Title: Fault-tolerant quantum computation with high threshold in two dimensions

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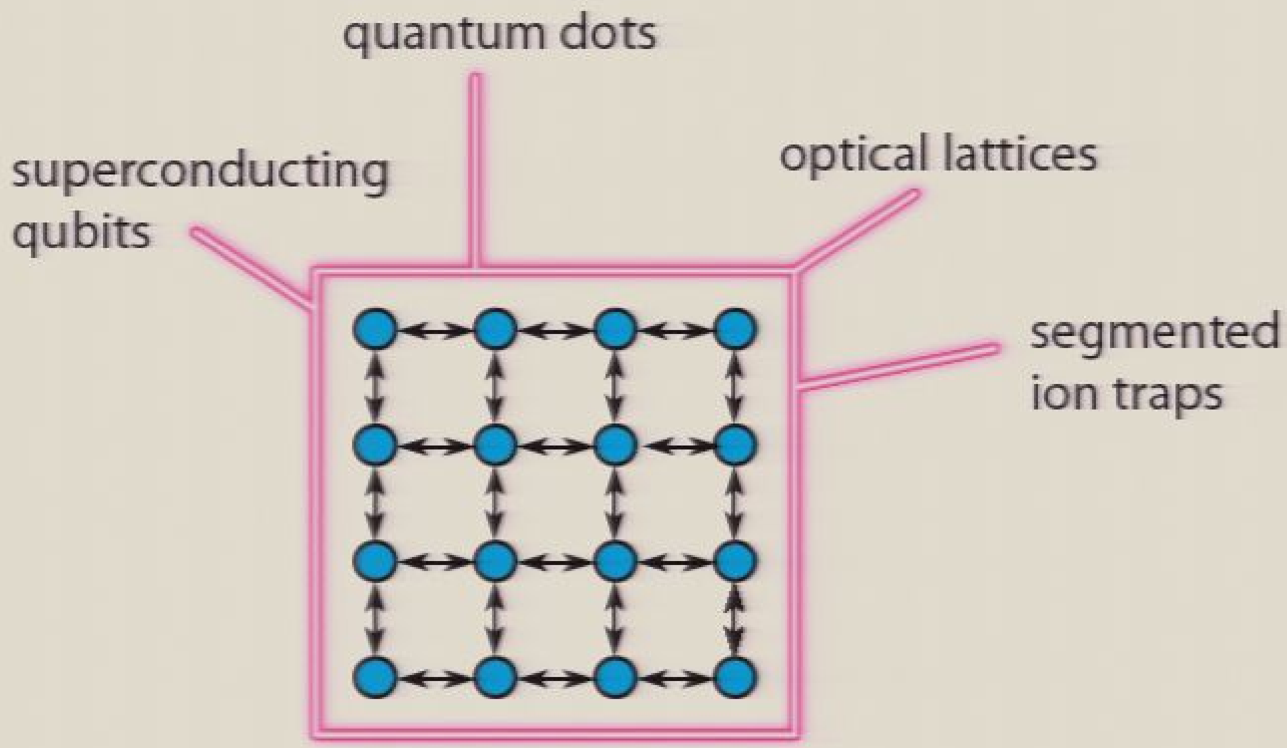
Abstract:

Motivation

Now that the threshold theorem for fault-tolerant quantum computation has been established, we are interested in experimentally viable methods for fault-tolerance. One requires:

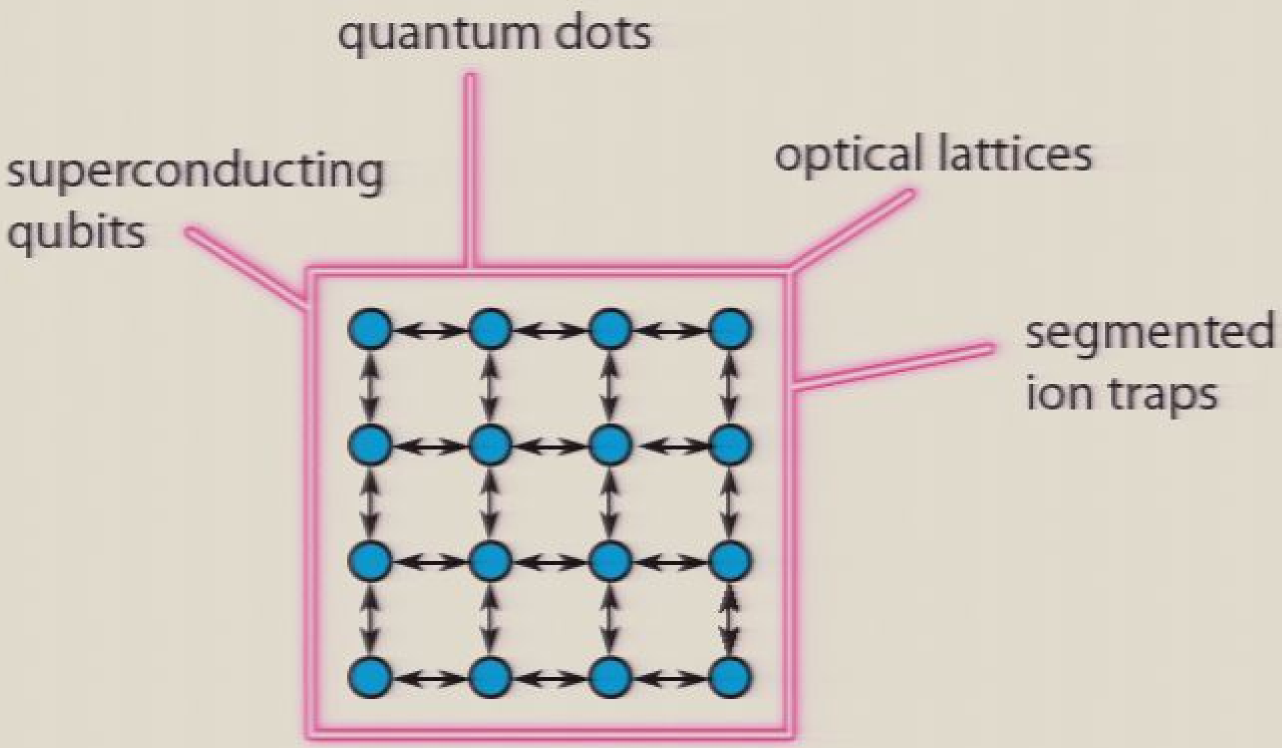
- *Large threshold value.*
- *Robustness of threshold against variations of the error model.*
- *Moderate overhead.*
- *Simple architecture (e.g. no long-range interaction).*

Motivation



- 2D, *nearest-neighbor translation-invarant interaction.*

Motivation



- 2D, *nearest-neighbor translation-invarant interaction.*
- FT threshold 0.75% each source
  (gate, preparation, measurement and storage error.)

Talk outline

1. The one-way quantum computer ($QC_{\mathcal{C}}$).

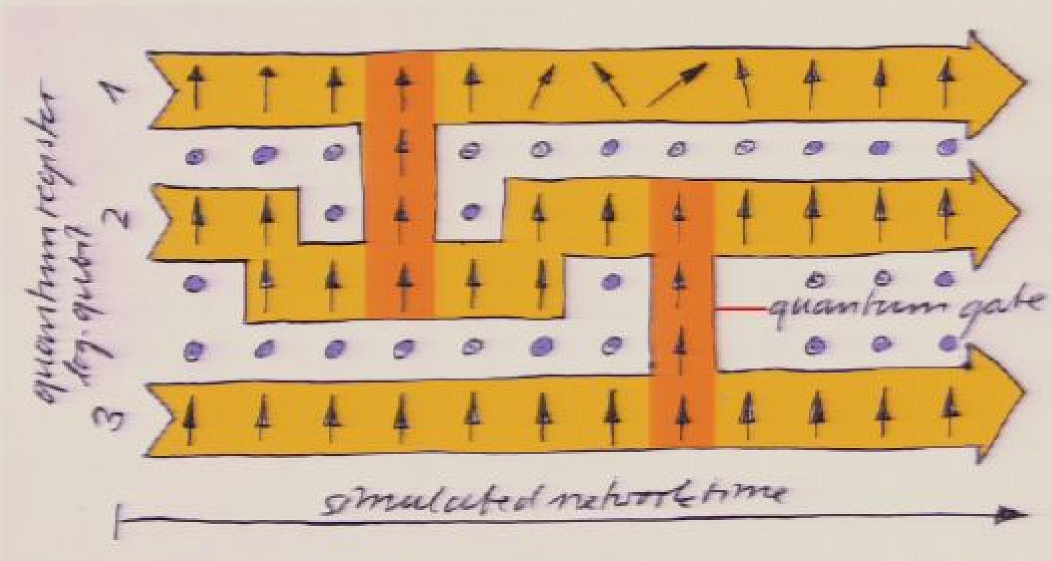
2. Making the $QC_{\mathcal{C}}$ fault-tolerant. Dimensionality $2 \rightarrow 3$.

2b. Reduction to 2D + time.

Part I:

The one-way quantum computer and cluster states

The one-way quantum computer



measurement of Z ($\odot$), X ($\uparrow$), $\cos\alpha\, X + \sin\alpha\, Y$ ($\nearrow$)

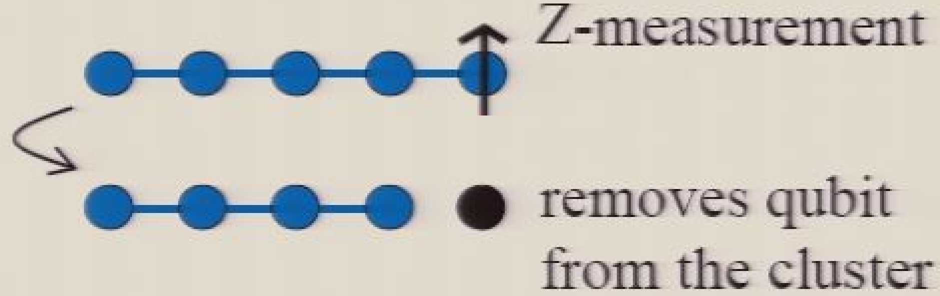
- Universal computational resource: cluster state.
- Information written onto the cluster, processed and read out by one-qubit measurements only.

Cluster states - creation

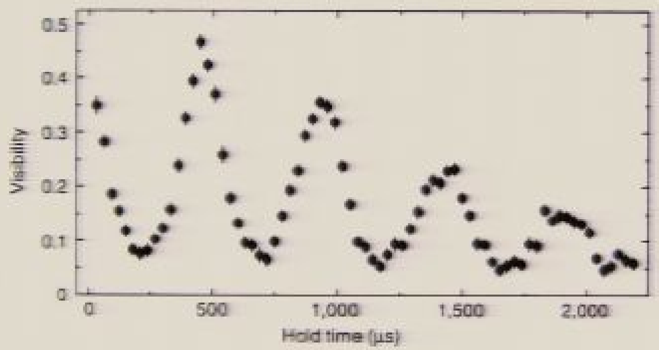
1. Prepare product state $\bigotimes_{a\in\mathcal{C}} \frac{|0\rangle_a + |1\rangle_a}{\sqrt{2}}$ on d-dimensional qubit lattice $\mathcal{C}$.

2. Apply the Ising interaction for a fixed time T (conditional phase of π accumulated).

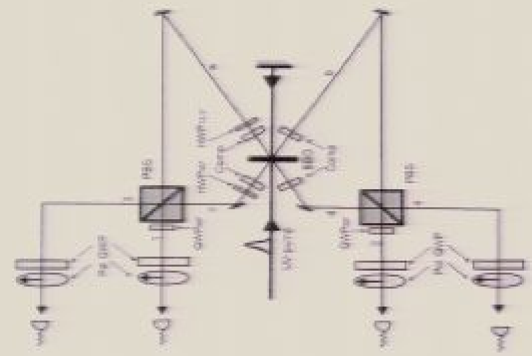
Z-**Rule:**



Cluster states - experiment



Cold atoms in optical lattices [1,2]



The $QC_{\mathcal{C}}$ with photons [3].

1: Greiner, Mandel, Esslinger, Hänsch, and Bloch, *Nature* 415, 39-44 (2002),

2: Greiner, Mandel, Hänsch and Bloch, *Nature*, 419, 51-54 (2002).

3: P. Walther *et al.*, *Nature* 434, 169 (2005).

Part II:

Fault-tolerance

Questions

After the proof of the threshold theorem for fault-tolerant quantum compuation:

- *What is the threshold value?*
- *What is the overhead?*
- *What are the requirements on interaction?*

Known threshold values

no constraint	geometric constraint 2D	geometric constraint 1D
[1] — 0.03, est.	[5] — $7 \cdot 10^{-3}$, est.	
[2] — 10^{-3}, est.		
[3] — 10^{-4}, est.		
[4] — 10^{-5}, bd.	[6] — $2 \cdot 10^{-5}$, bd.	
		[7] — 10^{-8}, bd.

