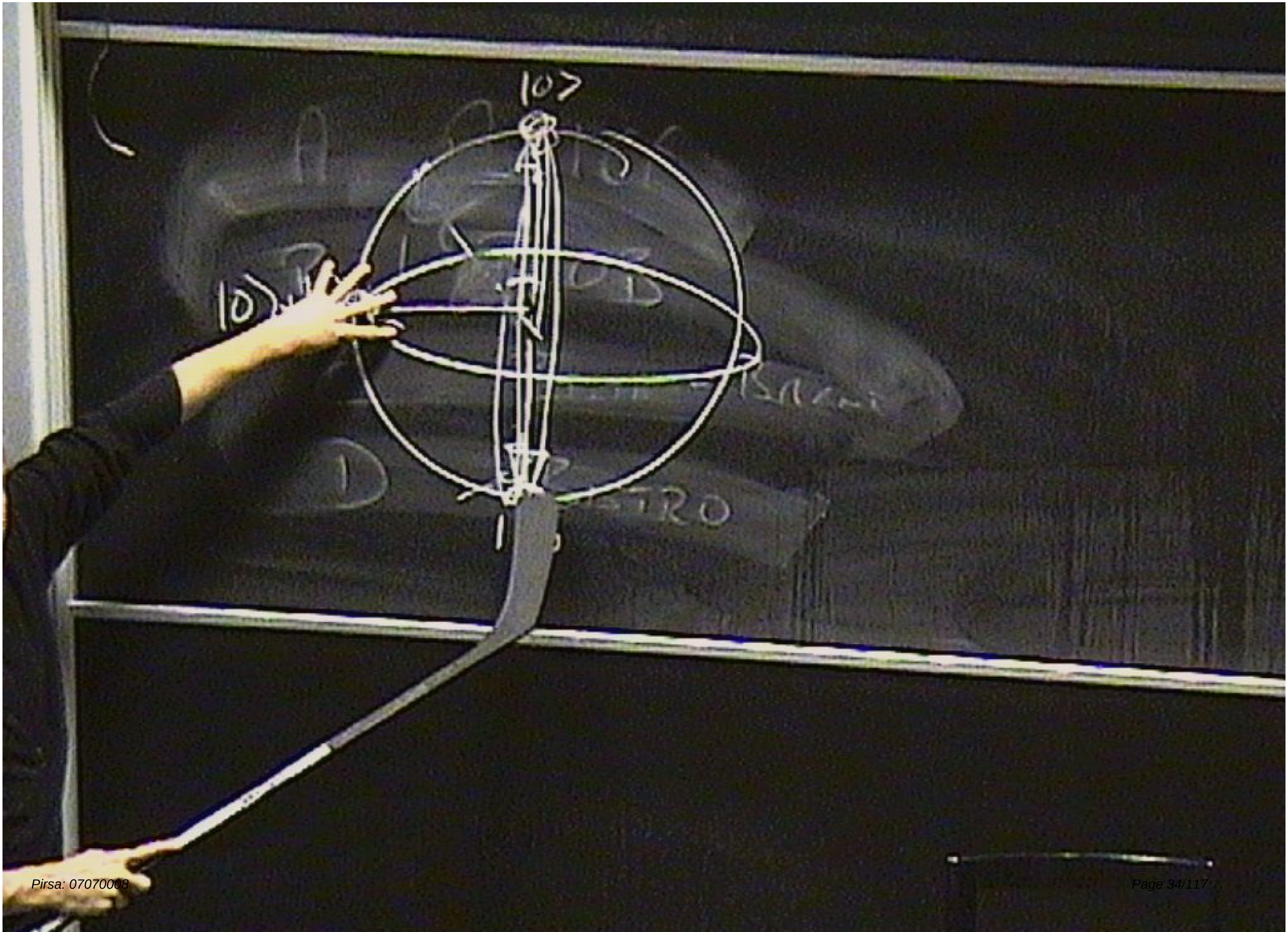

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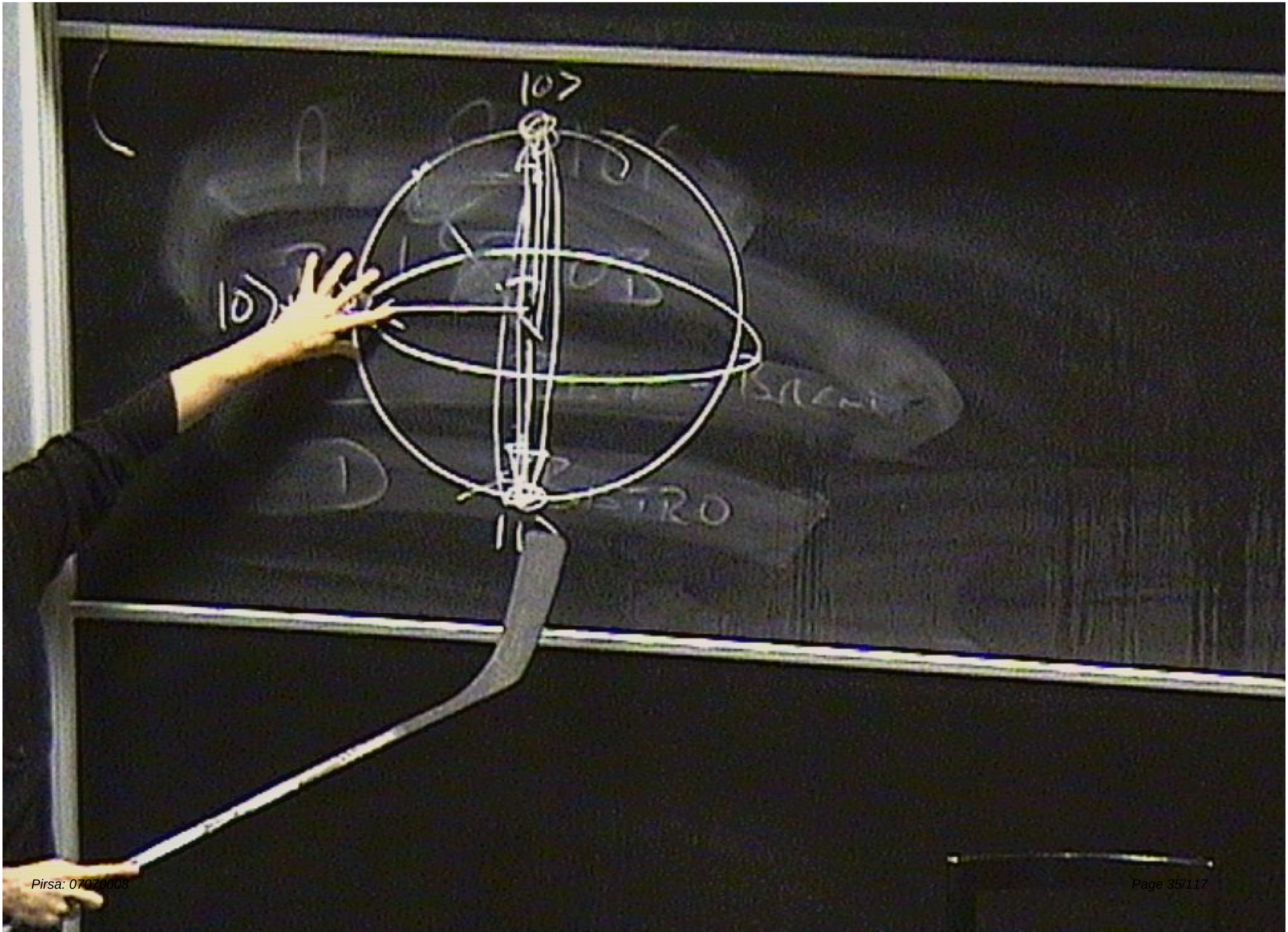
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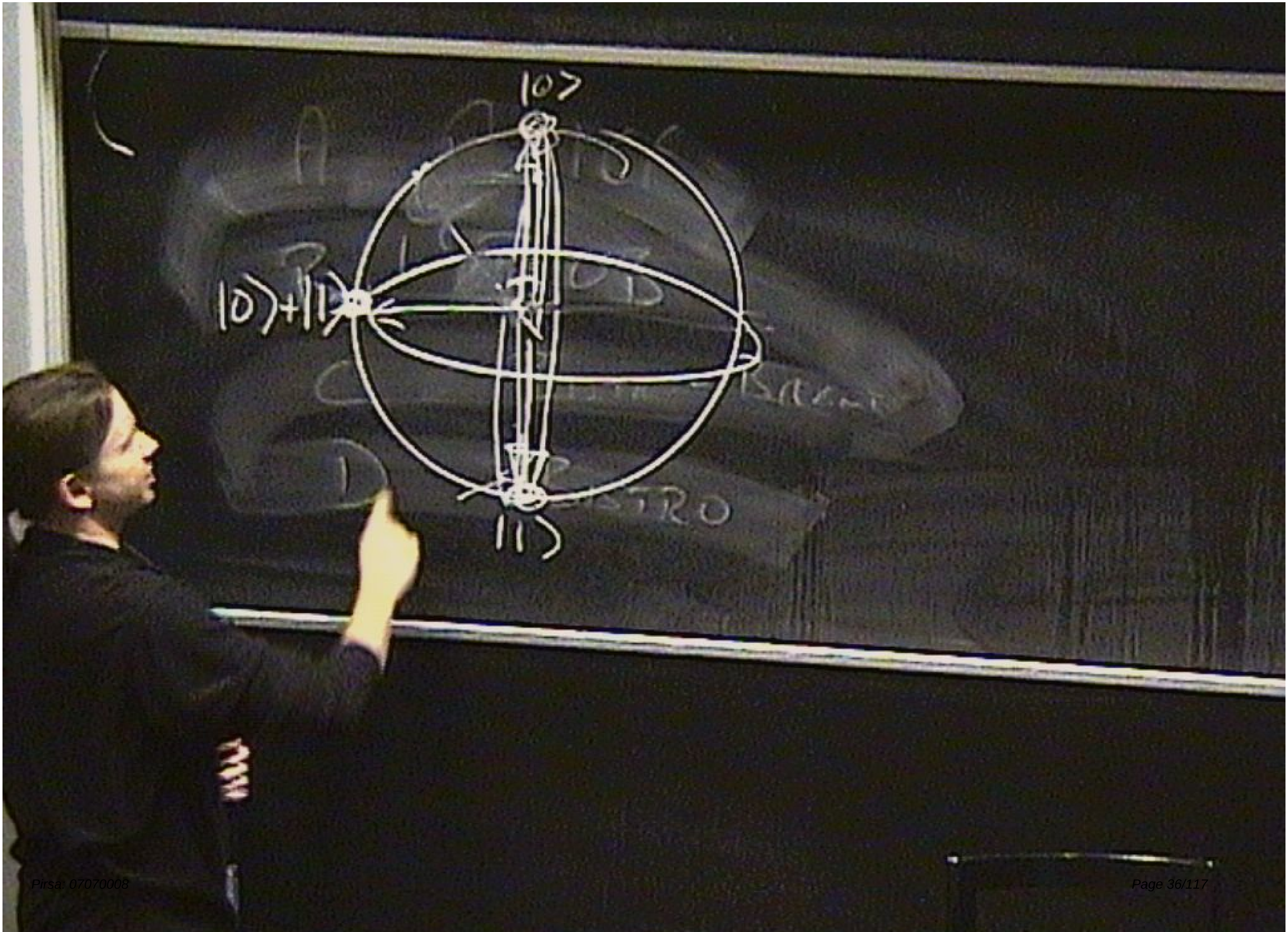
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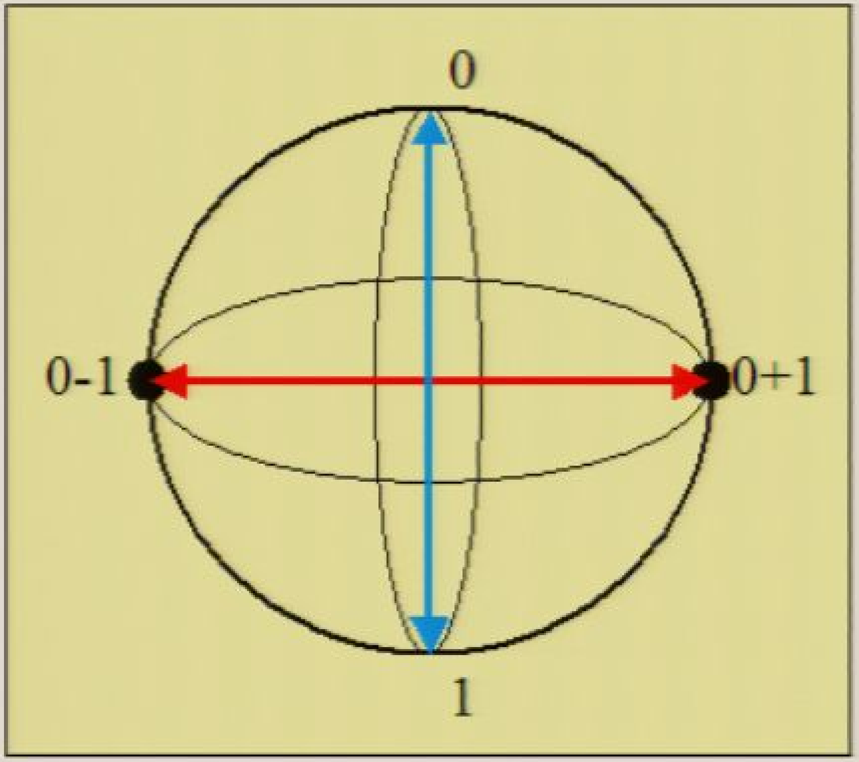

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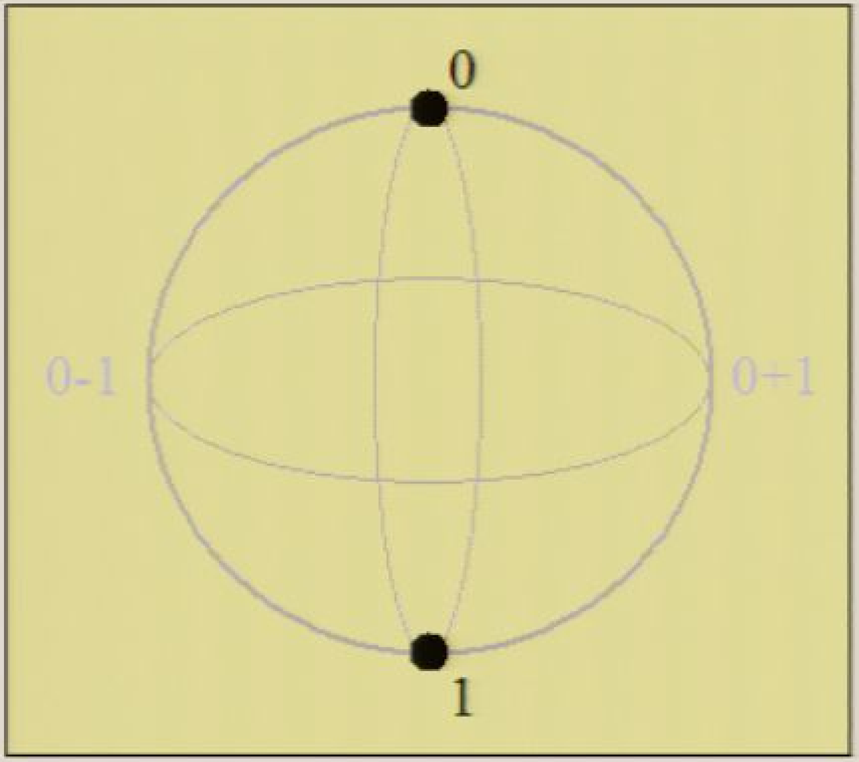
Classical from Quantum



If we stick defiantly to the states "0" and "1"…

…never look for any other possible states…

…and never enact any operation other than a simple 'flip' from 0 to 1…

we get standard classical theory, where cats are either alive or dead, and never inbetween.

But reality doesn't actually seem to be restricted in this way – it seems to be able to do more!

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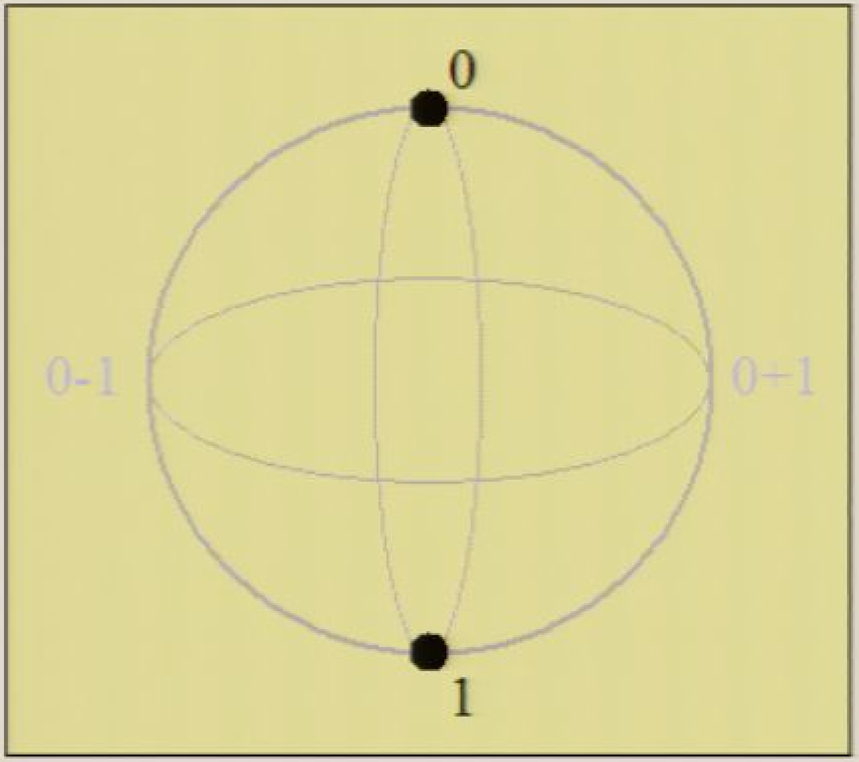

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Entanglement

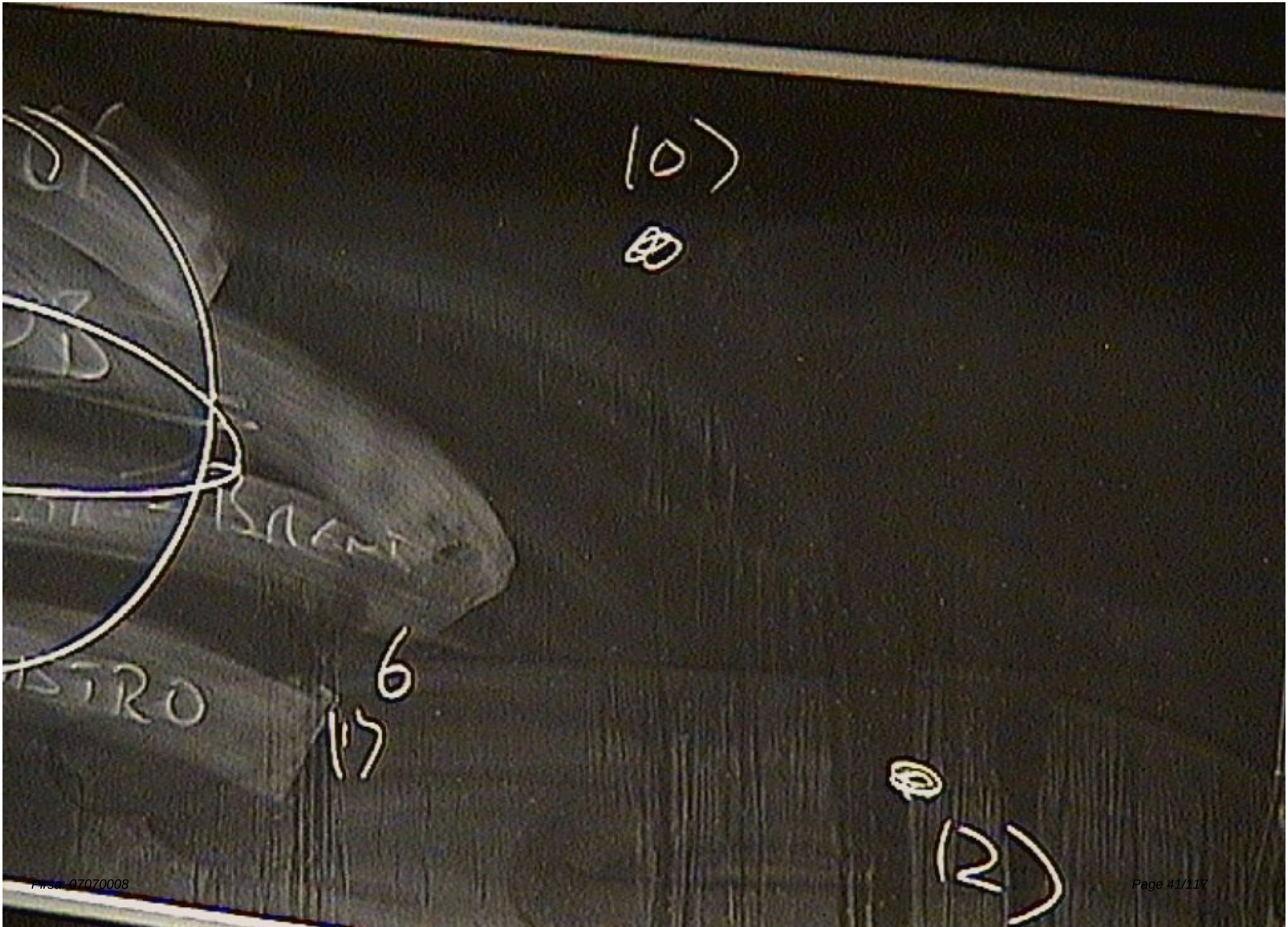
Entanglement is one of the stranger concepts in physics.

It's a direct consequence of the superposition rule for more than one object.

It seems to suggest that distant events can be 'spookily' correlated, even when there's no way for them to communicate with one another.

For this reason, entanglement is probably the most pseudoscientifically abused concept in physics. It is used to 'explain':

Clairvoyance, remote healing, homeopathy, telepathy, ESP, telekenisis, etc…etc…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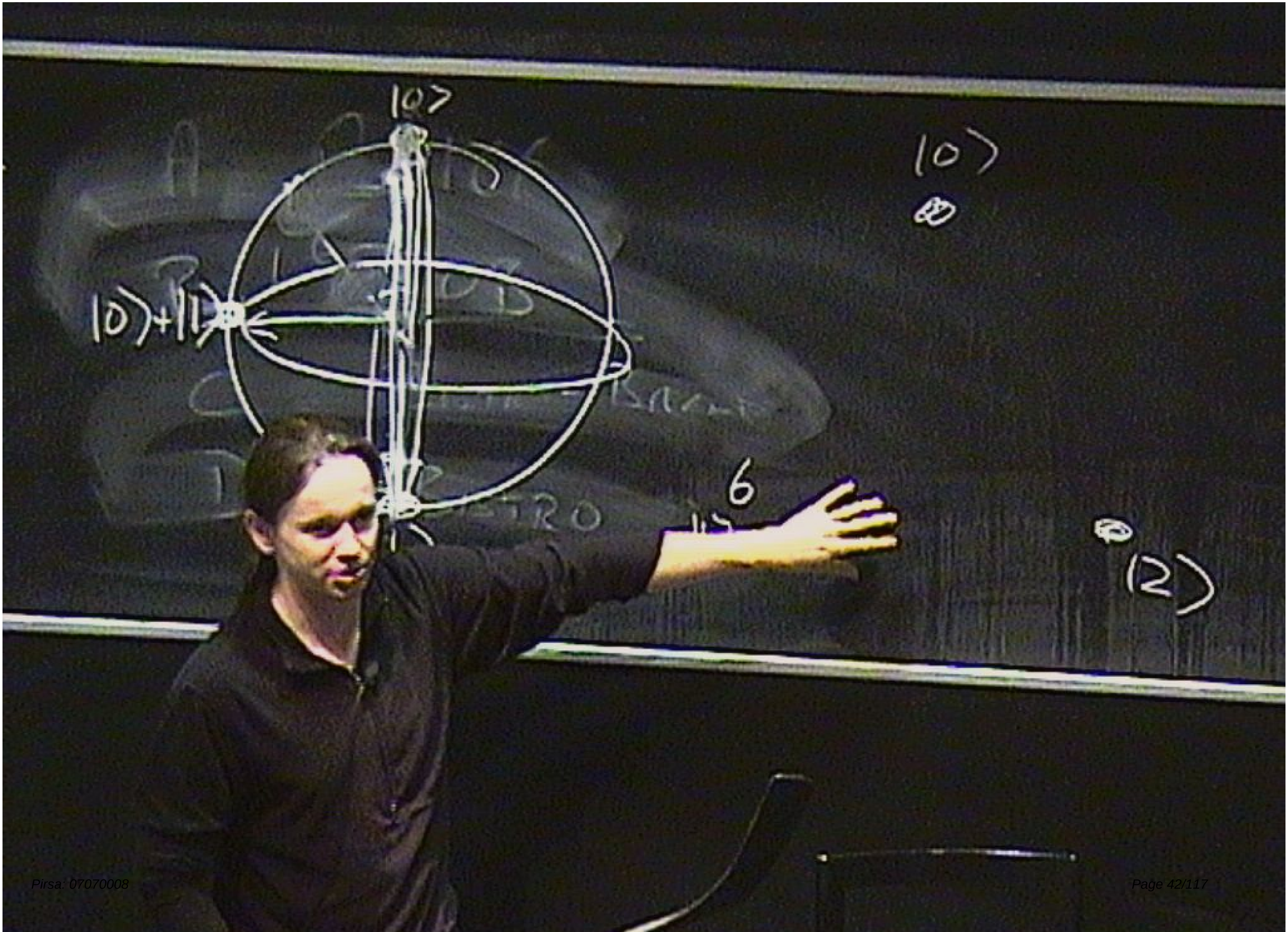
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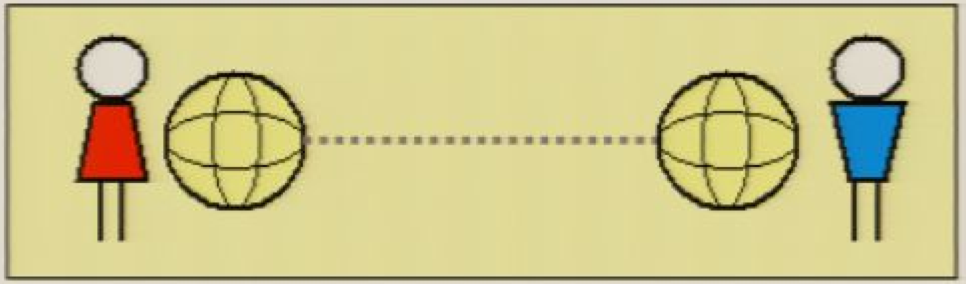


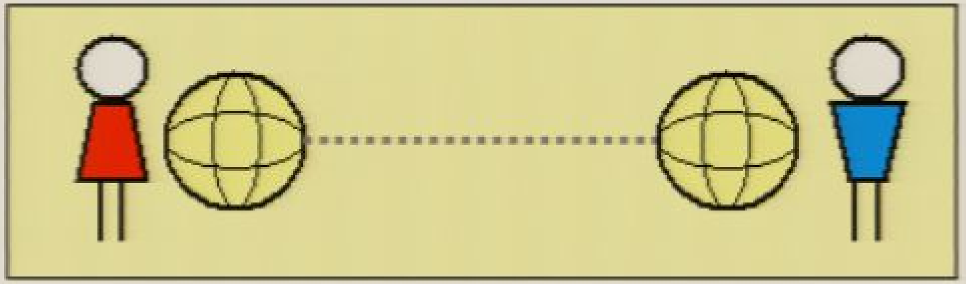
Consider not one, but *two* elementary quantum systems.

