Title: Latency in fault-tolerant quantum computing with local, two-dimensional architecture

Date: Jun 16, 2007 04:20 PM

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Abstract:

Fault-tolerant quantum computing

Fault tolerant design allows for arbitrarily long quantum computation

Key elements:

- Quantum error correcting codes
- Concatenation

If the physical error is below a certain threshold, the logical error is strongle suppressed

$$\epsilon_l = \epsilon_{th} \left(\frac{\epsilon_{phys}}{\epsilon_{th}} \right)^{2^k}$$

for *k* levels of concatenation

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for *k* levels of concatenation

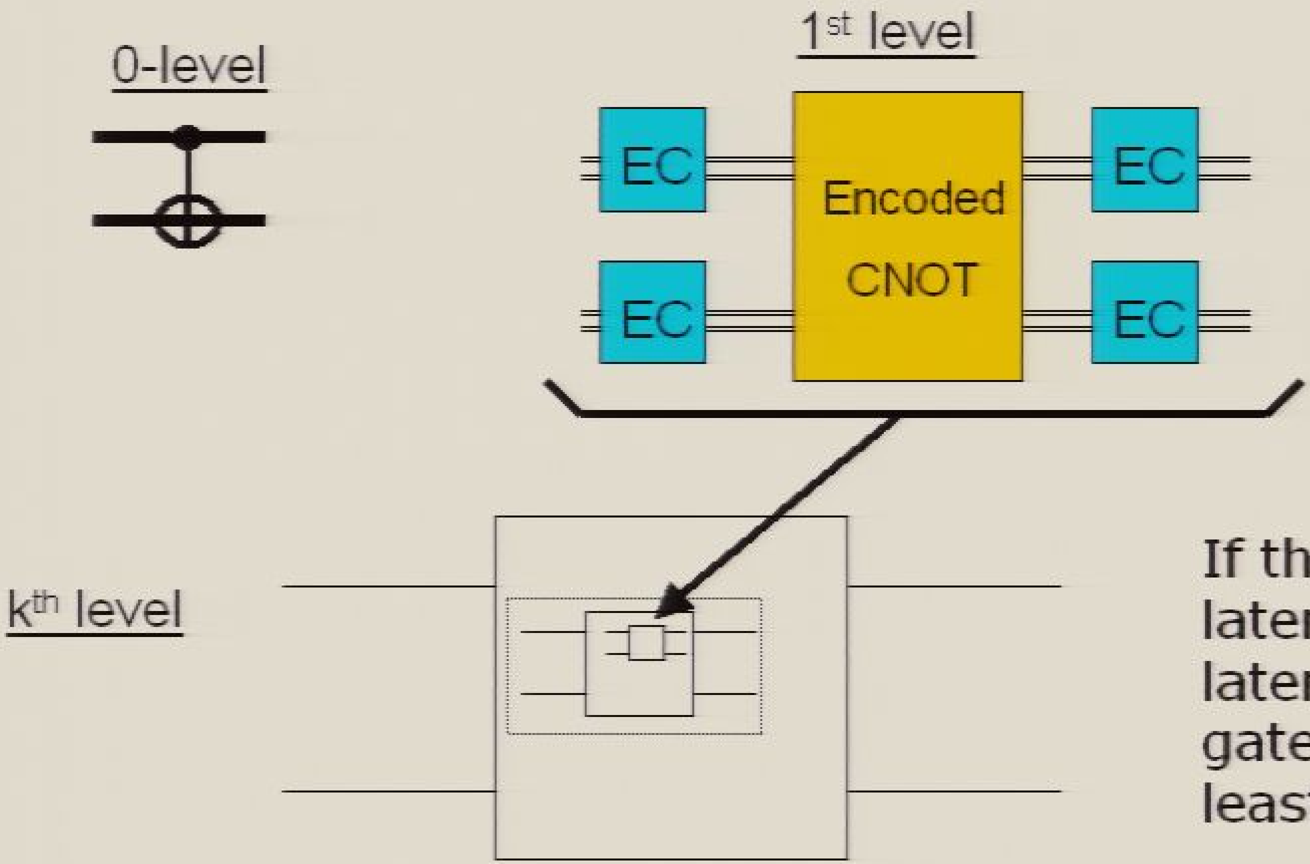
Latency

Concatenation reduces the error rate, but the price is paid in size and *latency*

