Title: Fault-tolerant quantum architecture: a comparative code analysis

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Abstract:

Fault-tolerant quantum computer architecture: a comparative code survey

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16 June 2007 / FTQC II

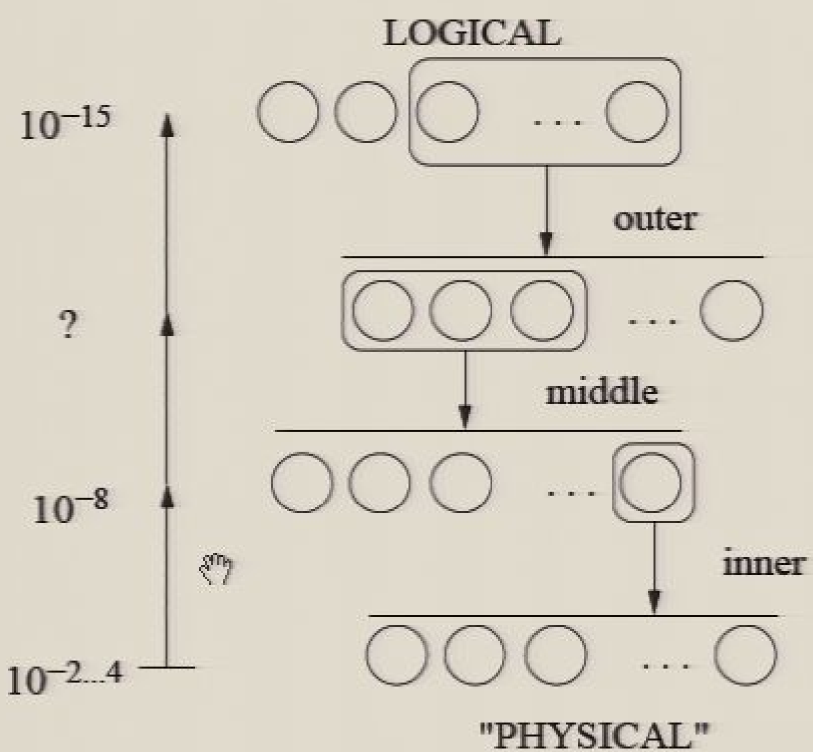
Outline

1. **Introduction**
   - Motivation
   - Relation to Prior Work
   - Goals
2. **Approach**
   - Rectangle Construction
   - Simulation Method
   - Code Zoo
3. **Results**
   - Threshold vs Block Size
   - Threshold vs Overhead

Motivation

- FTQC schemes use concatenated codes.
- Goals –
  - **(a)** achieve noise rates comparable to digital computers,
  - **(b)** balance noise tolerance with overhead,
  - **(c)** obtain acceptable noise threshold
- Several codes needed in practice to achieve goals.

Code Hierarchy



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  - high rate, low distance
- middle code(s) ?
- inner code
  - low rate, high distance
- desire *few* levels

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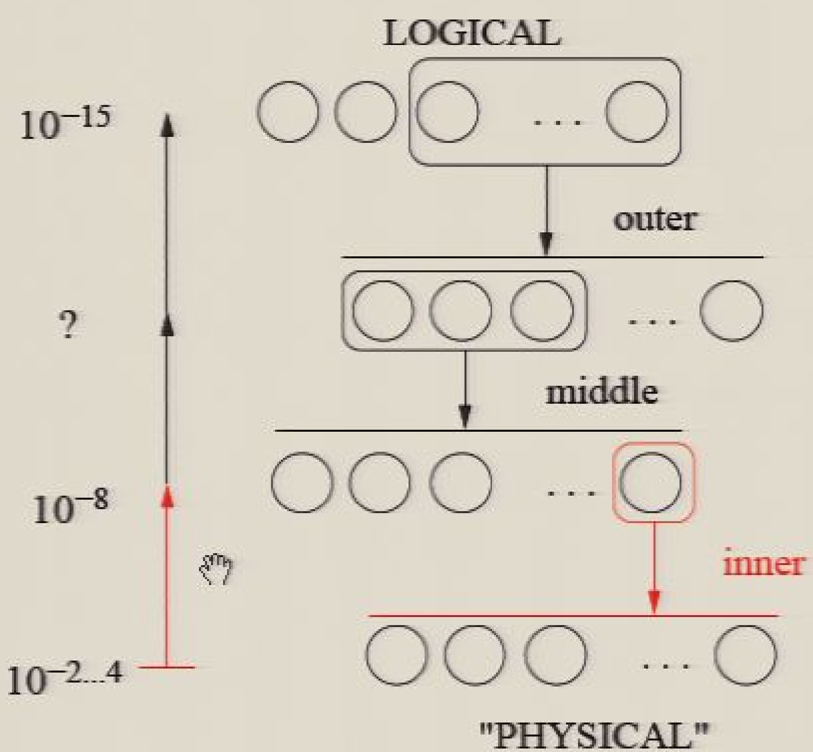

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Prior Work

- Steane, "Overhead and noise threshold of FTQEC" (2002)
  - only detailed survey of *many* codes for FTQEC
  - Steane and Golay codes numerically simulated
  - model developed – applicable to many other CSS codes

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Observations

- Threshold increases Steane to Golay, decreases thereafter
- "One level" architecture, balance of rate/distance
- [[127, 29, 15]] for subsequent ion-trap architecture (2004)

Why Another Code Survey?

- There is a rigorous theory based on *extended rectangles* for any minimum distance [AGP]; prior work not directly compatible with this theory
- Other families of codes to consider
- More codes can now be simulated

1) Threshold vs Block Size of Inner Codes

Threshold Lower Bound

$$\epsilon_{th} \geq \binom{A(n)}{t_0(n)+1}^{-1/t_0(n)}$$

- Suppose $t_0(n)$ linear
- $A(n)$ linear: limit nonzero
- $A(n)$ quadratic: limit zero

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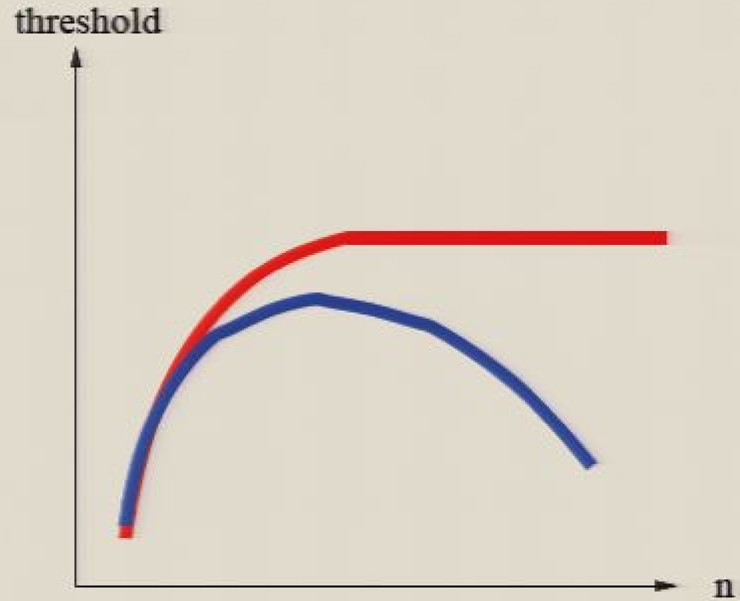

2) Threshold versus Overhead of Inner Codes

- overhead: CNOTs/Rectangle
- grows with
  - n = block size
  - $t_0(n)$ = effective error-correcting capability
  - **L = number of ancilla preparation attempts**
- restrict to "low" overhead

What code has largest threshold at given overhead?

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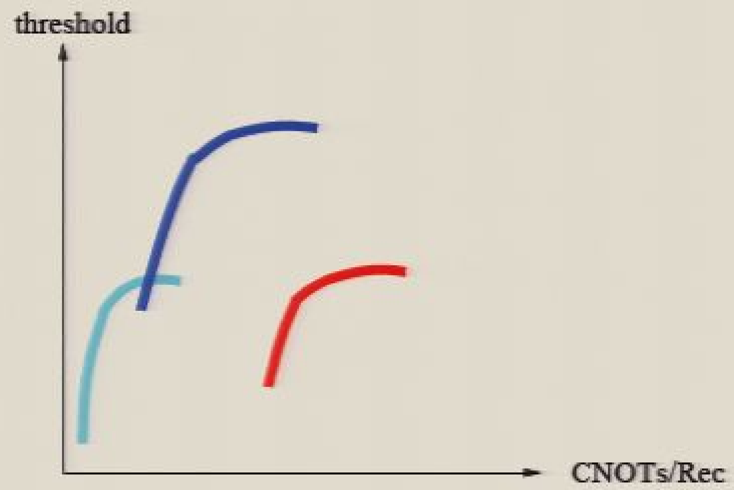

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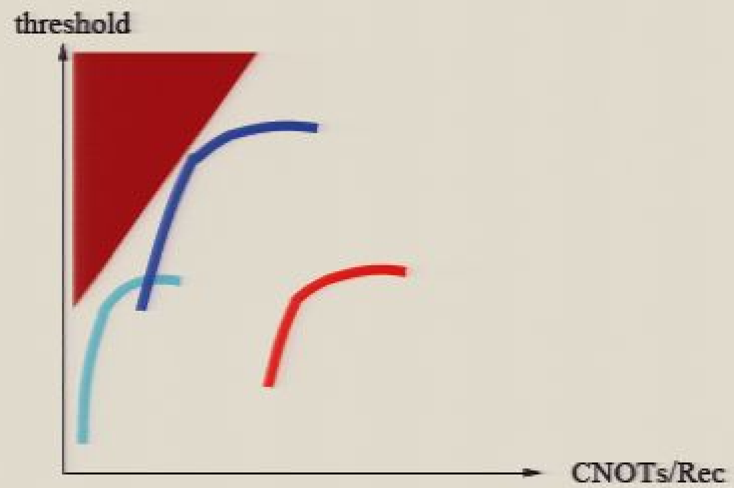

