

Title: Scalable quantum computer architecture for superconducting flux qubits

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Abstract:

# Long-range coupling mechanism and architecture for superconducting flux qubits

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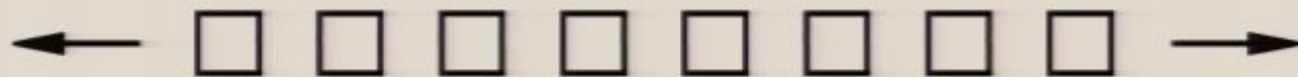
cond-mat/0702620

# Overview

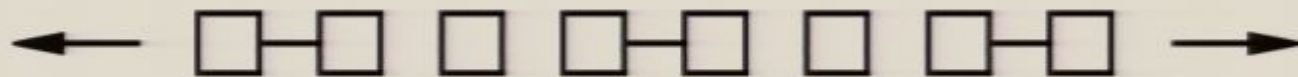
- What is scalability?
- Coupling flux qubits
- Why long-range coupling?
- Long-range coupling of flux qubits
- Interlude: error correction
- Universal set of gates
- Designing the flux qubit coupler network
- Calculating thresholds
- Conclusion and further work

# Scalability

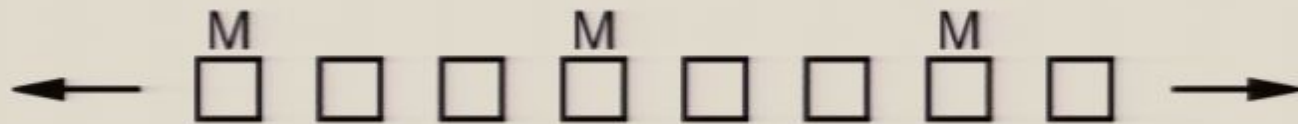
- Arbitrarily large number of qubits



- Number of simultaneous gates proportional to number of qubits