The recursive structure of concatenated codes yields an exponential growth in resource usage

The exact overhead will depend on the details of the implementation, such as the quantum code used and the architectural constraints

Concatenation and latency



Error correction is performed after every encoded operation

If the error correction latency is L_{EC} the latency of an encoded gate at level k is at least $(L_{EC})^k$

It is important to design efficient EC to reduce the latency

Latency in Error Correction

Basic form:

- prepare ancilla qubits
- interact with data qubits
- measure ancilla

} 3 timesteps

Typically:

- ancilla preparation requires at least 2 or more timesteps
- interaction with data qubits may require more than 1 timestep (e.g., CSS codes)

EC latency: at least 3, but more likely at least 5

9-qubit Bacon Shor code

$q1$	$q2$	$q3$
$q4$	$q5$	$q6$
$q7$	$q8$	$q9$

The state of a qubit is encoded in a subsystem of the subspace stabilized by the operators

Stabilizer operators

$$S_1 = X_1X_2X_3X_4X_5X_6$$
$$S_2 = X_4X_5X_6X_7X_8X_9$$
$$S_3 = Z_1Z_2Z_4Z_5Z_7Z_8$$
$$S_4 = Z_2Z_3Z_5Z_6Z_8Z_9$$

Logical operators

$$Z_L = Z_1Z_4Z_7$$
$$X_L = X_1X_2X_3$$

- This code protects against arbitrary one qubit errors

- Furthermore, the subsystem encoding feature allows for certain two qubit errors to act trivially on the encoded information

Error correction

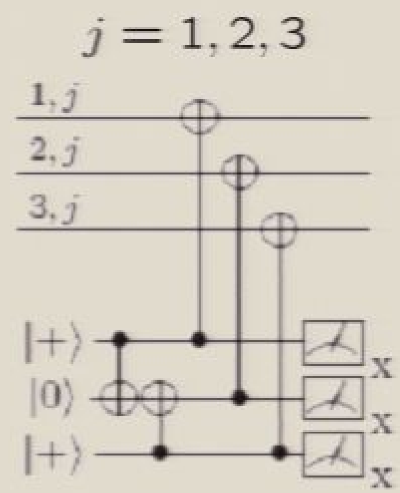
Aliferis, Cross

Phys. Rev. Lett. 98, 220502 (2007)

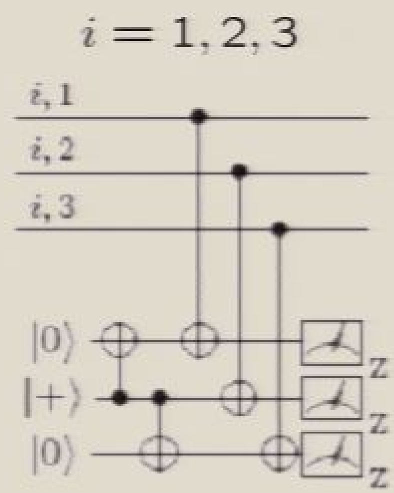
- The error information (error syndrome) can be fault tolerantly extracted using simple circuits
- Ancilla state does not require verification

Ran in parallel for all 3 columns

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



Extracts information about Z errors



Extracts information about X errors

Ran in parallel for all 3 rows

EC latency: 6 timesteps

Local, 2-d architecture

- Svore, DiVincenzo, Terhal: 7-qubit Steane code, embedded in a 6x8 qubit array (EC requires 27 timesteps)
  (Quantum. Inf. Comput. 7, 297(2007))

- 9-qubit code: embedded in a 7x7 array, dummy qubits used for ancilla preparation and measurement, and qubit transport.

- EC is performed in 7 timesteps

- Encoded gates:

CNOT	9 timesteps
SWAP	7 timesteps
Hadamard	5 timesteps
$\{\|0\rangle, \|+\rangle\}$ – prep.	9 timesteps

