

Title: Quantum Engineering meets Optics

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Abstract:

Quantum Engineering (with Josephson Qubits) Meets Optics

Gerd Schön

<http://www.tfp.uni-karlsruhe.de/>

work with:

Alexander Shnirman

Yuriy Makhlin

Julian Hauss

Arkady Fedorov

Carsten Hutter



Pirsa: 07/60069

Universität Karlsruhe (TH)
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- Josephson qubits
 - charge, flux, phase qubit
 - experiments: spectroscopy, coherent oscillations, Ramsey fringes, Rabi, ...
 - qubit-qubit coupling
 - decoherence/relaxation
- Josephson qubits coupled to oscillators
 - LC-oscillators, transmission lines, nano-mechanical oscillators
 - strong coupling regime
 - vacuum Rabi oscillations
 - dispersive regime, read-out
- Single- and two photon lasing and cooling at the Rabi frequency
- Superconducting SET transistor and resonator

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Josephson Charge Qubits

$$H = E_C \left(2n - \frac{C_g V_g}{e} \right)^2 - E_J \cos\left(\pi \frac{\Phi_x}{\Phi_0}\right) \cos\theta$$

2 degrees of freedom

$$[n, \theta] = -i \text{ charge and phase}$$

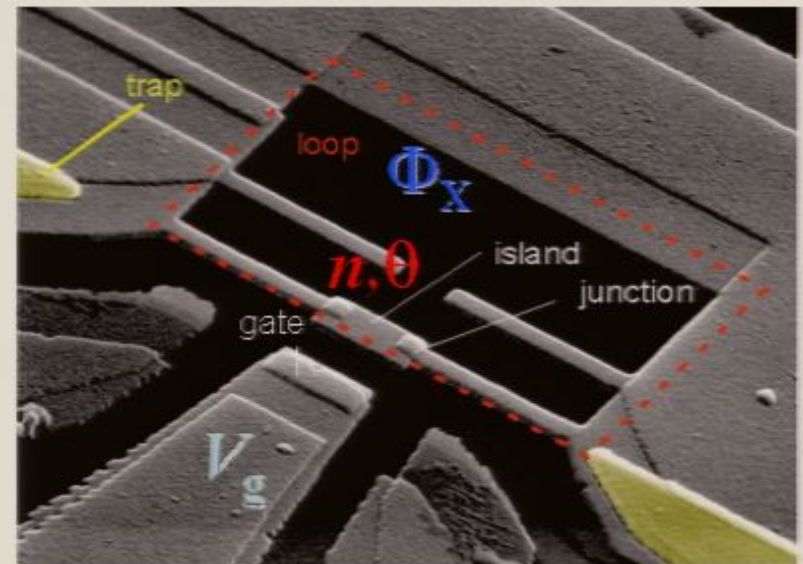
2 energy scales E_C, E_J

charging energy, Josephson coupling

2 control fields: V_g and Φ_x
gate voltage, flux

theory: Caldeira, Leggett (1981)
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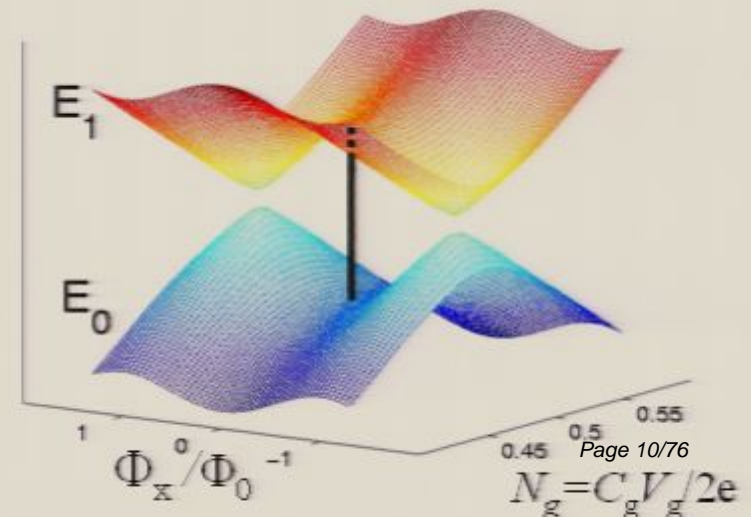
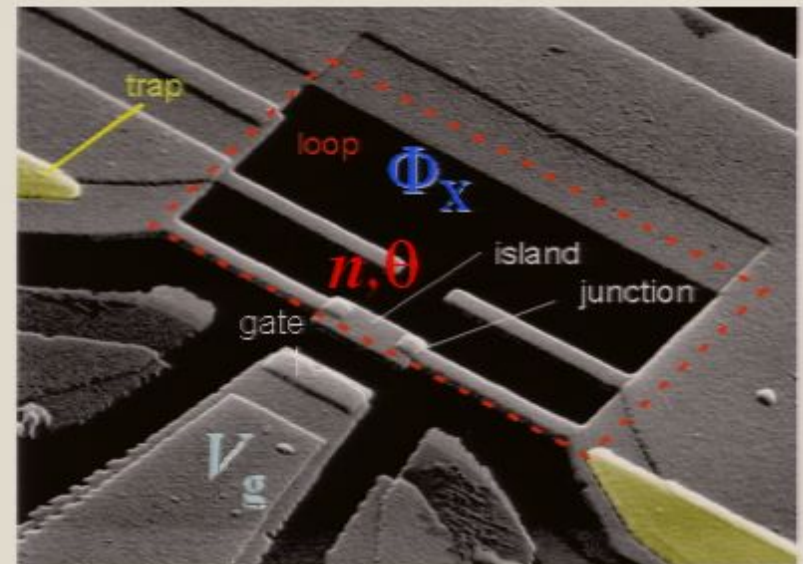
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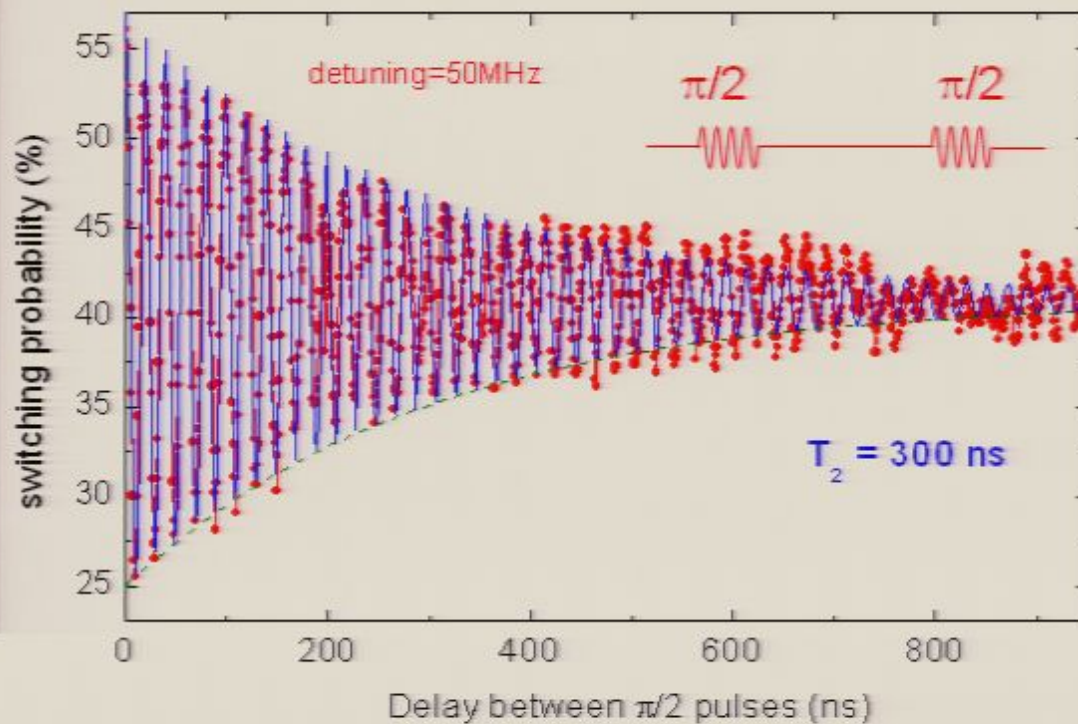
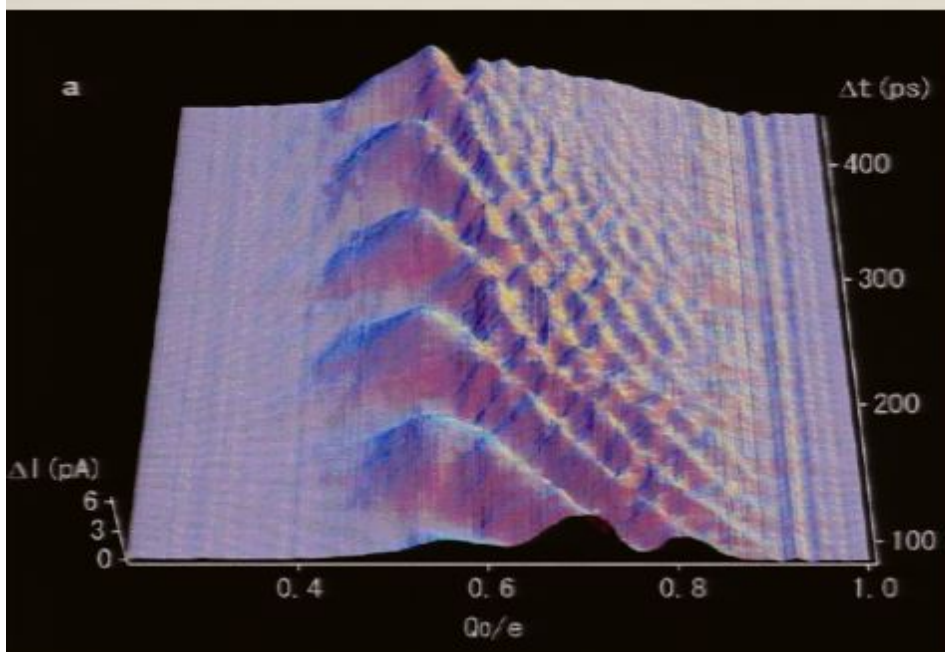
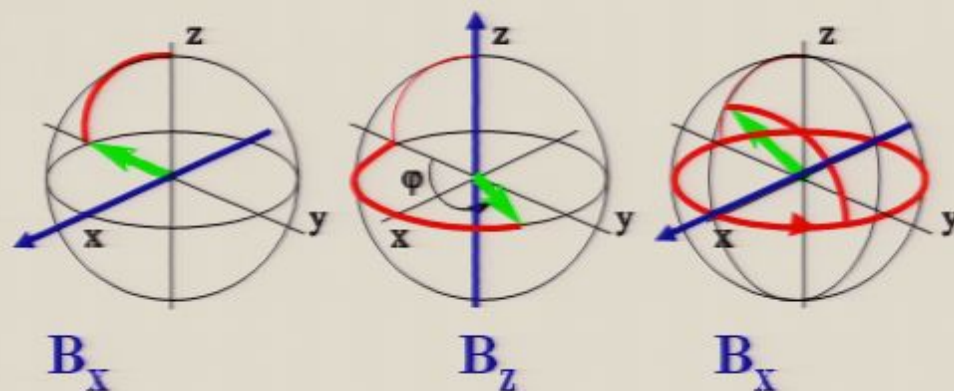
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Quantum coherent oscillations (Ramsey fringes)

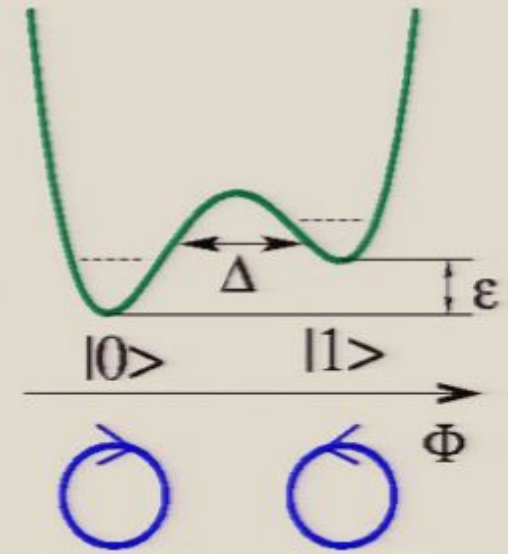
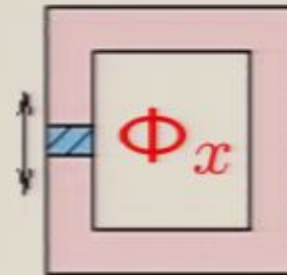
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Josephson Flux Qubits

rf-SQUID

Legget et al., 81

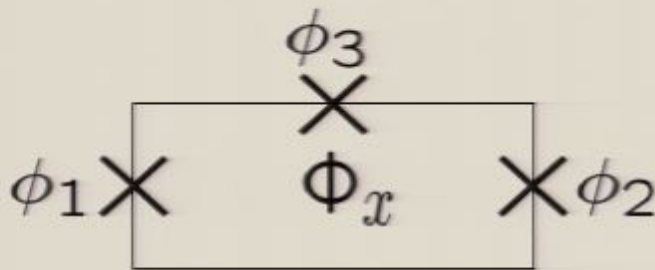


$$H = \frac{Q^2}{2C} - E_J \cos\left(2\pi \frac{\Phi}{\Phi_0}\right) + \frac{(\Phi - \Phi_x)^2}{2L}$$

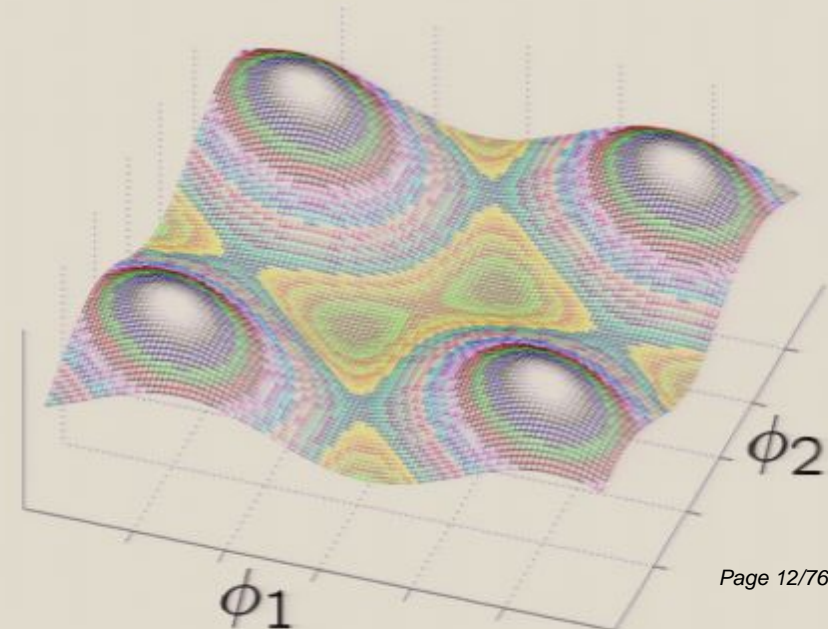
2 states, macroscopically distinct:
clockwise and counter-clockwise currents

$$H \approx -\frac{1}{2}\epsilon(\Phi_x) \sigma_z - \frac{1}{2}\Delta \sigma_x$$

3 jct flux qubit: Mooij et al. '99

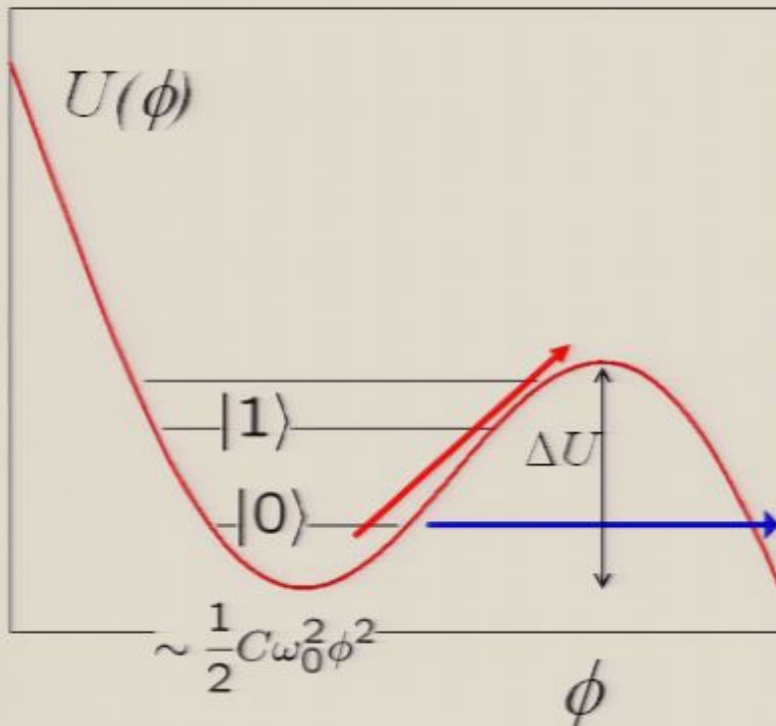


$$\phi_3 = \Phi_x - \phi_1 - \phi_2$$



Phase Qubits

$$H = \frac{Q^2}{2C} - U(\phi) \quad Q = \frac{\hbar}{i} \frac{d}{d\hbar\phi/2e}$$



Thermal activation
Fulton & Dolan '70s

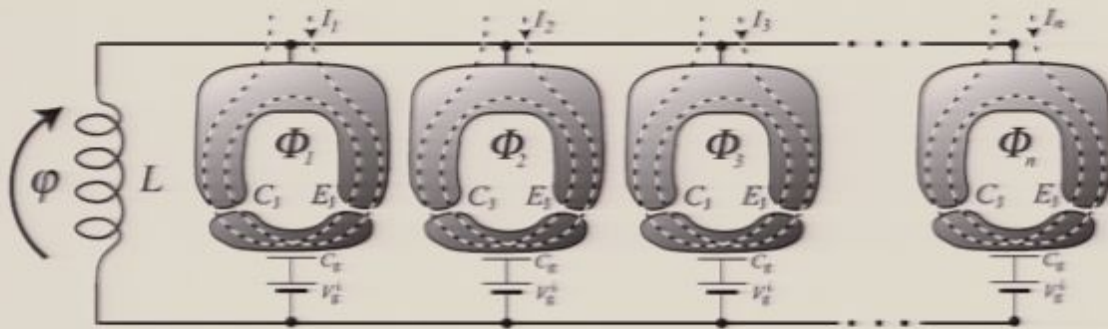
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Phase qubit
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Coupling of Qubits



**Qubits coupled
by an LC – oscillator**

Makhlin, Shnirman, G.S. (99)