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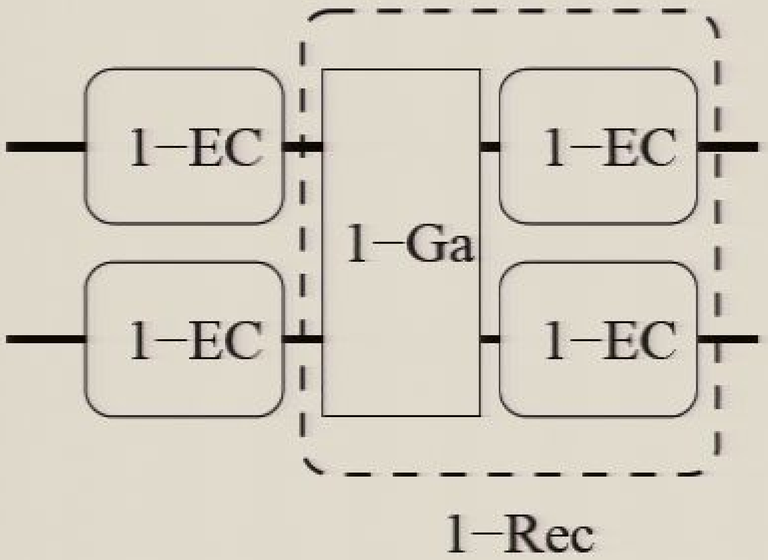
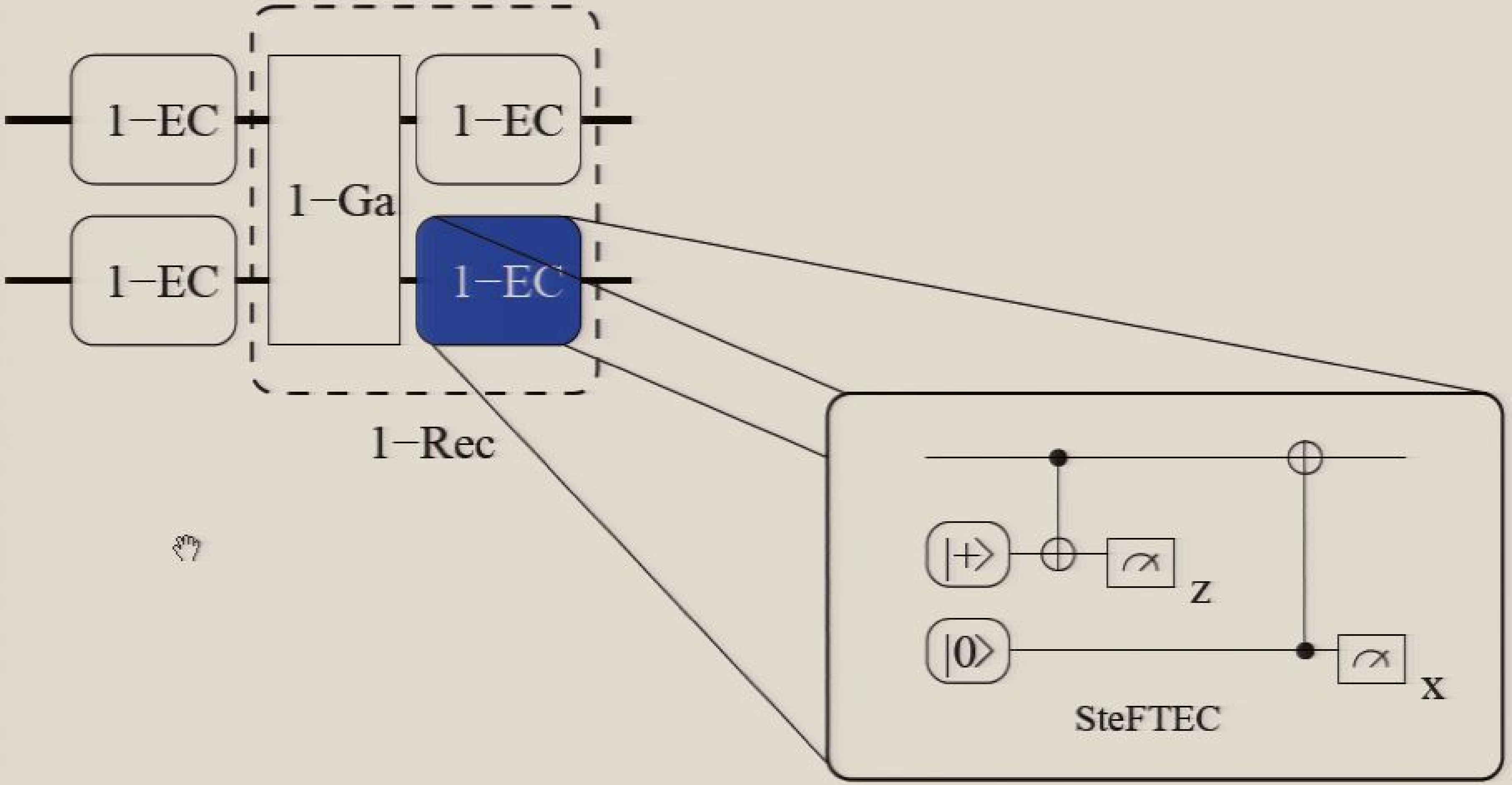
Extended Rectangle



Rectangle Construction

Extended Rectangle



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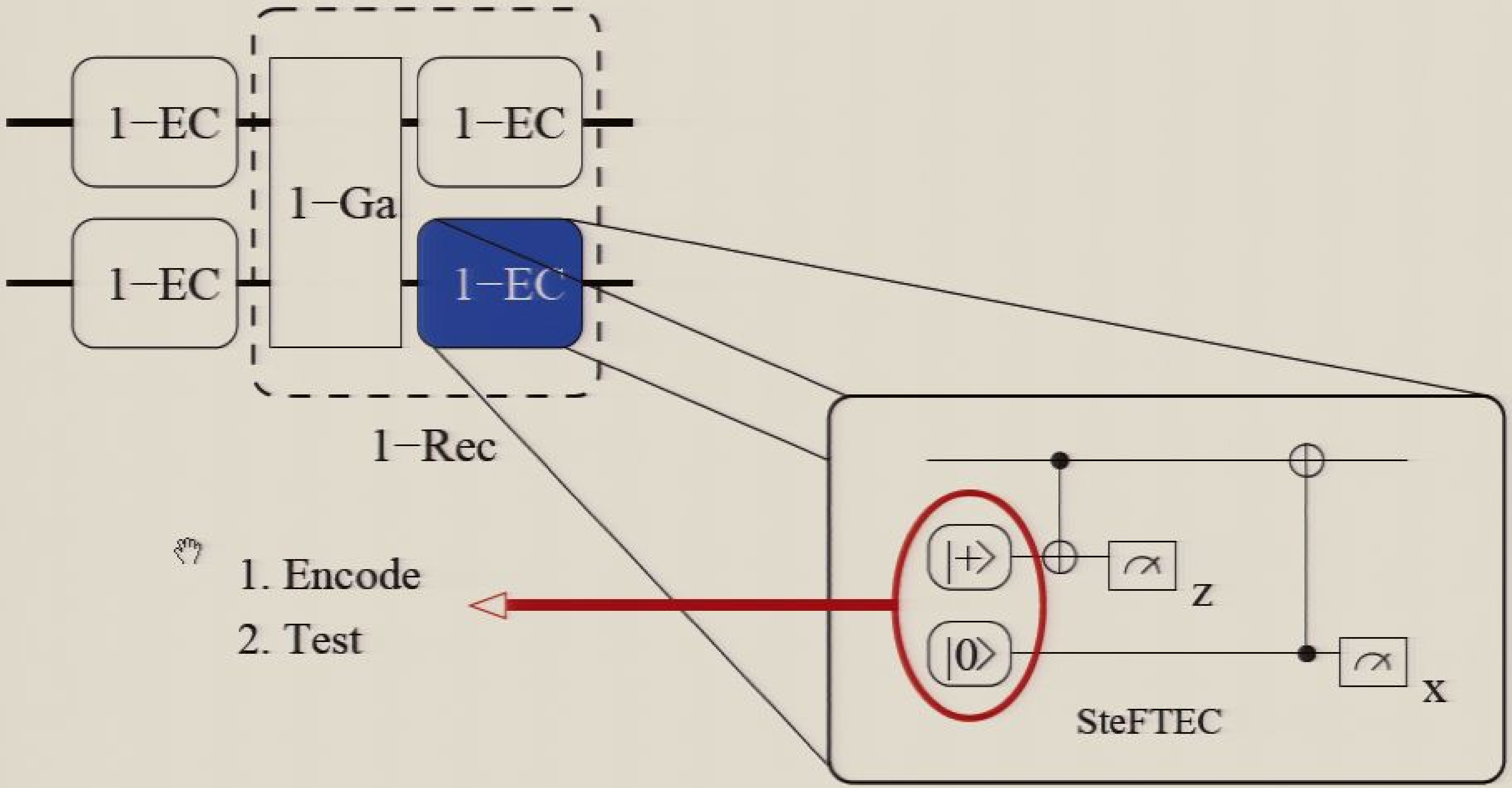


