

Title: Scalable quantum computer architecture for superconducting flux qubits

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URL: <http://pirsa.org/07060059>

Abstract:

Long-range coupling mechanism and architecture for superconducting flux qubits

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3: Department of Physics,
Syracuse University, Syracuse, NY, USA

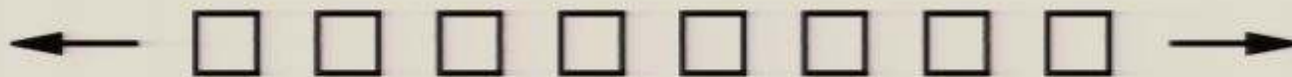
cond-mat/0702620

Overview

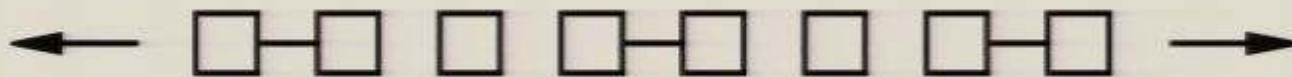
- What is scalability?
- Coupling flux qubits
- Why long-range coupling?
- Long-range coupling of flux qubits
- Interlude: error correction
- Universal set of gates
- Designing the flux qubit coupler network
- Calculating thresholds
- Conclusion and further work

Scalability

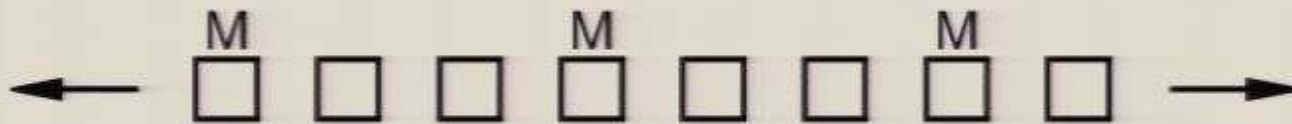
- Arbitrarily large number of qubits



- Number of simultaneous gates proportional to number of qubits



- Number of simultaneous measurements proportional to number of qubits

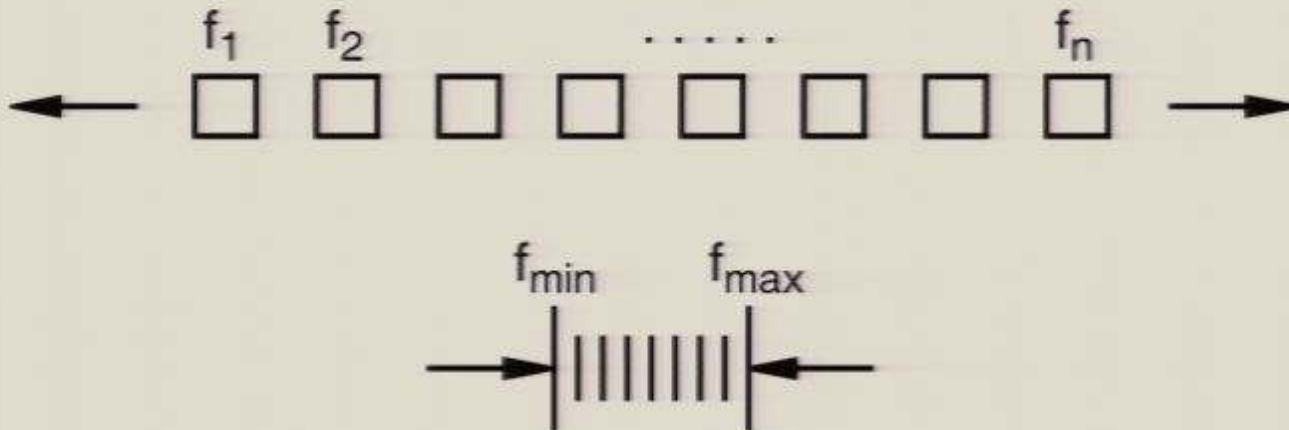


- Physics of gates and measurements independent of number of qubits



Common unscalable examples

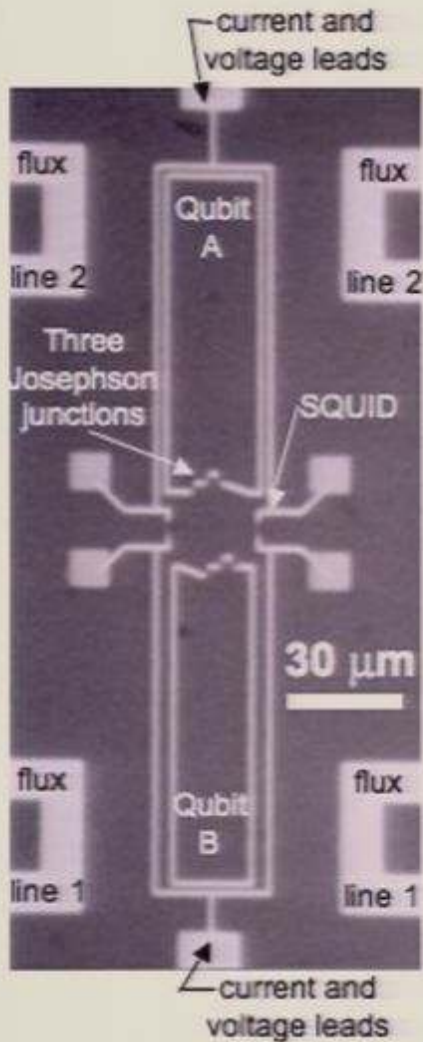
- Anything with frequency crowding



- Anything with a single shared device for gates or measurement

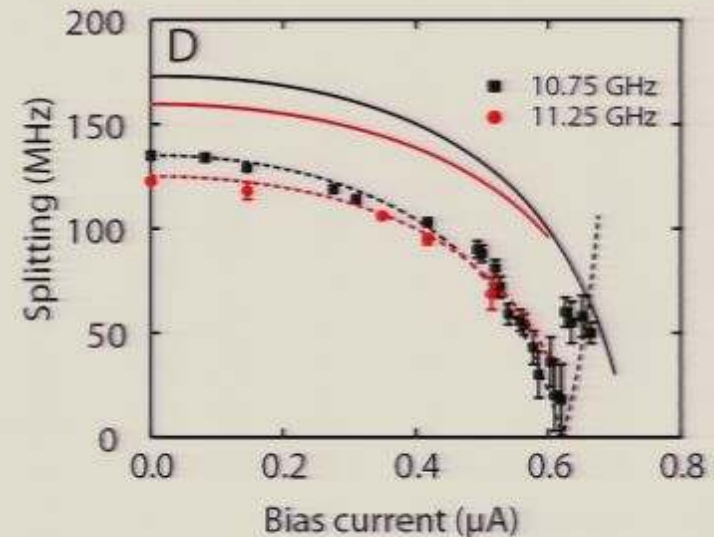
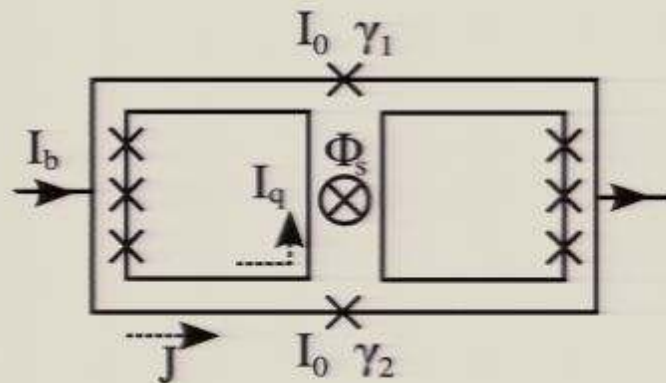


Our starting point

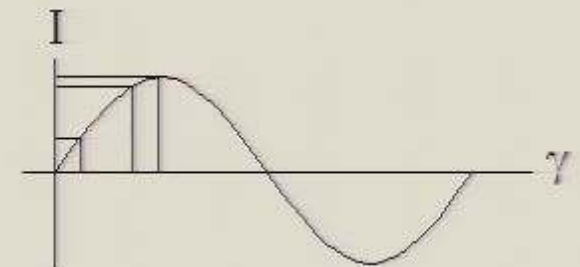


$$\frac{I_0 \gamma}{\times}$$

$$I = I_0 \sin \gamma$$



- $I_b = I_0 \sin \gamma_1 + I_0 \sin \gamma_2$
- $2J = I_0 \sin \gamma_2 - I_0 \sin \gamma_1$
- $\gamma_1 - \gamma_2 + \frac{2\pi}{\Phi_0} (\Phi_s - LJ) = 0$



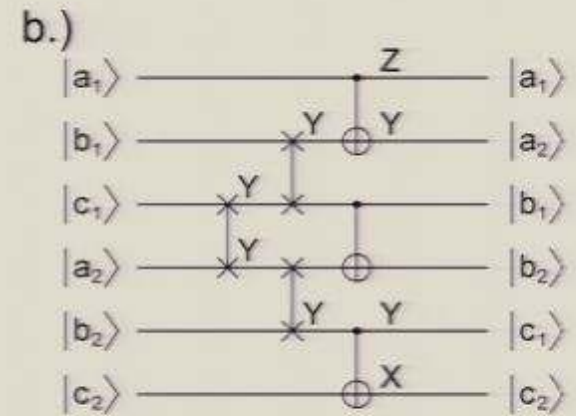
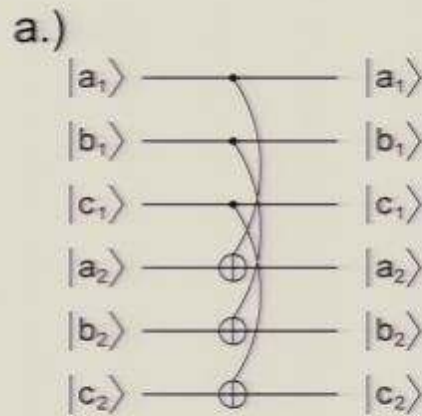
Shortcomings

- Need long-range interactions
- Thresholds
 - unlimited range, unlimited qubits: $\sim 10^{-2}$
Knill, quant-ph/0410199
 - unlimited range, many qubits: $\sim 10^{-3}$ – 10^{-4}
Steane, Phys. Rev. A 68, 042322 (2003)
 - 2D lattice, nearest neighbor: $\sim 10^{-5}$
Svore, QIC 7, 297 (2007)
 - bilinear nearest neighbor: $\sim 10^{-6}$
Stephens, quant-ph/0702201
 - linear nearest neighbor: $\sim 10^{-8}$
Stephens, in preparation

Why long-range coupling?

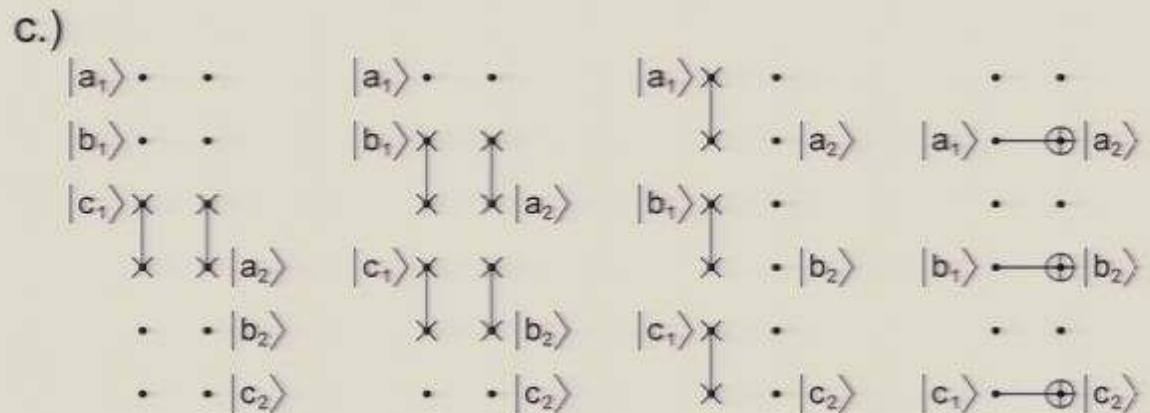
- Case a.)

- qubits: $2n$
- gates: n
- idle: 0
- depth: 1

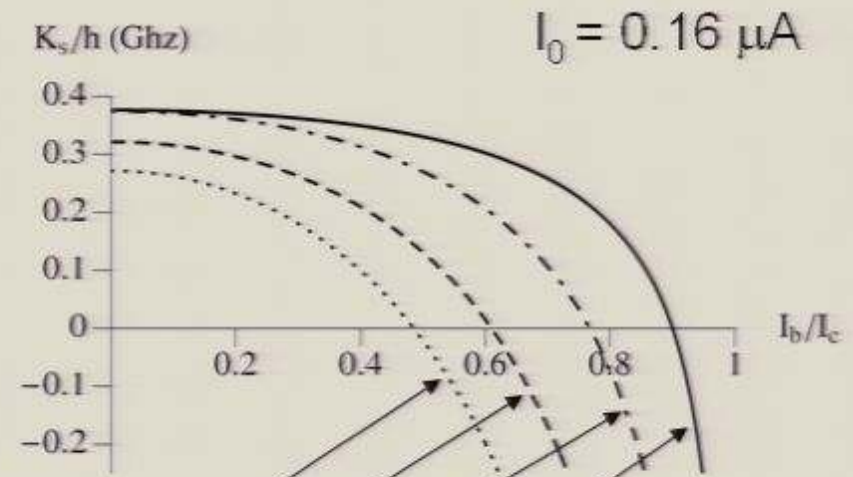
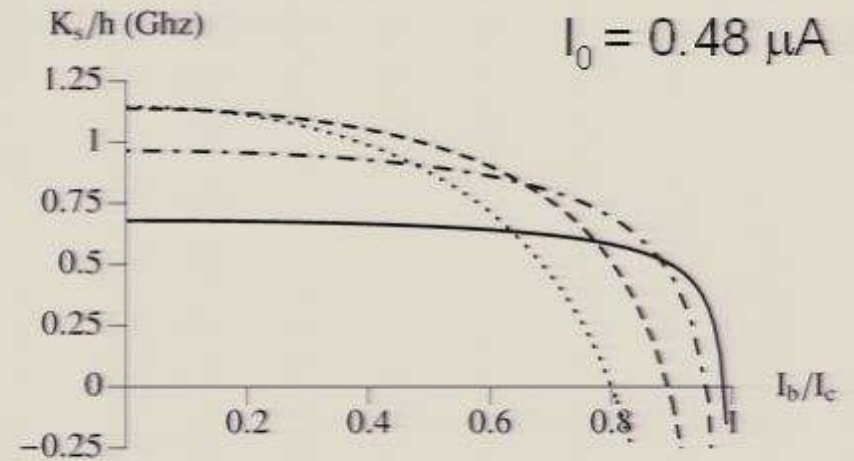
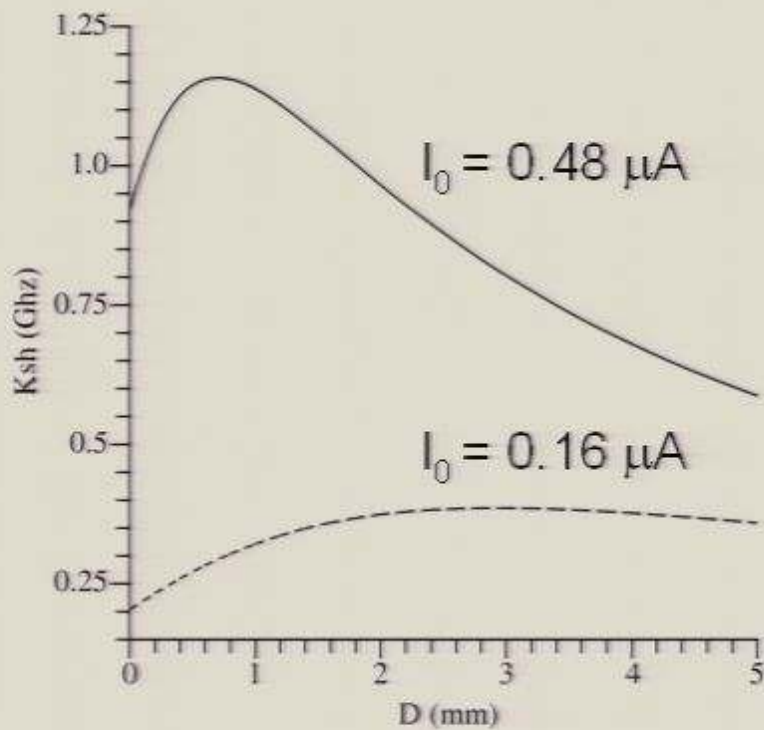
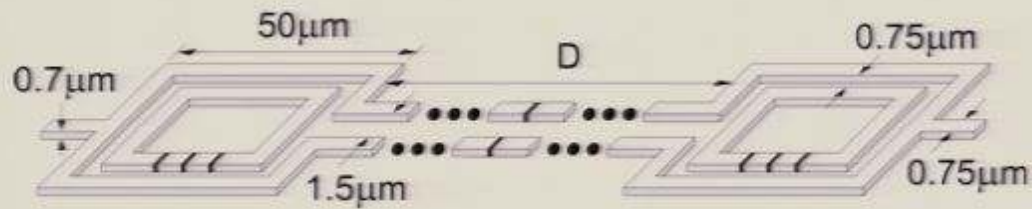