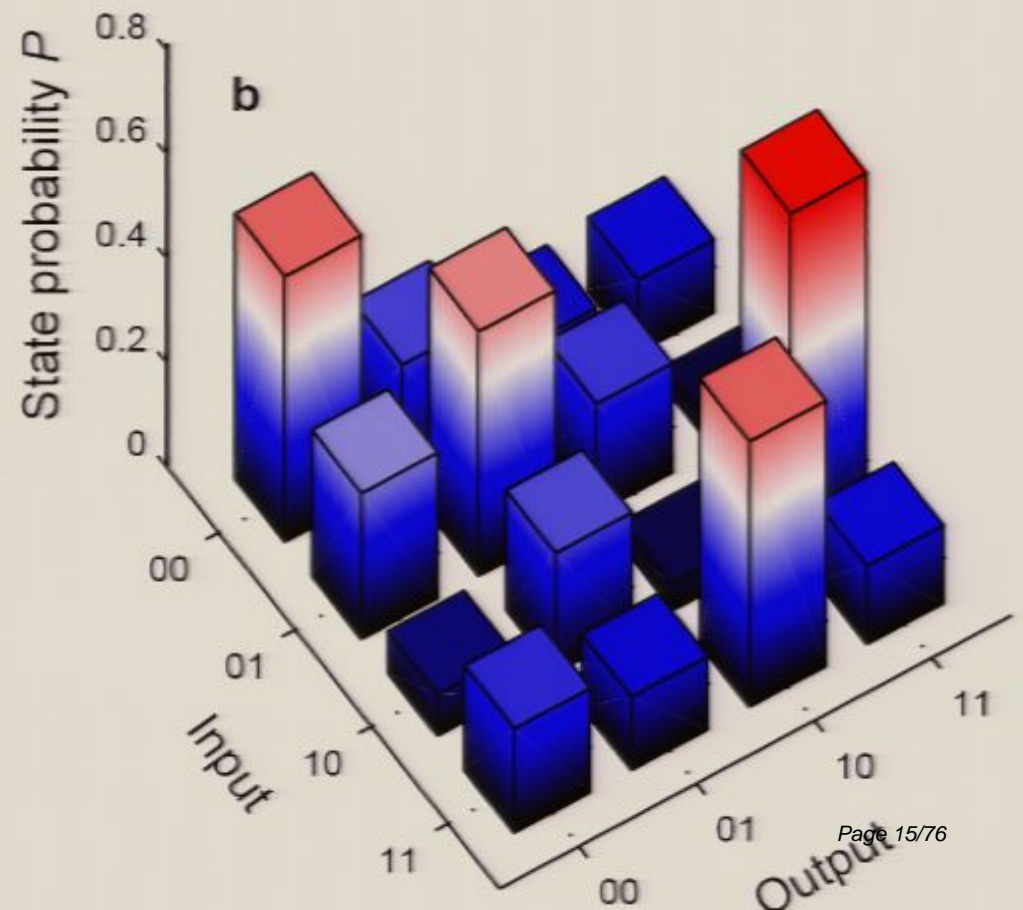
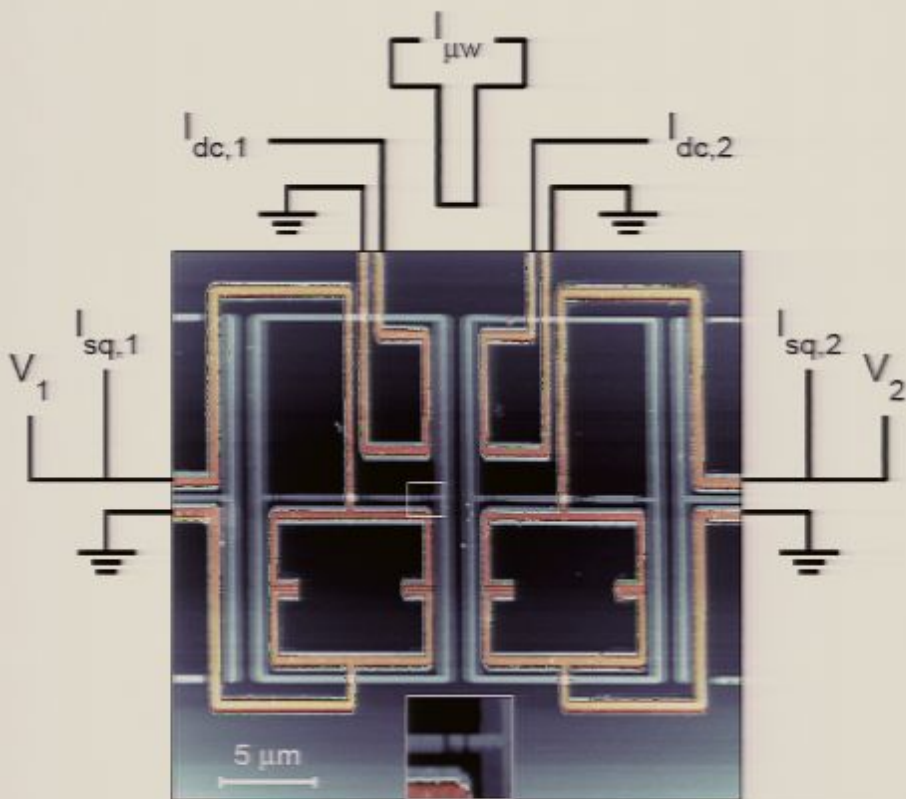
$$\mathcal{H} = \sum_{i=1}^N [\Delta E_{\text{ch}}(Q_{gi}) - E_J(\Phi_{xi}) \cos \varphi_i] - \frac{L}{2} \left(\sum_i I_i \right)^2$$

$$I_i \propto E_J(\Phi_{xi}) \sin \varphi_i$$

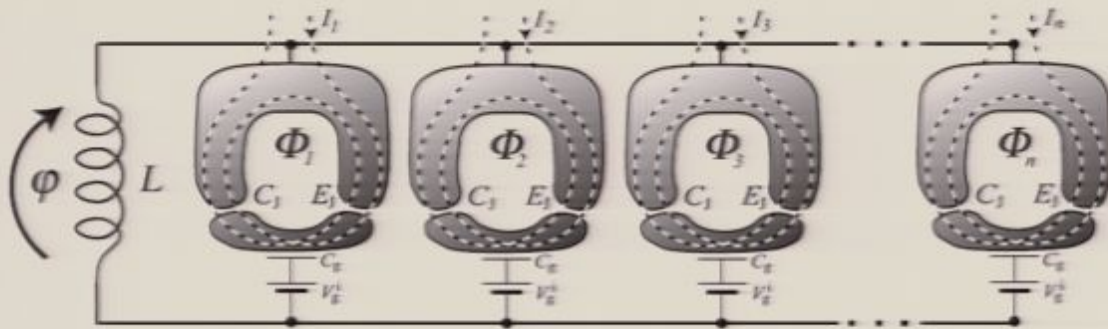
LETTERS

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J. H. Plantenberg¹, P. C. de Groot¹, C. J. P. M. Harmans¹ & J. E. Mooij¹



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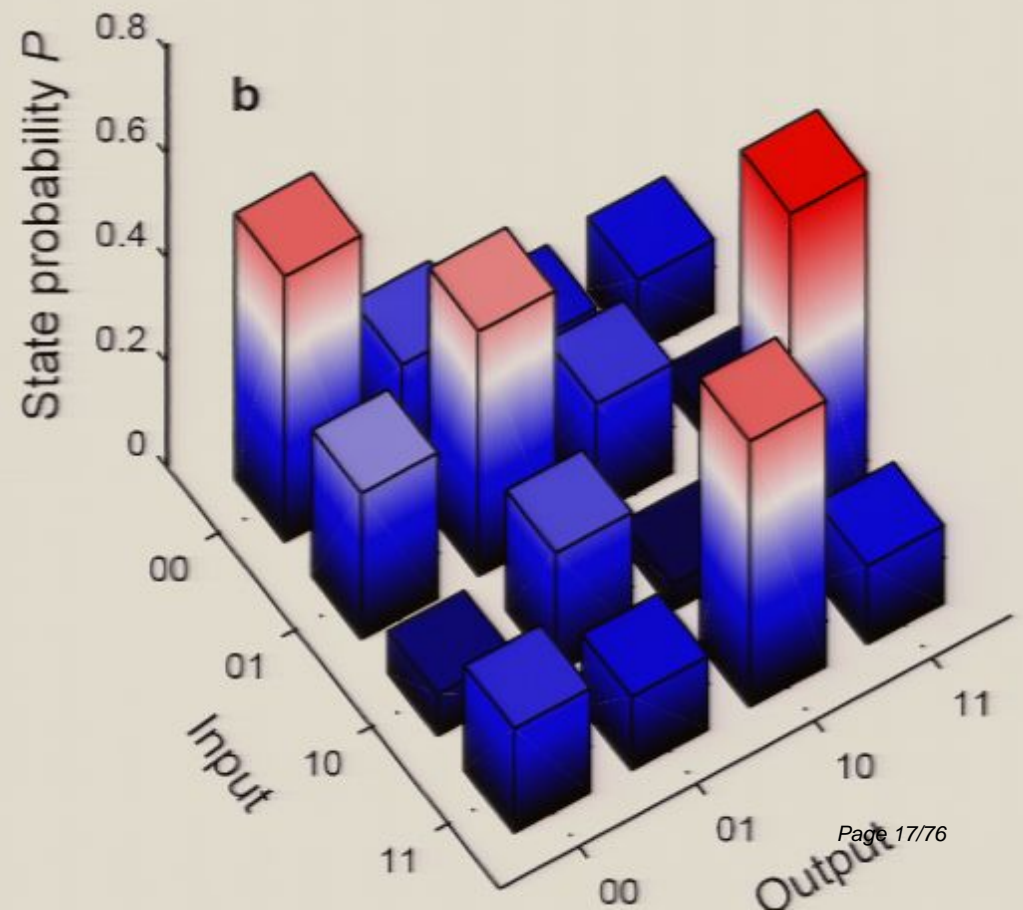
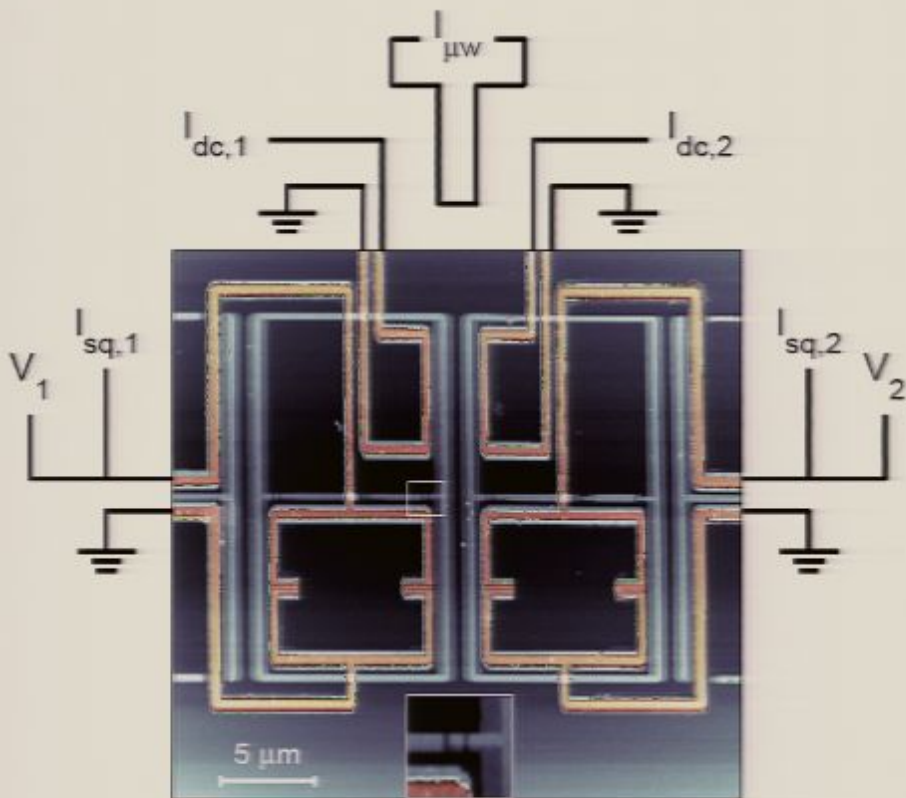
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Noise and Decoherence

Sources of noise

- noise from control and measurement circuit, $Z(\omega)$
- background charge fluctuations
- ...

Properties of noise

- spectrum: Ohmic (white), $1/f$, ...
- Gaussian, 2-level systems, ...
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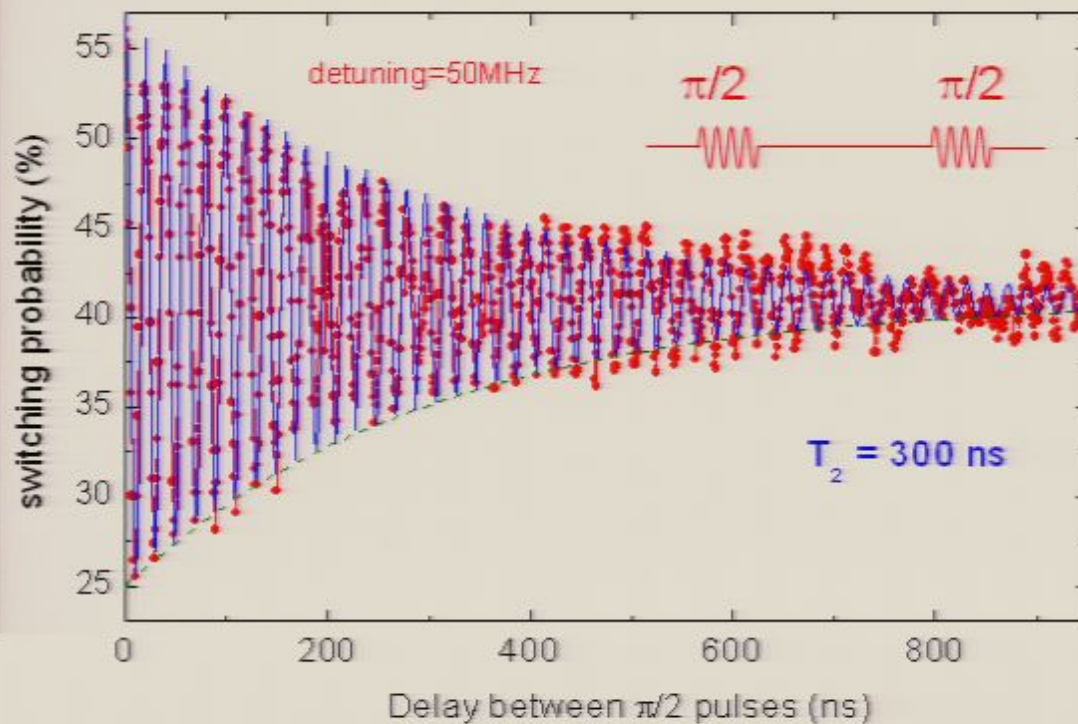
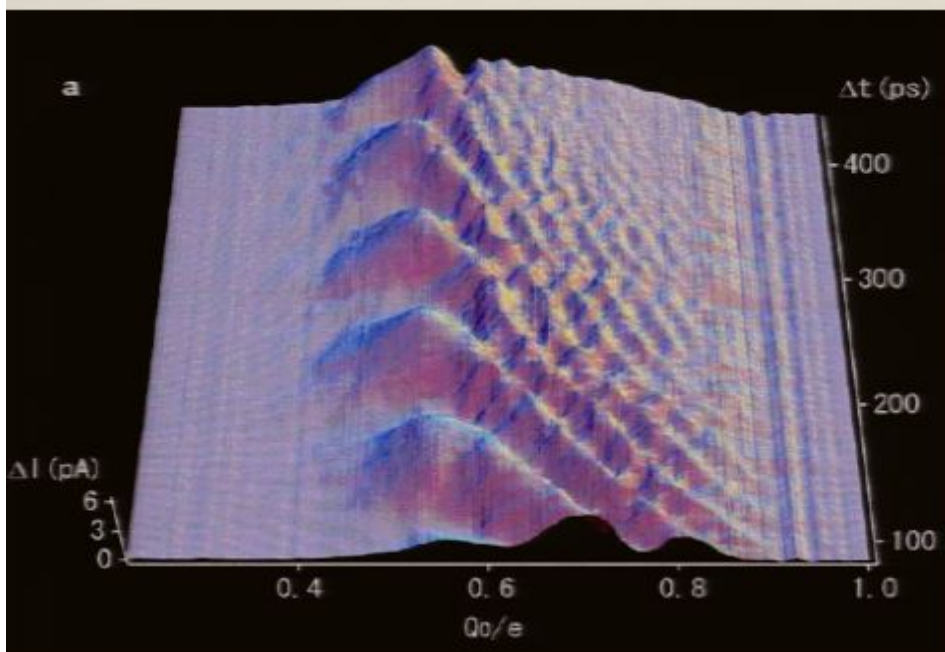
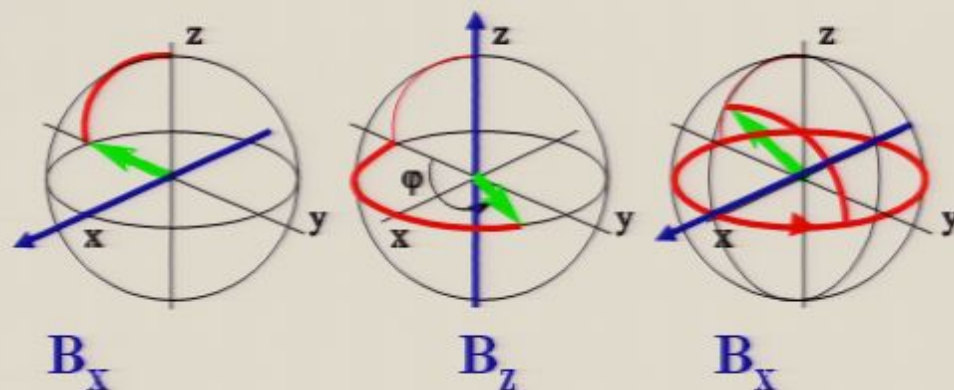
$$\propto \omega \coth \frac{\hbar\omega}{2k_B T}, \quad 1/\omega, \dots$$