Each system really corresponds to some possible binary question we ask of reality. For example, these could be two atoms, in two different places, and of each atom we could ask “are you in the ground state or the exited state”?

There are four possible answers:

- A and B are both in the ground state.
- A is in the excited state and B is in the ground state.
- A is in the ground state B is in the excited state.
- A and B are both in the the excited state.

Entanglement



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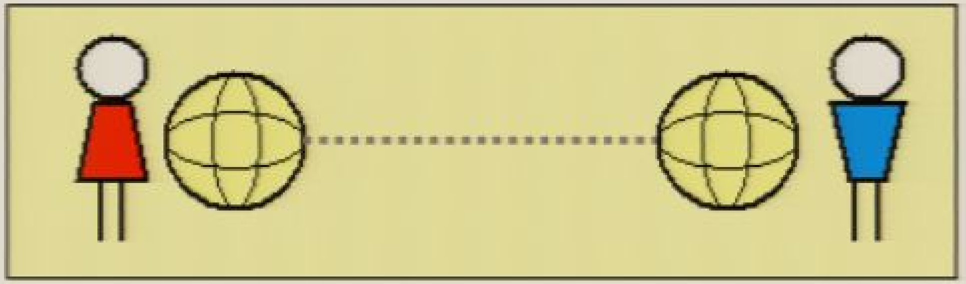
\begin{aligned}
|\psi_1\rangle &= |0\rangle_A \; |0\rangle_B \\
|\psi_2\rangle &= |1\rangle_A \; |0\rangle_B \\
|\psi_3\rangle &= |0\rangle_A \; |1\rangle_B \\
|\psi_4\rangle &= |1\rangle_A \; |1\rangle_B
\end{aligned}
$$

Entanglement



We can apply the superposition principle exactly as before, but now the space of possible states has *four* complex dimensions.

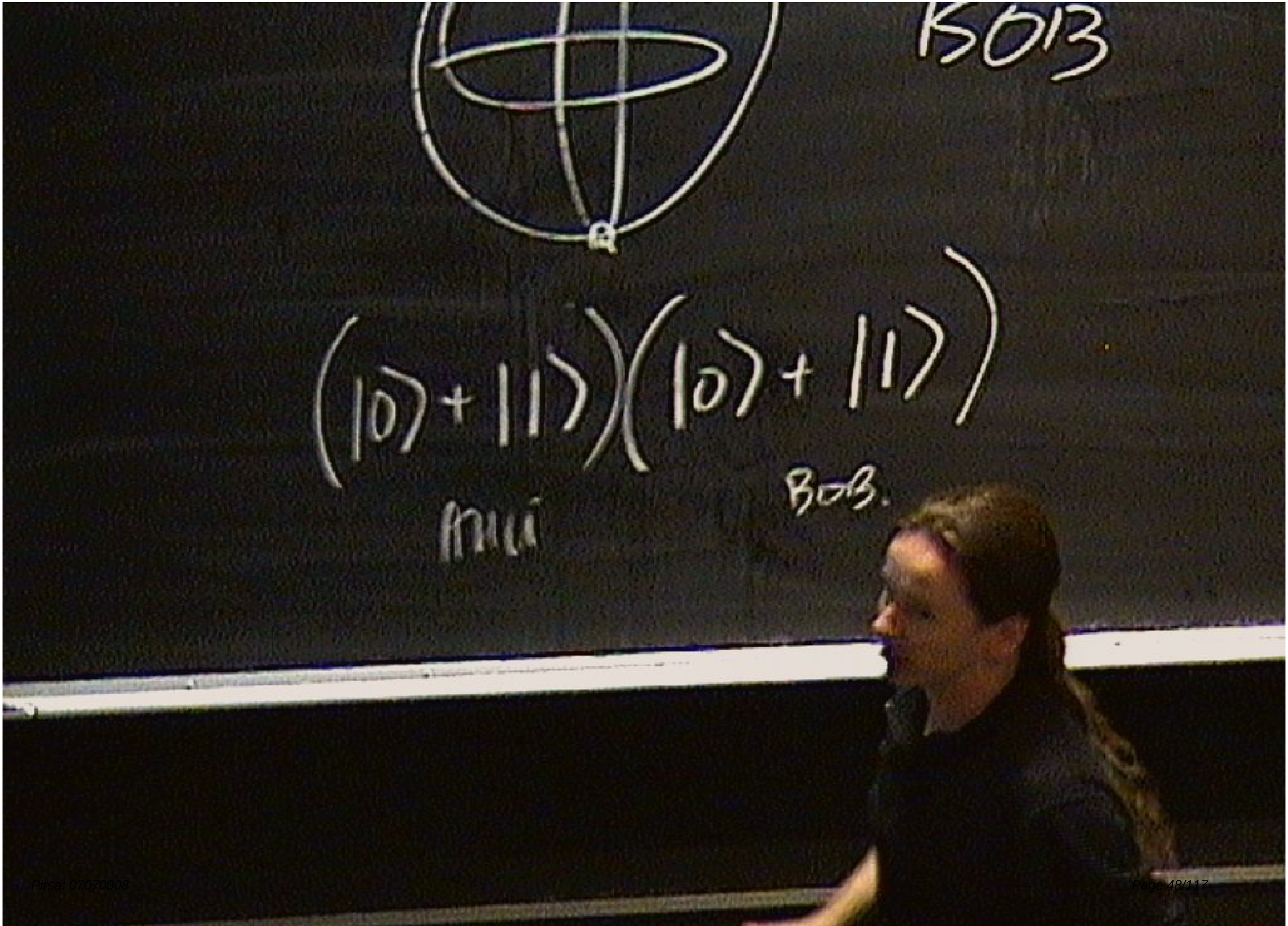
Possible state:

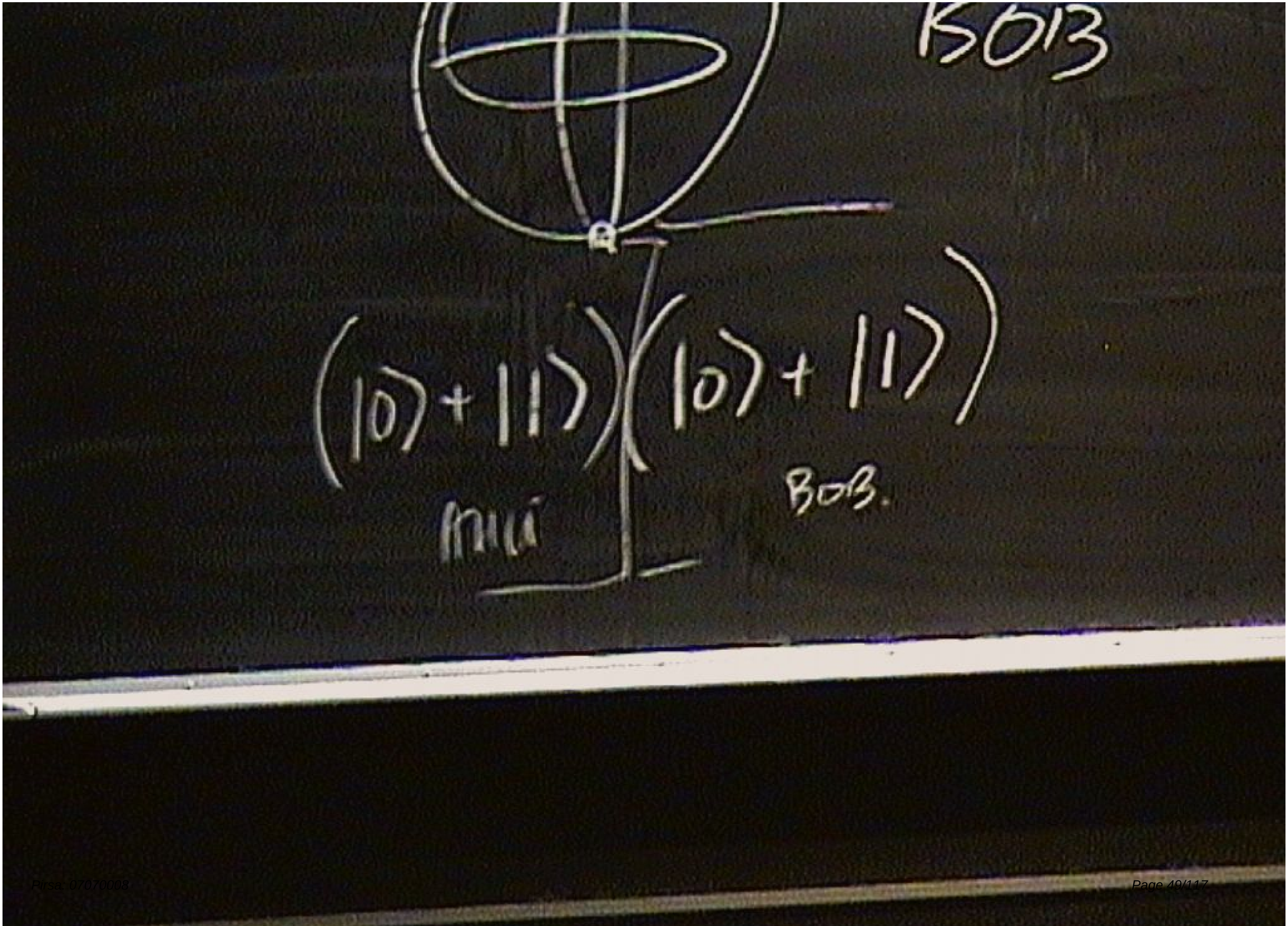
$$\alpha|0\rangle_A|0\rangle_B + \beta|0\rangle_A|1\rangle_B + \gamma|1\rangle_A|0\rangle_B + \delta|1\rangle_A|1\rangle_B$$

Consider the state:

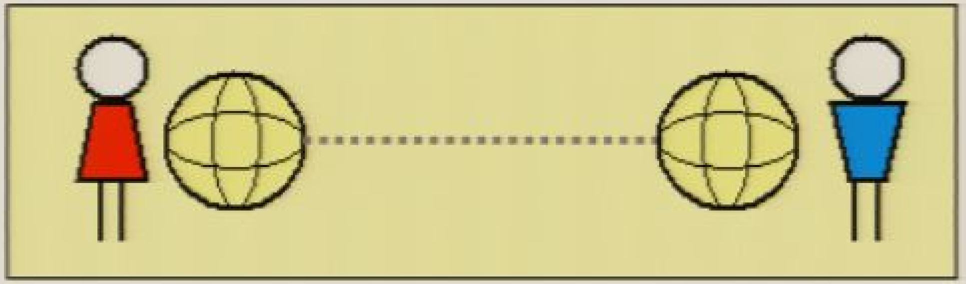
$$\frac{1}{2}|0\rangle_A|0\rangle_B + \frac{1}{2}|0\rangle_A|1\rangle_B + \frac{1}{2}|1\rangle_A|0\rangle_B + \frac{1}{2}|1\rangle_A|1\rangle_B$$

There is an equal chance of all four results. Seeing one atom in a given state yields no information about the other atom at all. The two systems are not at all entangled.


BOB
(|0> + |1>)(|0> + |1>)
BOB.


BOB
(|0⟩ + |1⟩)(|0⟩ + |1⟩)
BOB.

Entanglement



We can apply the superposition principle exactly as before, but now the space of possible states has *four* complex dimensions.

Possible state:

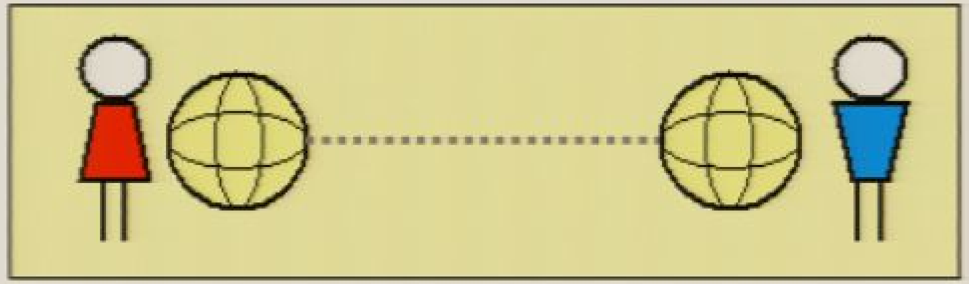
$$\alpha|0\rangle_A|0\rangle_B + \beta|0\rangle_A|1\rangle_B + \gamma|1\rangle_A|0\rangle_B + \delta|1\rangle_A|1\rangle_B$$

Consider the state:

$$\frac{1}{\sqrt{2}}|0\rangle_A|0\rangle_B + \frac{1}{\sqrt{2}}|1\rangle_A|1\rangle_B$$

The two atoms are correlated. They are either both in the ground state, or both in the excited state. When measuring their energy, we will never see them in different states.

Entanglement



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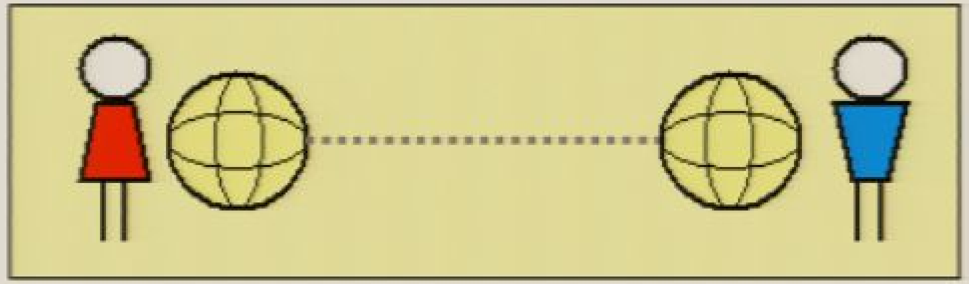
Entanglement



$$\frac{1}{\sqrt{2}}|0\rangle_A|0\rangle_B + \frac{1}{\sqrt{2}}|1\rangle_A|1\rangle_B$$

This kind of correlation isn't too impressive at first. Many objects seem to be able to do this. For example, socks:

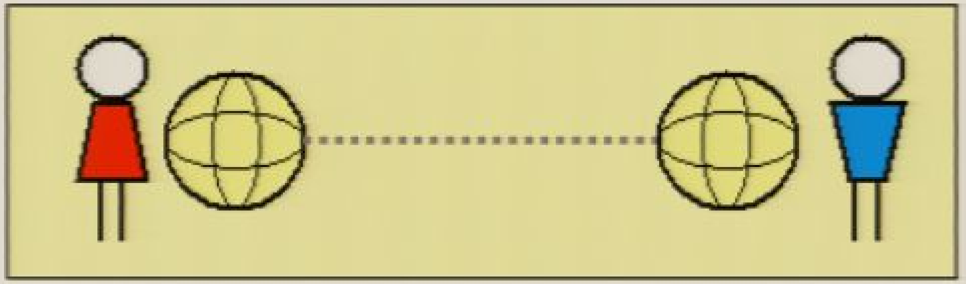
Probability (1/2) =  = | 0 > | 0 >

Probability (1/2) =  = | 1 > | 1 >

And this is true, so long as we stick to asking the same classical question "Is it **0** or is it **1** ? "

But what if we ask a different question?

Entanglement



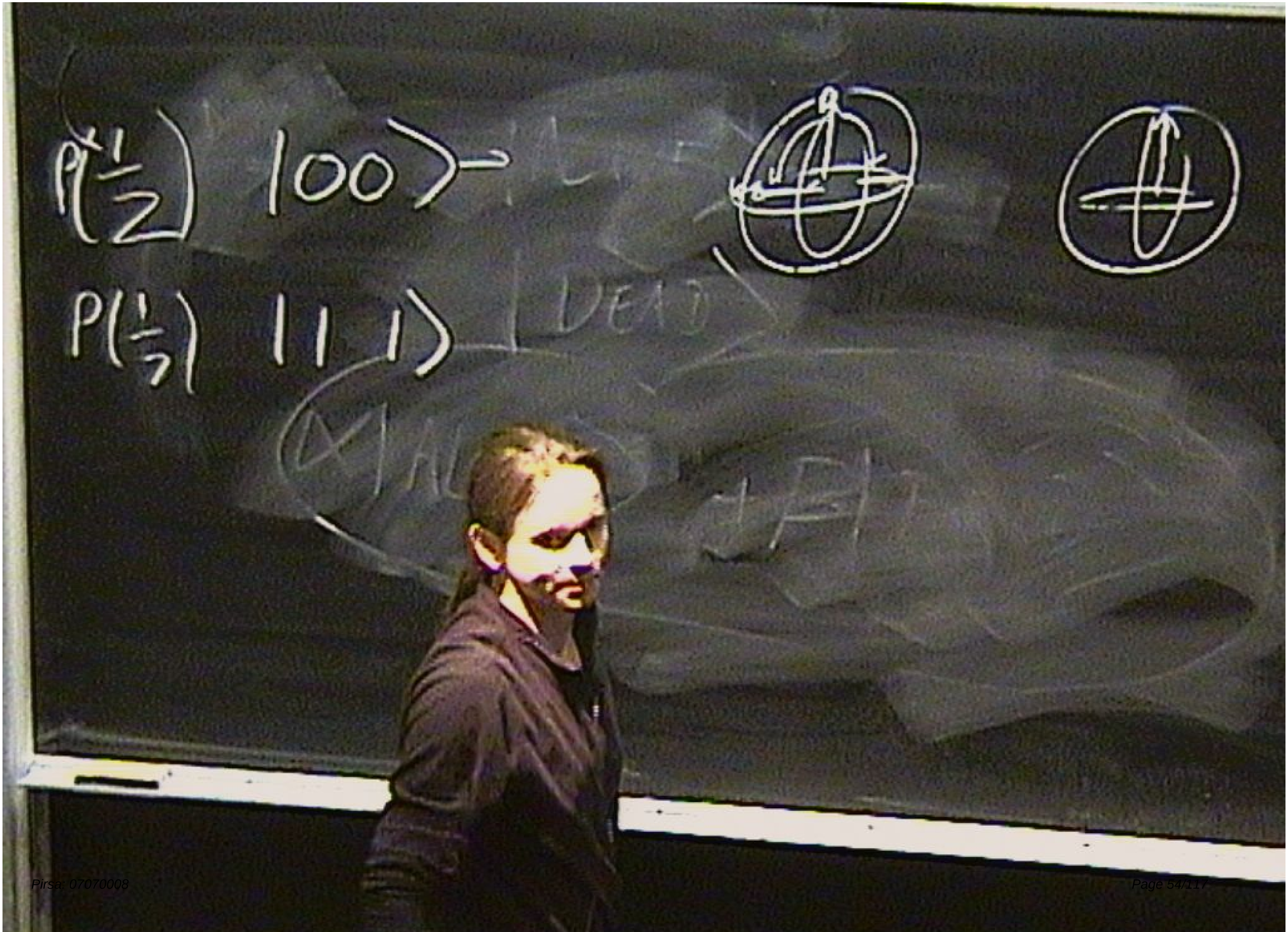
$$\frac{1}{\sqrt{2}}|0\rangle_A|0\rangle_B + \frac{1}{\sqrt{2}}|1\rangle_A|1\rangle_B$$

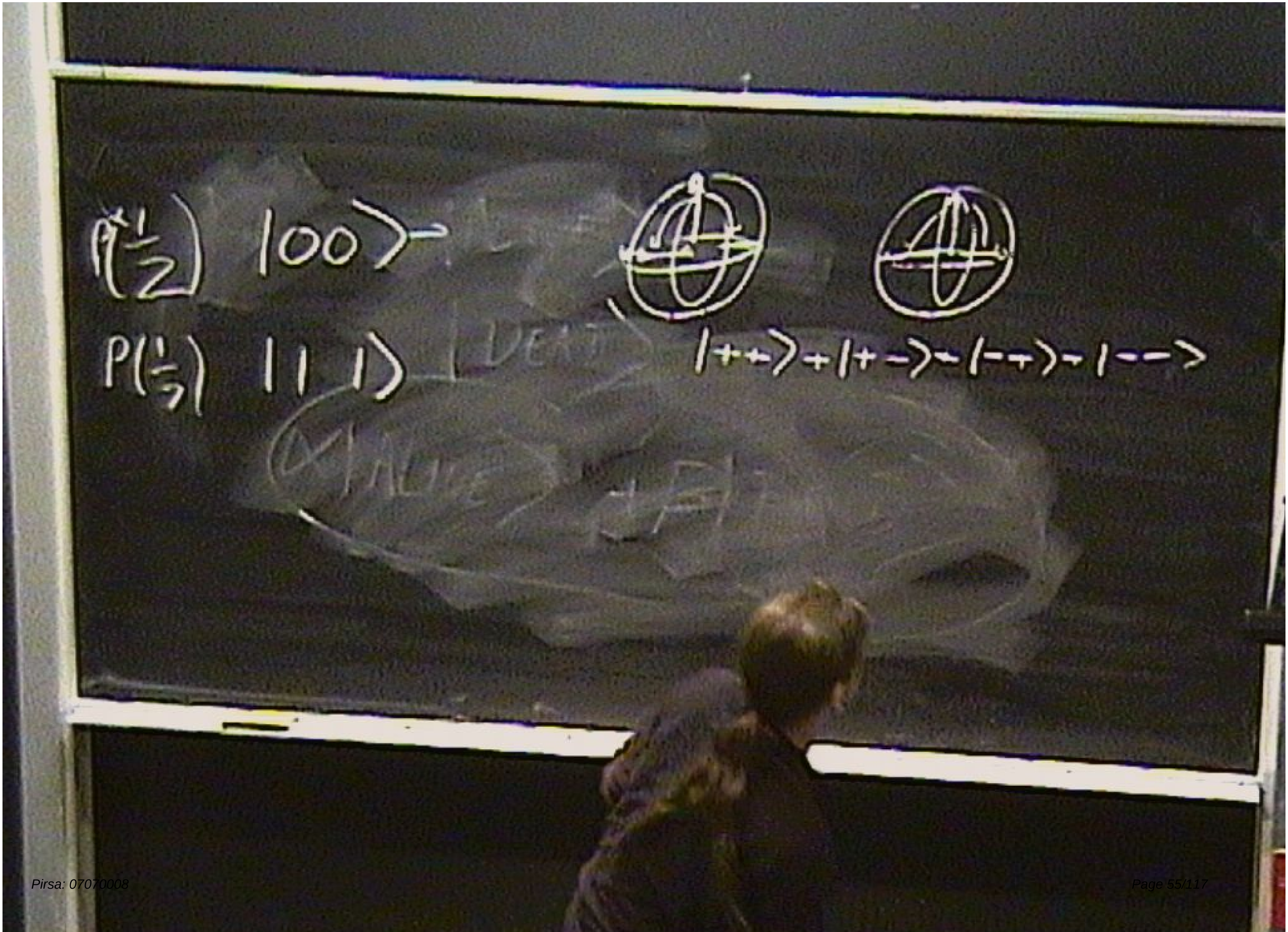
According to quantum mechanics, we should be able to ask each system

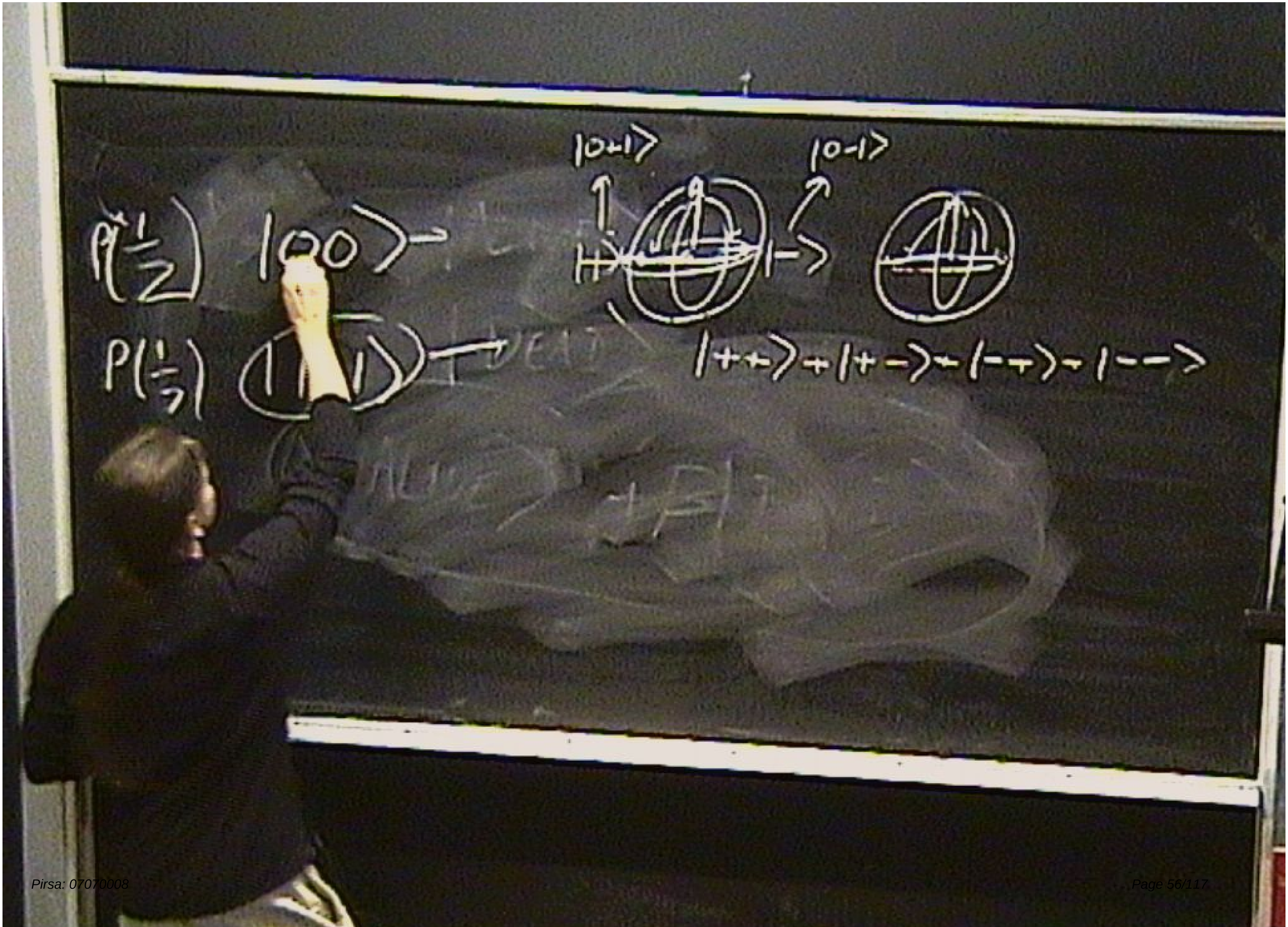
"Are you in state $|0\rangle + |1\rangle$ or in state $|0\rangle - |1\rangle$?"