- Case c.)

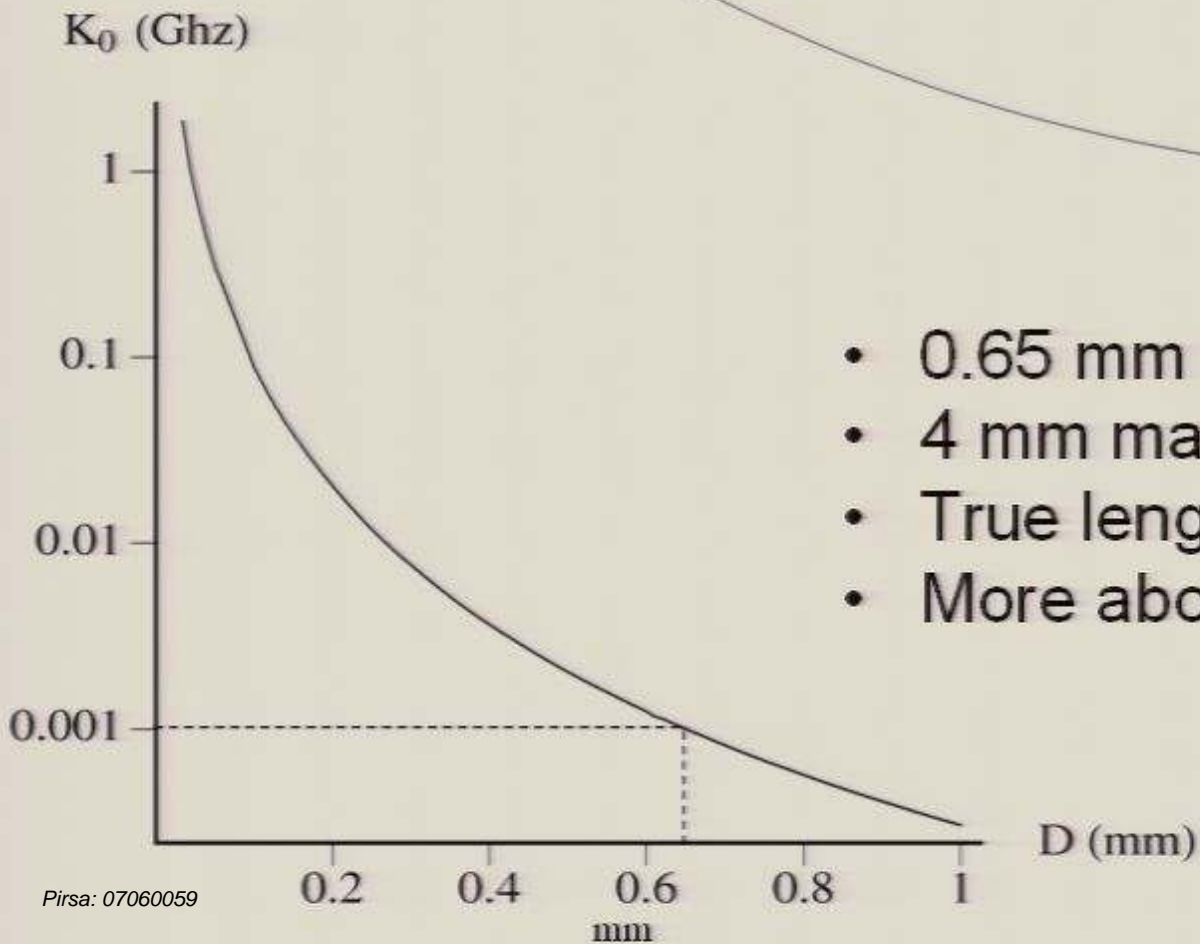
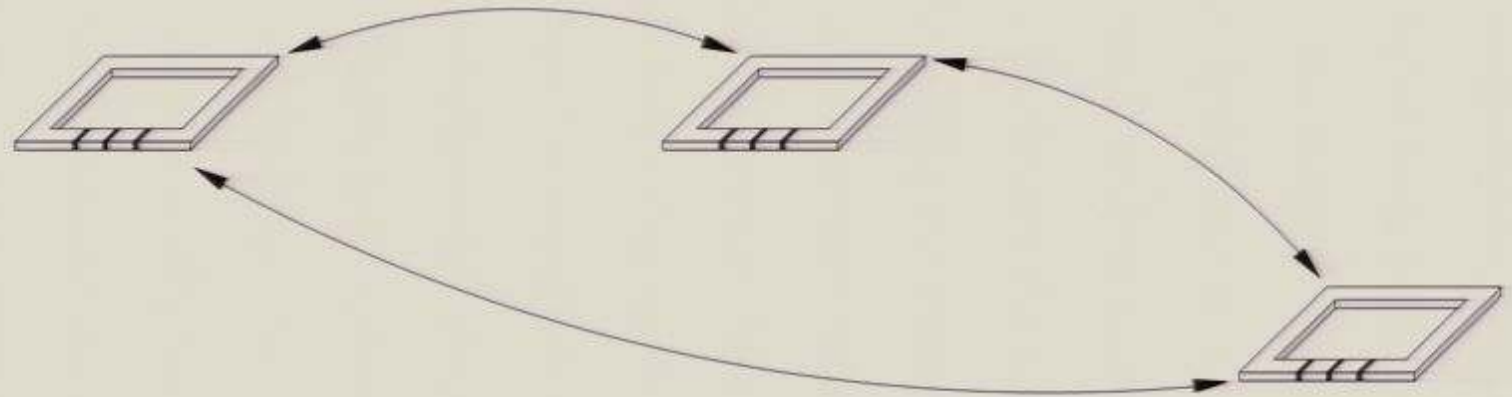
- qubits: $4n$
- gates: $2n^2 + n$
- idle: $2n^2$
- depth: $2n + 1$



Extending the coupler



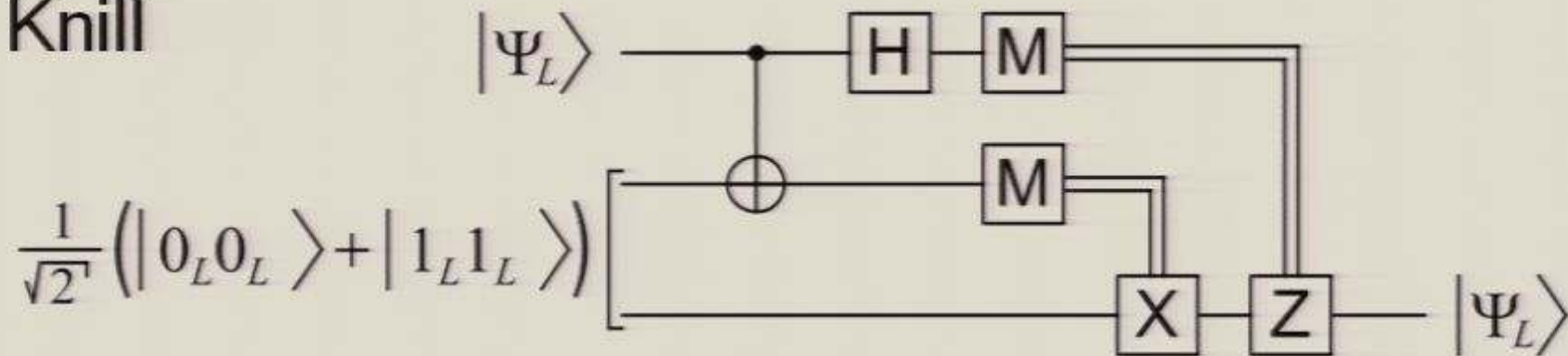
True coupler length: crosstalk



- 0.65 mm minimum separation
- 4 mm maximum separation
- True length: 5-6 qubits
- More about this later...

Interlude: error correction

- Knill



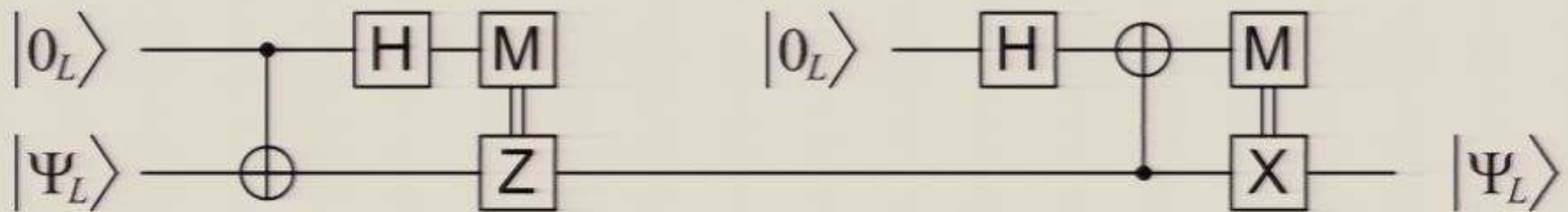
$$|0_L\rangle = \frac{1}{\sqrt{8}} (|00000000\rangle + |10101010\rangle + |01100111\rangle + |11001110\rangle + |00011111\rangle + |10110101\rangle + |01111100\rangle + |11010001\rangle)$$

$$|1_L\rangle = \frac{1}{\sqrt{8}} (|11111111\rangle + |01010101\rangle + |10011100\rangle + |00110001\rangle + |11100000\rangle + |01001010\rangle + |10000111\rangle + |00101110\rangle)$$

- Best approach for high threshold $\sim 10^{-2}$

Interlude: error correction

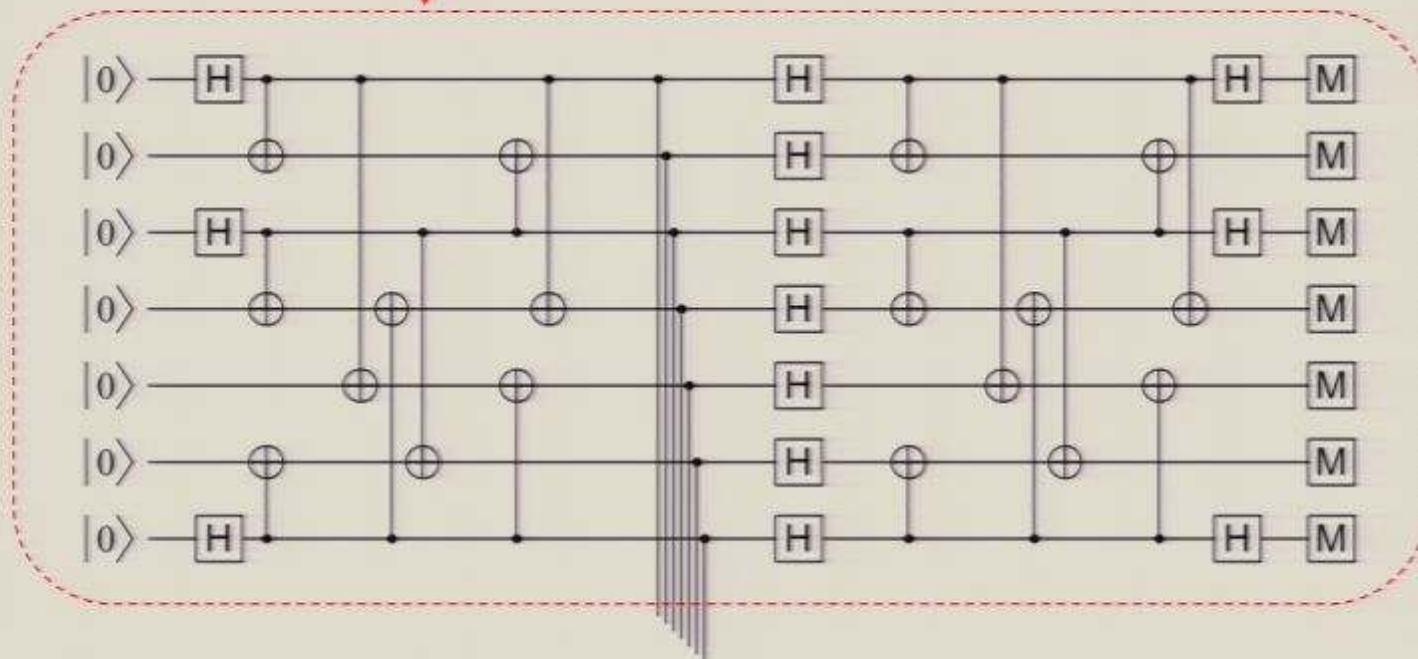
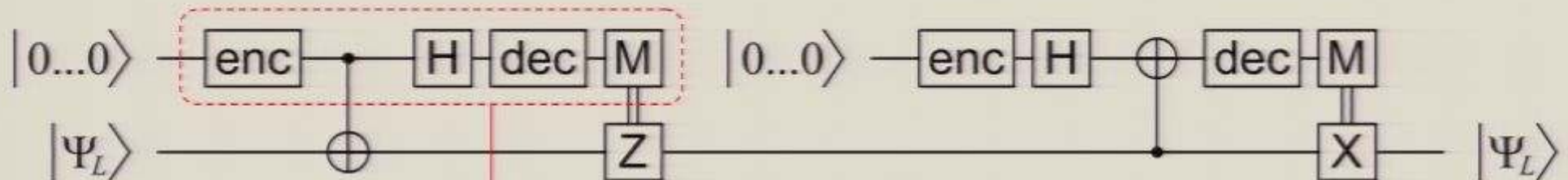
- Steane: threshold $\sim 10^{-3}$ to 10^{-4}