Rectangle Construction

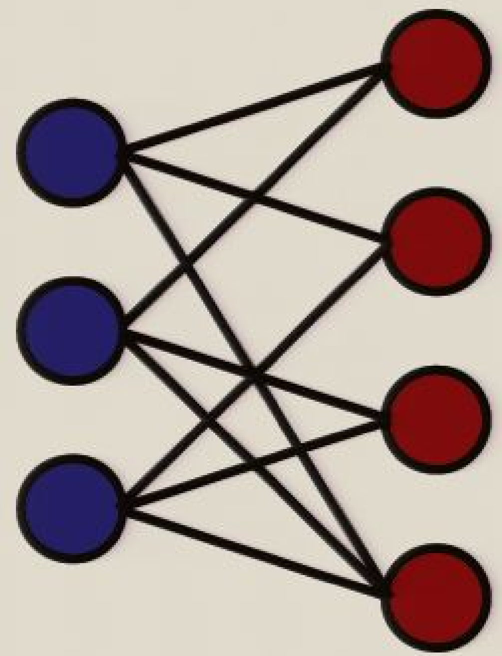
Preparing Ancilla: Encoding

Systematic Form

$$G = \left(\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right)$$

Latin Rectangle

$$G = \left(\begin{array}{ccc|cccc} * & * & * & * & * & - & * \\ * & * & * & * & - & * & * \\ * & * & * & - & * & * & * \end{array}\right)$$



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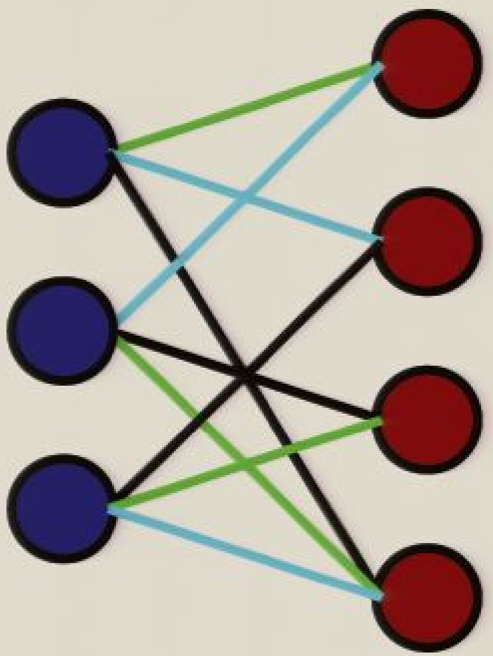
Latin Rectangle

$$G = \left(\begin{array}{ccc|cccc} * & * & * & 1 & 2 & - & * \\ * & * & * & 2 & - & * & 1 \\ * & * & * & - & * & 1 & 2 \end{array}\right)$$



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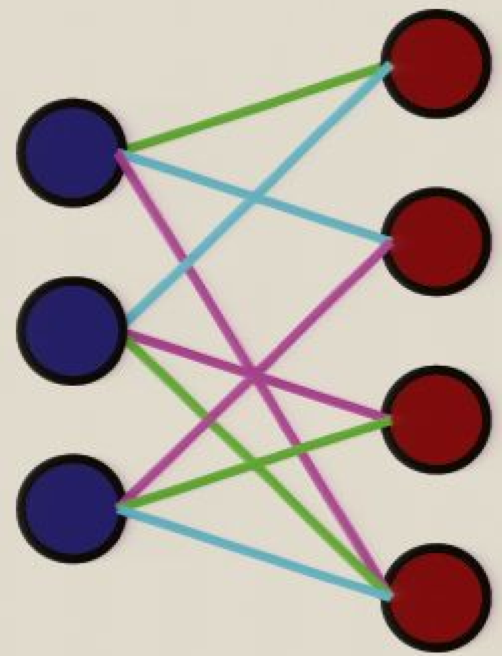
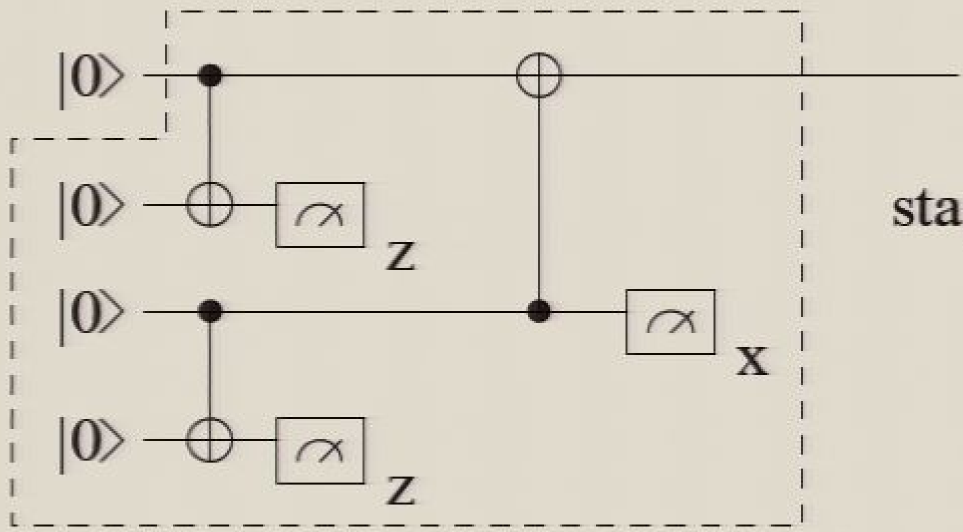


Preparing Ancilla: Testing



stages of error–detectio[n]

R

parallel preparation attempts

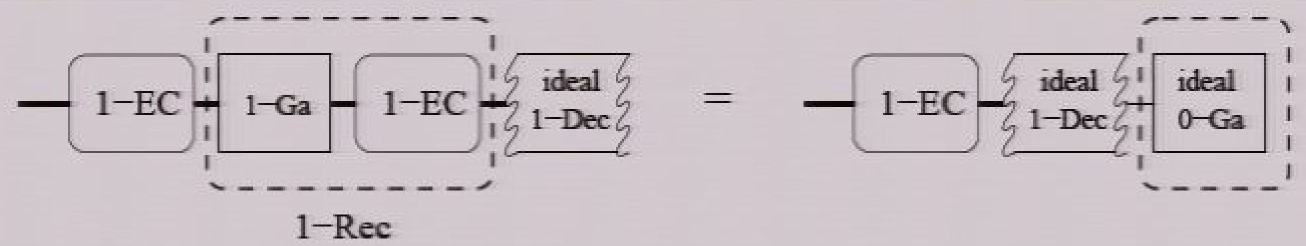
L

Construction Summary

- CNOT ex-Recs of codes encoding $k \approx 1$ qubits
- SteFTEC with
  - R stages of error-detection
  - L parallel preparation attempts
- Notes:
  - fault-tolerant but not strictly fault-tolerant
  - ex-Recs not optimized
  - No penalty for non-local gates

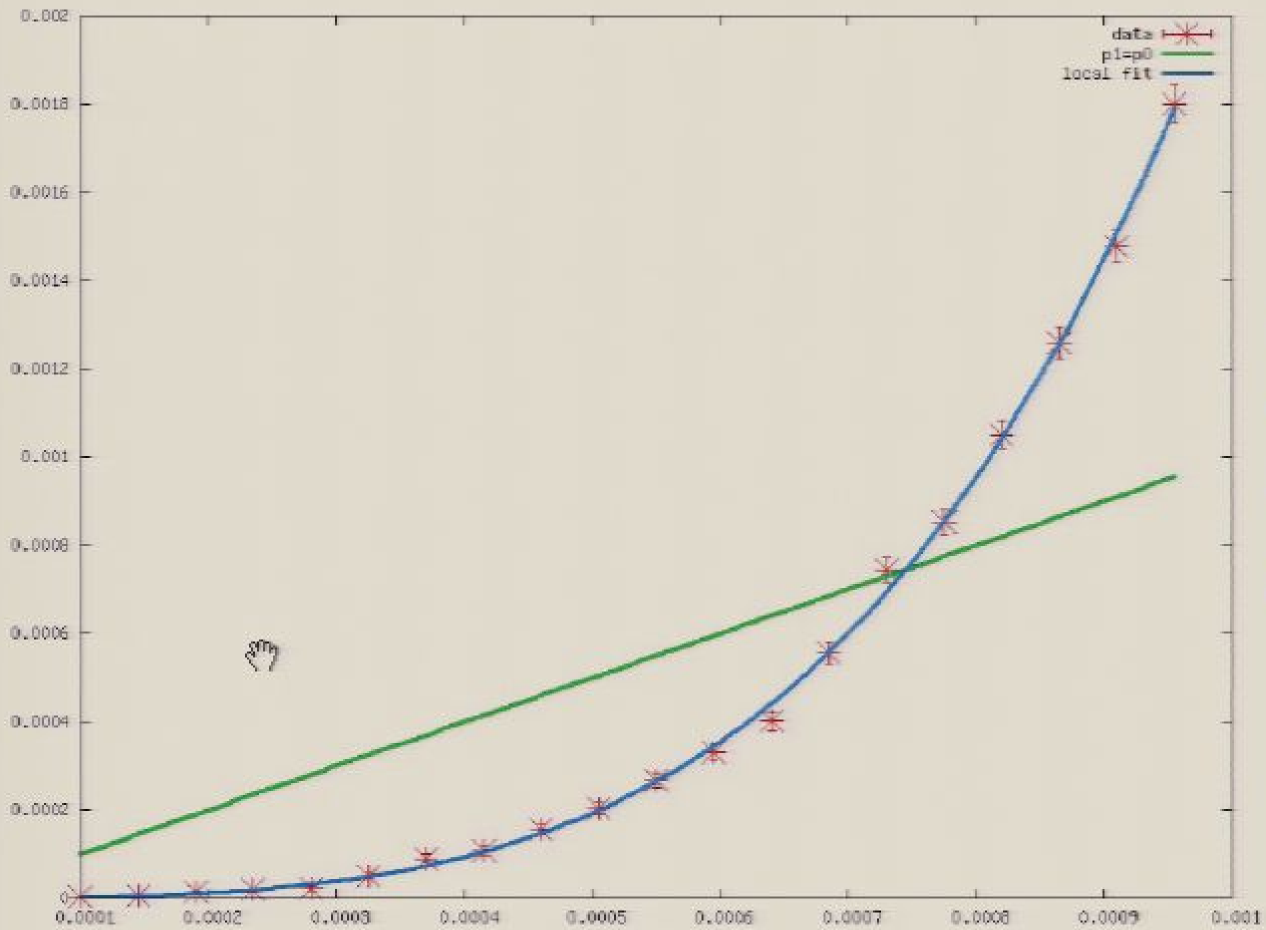
"ex-Rec aware" Monte-Carlo Method

Correctness [AGP]



- Estimate $\mathbb{P}(\text{1-Rec not correct})$ for dep. noise
- No need to repeat gates for steady-state distribution

Example



- p_0 is prob. of dep. fault
- $p_1 = \mathbb{P}(\text{1-Rec incorrect})$
- 1-σ error bars
- sample local poly. fits; estimate quasithreshold

Families of Codes

CSS Codes

Only consider binary codes whose generators can each be written as a product of only X's or only Z's – CNOT transversal.