- Number of simultaneous measurements proportional to number of qubits

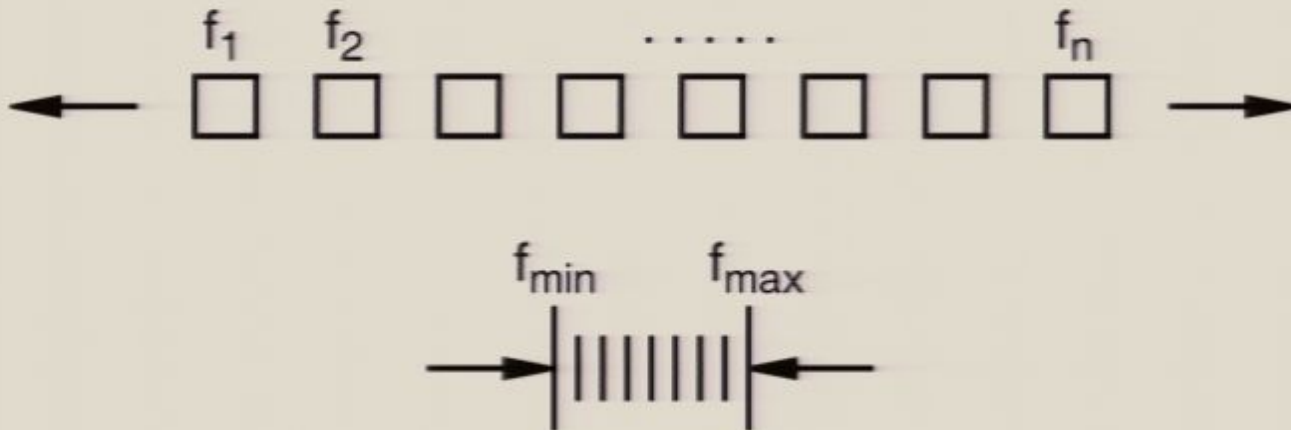


- Physics of gates and measurements independent of number of qubits

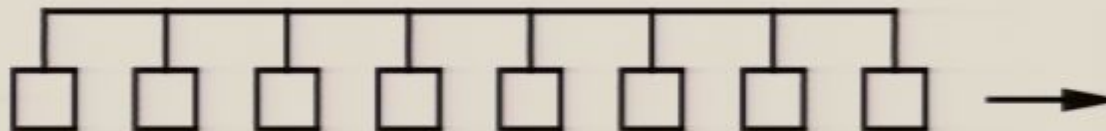


# Common unscalable examples

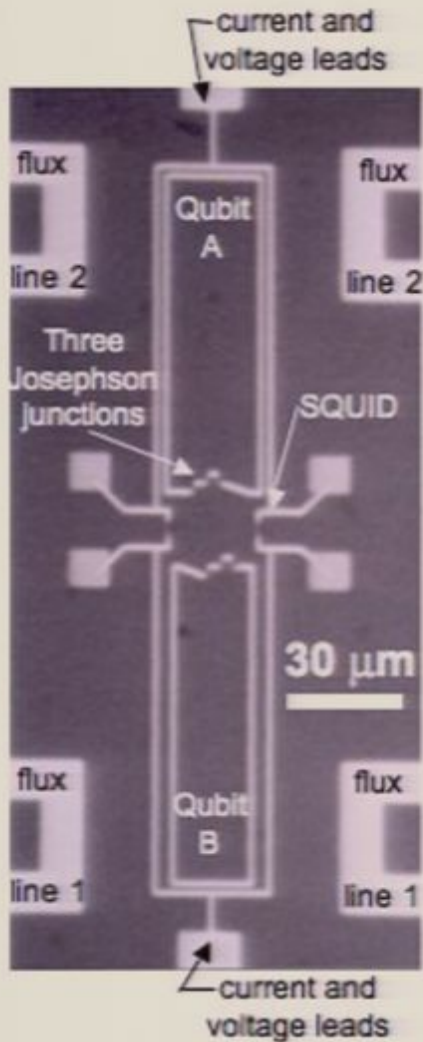
- Anything with frequency crowding



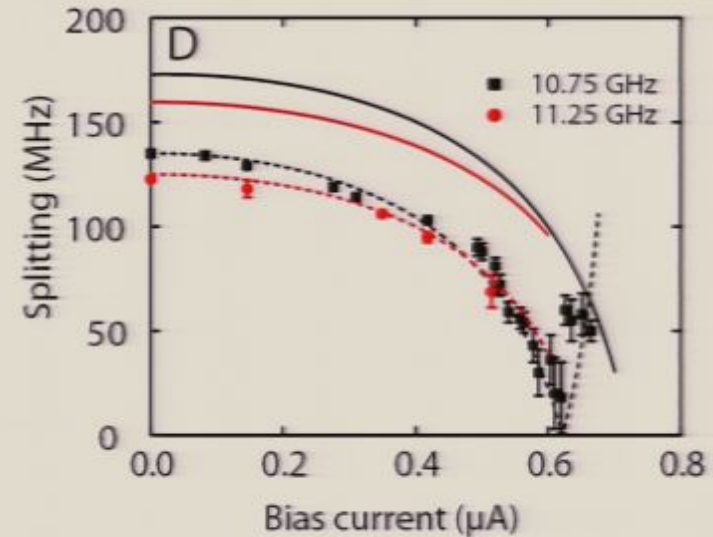
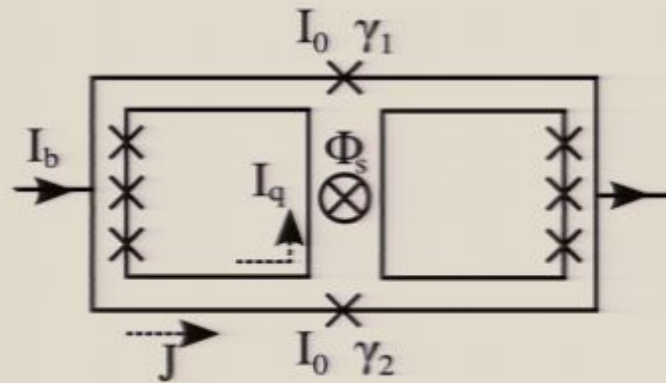
- Anything with a single shared device for gates or measurement



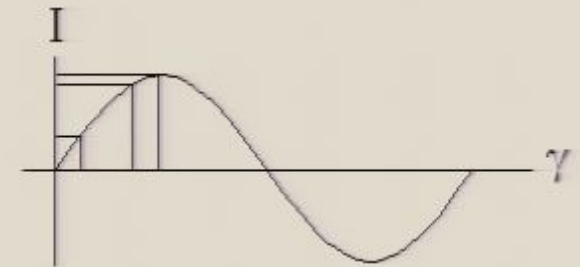
# Our starting point



$$I = I_0 \sin \gamma$$



- $I_b = I_0 \sin \gamma_1 + I_0 \sin \gamma_2$
- $2J = I_0 \sin \gamma_2 - I_0 \sin \gamma_1$
- $\gamma_1 - \gamma_2 + \frac{2\pi}{\Phi_0} (\Phi_s - LJ) = 0$

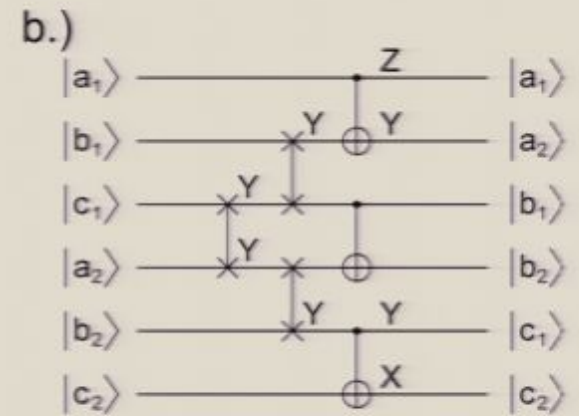
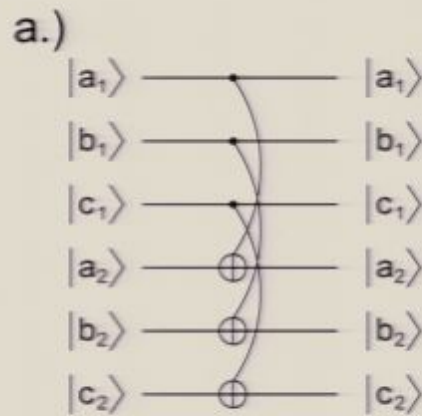