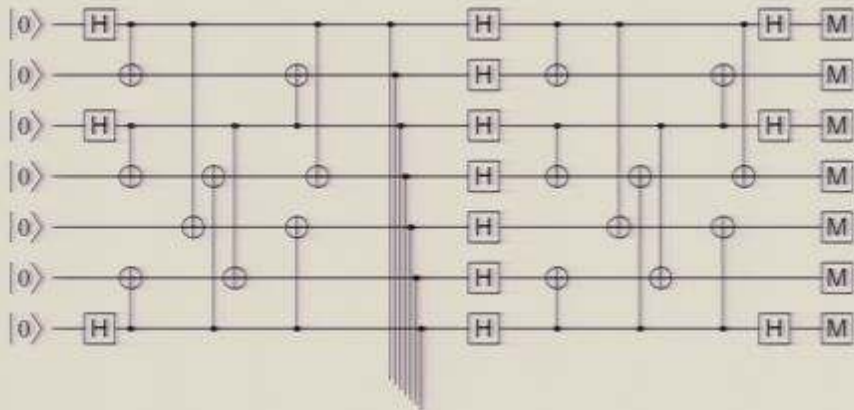
- Still need to repeat logical state preparation
- Still need to repeat measurements
- Still need large ancilla factories
- Still need very long-range interactions

Interlude: error correction

- Steane/DiVincenzo PRL 98, 020501 (2007)

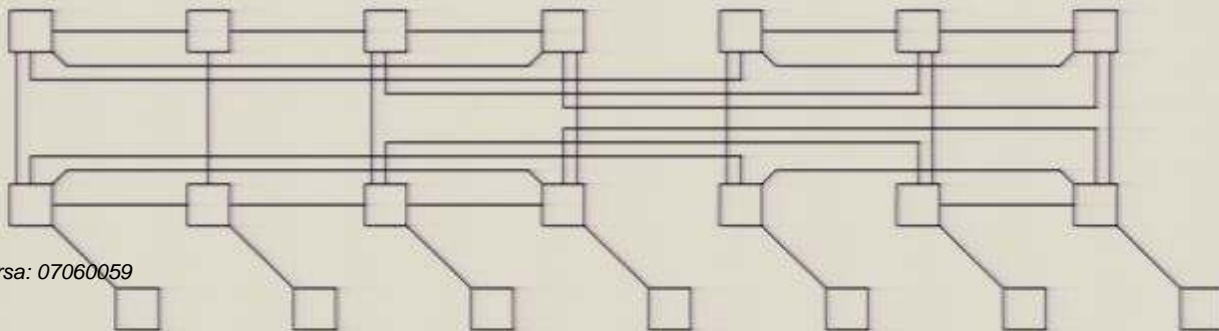
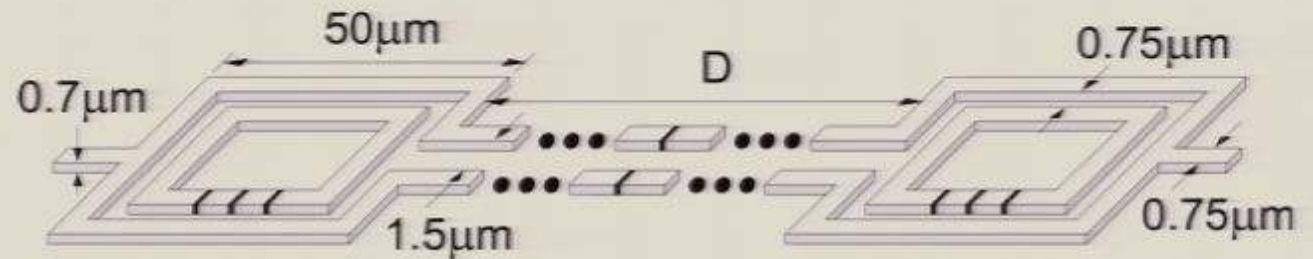


Laying out the circuitry



- Error correction

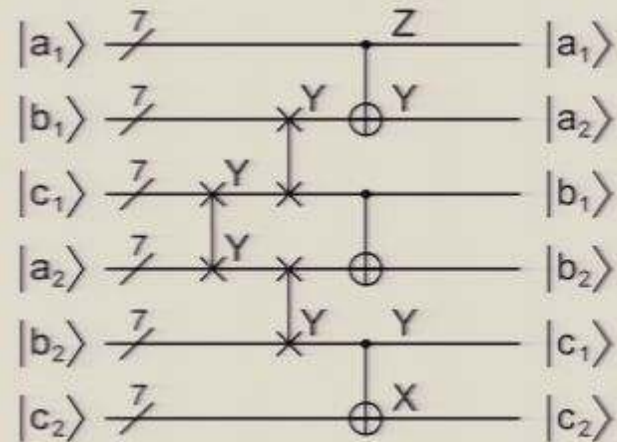
- Coupler



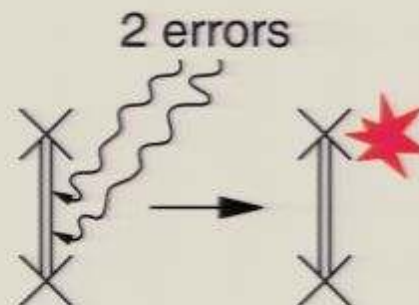
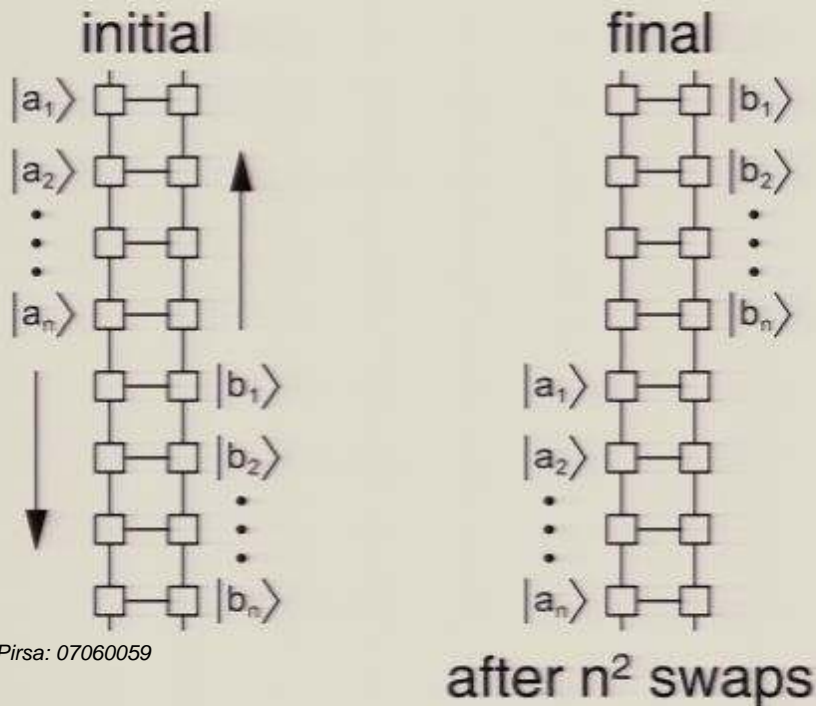
- Network

Level 2 circuitry

- Must still avoid linear nearest neighbor

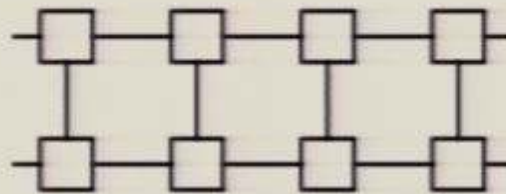


- Need logical bilinear network
- Permits fault-tolerant swap

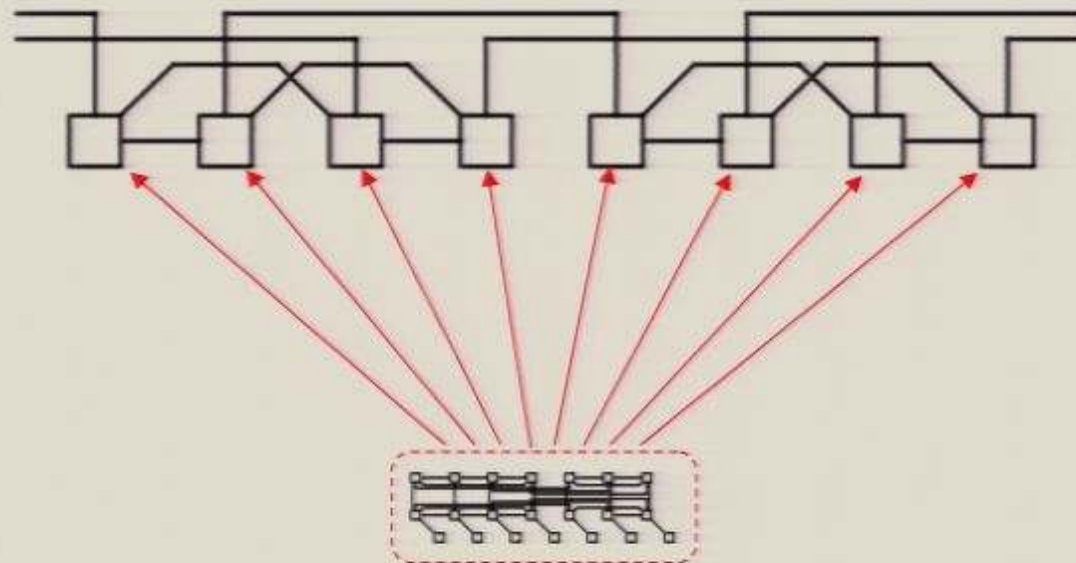


Level 2 circuitry

- Can't do logical bilinear directly

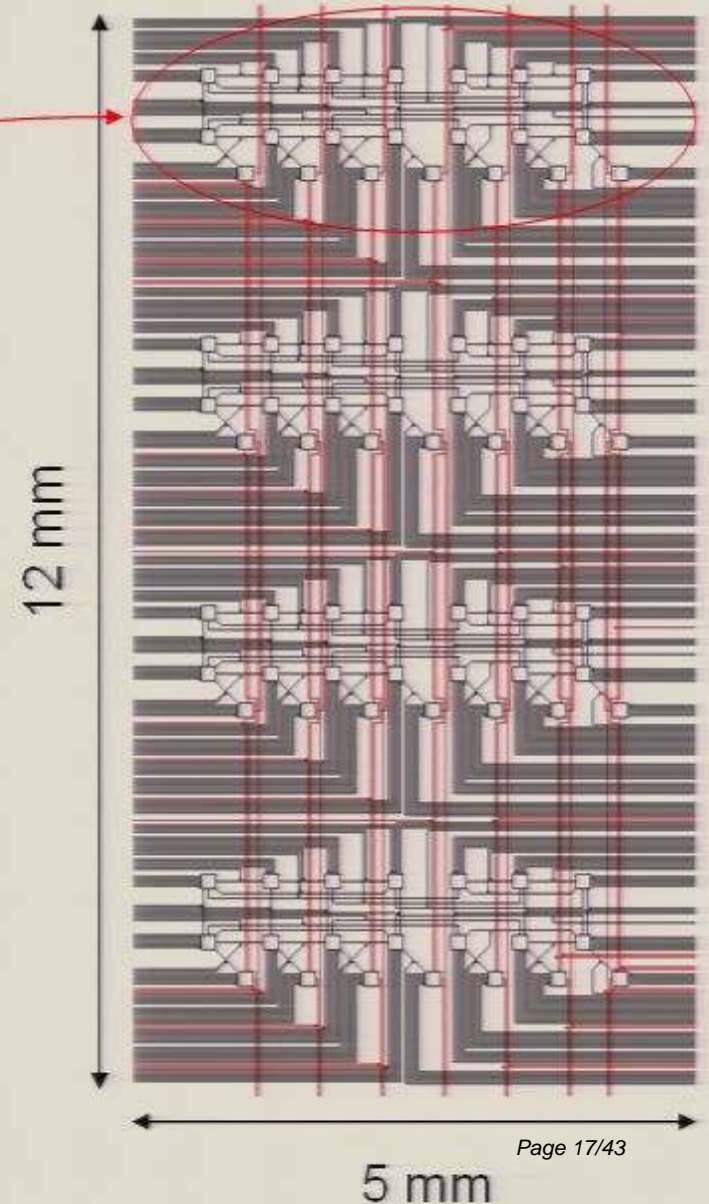
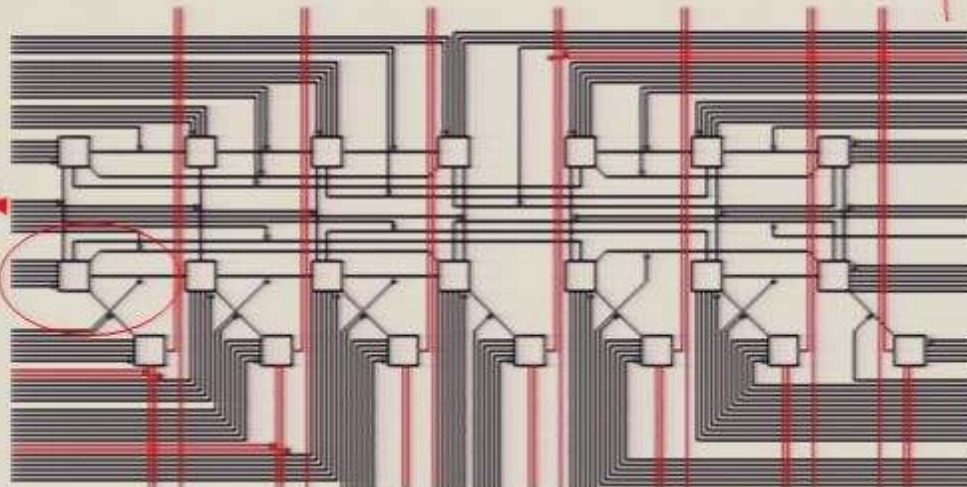
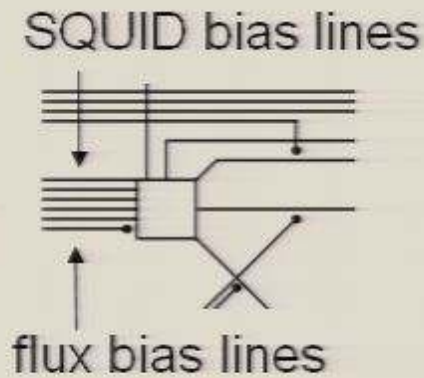