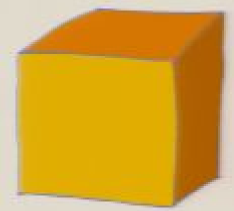
- Error sources:
  $|+\rangle$-Preparation, $\Lambda(Z)$-gates, Hadamard gates, measurement.

[1] Knill, (2005); [2] Zalka (1999); [3] Dawson & Nielsen (2005); [4] Aliferis & Gottesman & Preskill (2005), [5] Raussendorf & Harrington, quant-ph/0610062; [6] Svore & DiVincenzo & Terhal, quant-ph/0604090, [7] Aharonov & Ben-Or (1999)

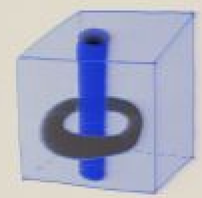
Fault-tolerant $QC_{\mathcal{C}}$

Main idea: *Replace 2D cluster state by 3D cluster state!*

1. The 3D cluster state is a fault-tolerant substrate.

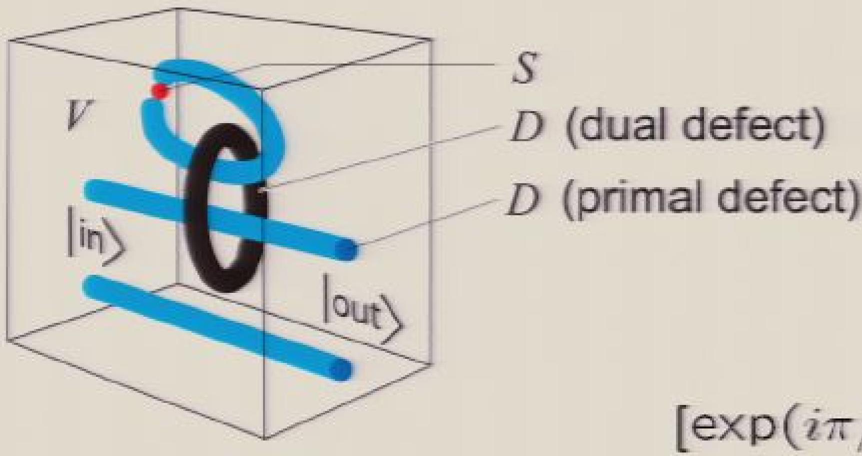


2. Topological quantum logic through choice of boundary conditions.



- Can be mapped to 2D physical setting.
- Threshold value: 7.5×10^{-3} each source (preparation, gate, measurement, storage error).

2.1 Macroscopic view



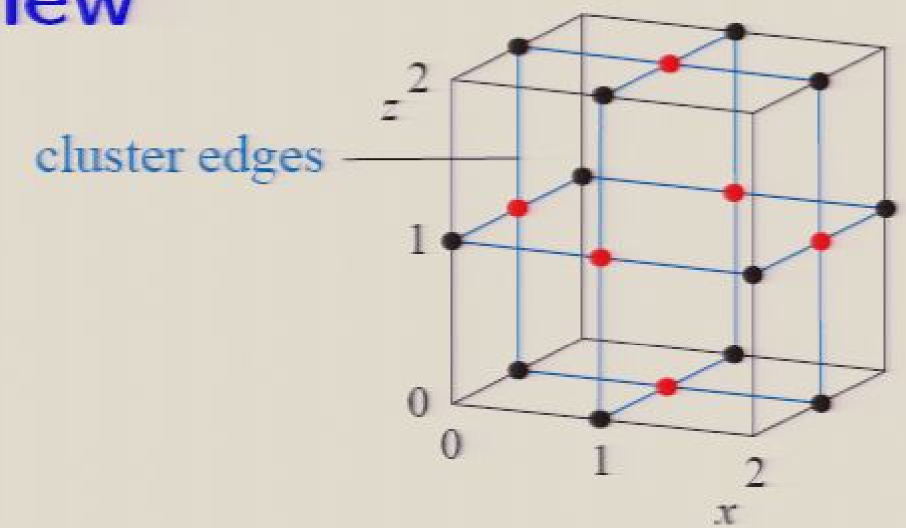
[$\exp(i\pi/8\,X)$-gate]

- Three cluster regions:
  V (vacuum), D (defect) and S (singular).

Qubits $q \in V$:	local X-measurements,
Qubits $q \in D$:	local Z-measurements,
Qubits $q \in S$:	local measurements of $\frac{X \pm Y}{\sqrt{2}}$.

- Defect region D is string-like. Can understand quantum circut in terms of the *topology* of D.

2.1 Microscopic view



elementary cell of $\mathcal{L}$

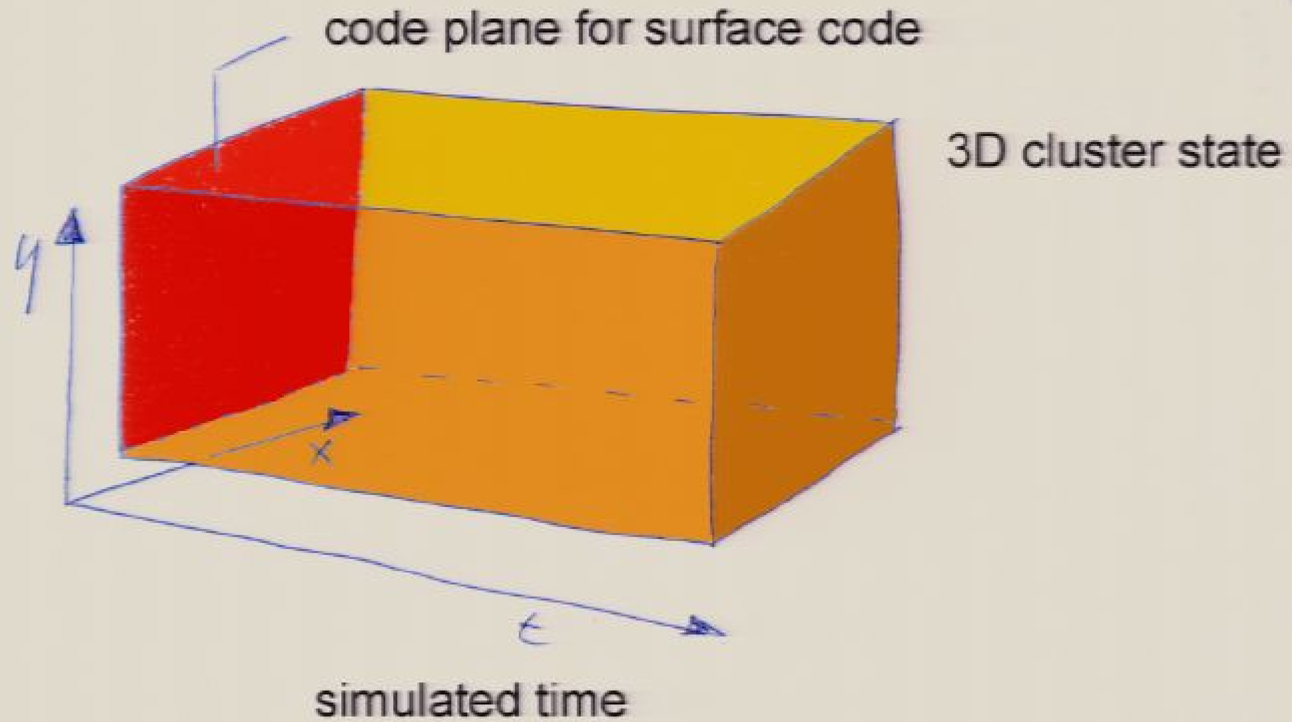
qubit location: (even, odd, odd) - face of $\mathcal{L}$,

qubit location: (odd, odd, even) - edge of $\mathcal{L}$,

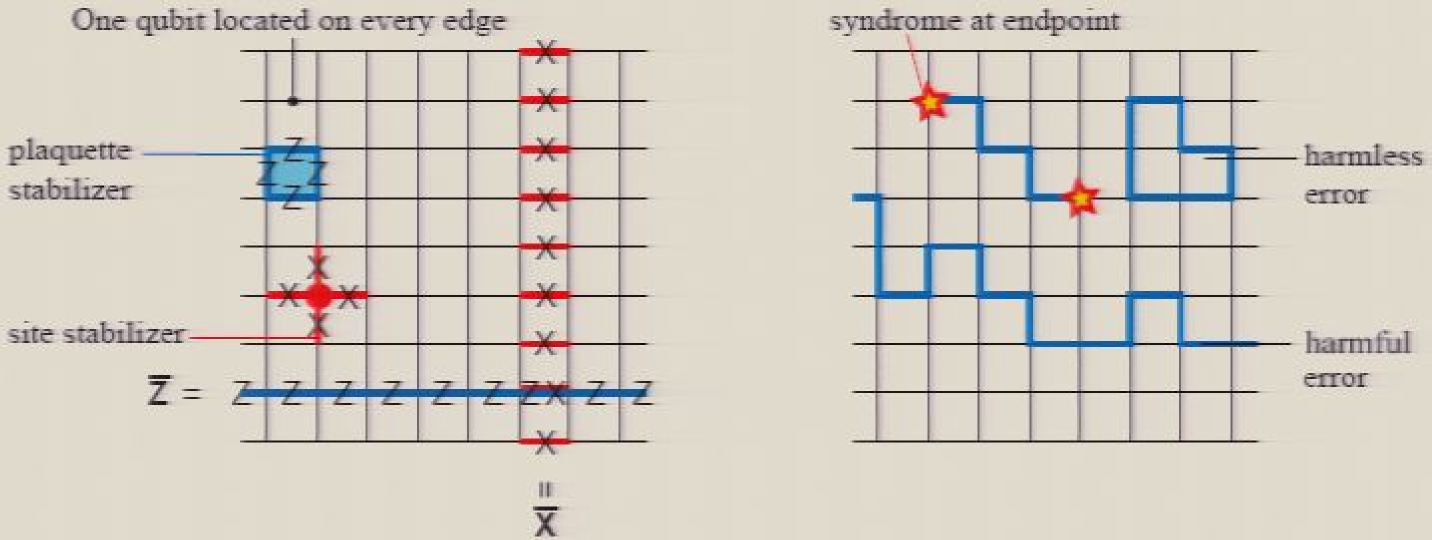
syndrome location: (odd, odd, odd) - cube of $\mathcal{L}$,

syndrome location: (even, even, even) - site of $\mathcal{L}$.

2.1 Key to scheme



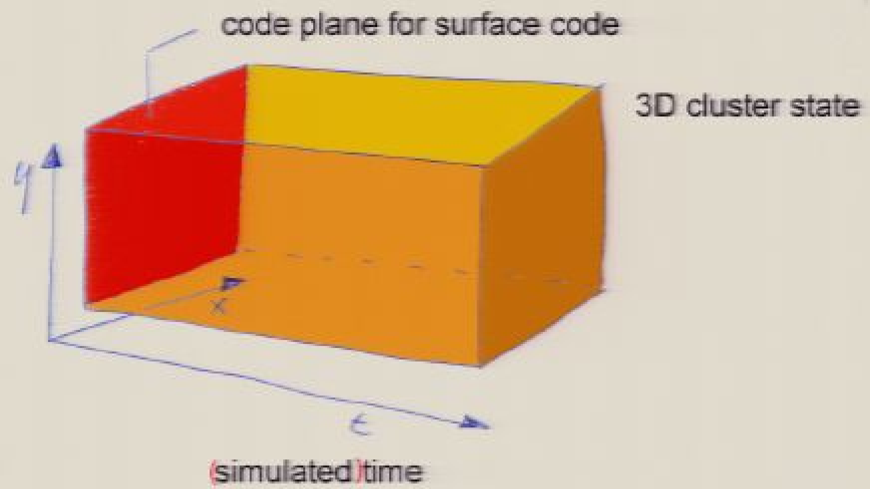
2.1 Surface codes



- Surface codes are special CSS codes associated with planar graphs/ lattices.
- Harmfull errors stretch across the entire lattice (rare events).

A. Kitaev,quant-ph/9707021 (1997).

2.1 $QC_{\mathcal{C}}$: topological error correction in V

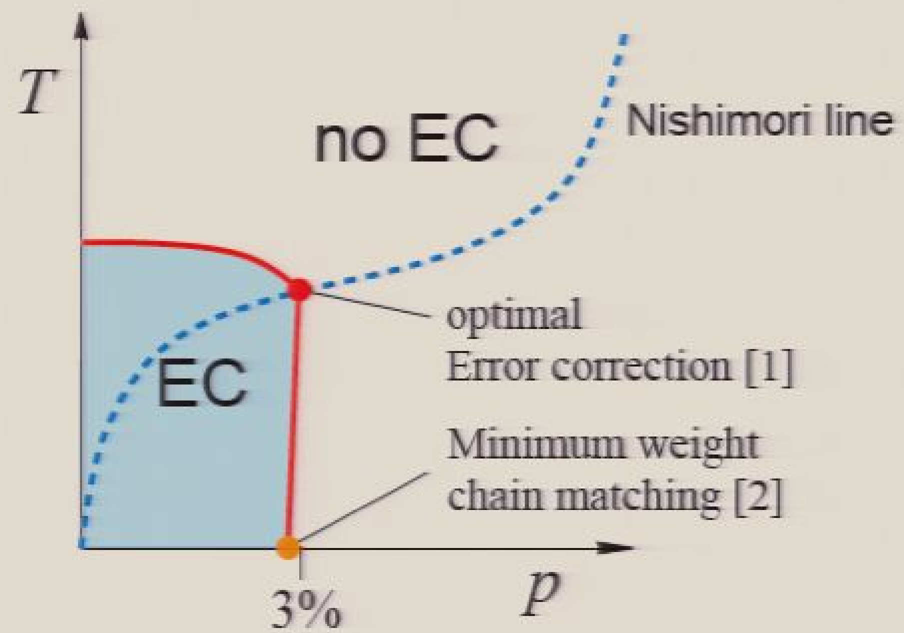


- Fault-tolerant data storage with planar code described by *Random plaquette* Z_2*-gauge model* (RPGM) [1].
- Same error-correction applies in 3D cluster states.