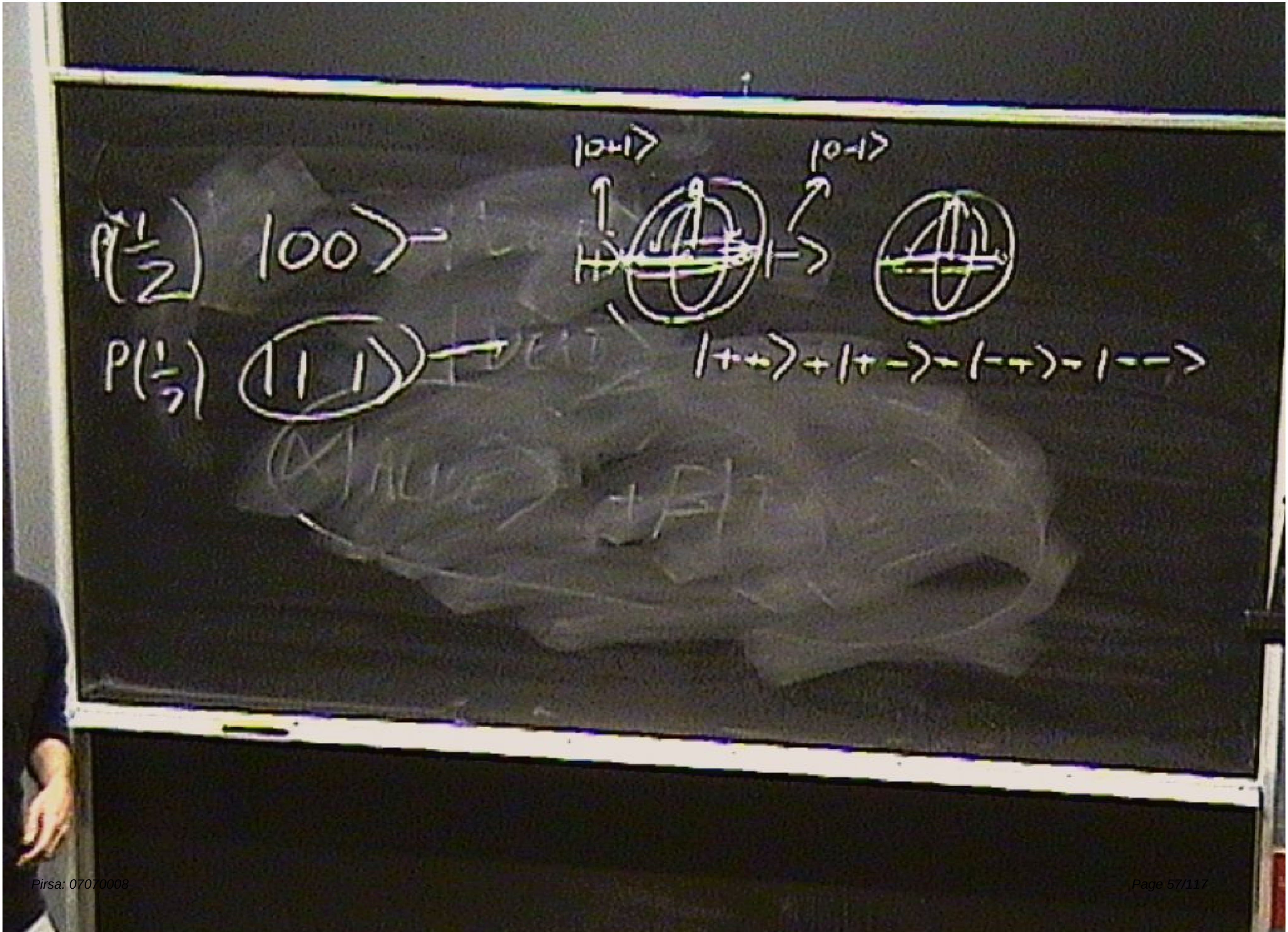
What happens then?

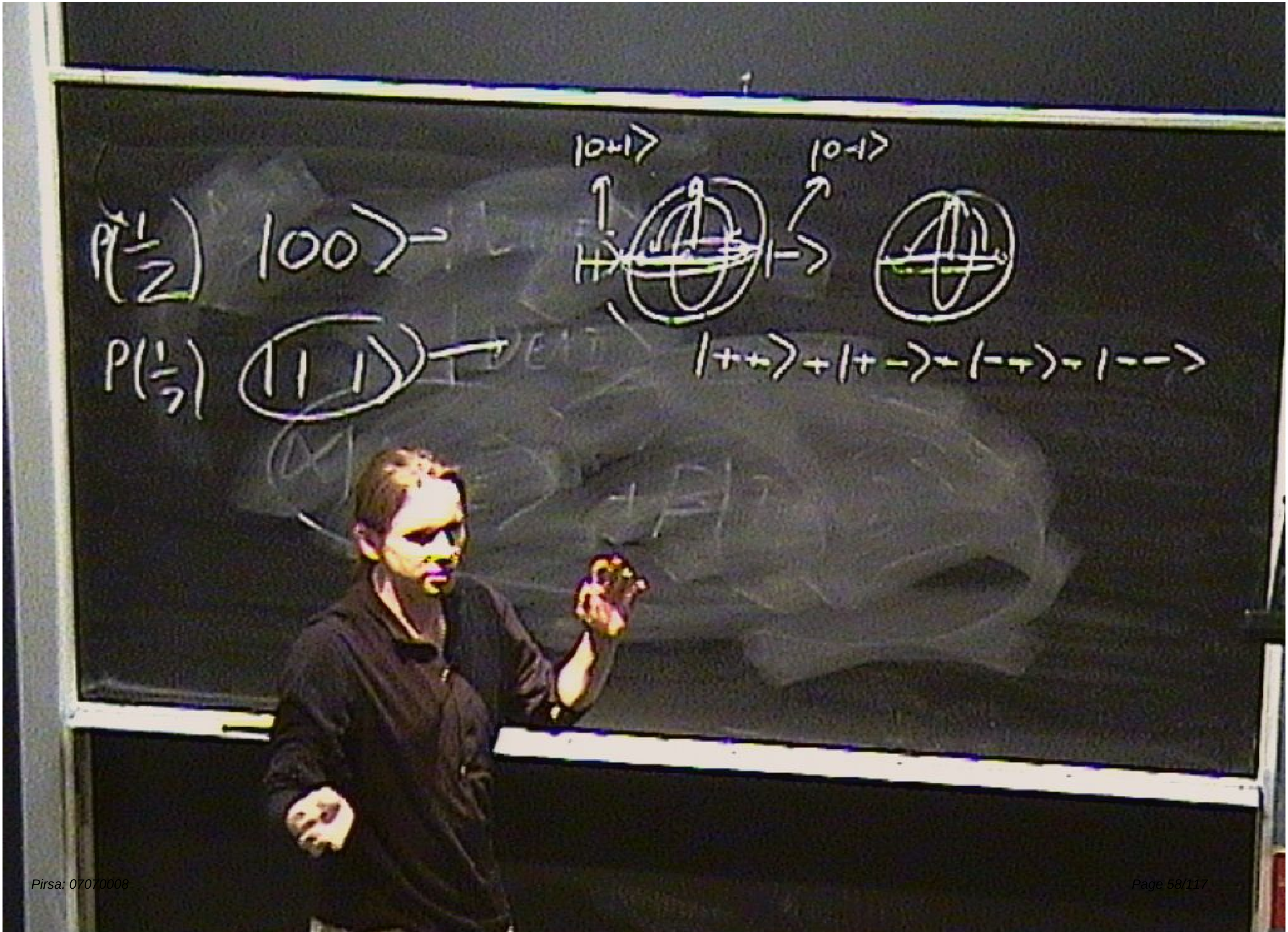
To see what would happen, we just need to do some very simple mathematics, and reexpress our state in terms of $|0\rangle + |1\rangle$ and $|0\rangle - |1\rangle$.

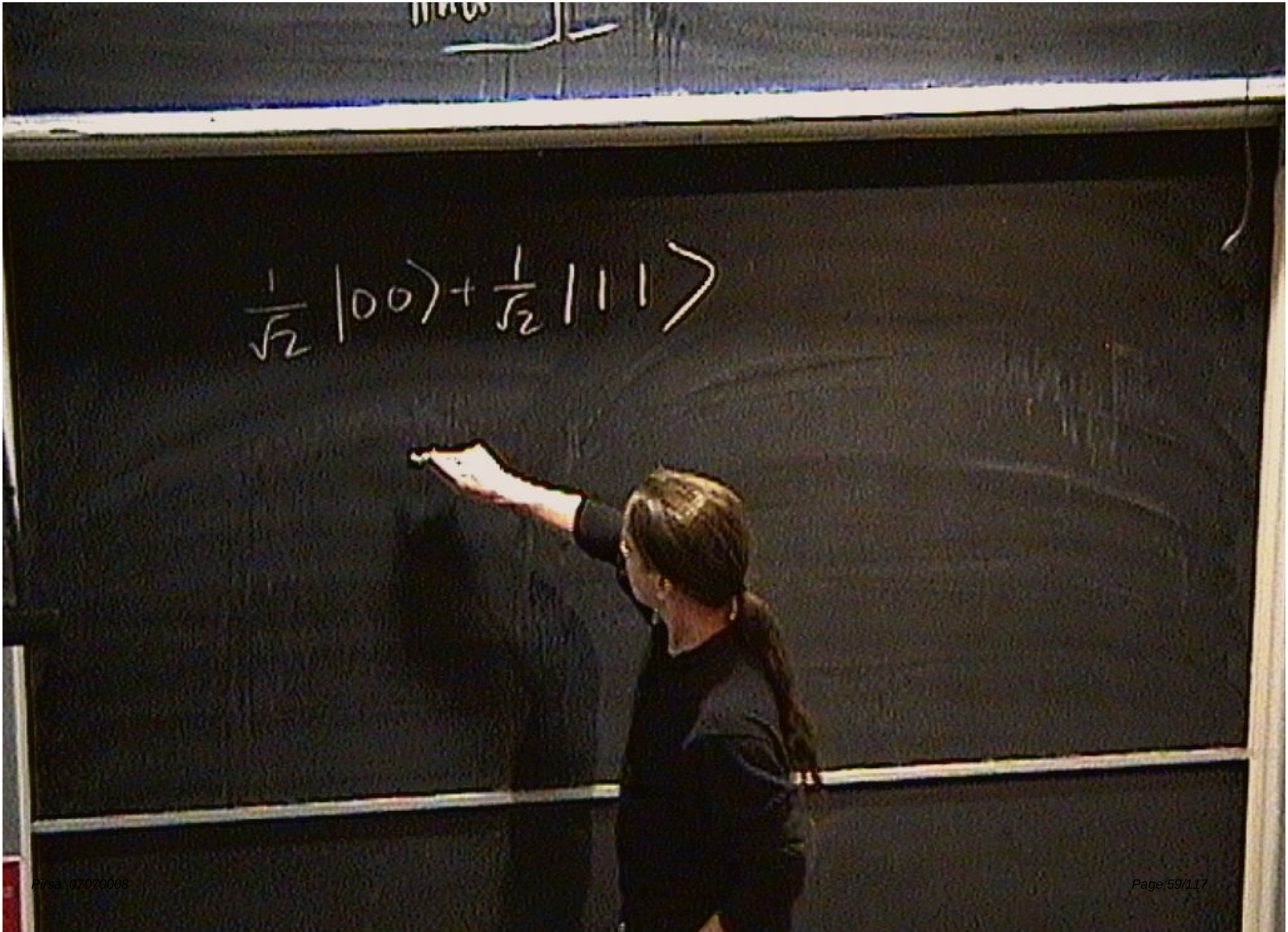

P(1/2) |00>
P(1/2) |11>
|DEAD>

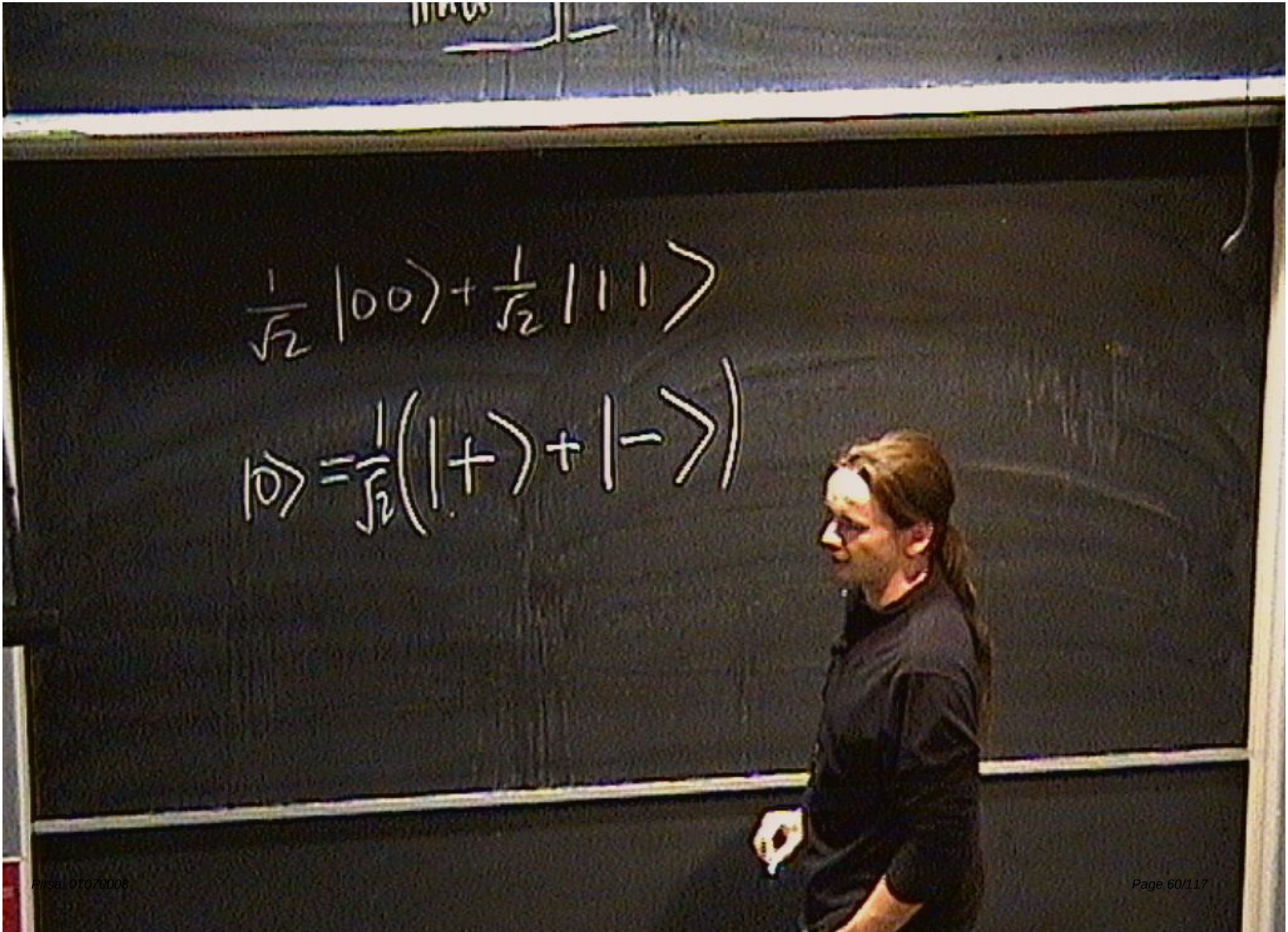

P(½) |00>
P(½) |11>
|++>+|+−>+|−+>+|−−>


|0+1>
|0-1>
|00>
|11>

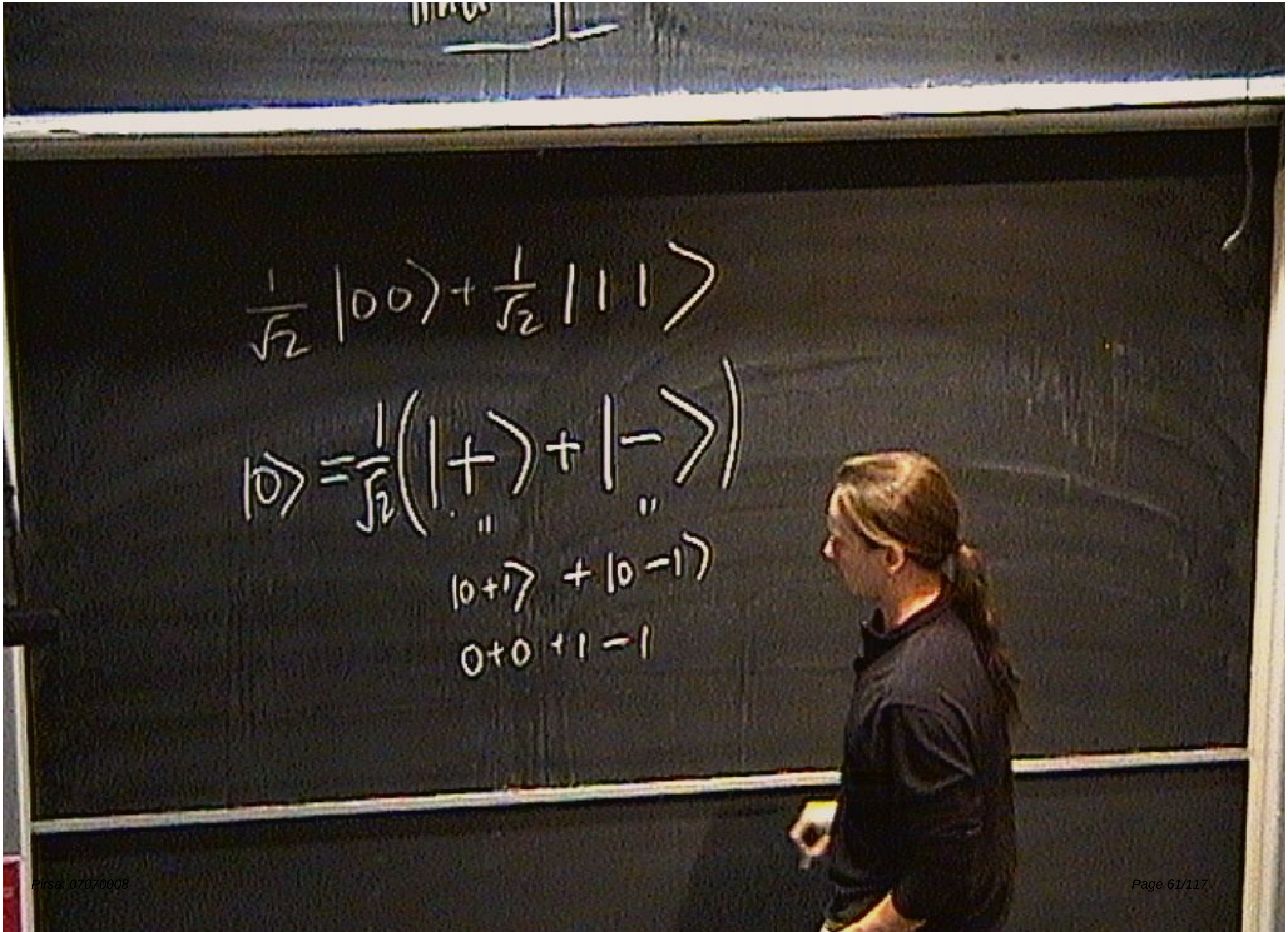

|0+1>
|0-1>
P(1/2) |00>
P(1/2) |11>
|++>+|+->+|-+>-|-->


|0+1>
|0-1>
P(1/2) |00>
P(1/2) |11>
|++> + |+-> + |-+> + |-->

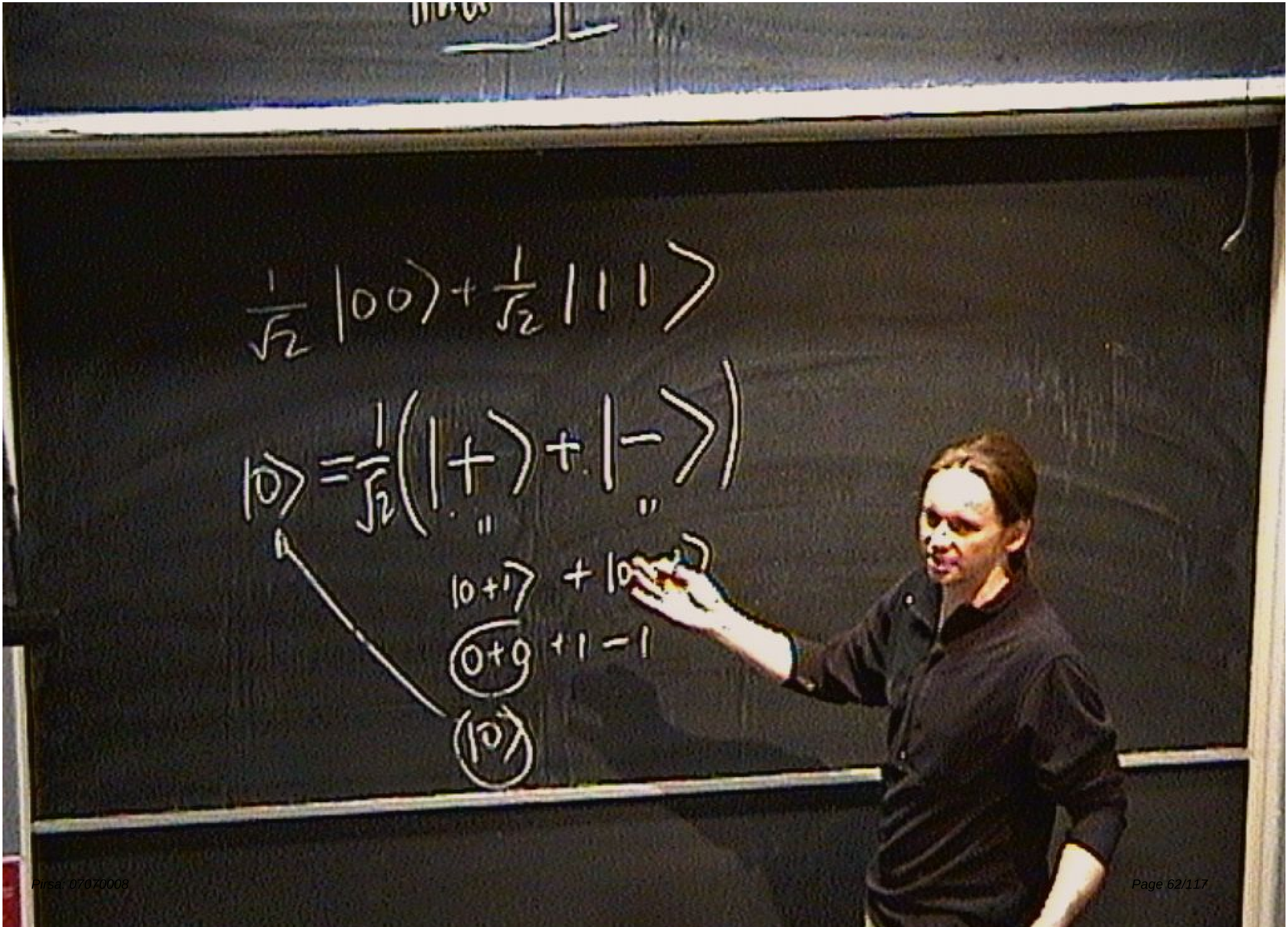

$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

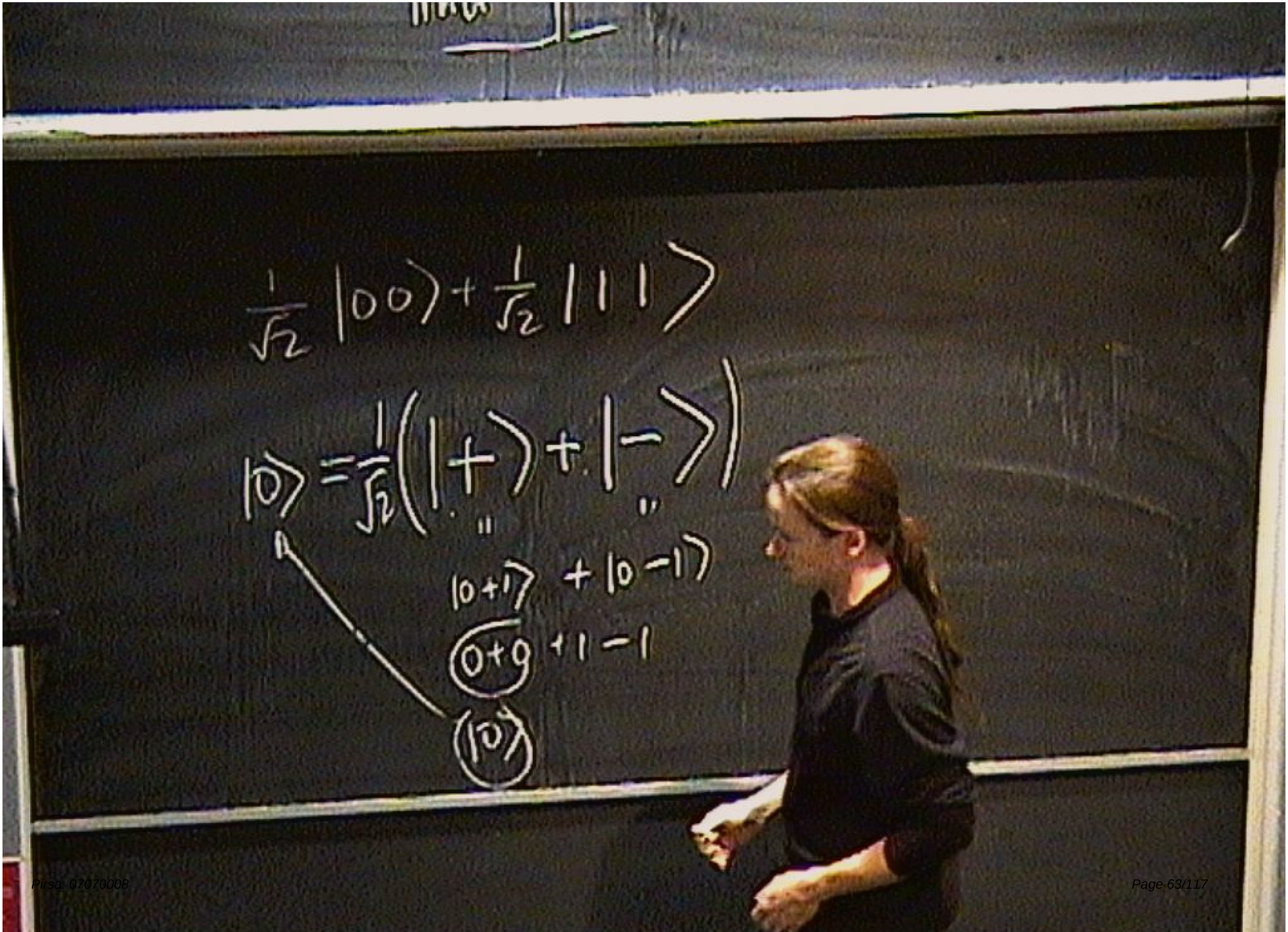


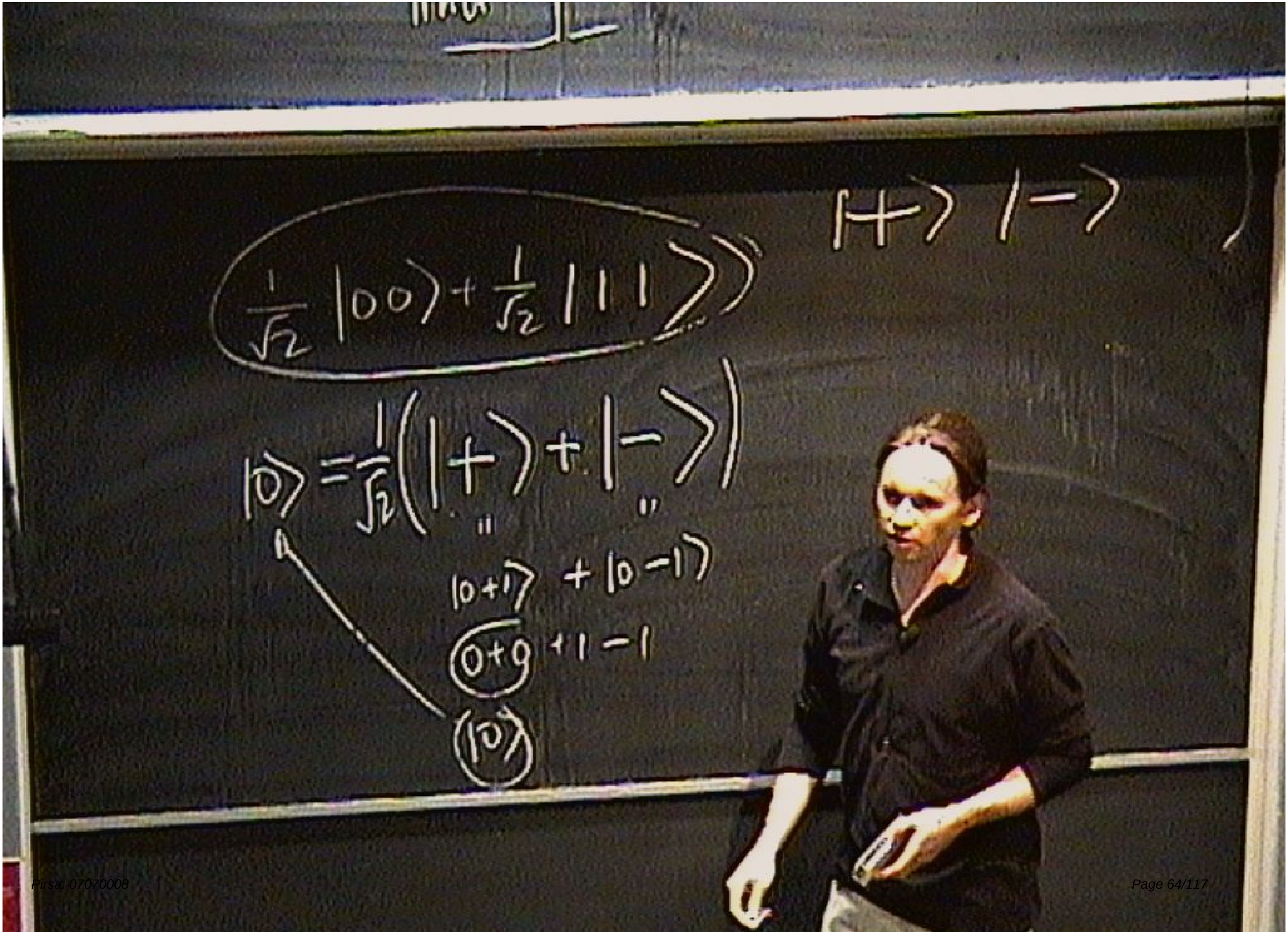
$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$
$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$