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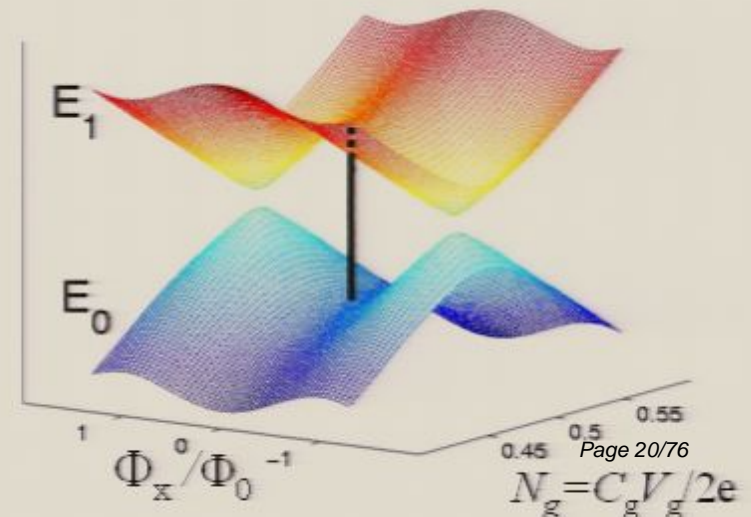
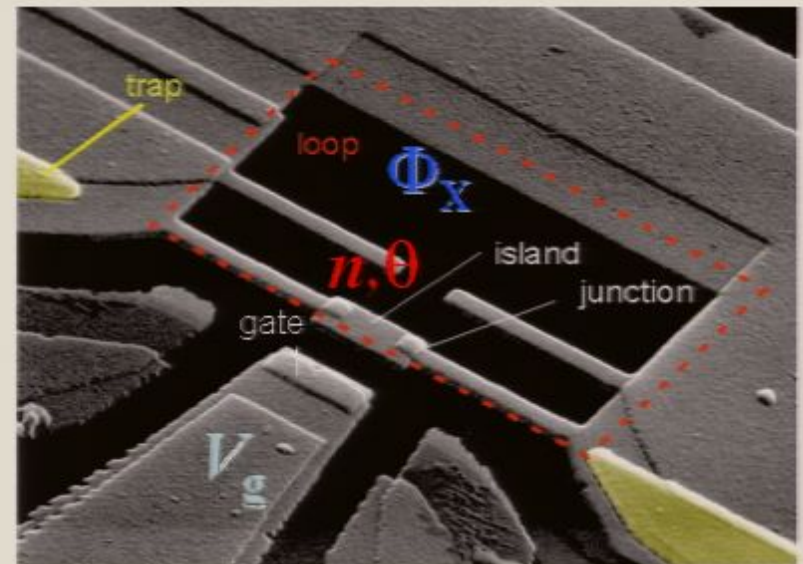
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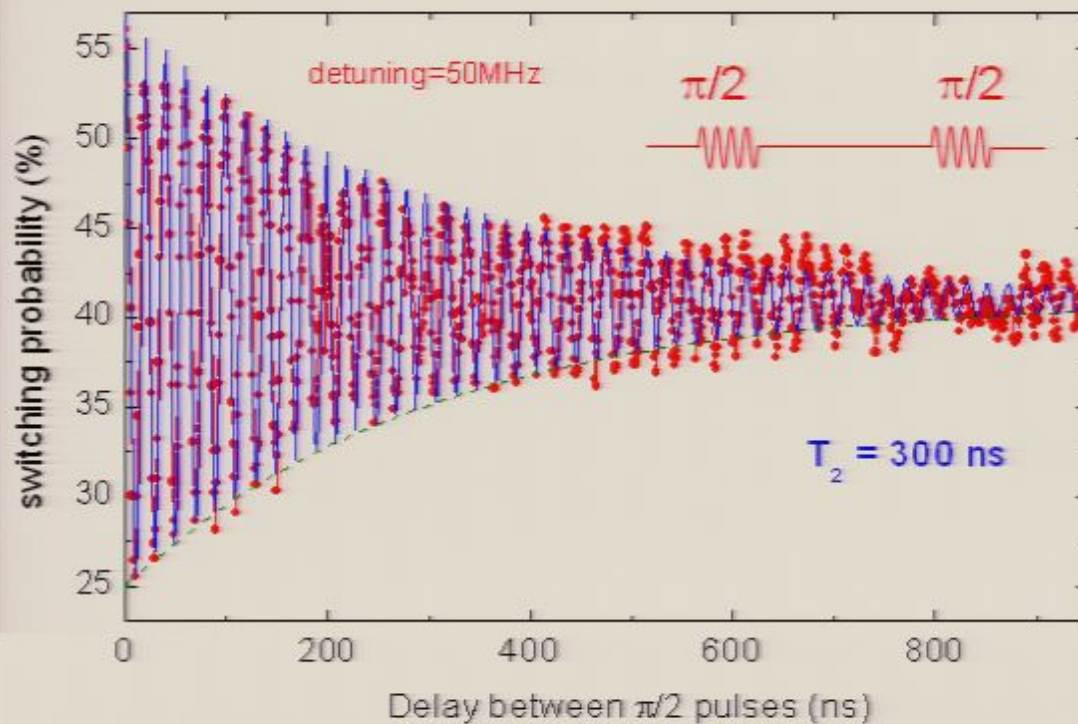
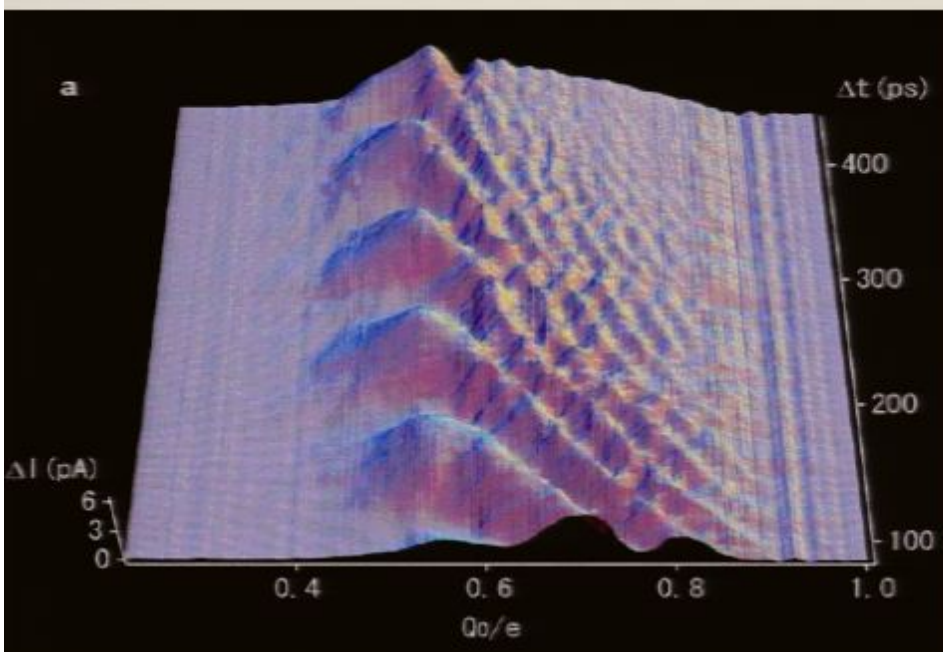
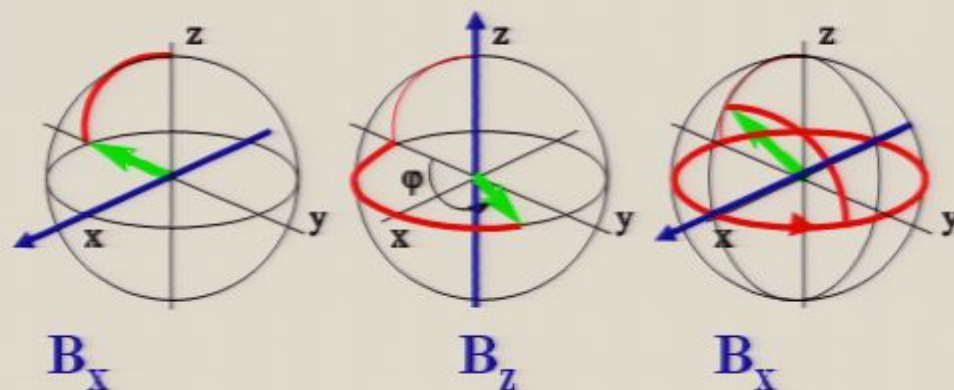
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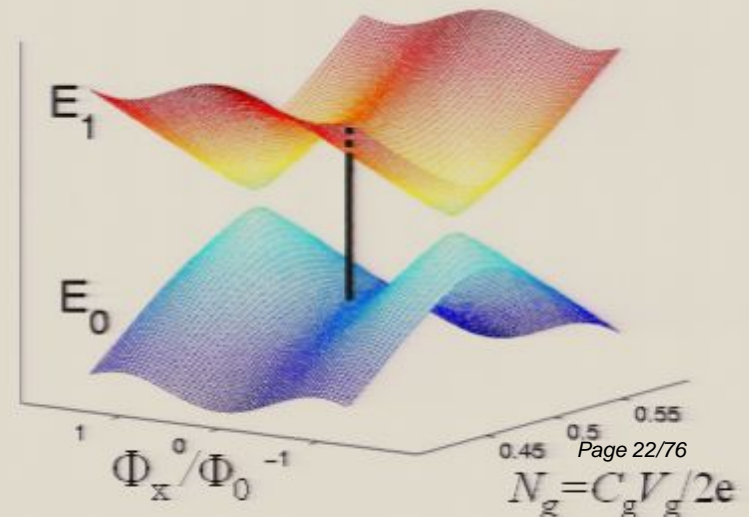
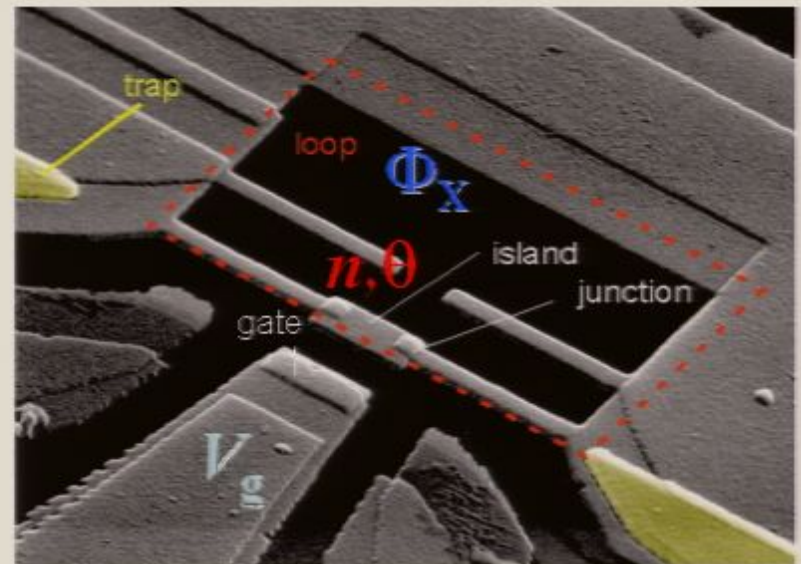
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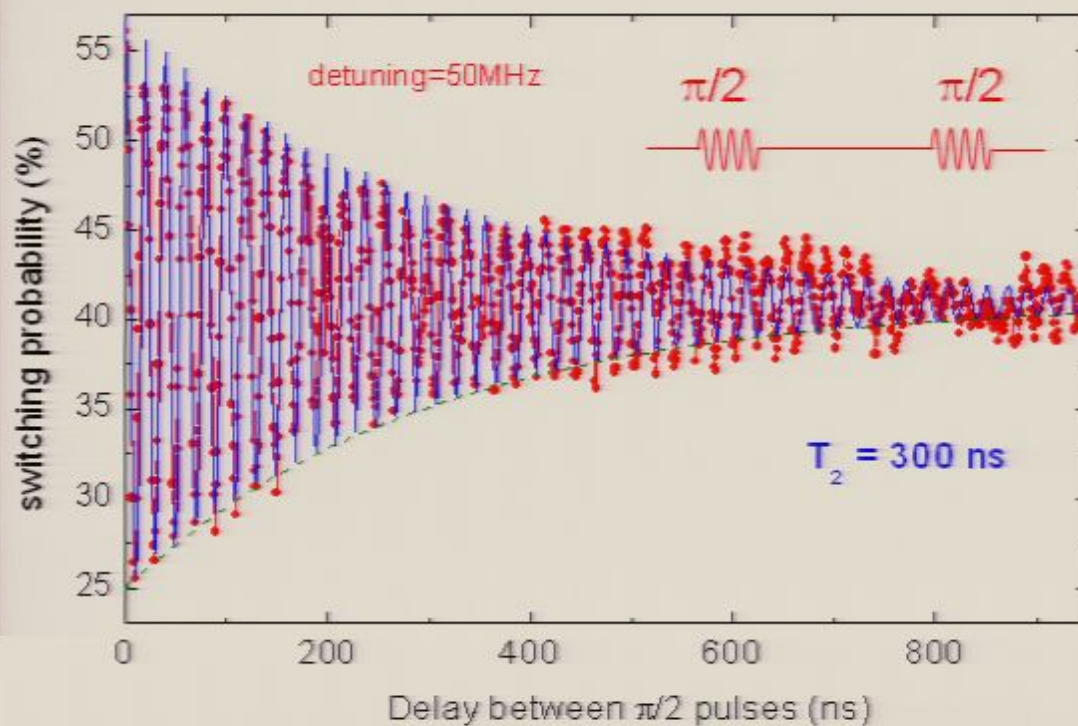
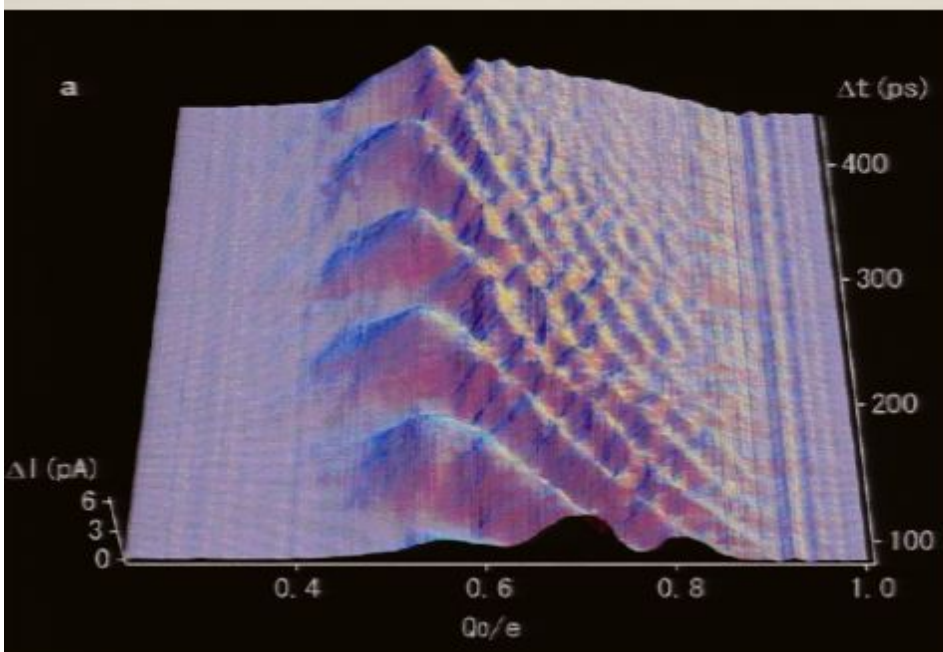
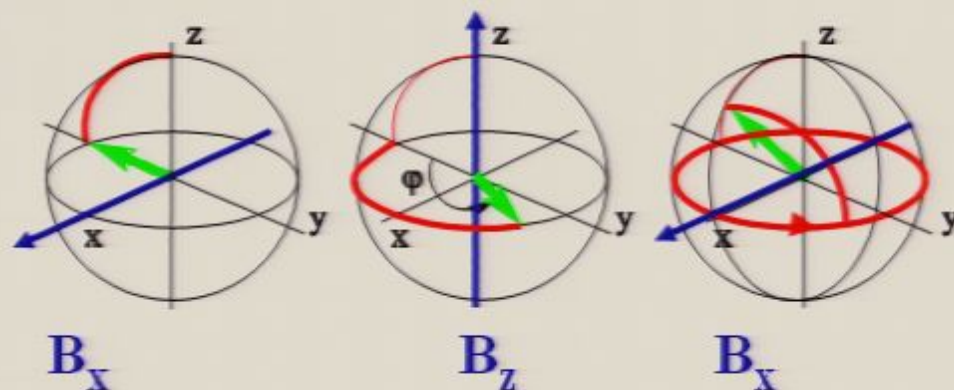
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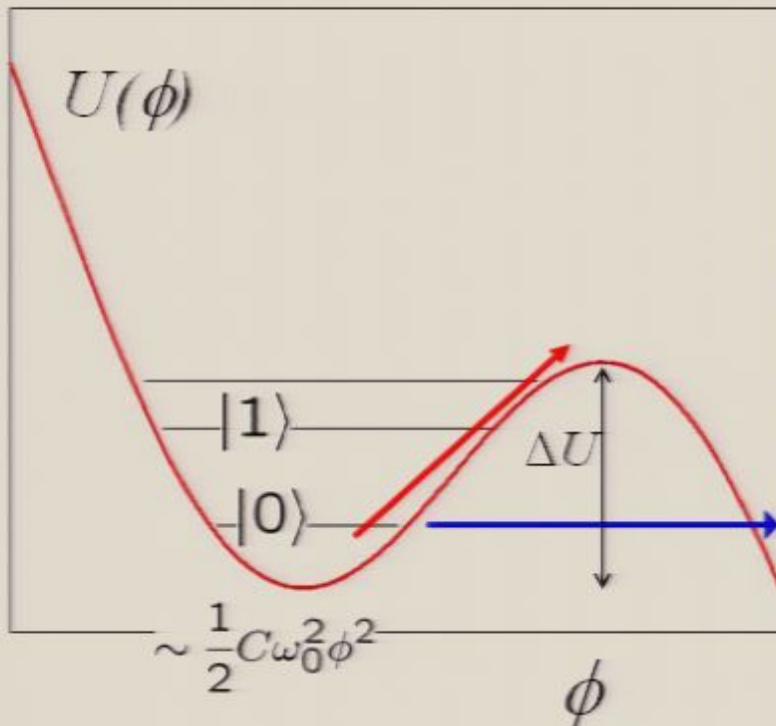
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Fulton & Dolan '70s

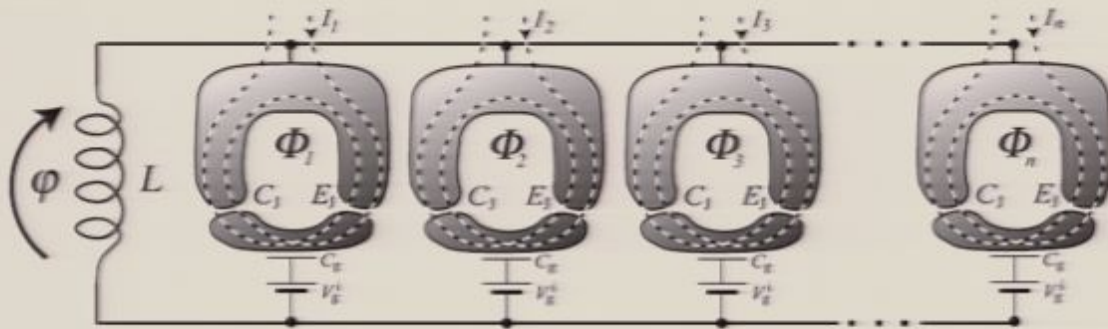
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longitudinal – transverse – quadratic (longitudinal) ...

Linear coupling, regular power spectrum

$$H = -\frac{1}{2}(\Delta E \tau_z + X \cos\eta \tau_z + X \sin\eta \tau_x) + H_{Bath}$$

Golden rule (Bloch – Redfield) \Rightarrow exponential decay
(Leggett *et al.* 87, Weiss 99)

$$|\rho_{ij}(t)| \propto e^{-\Gamma t}$$

$$\frac{1}{T_1} = \Gamma_{\text{rel}} = \frac{1}{2} S_X(\omega = \Delta E) \sin^2\eta$$

$$\frac{1}{T_2} = \Gamma_{\varphi} = \frac{1}{2} \frac{1}{T_1} + \frac{1}{2} S_X(\omega \approx 0) \cos^2\eta$$

pure dephasing: Γ_{φ}^*

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Nyquist noise due to R

$$S_{\delta V}(\omega) = \hbar \omega R \coth \frac{\hbar \omega}{2k_B T}$$

$$\Rightarrow \Gamma_{rel} \propto (e^2 / h) R \Delta E$$

$$\Gamma_\varphi^* \propto (e^2 / h) R k_B T$$

Dephasing due to 1/f noise, nonlinear coupling, ... ?

longitudinal linear coupling, $1/f$ noise

$$H = -\frac{1}{2}[\Delta E + X(t)]\tau_z$$

$$S_X(\omega) = \frac{E_{1/f}^2}{|\omega|}$$

beyond Golden rule:
non-exponential decay

$$|\rho_{01}(t)| = \exp\left(-\frac{E_{1/f}^2}{2\pi} t^2 \ln|\omega_{ir}t|\right)$$

$$\Gamma_\varphi^* \propto E_{1/f}$$

Cottet et al. (01)

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At symmetry point: Quadratic longitudinal 1/f noise

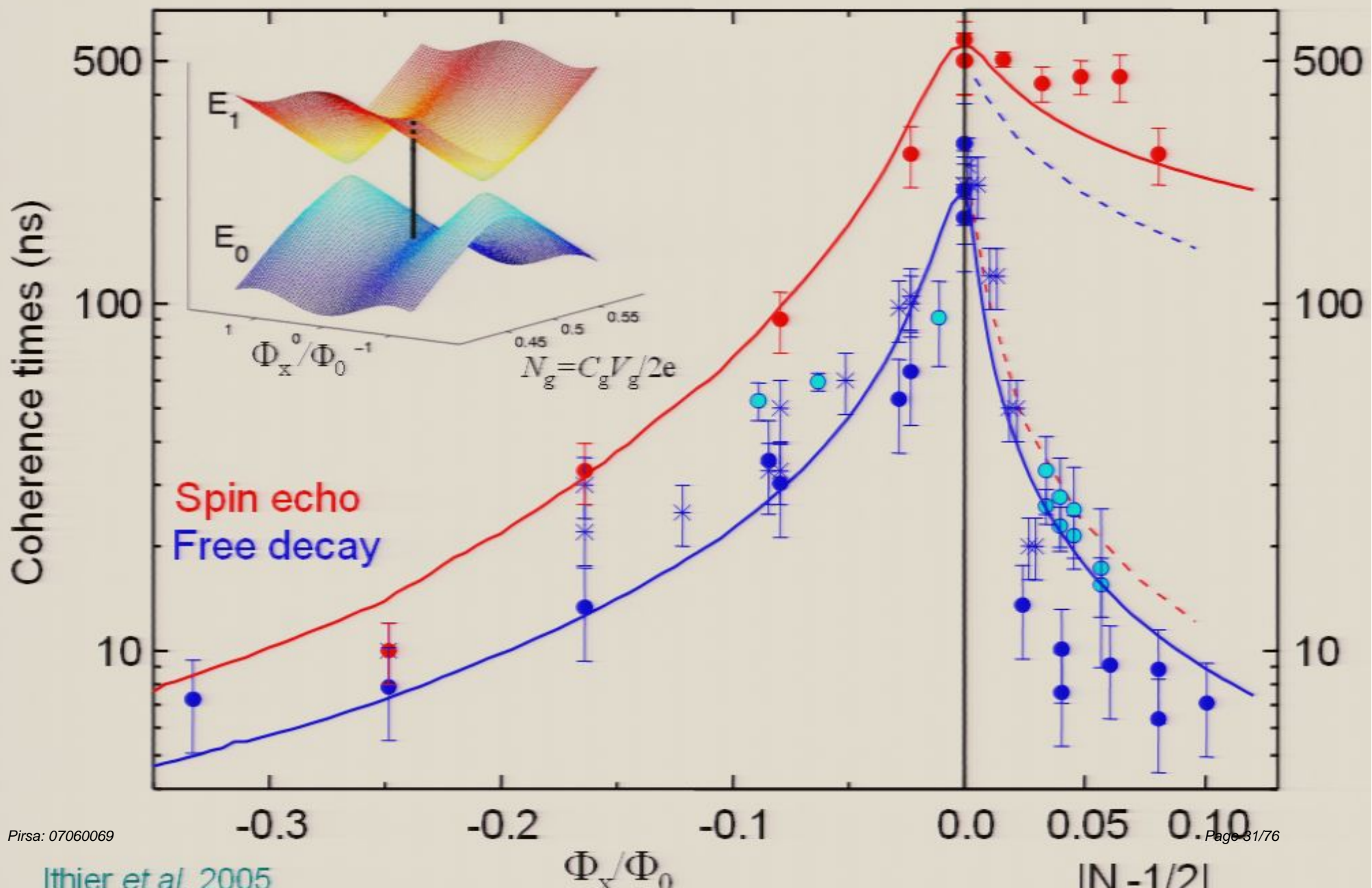
$$H = -\frac{1}{2}(\Delta E + \lambda X^2)\tau_z$$

power-law decay

$$\rho_{01}(t) \propto \left[1 + i\frac{2}{\pi}\Gamma_{1/f}t \ln(\omega_{ir}t)\right]^{-1/2}$$