# Shortcomings

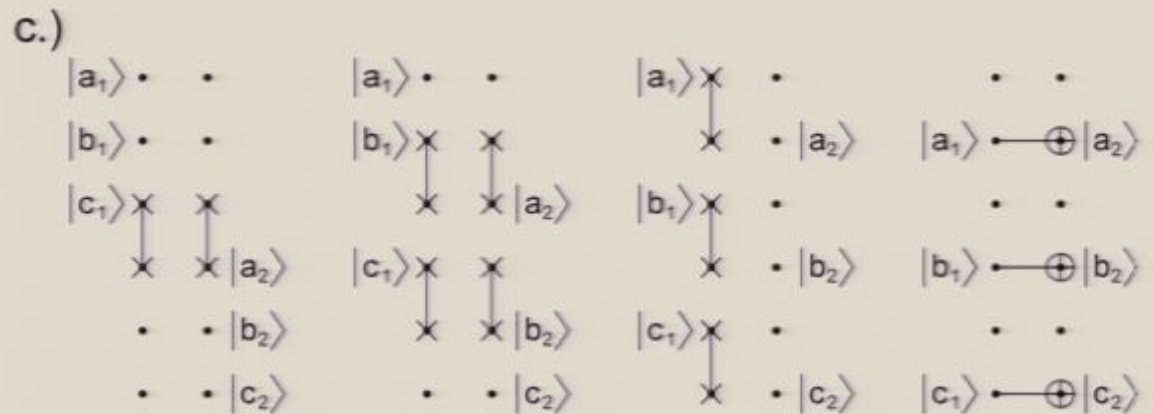
- Need long-range interactions
- Thresholds
  - unlimited range, unlimited qubits:  $\sim 10^{-2}$   
Knill, quant-ph/0410199
  - unlimited range, many qubits:  $\sim 10^{-3}$ – $10^{-4}$   
Steane, Phys. Rev. A 68, 042322 (2003)
  - 2D lattice, nearest neighbor:  $\sim 10^{-5}$   
Svore, QIC 7, 297 (2007)
  - bilinear nearest neighbor:  $\sim 10^{-6}$   
Stephens, quant-ph/0702201
  - linear nearest neighbor:  $\sim 10^{-8}$   
Stephens, in preparation

# Why long-range coupling?

- Case a.)
  - qubits:  $2n$
  - gates:  $n$
  - idle: 0
  - depth: 1

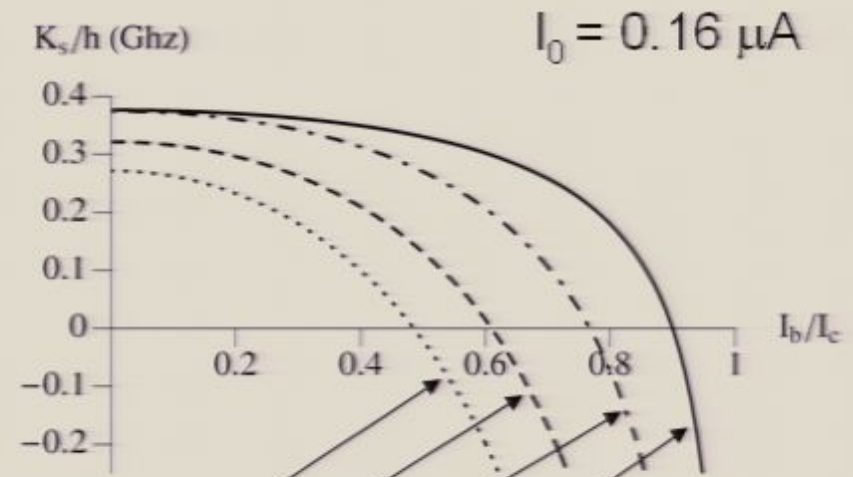
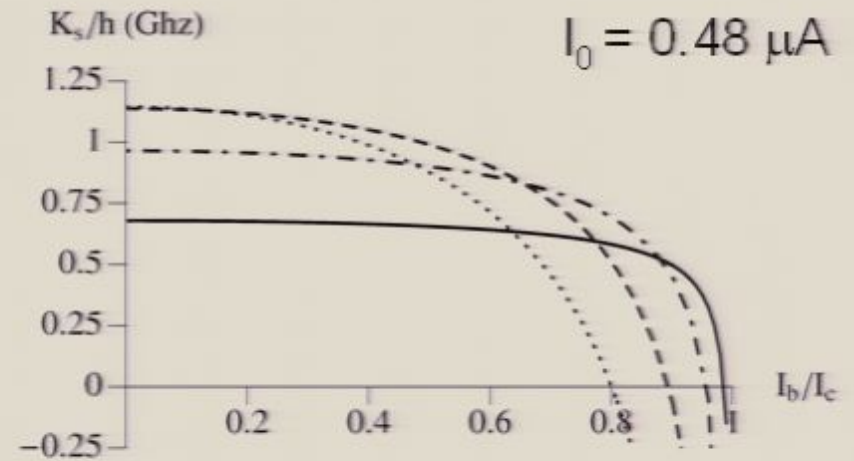
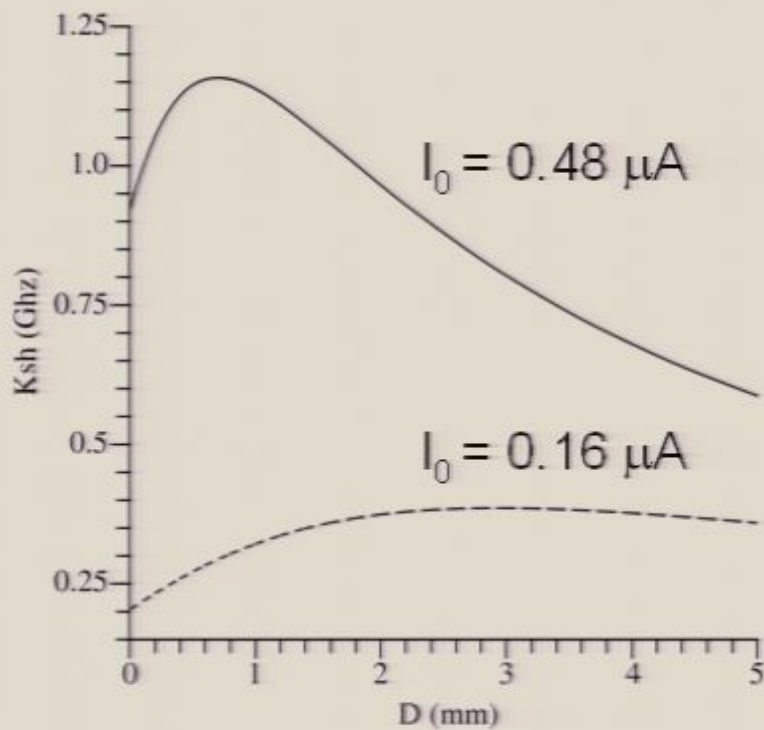
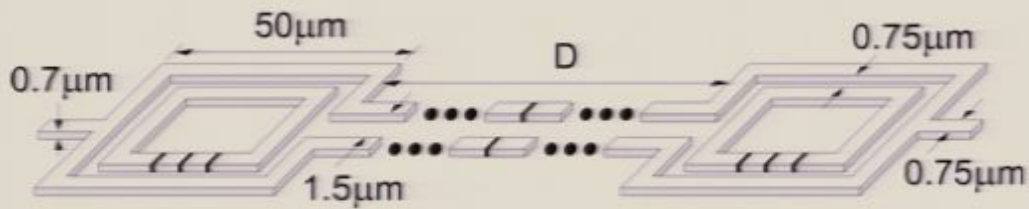