$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$
$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$
$|0+1\rangle + |0-1\rangle$
$0+0 + 1-1$


$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$
$|0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$
$|0+\rangle + |0-\rangle$
$0+0 \quad +1 \quad -1$
$|0\rangle$


1/√2 |00⟩ + 1/√2 |11⟩
|0⟩ = 1/√2 (|+⟩ + |−⟩)
|0+1⟩ + |0−1⟩
0+0 +1−1
|0⟩



$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$
$|+\rangle$ $|-\rangle$
$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$
"   "
$|0+1\rangle + |0-1\rangle$
$0+0$ $+1-1$
$|0\rangle$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$$|0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$$

$$|0+1\rangle + |0-1\rangle$$

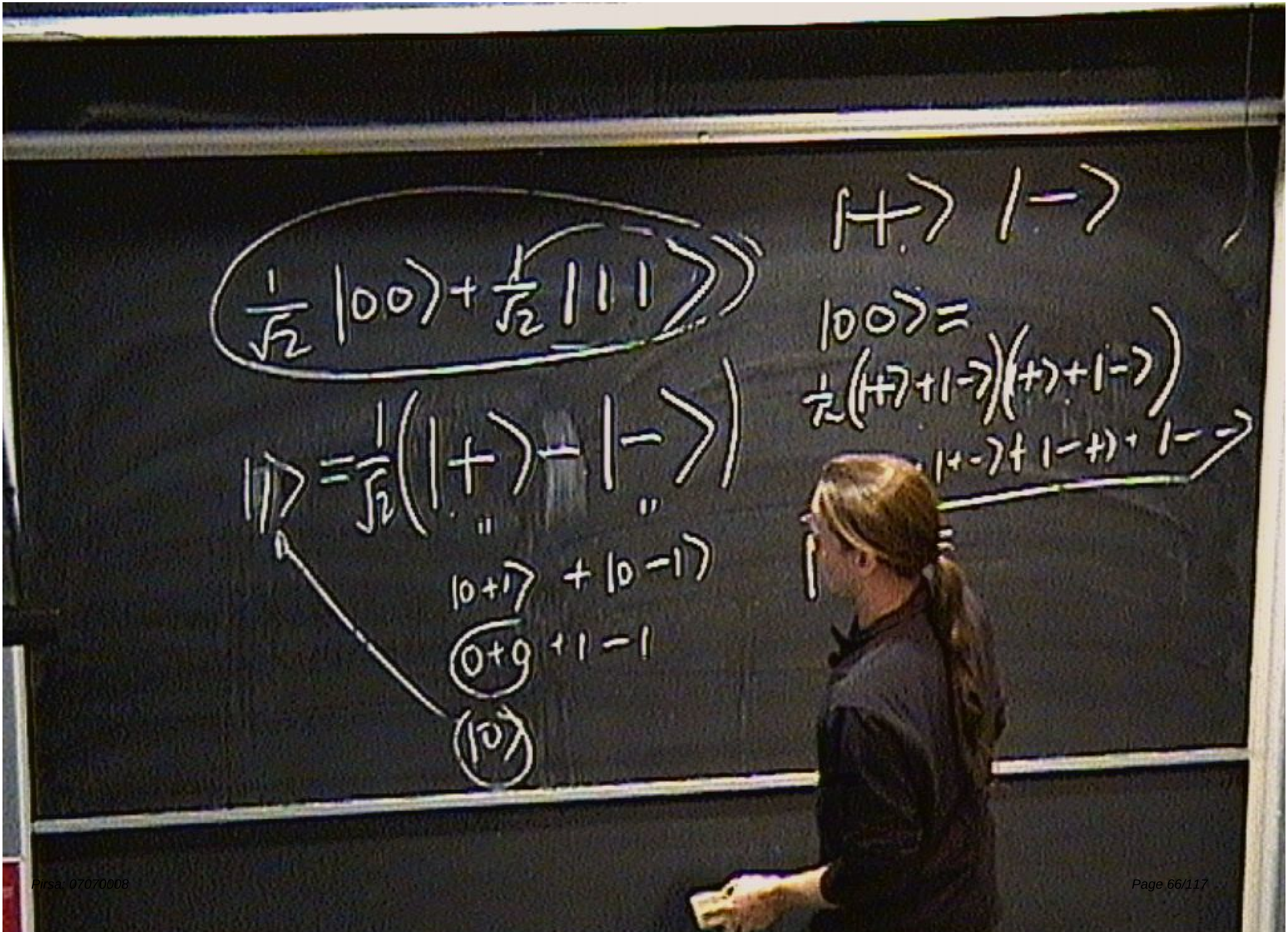
$$0+0+1-1$$

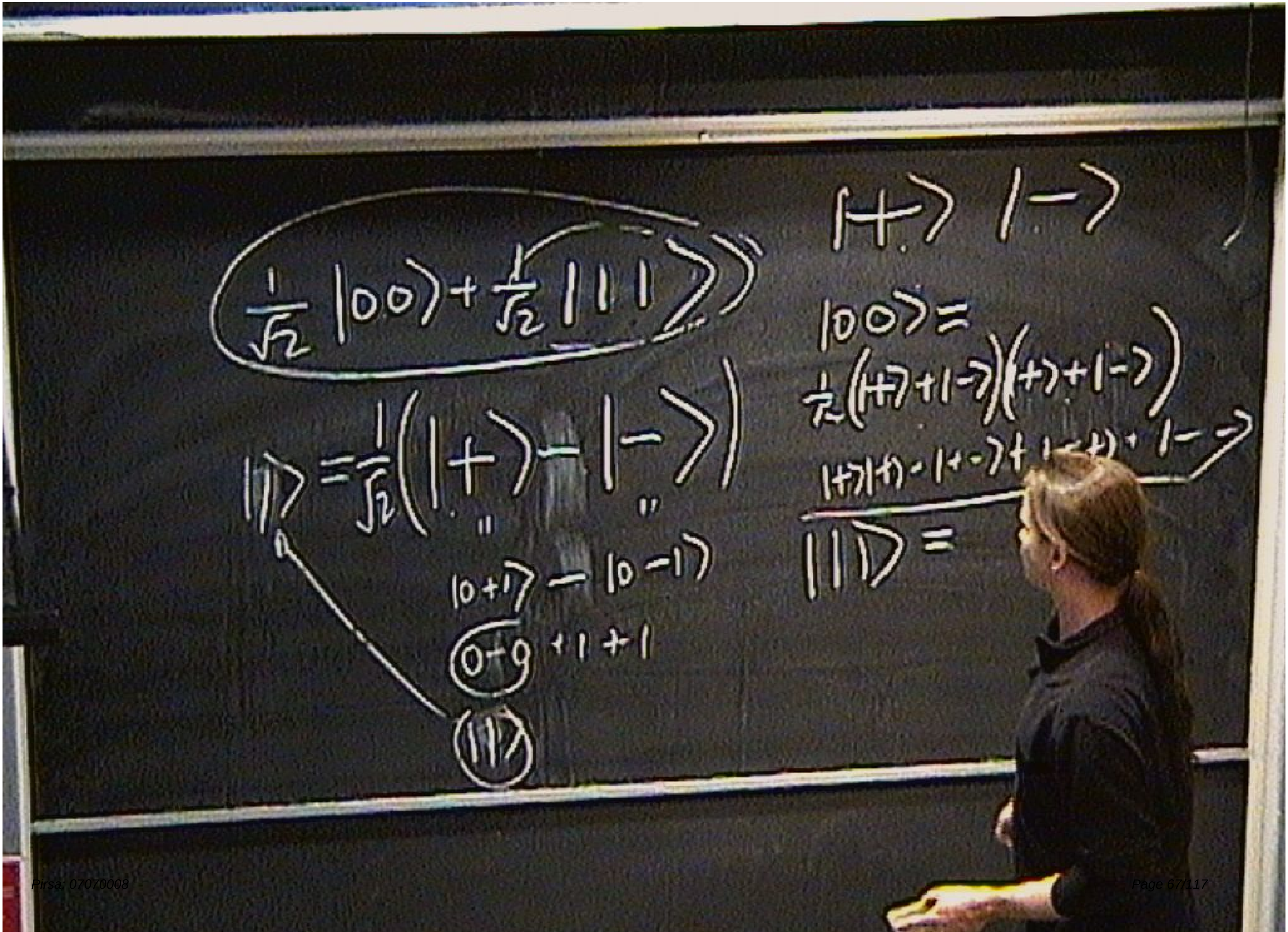
$$|0\rangle$$

$$|+\rangle \quad |-\rangle$$

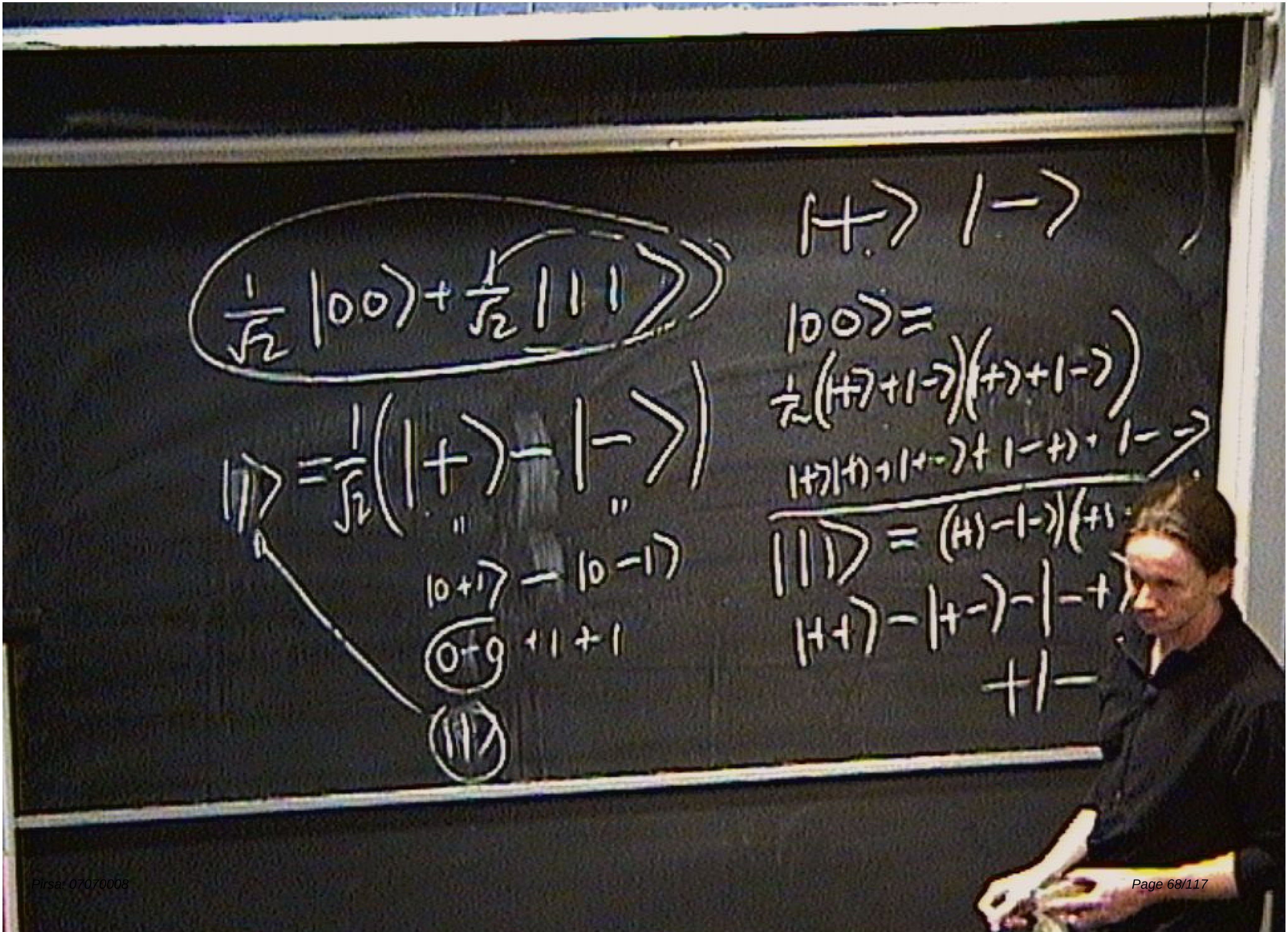
$$|00\rangle$$

$$\frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)\left(|+\rangle + |-\rangle\right)$$

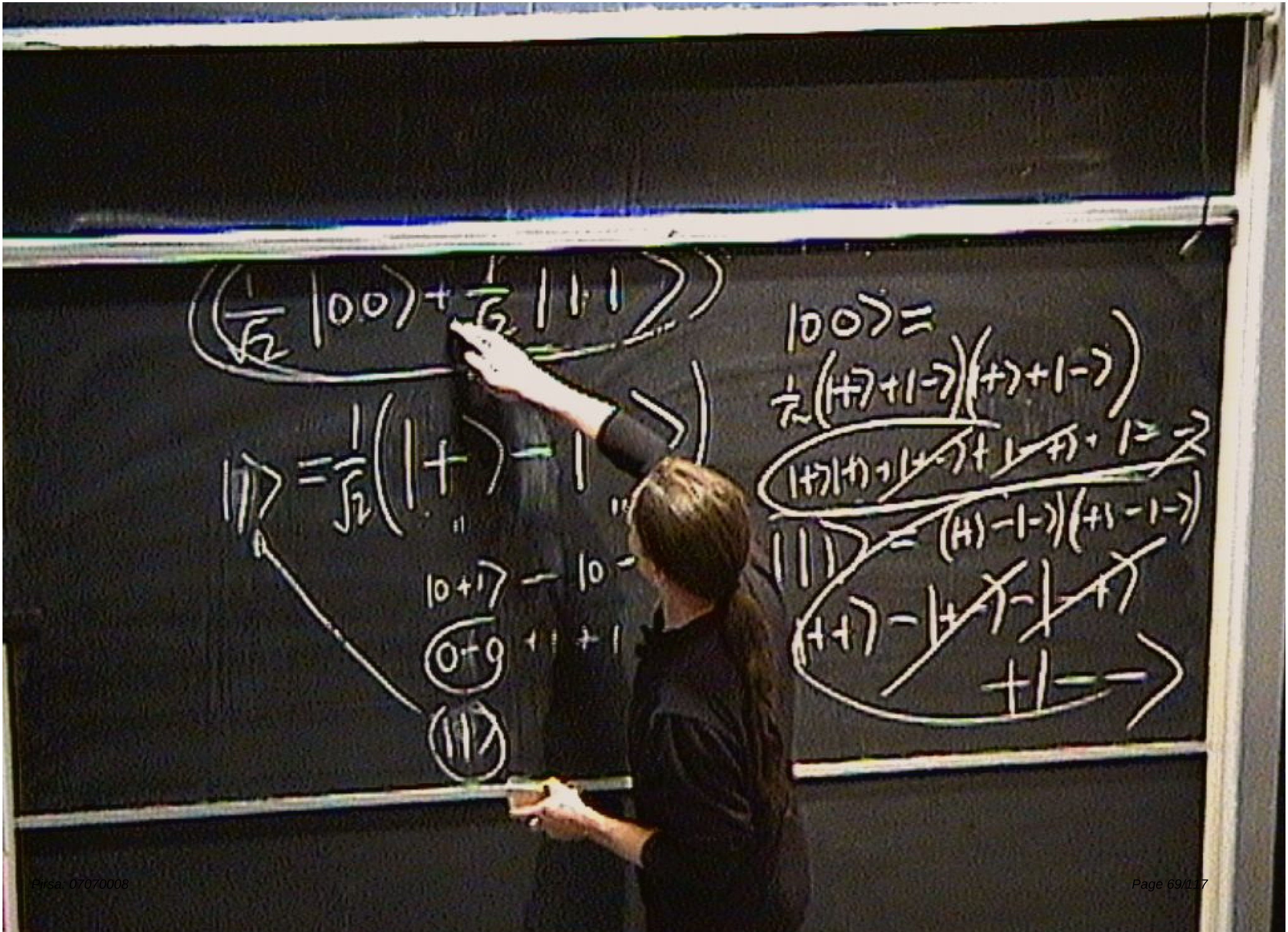

1/√2 |00> + 1/√2 |11>
|+> |->
|00> =
1/2 (|+> + |->)(|+> + |->)
|+-> + |-+> + |-->
|1> = 1/√2 (|+> - |->)
|0+1> + |0-1>
0+0 +1 -1
|0>

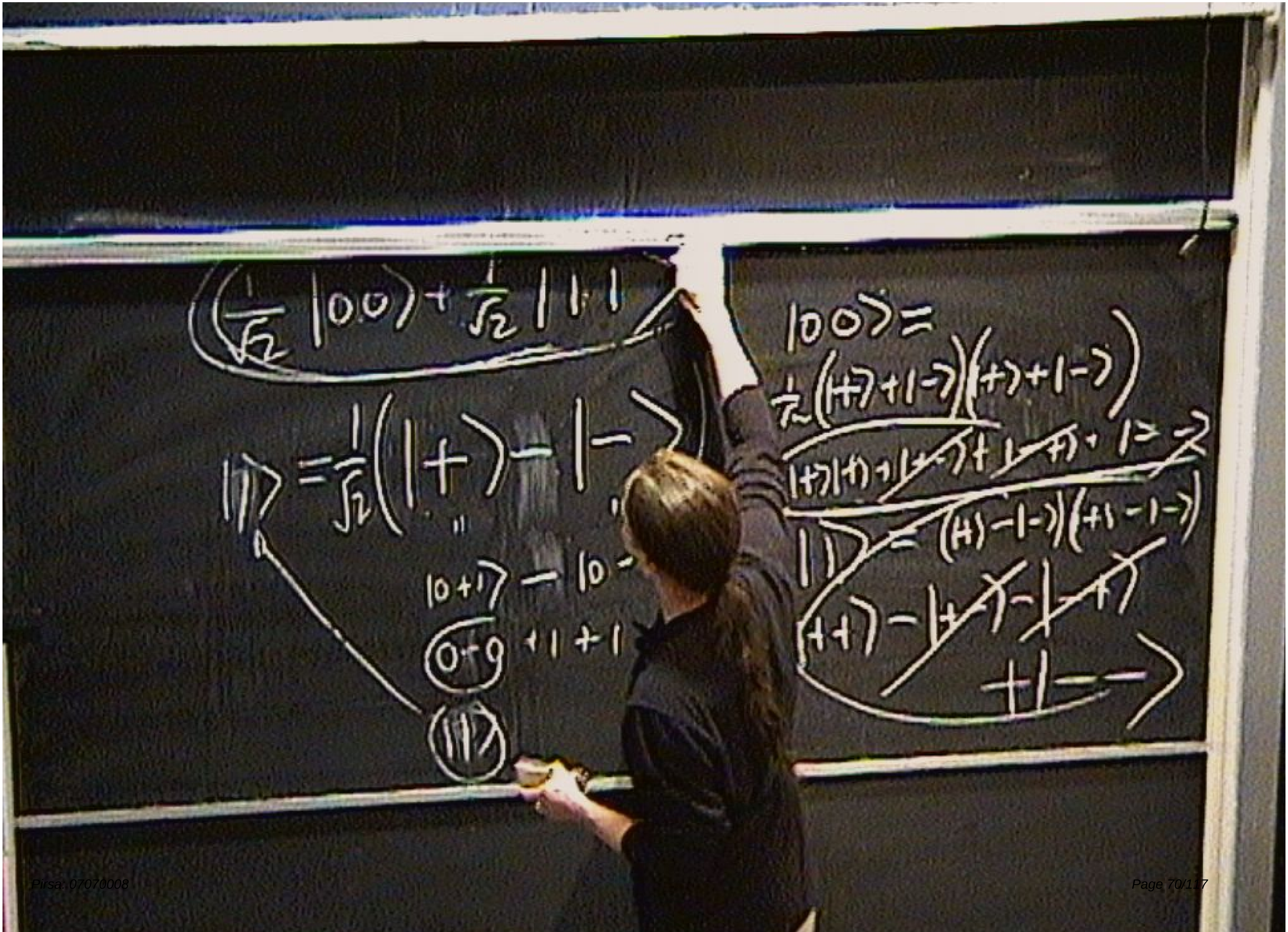


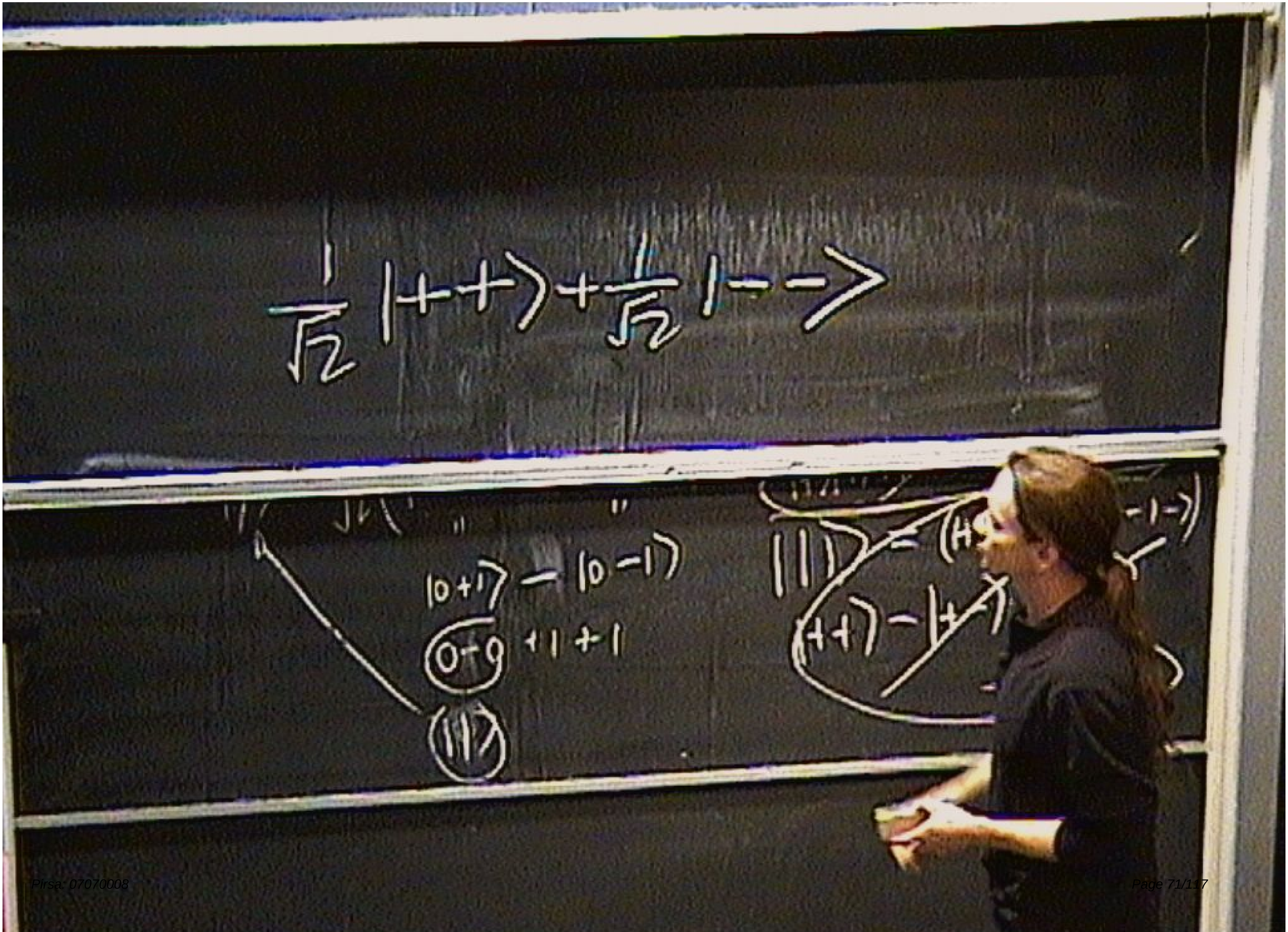
$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$
$|11\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$
$|0+1\rangle - |0-1\rangle$
$0+0 +1 +1$
$|11\rangle$
$|+\rangle \quad |-\rangle$
$|00\rangle =$
$\frac{1}{2}(|+\rangle + |-\rangle)(|+\rangle + |-\rangle)$
$|11\rangle =$



|+> |->
1/√2 |00> + 1/√2 |11>
|00> =
1/2 (|+>+|->)(|+>+|->)
|++>+|+->+|-+>+|-->
|1> = 1/√2 (|+> − |->)
|0+1> − |0−1>
0+0 +1 +1
|1>
|11> = (|+>−|->)(|+>
|++> − |+-> − |-+>
+|−