Error correction sequence

Timestep: 1

O	O	$P_Z(a1)$	$P_X(a2)$	O	O	O
O	$d1$	O	$d2$	O	$d3$	O
O	O	$P_Z(a4)$	$P_X(a5)$	O	O	$P_X(a16)$
$P_Z(a11)$	$d4$	O	$d5$	O	$d6$	$P_Z(a17)$
$P_X(a12)$	O	$P_Z(a14)$	O	O	O	O
O	$d7$	$P_X(a15)$	$d8$	O	$d9$	O
O	O	O	$P_X(a8)$	$P_Z(a9)$	O	O

a_i	:	ancilla qubit
$\Leftrightarrow$	:	$SWAP$ gate
$\rightarrow$	:	$CNOT$ gate
P_X	:	preparation of qubit in state $\|+\rangle$
P_Z	:	preparation of qubit in state $\|0\rangle$
M_X	:	measurement of qubit in X basis
M_Z	:	measurement of qubit in Z basis

Error correction sequence

Timestep: 2

O	O	$a1$	$\leftarrow$	$a2$		$P_Z(a3)$	O	O
O	$d1$	O		$d2$		O	$d3$	O
$P_X(a10)$	O	$a4$	$\leftarrow$	$a5$		$P_Z(a6)$	O	$a16$
								$\downarrow$
$a11$	$d4$	$P_Z(a13)$		$d5$		O	$d6$	$a17$
$\uparrow$								
$a12$	O	$a14$		O		O	O	$P_X(a18)$
		$\uparrow$						
O	$d7$	$a15$		$d8$		O	$d9$	O
O	O	$P_Z(a7)$		$a8$	$\rightarrow$	$a9$	O	O

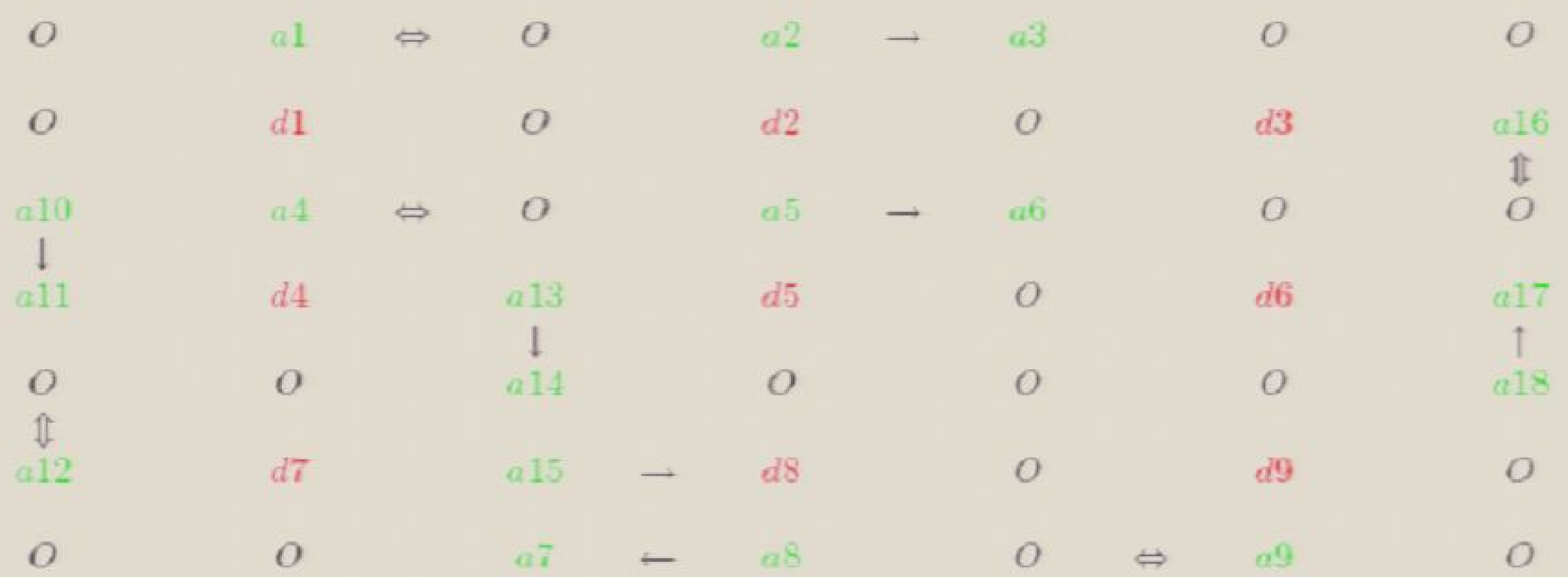
---

a_i : ancilla qubit
$\Leftrightarrow$: $SWAP$ gate
$\rightarrow$: $CNOT$ gate

P_X : preparation of qubit in state $|+\rangle$
P_Z : preparation of qubit in state $|0\rangle$
M_X : measurement of qubit in X basis
M_Z : measurement of qubit in Z basis