- Case c.)
  - qubits:  $4n$
  - gates:  $2n^2 + n$
  - idle:  $2n^2$
  - depth:  $2n + 1$



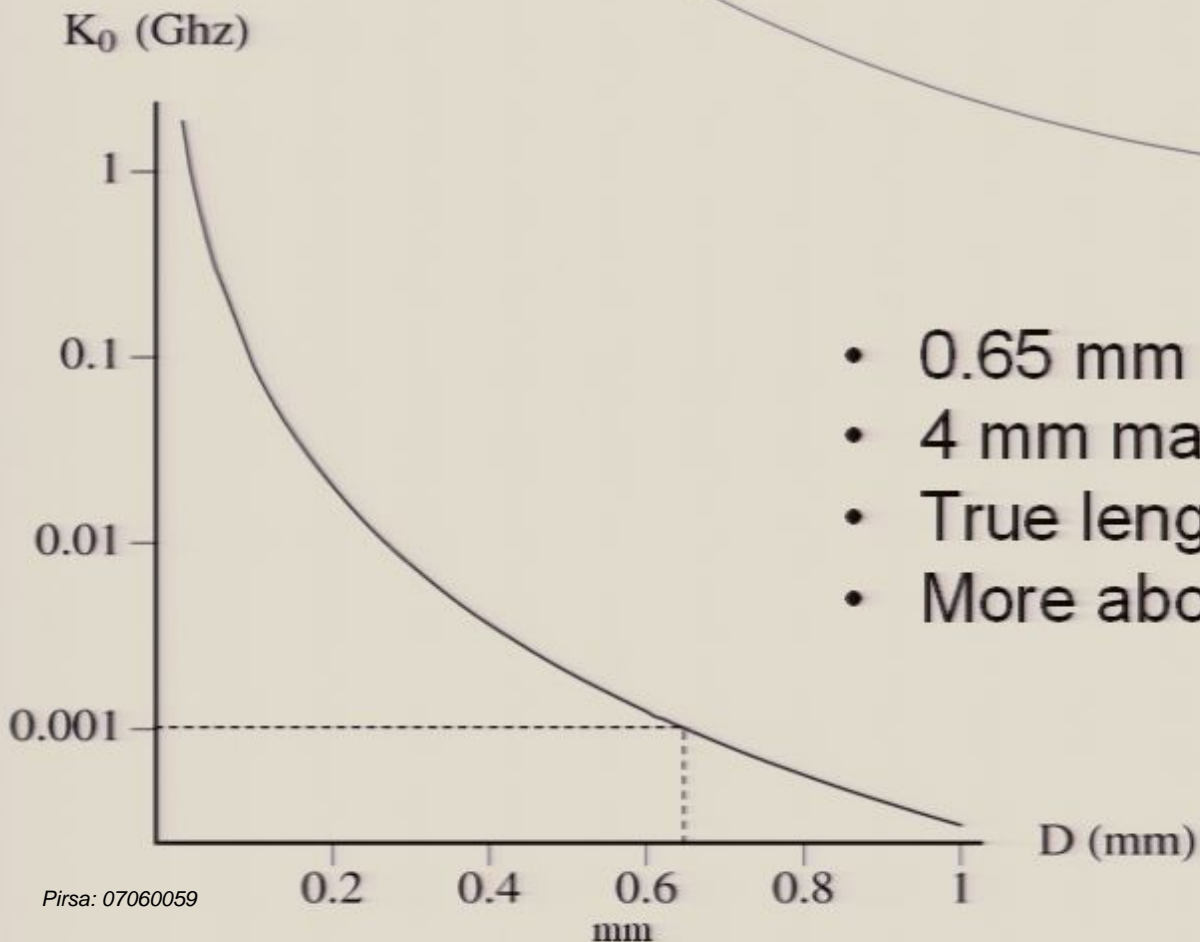
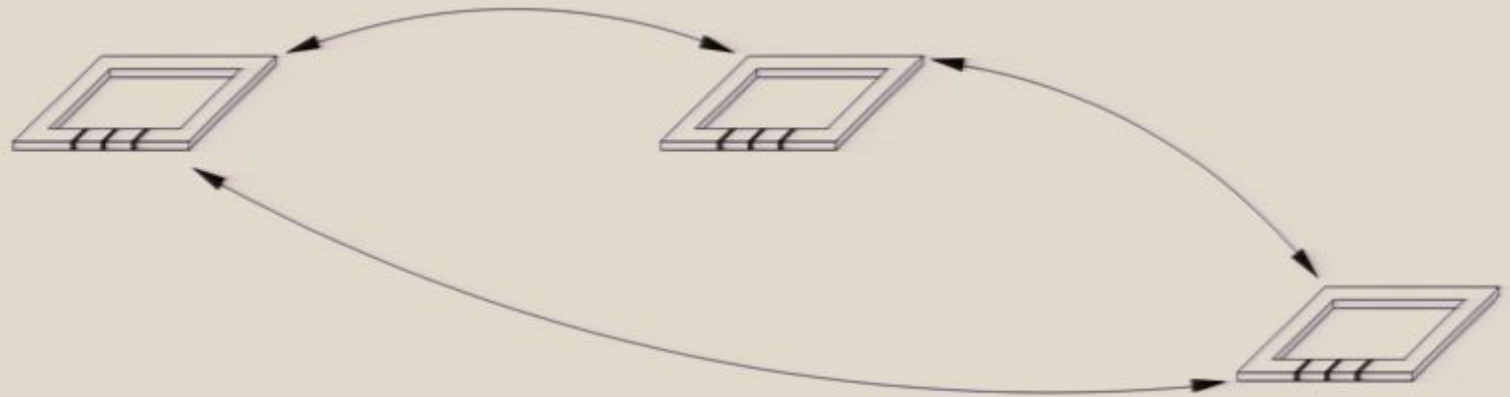