|00⟩ =
|0+⟩ − |0−⟩


$\frac{1}{\sqrt{2}}|{+}{+}\rangle+\frac{1}{\sqrt{2}}|{-}{-}\rangle$

$$\frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle$$

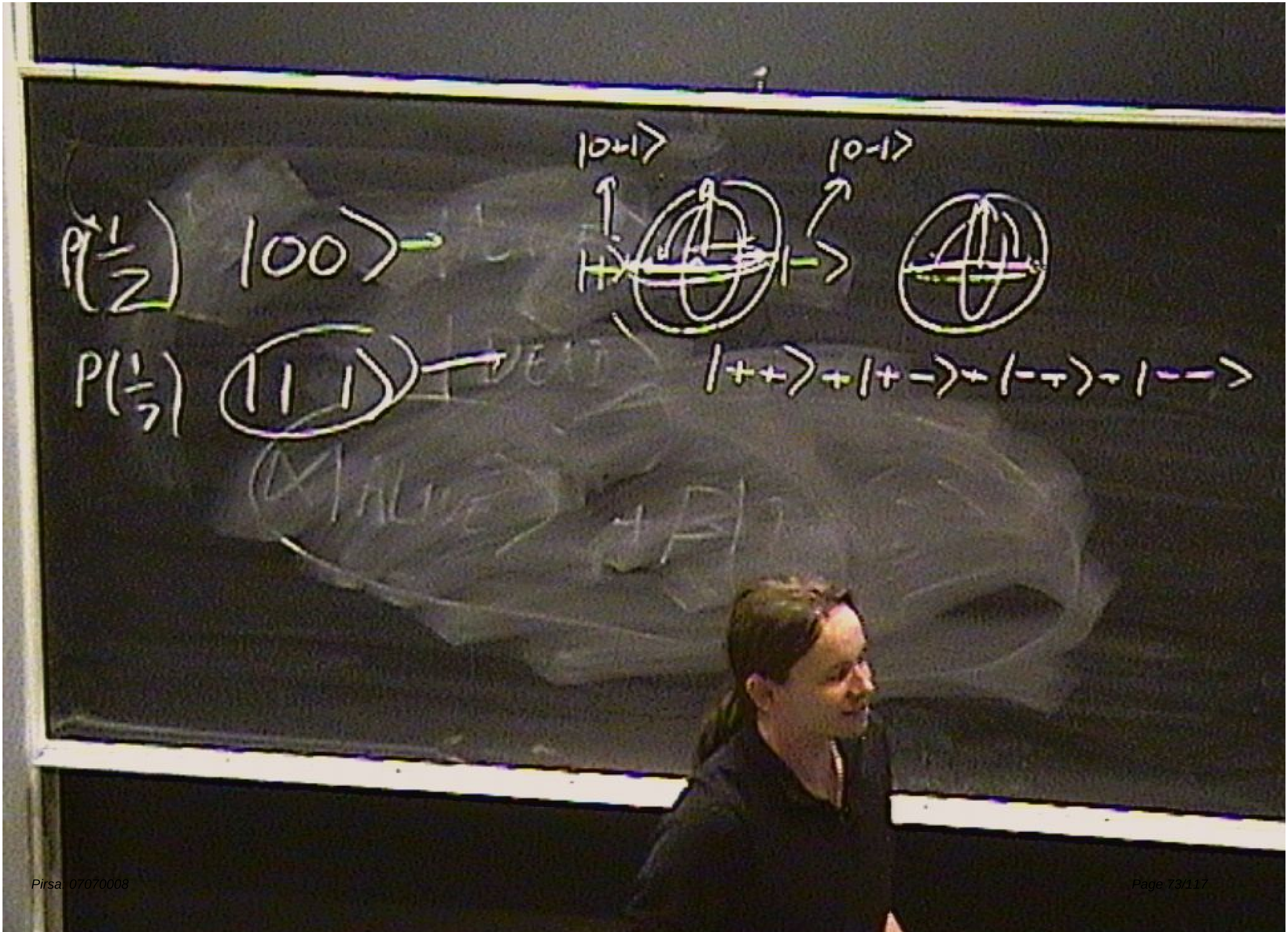
$$|0+\rangle - |0-\rangle$$

$0-0 \quad +1 \quad +1$

$|11\rangle$

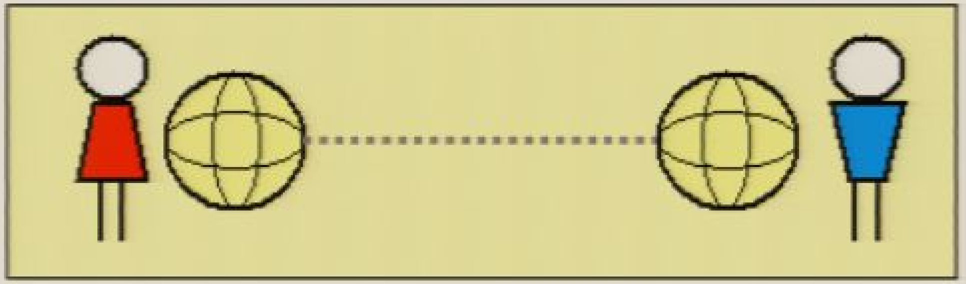
$$|11\rangle = (|+\rangle - |-\rangle)(|+\rangle - |-\rangle)$$

$$|++\rangle - \cancel{|+-\rangle} - \cancel{|-+\rangle} + |--\rangle$$



P(1/2) |00>
P(1/2) |11>
|0+1>
|0-1>
|+>
|->
|++> + |+-> + |-+> - |-->

Entanglement



$$\frac{1}{\sqrt{2}}|0\rangle_A|0\rangle_B + \frac{1}{\sqrt{2}}|1\rangle_A|1\rangle_B$$

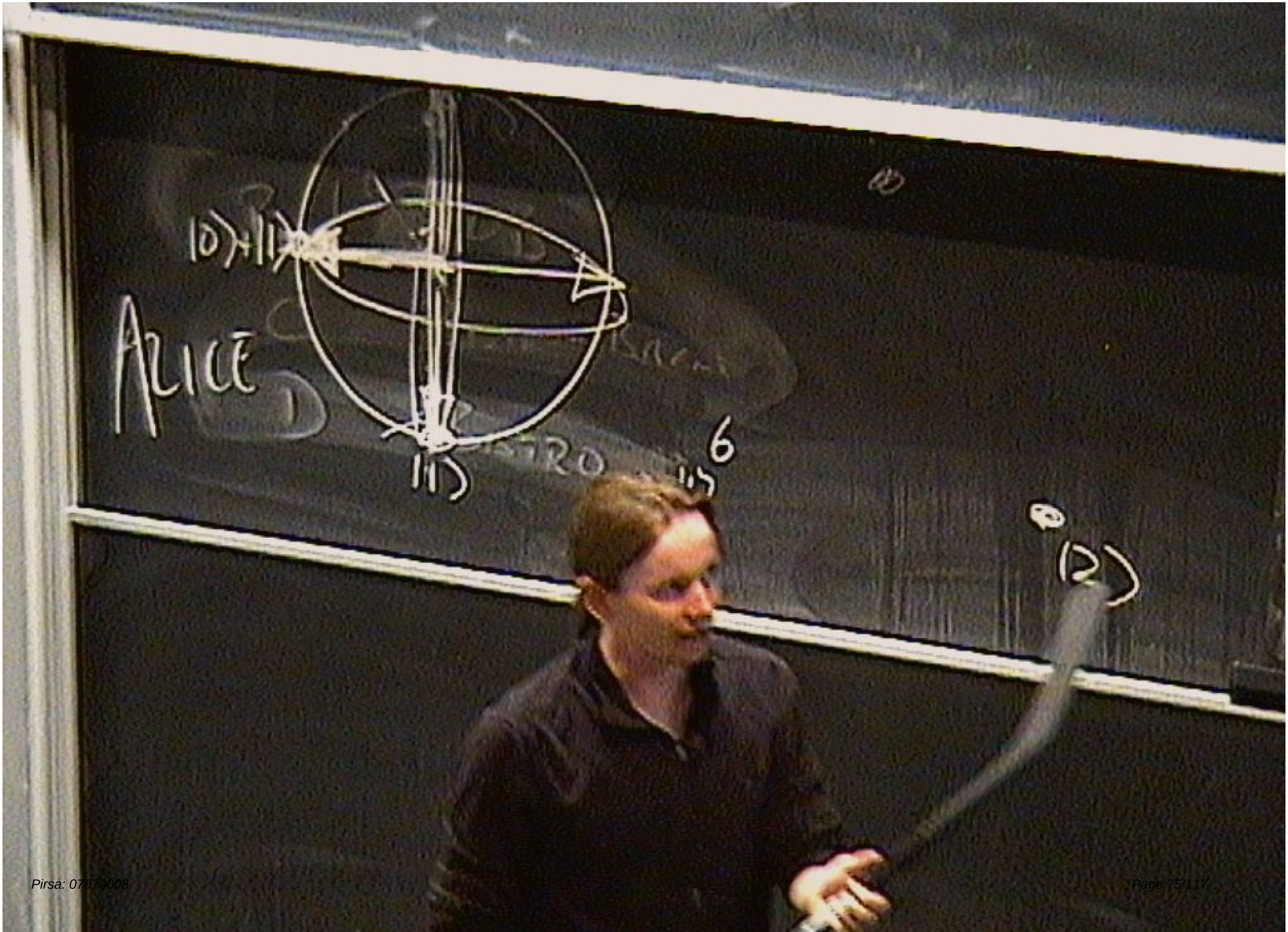
If we have a classical mixture, the correlations are lost when we ask our system a different question.

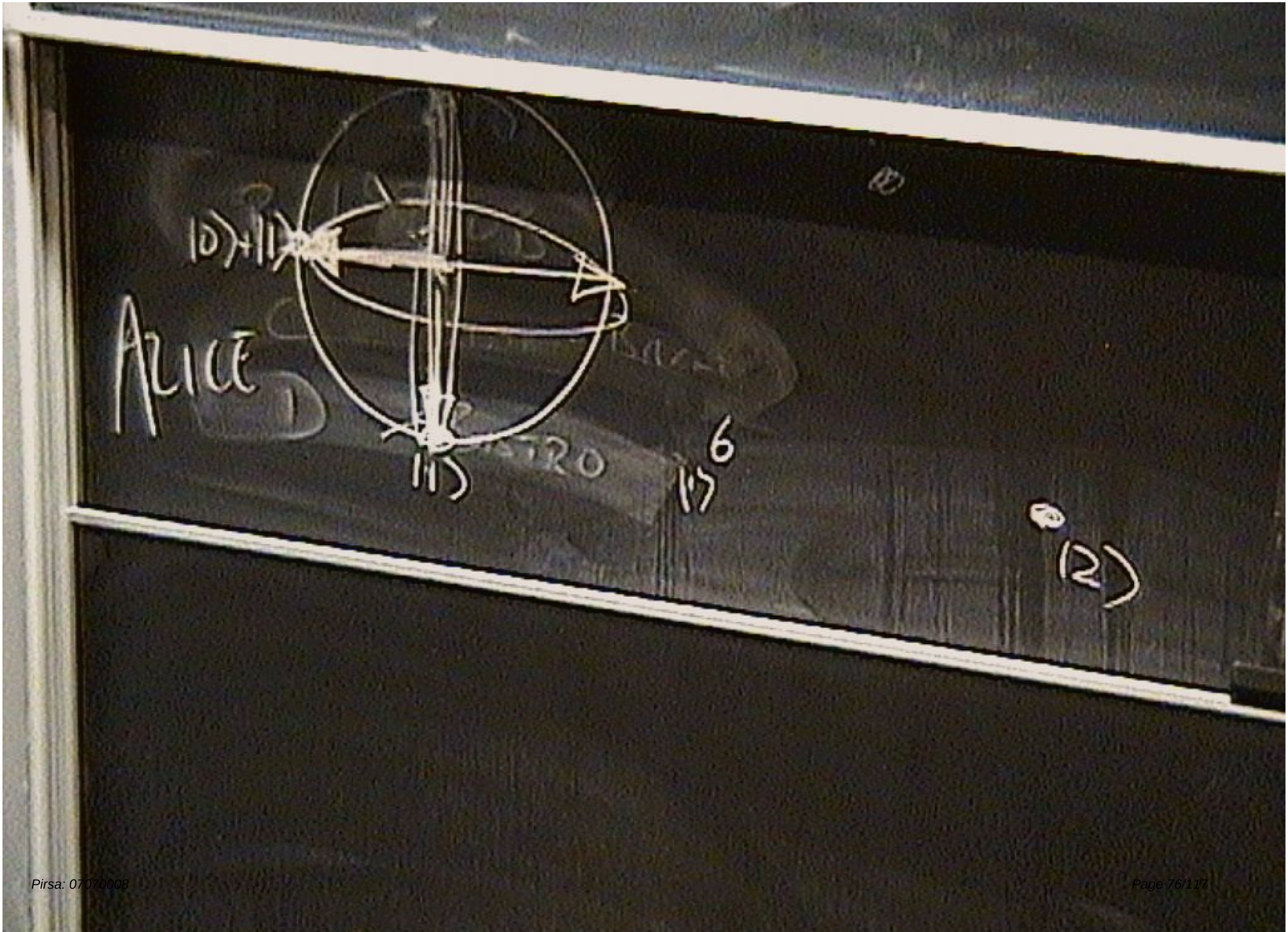
With an entangled state, the correlations are so strong that they exist *for all possible questions we might ask*.

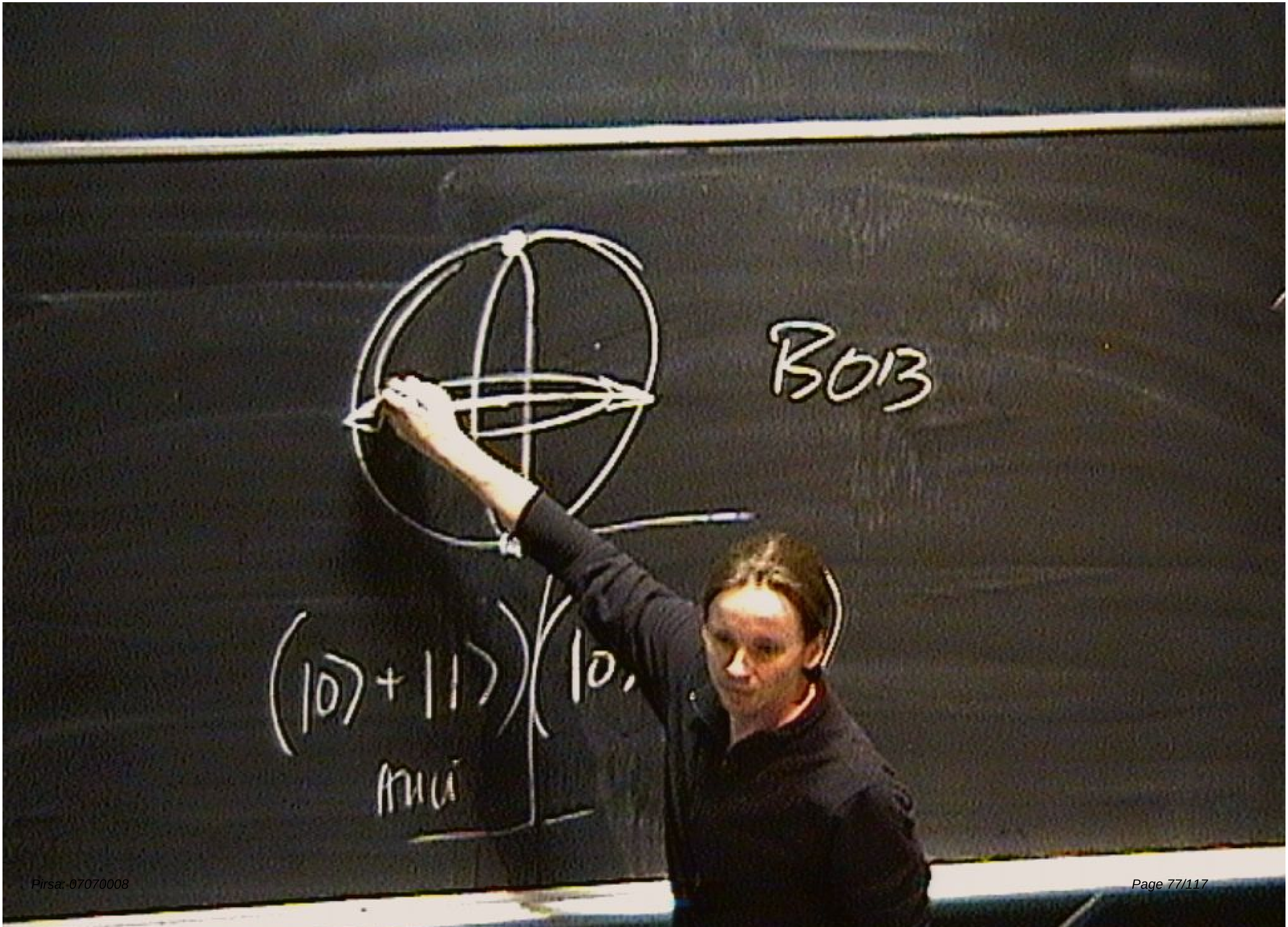
Whatever question we ask system A, and whatever the result, system B will always behave as if it were in exactly the same state.

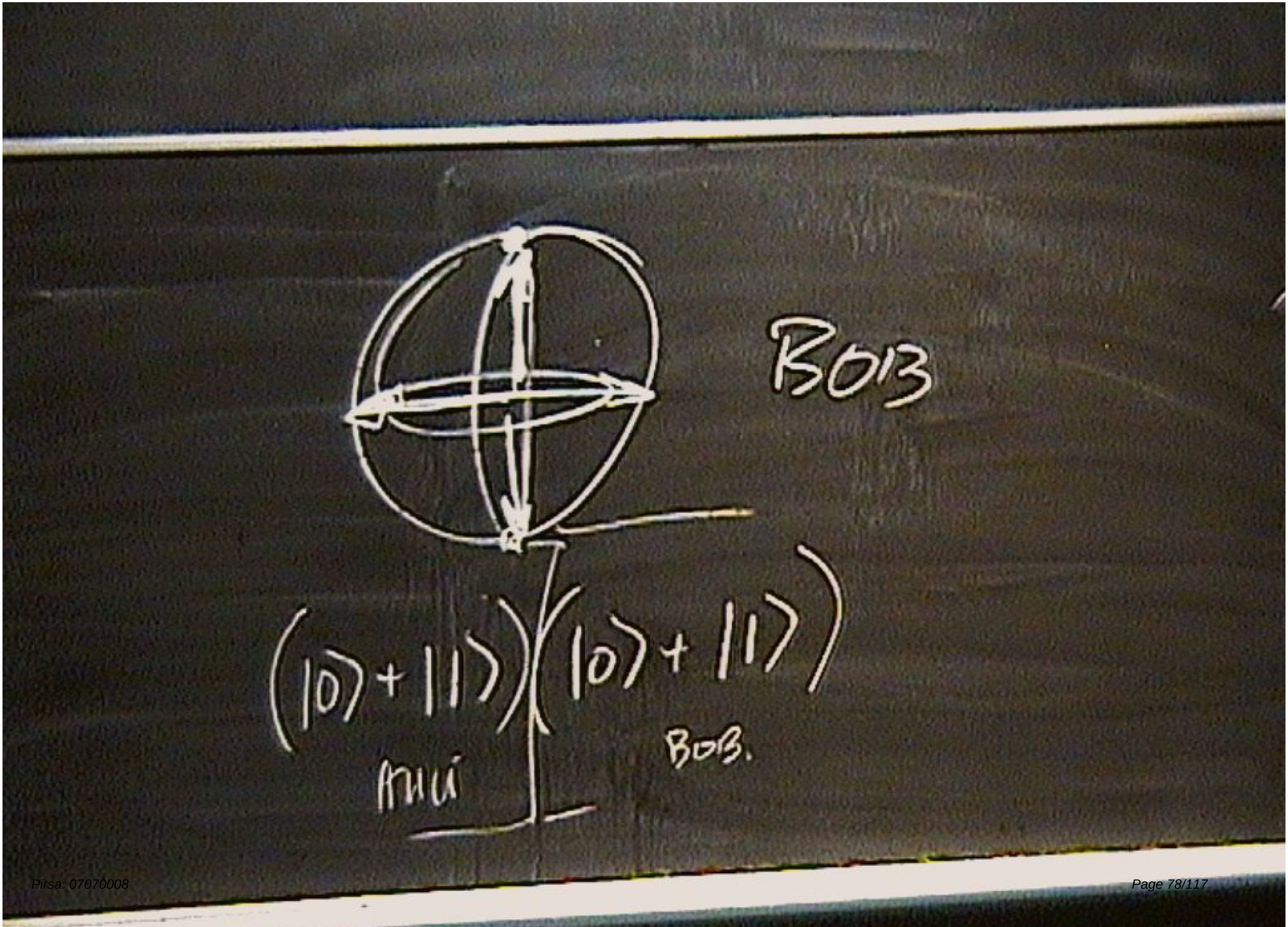
But B couldn't possibly know what measurement we performed on A. How does B know whether to act like $|0\rangle$ or act like $|0\rangle + |1\rangle$?

But how does one system seem to know what question we asked the other?

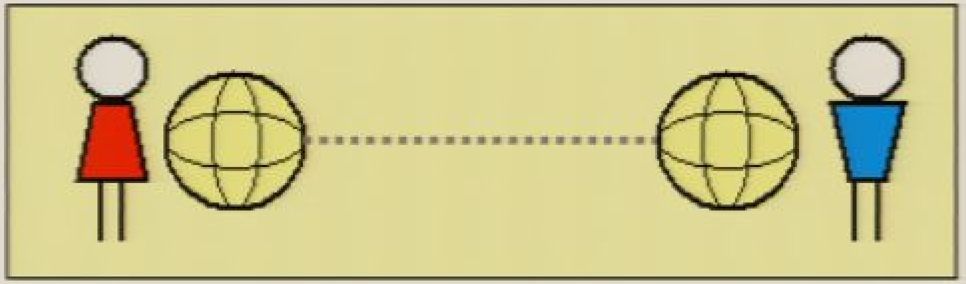

|0⟩+|1⟩
ALICE
6
|1⟩
|2⟩


|0⟩+|1⟩
ALICE
|1⟩
|7⟩
6
|2⟩


BOB
$(|0\rangle + |1\rangle)$


BOB
(|0⟩ + |1⟩)(|0⟩ + |1⟩)
BOB.

Randomness



$$\frac{1}{\sqrt{2}}|0\rangle_A|0\rangle_B + \frac{1}{\sqrt{2}}|1\rangle_A|1\rangle_B$$

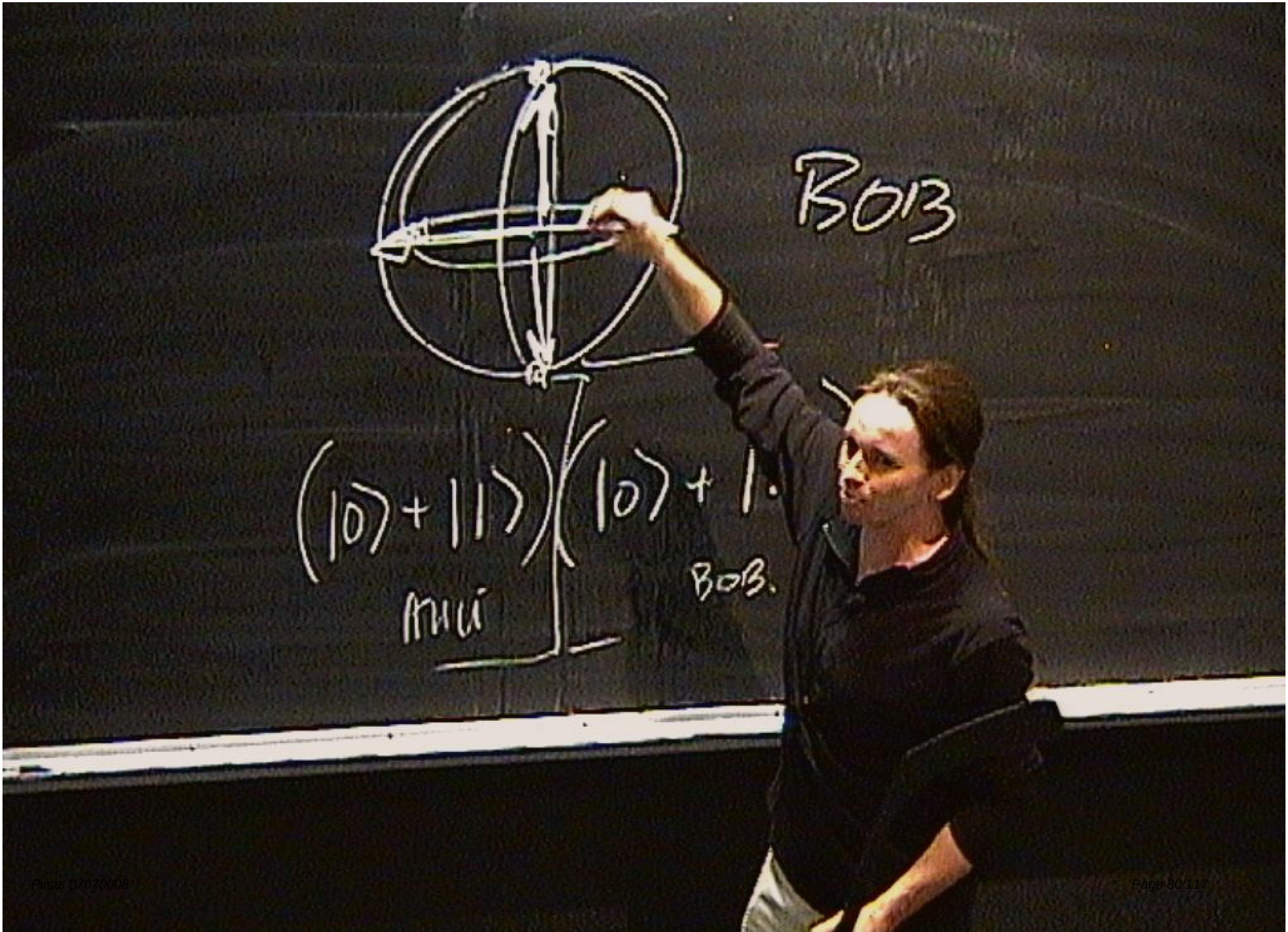
Despite the very strong correlations, no information can be transmitted.