[1] Dennis et *al.*, quant-ph/0110143 (2001).

2.1 Phase diagram of the RPGM

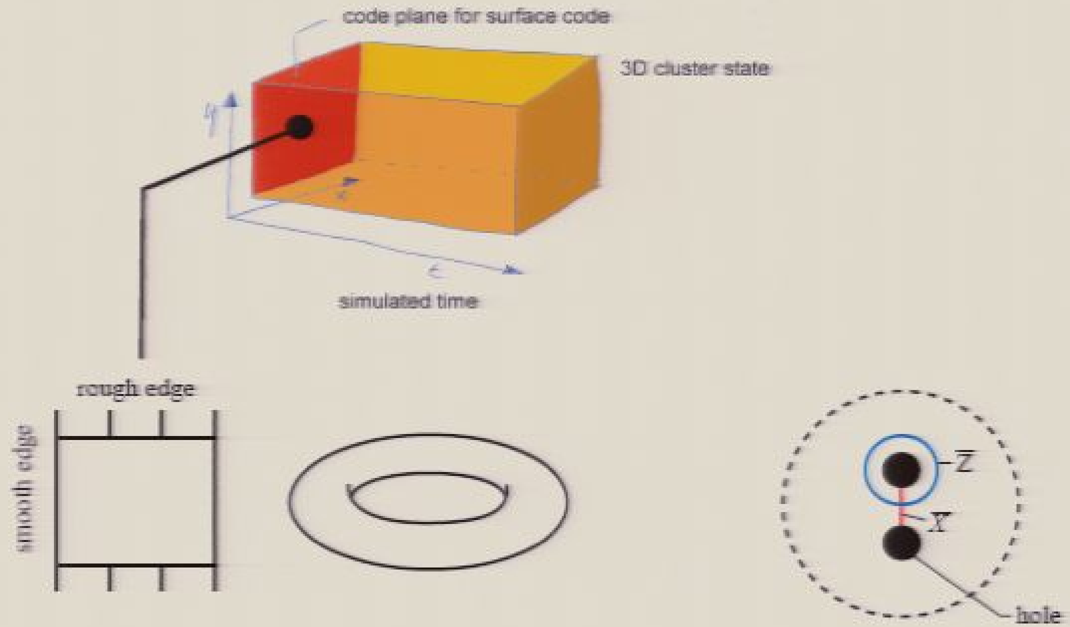
Map error correction to statistical mechanics:



- Have an error budget of 3%.

[1] T. Ohno et *al.*, quant-ph/0401101 (2004). [2] E. Dennis et *al.*, quant-ph/0110143 (2001); J. Edmonds, Canadian J. Math. **17**, 449 (1965).

2.2 Fault-tolerant quantum logic



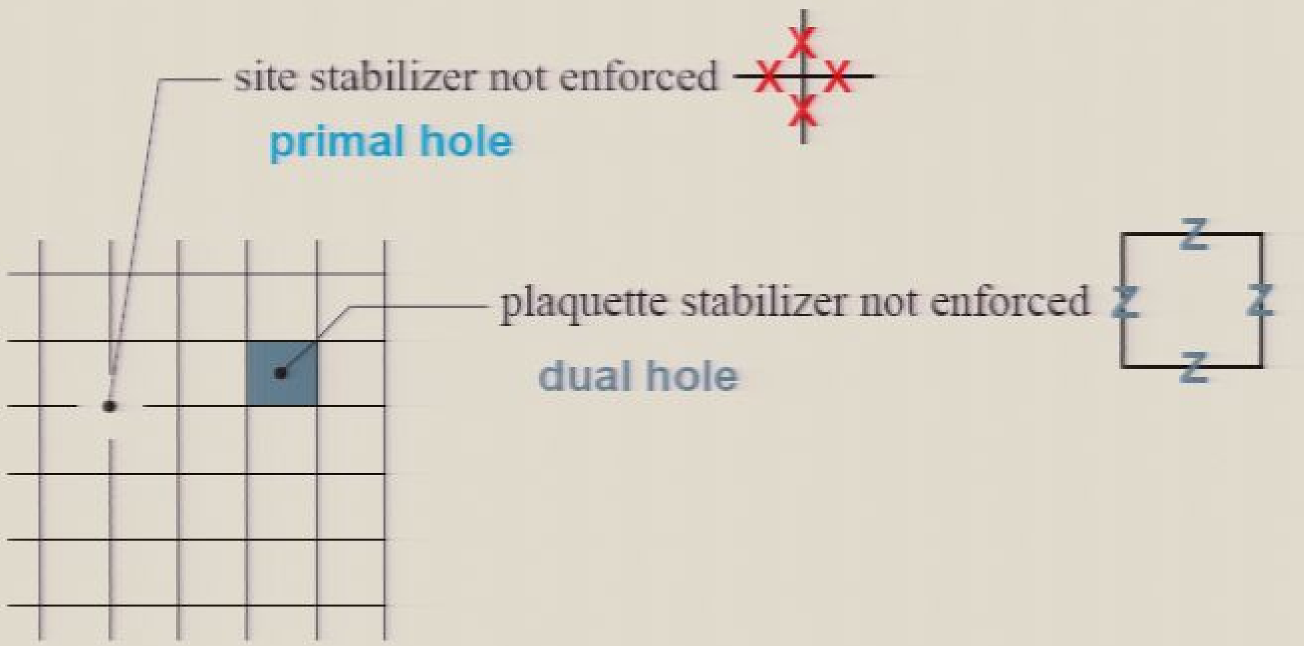
Plane segment — 1 Qubit

Torus — 2 Qubits

Plane with 2 holes — 1 Qubit

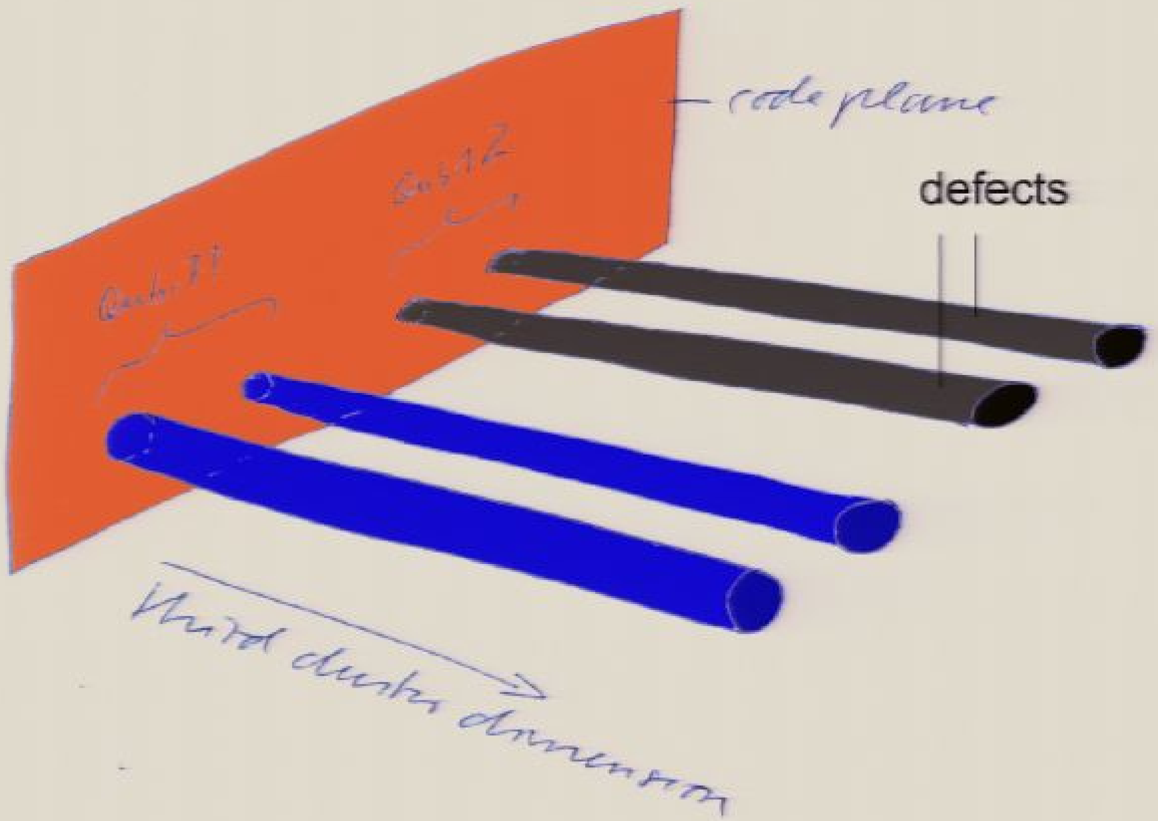
- Storage capacity of the code depends upon the topology of the code surface.

2.2 Surface code on plane with holes



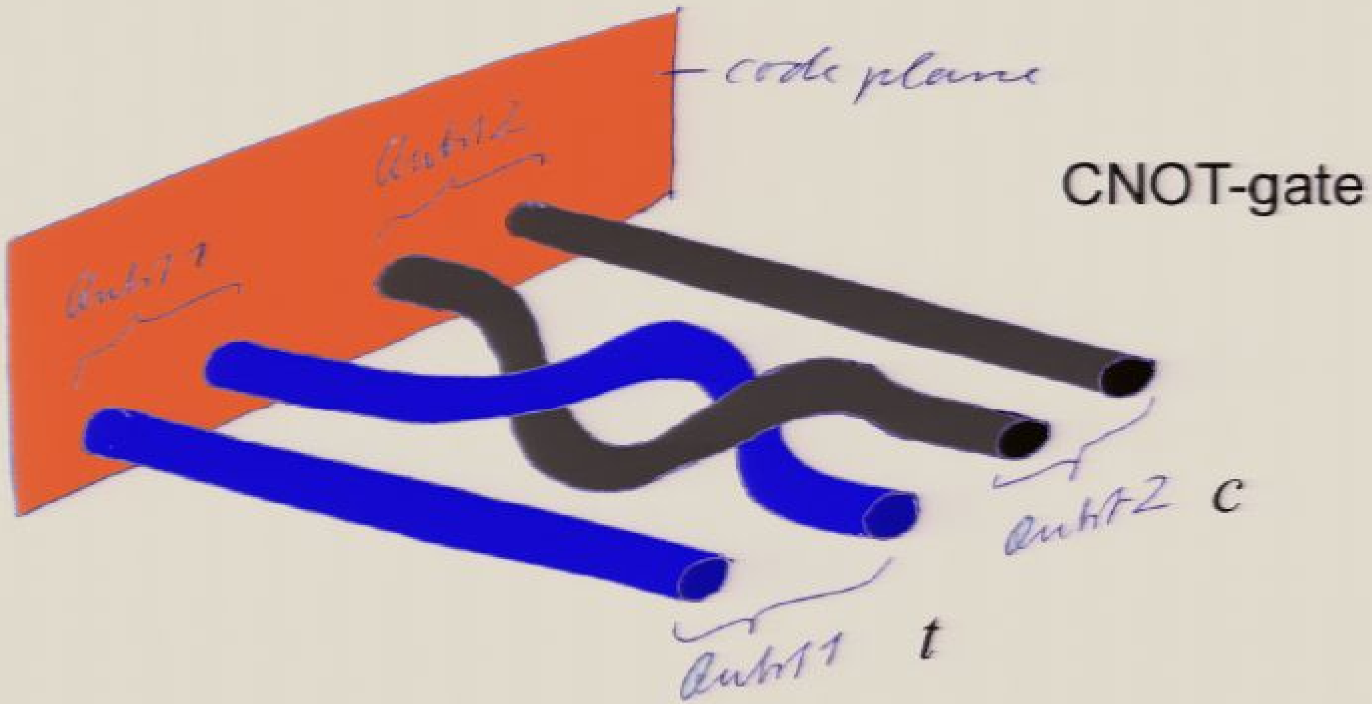
- There are two types of holes: primal and dual.
- A pair of same-type holes constitutes a qubit.