- Bacon-Shor codes
- Bravyi-Kitaev planar codes
- Doubly-even dual-containing (DC) codes
- Binary subfield DC codes (polynomial codes)

Code Zoo Manifest

NAME	PARAM.	DECODER
Bacon-Shor, 3×3	$[[9, 1, 3]]$	Majority
Bacon-Shor, 5×5	$[[25, 1, 5]]$	Majority
Bacon-Shor, 7×7	$[[49, 1, 7]]$	Majority
Bacon-Shor, 9×9	$[[81, 1, 9]]$	Majority
Planar, 3×3	$[[13, 1, 3]]$	Min. Wt. Max. Match
Planar, 5×5	$[[41, 1, 5]]$	Min. Wt. Max. Match
Planar, 7×7	$[[85, 1, 7]]$	Min. Wt. Max. Match
Steane	$[[7, 1, 3]]$	Meggitt
Golay	$[[23, 1, 7]]$	Meggitt
QR47	$[[47, 1, 11]]$	Algebraic
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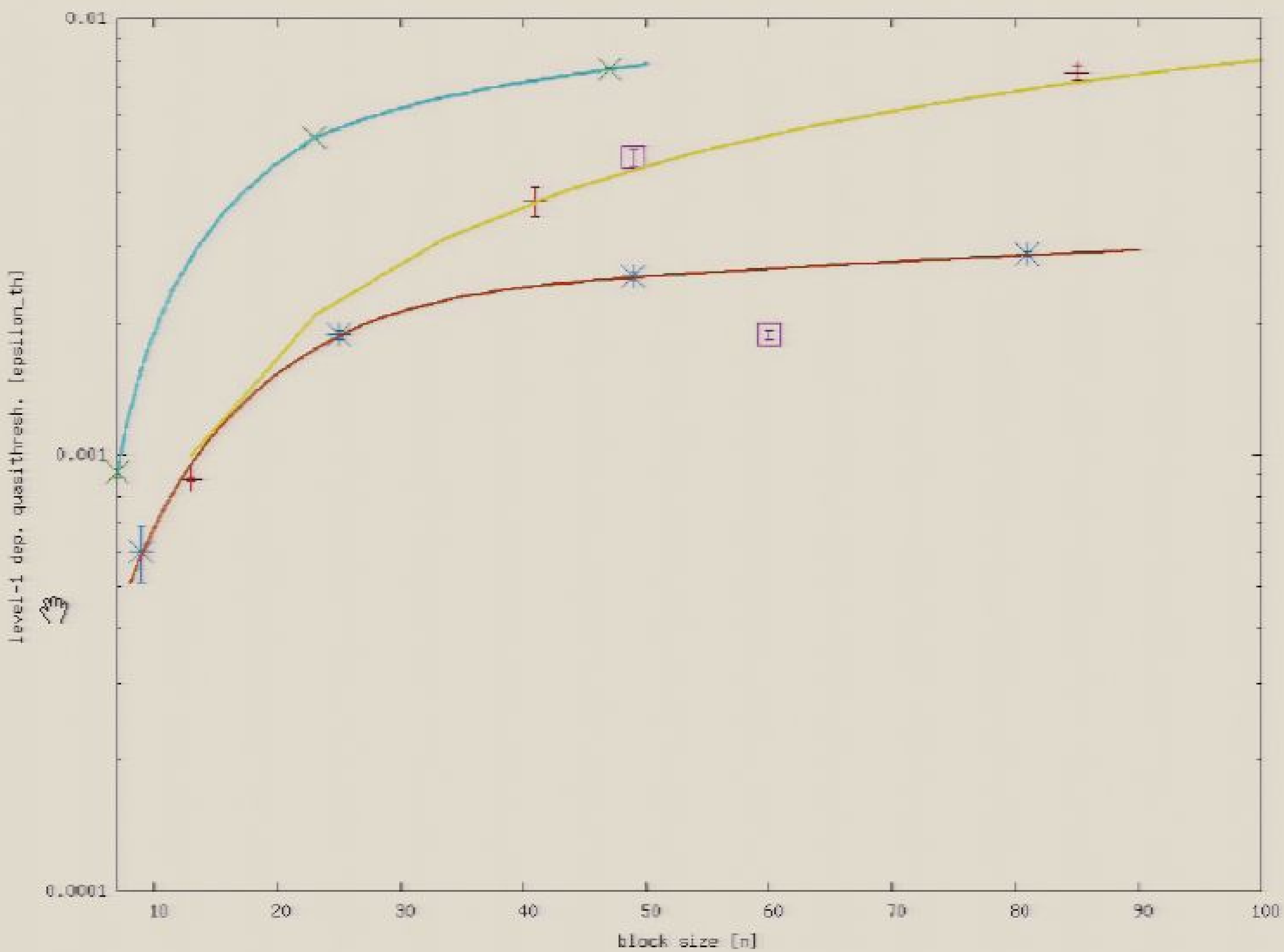
Threshold vs Block Size

Perfect Ancilla



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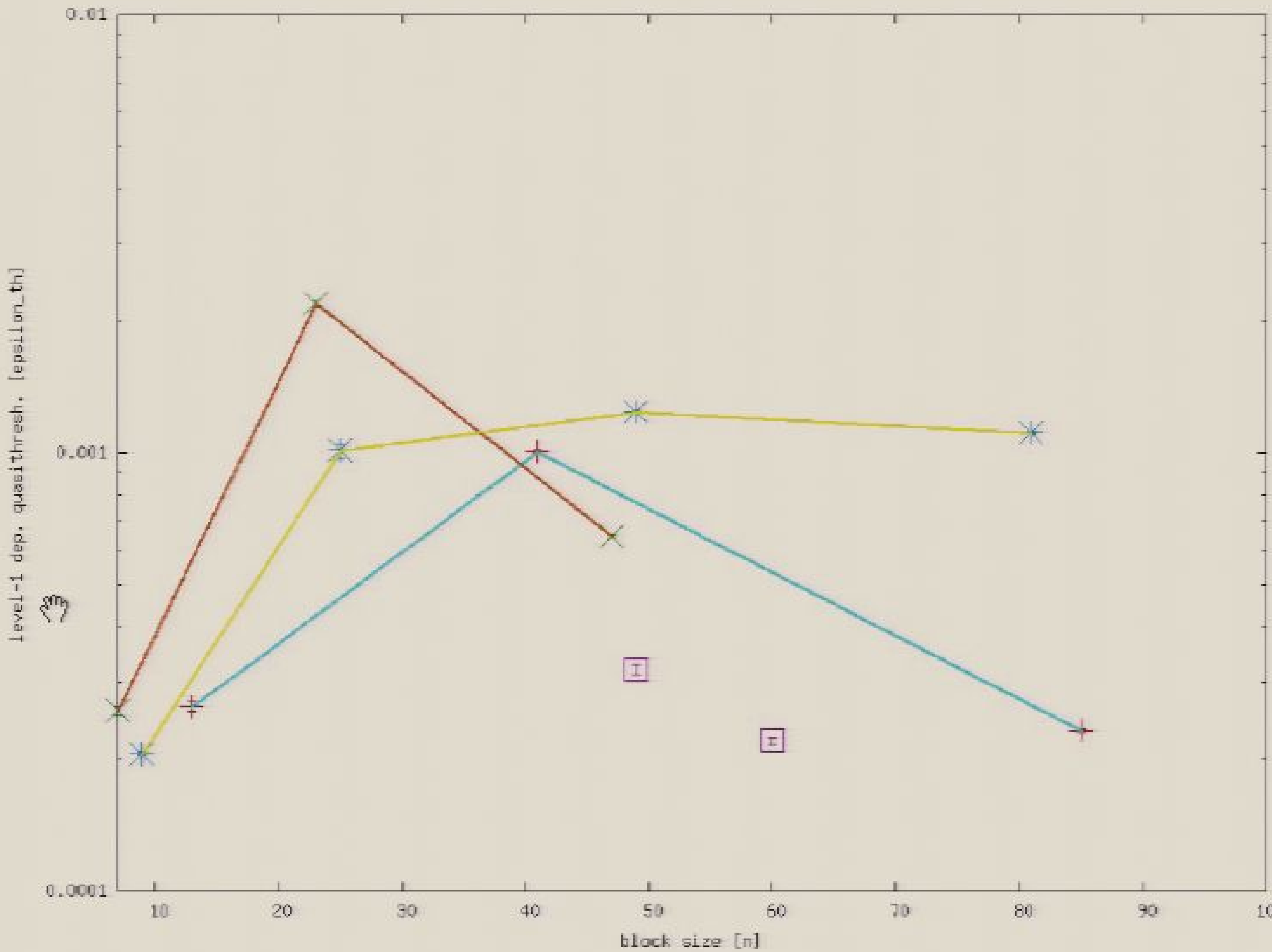