# Extending the coupler



$D = 500, 1000, 2000, 4000 \mu\text{m}$

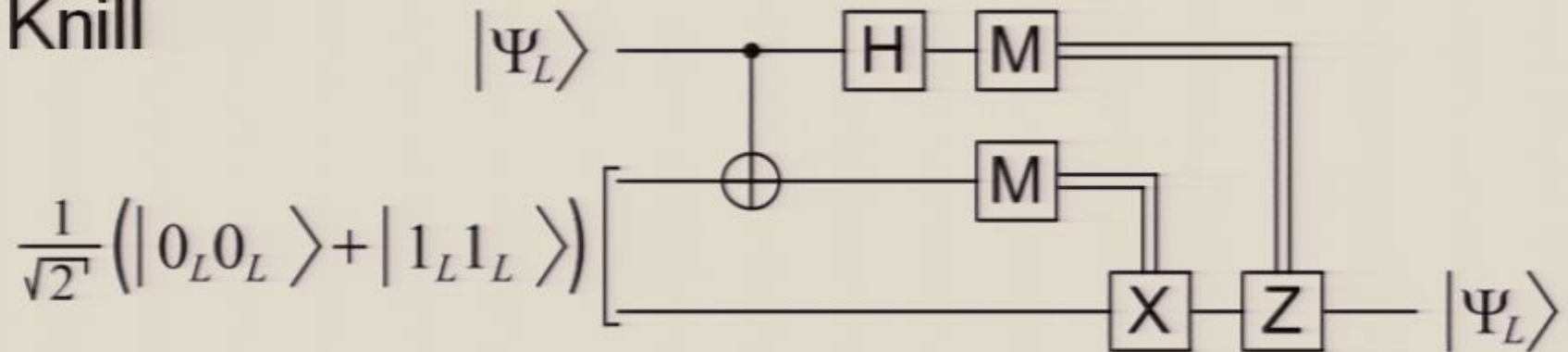
# True coupler length: crosstalk



- 0.65 mm minimum separation
- 4 mm maximum separation
- True length: 5-6 qubits
- More about this later...

# Interlude: error correction

- Knill



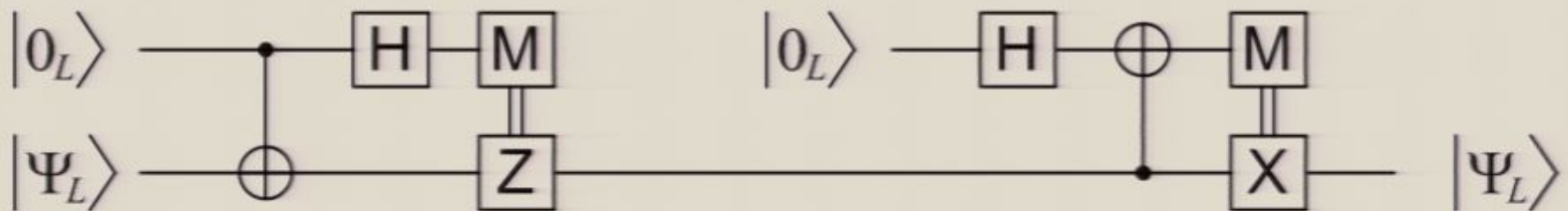
$$|0_L\rangle = \frac{1}{\sqrt{8}} (|0000000\rangle + |1010101\rangle + |0110011\rangle + |1100110\rangle + |0001111\rangle + |1011010\rangle + |0111100\rangle + |1101001\rangle)$$

$$|1_L\rangle = \frac{1}{\sqrt{8}} (|1111111\rangle + |0101010\rangle + |1001100\rangle + |0011001\rangle + |1110000\rangle + |0100101\rangle + |1000011\rangle + |0010110\rangle)$$