- Need to stretch the design



Level 2 circuitry

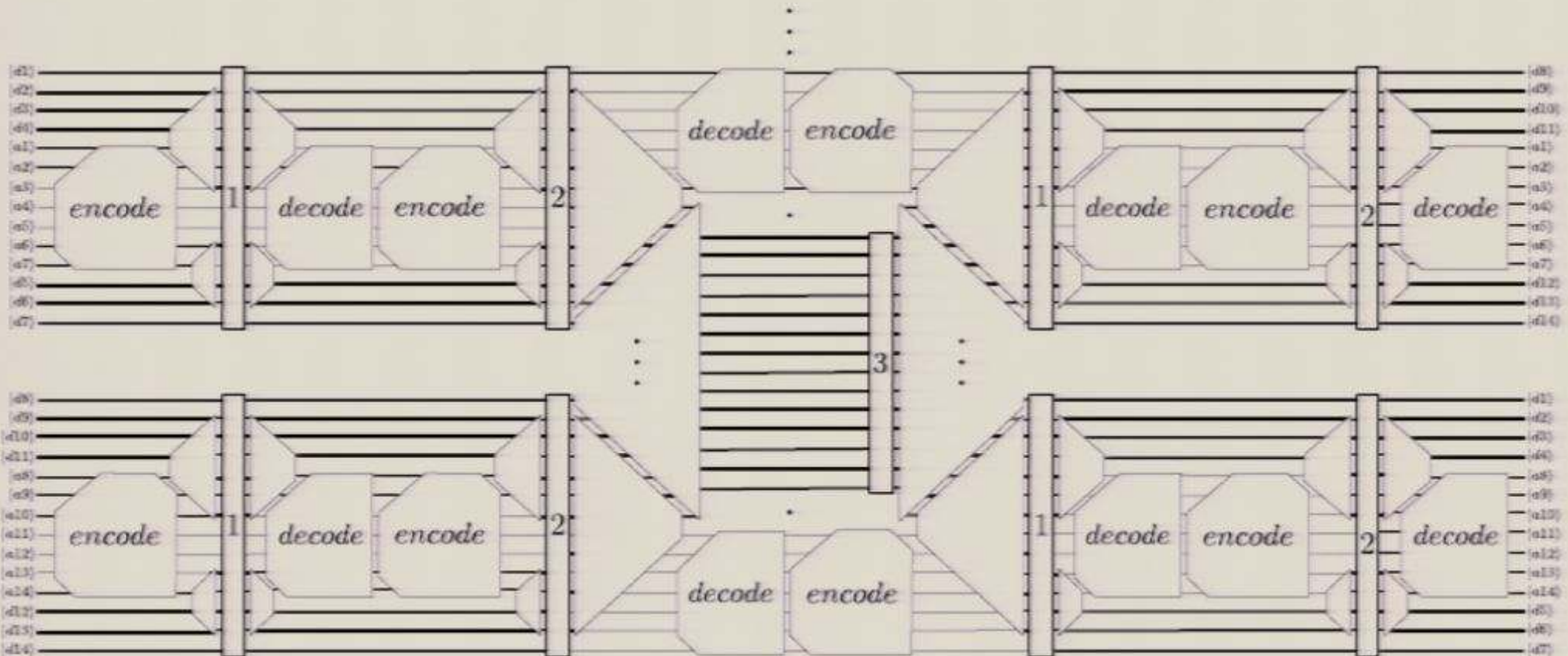
- Must control each qubit



- 1 wire per $40 \mu\text{m}$

Level 3 circuitry

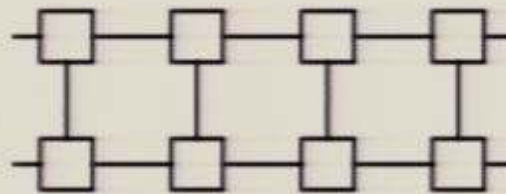
- Linear nearest neighbor with fault-tolerant swap



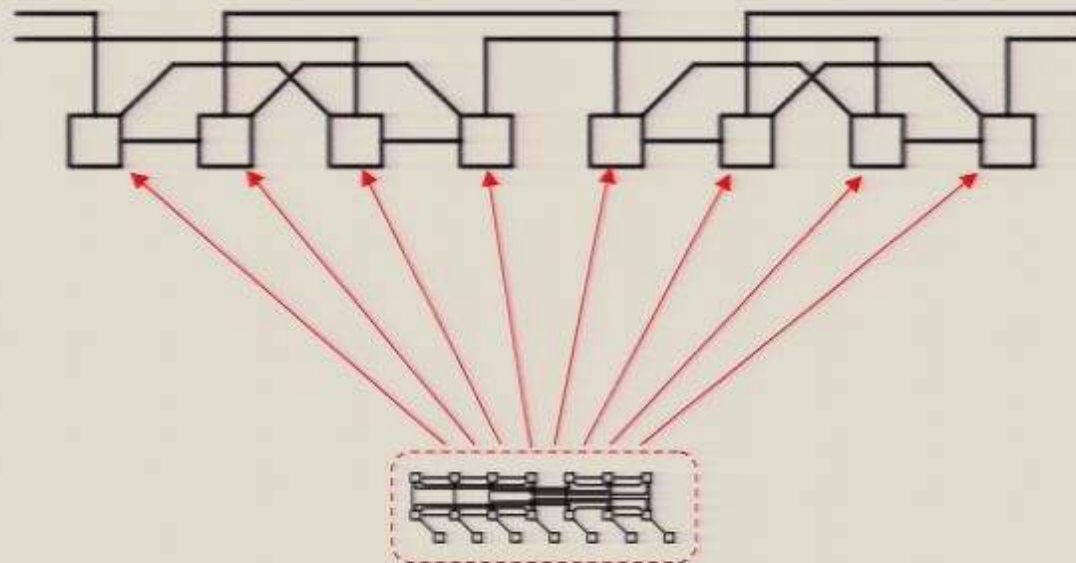
- Each horizontal line represents 2×21^2 qubits

Level 2 circuitry

- Can't do logical bilinear directly

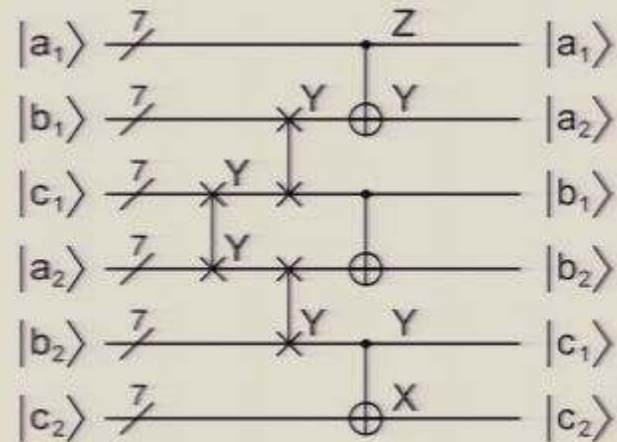


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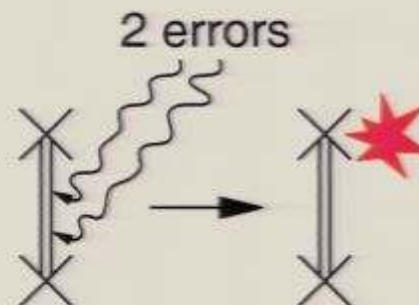
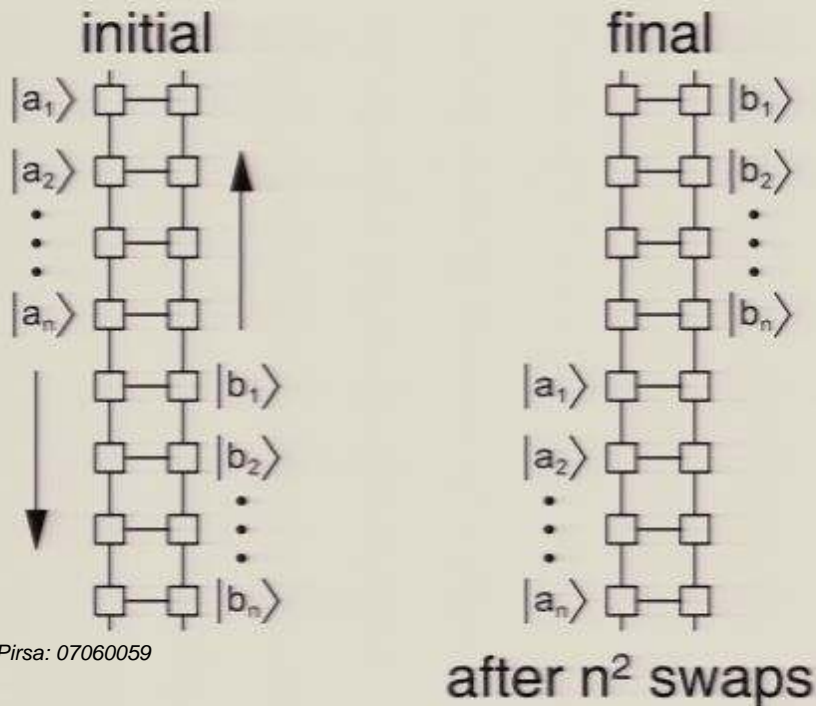


Level 2 circuitry

- Must still avoid linear nearest neighbor

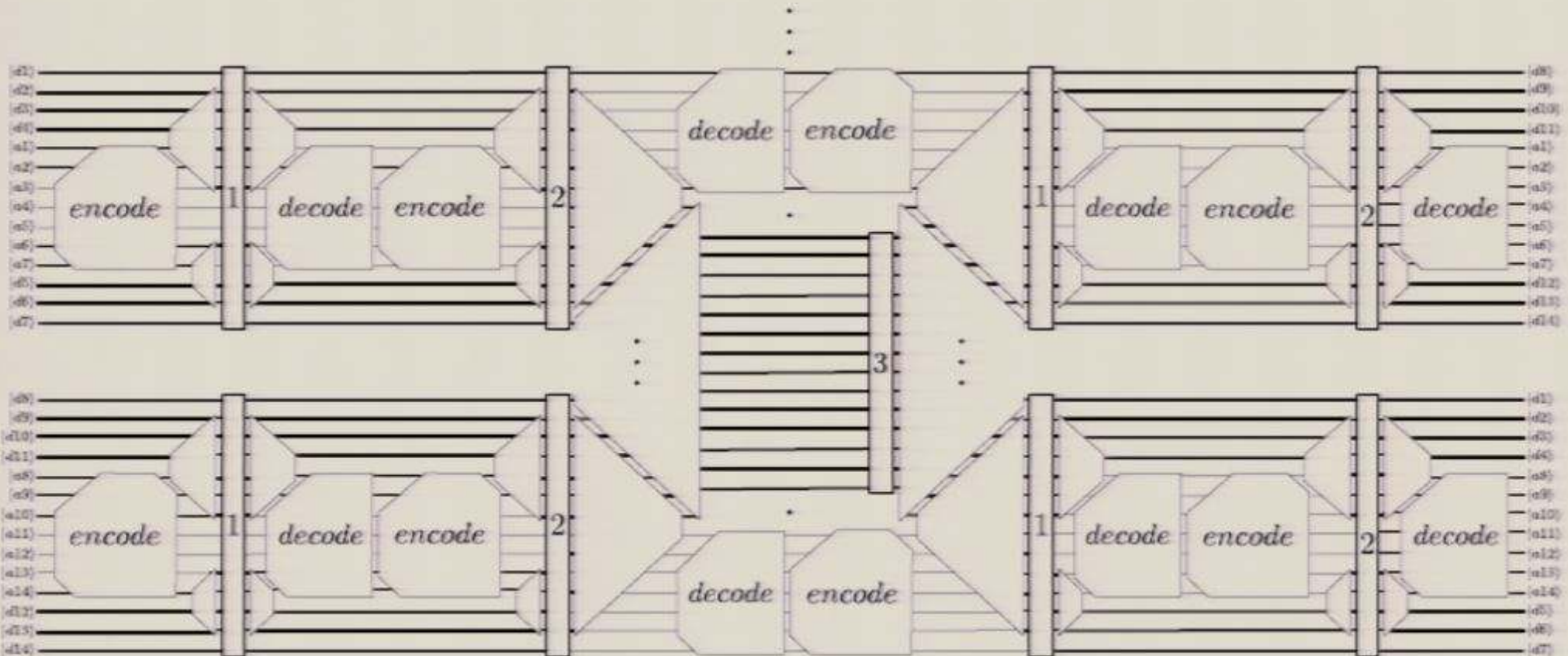


- Need logical bilinear network
- Permits fault-tolerant swap



Level 3 circuitry

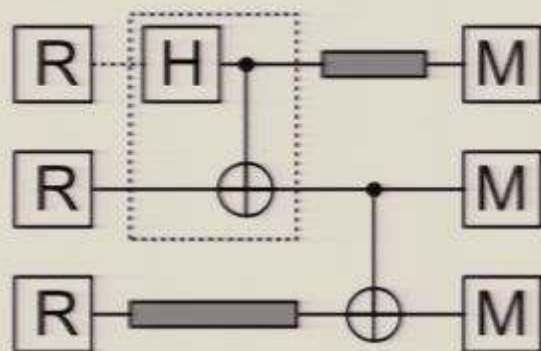
- Linear nearest neighbor with fault-tolerant swap