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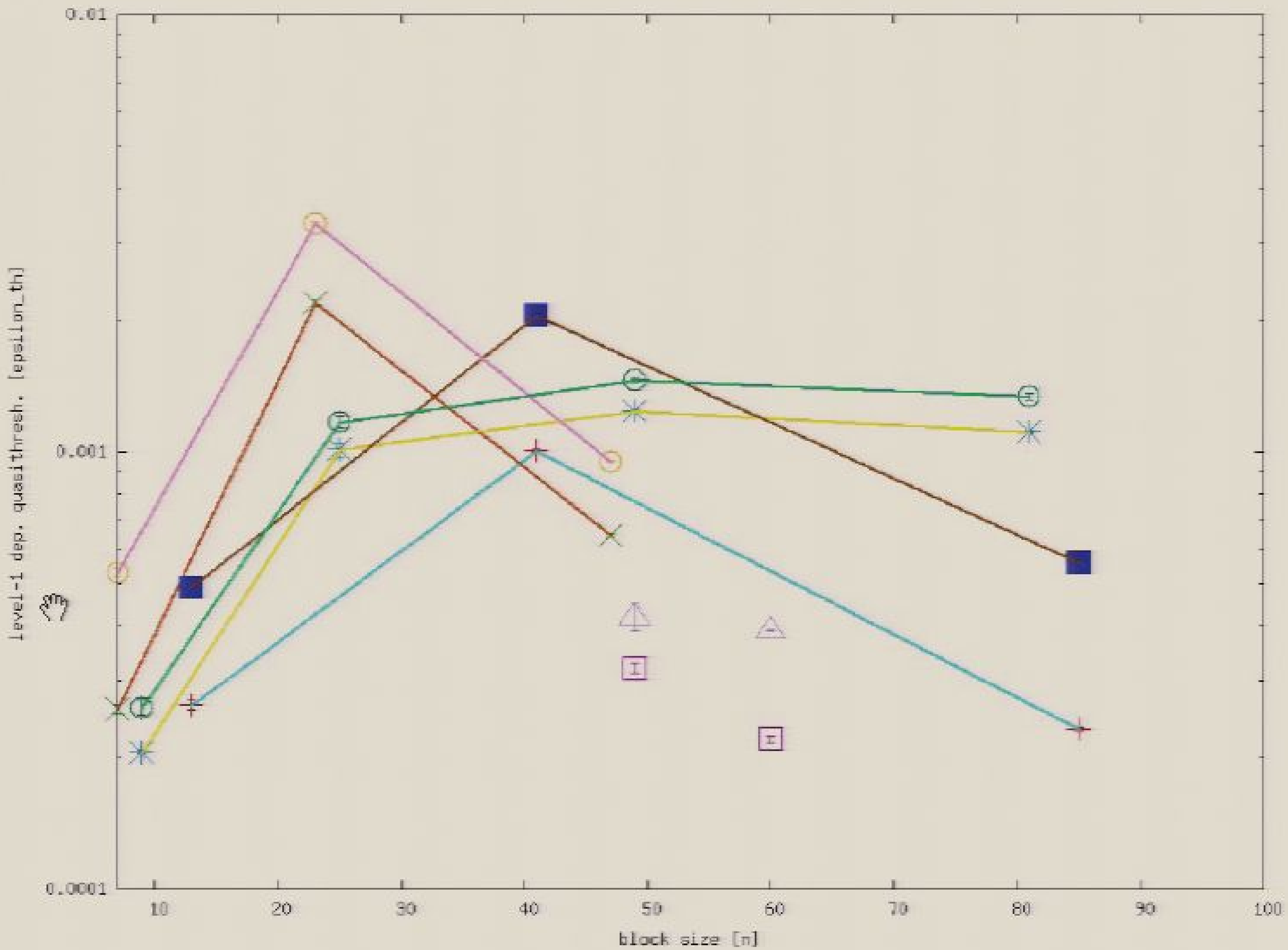

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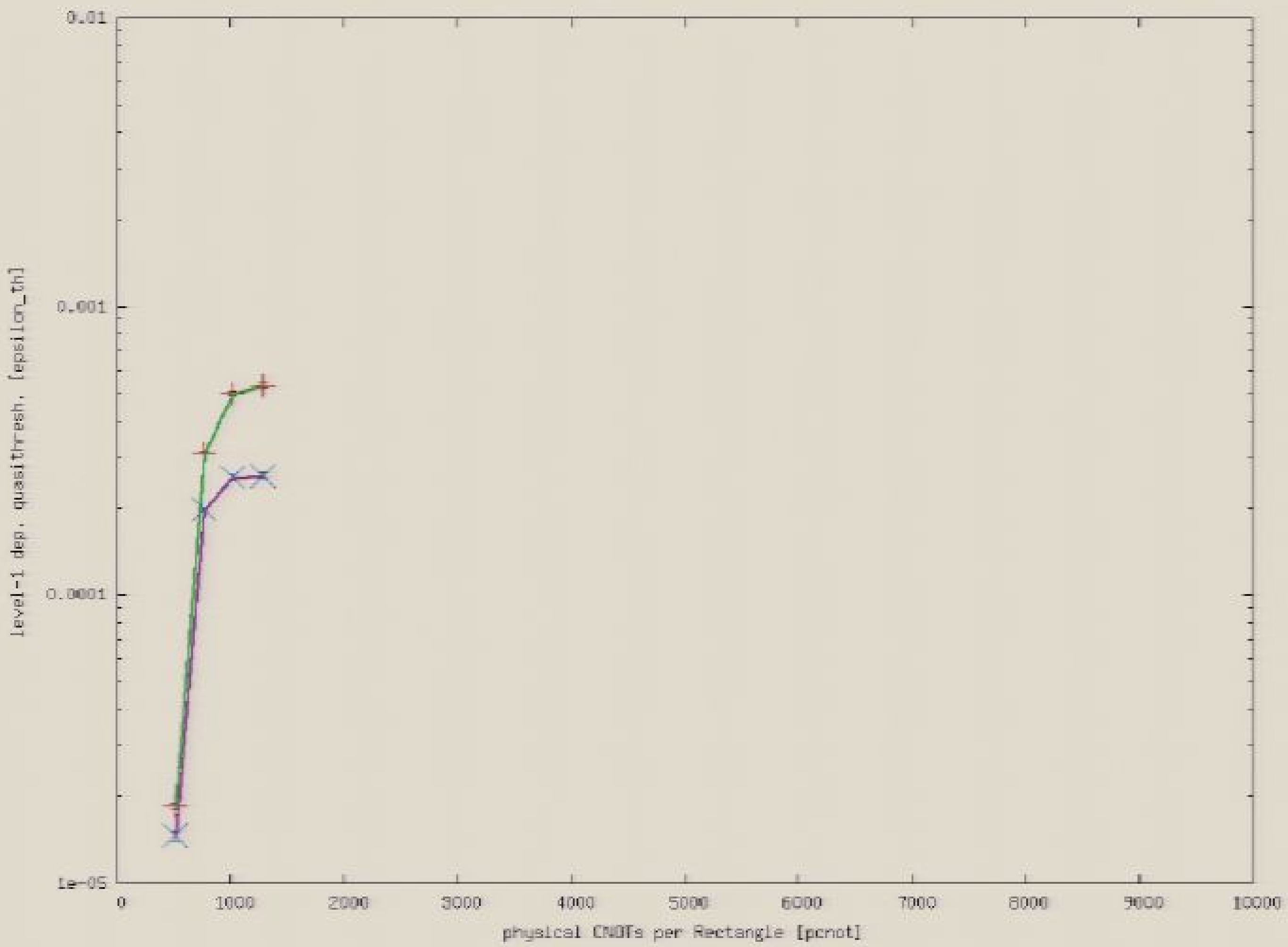

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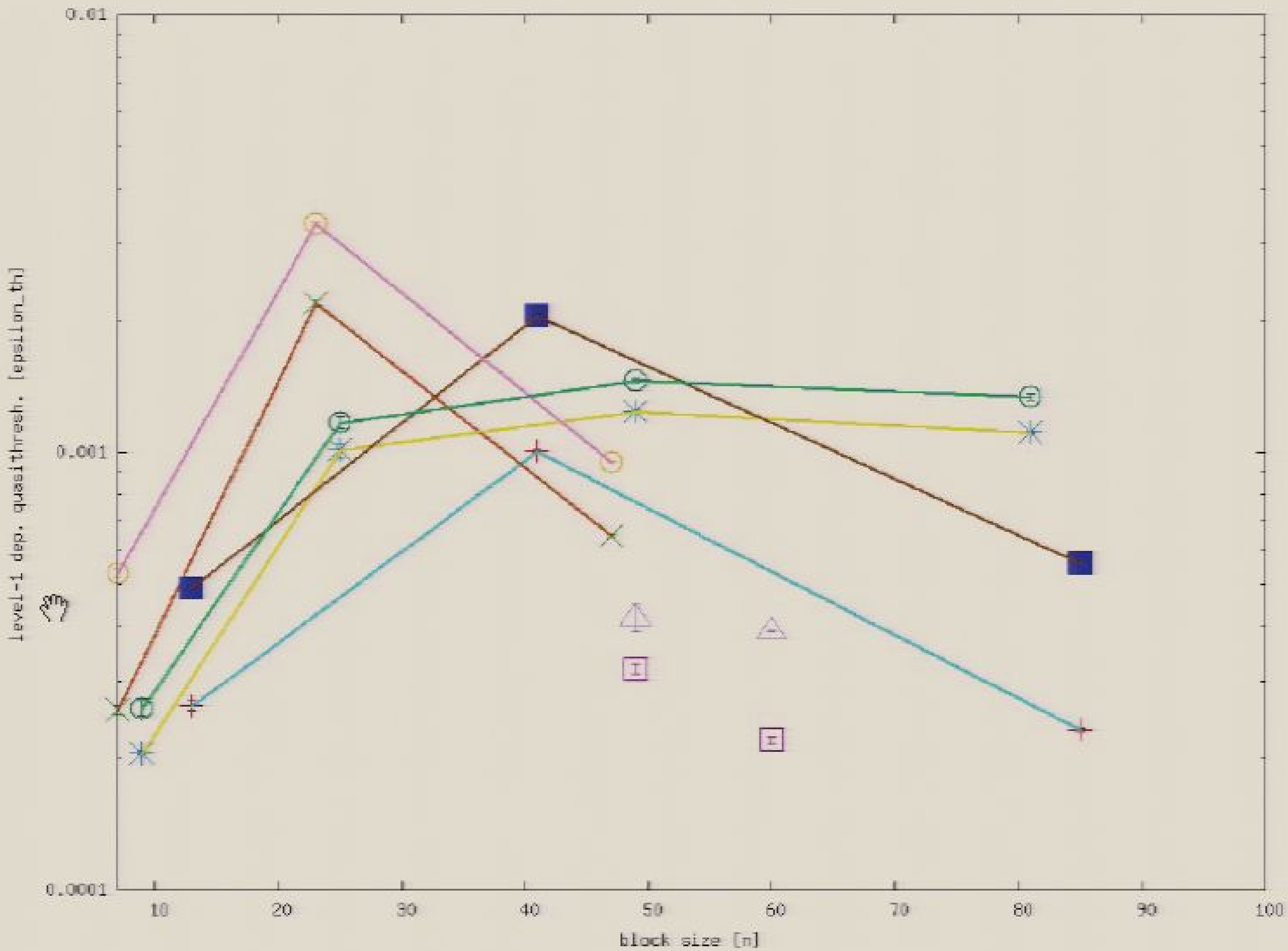