Error correction sequence

Timestep: 3



a_i : ancilla qubit
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P_X : preparation of qubit in state $|+\rangle$
P_Z : preparation of qubit in state $|0\rangle$
M_X : measurement of qubit in X basis
M_Z : measurement of qubit in Z basis

Error correction sequence

Timestep: 4

O		$a1$		O		$a2$	O	$\Leftrightarrow$	$a3$		O
		$\uparrow$				$\uparrow$					
$a10$		$d1$		O		$d2$	O		$d3$	$\leftarrow$	$a16$
$\Updownarrow$											
O		$a4$		$a13$		$a5$	O	$\Leftrightarrow$	$a6$		O
		$\uparrow$		$\Updownarrow$		$\uparrow$					
$a11$		$d4$		O		$d5$	O		$d6$	$\leftarrow$	$a17$
O		O		O	$\Leftrightarrow$	$a14$	O		O		O
											$\Updownarrow$
$a12$	$\rightarrow$	$d7$		$M_X(a15)$		$d8$	O		$d9$		$a18$
						$\downarrow$			$\downarrow$		
O		$a7$	$\Leftrightarrow$	O		$a8$	O		$a9$		O

---

a_i : ancilla qubit
$\Leftrightarrow$: $SWAP$ gate
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P_X : preparation of qubit in state $|+\rangle$
P_Z : preparation of qubit in state $|0\rangle$
M_X : measurement of qubit in X basis
M_Z : measurement of qubit in Z basis

Error correction sequence

Timestep: 5

O	$M_Z(a1)$	O	$M_Z(a2)$	O	$a3$	O
					$\uparrow$	
$a10$ $\rightarrow$	$d1$	$a13$	$d2$	O	$d3$	$M_X(a16)$
		$\Updownarrow$				
O	$M_Z(a4)$	O	$M_Z(a5)$	O	$a6$	O
					$\uparrow$	
$a11$ $\rightarrow$	$d4$	O	$d5$	O	$d6$	$M_X(a17)$
			$\uparrow$			
O	O	O	$a14$	O	O	O
$M_X(a12)$	$d7$	O	$d8$	O	$d9$	$\leftarrow$ $a18$
	$\downarrow$					
O	$a7$	O	$M_Z(a8)$	O	$M_Z(a9)$	O

a_i : ancilla qubit
$\Leftrightarrow$: $SWAP$ gate
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P_X : preparation of qubit in state $|+\rangle$
P_Z : preparation of qubit in state $|0\rangle$
M_X : measurement of qubit in X basis
M_Z : measurement of qubit in Z basis

Error correction sequence

Timestep: 6

O	O	O	O	O	$M_Z(a3)$	O
$M_X(a10)$	$d1$	$a13$ $\rightarrow$	$d2$	O	$d3$	O
O	O	O	O	O	$M_Z(a6)$	O
$M_X(a11)$	$d4$	O	$d5$	O	$d6$	O
O	O	O	$M_X(a14)$	O	O	O
O	$d7$		$d8$	O	$d9$	$M_X(a14)$
O	$M_Z(a7)$	O	O	O	O	O

---

a_i : ancilla qubit
$\Leftrightarrow$: $SWAP$ gate
$\rightarrow$: $CNOT$ gate

P_X : preparation of qubit in state $|+\rangle$
P_Z : preparation of qubit in state $|0\rangle$
M_X : measurement of qubit in X basis
M_Z : measurement of qubit in Z basis