Shnirman, Makhlin (03)

Optimum-point strategy for noise reduction



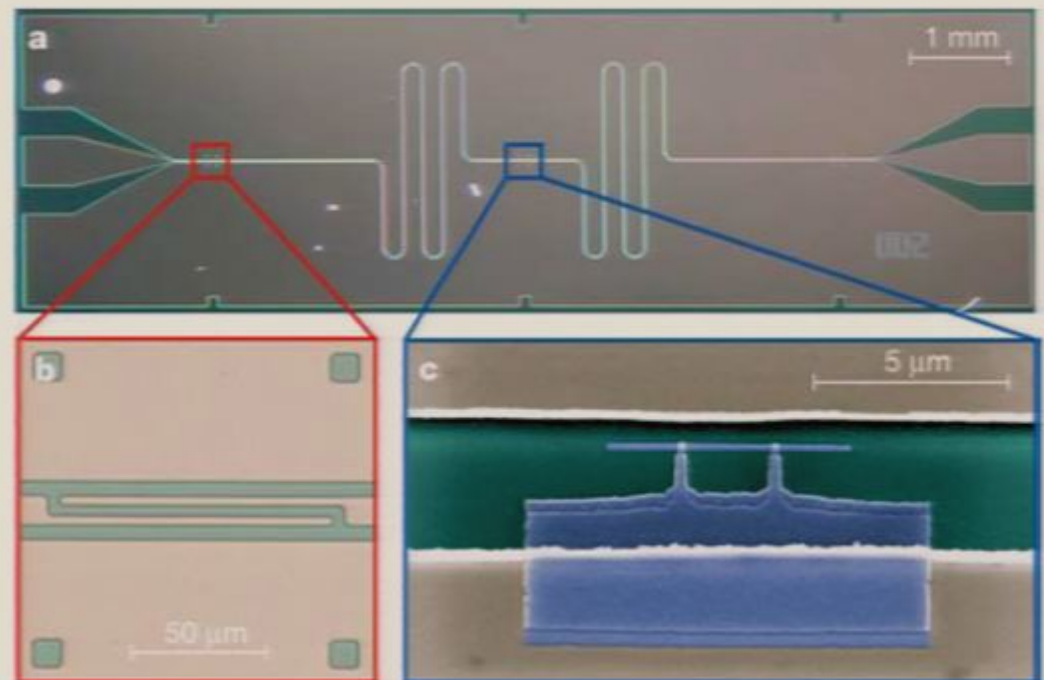
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Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics

A. Wallraff¹, D. I. Schuster¹, A. Blais¹, L. Frunzio¹, R.-S. Huang^{1,2}, J. Majer¹, S. Kumar¹, S. M. Girvin¹ & R. J. Schoelkopf¹

NATURE | VOL 431 | 9 SEPTEMBER 2004 |

- superconducting two-level system, playing the role of an artificial atom
- coupled to an on-chip cavity consisting of a superconducting transmission line resonator
- strong coupling regime can be attained
- coherent interaction of a superconducting two-level system with a single microwave photon.

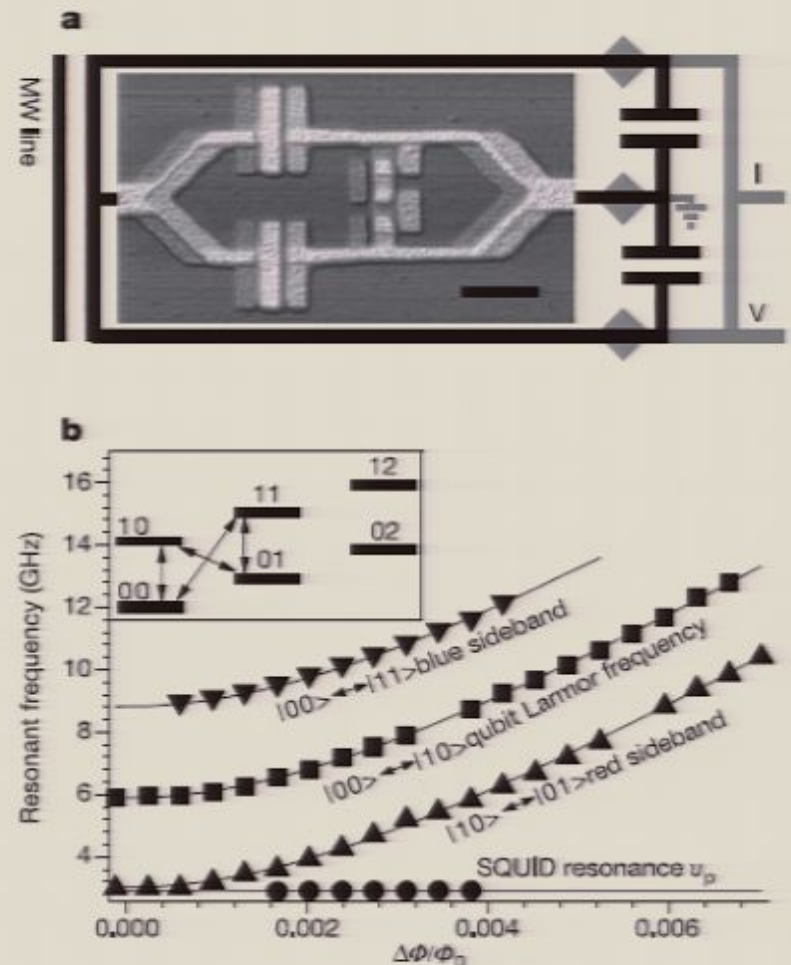


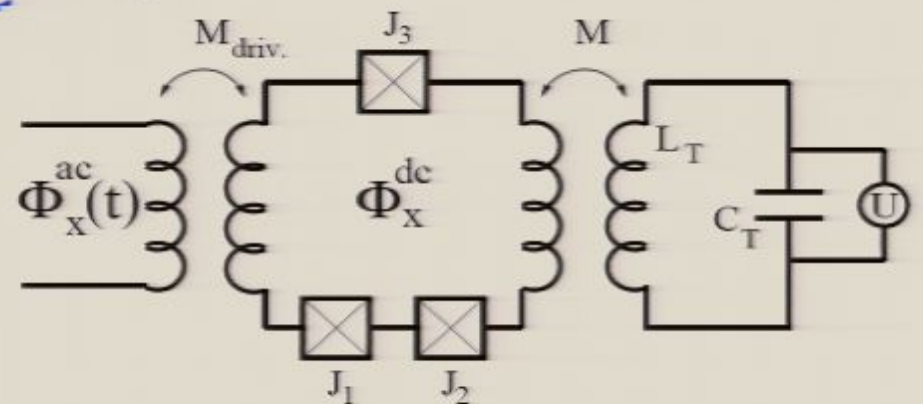
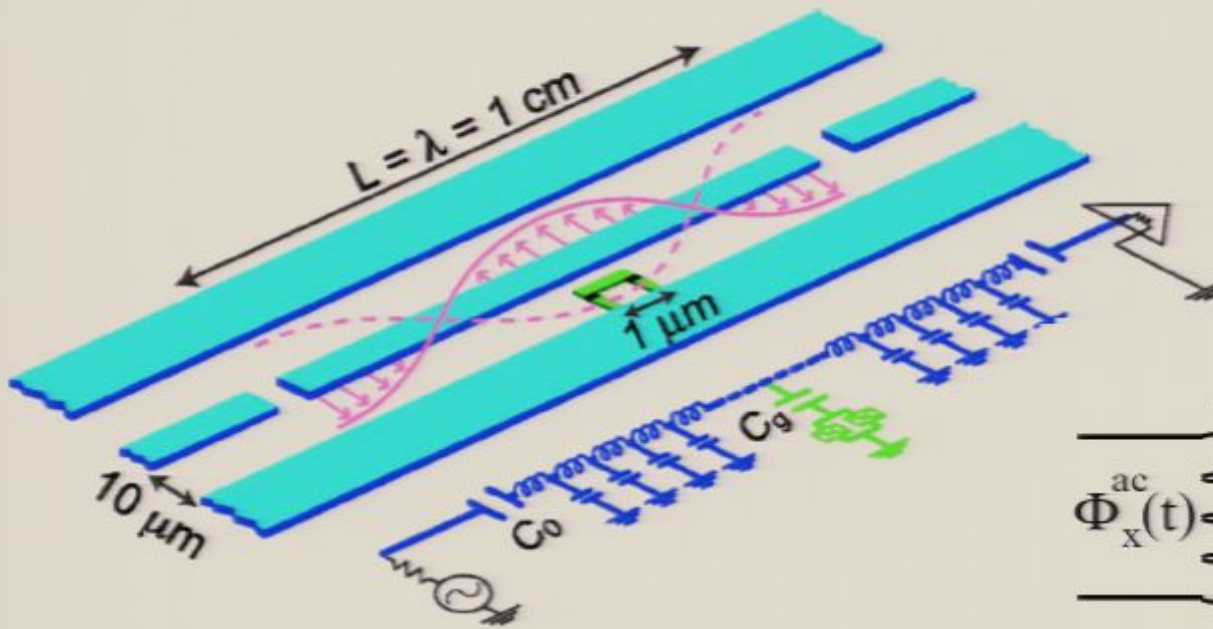
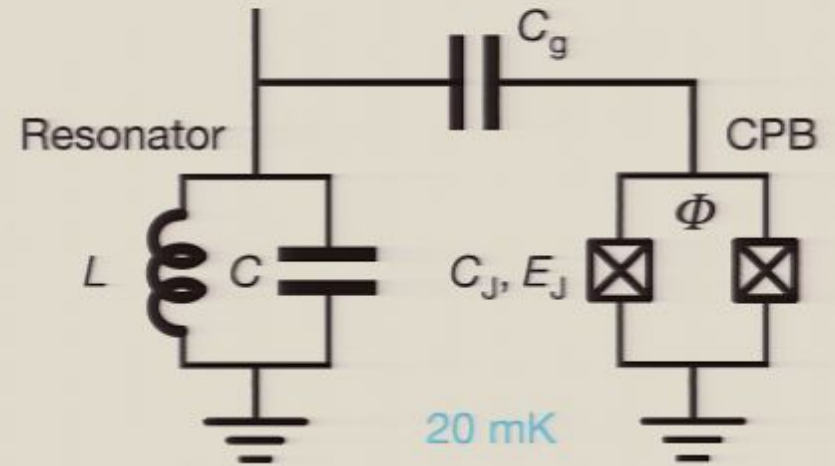
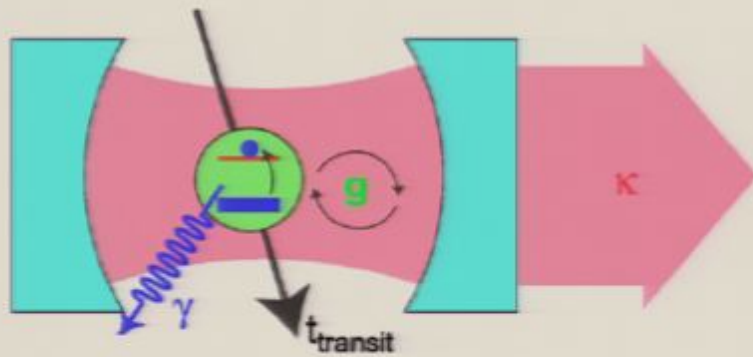
Coherent dynamics of a flux qubit coupled to a harmonic oscillator

I. Chiorescu^{1,*}, P. Bertet¹, K. Semba^{1,2}, Y. Nakamura^{1,3},
C. J. P. M. Harmans¹ & J. E. Mooij¹

NATURE | VOL 431 | 9 SEPTEMBER 2004 |

- entanglement between a superconducting flux qubit (a two-level system) and a superconducting quantum interference device (SQUID)
- The latter provides the measurement system for detecting the quantum states
- It is also an effective inductance that, in parallel with an external shunt capacitance, acts as a harmonic oscillator.
- We achieve generation and control of the entangled state by performing microwave spectroscopy and detecting the resultant Rabi oscillations of the coupled system.

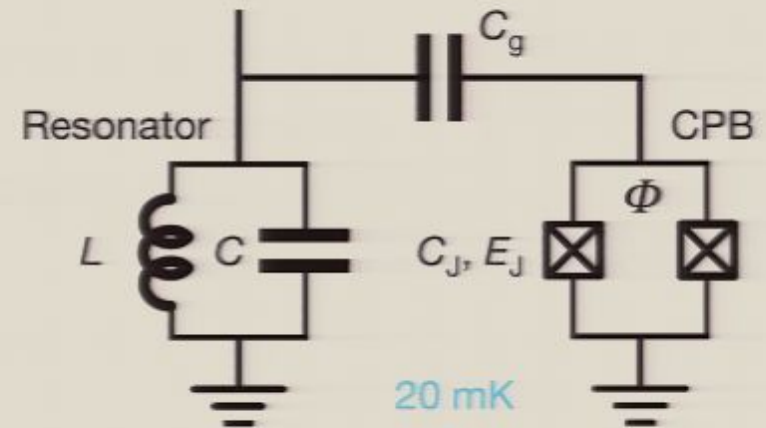




Resonator coupled capacitively to charge qubit

Couples like gate voltage. Quantize oscillator.

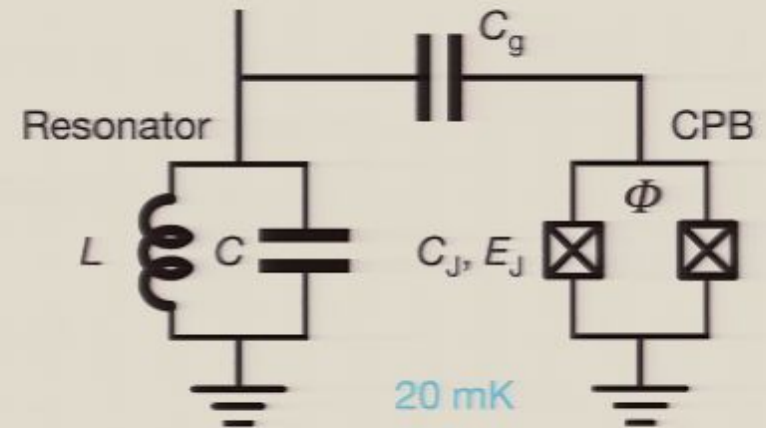
$$H = -\frac{1}{2}\Delta E_{\text{ch}}(V_g)\sigma_z - \frac{1}{2}E_J(\Phi)\sigma_x + \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + e\frac{C_g}{C_\Sigma}\sqrt{\frac{\hbar\omega_r}{C}}(a + a^\dagger)\sigma_z$$



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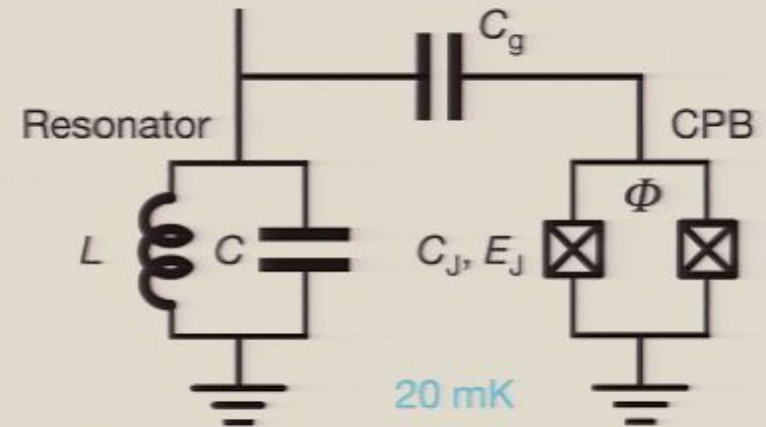
in eigenbasis of qubit + RWA \Rightarrow Jaynes-Cummings Hamiltonian

$$H = \frac{1}{2}\hbar\Omega \sigma_z + \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) + \hbar g (a \sigma_+ + a^\dagger \sigma_-) + H_\gamma + H_\kappa$$

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$$\Omega = \frac{1}{\hbar}\sqrt{\Delta E_{\text{ch}}^2 + E_J^2} \sim 10\text{GHz}$$

Transition frequency

$$g = \frac{e C_g}{\hbar C_\Sigma}\sqrt{\frac{\hbar\omega_r}{C}} \sim 100\text{MHz}$$

Vacuum Rabi rate

large dipole moment

$$d = \hbar g / E_{\text{rms}} \sim 10^4 e a_0$$

Strong detuning: dispersive regime

$$H = \frac{1}{2}\hbar\Omega\sigma_z + \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \hbar\frac{g^2}{\Omega - \omega_r}\left(a^\dagger a + \frac{1}{2}\right)\sigma_z$$

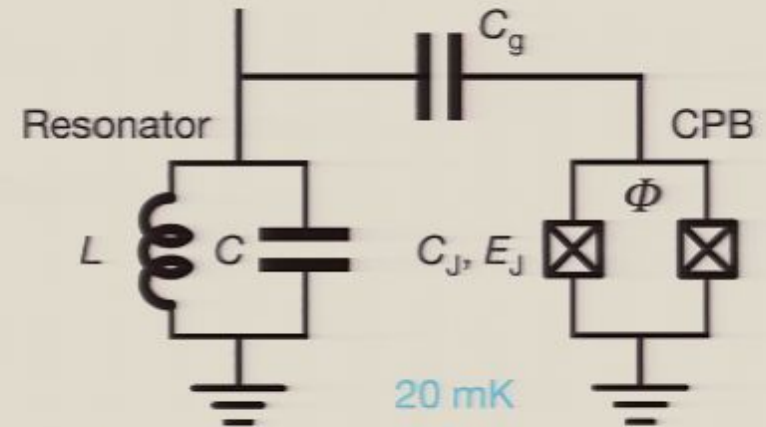
ac Stark effect and Lamb

- resonator frequency (and response) shifted by qubit, state-dependent
- qubit transition frequency shifted by resonator, state-dependent
- used for read-out