2.2 Defects for quantum logic



Defects are the extension of holes in the code plane to the third dimension.

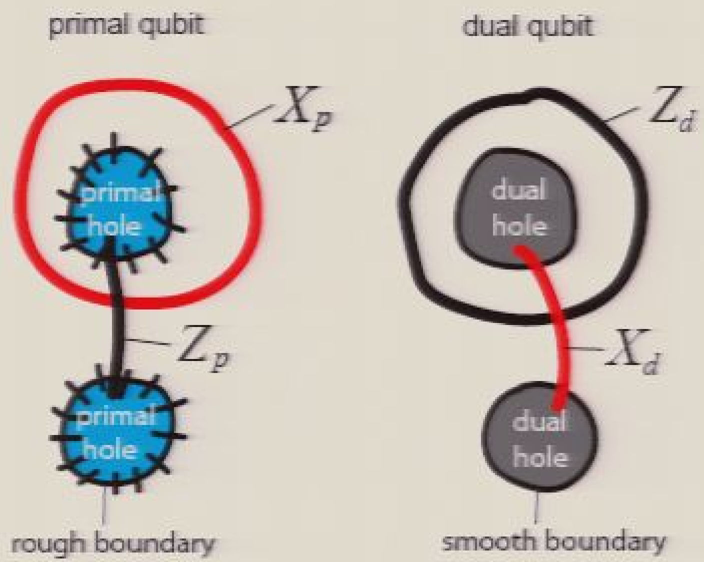
2.2 Defects for quantum logic



Topological quantum gates are encoded in the way primal and dual defects are wound around another.

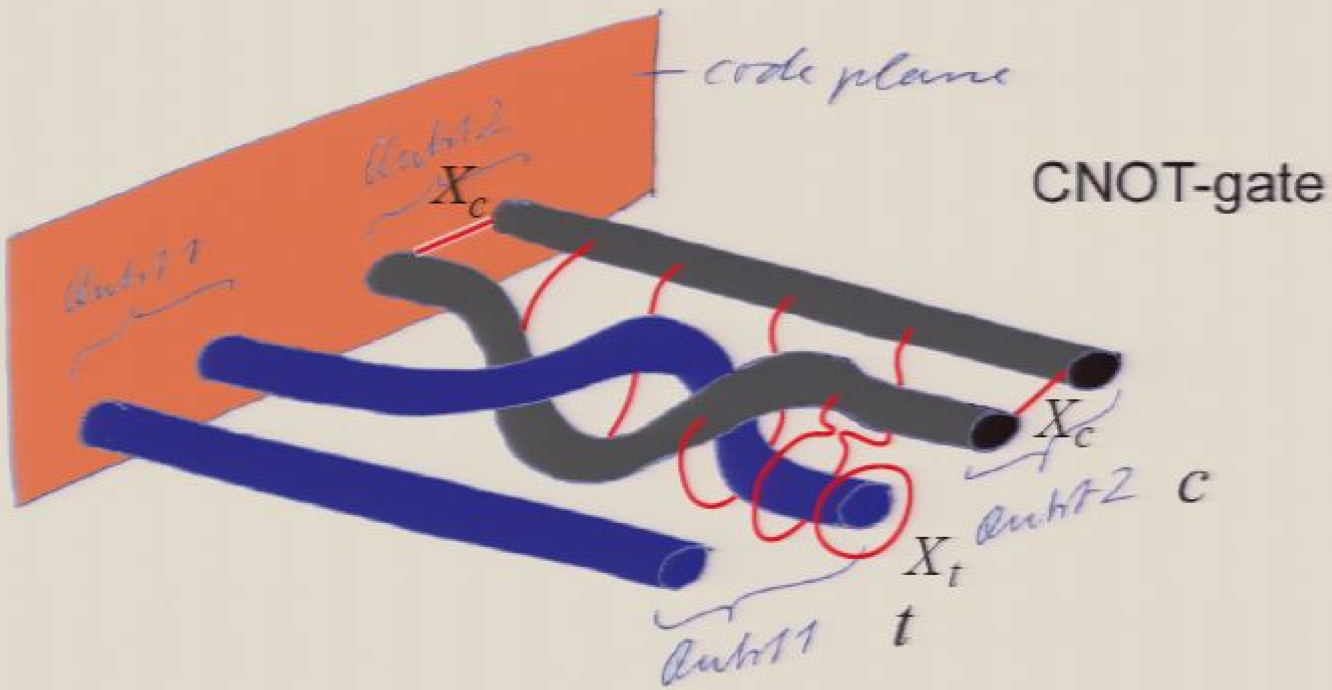
2.2 Explanation of the CNOT-gate

Surface code with boundary:



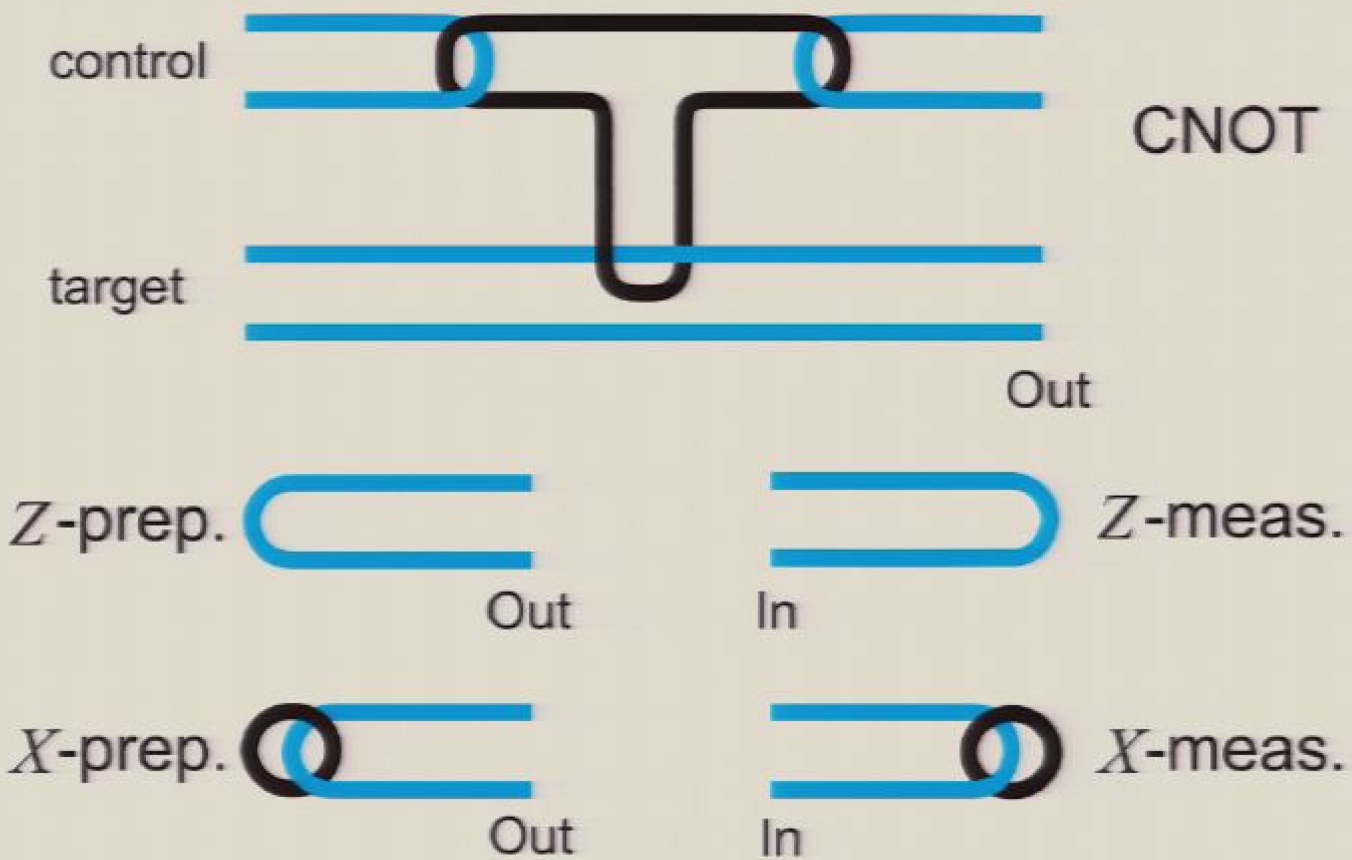
- X-chain cannot end in primal hole, can end in dual hole.
- Z-chain can end in primal hole, cannot end in dual hole.

2.2 Eplanation of the CNOT-gate



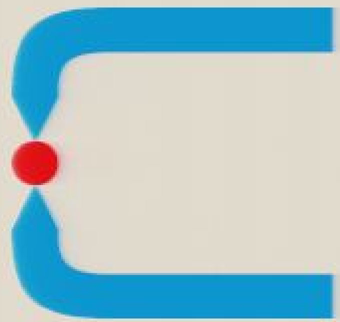
- Propagation relation: $X_c \longrightarrow X_c \otimes X_t$.
- Remaining prop rel $Z_c \rightarrow Z_c$, $X_t \rightarrow X_t$, $Z_t \rightarrow Z_c \otimes Z_t$ for CNOT derived analogously.

2.2 Quantum gates



2.2 Quantum gates

- Need one non-Clifford element:
  fault-tolerant preparation of $|A\rangle := \frac{X+Y}{\sqrt{2}}|A\rangle$.

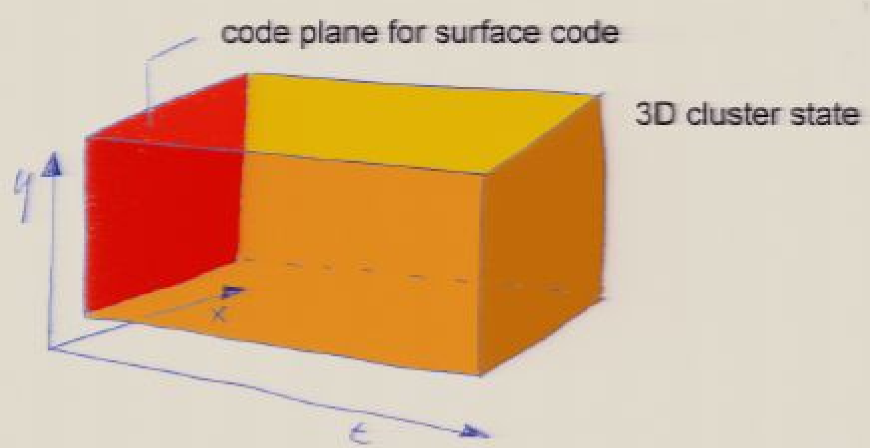


Singular Qubit

- FT prep. of $|A\rangle$ provided through realization of *magic state distillation** on the cluster state.

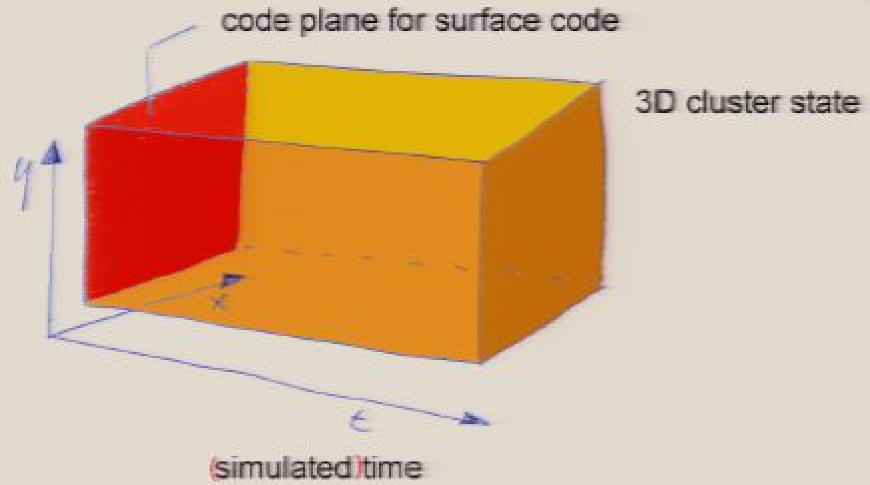
*: S. Bravyi and A. Kitaev, Phys. Rev. A 71, 022316 (2005).

2.3 Mapping to 2D



- *Turn simulated time into real time!*
- Require a *2D single-layer structure.*

2.3 Mapping to 2D



- After mapping back in the circuit model
- *Still require only nearest-neighbor translation-invariant interaction.*

2.4 Fault-tolerance threshold

Error sources (after mapping to 2D):

1. $|+\rangle$-preparation: Perfect preparation followed by 1-qubit partially depolarizing noise with probability p_P.
2. $\Lambda(Z)$-gates (space-like edges of $\mathcal{L}$): Perfect gates followed by 2-qubit partially depolarizing noise with probability p_2.
3. Hadamard-gates (time-like edges of $\mathcal{L}$): Perfect gates followed by 1-qubit partially depolarizing noise with probability p_1.
4. Measurement: Perfect measurement preceeded by 1-qubit partially depolarizing noise with probability p_M .