Error correction sequence

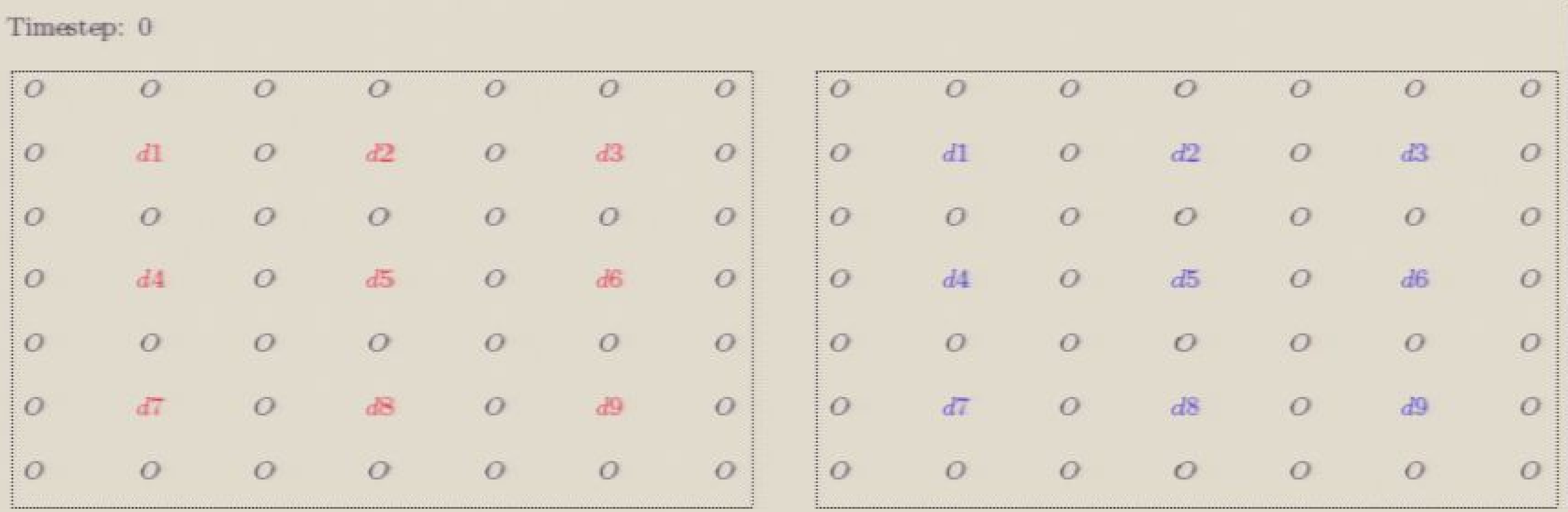
Timestep: 7

O	O	O	O	O	O	O
O	$d1$	$M_X(a13)$	$d2$	O	$d3$	O
O	O	O	O	O	O	O
O	$d4$	O	$d5$	O	$d6$	O
O	O	O	O	O	O	O
O	$d7$	O	$d8$	O	$d9$	O
O	O	O	O	O	O	O

a_i : ancilla qubit
$\Leftrightarrow$: $SWAP$ gate
$\rightarrow$: $CNOT$ gate

P_X : preparation of qubit in state $|+\rangle$
P_Z : preparation of qubit in state $|0\rangle$
M_X : measurement of qubit in X basis
M_Z : measurement of qubit in Z basis

Encoded CNOT



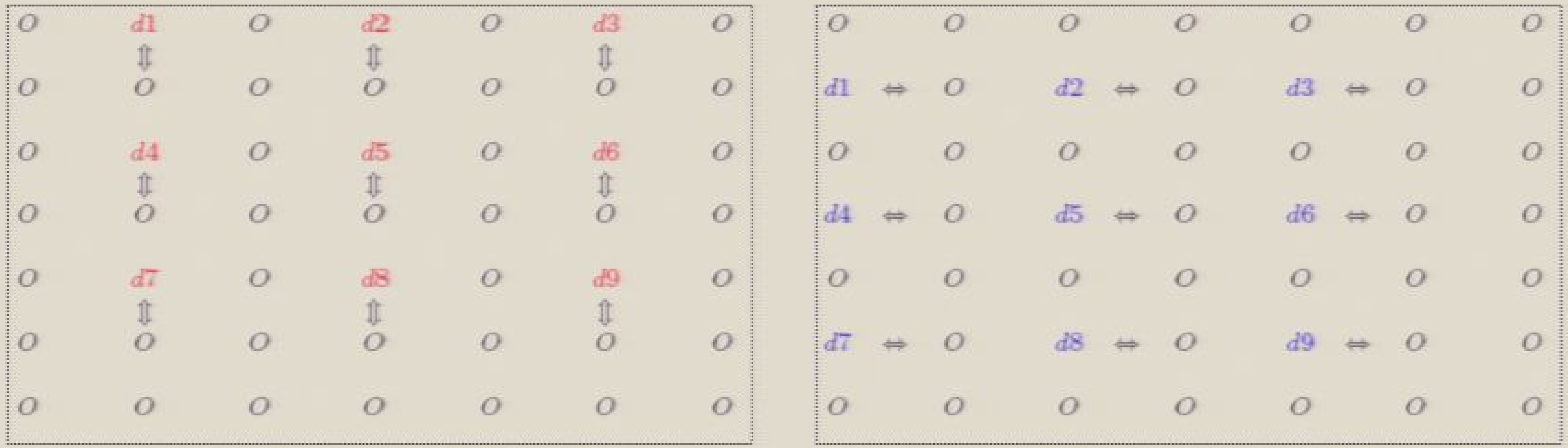
Block 1
(control)