- Best approach for high threshold  $\sim 10^{-2}$

# Interlude: error correction

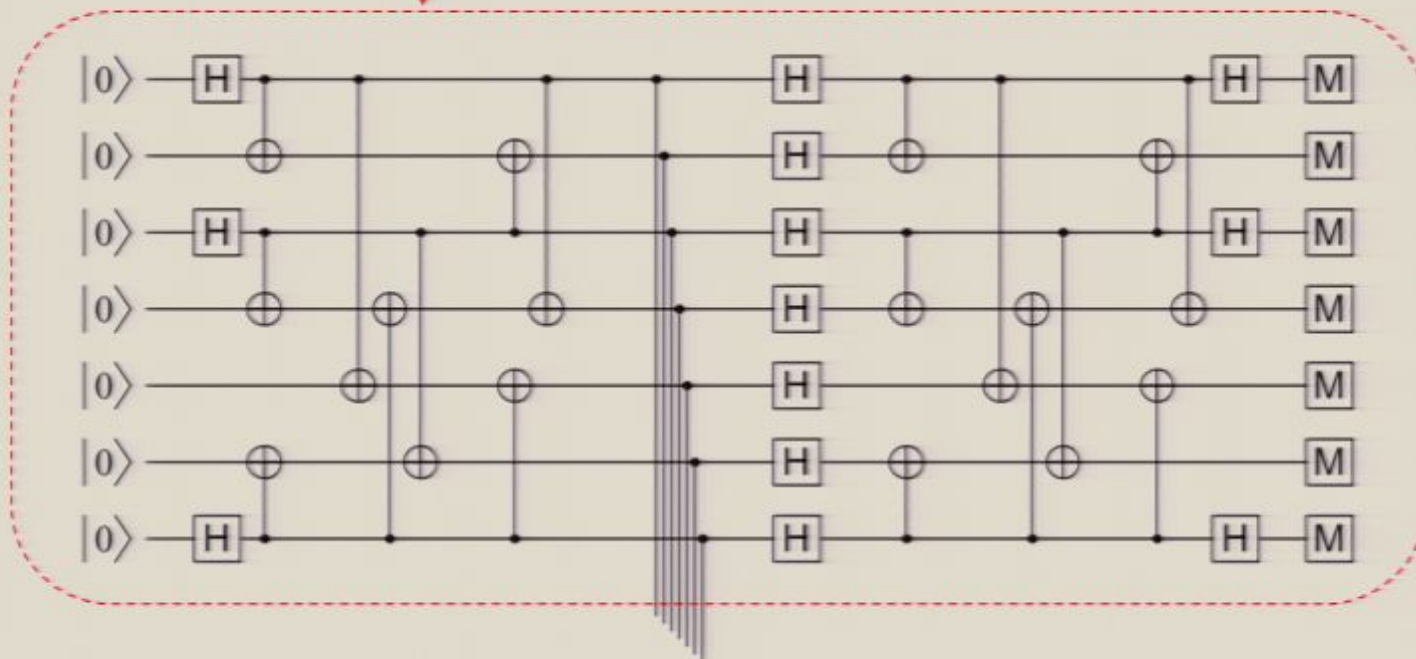
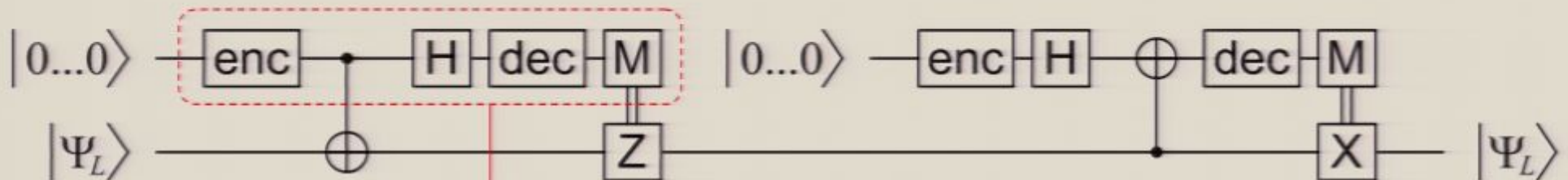
- Steane: threshold  $\sim 10^{-3}$  to  $10^{-4}$



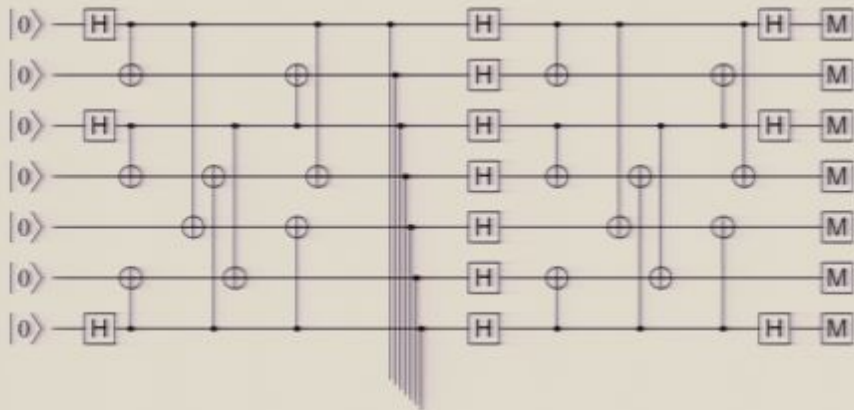
- Still need to repeat logical state preparation
- Still need to repeat measurements
- Still need large ancilla factories
- Still need very long-range interactions

# Interlude: error correction

- Steane/DiVincenzo PRL 98, 020501 (2007)