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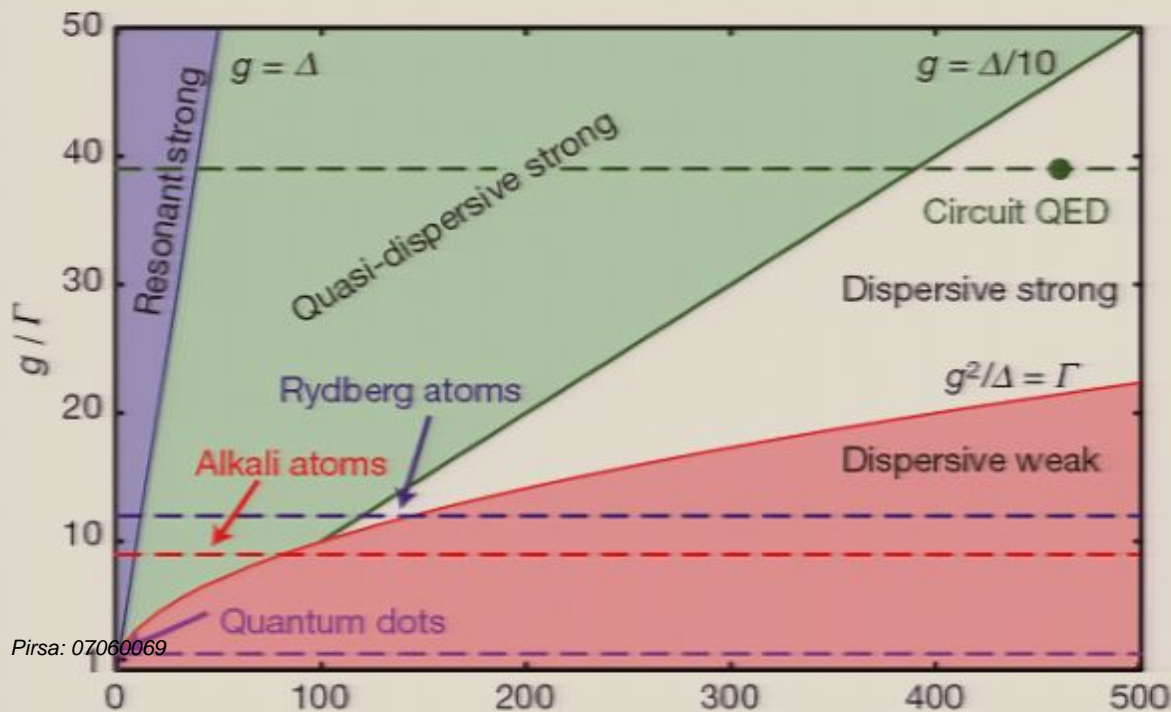
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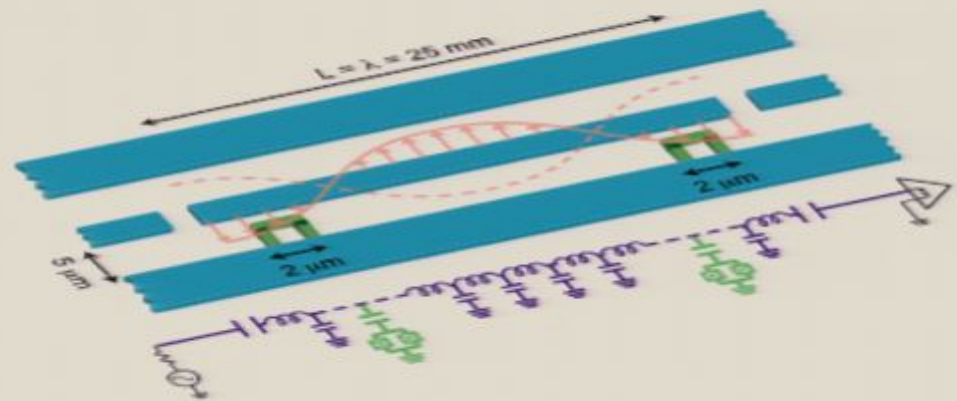
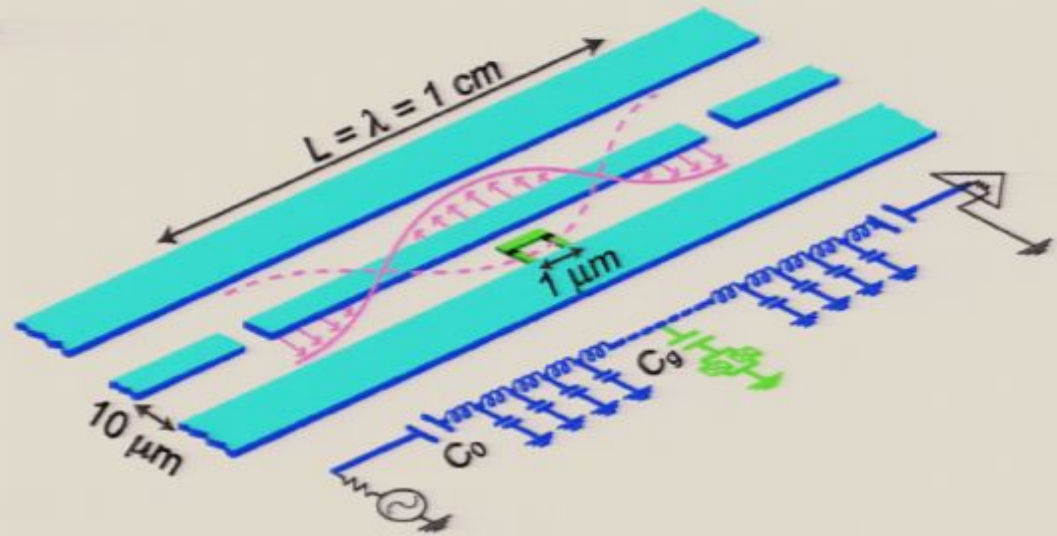
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Schuster et al., Nature 07

Coupling to transmission line

- small effective volume
 $\approx 10^{-5}$ wavelenghts³
- large zero-point fluctuations
 $V_{\text{rms}} \approx 2 \mu\text{V}$, $V_{\text{rms}} \approx 0.2 \text{ V/m}$
100 time larger than in 3D cavity
- high quality $Q \approx 10^6$ achieved
for better coupling $Q \approx 10^4$
- also for coupling qubits

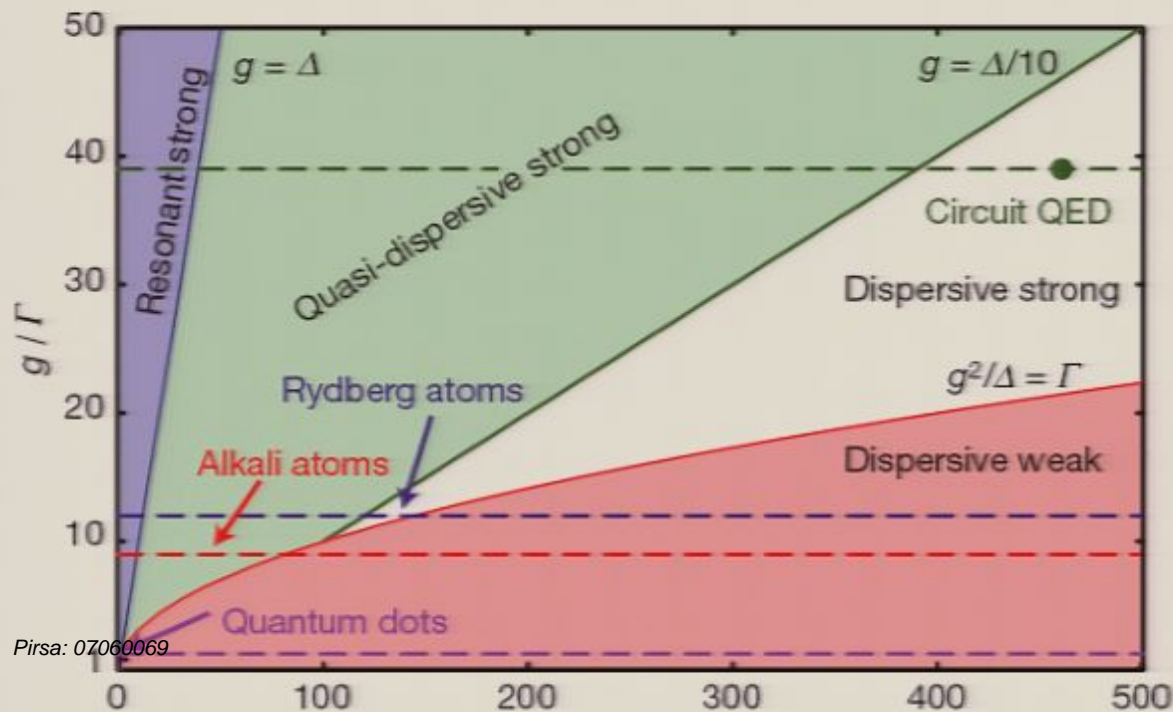


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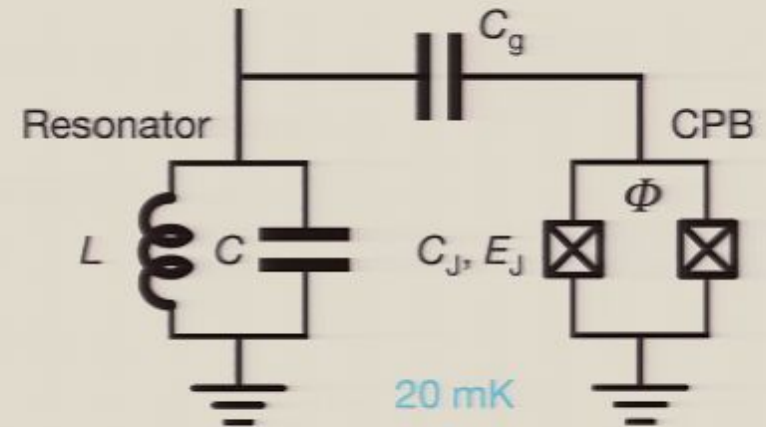


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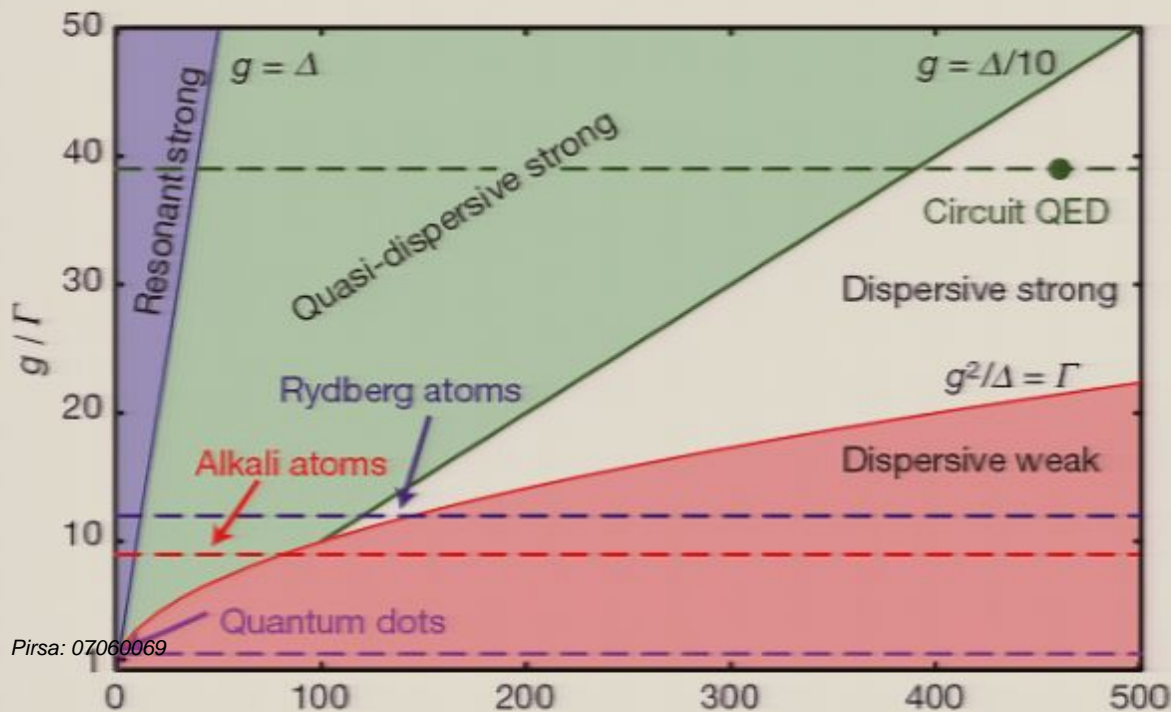
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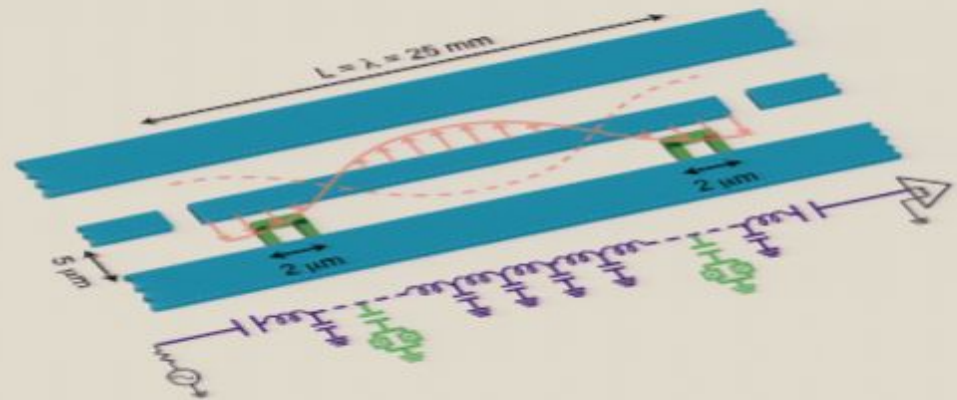
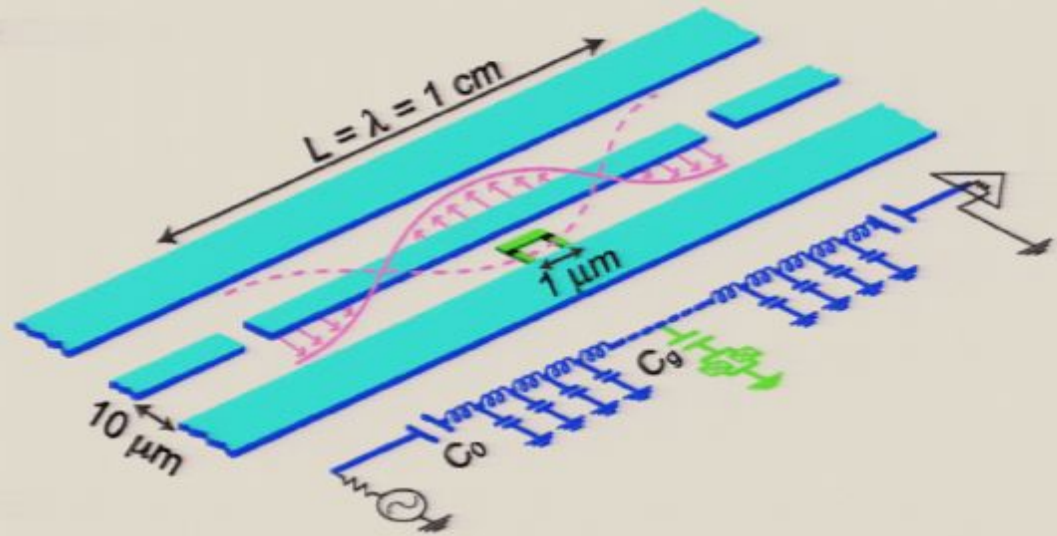
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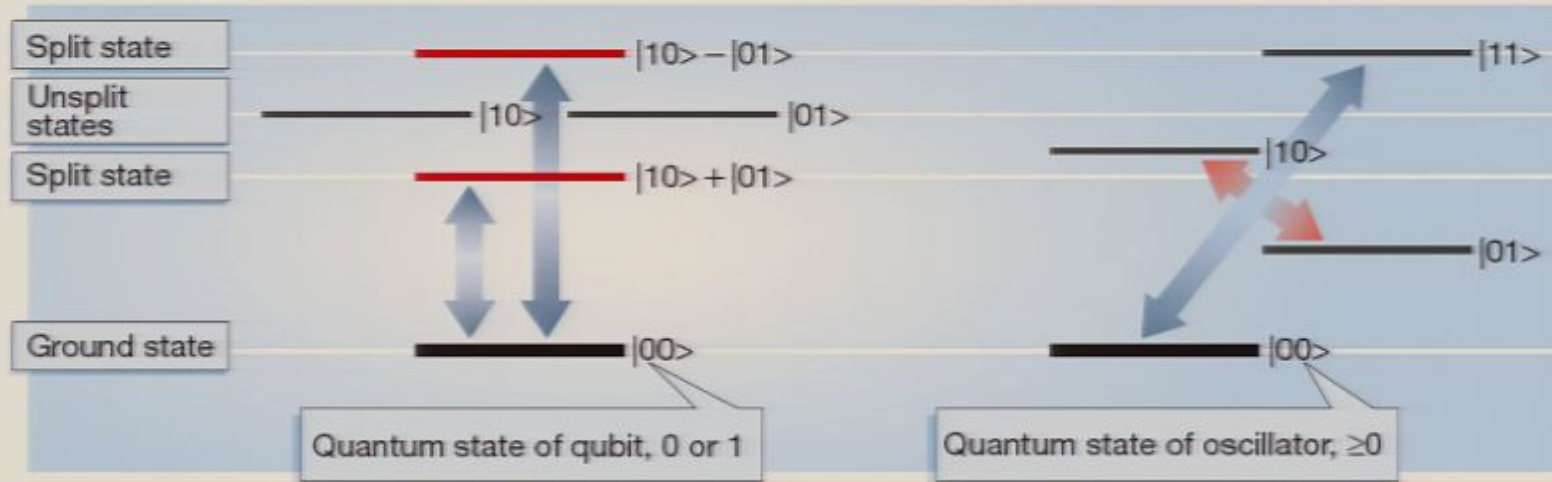


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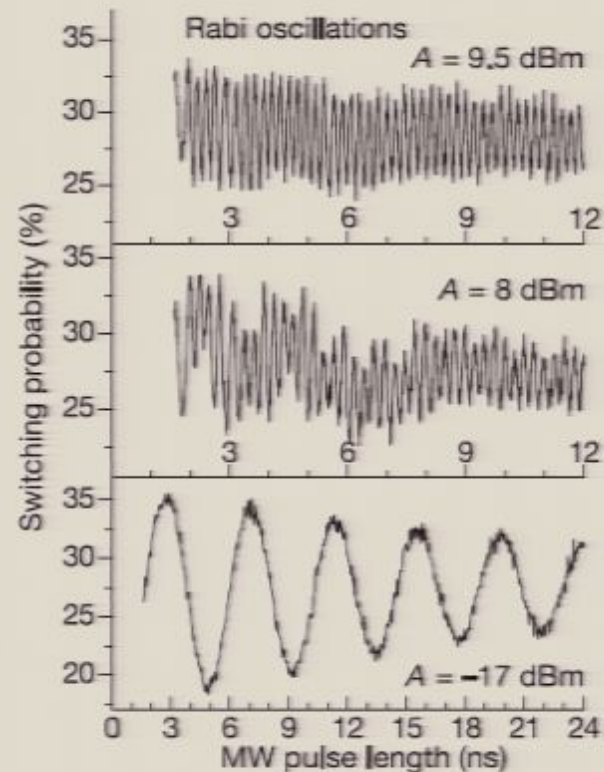
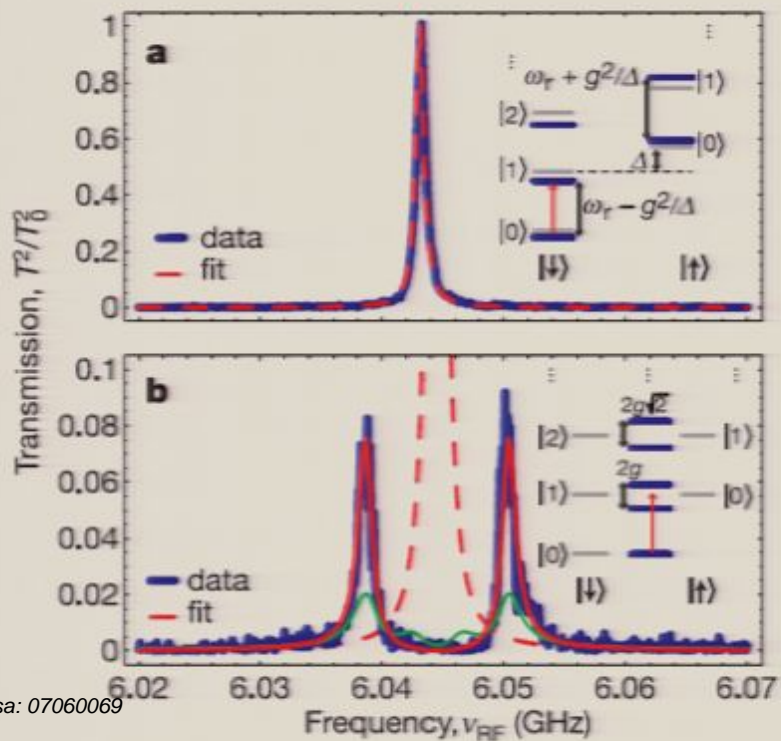




The qubit and the cavity

Makhlin, G.S.
Shnirman,
Nature 04
News and Views

Figure 1 The structure of the energy levels in a coupled qubit–oscillator system, as studied by Wallraff *et al.*² (left) and Chiorescu *et al.*³ (right).