Block 2
(target)

$\Leftrightarrow$: $SWAP$ gate
$\rightarrow$: $CNOT$ gate

Encoded CNOT

Timestep: 1

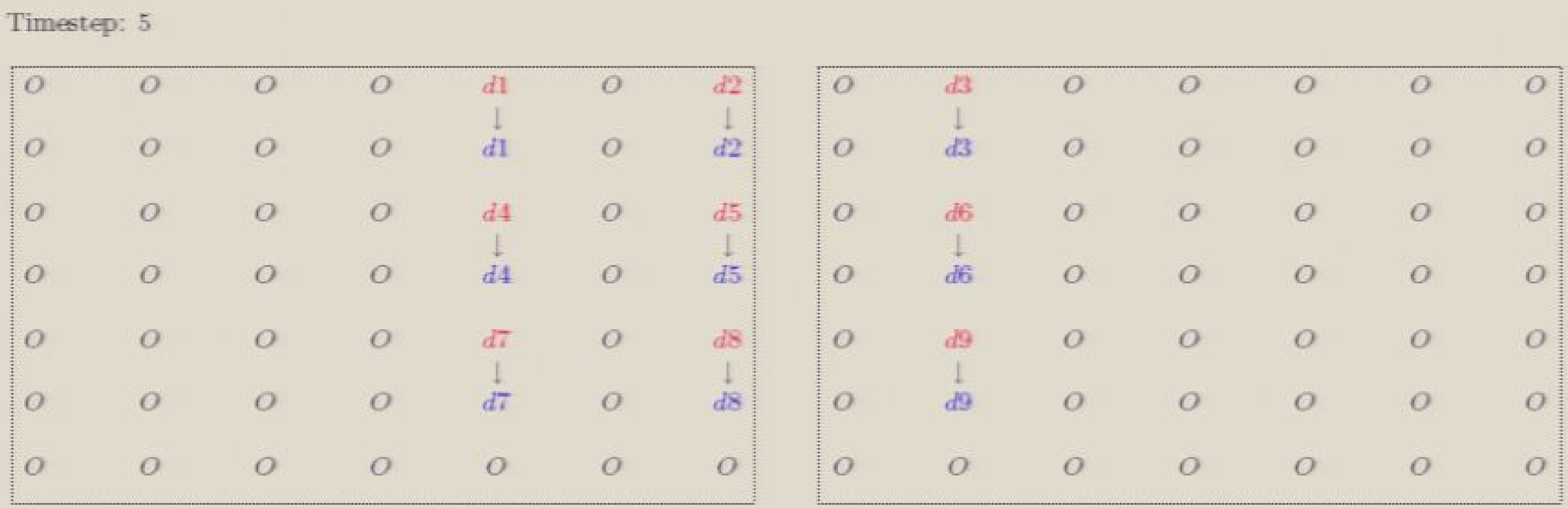


Block 1
(control)

Block 2
(target)

$\Leftrightarrow$: $SWAP$ gate
$\rightarrow$: $CNOT$ gate

Encoded CNOT



Block 1
(control)

Block 2
(target)