- No qubit is ever idle. (Additional memory error - same threshold)
- For threshold set $p_1 = p_2 = p_P = p_M =: p$.

2.4 Fault-tolerance threshold

Topological threshold in cluster region V:

$$p_c = 7.5 \times 10^{-3}. \tag{1}$$

Purification threshold for fault-tolerant $|A\rangle$-preparation:

$$p_c = 3.7 \times 10^{-2}. \tag{2}$$

Topological EC sets the overall threshold.

Overhead and robustness of threshold

- Denote by S (S') the bare (encoded) size of a quantum circuit. Then, for the described method:

$$S' \sim S \log^3 S. \quad (3)$$

- The threshold is robust against variations in the error model such as higher weight elemetary errors.

Summary

Phys. Rev. Lett. 98, 190504
quant-ph/0703143

Scenario:

- Local and next-neighbor gates in 2D single layer structure.

Numbers:

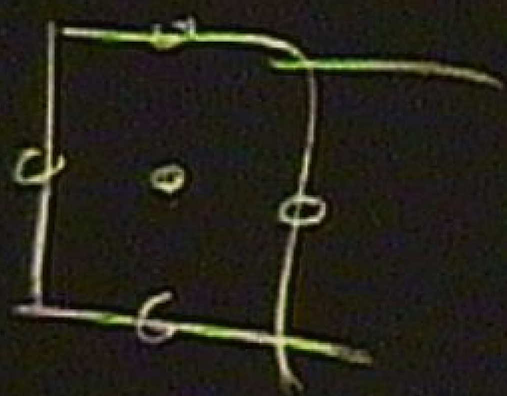
- Fault-tolerance threshold of 7.5×10^{-3} for preparation, gate, memory and measurement error (each source).

Methods:

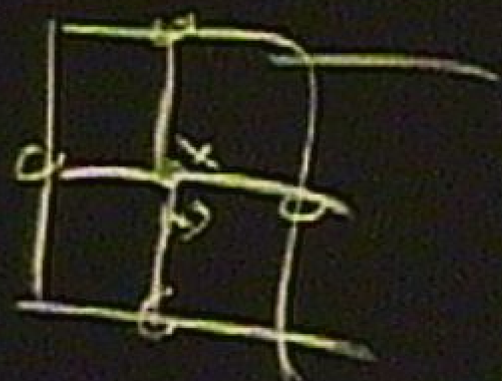
- 3D cluster states provide intrinsic topological error correction and topologically protected quantum gates.

Suitable systems for realization:

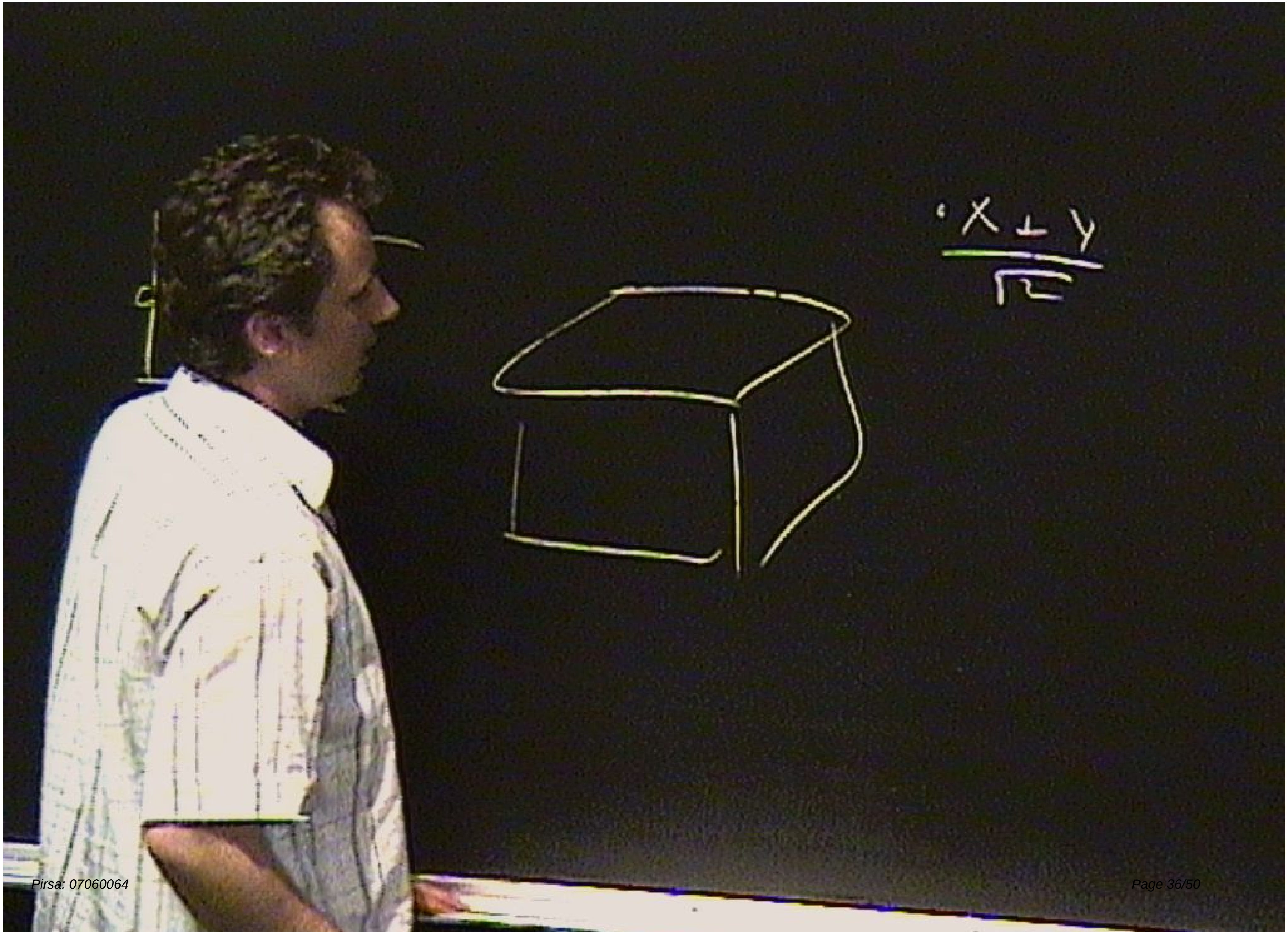
- Cold atoms in optical lattices, Josephson junction arrays, segmented ion traps, ...