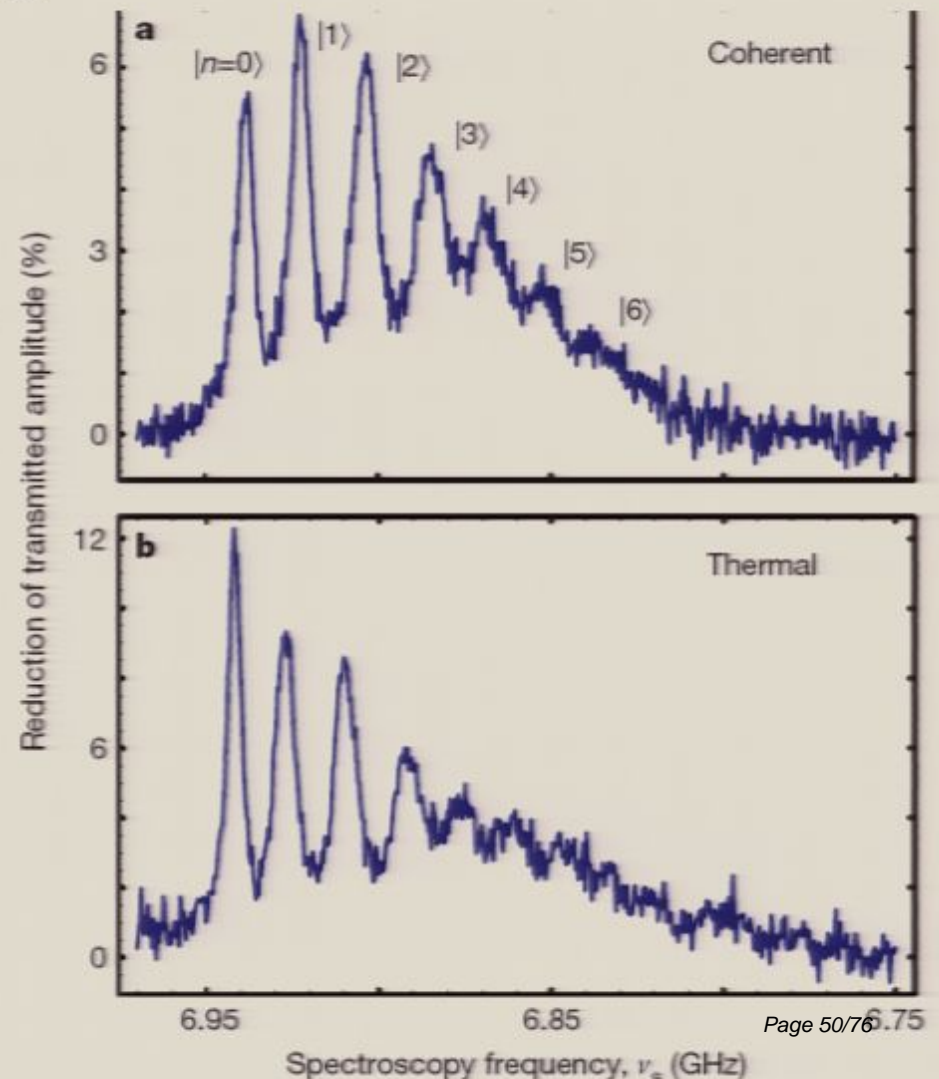
Resolving photon number states in a superconducting circuit

D. I. Schuster^{1*}, A. A. Houck^{1*}, J. A. Schreier¹, A. Wallraff^{1†}, J. M. Gambetta¹, A. Blais^{1†}, L. Frunzio¹, J. Majer¹, B. Johnson¹, M. H. Devoret¹, S. M. Girvin¹ & R. J. Schoelkopf¹

NATURE | Vol 445 | 1 February 2007

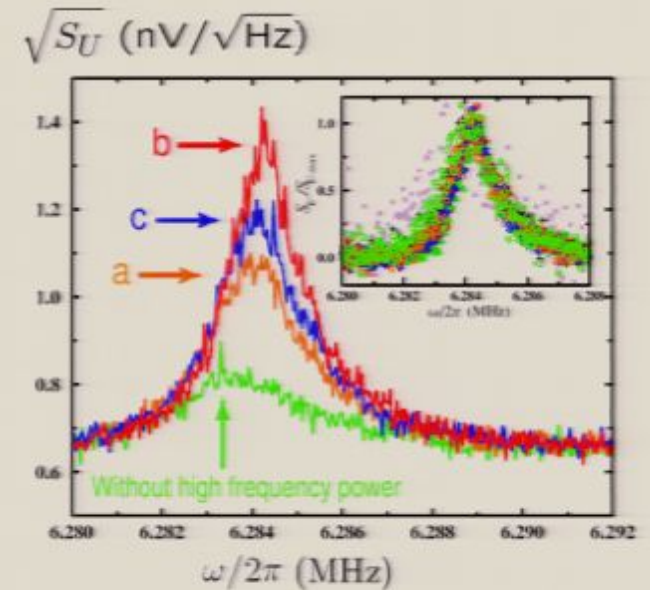
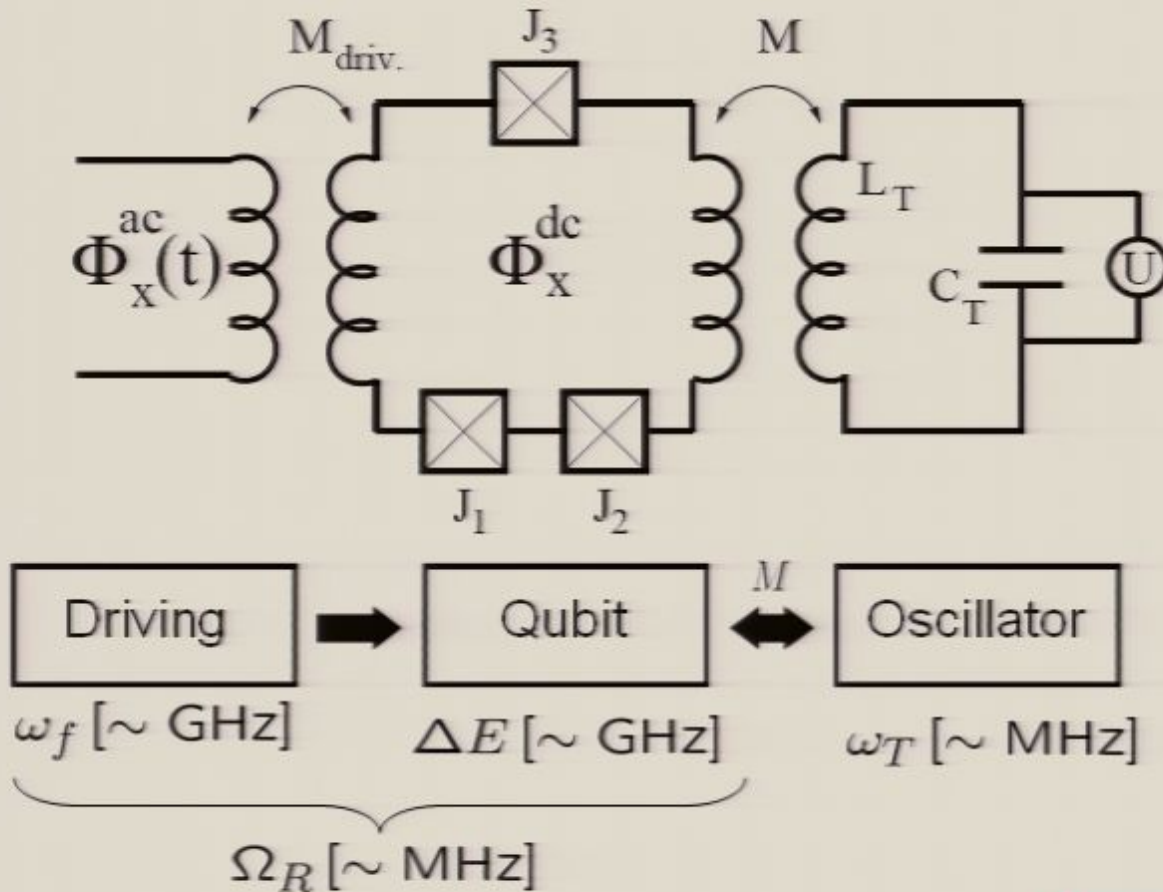
$$H = \hbar\omega_r(a^\dagger a + 1/2) + \hbar\omega_a\sigma_z/2 + \hbar\chi(a^\dagger a + 1/2)\sigma_z$$

- circuit QED in strong dispersive regime
- a single photon has a large effect on the qubit without ever being absorbed.
- the qubit transition energy can be resolved into a separate spectral line for each photon number state of the microwave field.
- This effect is used to distinguish between coherent and thermal fields, ... create a photon statistics analyser.



- Josephson qubits
 - charge, flux, phase qubit
 - experiments: spectroscopy, coherent oscillations, Ramsey fringes, Rabi, ...
 - qubit-qubit coupling
 - decoherence/relaxation
- Josephson qubits coupled to oscillators
 - LC-oscillators, transmission lines, nano-mechanical oscillators
 - strong coupling regime
 - vacuum Rabi oscillations
 - dispersive regime, read-out
- Single- and two photon lasing and cooling at the Rabi frequency
- Superconducting SET transistor and resonator

Lasing and cooling at the Rabi frequency



E. Il'ichev et al., PRL (2003)

Idea: Drive qubit to perform Rabi oscillations
in resonance with slow oscillator

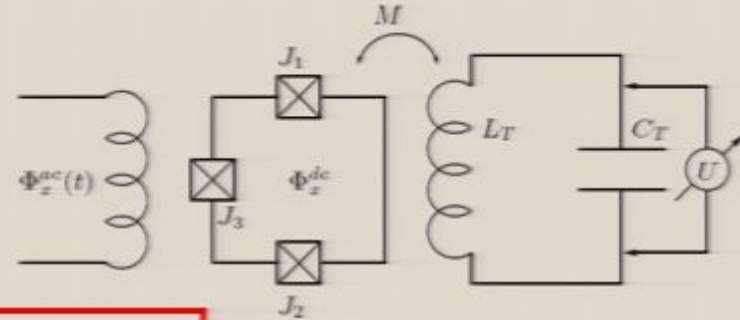
$$\hbar\omega_f \approx \Delta E$$

$$\Omega_R \approx \omega_T$$

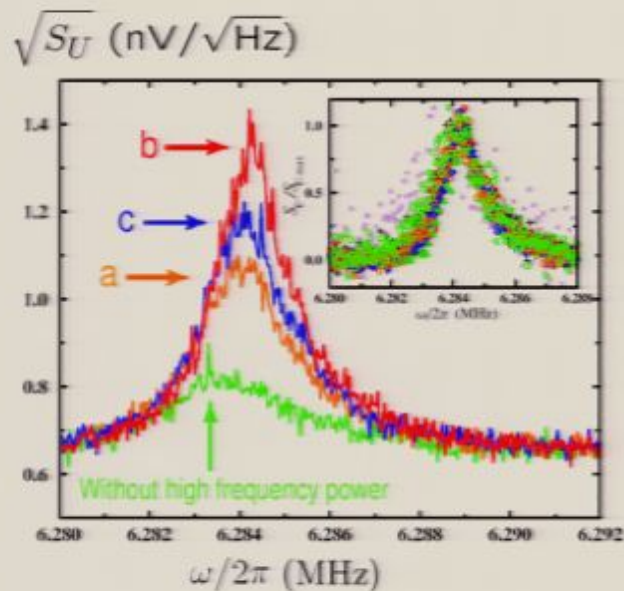
⇒ Qubit excites $\bar{n} > n_{th}$ or cools oscillator $\bar{n} < n_{th}$

System and model

3-junction flux qubit + Rabi driving
+ oscillator



$$H = -\frac{\epsilon(\Phi_x^{dc})}{2} \sigma_z - \frac{\Delta}{2} \sigma_x + \hbar \omega_T a^\dagger a - \hbar \Omega_{R0} \cos(\omega_f t) \sigma_z + g \sigma_z (a + a^\dagger)$$



For blue detuning

$$\delta\omega = \omega_f - \Delta E / \hbar$$

the qubit has “**negative temperature**”
(population inversion) at Rabi frequency:
laser-like pumping of the oscillator

For red detuning: cooling

→ Rotating frame, diagonalization, RWA

$$\Rightarrow H_0^R = \hbar\omega_T a^\dagger a + \frac{1}{2}\hbar\Omega_R\sigma_z$$

$$H_I^R = g_1 \left(a^\dagger \sigma_- e^{-i(\Omega_R - \omega_T)t} + h.c. \right) \\ + g_2 \left(a^{\dagger 2} \sigma_- e^{-i(\Omega_R - 2\omega_T)t} + h.c. \right) \\ + g_3 \left(a^\dagger a \sigma_z + h.c. \right)$$

$$g_1 = g \sin \zeta \cos \beta$$

$\tan \zeta = \epsilon / \Delta \leftrightarrow$ deviation from symmetry point

$$g_2 = -(g^2 / \Delta) \cos \beta$$

$\tan \beta = \delta\omega / \Omega_{R0} \leftrightarrow$ detuning
 $\delta\omega = \omega_f - \Delta E / \hbar$

$$g_3 = -(g^2 / \Delta) \cos^2 \zeta \sin \beta$$

$$g_1 \approx 2\pi \times 30\text{kHz for } \epsilon = 0.01\Delta$$

vanishes at symmetry point

$$g_2 \approx 2\pi \times 10\text{kHz}$$

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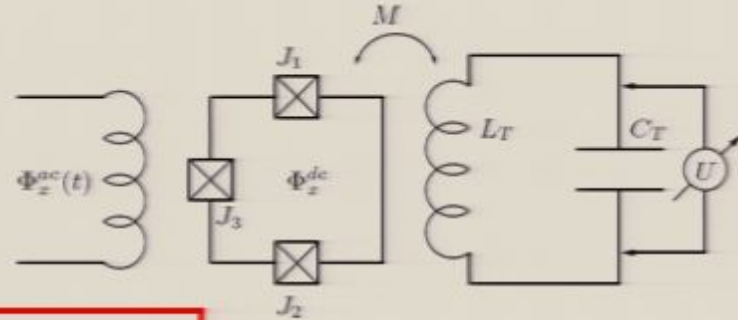
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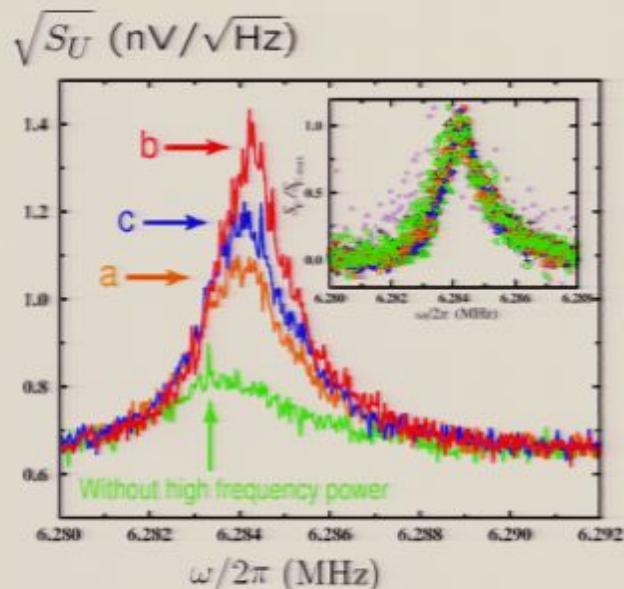


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$$g \approx M I_p I_{T,0} \sim 2\pi \times 3\text{MHz}$$

Persistent current $I_p \sim 300\text{ nA}$
of qubit

Zero motion current in the LC circuit $I_{T,0} = \sqrt{\frac{\hbar \omega_T}{2L_T}}$



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Dissipation

$$\dot{\rho} = -\frac{i}{\hbar} [H, \rho] + L_Q \rho + L_R \rho$$

Qubit:
$$L_Q \rho = \frac{\Gamma_0}{2} (2\sigma_- \rho \sigma_+ - \rho \sigma_+ \sigma_- - \sigma_+ \sigma_- \rho)$$

assume only spontaneous emission

no upward transition ($T \approx 0$), no pure dephasing ($T_2 \approx 2T_1$)

Resonator: Lindblad form

$$\begin{aligned} L_R \rho &= \frac{\kappa}{2} (n_{th} + 1) (2a \rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a) \\ &+ \frac{\kappa}{2} n_{th} (2a^\dagger \rho a - a a^\dagger \rho - \rho a a^\dagger) \end{aligned}$$

Dissipation in the rotating frame

$$\begin{aligned} L_Q^R \rho_I^R &= \frac{\Gamma_{\downarrow}}{2} \left(2\sigma_- \rho_I^R \sigma_+ - \rho_I^R \sigma_+ \sigma_- - \sigma_+ \sigma_- \rho_I^R \right) \\ &+ \frac{\Gamma_{\uparrow}}{2} \left(2\sigma_+ \rho_I^R \sigma_- - \rho_I^R \sigma_- \sigma_+ - \sigma_- \sigma_+ \rho_I^R \right) \\ &+ \frac{\Gamma_{\varphi}^*}{2} \left(\sigma_z \rho_I^R \sigma_z - \rho_I^R \right) \end{aligned}$$

$$\Gamma_{\downarrow, \uparrow} = \frac{\Gamma_0}{4} (1 + \sin^2 \beta \pm 2 \sin \beta)$$

$$\Gamma_{\varphi}^* = \frac{\Gamma_0}{2} \cos^2 \beta$$

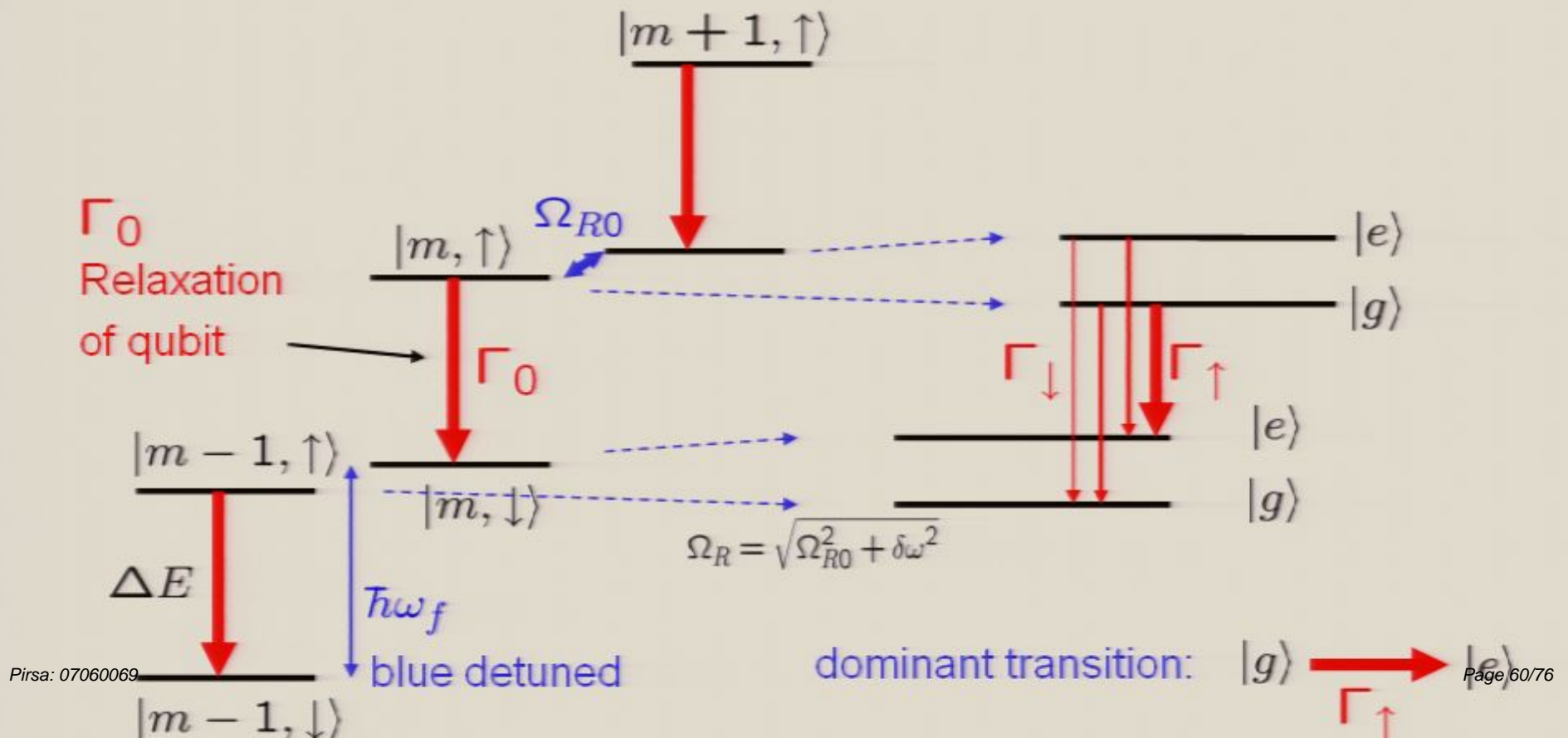
**Transformations introduce upward transition = Pumping
plus weaker relaxation
plus pure dephasing**

Population inversion with dressed states

- Physical transitions in lab frame only down in energy
- Population inversion in the dressed states basis (Mollow ..., Zakrewski et al., 91)

quantize driving field: $H_{\text{driving}} = \hbar\omega_f d^\dagger d + \lambda\sigma_x(d^\dagger + d)$

coherent state $(d^\dagger + d) \rightarrow \sqrt{m} \cos(\omega_f t)$ $\Omega_{R0} = \lambda\sqrt{m}$