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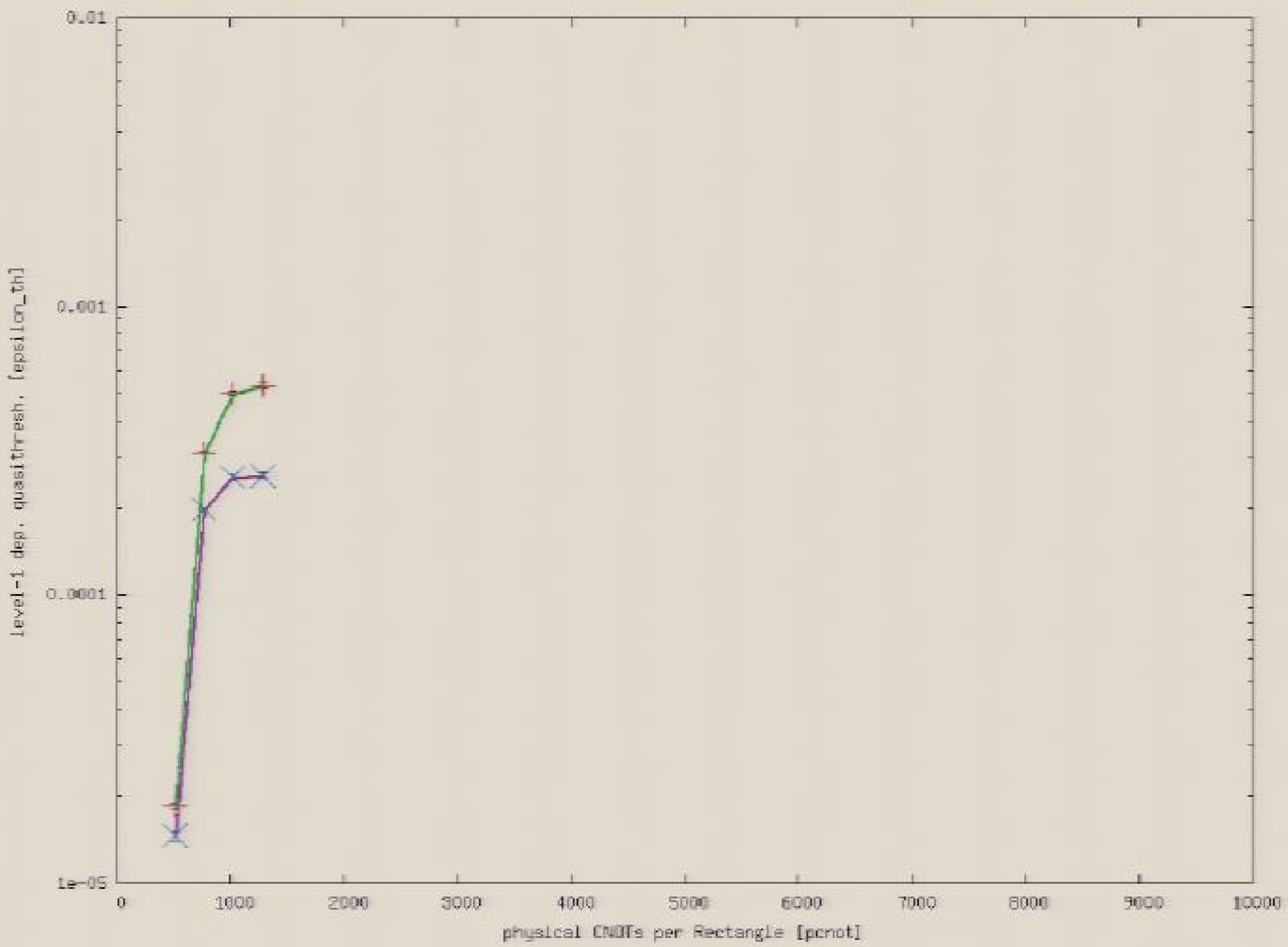

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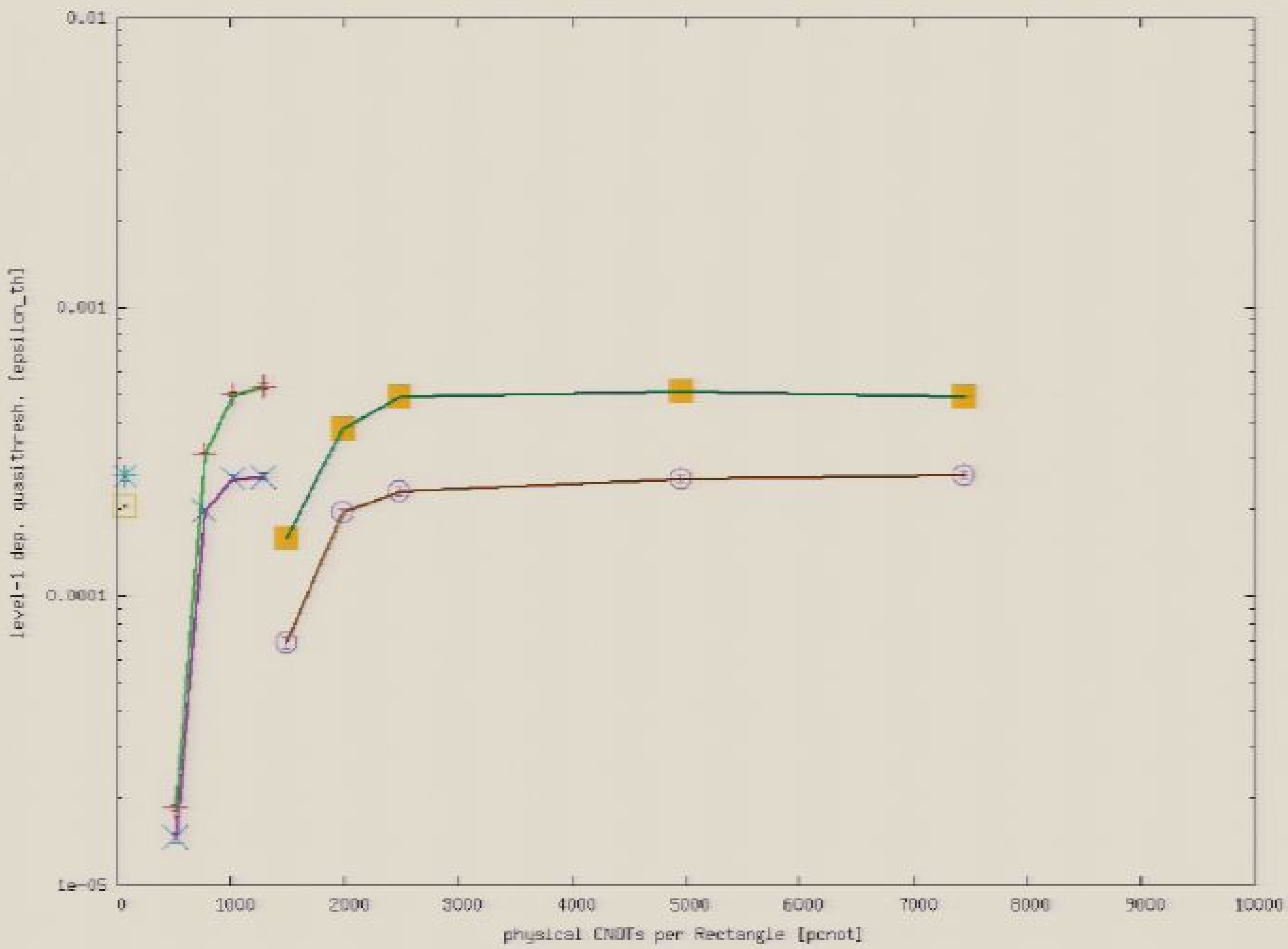