- Each horizontal line represents 2×21^2 qubits

Calculating thresholds

- Random example pretending to cope with one error



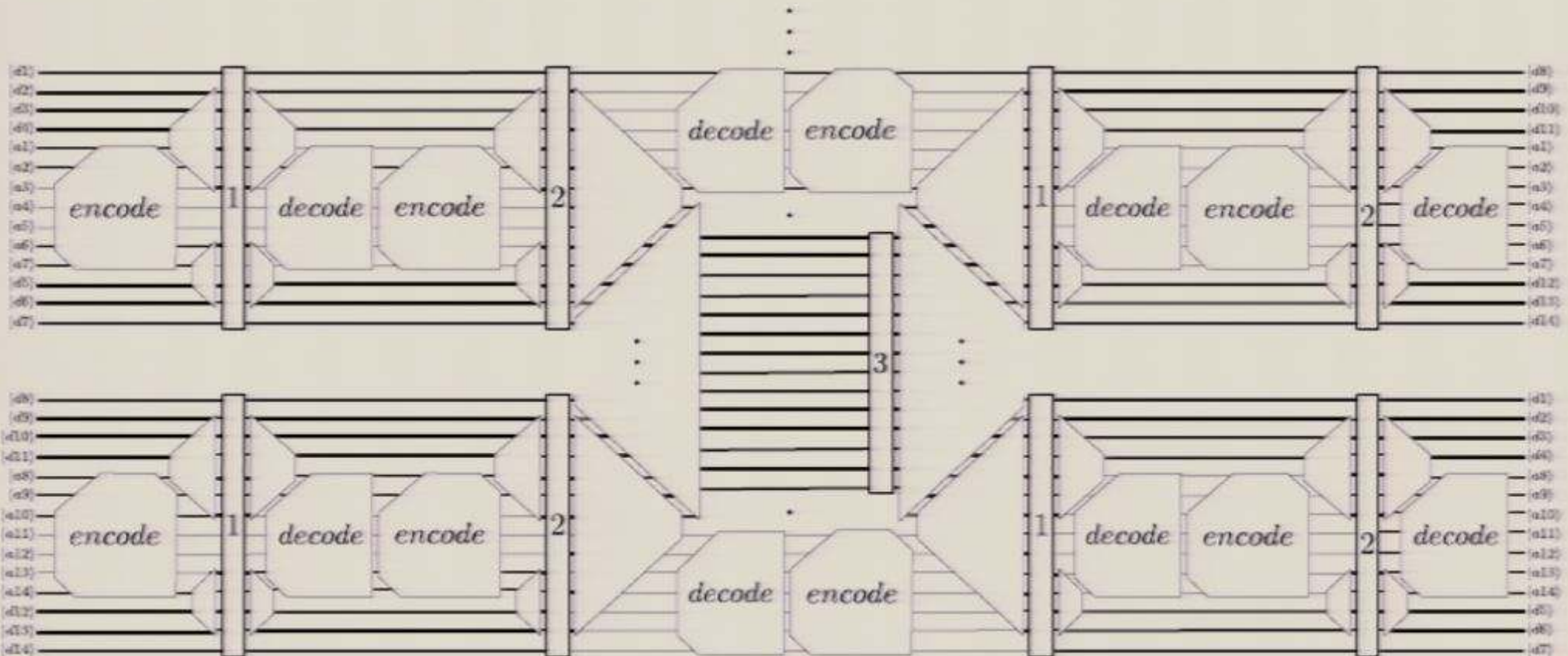
- Resets: 3
- Gates: 2
- Waits: 2
- Measurements: 3

$$P_{fail} = 1 - P_{success}$$

$$\begin{aligned}
 &= 1 - (1 - P_{reset})^3 (1 - P_{gate})^2 (1 - P_{wait})^2 (1 - P_{meas})^3 \\
 &\quad - P_{reset} (1 - P_{reset})^2 (1 - P_{gate})^2 (1 - P_{wait})^2 (1 - P_{meas})^3 \\
 &\quad - P_{gate} (1 - P_{reset})^3 (1 - P_{gate}) (1 - P_{wait})^2 (1 - P_{meas})^3 \\
 &\quad - P_{wait} (1 - P_{reset})^3 (1 - P_{gate})^2 (1 - P_{wait}) (1 - P_{meas})^3 \\
 &\quad - P_{meas} (1 - P_{reset})^3 (1 - P_{gate})^2 (1 - P_{wait})^2 (1 - P_{meas})^2
 \end{aligned}$$

Level 3 circuitry

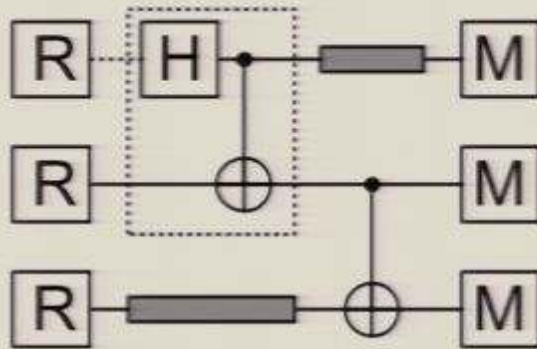
- Linear nearest neighbor with fault-tolerant swap



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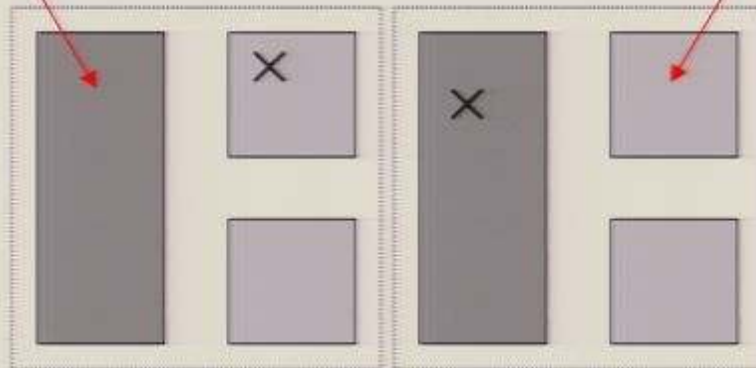
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 \end{aligned}$$

Extended rectangles

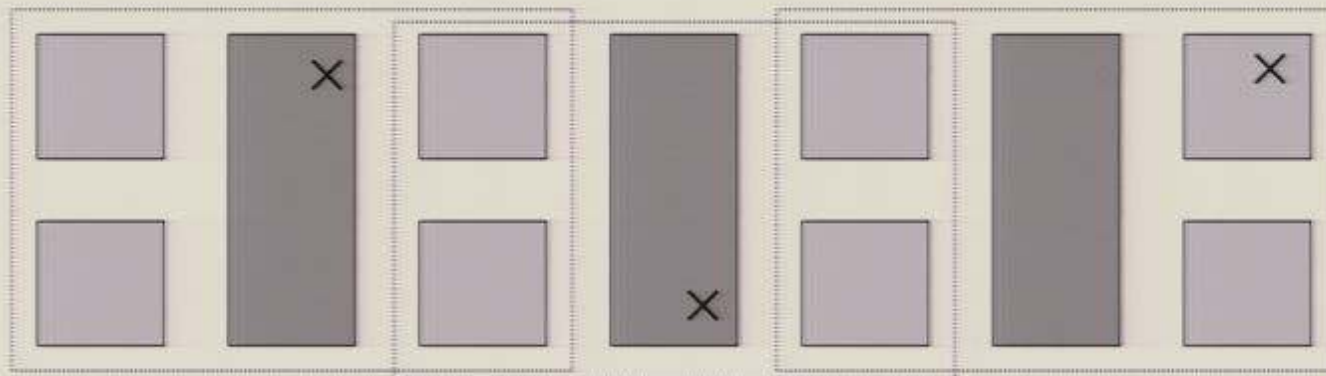
- How much circuitry needs to be included?

Logical interaction

Error correction



X



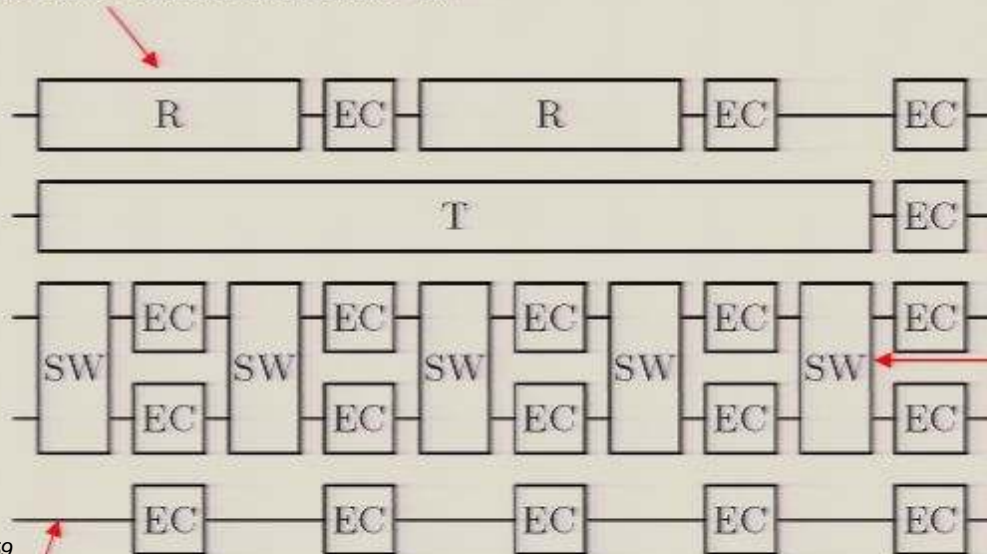
Which threshold?

- Must calculate threshold of most complex gate
- Universal gate set:
 - H, X, Z, S, S[†], and all combinations (23)
 - CNOT, SWAP
 - Measure, initialize, wait
 - T-gate ($\pi/8$ -gate)

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

measure and initialize



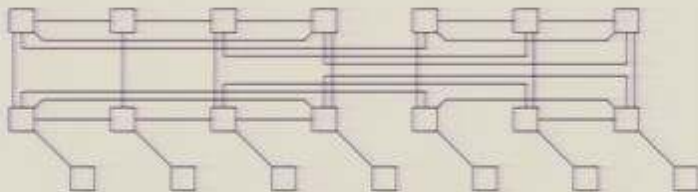
Pirsa: 07060059



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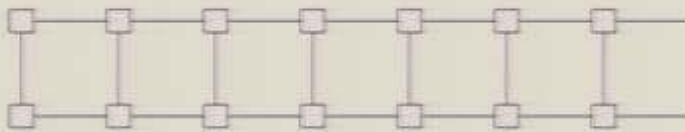
Three universal gate sets

- Non-local network



	$i = m$	$i = S$	$i = r$	depth
$j = m$	$206 + 28t_r$	100	28	$15 + 2t_r$
$j = S$	$398 + 56t_r$	207	56	$15 + 2t_r$
$j = T$	$1067 + 133t_r$	289	98	$75 + 10t_r$
$j = r$	$366 + 42t_r$	125	42	$30 + 4t_r$

- Bilinear network

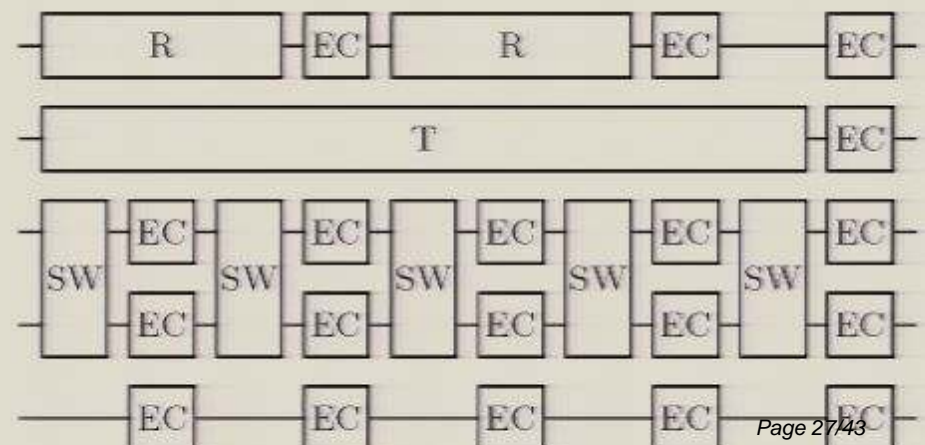


	$i = m$	$i = S$	$i = T$	$i = r$	depth
$j = m$	710	408	0	40	45
$j = S$	1114	1122	0	80	45
$j = T$	3171	1330	28	137	225
$j = r$	1129	510	0	57	90

- Linear nearest neighbor



	$i = m$	$i = S$	$i = T$	$i = r$	depth
$j = m$	558	204	0	28	38
$j = S$	824	603	0	56	38
$j = T$	2496	670	28	98	190
$j = r$	974	255	0	42	76



Thresholds for the architecture

- Four variables: p_{swap} , p_{memory} , p_{readout} , t_{readout}
- Set: $p_{\text{memory}} = 0.1 p_{\text{swap}}$, $p_{\text{readout}} = p_{\text{swap}}$, $t_{\text{readout}} = 10$

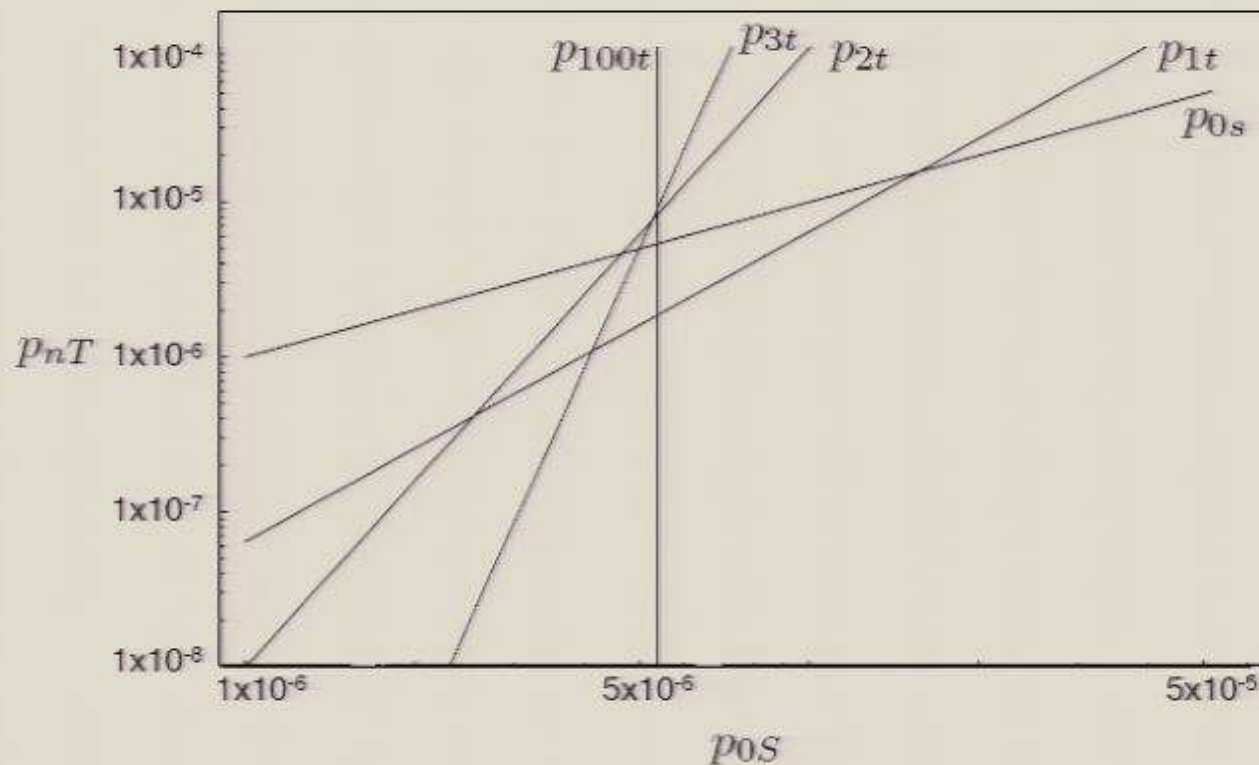
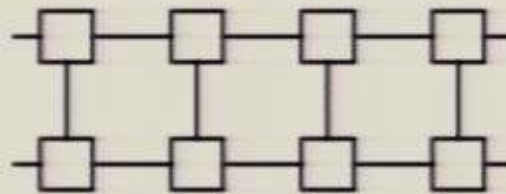


FIG. 1: p_0S and $p_nT(p_0S, 0.1, 1.0, 10.0)$ for $n = \{1, 2, 3, 100\}$.
The lower bound to the 100 T threshold is 5.36×10^{-6} .