Laser theory

Zakrewski *et al.*, "Theory of dressed state lasers" (1991)

Lasing at $\omega_T = \Delta E \pm \Omega_R$

Here: Lasing (and cooling) for $\omega_T \approx \Omega_R$

Langevin equation in the coherent-state representation

(two-photon regime)

$$\dot{\alpha} = - \left[\kappa - \frac{4g_2^2 |\alpha|^2 \Gamma_\phi}{\Gamma_\phi^2 + \delta\Omega^2} s_z^{st} + i \left(4g_3 + \frac{4g_2^2 |\alpha|^2 \delta\Omega}{\Gamma_\phi^2 + \delta\Omega^2} \right) s_z^{st} \right] \frac{\alpha}{2} + \xi(t)$$

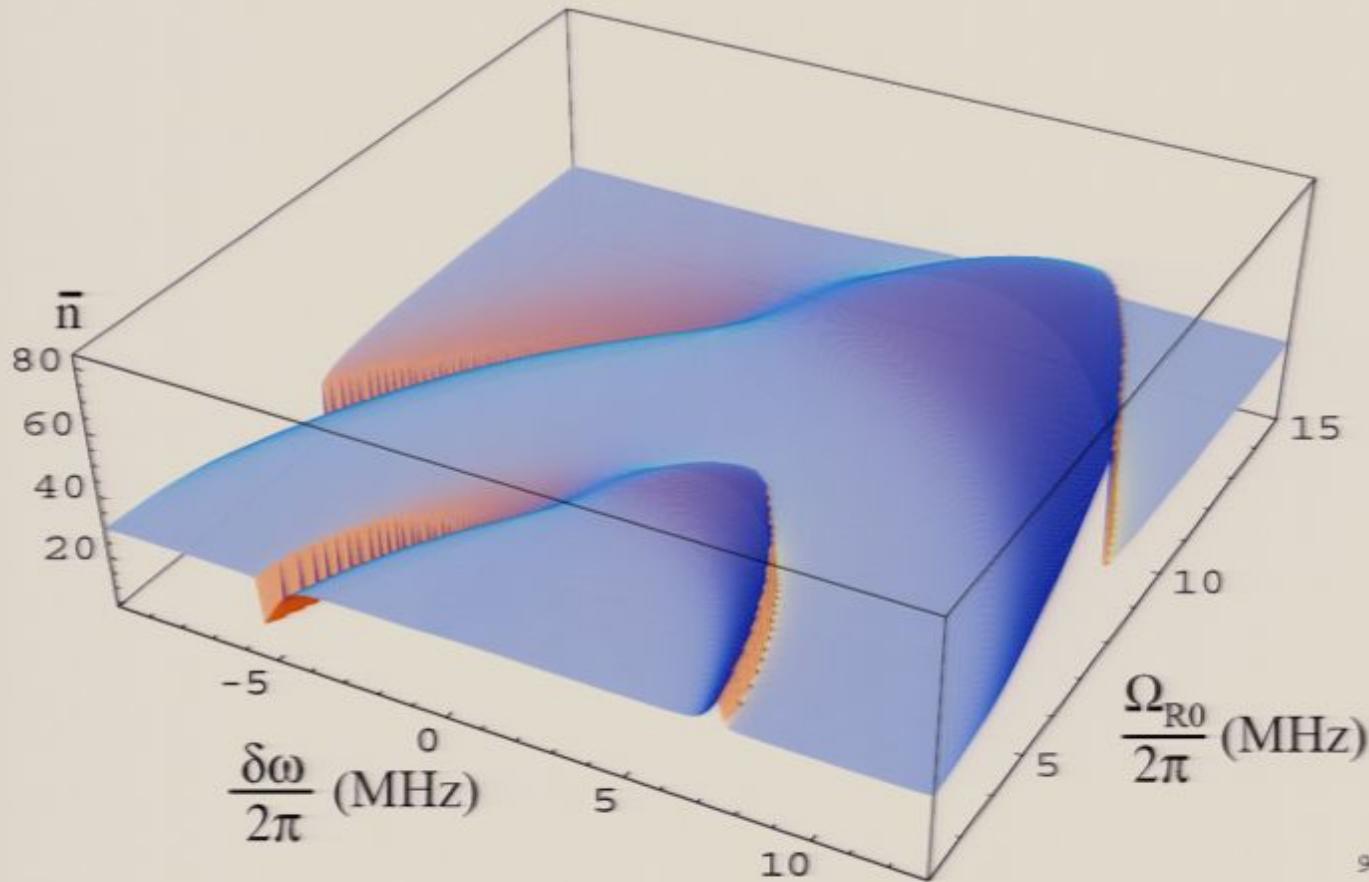
$$s_z^{st} = -D_0 / \left(1 + (|\alpha|^2 / \tilde{n}_0)^2 \right)$$

$$D_0 = \frac{\Gamma_\downarrow - \Gamma_\uparrow}{\Gamma_\downarrow + \Gamma_\uparrow} \quad \text{population inversion}$$

$$\delta\Omega = \Omega_R - \omega_T + 4g_3 |\alpha|^2$$

n_0 = saturation number

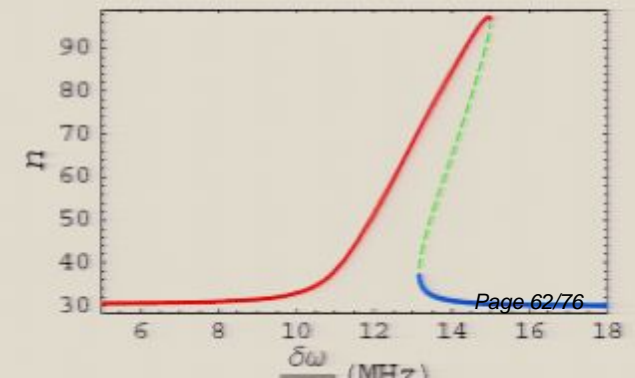
Lasing and cooling at the Rabi frequency, 1 and 2 Photon processes



$$n - n_{th} \sim \frac{D_0 \Gamma_1}{\kappa}$$

$$\kappa \sim \omega_T / Q$$

Bistability



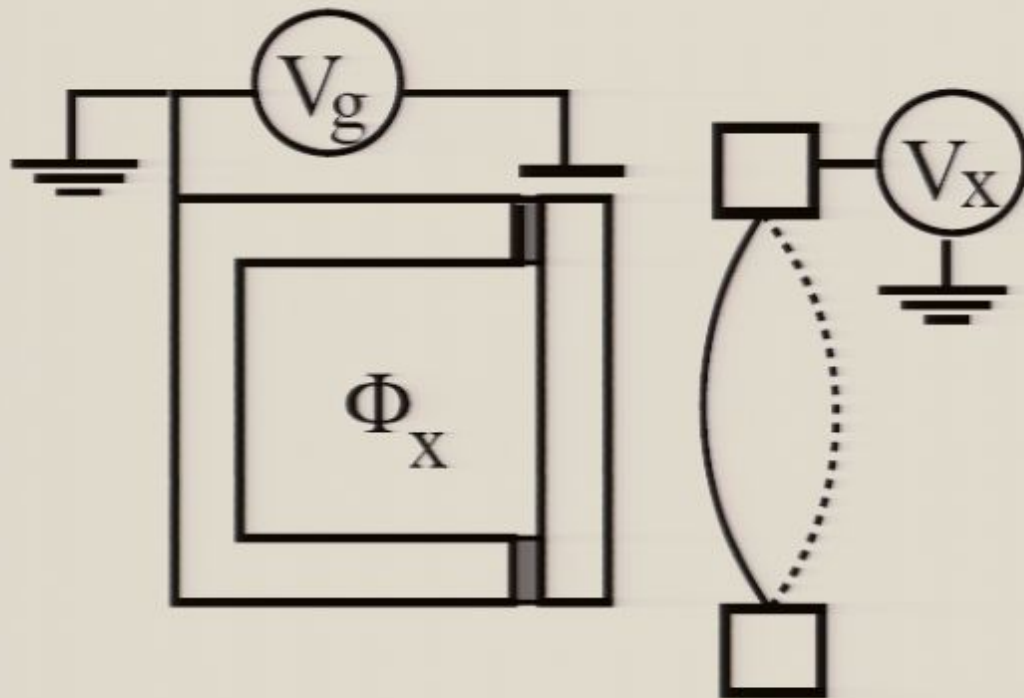
Couple to nanomechanical system

Charge qubit coupled capacitively to mechanical oscillator.

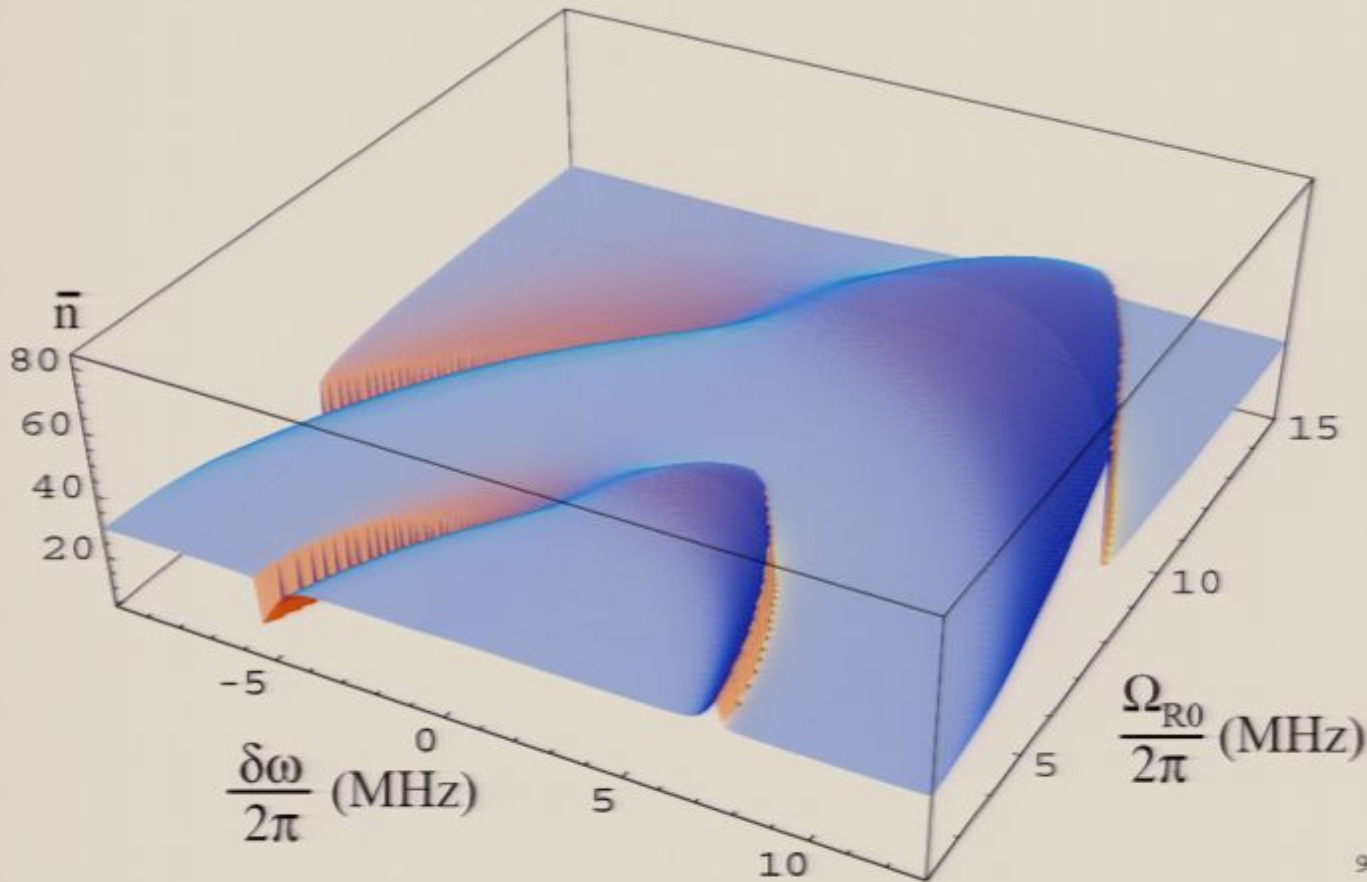
Same physics but higher quality: $Q \sim 100$

⇒ proper lasing state possible (SASER)

strong cooling $\bar{n} \sim 1$ appears possible (?)



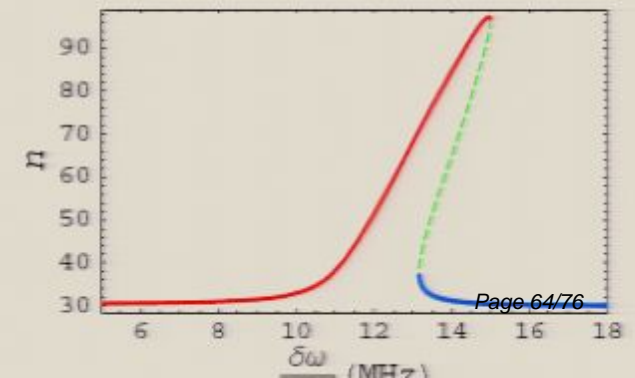
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$$\kappa \sim \omega_T / Q$$

Bistability



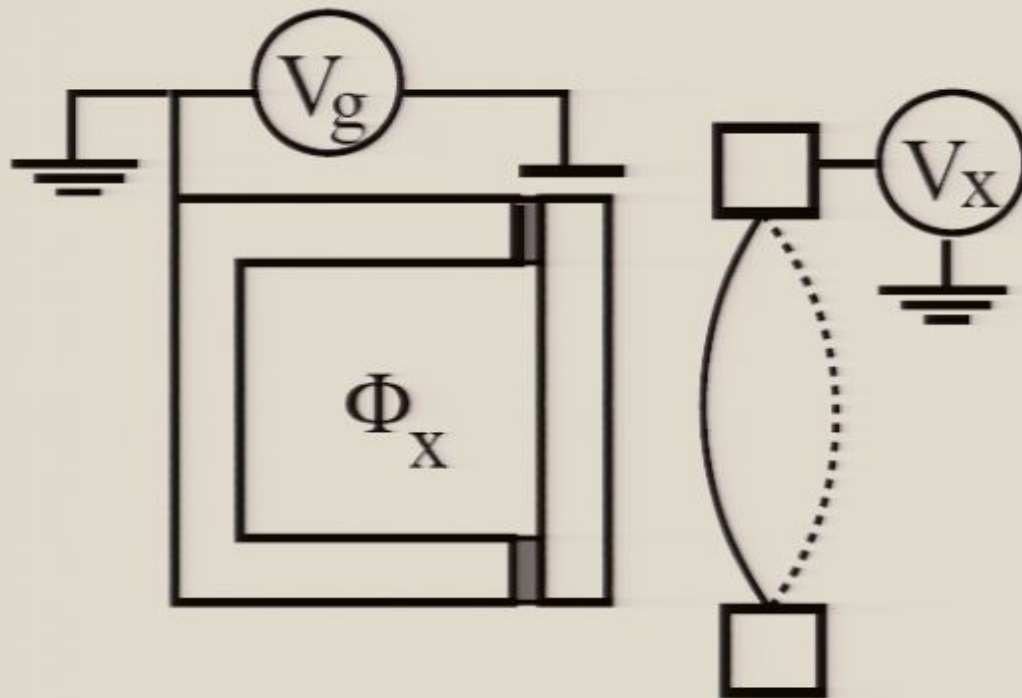
Couple to nanomechanical system

Charge qubit coupled capacitively to mechanical oscillator.

Same physics but higher quality: $Q \sim 100$

\Rightarrow proper lasing state possible (SASER)

strong cooling $\bar{n} \sim 1$ appears possible (?)



Laser theory

Zakrewski *et al.*, "Theory of dressed state lasers" (1991)

Lasing at $\omega_T = \Delta E \pm \Omega_R$

Here: Lasing (and cooling) for $\omega_T \approx \Omega_R$

Langevin equation in the coherent-state representation

(two-photon regime)

$$\dot{\alpha} = - \left[\kappa - \frac{4g_2^2 |\alpha|^2 \Gamma_\phi}{\Gamma_\phi^2 + \delta\Omega^2} s_z^{st} + i \left(4g_3 + \frac{4g_2^2 |\alpha|^2 \delta\Omega}{\Gamma_\phi^2 + \delta\Omega^2} \right) s_z^{st} \right] \frac{\alpha}{2} + \xi(t)$$

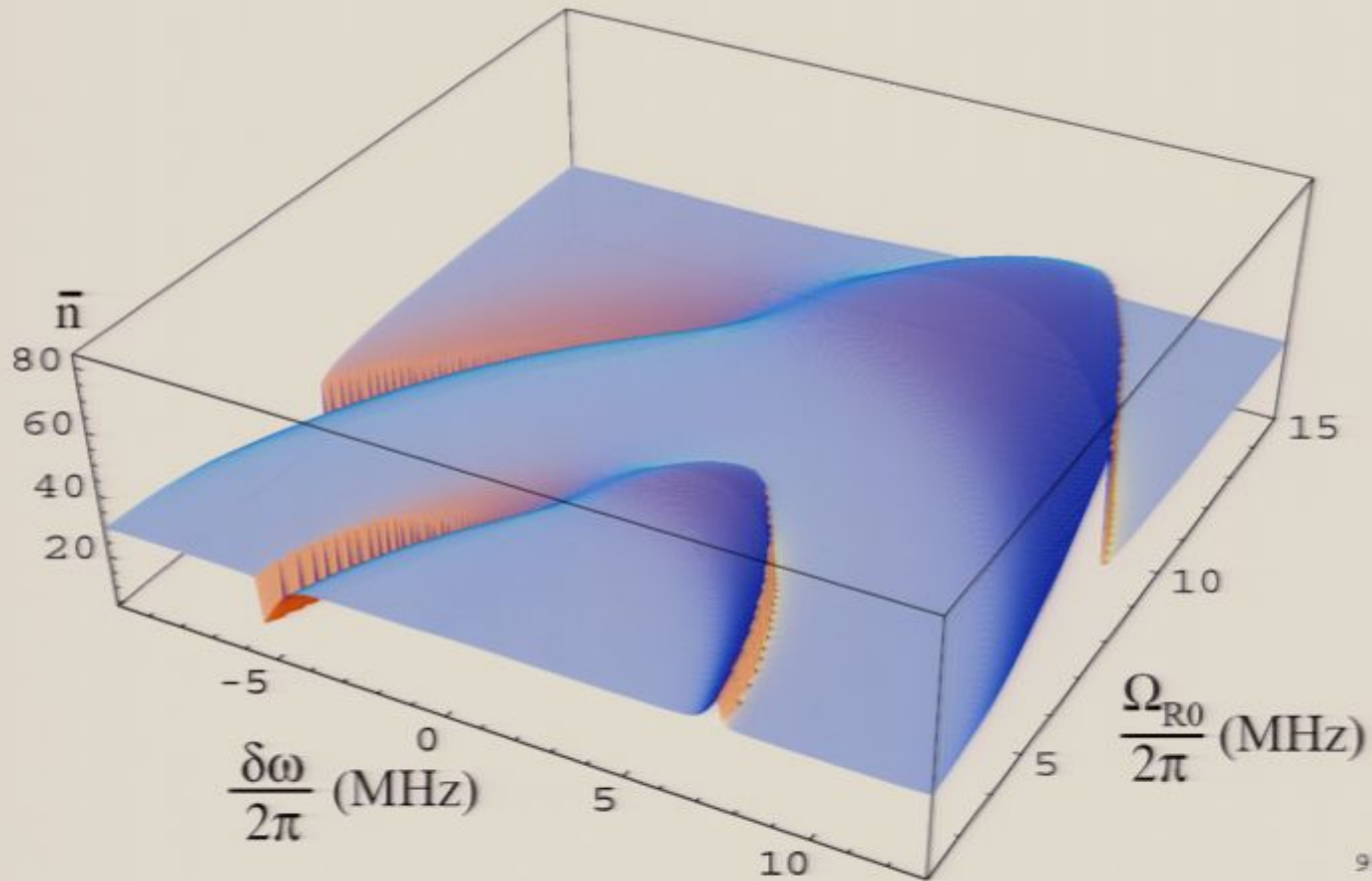
$$s_z^{st} = -D_0 / \left(1 + (|\alpha|^2 / \tilde{n}_0)^2 \right)$$

$$D_0 = \frac{\Gamma_\downarrow - \Gamma_\uparrow}{\Gamma_\downarrow + \Gamma_\uparrow} \quad \text{population inversion}$$