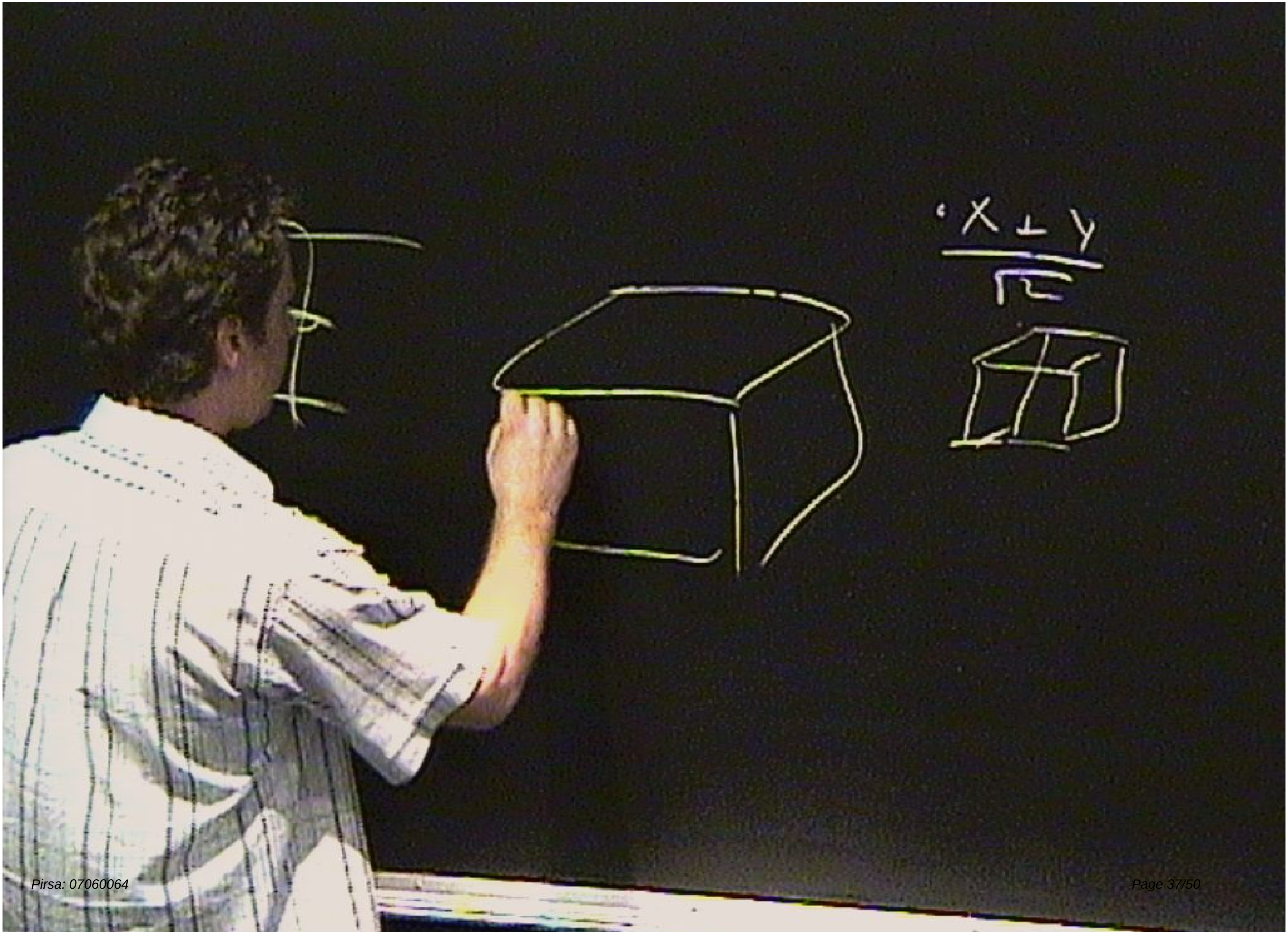
$$\frac{X \perp Y}{\sqrt{2}}$$

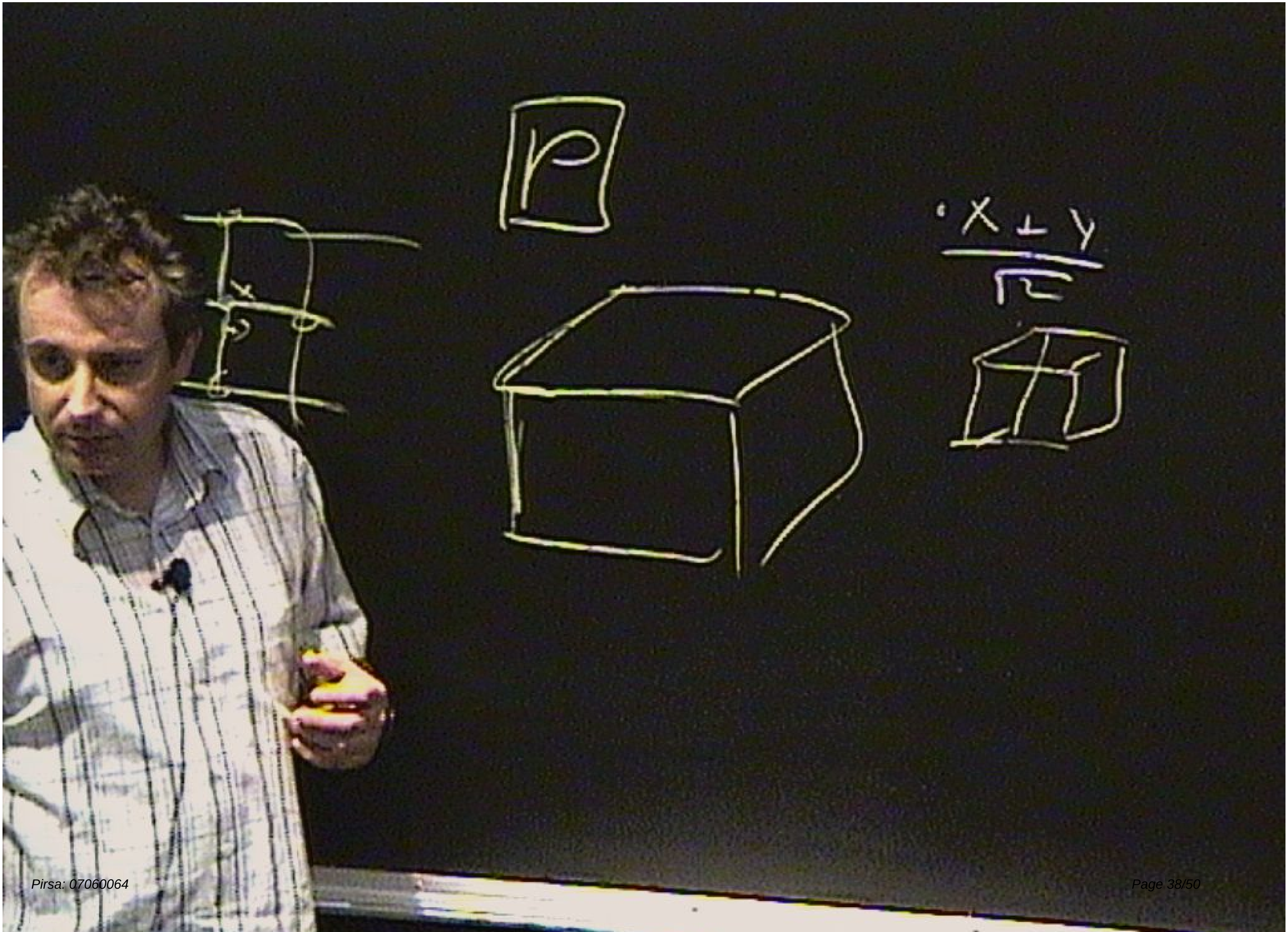


$$\frac{X + Y}{\sqrt{2}}$$



X ⊥ Y

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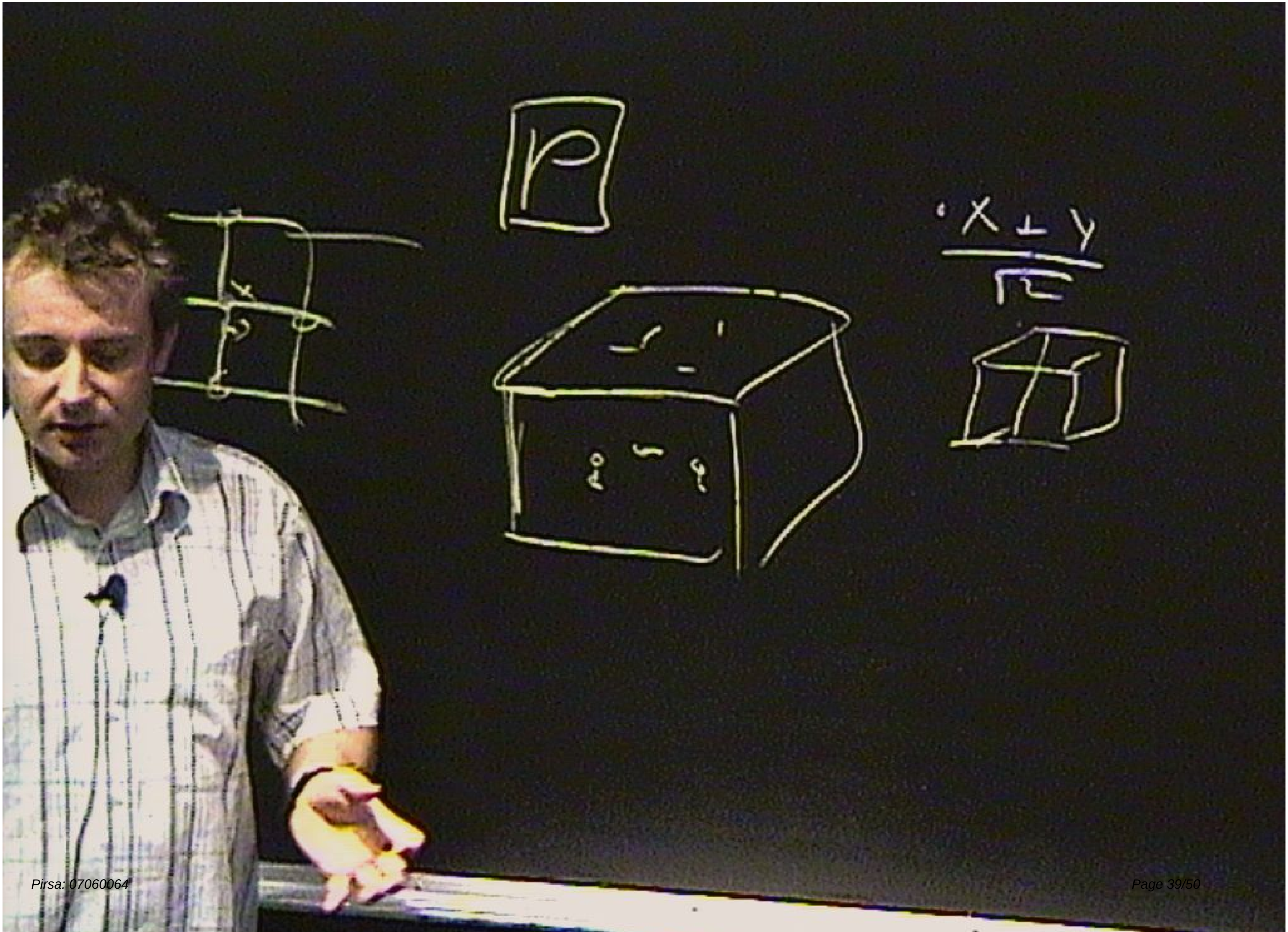

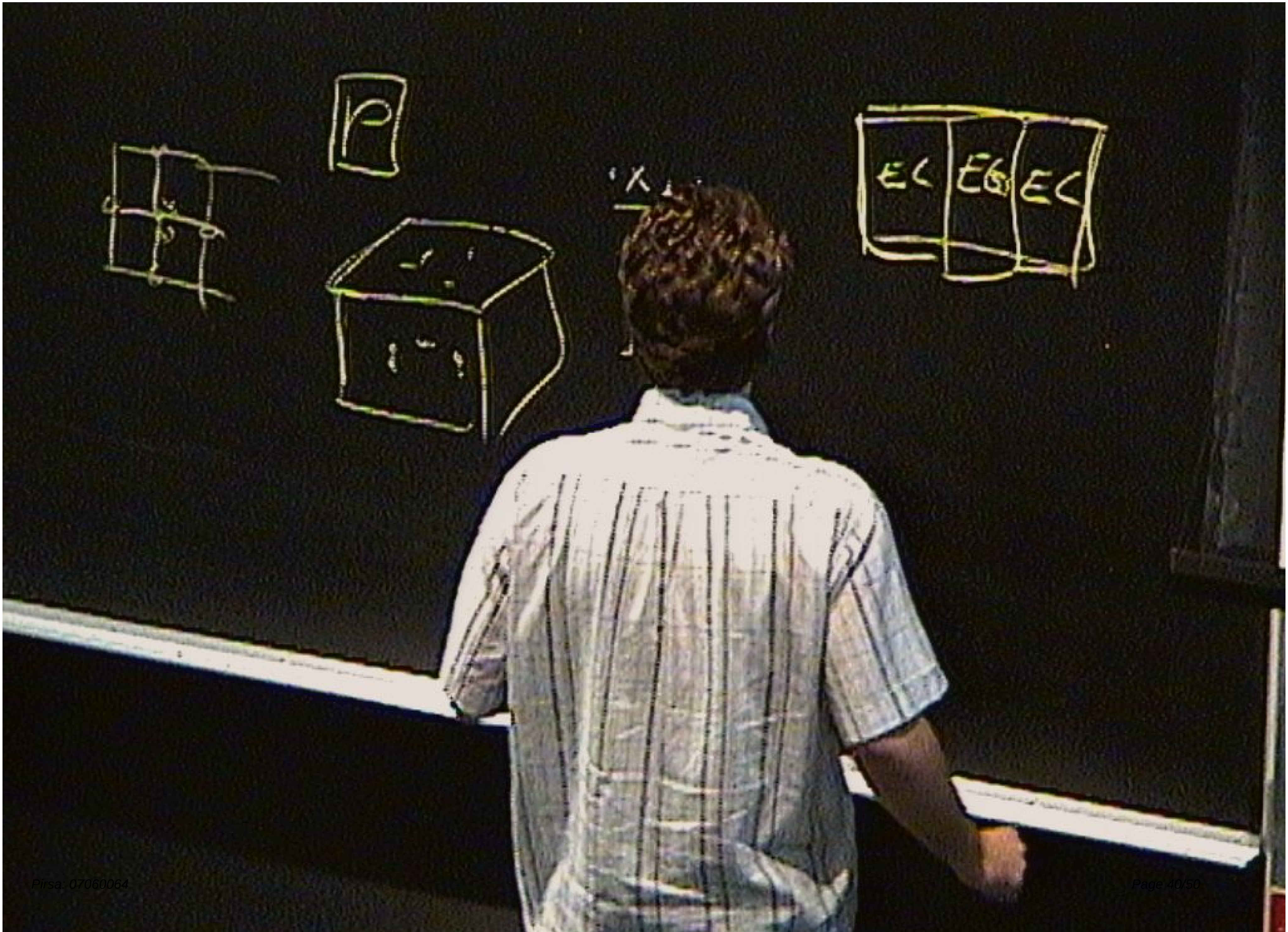
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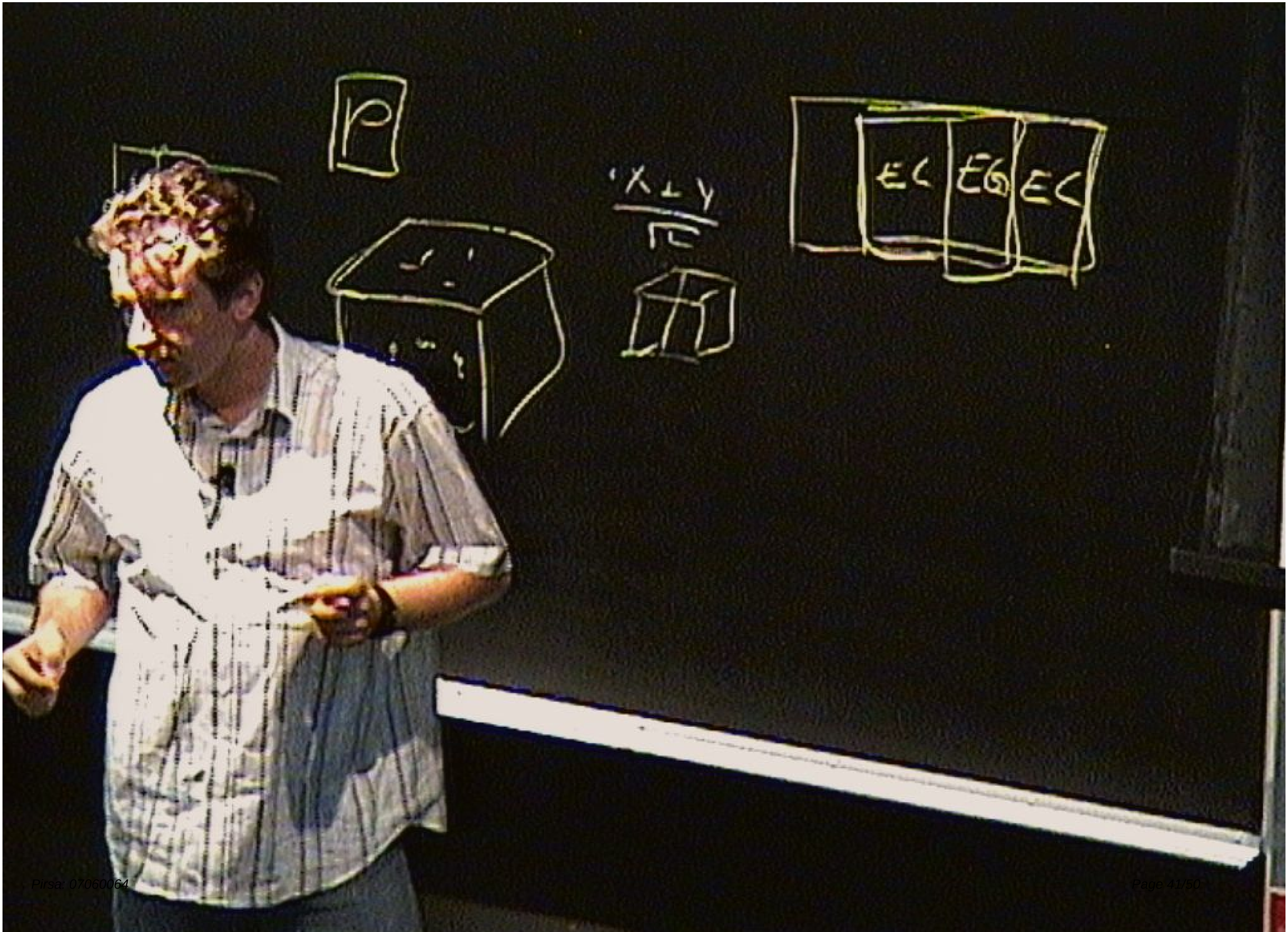
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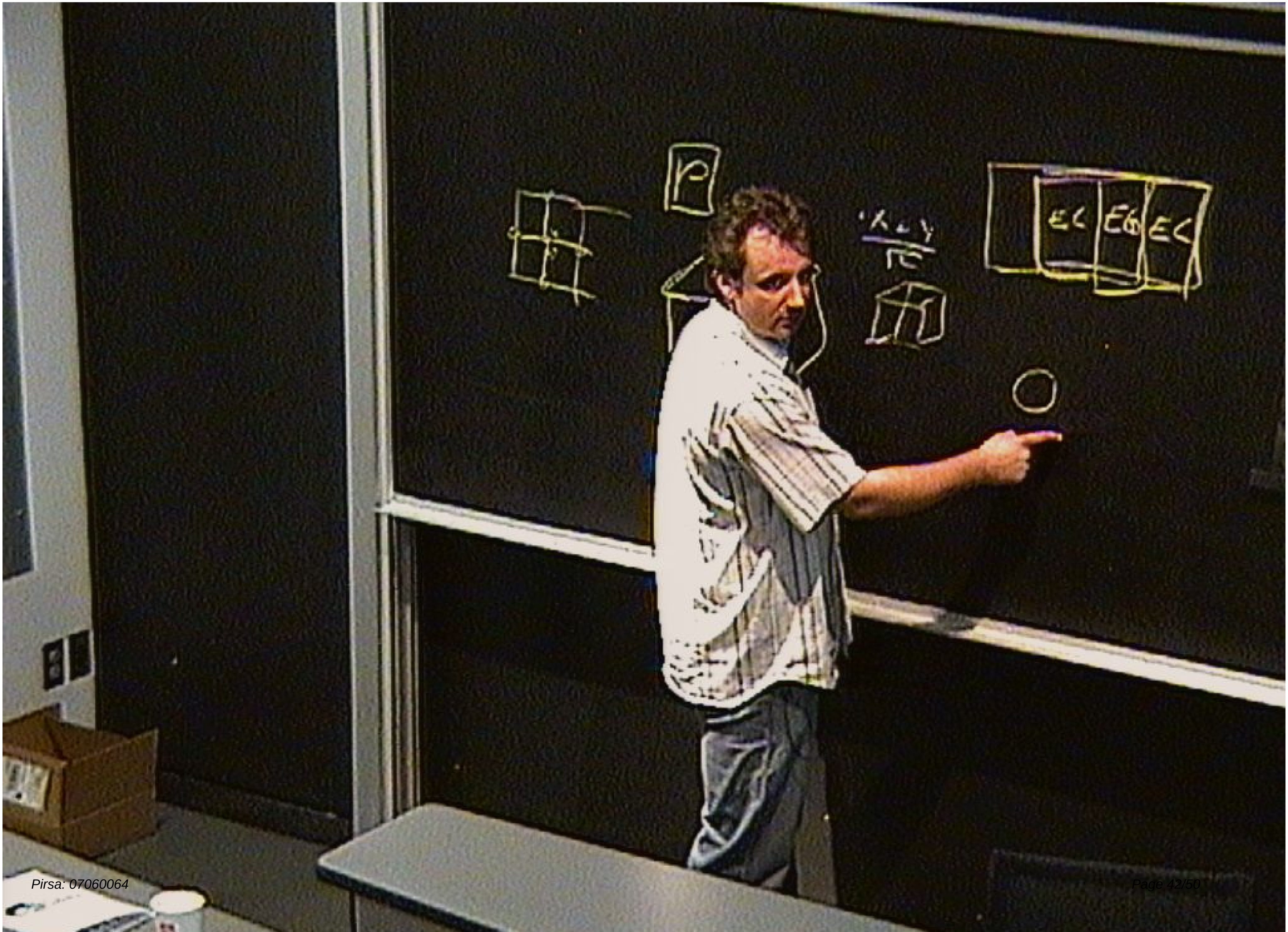