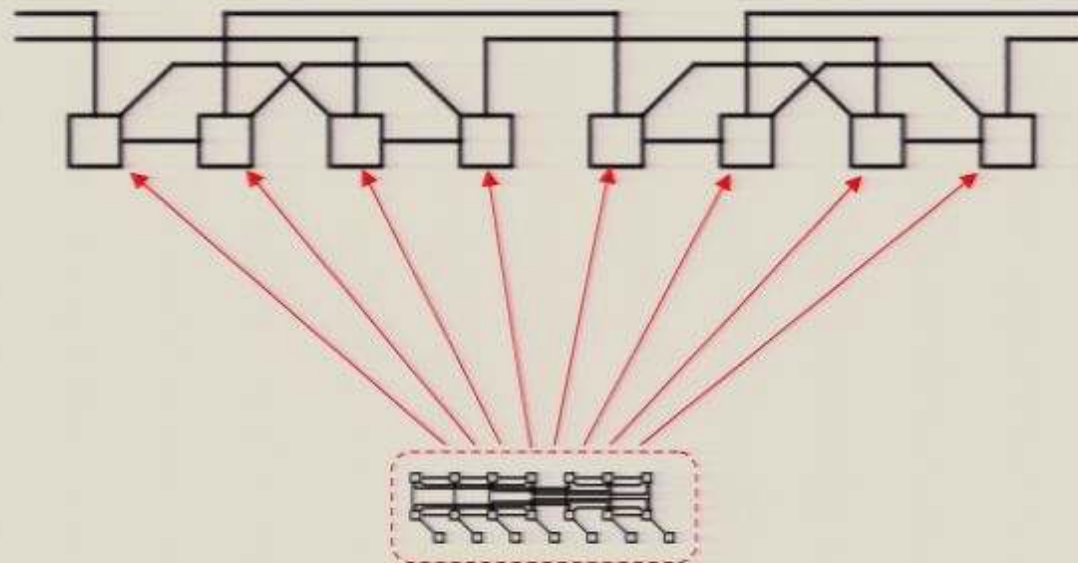
- Infinite level threshold: $\sim 5 \times 10^{-6}$, level-1 threshold $\sim 10^{-5}$

Level 2 circuitry

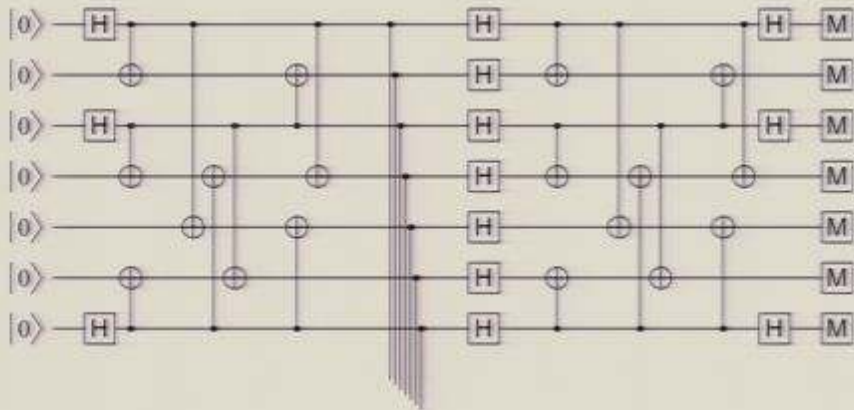
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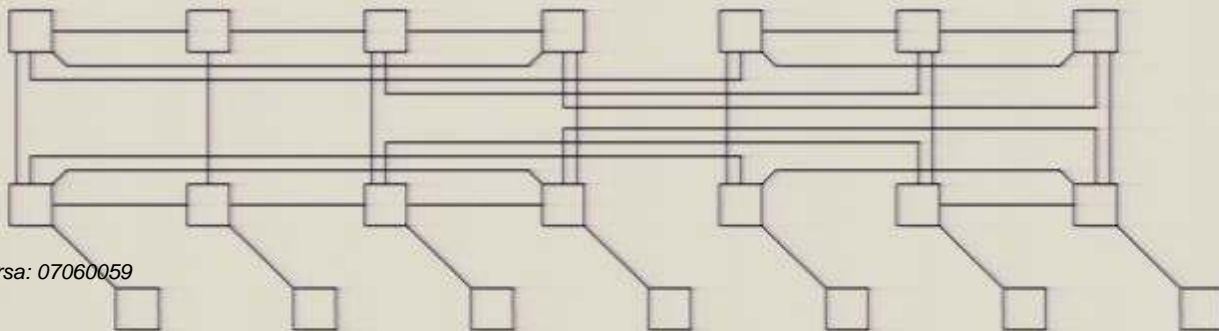
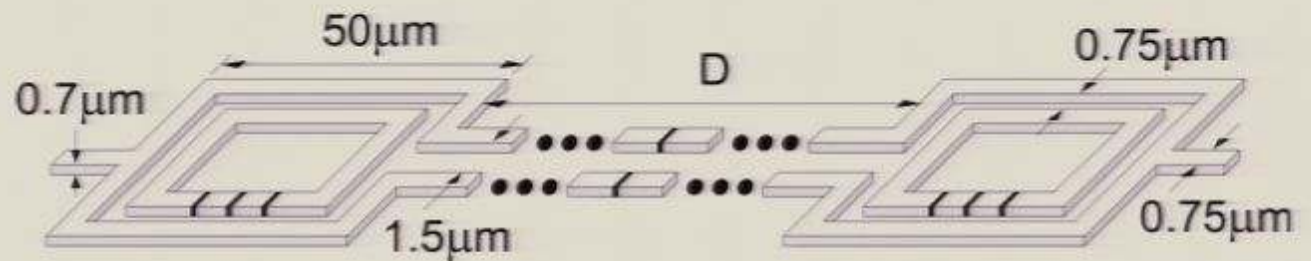


Laying out the circuitry



- Error correction

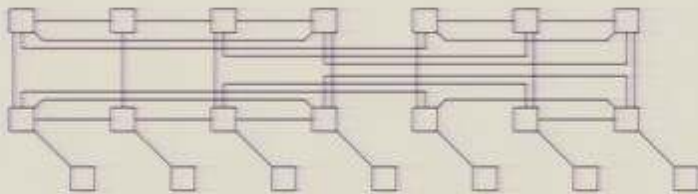
- Coupler



- Network

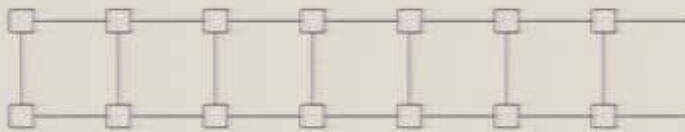
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- Bilinear network

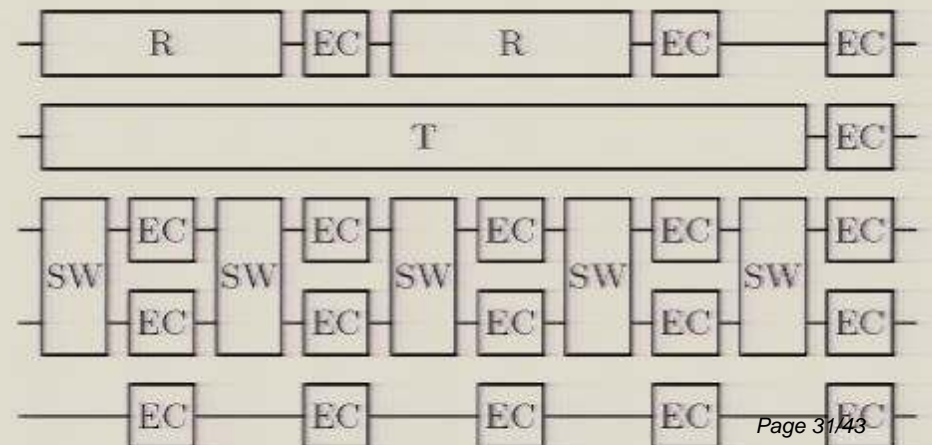


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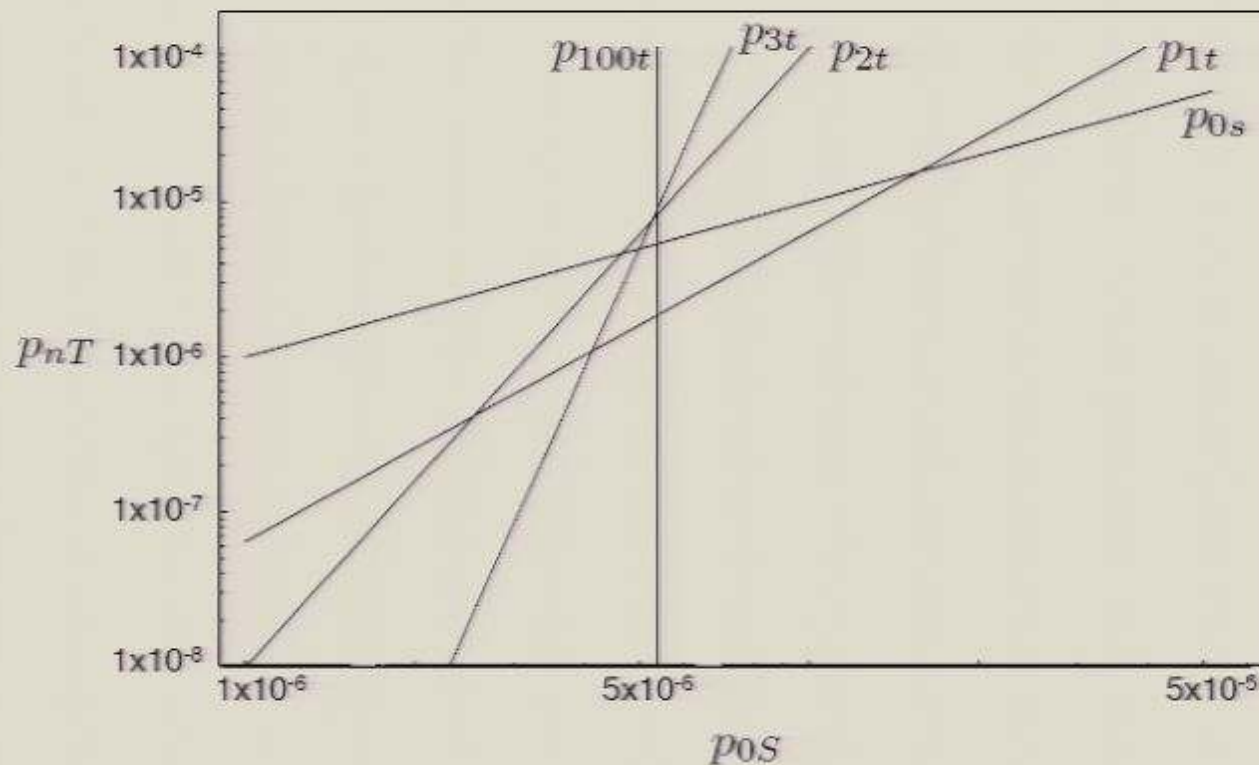
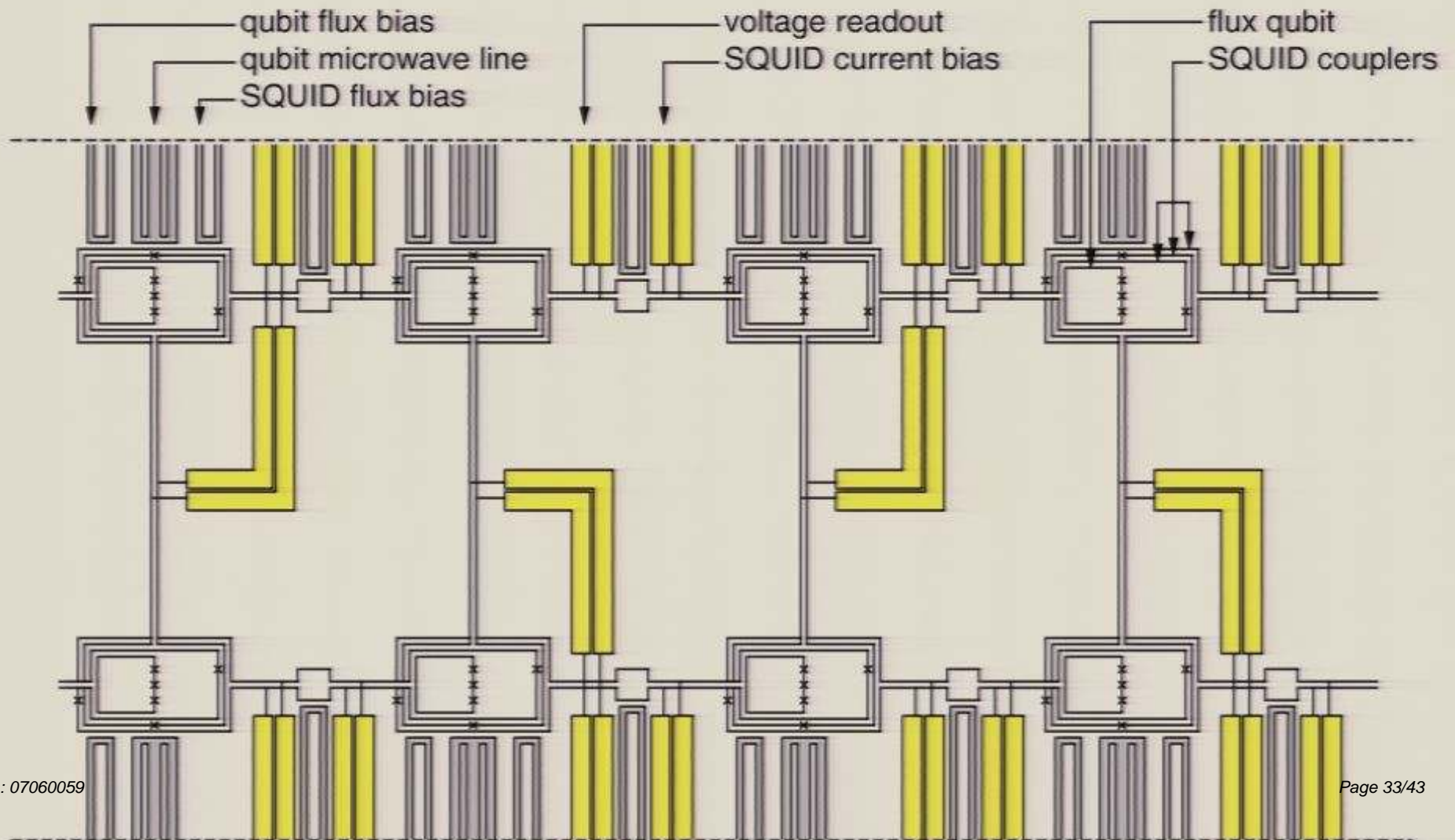