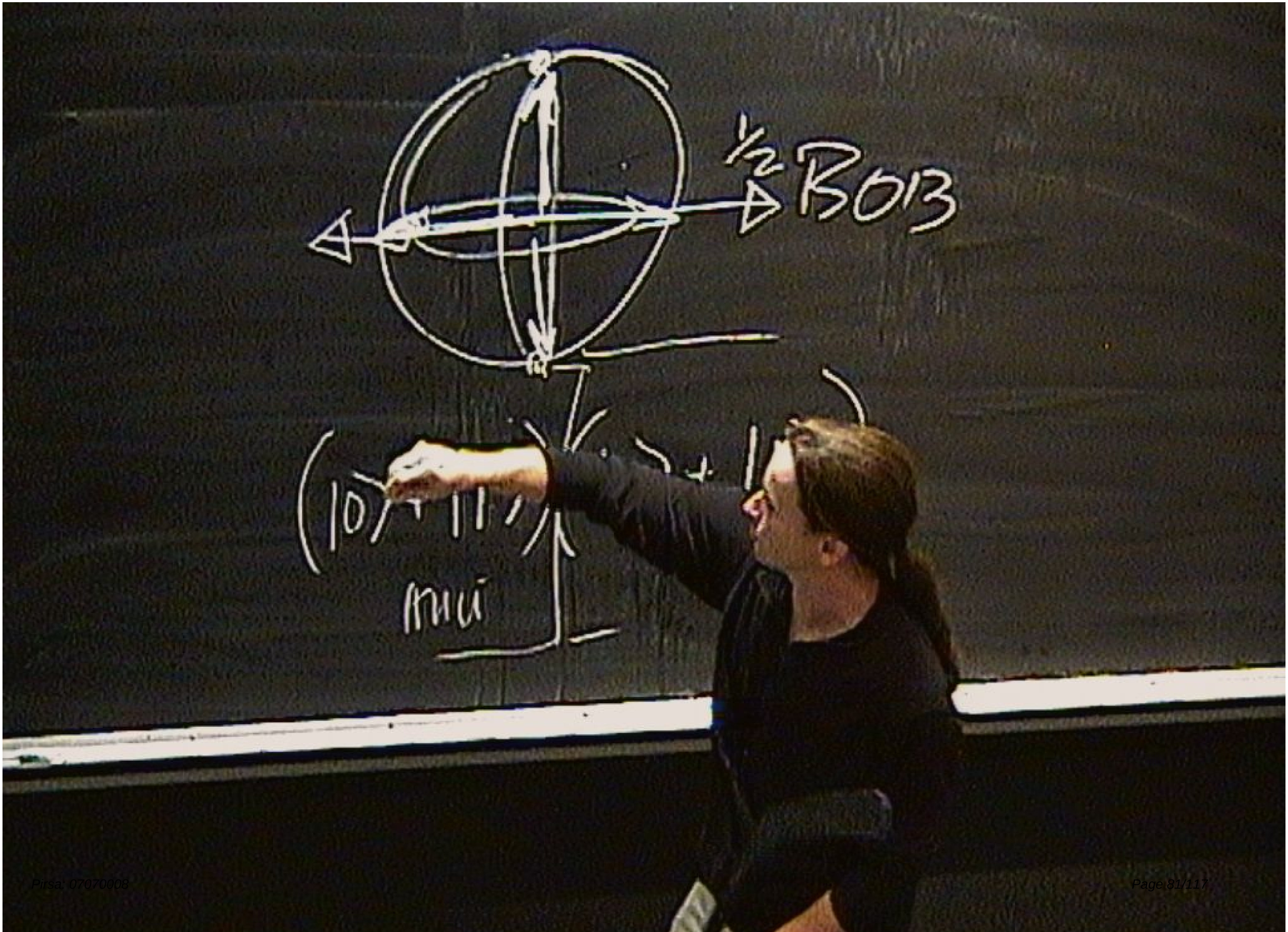
Alice's measurement produces a *completely random* result.

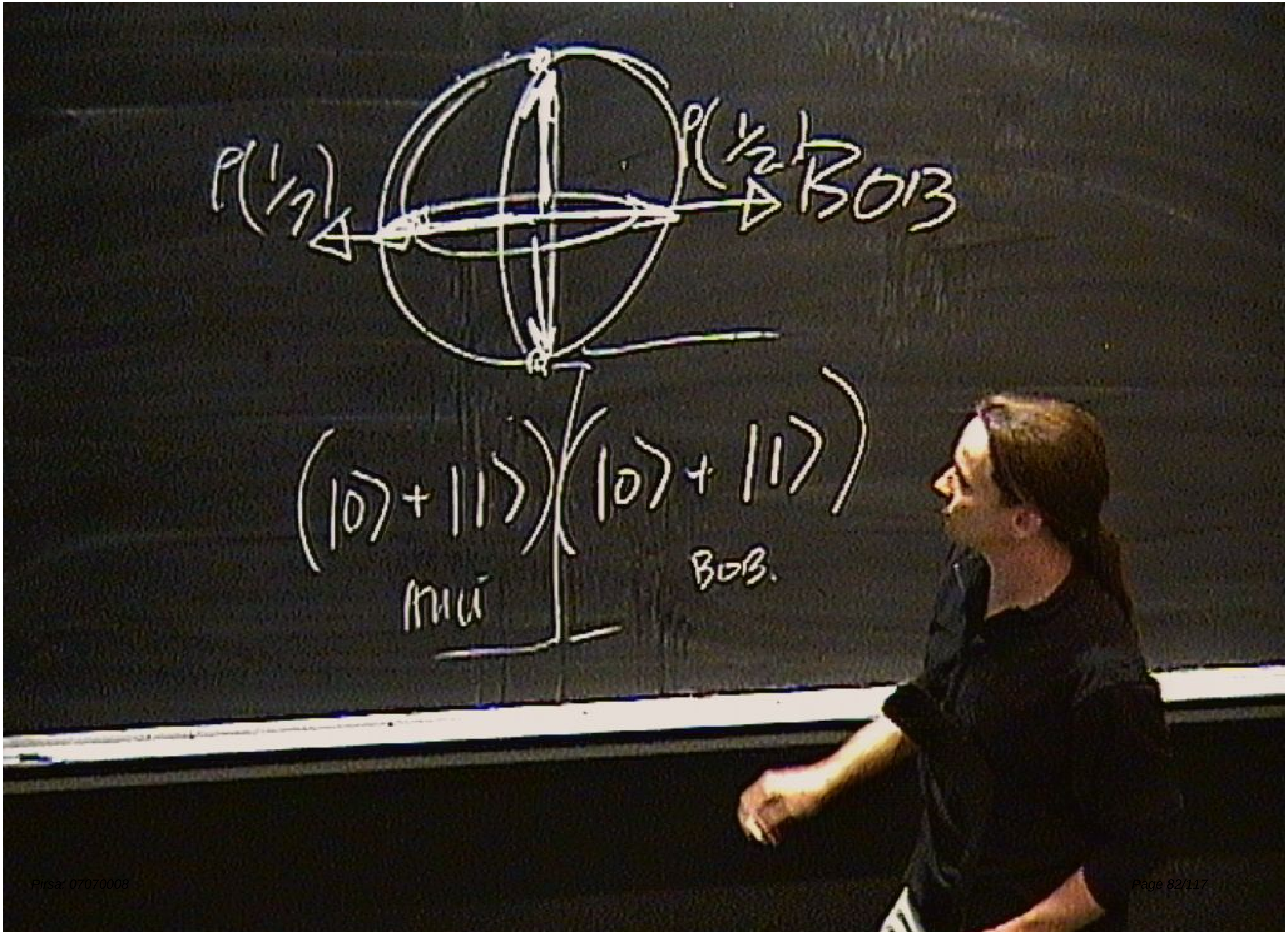
Although she now knows what Bob's system will do next, as far as Bob is concerned his system is behaving completely randomly too.

The correlations only appear when they compare notes later.

So relativity survives!


BOB
BOB.


½ BOB

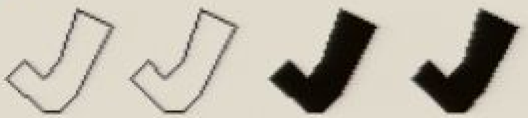

ρ(1/2)
ρ(1/2) BOB
(|0⟩ + |1⟩)(|0⟩ + |1⟩)
BOB.

Nonlocality Tests

So quantum entanglement produces:

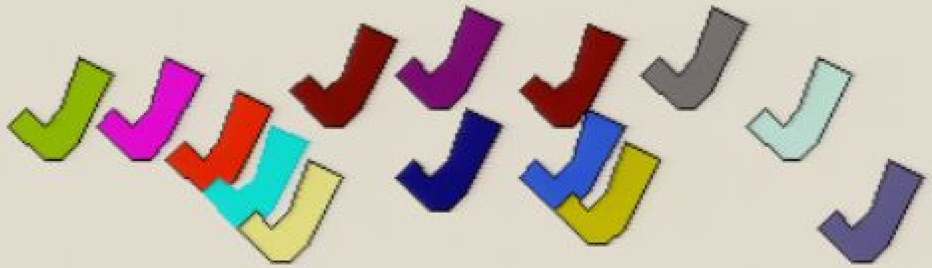
- Very strong correlations, more powerful than any classical correlations.
- No way to exploit the correlations for information transfer.

Is this really true? Maybe we can't get equivalent correlations with a couple of pairs of socks…



But what about if we use…

Nonlocality Tests



This is actually a *very* fundamental question.

Our intuition tells us that in we have a system at point A, all its behaviour will be determined by two things.

What we do to it at point A.

What its individual physical properties are. (The colour of its socks.)

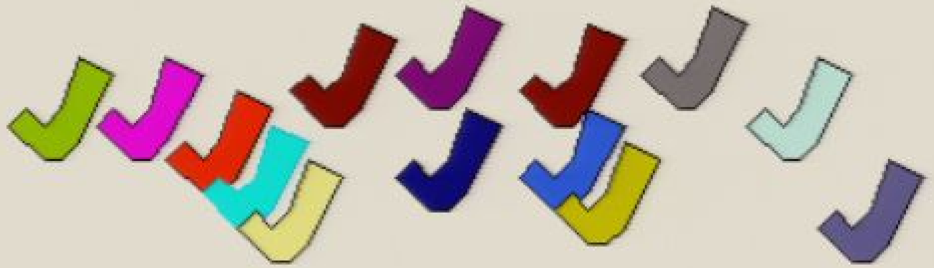
Intuitively, it shouldn't matter…

What someone else may or may not be doing miles away.

What something else's physical properties are.

This intuition is called '*local realism*', and it turn out to be *wrong*.

Nonlocality Tests



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Nonlocality Tests

Proving that local realism is wrong is not easy.

Two measurements have to be performed at spacelike separated locations (i.e. so far apart that light could not travel from one experiment to the other while they are being performed.)

Until recently the proofs have been highly statistical, and very vulnerable to experimental errors.

But over the past 10 years, a new kind of proof has been developed:

Pseudotelepathy Games.

Pseudotelepathy Games

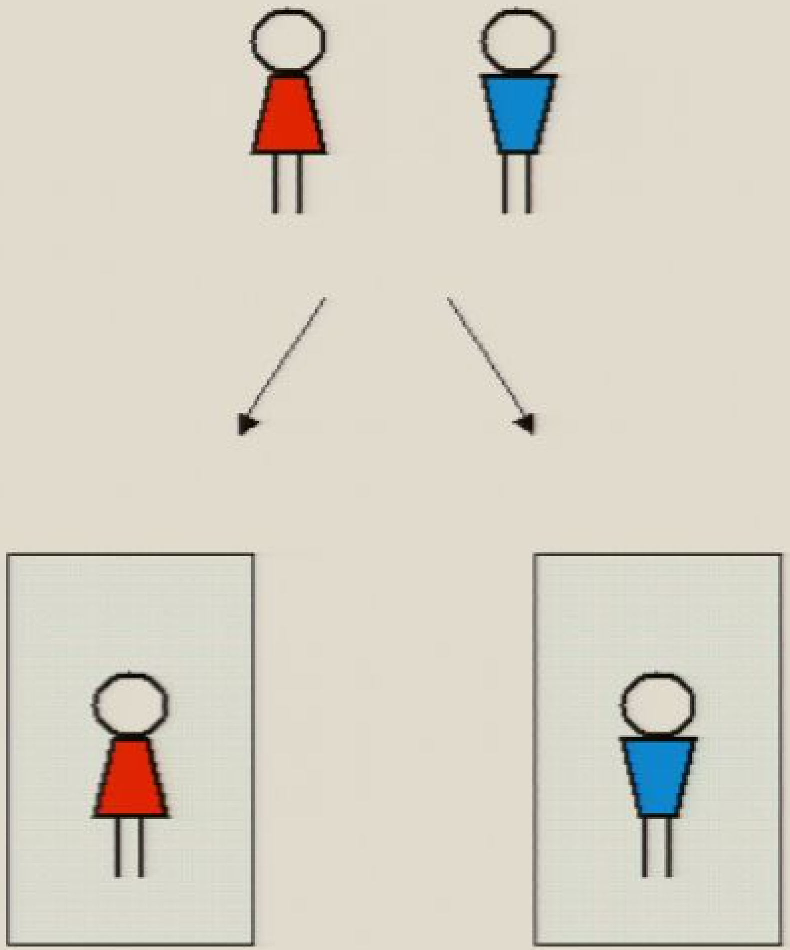
A game show with two contestants, Alice and Bob, cooperating to try to win a prize.

They will be asked certain specific questions, and their answers have to satisfy a certain winning condition – they have to be correlated in a certain way.

They know what the questions might be, and what the winning condition will be, and can discuss strategy beforehand.

Then they step into glass booths, and can no longer talk to one another.

The host asks them the questions. Can they win?

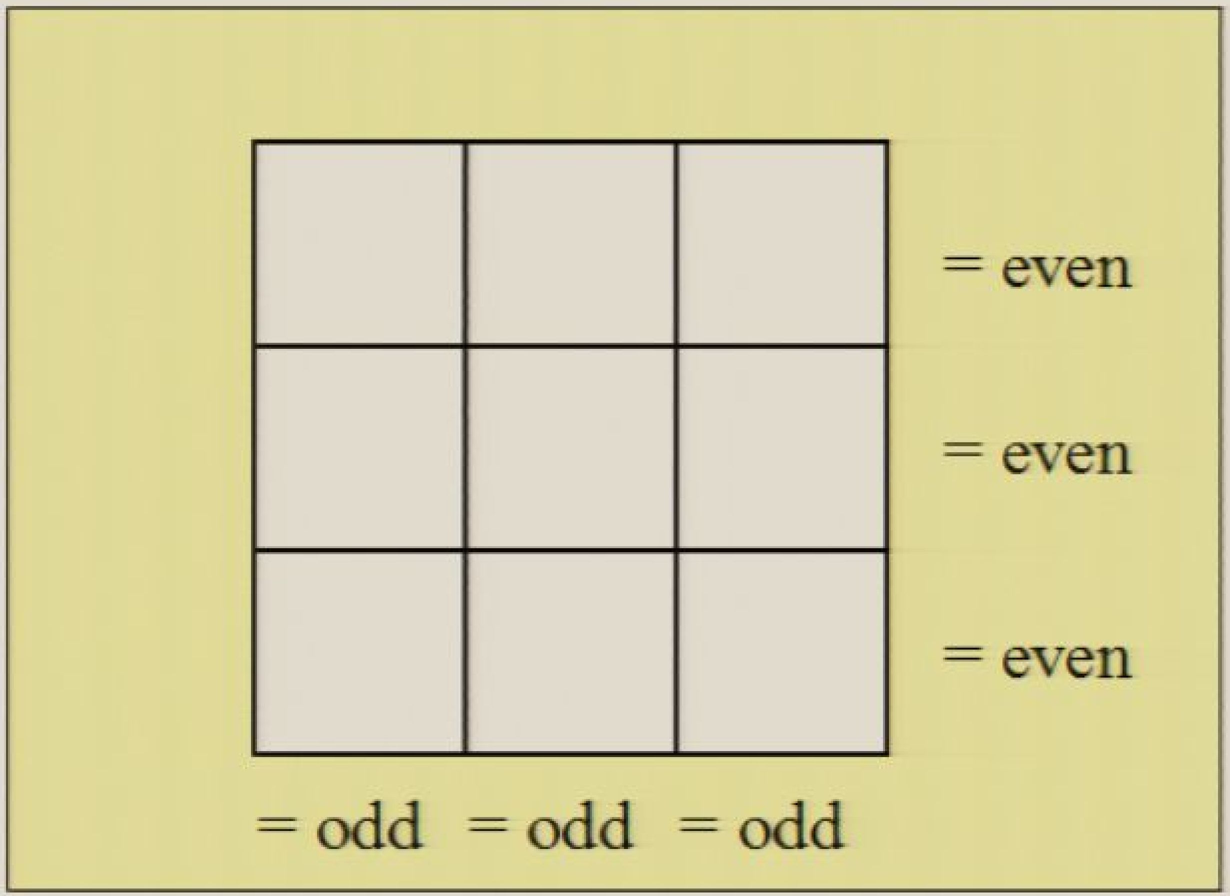


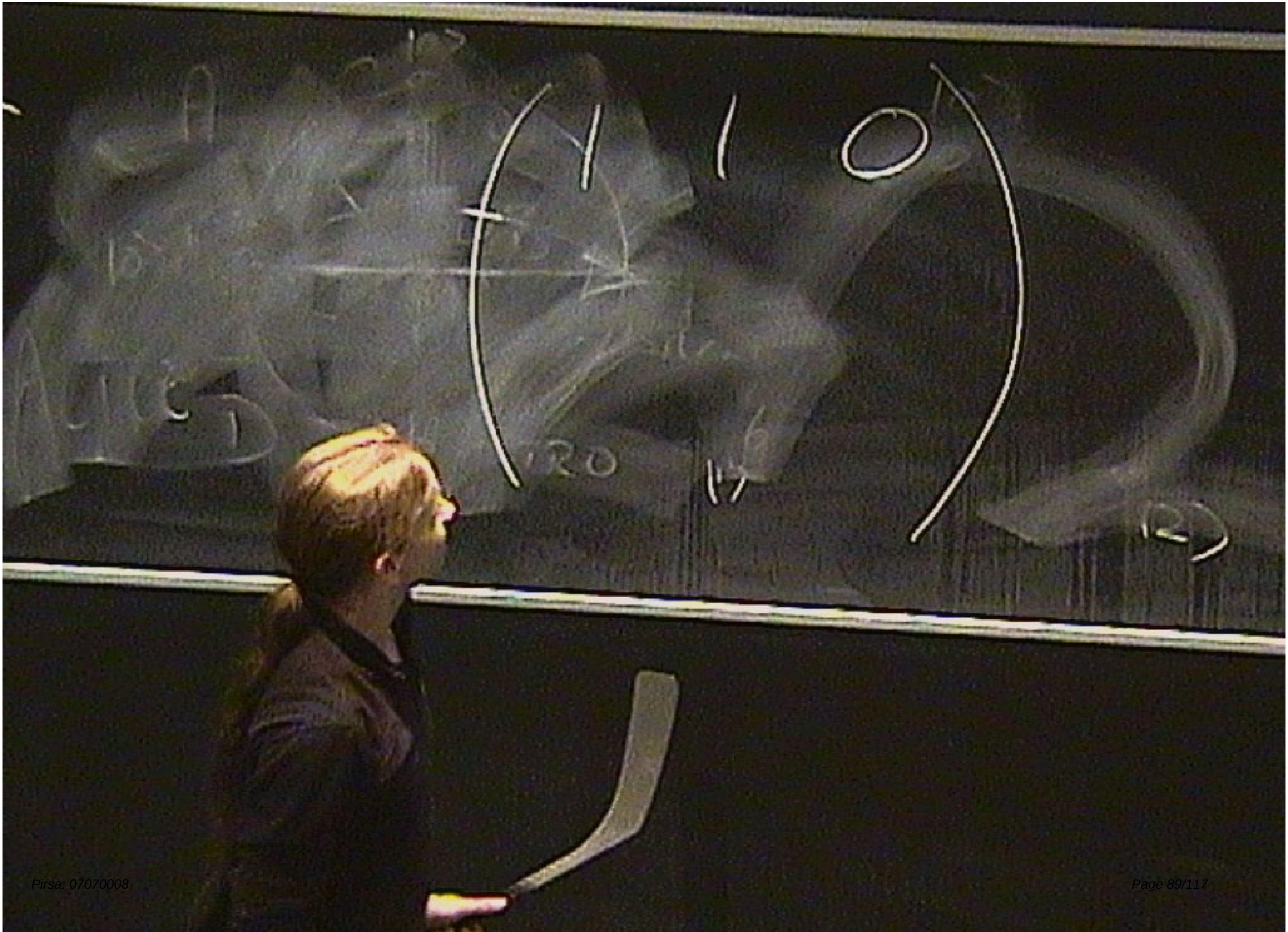
Magic Square Game

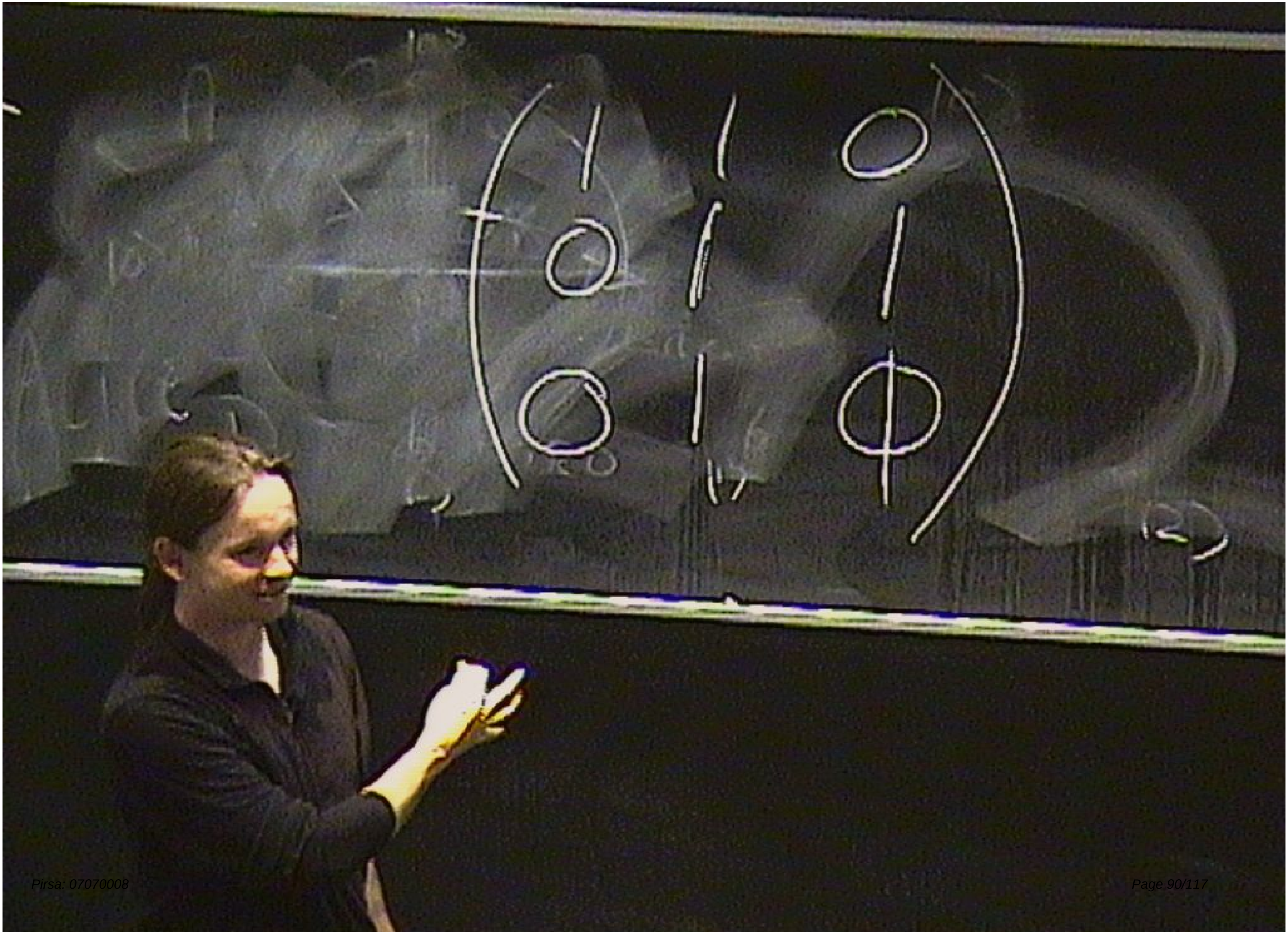
This game show is built around the ‘Magic Square’.

A magic square has three rows and three columns of binary digits – either **0** or **1**.

Every row adds to an even total, and every column to an odd total.






1 1 0
0 1 1
0 1 ϕ

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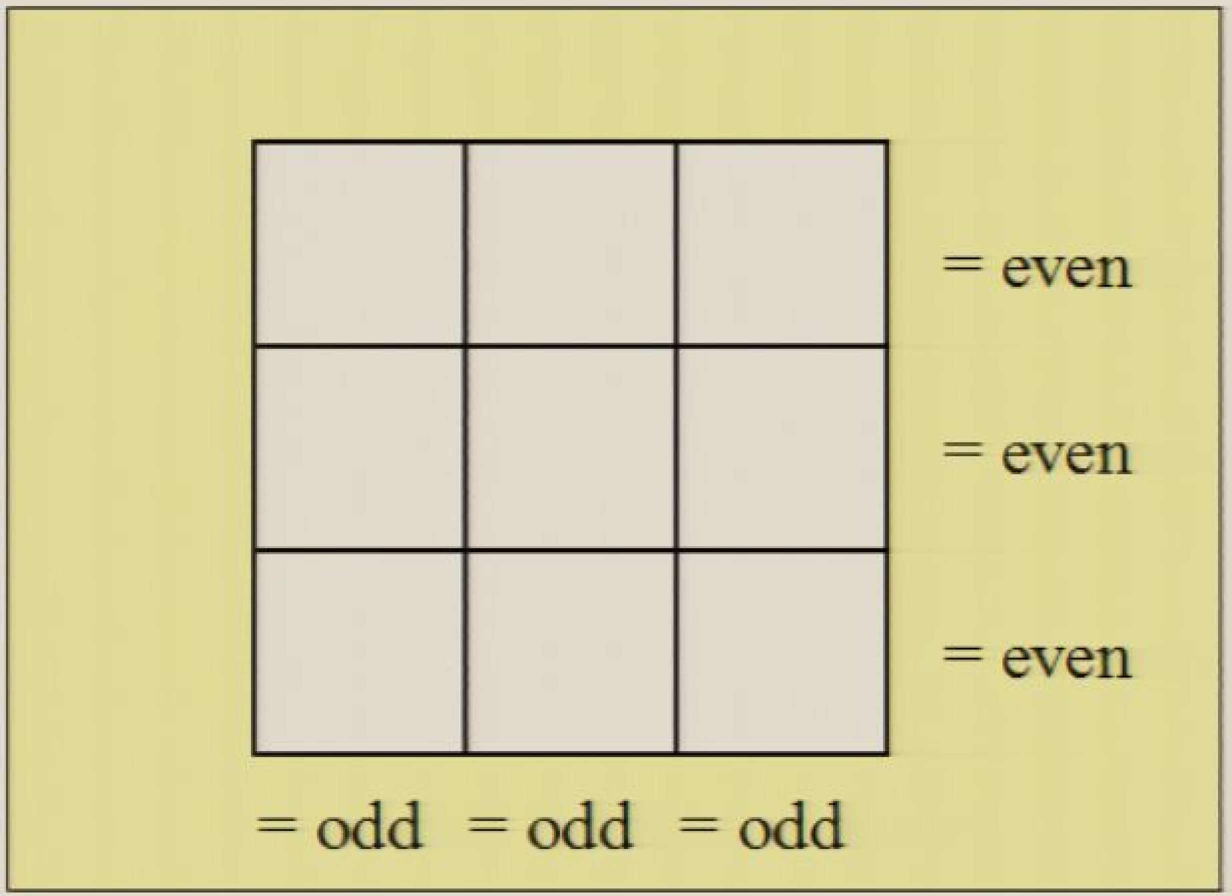
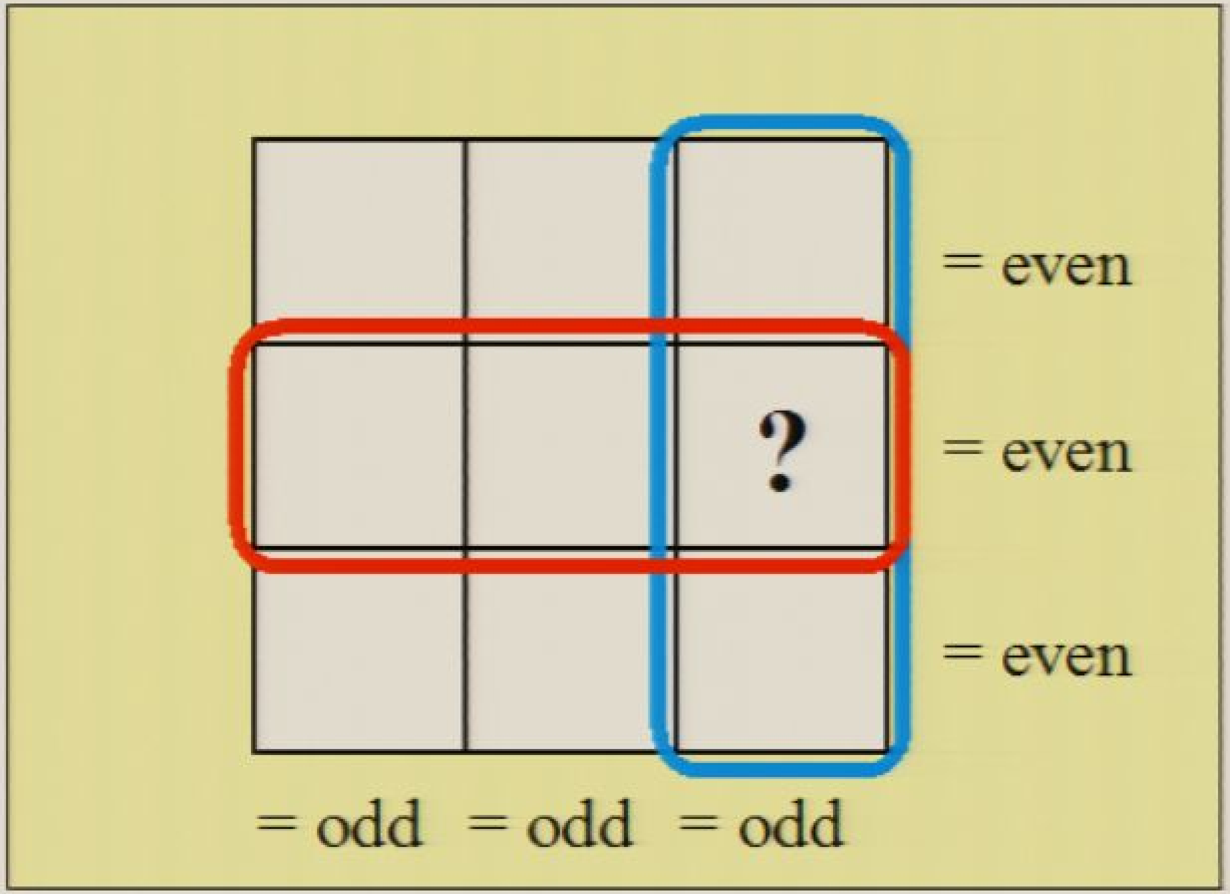


Magic Square Game

Alice and Bob have to convince us that they **are** able to build magic squares.