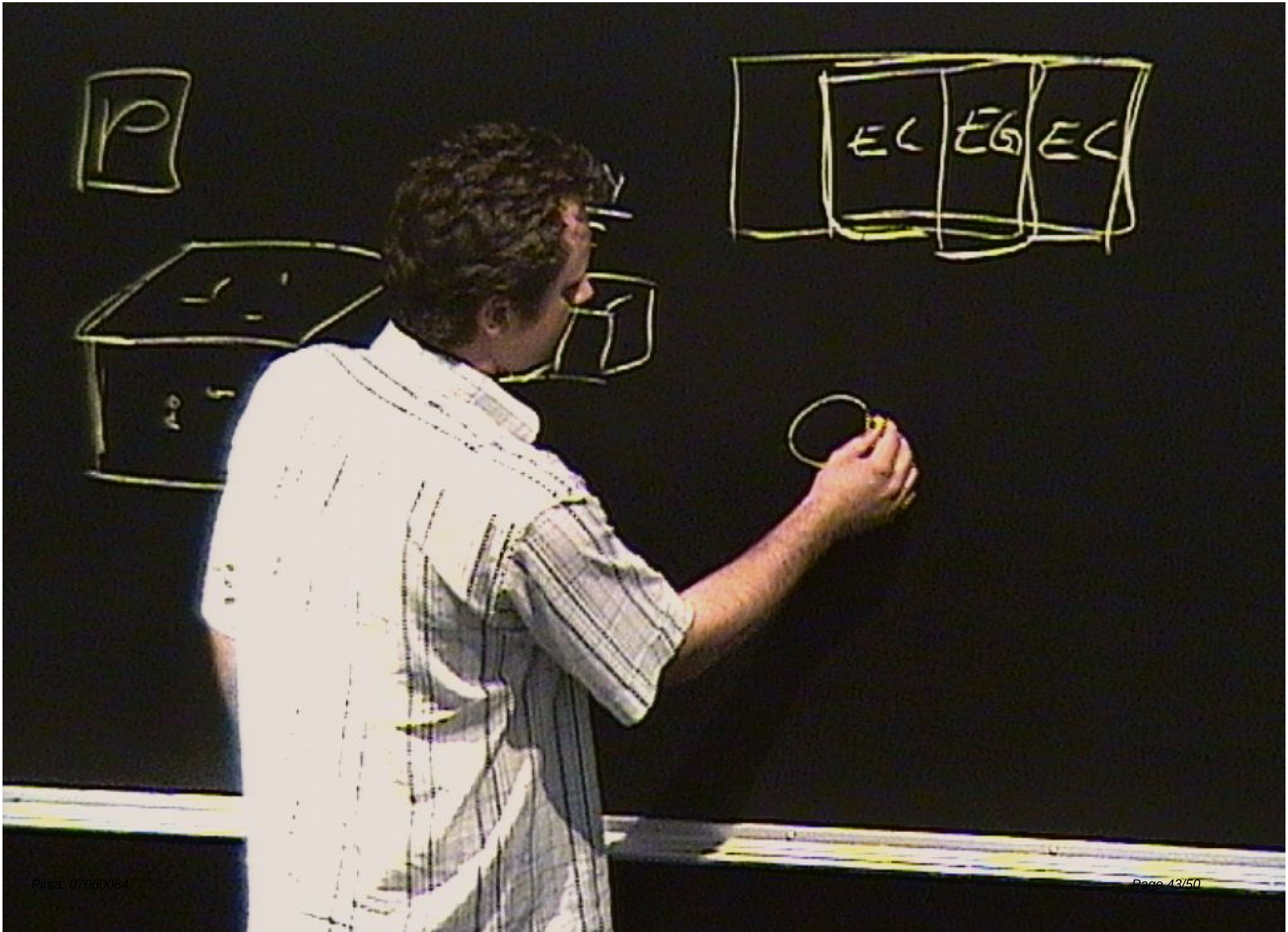
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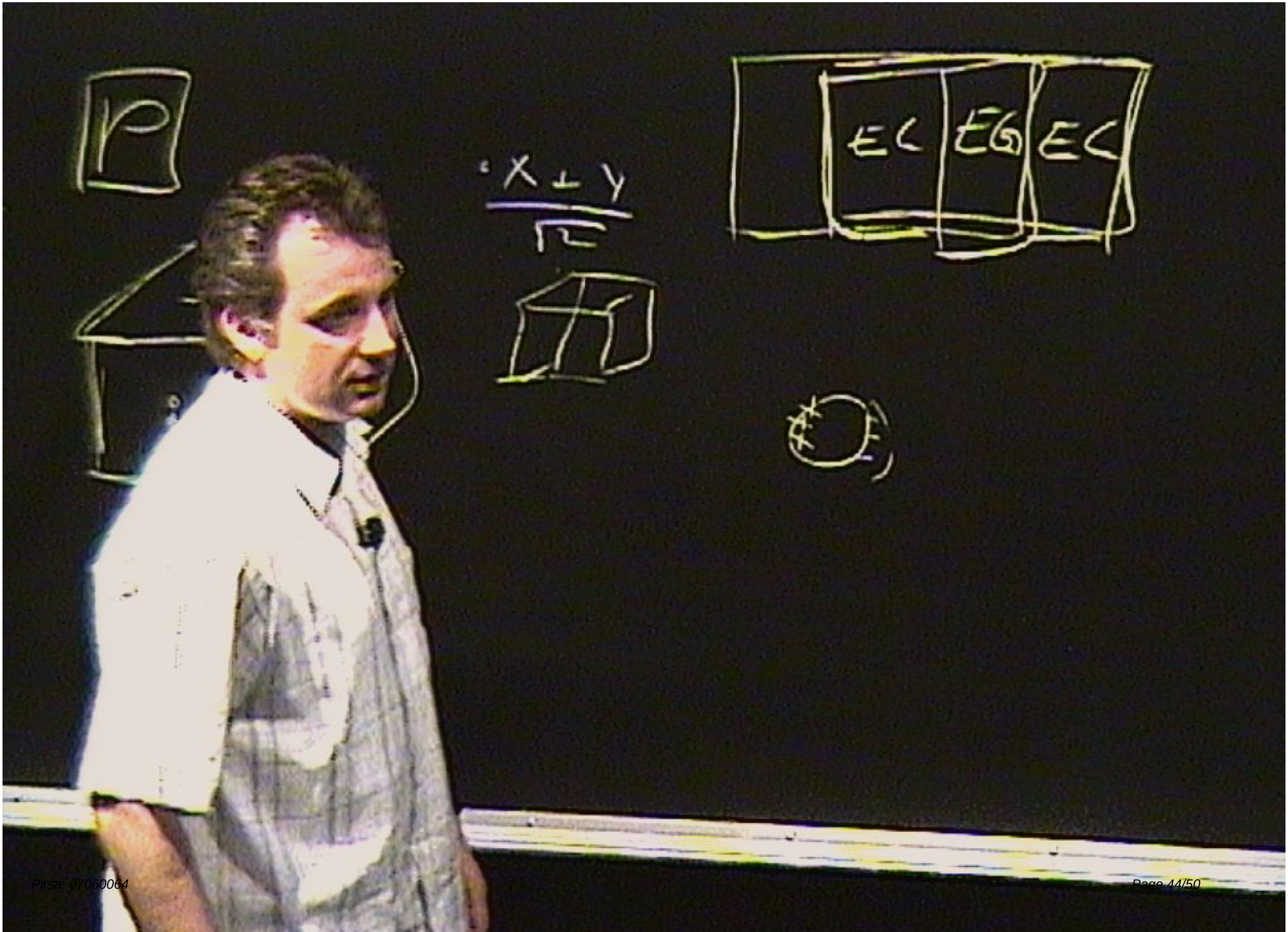