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- Infinite level threshold: $\sim 5 \times 10^{-6}$, level-1 threshold $\sim 10^{-5}$

Bilinear details

- Comparable threshold 2×10^{-6} (Stephens, quant-ph/0702201)
- Simpler construction likely to outweigh lower threshold



Thresholds for the architecture

- Four variables: p_{swap} , p_{memory} , p_{readout} , t_{readout}
- Set: $p_{\text{memory}} = 0.1 p_{\text{swap}}$, $p_{\text{readout}} = p_{\text{swap}}$, $t_{\text{readout}} = 10$

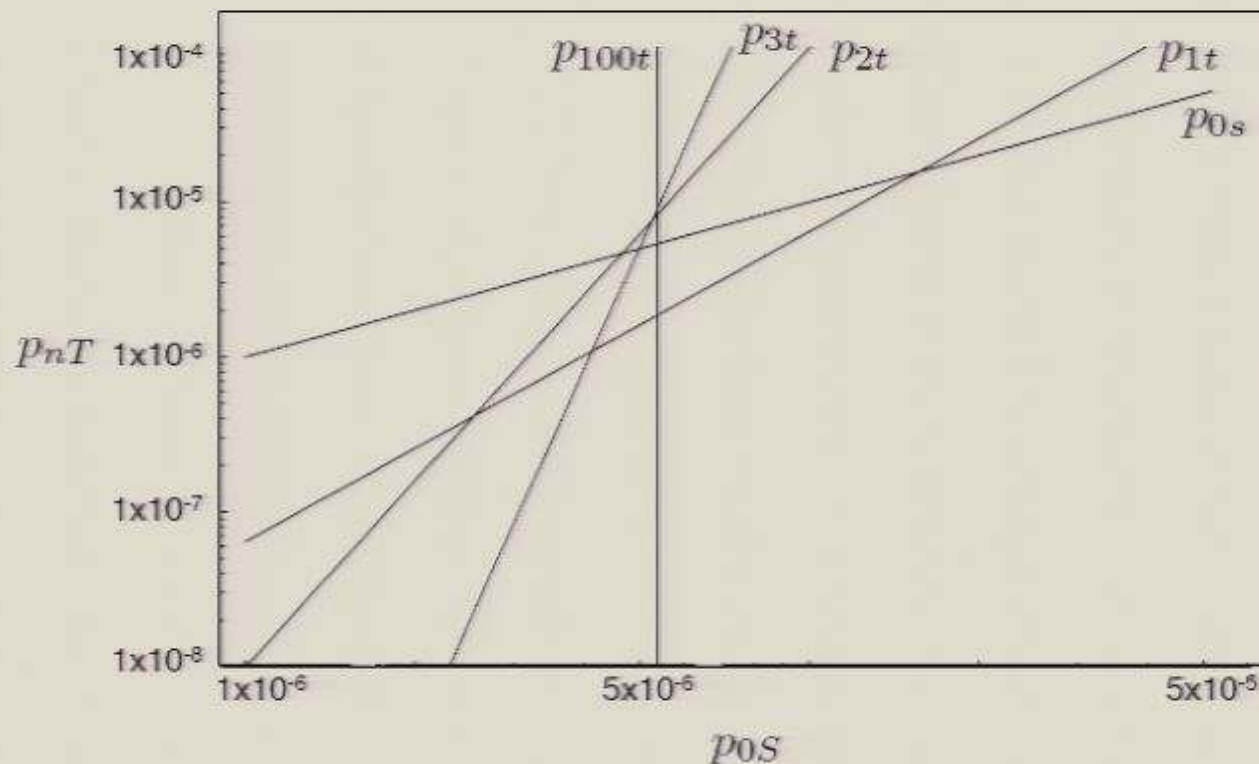
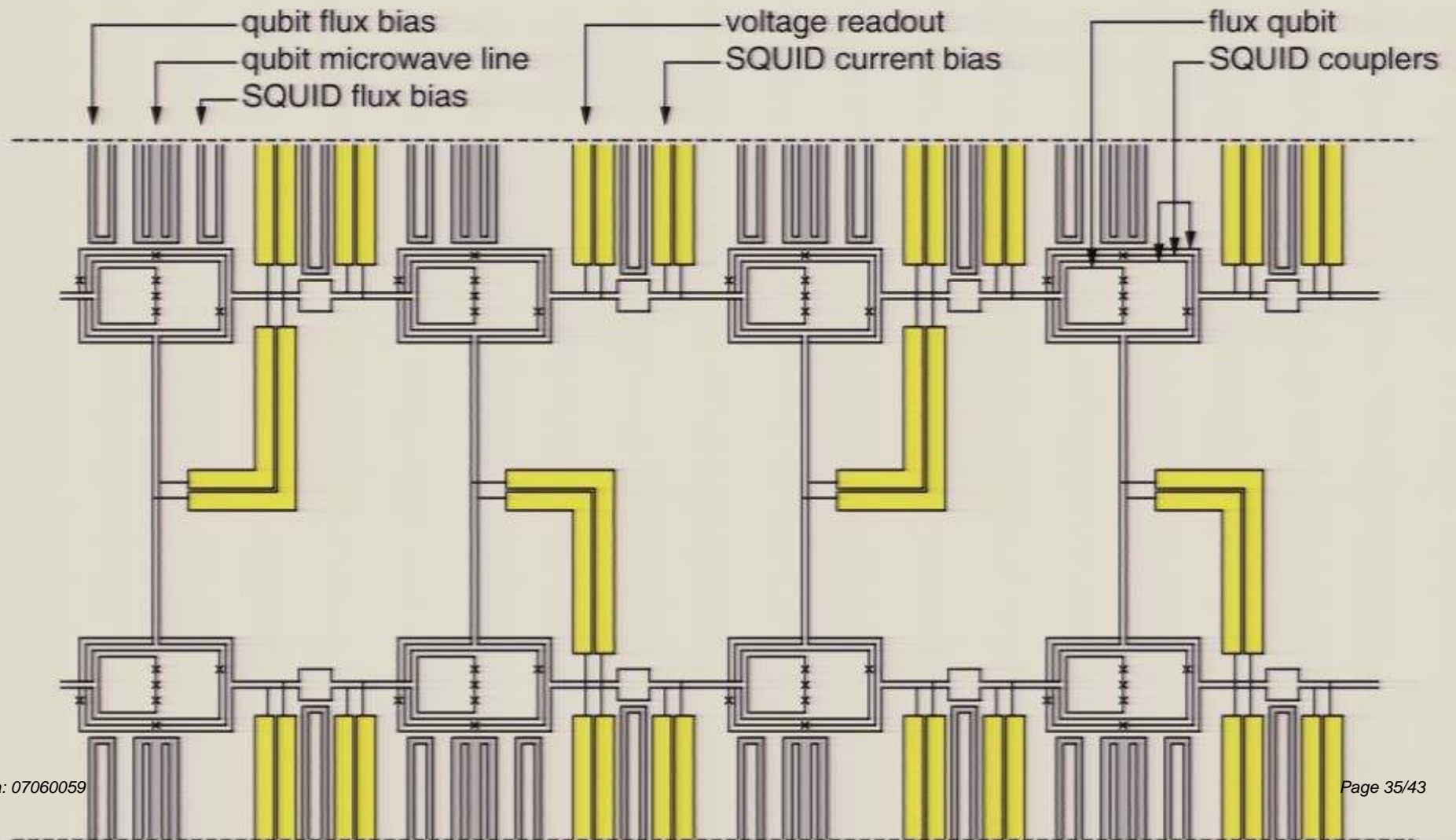


FIG. 1: p_0S and $p_nT(p_0S, 0.1, 1.0, 10.0)$ for $n = \{1, 2, 3, 100\}$.
The lower bound to the 100 T threshold is 5.36×10^{-6} .

- Infinite level threshold: $\sim 5 \times 10^{-6}$, level-1 threshold $\sim 10^{-5}$

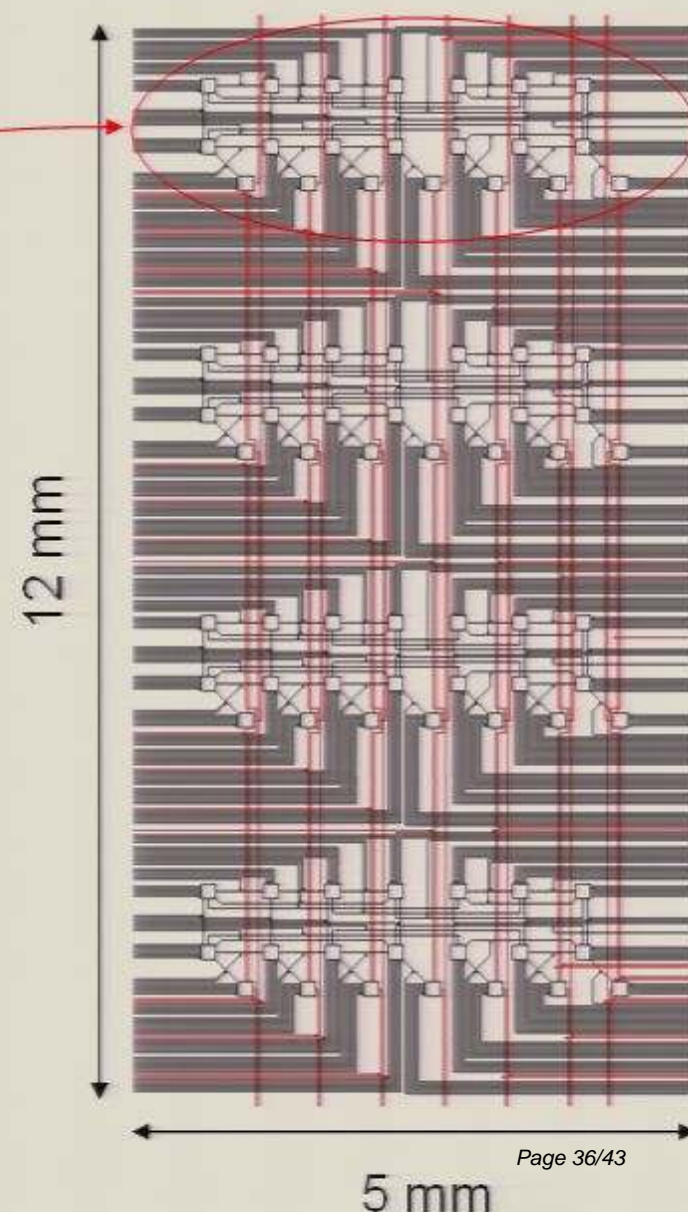
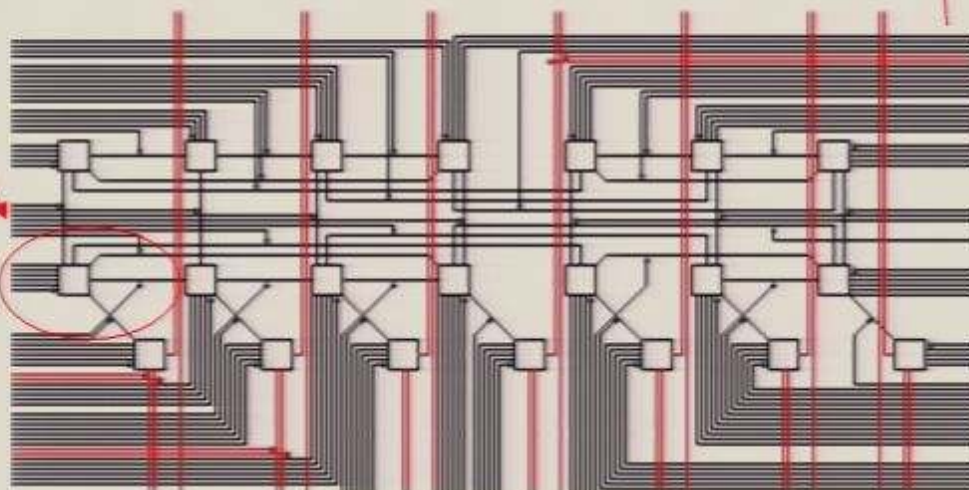
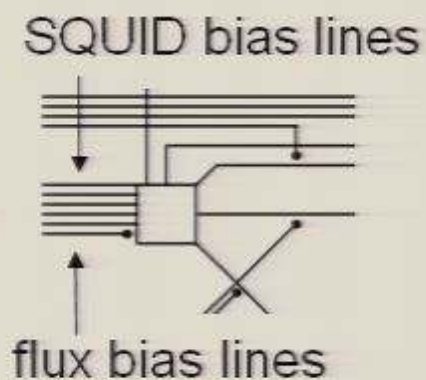
Bilinear details

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Level 2 circuitry

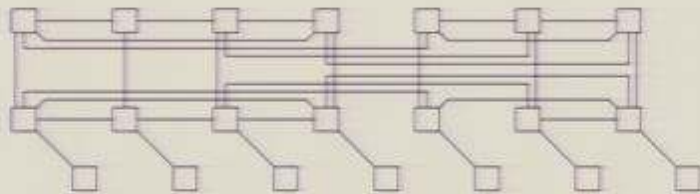
- Must control each qubit



- 1 wire per 40 μm

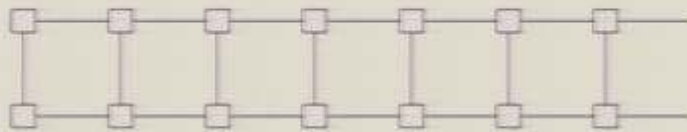
Three universal gate sets

- Non-local network



	$i = m$	$i = S$	$i = r$	depth
$j = m$	$206 + 28t_r$	100	28	$15 + 2t_r$
$j = S$	$398 + 56t_r$	207	56	$15 + 2t_r$
$j = T$	$1067 + 133t_r$	289	98	$75 + 10t_r$
$j = r$	$366 + 42t_r$	125	42	$30 + 4t_r$

- Bilinear network

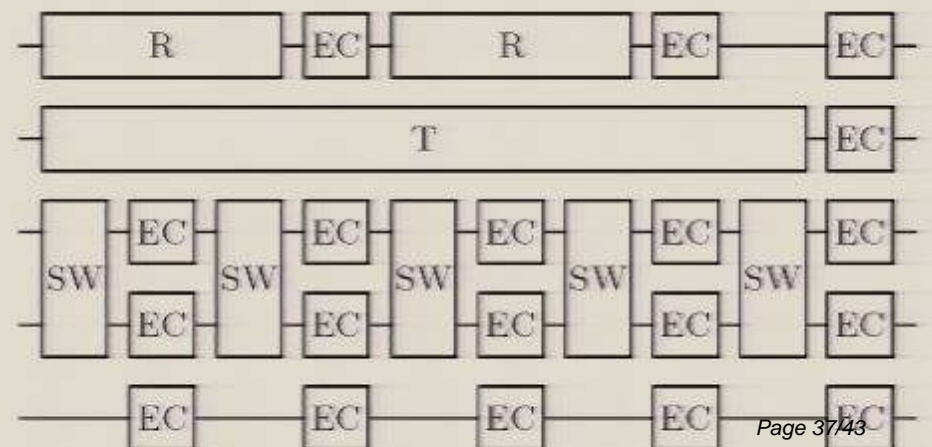


	$i = m$	$i = S$	$i = T$	$i = r$	depth
$j = m$	710	408	0	40	45
$j = S$	1114	1122	0	80	45
$j = T$	3171	1330	28	137	225
$j = r$	1129	510	0	57	90