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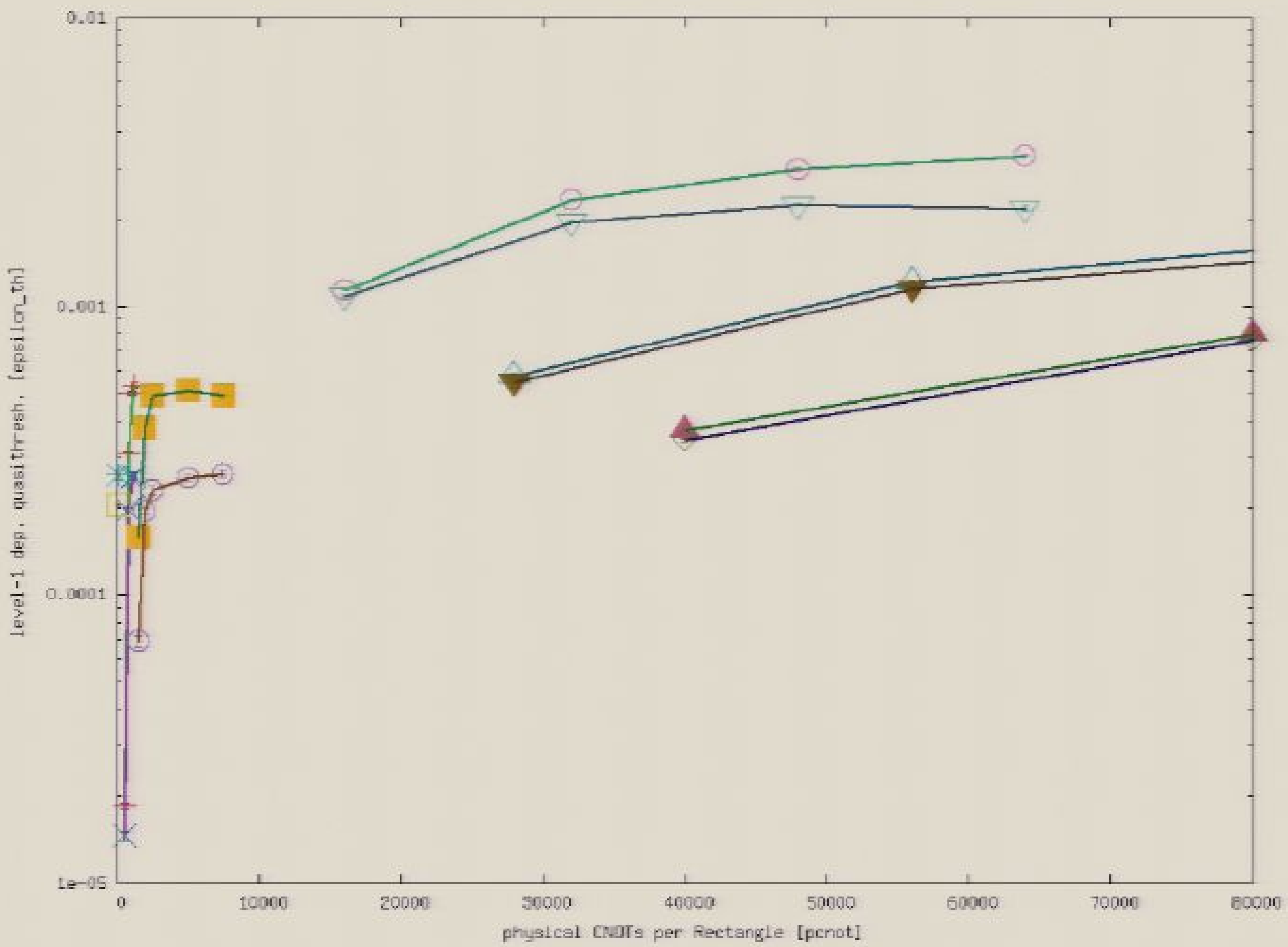

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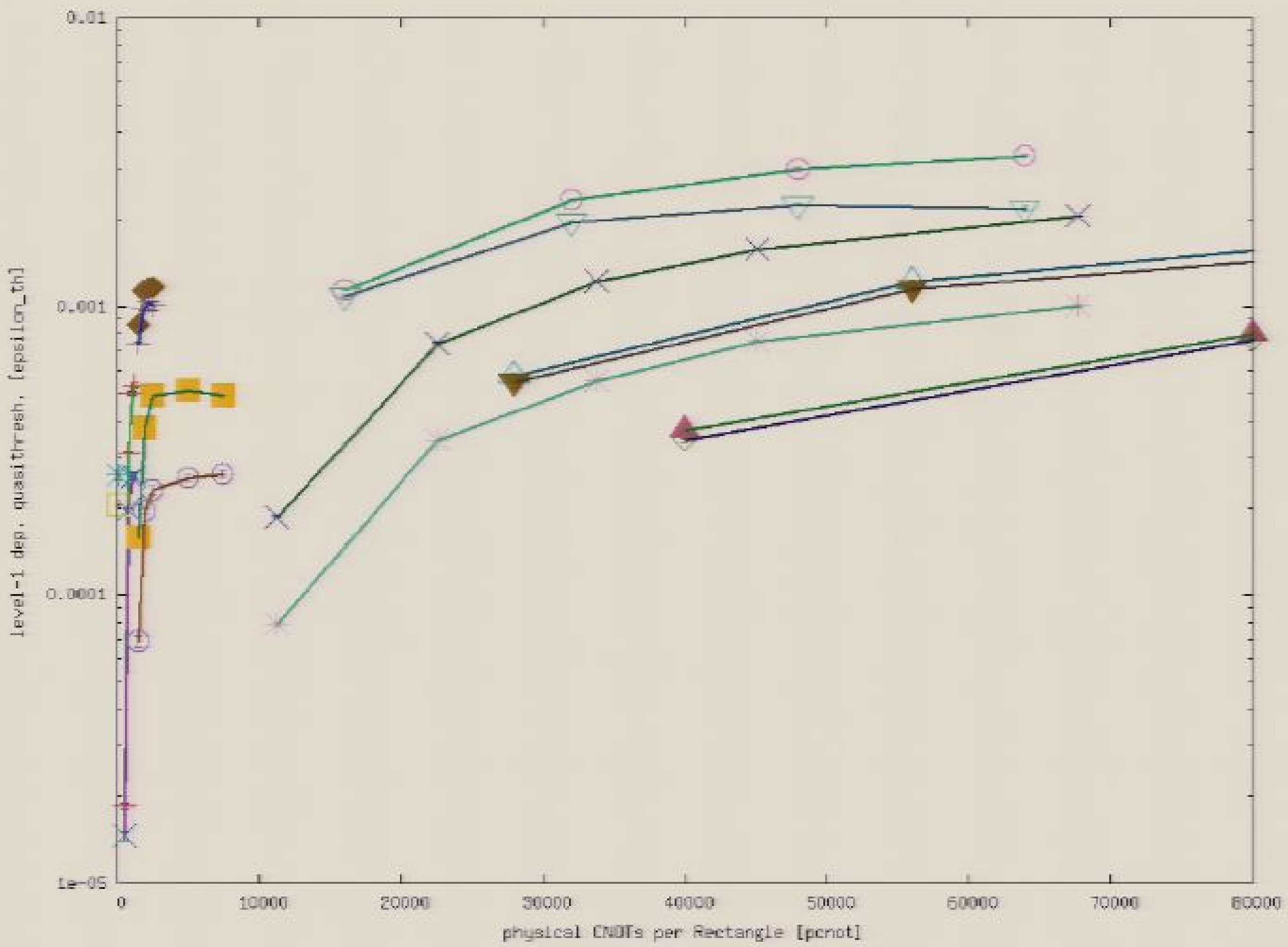

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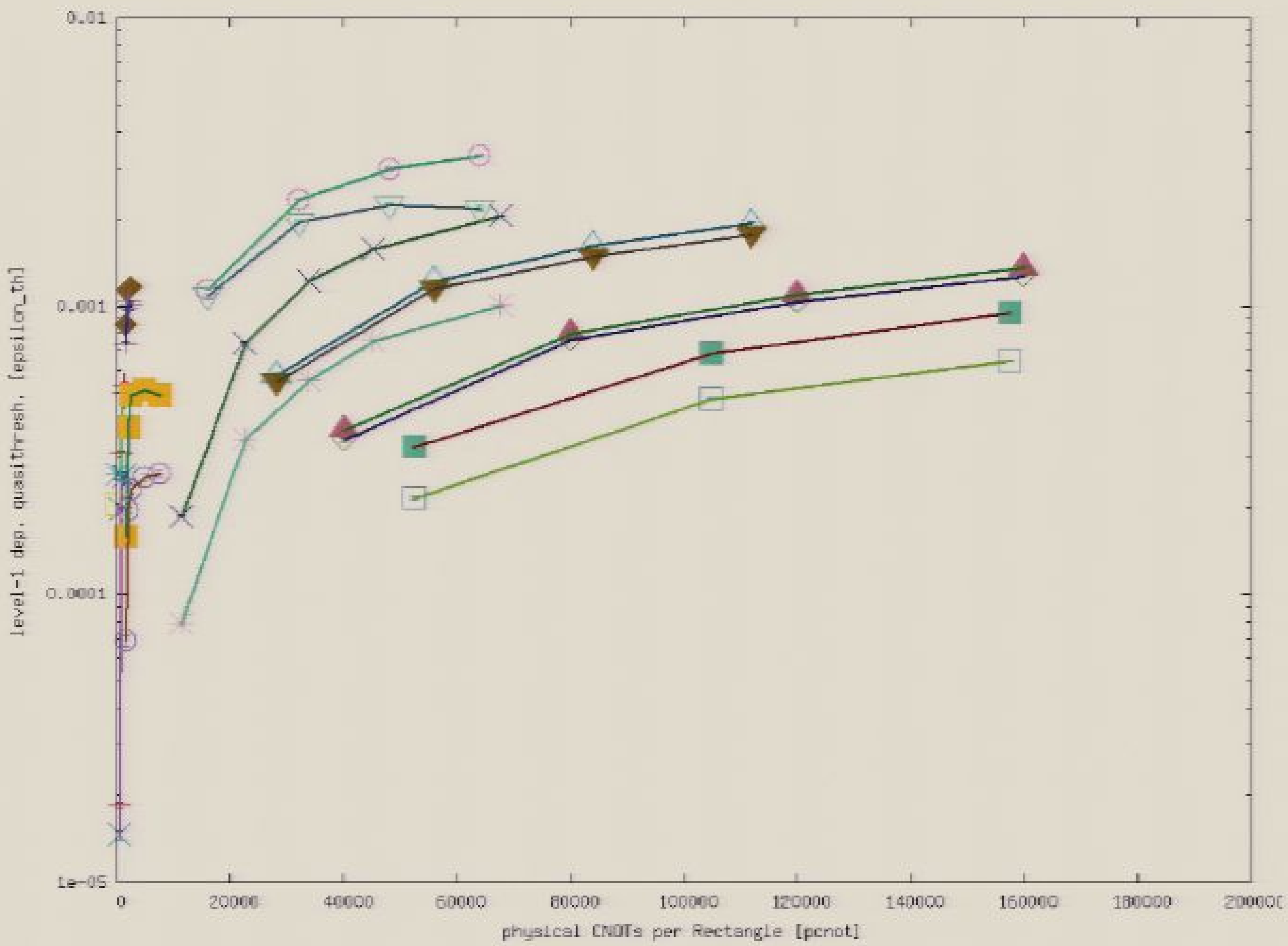