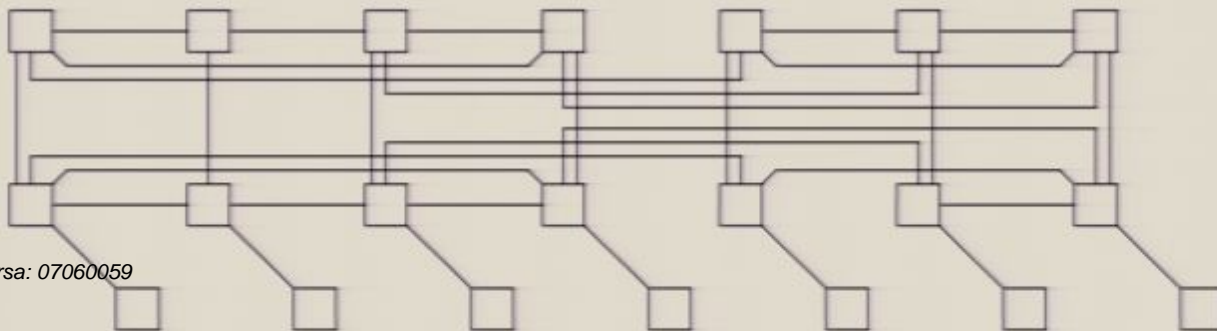
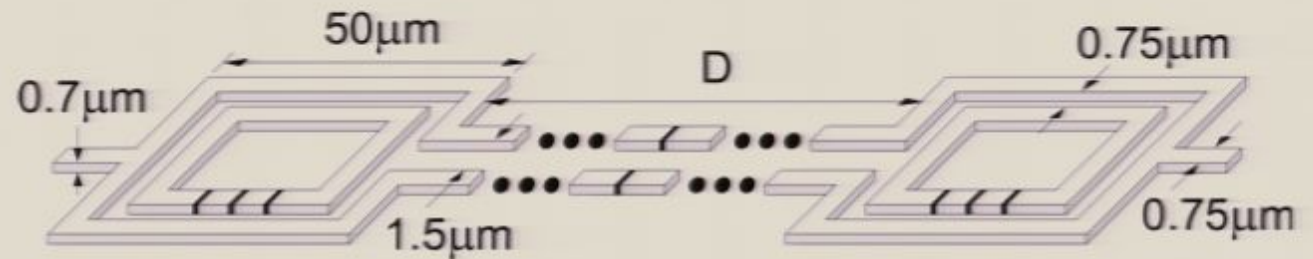


# Laying out the circuitry



- Error correction

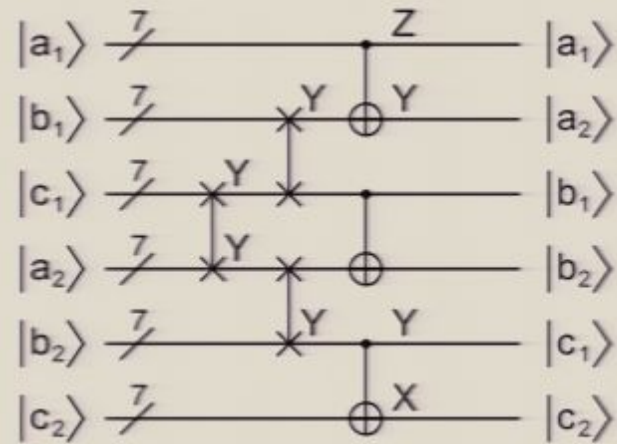
- Coupler



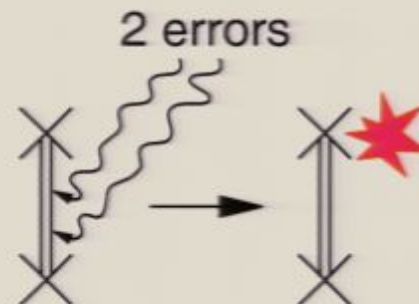
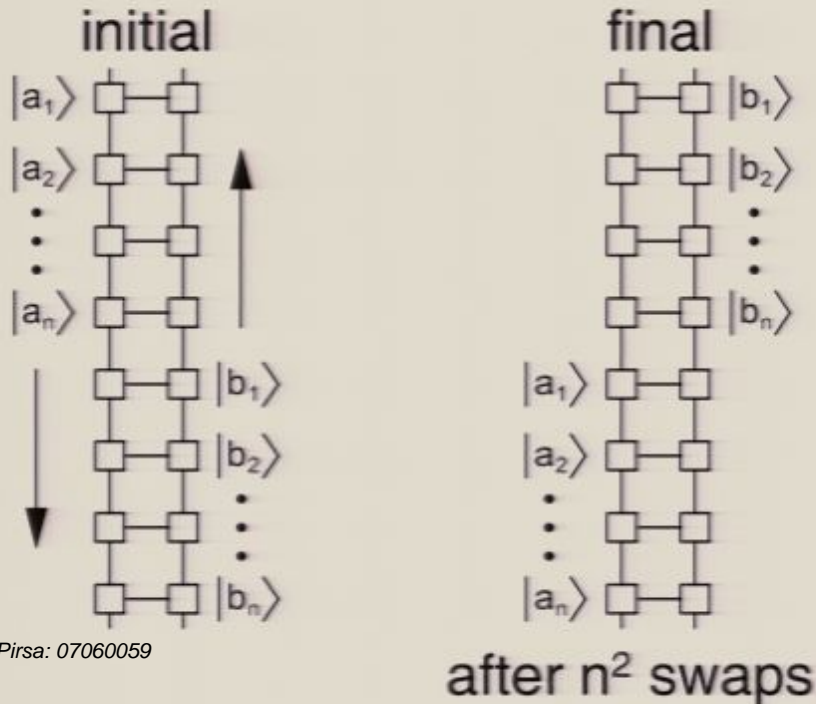
- Network