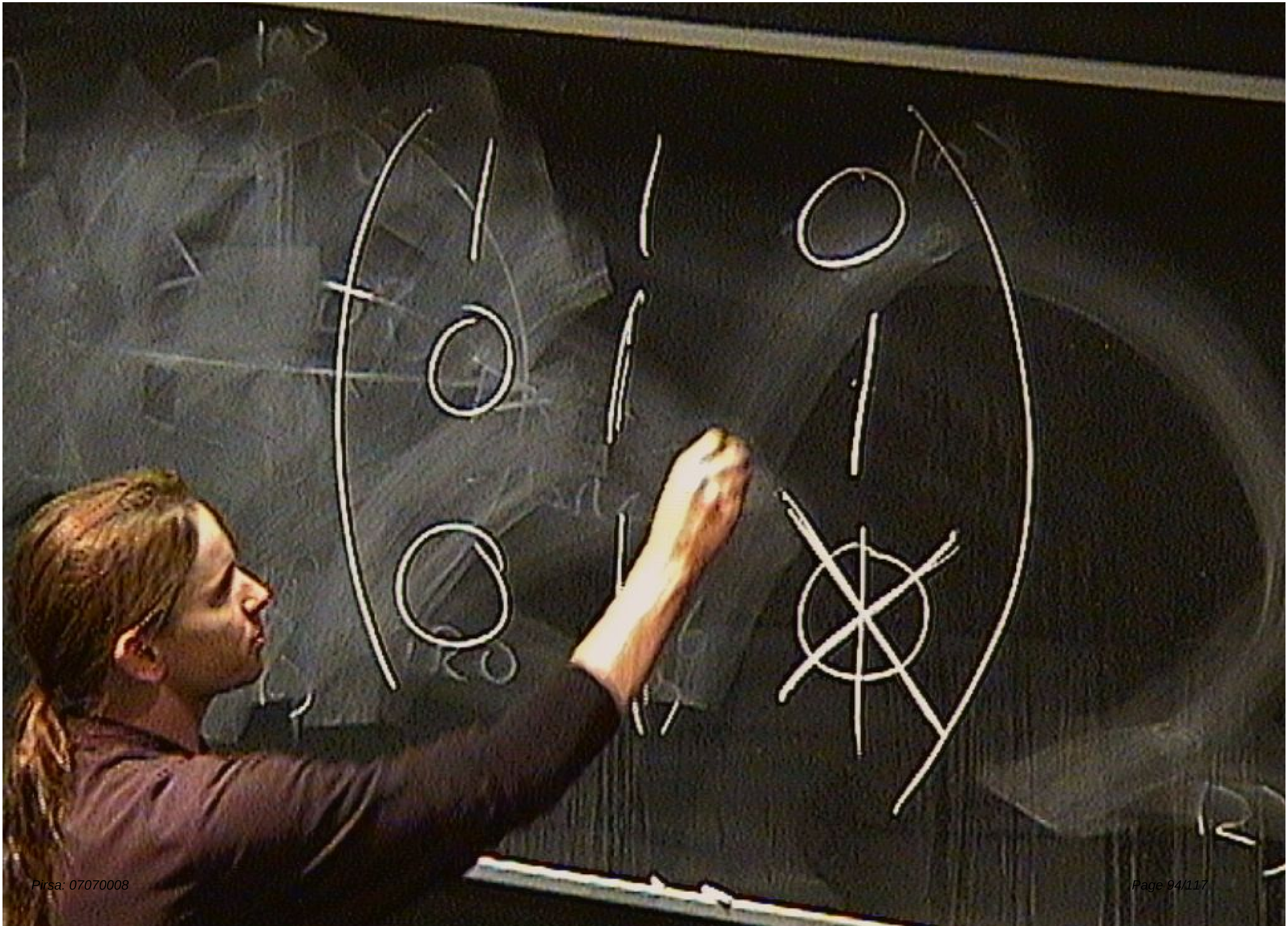
We're going pick a row number (1, 2 or 3) at random and ask Alice to tell us what's in it. Bob has to announce a random column.

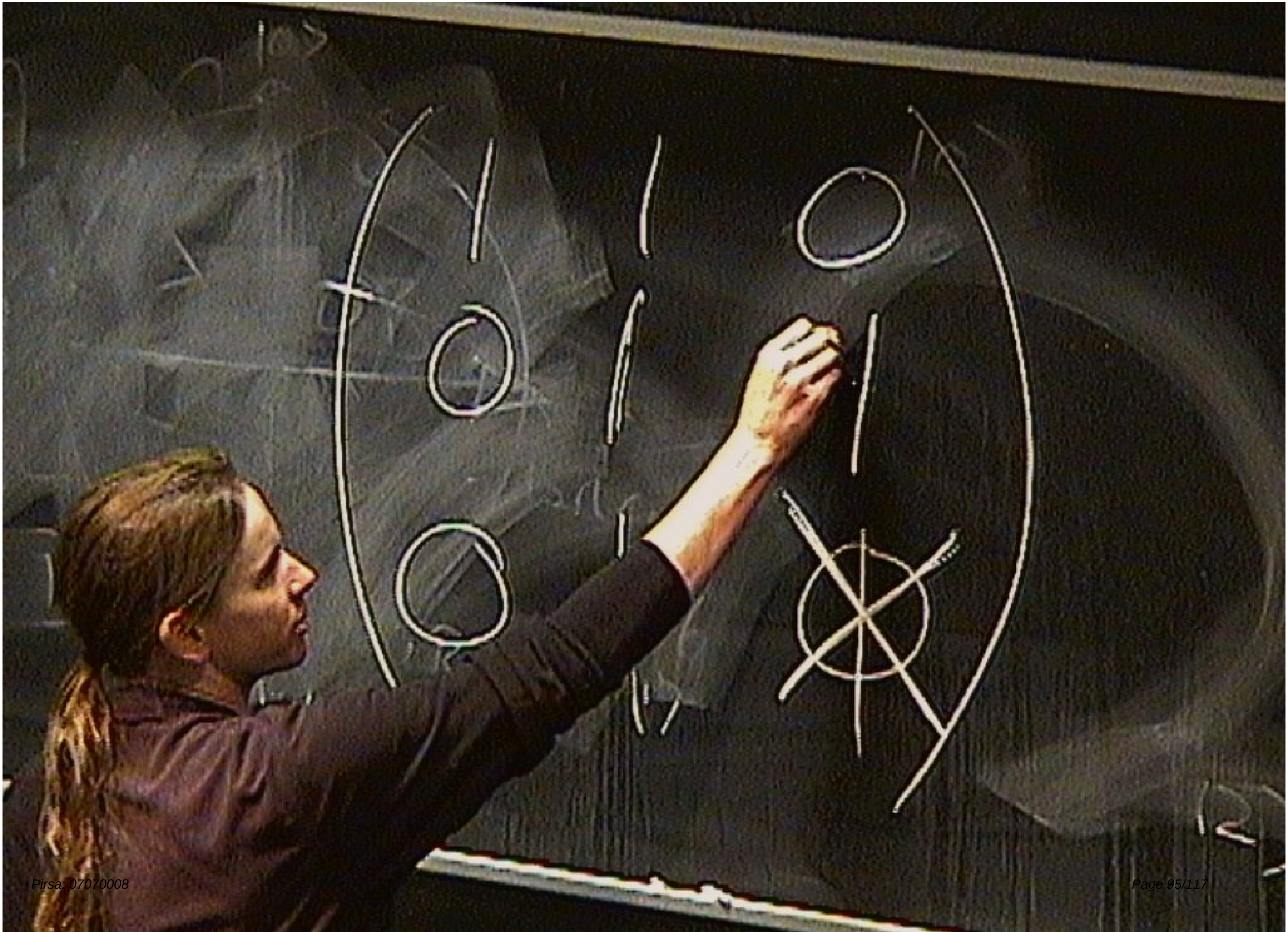


Magic Square Game

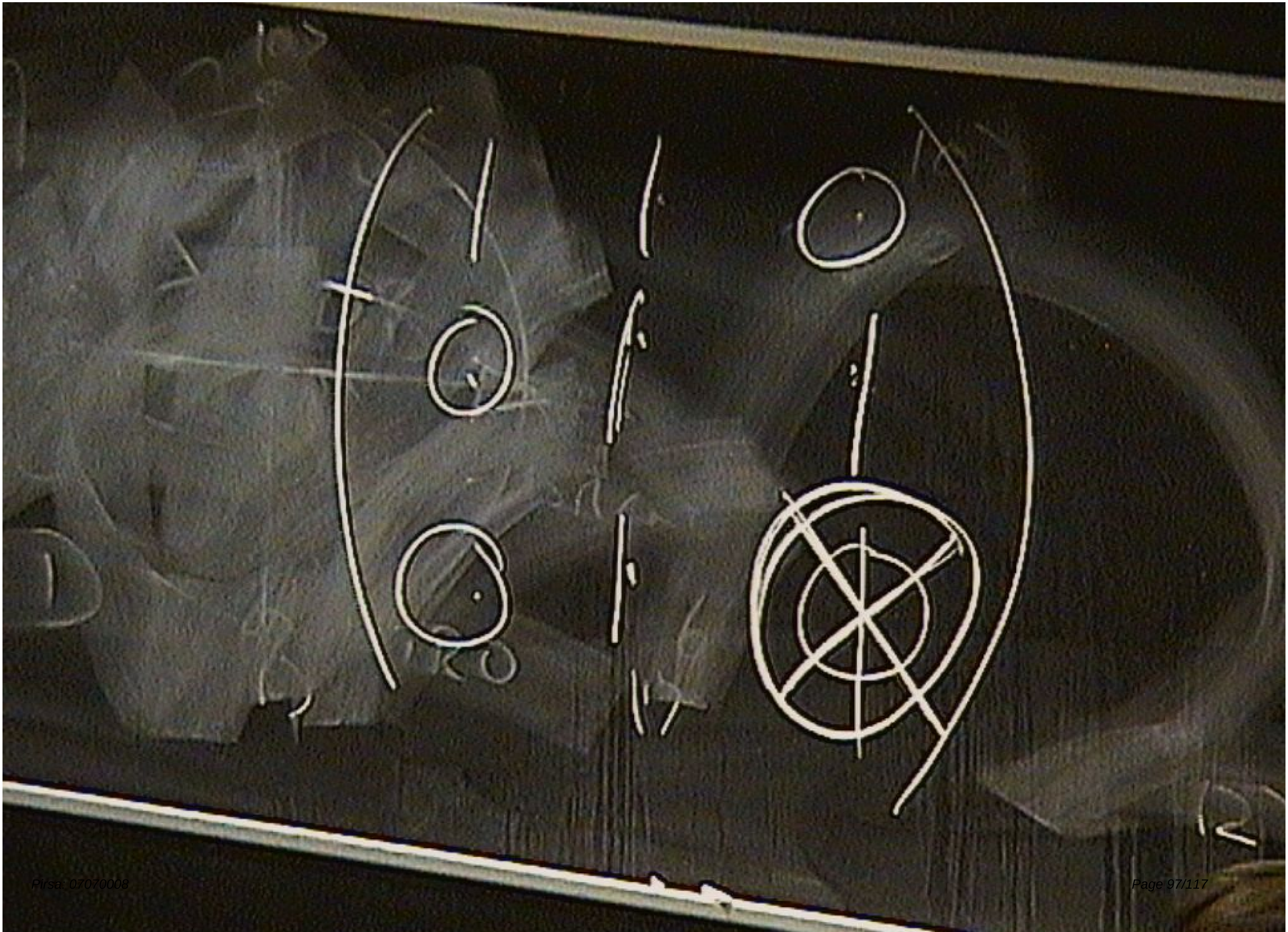
Classically, even sharing infinitely many socks, they best Alice and Bob can do is an **8/9** probability of getting each question right. It won't take many questions to prove they're bluffing.

1	1	0	= even
0	1	1	= even
0	1	x	= even
= odd	= odd	= odd	


1 1 0
0 1 1
0



$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & \cancel{0} \end{pmatrix}$$



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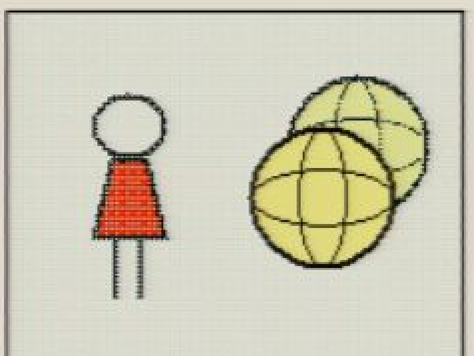
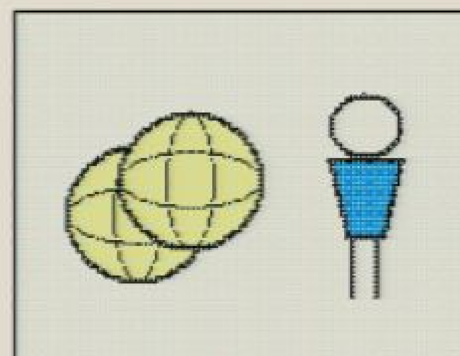
Classically, even sharing infinitely many socks, they best Alice and Bob can do is an **8/9** probability of getting each question right. It won't take many questions to prove they're bluffing.

1	1	0	= even
0	1	1	= even
0	1	x	= even
= odd	= odd	= odd	

Magic Square Game

$$|\psi\rangle = \frac{1}{2}|0011\rangle - \frac{1}{2}|0110\rangle - \frac{1}{2}|1001\rangle + \frac{1}{2}|1100\rangle$$





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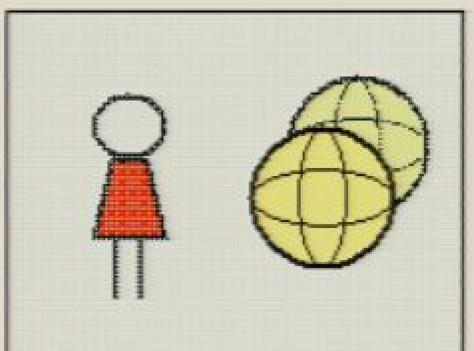
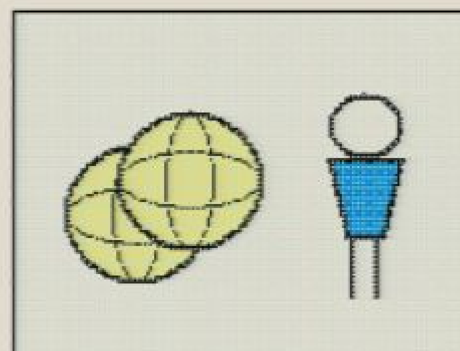


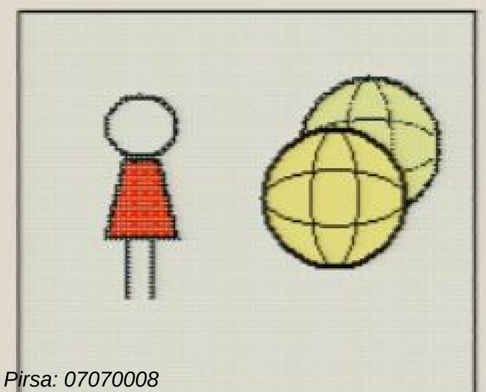


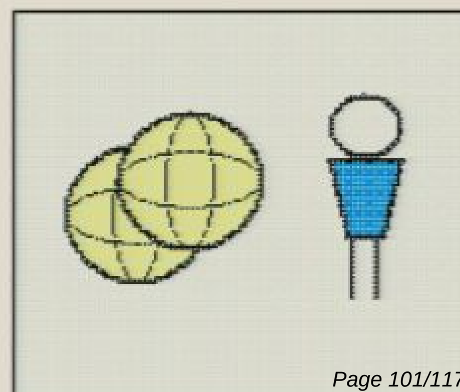
Magic Square Game

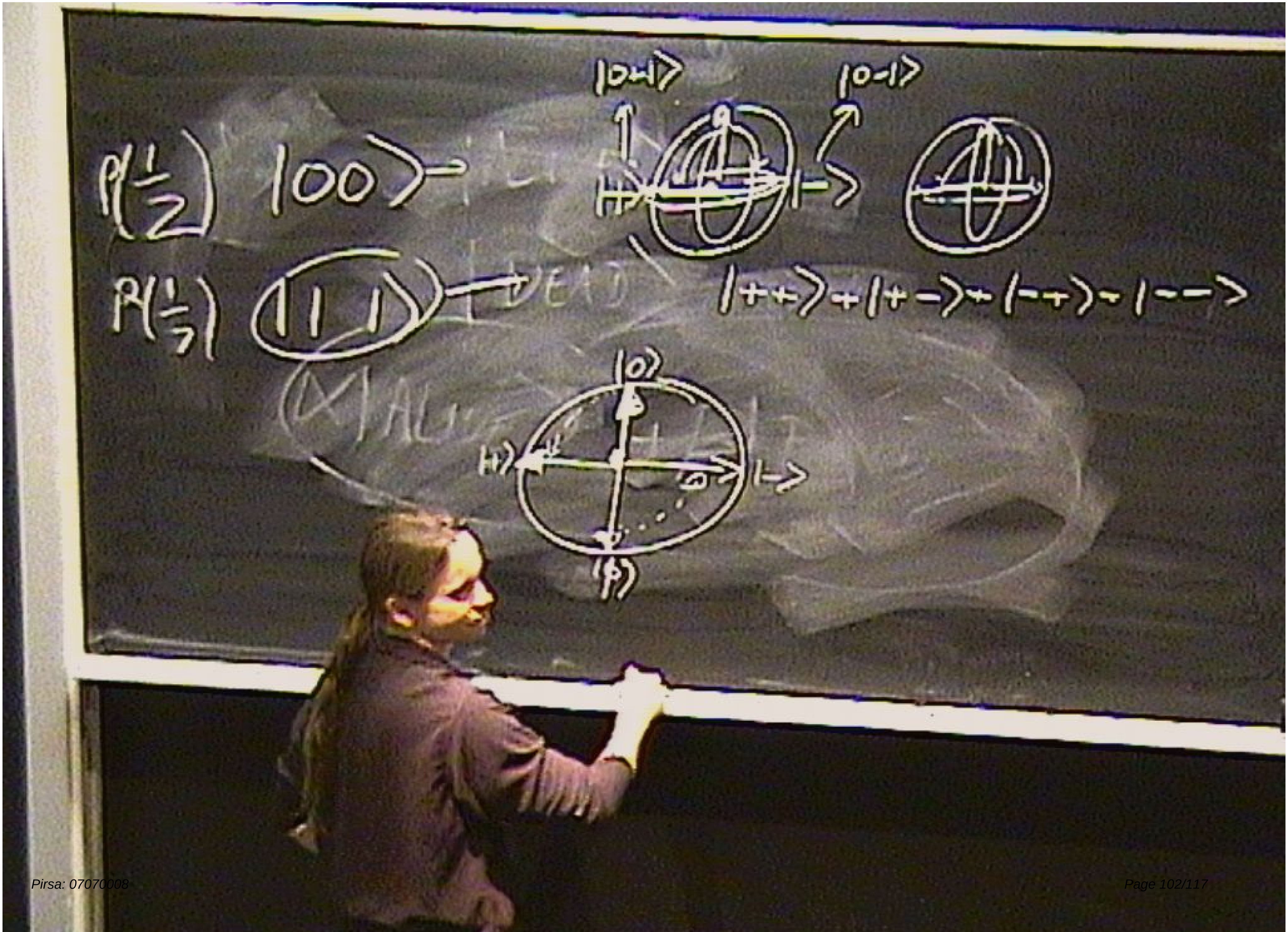
$$|\psi\rangle = \tfrac{1}{2}|0011\rangle - \tfrac{1}{2}|0110\rangle - \tfrac{1}{2}|1001\rangle + \tfrac{1}{2}|1100\rangle$$

$$A_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} i & 0 & 0 & 1 \\ 0 & -i & 1 & 0 \\ 0 & i & 1 & 0 \\ 1 & 0 & 0 & i \end{bmatrix},\quad A_2 = \frac{1}{2}\begin{bmatrix} i & 1 & 1 & i \\ -i & 1 & -1 & i \\ i & 1 & -1 & -i \\ -i & 1 & 1 & -i \end{bmatrix},\quad A_3 = \frac{1}{2}\begin{bmatrix} -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 \end{bmatrix}$$

$$B_1 = \frac{1}{2}\begin{bmatrix} i & -i & 1 & 1 \\ -i & -i & 1 & -1 \\ 1 & 1 & -i & i \\ -i & i & 1 & 1 \end{bmatrix},\quad B_2 = \frac{1}{2}\begin{bmatrix} -1 & i & 1 & i \\ 1 & i & 1 & -i \\ 1 & -i & 1 & i \\ -1 & -i & 1 & -i \end{bmatrix},\quad B_3 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$






$|0{+}1\rangle$
$|0{-}1\rangle$
$P(\frac{1}{2})$ $|00\rangle$
$P(\frac{1}{2})$ $|11\rangle$
$|{+}\rangle$ $|{-}\rangle$
$|{+}{+}\rangle + |{+}{-}\rangle + |{-}{+}\rangle - |{-}{-}\rangle$
$|0\rangle$
$|{+}\rangle$ $|{-}\rangle$
$|\phi\rangle$

Magic Square Game

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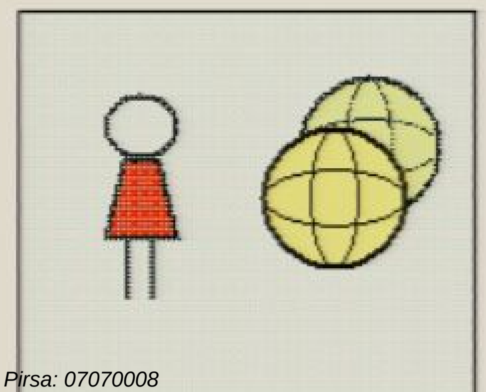

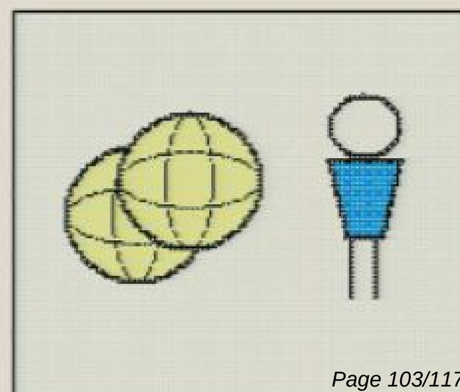

Magic Square Game

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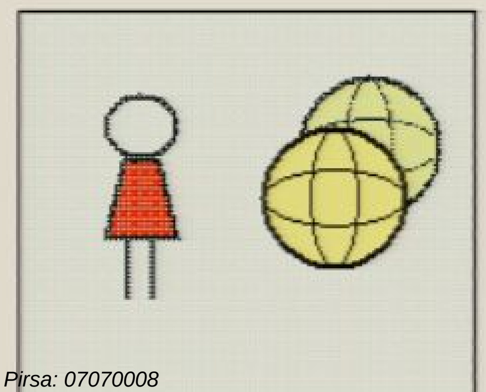

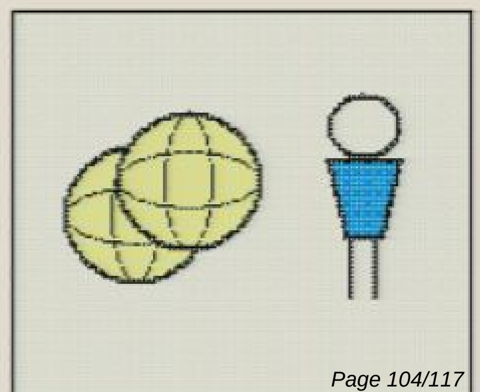

Magic Square Game

$$(A_2 \otimes B_3)|\psi\rangle = \frac{1}{2\sqrt{2}}\big[|0000\rangle - |0010\rangle - |0101\rangle + |0111\rangle + |1001\rangle + |1011\rangle - |1100\rangle - |1110\rangle\big]$$