$$\delta\Omega = \Omega_R - \omega_T + 4g_3 |\alpha|^2$$

n_0 = saturation number

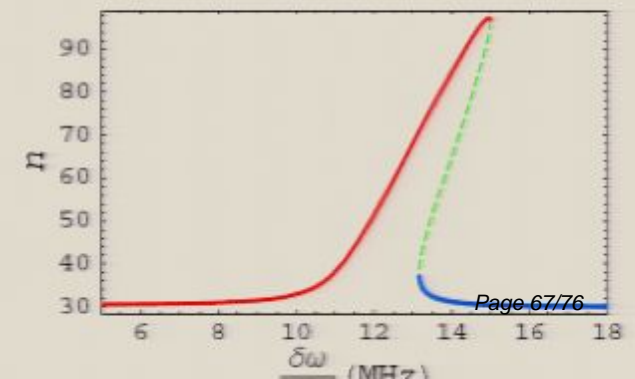
Lasing and cooling at the Rabi frequency, 1 and 2 Photon processes



$$n - n_{th} \sim \frac{D_0 \Gamma_1}{\kappa}$$

$$\kappa \sim \omega_T / Q$$

Bistability



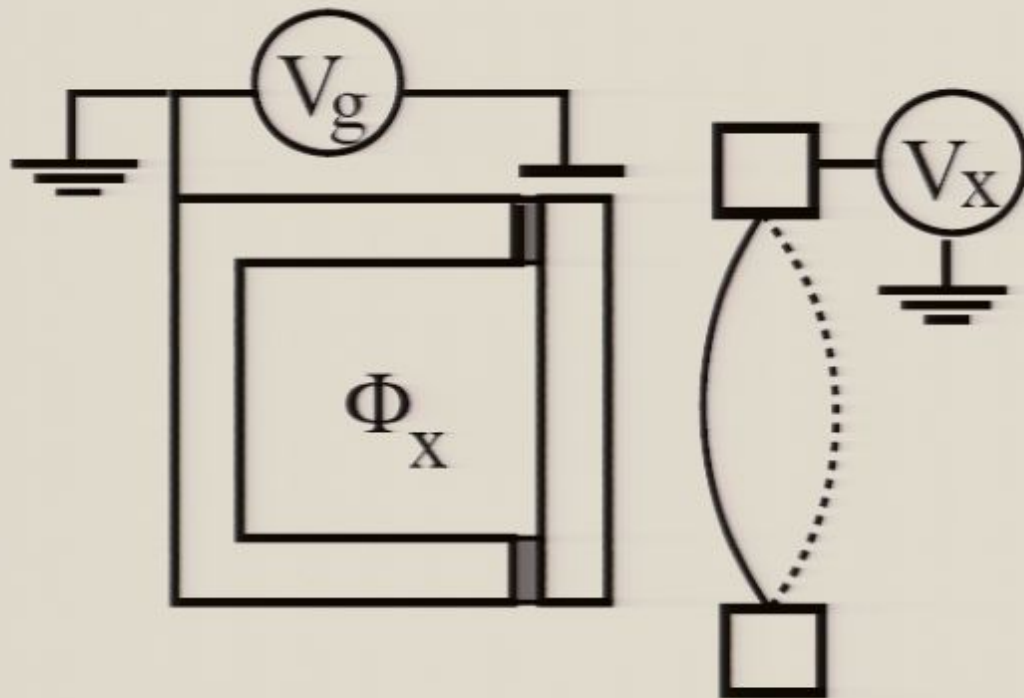
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- Josephson qubits
 - charge, flux, phase qubit
 - experiments: spectroscopy, coherent oscillations, Ramsey fringes, Rabi, ...
 - qubit-qubit coupling
 - decoherence/relaxation
- Josephson qubits coupled to oscillators
 - LC-oscillators, transmission lines, nano-mechanical oscillators
 - strong coupling regime
 - vacuum Rabi oscillations
 - dispersive regime, read-out
- Single- and two photon lasing and cooling at the Rabi frequency
- Superconducting SET transistor and resonator

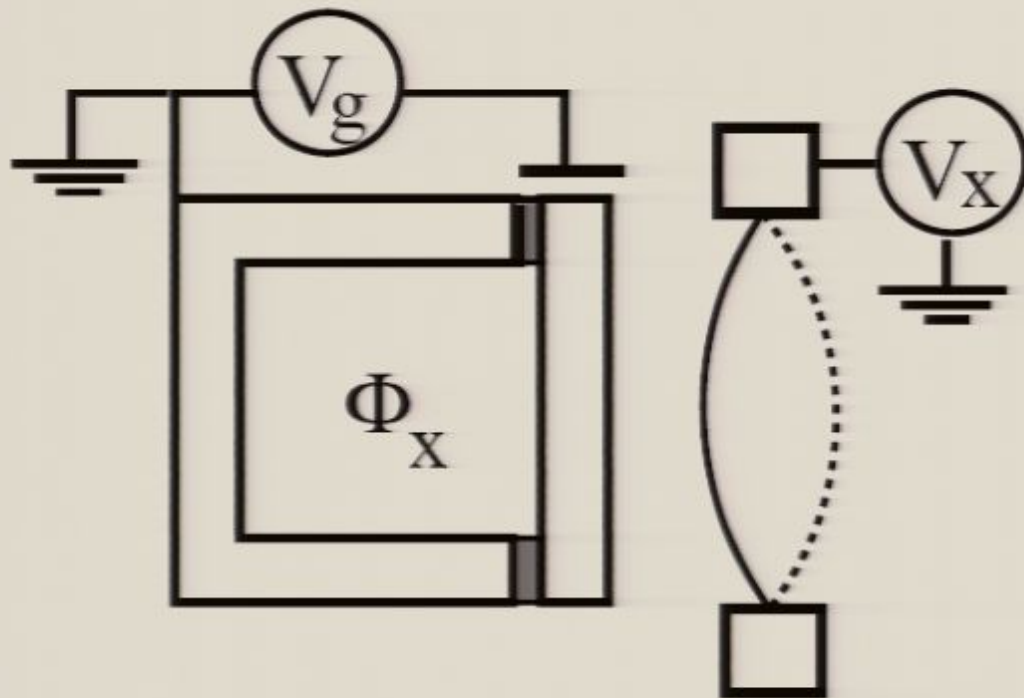
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Quantum Dynamics of a Resonator Driven by a Superconducting Single-Electron Transistor: A Solid-State Analogue of the Micromaser

Armour, Blencowe, Schwab, PRL 2002

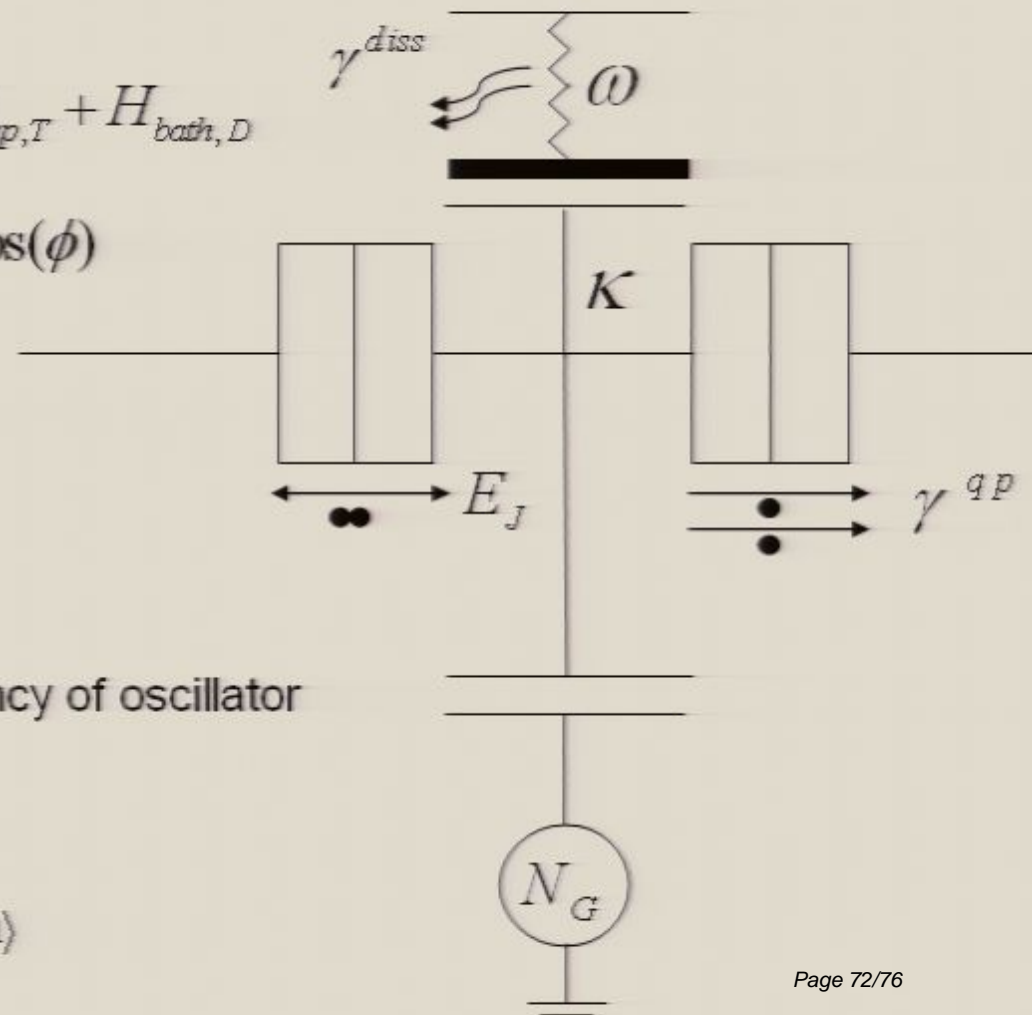
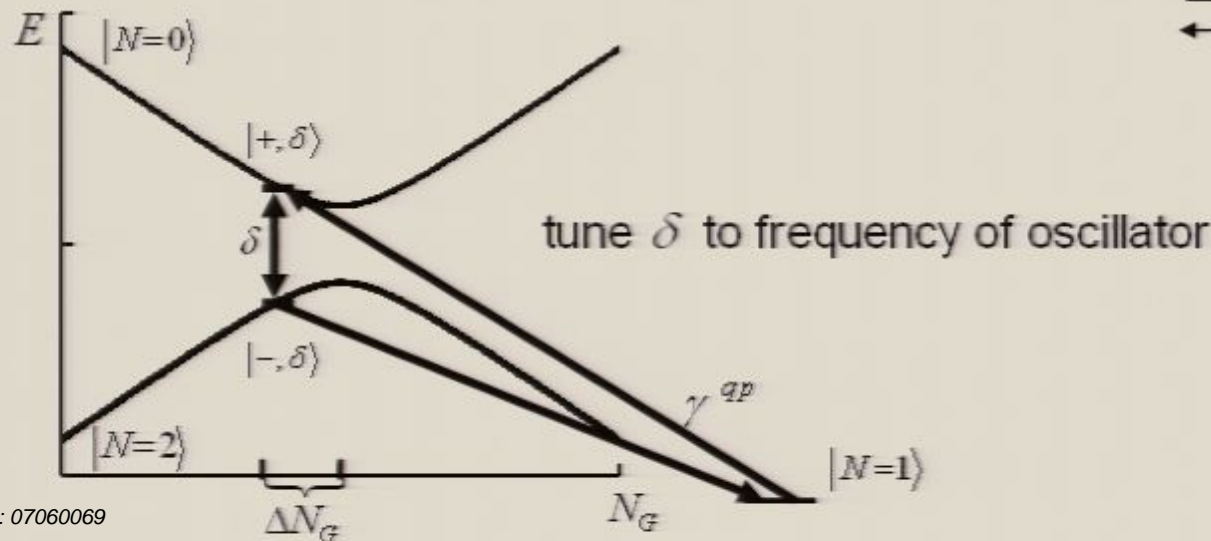
Rodrigues, Imbers, Armour, RRL 2007

Marthaler *et al.*, in prep.

$$H_{Tot} = H_0 + \kappa(\hat{N} - 1)(a + a^\dagger) + \omega a^\dagger a + H_{qp,T} + H_{bath,D}$$

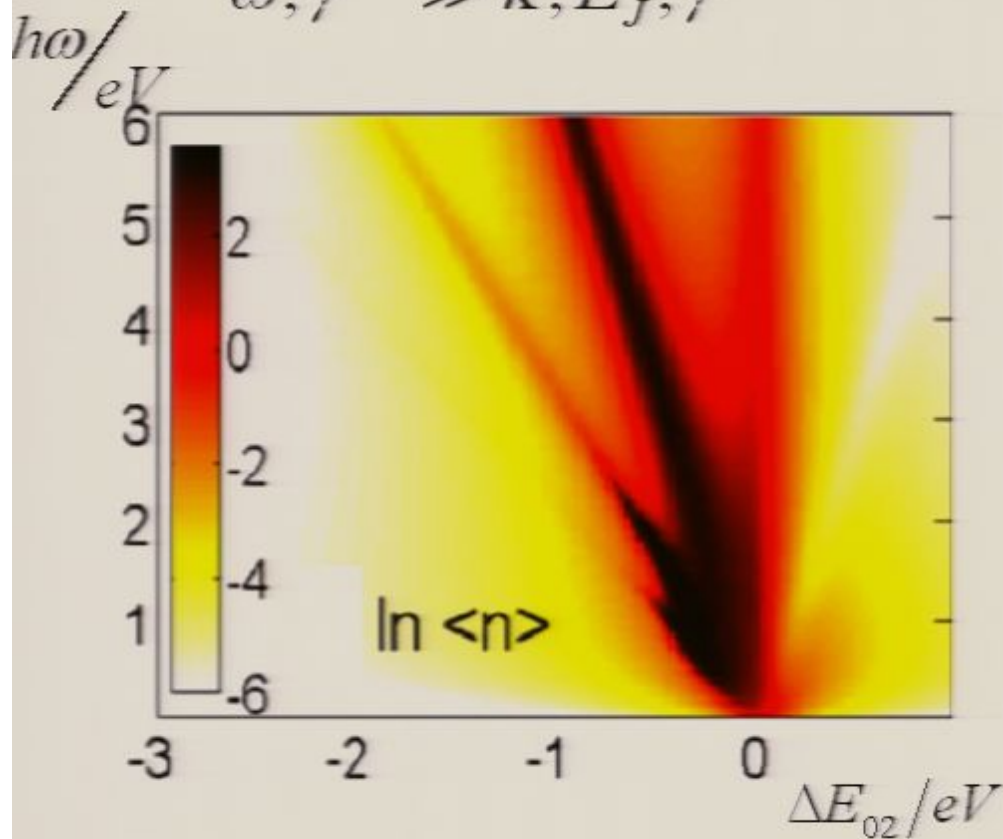
$$H_0 = E_C(\hat{N} + N_G) - \frac{1}{2}eV\hat{N} - 2E_J \cos(\bar{\phi}) \cos(\phi)$$

Josephson-quasiparticle cycle



Oszillator quanta $\langle n \rangle$:

$$\omega, \gamma^{qp} \gg \kappa, E_J, \gamma^{diss}$$

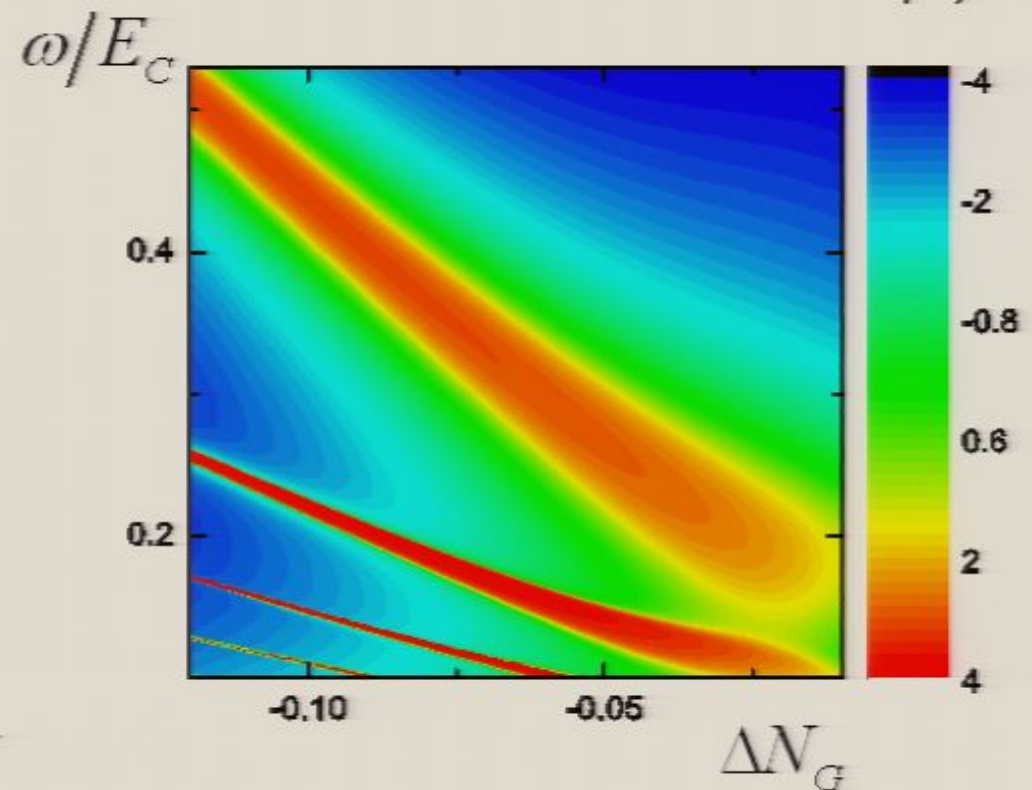


$$\gamma^{qp} = eV/h; E_J = eV/16; \kappa/eV = 0.01$$

$$\gamma^{diss}/\gamma^{qp} = 0.002; \bar{n} = 0; \Delta E_{02} = 4E_C \Delta N_G$$

$$\omega, E_J, \kappa \gg \gamma^{qp}, \gamma^{diss}$$

$\ln \langle n \rangle$



$$E_J/E_C = 0.18; \Delta/E_C = 22; eV/E_C = 7$$

$$\kappa/E_C = 0.01; \gamma^{diss}/\gamma^{qp} = 0.1$$

Josephson qubit coupled to oscillator

Summary

- Use oscillator to couple and manipulate qubit, and for communication
Theory: (compare: Cirac & Zoller), Shnirman *et al.*; Blais *et al.*
Experiment: ?
- Use qubit to manipulate and measure oscillator (electric or mechanical)
Theory: Tian *et al.*; Clerk *et al.*
Experiment: Wallraff *et al.*; Ilichev *et al.*; Schwab *et al.*
- Coupled system (qubit and oscillator on equal footing)
entanglement
Experiment: Chiorescu *et al.*
other quantum systems coupled to oscillator
Theory: Armour, Blencowe, Schwab; Marthaler *et al.*

Josephson qubit coupled to oscillator

Open problems / potential future directions

- Decoherence, Relaxation,
 - sources, rates, decay laws, ...
 - reduction in experiment
- Couple qubits (controlled gates, ...)
- Explore qubit-oscillator coupling
 - further possibilities for manipulation and measurement
 - information storage
- Explore dissipative quantum systems coupled to oscillators



Universität Karlsruhe (TH)
Research University - founded 1825



The Universität Karlsruhe (TH) and the Forschungszentrum Karlsruhe are joining forces to form the Karlsruhe Institute of Technology (KIT) and are planning to integrate their research activities both structurally and strategically. Within the framework of this award-winning concept for the future, funded by the German Excellence Initiative, we seek applications for the position of

Professor (W3) for Theoretical Quantum Optics

at the Department of Physics. This position includes resources equivalent to an endowed chair.

We are looking for an expert in the area of theoretical quantum optics with links to condensed-matter theory. The research of the applicant should focus on (i) the realization of quantum-optics concepts in solid-state or hybrid systems, (ii) many-particle effects in ultra-cold gases or (iii) quantum-information processing and quantum simulation.

The position offers excellent opportunities for collaboration within the Department of Physics and with other departments of the university, with the Research Center Karlsruhe (FZK) in the framework of KIT, within the "DFG-Center for Functional Nanostructures (CFN)" and the "Karlsruhe School of Optics and Photonics (KSOP)"

Teaching duties include participation in the education of physics students as well as of students majoring in other natural sciences or engineering.

The University of Karlsruhe aims to increase the number of female professors and especially welcomes applications from women. Handicapped persons with equal qualifications will be preferred.

In the case of a first appointment to a professorship the contract will not be tenured initially. Exceptions from this rule are possible.

Applications with the usual credentials, copies of the five most important publications and a statement about past and planned research and teaching activities should be sent before June 23, 2007 to the Dean, Department of Physics, Universität Karlsruhe (TH), D-76128 Karlsruhe, Germany.