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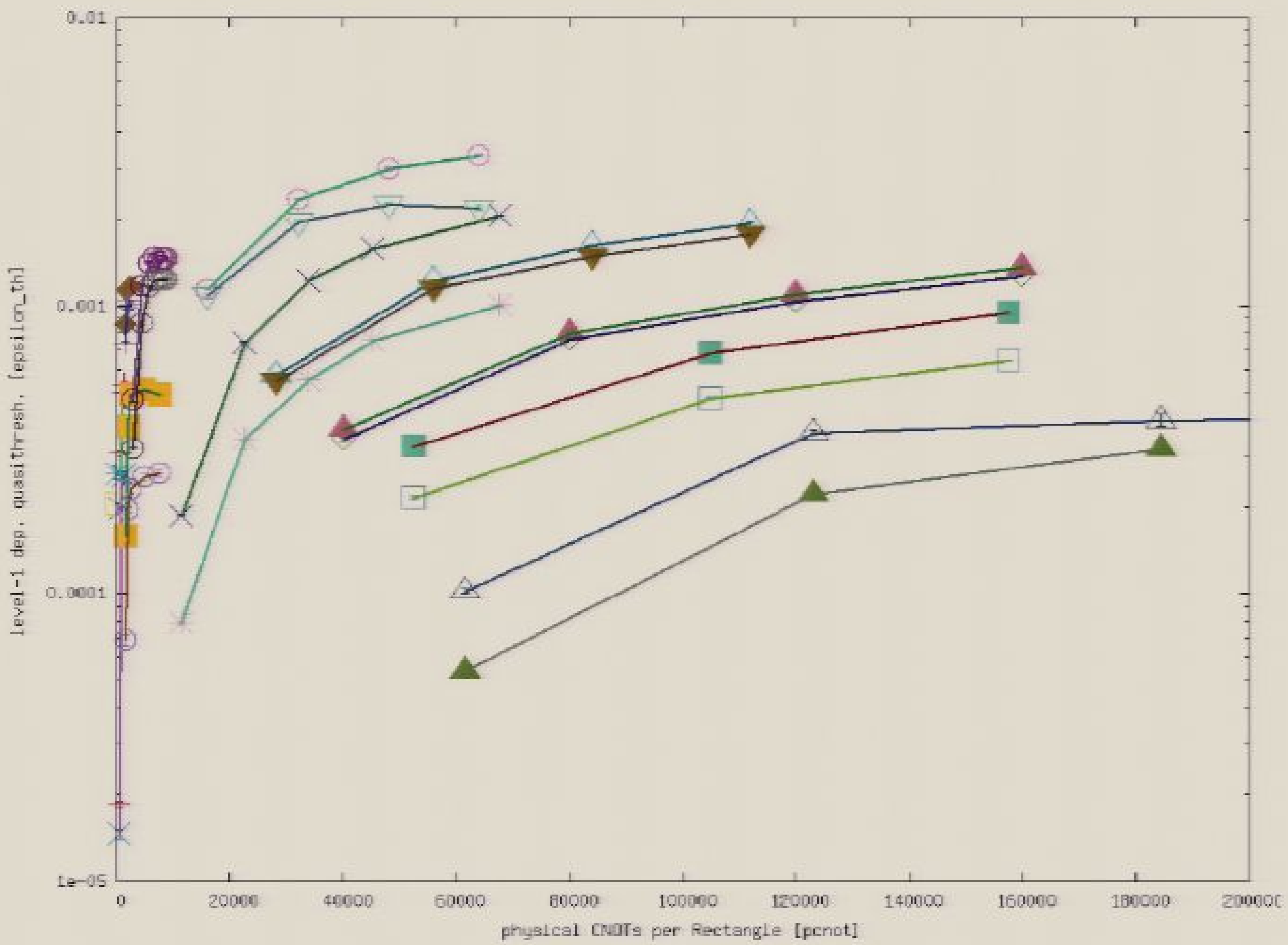

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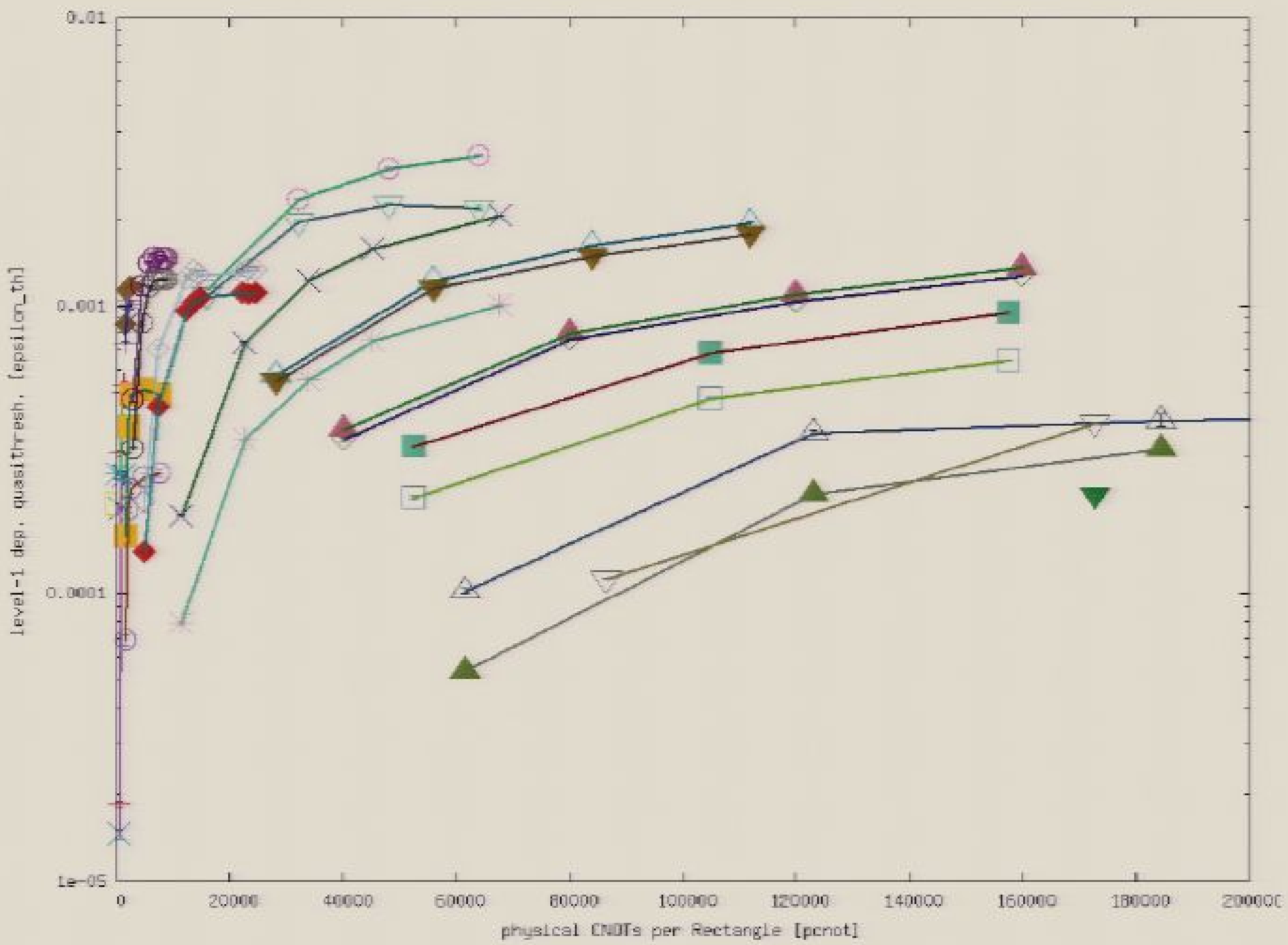

Summary

- The Golay code performs well in this survey: 3×10^{-3} depolarizing noise threshold at 3×10^{4} CNOTs/Rec.
- As Overhead increases from zero to 3×10^{4}, the following codes (in the survey) achieve highest thresholds in this setting: *Bacon-Shor* 3^2, *Steane*, *Bacon-Shor* 5^2, *Bacon-Shor* 7^2, *Golay*, . . .
- Planar 5×5 is comparable to Golay when memory locations are neglected.
- *Outlook:*
  - Failure rates p_1 at fixed p_0 vs Overhead
  - Comparison with Knill's scheme, Fibonacci scheme
  - Other levels of hierarchy: outer, intermediate, physical