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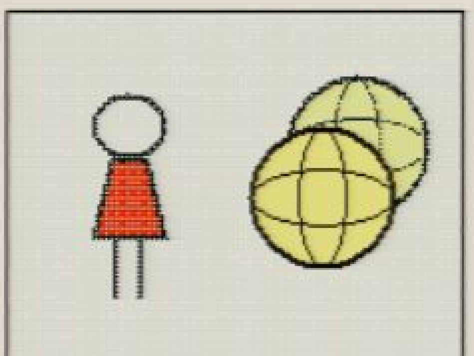
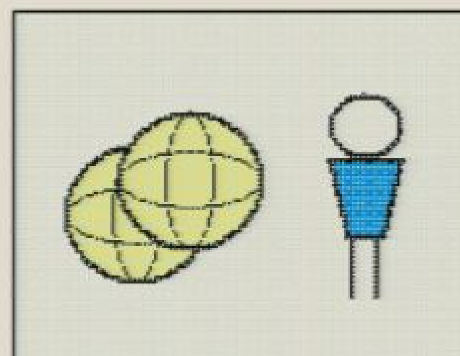
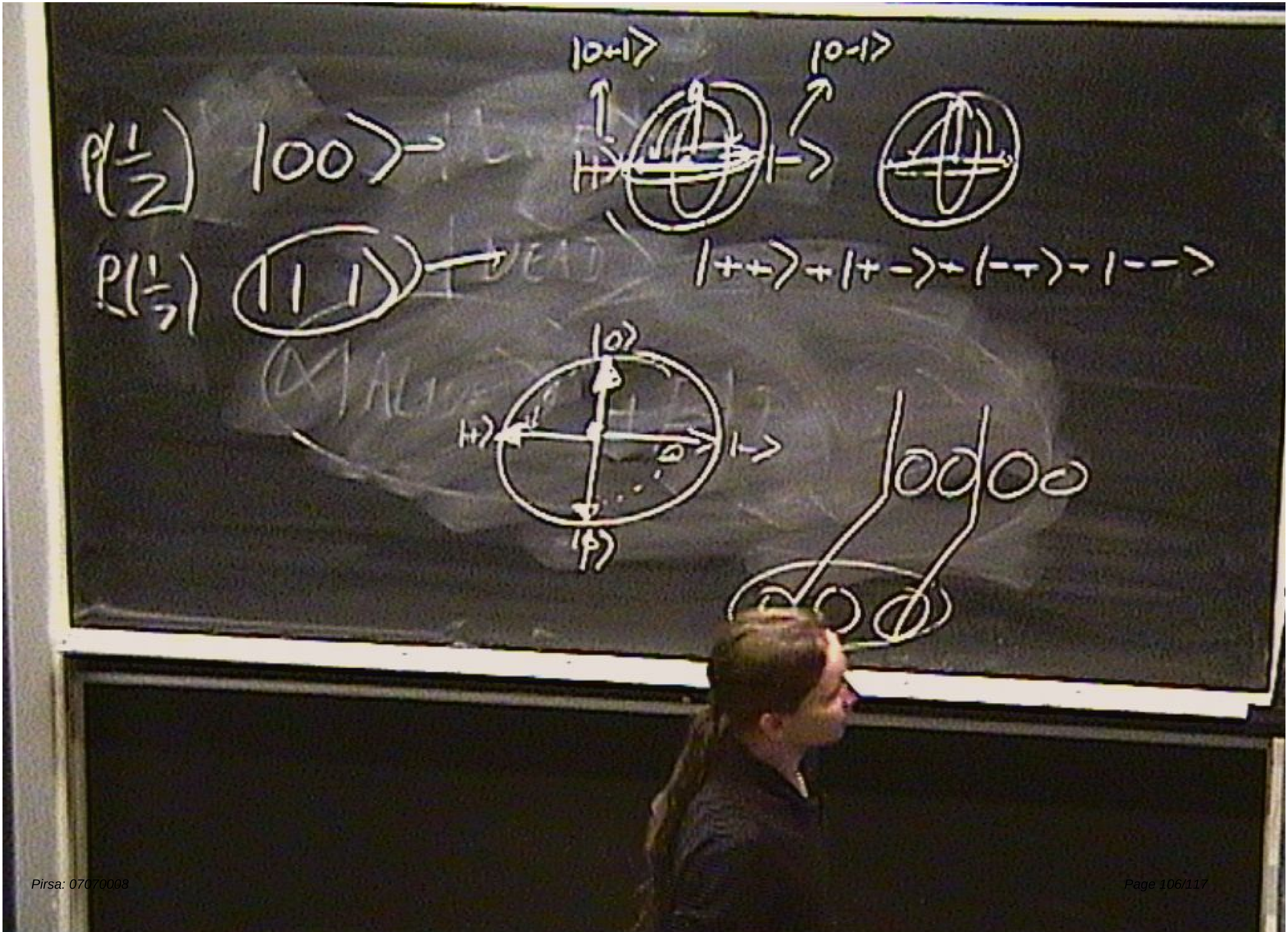
$$B_1 = \frac{1}{2}\begin{bmatrix} i & -i & 1 & 1 \\ -i & -i & 1 & -1 \\ 1 & 1 & -i & i \\ -i & i & 1 & 1 \end{bmatrix}, \quad B_2 = \frac{1}{2}\begin{bmatrix} -1 & i & 1 & i \\ 1 & i & 1 & -i \\ 1 & -i & 1 & i \\ -1 & -i & 1 & -i \end{bmatrix}, \quad B_3 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$



Alice and Bob always win!




|0+1>
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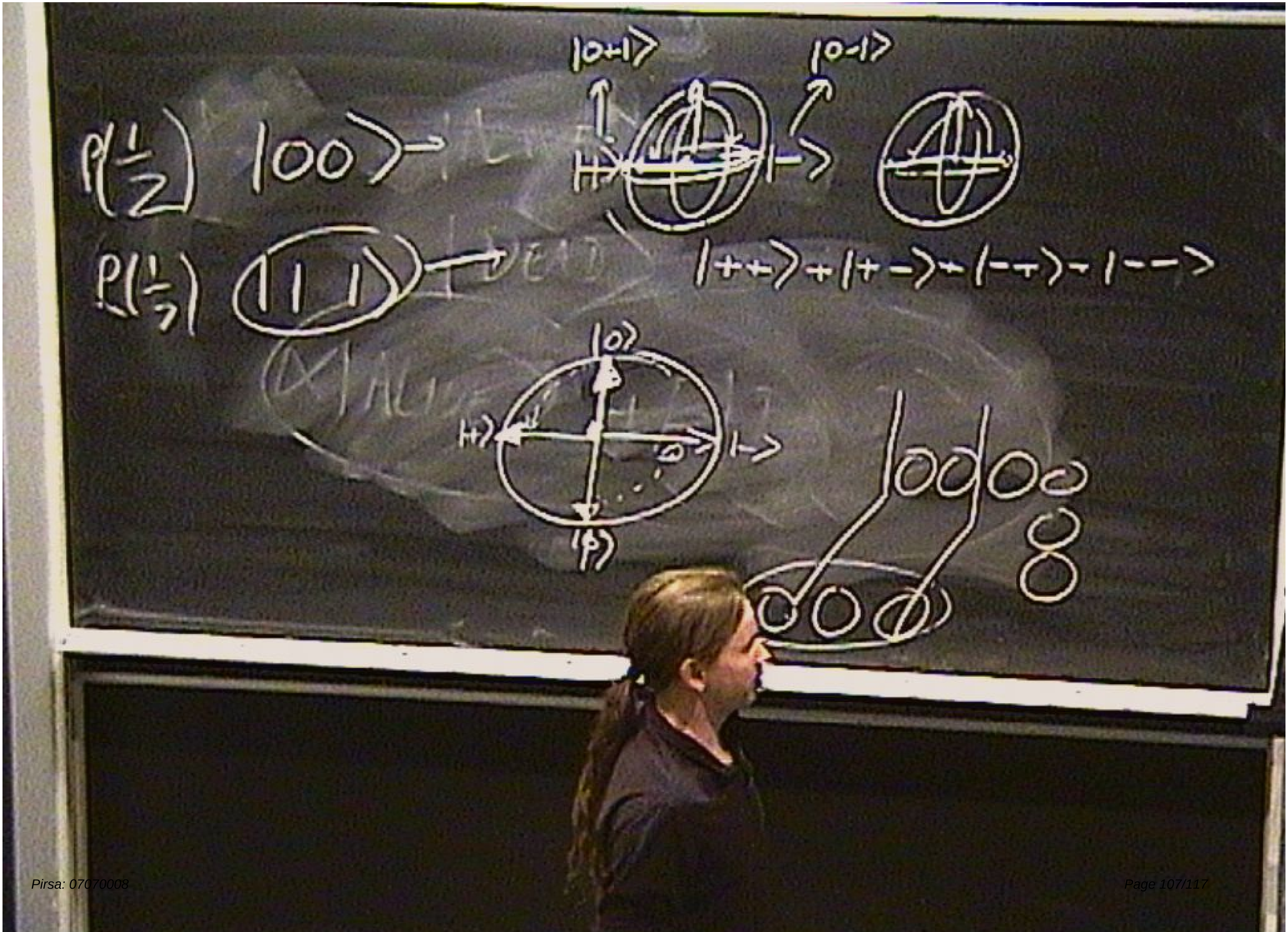
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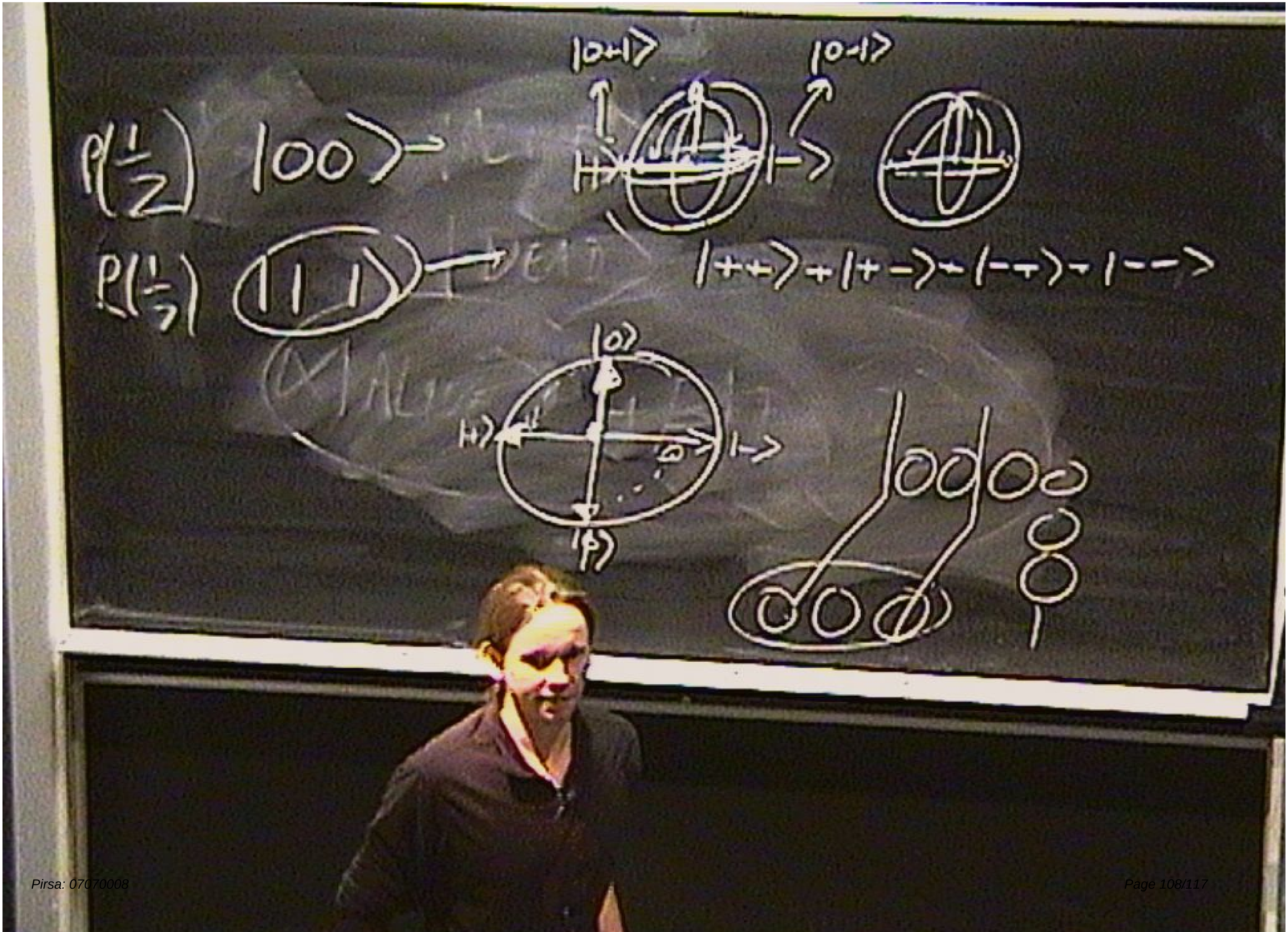
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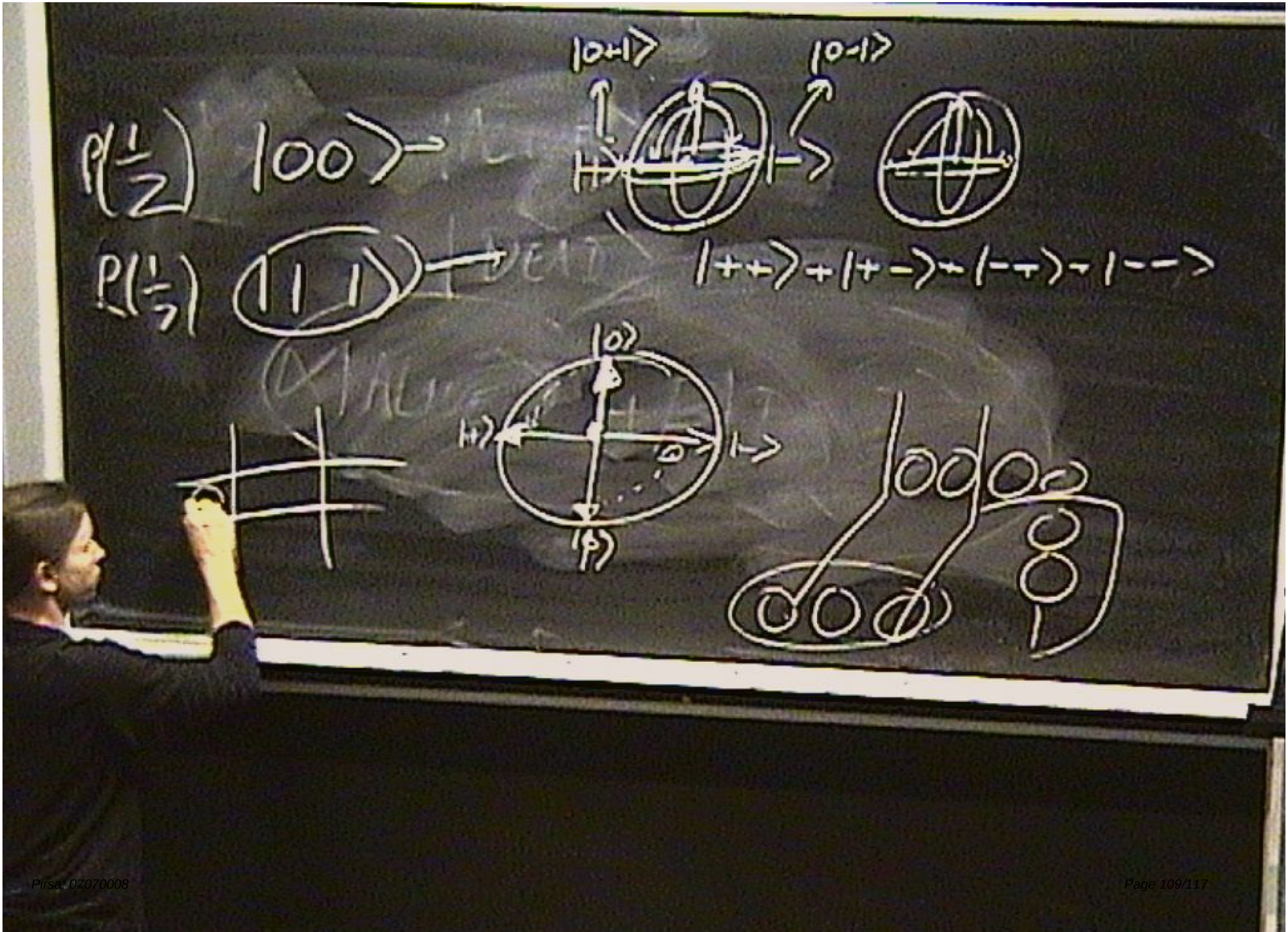
ALIVE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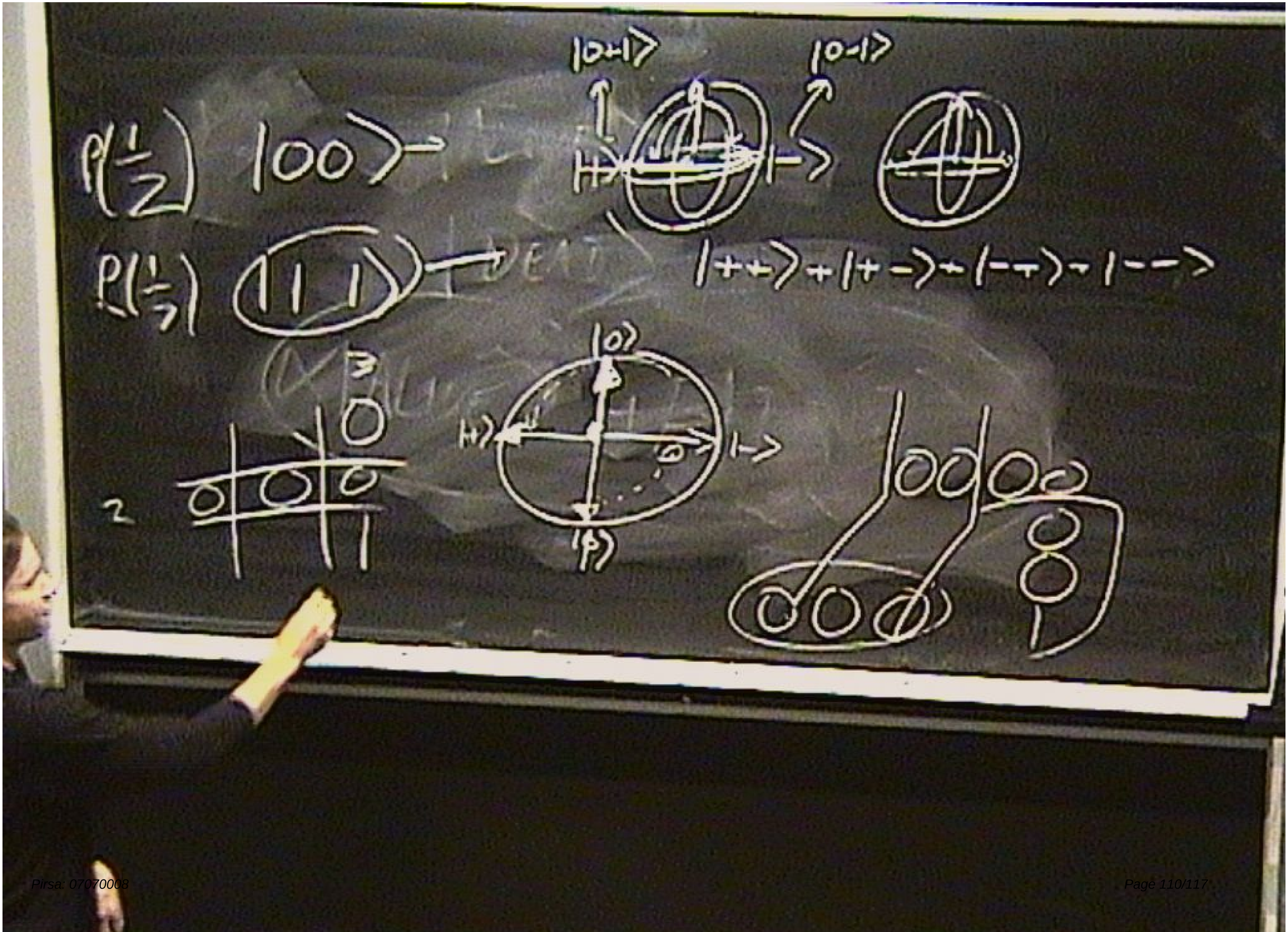

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Magic Square Game

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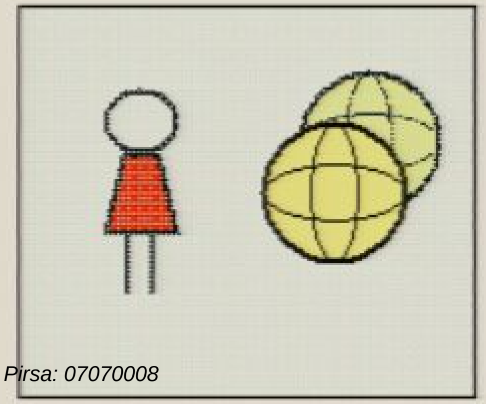
Alice and Bob always output random strings of 1s and 0s.

Alice and Bob never find out what question the other was asked.

The stuff in the glass booths is correlated in a way that is *impossible to account* for with local realism.



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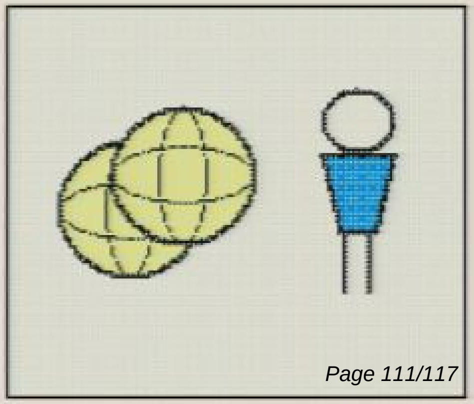

Magic Square Game

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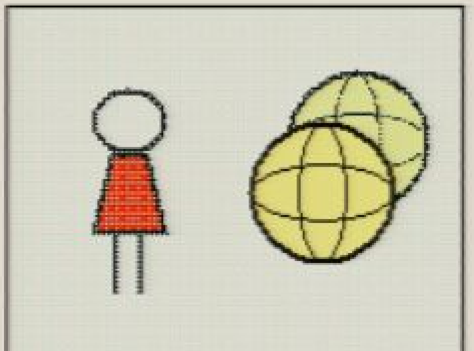

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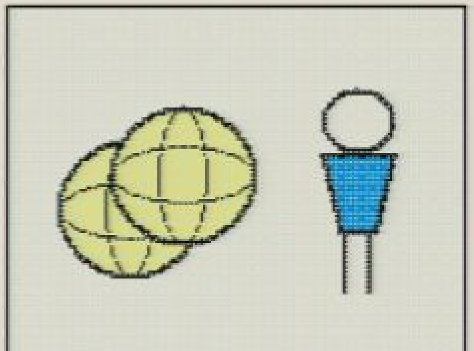
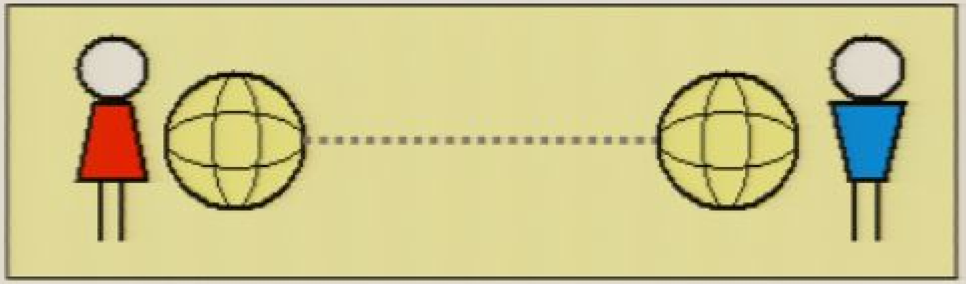


Entanglement



We can apply the superposition principle exactly as before, but now the space of possible states has *four* complex dimensions.

Possible state:

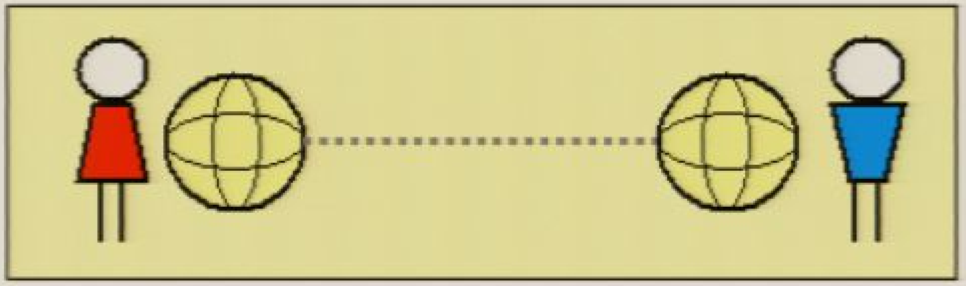
$$\alpha|0\rangle_A|0\rangle_B + \beta|0\rangle_A|1\rangle_B + \gamma|1\rangle_A|0\rangle_B + \delta|1\rangle_A|1\rangle_B$$

Consider the state:

$$\frac{1}{\sqrt{2}}|0\rangle_A|0\rangle_B + \frac{1}{\sqrt{2}}|1\rangle_A|1\rangle_B$$

The two atoms are correlated. They are either both in the ground state, or both in the excited state. When measuring their energy, we will never see them in different states.

Entanglement and Nonlocality



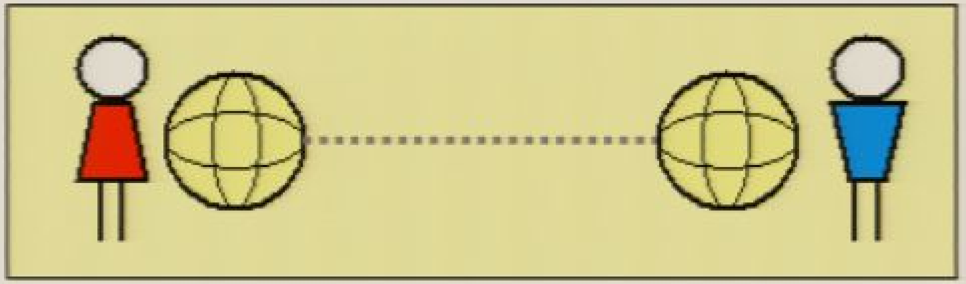
This is quantum nonlocality.

Correlated behaviour that is so strong as to be impossible to reproduce without entanglement, even on purpose.

Yet so *random* that no information passes between the systems, so we don't break the speed of light.

Why does the universe behave this way?

Entanglement



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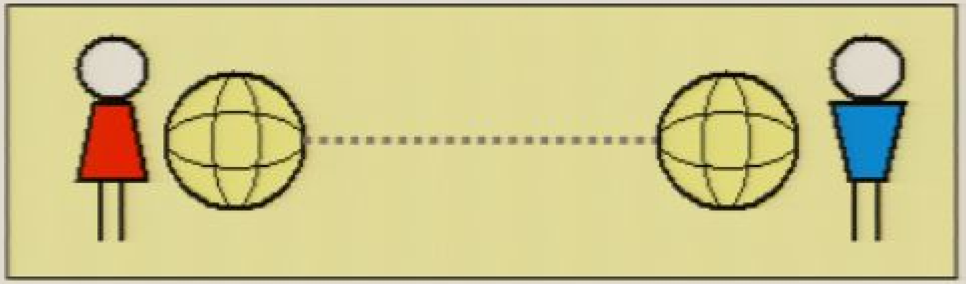
Entanglement



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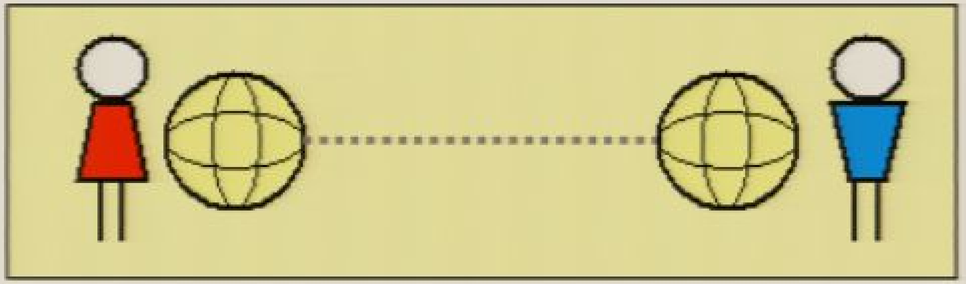

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