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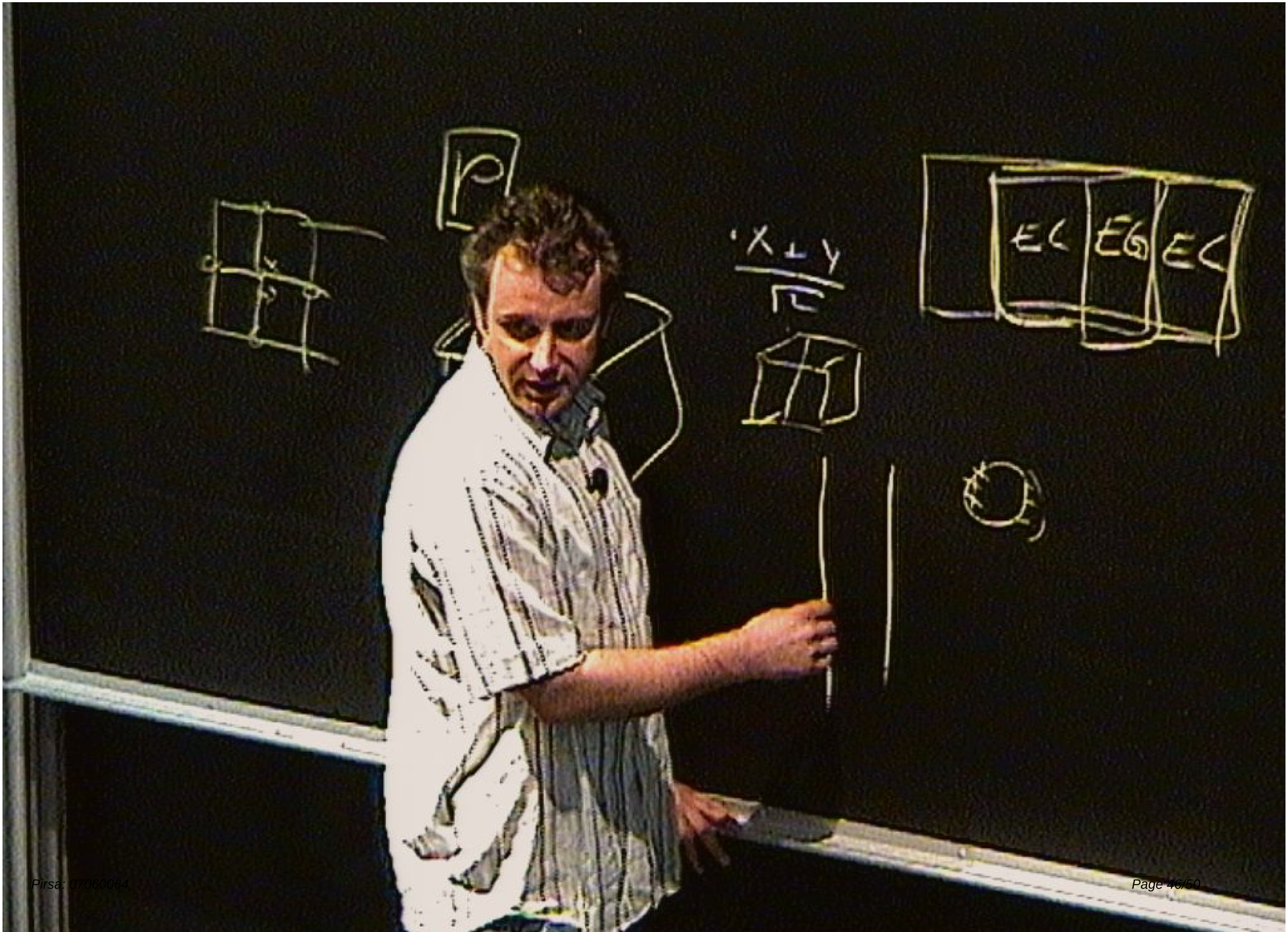

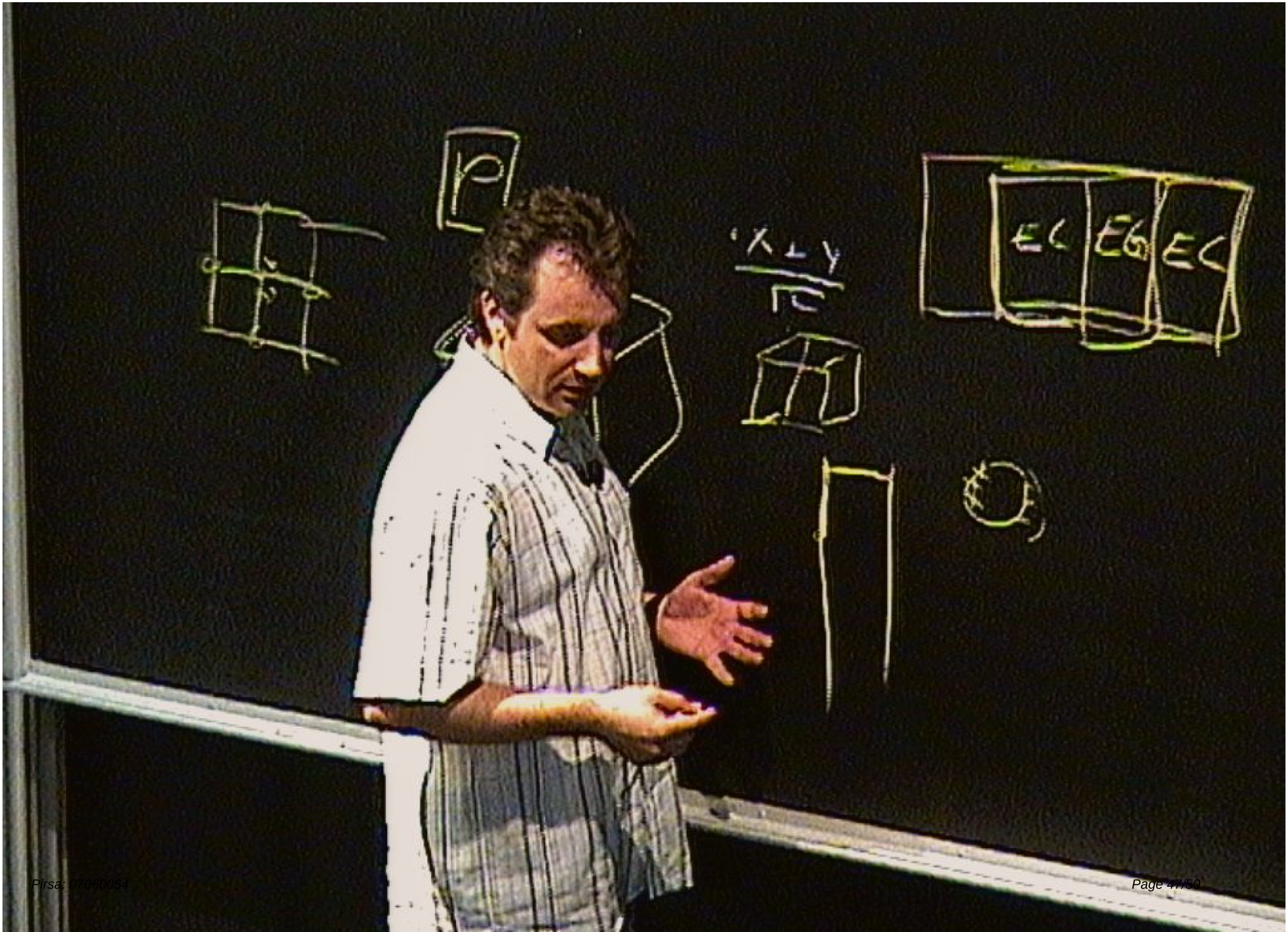
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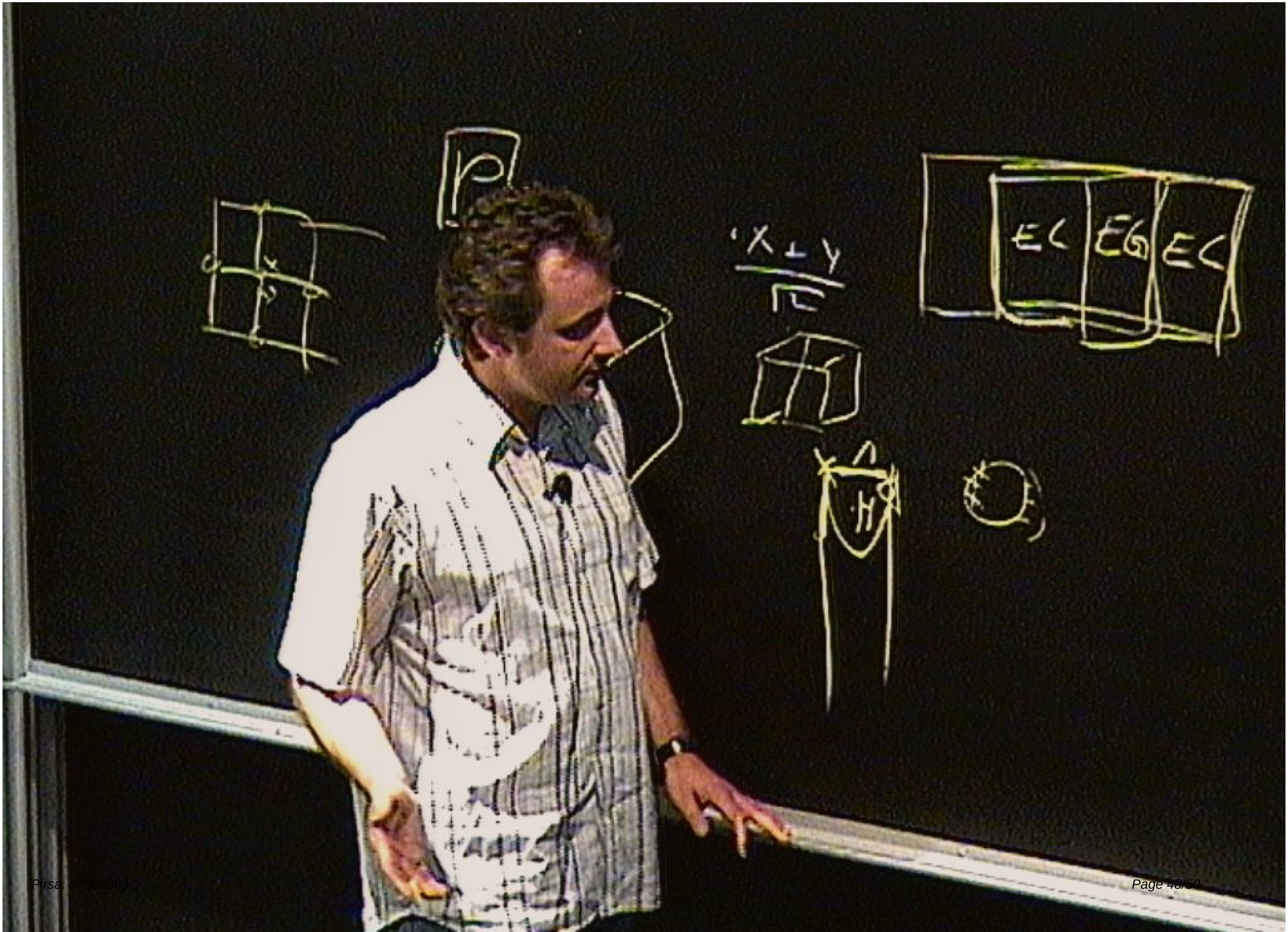
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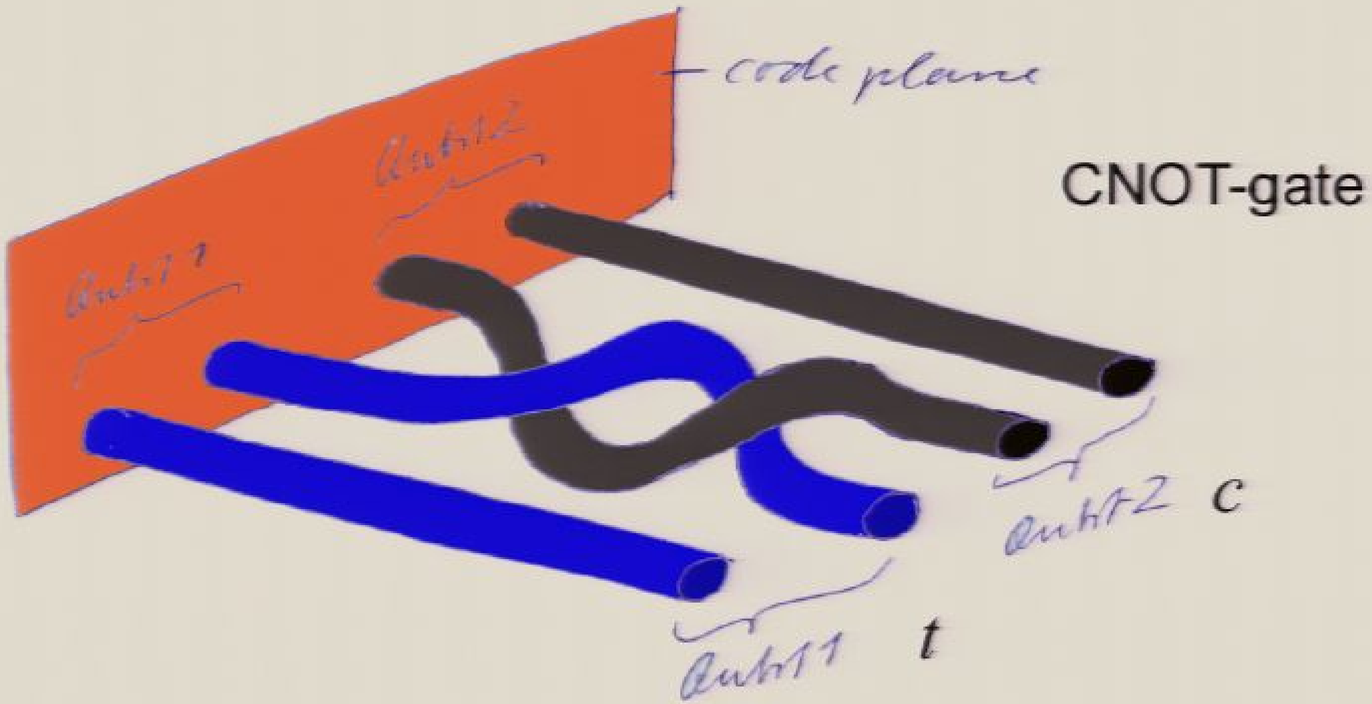
2.2 Defects for quantum logic



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