$\Leftrightarrow$: $SWAP$ gate
$\rightarrow$: $CNOT$ gate

Encoded CNOT

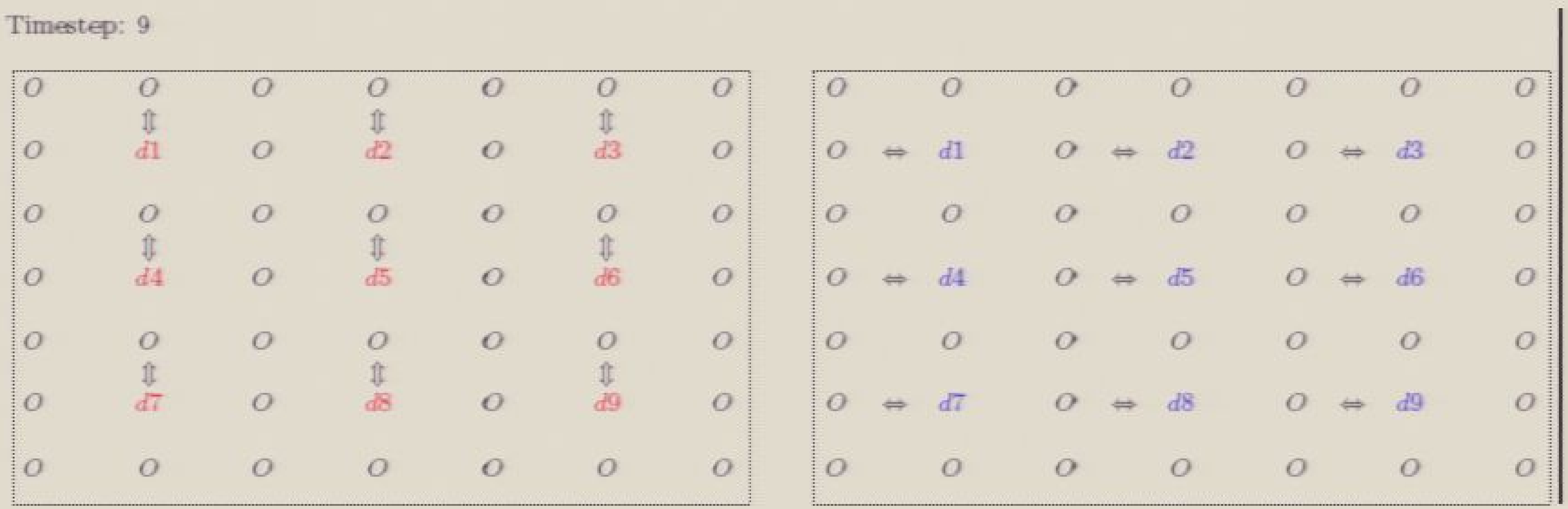
Timestep: 7

O	O	$d1$ ⇔	O	$d2$ ⇔	O	$d3$ ⇔	O	O	O	O	O	O	O
O	O	O	O	O	O ⇔	$d1$	O ⇔	$d2$	O ⇔	$d3$	O	O	O
O	O	$d4$ ⇔	O	$d5$ ⇔	O	$d6$ ⇔	O	O	O	O	O	O	O
O	O	O	O	O	O ⇔	$d4$	O ⇔	$d5$	O ⇔	$d6$	O	O	O
O	O	$d7$ ⇔	O	$d8$ ⇔	O	$d9$ ⇔	O	O	O	O	O	O	O
O	O	O	O	O	O ⇔	$d7$	O ⇔	$d8$	O ⇔	$d9$	O	O	O
O	O	O	O	O	O	O	O	O	O	O	O	O	O

Block 1 (control) | Block 2 (target)

$\Leftrightarrow$: $SWAP$ gate
$\rightarrow$: $CNOT$ gate

Encoded CNOT



Block 1
(control)

Block 2
(target)

⇔ : $SWAP$ gate
→ : $CNOT$ gate

Universality and threshold

- Need extra single qubit gates for universality

Use "quantum software":

(Aliferis, PhD Thesis, Caltech (2006))

- Noisy |+i> purification
- State injection at algorithmic level

- Threshold (for CSS gates)

$$\epsilon_0 = 1.83 \times 10^{-5}$$

Memory errors equal 1/10 of gate errors

Latency gain: $\left(\frac{16}{35}\right)^k \sim (0.45)^k$ for CNOT 1-Rec

Summary

- Error correction is the bottleneck for latency in concatenated FTQC
- EC latency depends on the code chosen
- When locality and dimensionality constraints are considered, codes with nice geometrical properties are very useful
- 9-qubit Bacon-Shor code turns out to be pretty optimal
- Total latency will depend on the number of concatenation levels, so codes with higher distance may be useful if they reduce the number of levels required
- Universality can be attained at the algorithmic level by sate injection (optimizing this part has not been studied yet for this scheme)