- Linear nearest neighbor

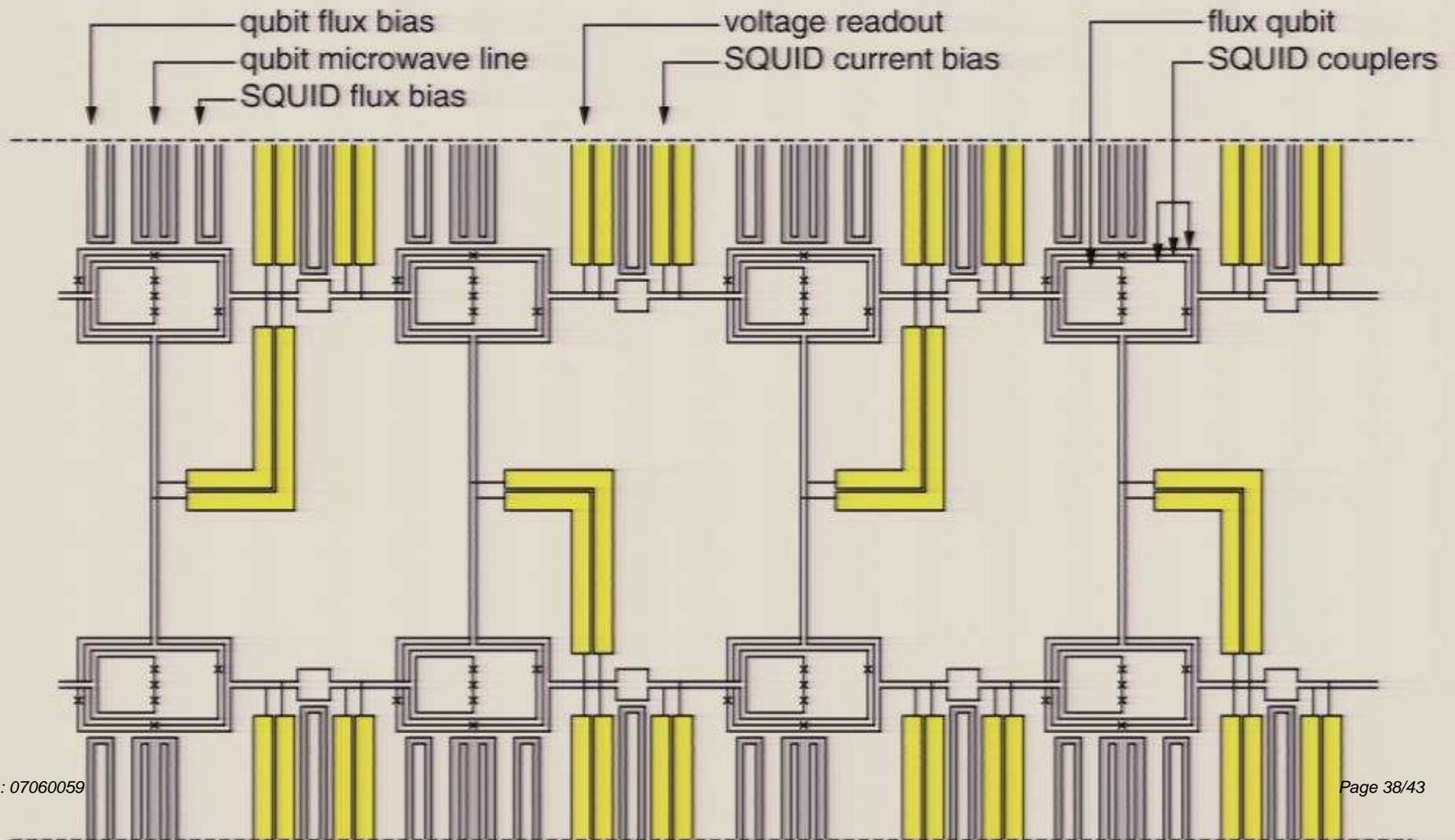


	$i = m$	$i = S$	$i = T$	$i = r$	depth
$j = m$	558	204	0	28	38
$j = S$	824	603	0	56	38
$j = T$	2496	670	28	98	190
$j = r$	974	255	0	42	76

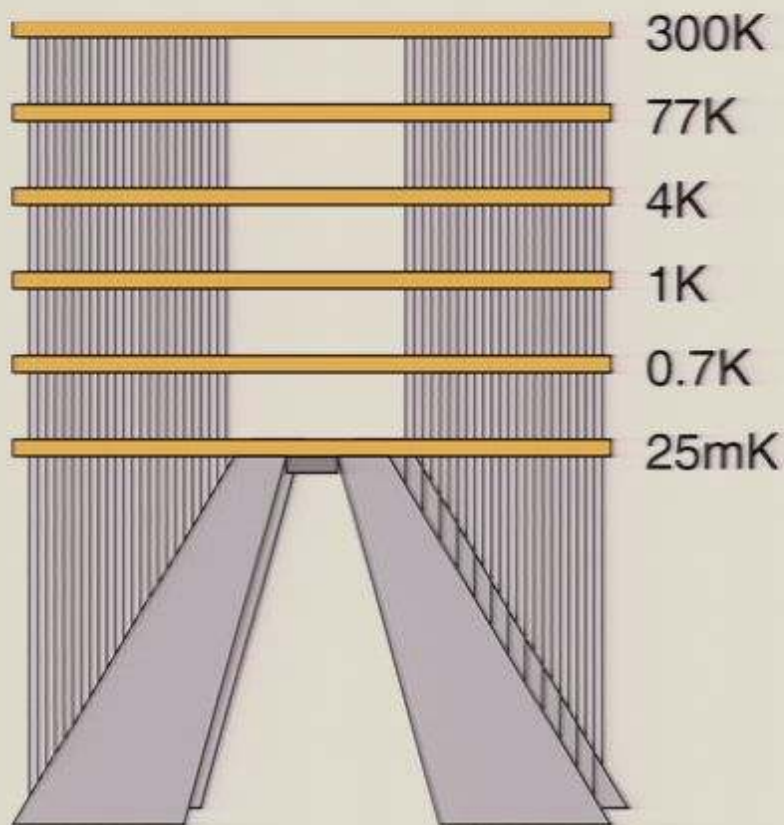


Bilinear details

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- Simpler construction likely to outweigh lower threshold



Further work



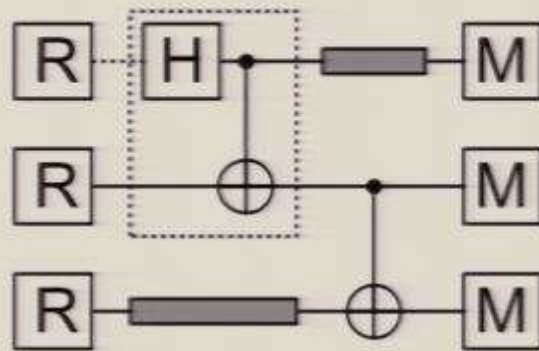
- Cooling – details
- Wire density – serious limitation
- Crosstalk and shielding
- Interchip spanning schemes
- Algorithms without error correction

Conclusion

- There is no universal threshold of 10^{-4}
- Without long-range interactions, threshold much lower
- For flux qubit architecture, threshold $\sim 5 \times 10^{-6}$
- Implies gate error rates of $\sim 10^{-7}$ needed
- May need to compute without error correction

Calculating thresholds

- Random example pretending to cope with one error



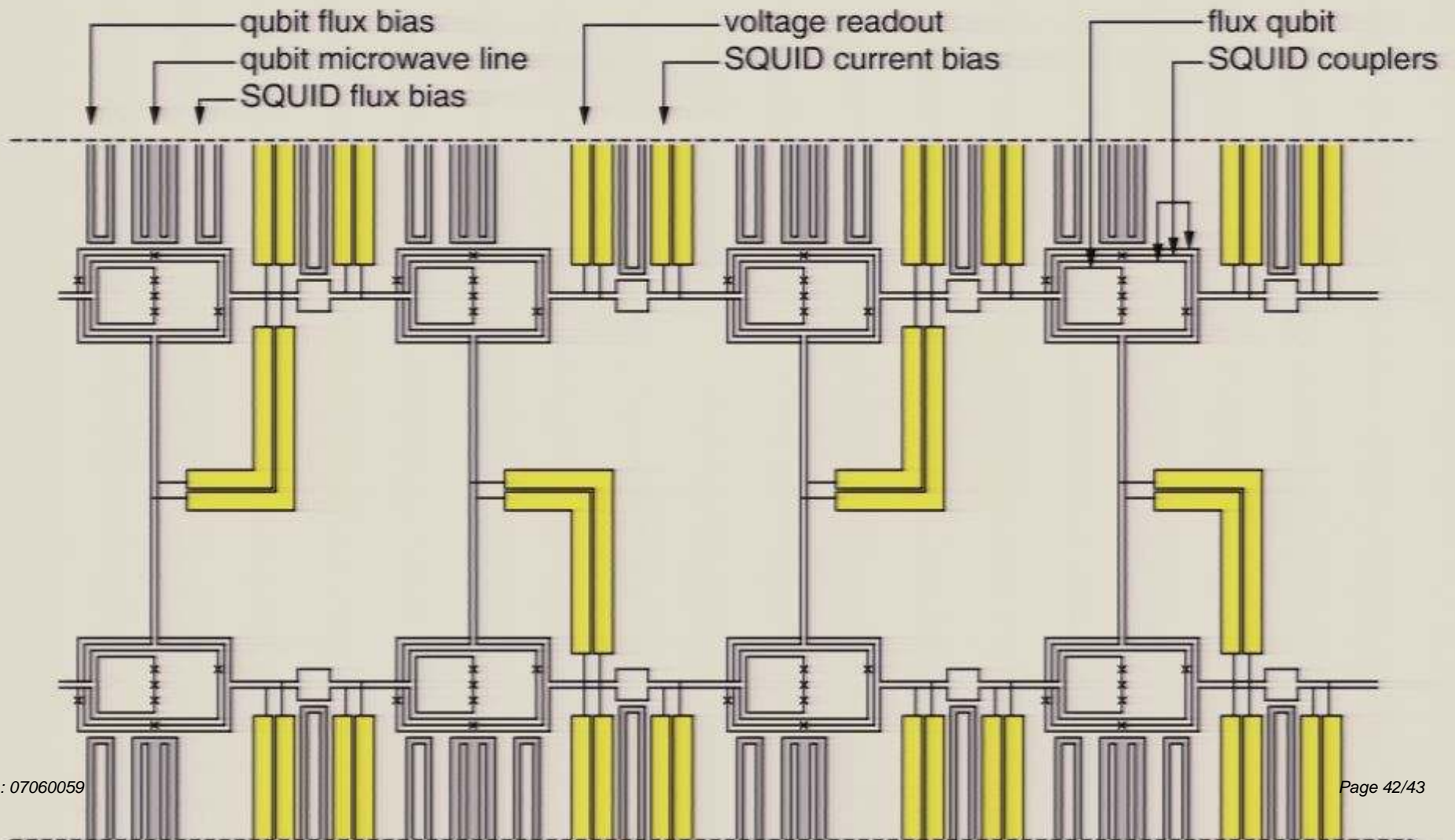
- Resets: 3
- Gates: 2
- Waits: 2
- Measurements: 3

$$P_{fail} = 1 - P_{success}$$

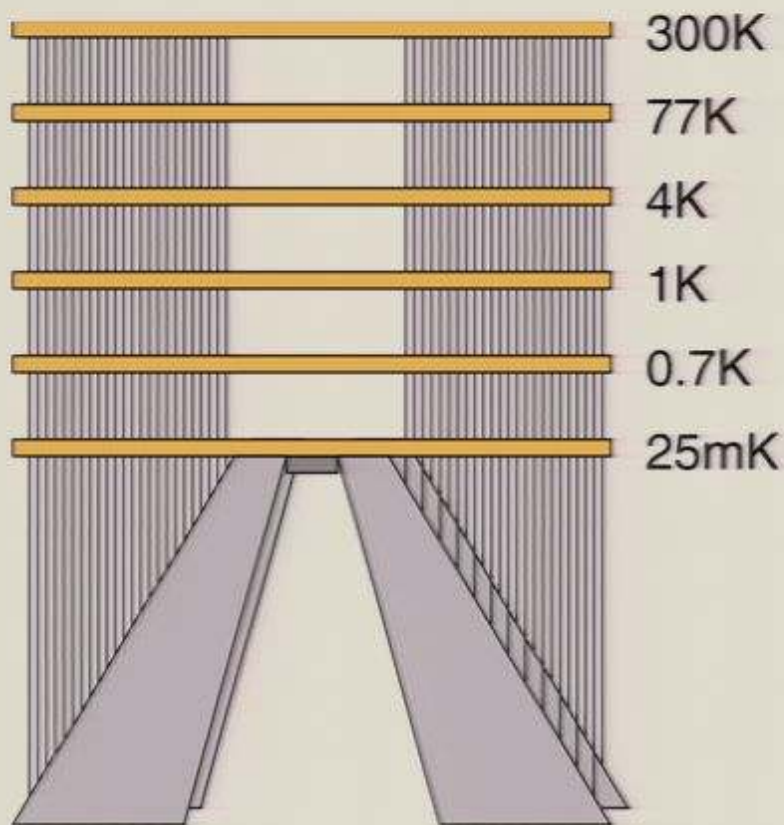
$$\begin{aligned}
 &= 1 - (1 - P_{reset})^3 (1 - P_{gate})^2 (1 - P_{wait})^2 (1 - P_{meas})^3 \\
 &\quad - P_{reset} (1 - P_{reset})^2 (1 - P_{gate})^2 (1 - P_{wait})^2 (1 - P_{meas})^3 \\
 &\quad - P_{gate} (1 - P_{reset})^3 (1 - P_{gate}) (1 - P_{wait})^2 (1 - P_{meas})^3 \\
 &\quad - P_{wait} (1 - P_{reset})^3 (1 - P_{gate})^2 (1 - P_{wait}) (1 - P_{meas})^3 \\
 &\quad - P_{meas} (1 - P_{reset})^3 (1 - P_{gate})^2 (1 - P_{wait})^2 (1 - P_{meas})^2
 \end{aligned}$$

Bilinear details

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Further work



- Cooling – details
- Wire density – serious limitation
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