# Level 2 circuitry

- Must still avoid linear nearest neighbor

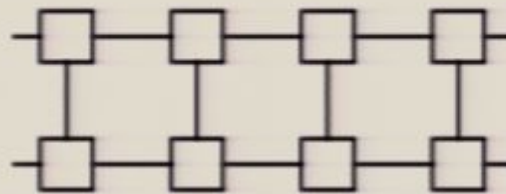


- Need logical bilinear network
- Permits fault-tolerant swap

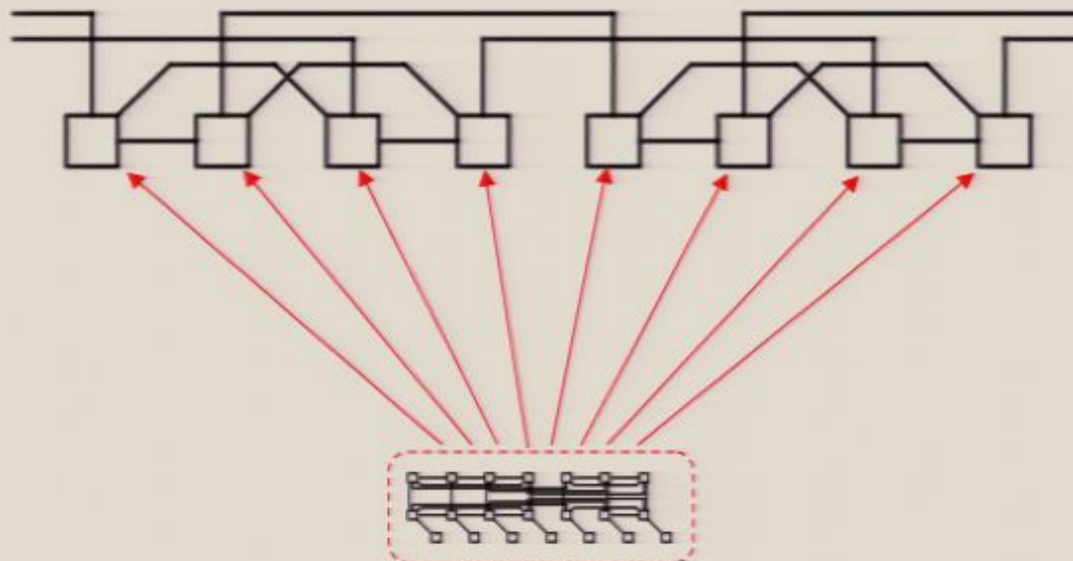


# Level 2 circuitry

- Can't do logical bilinear directly



- Need to stretch the design

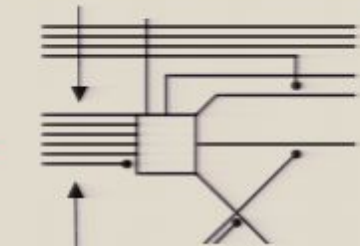




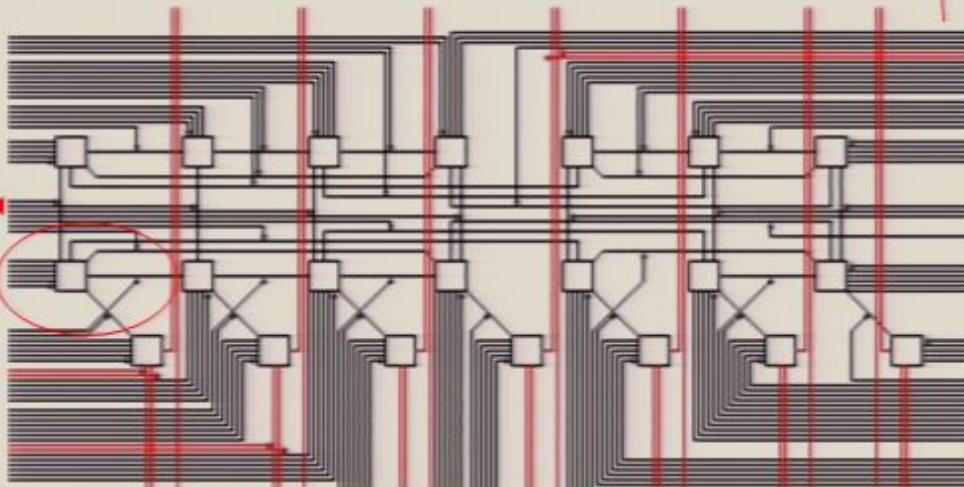
# Level 2 circuitry

- Must control each qubit

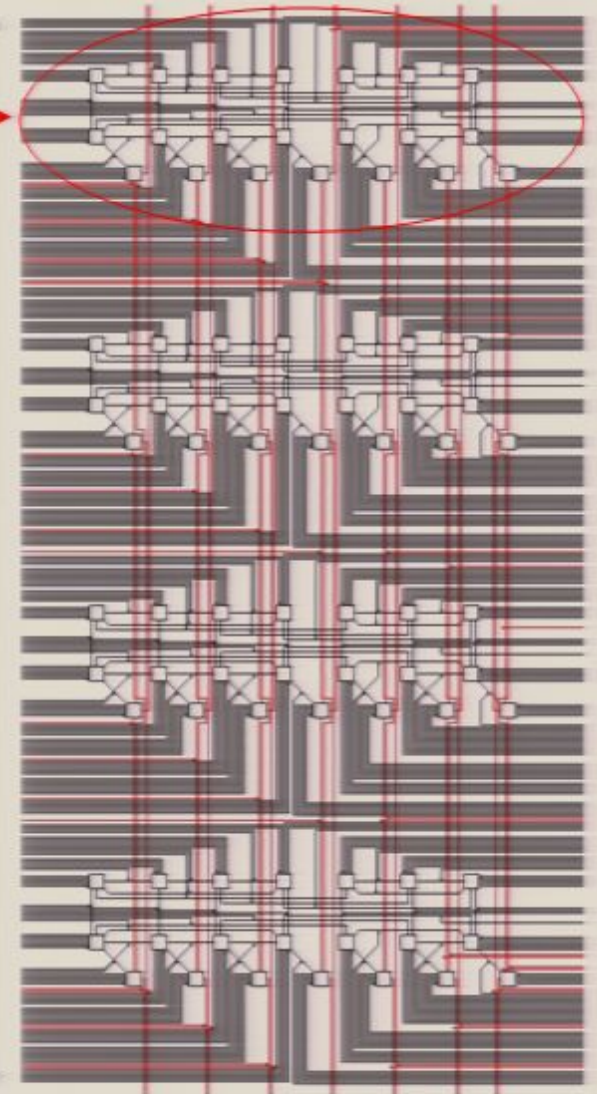
SQUID bias lines



flux bias lines



12 mm

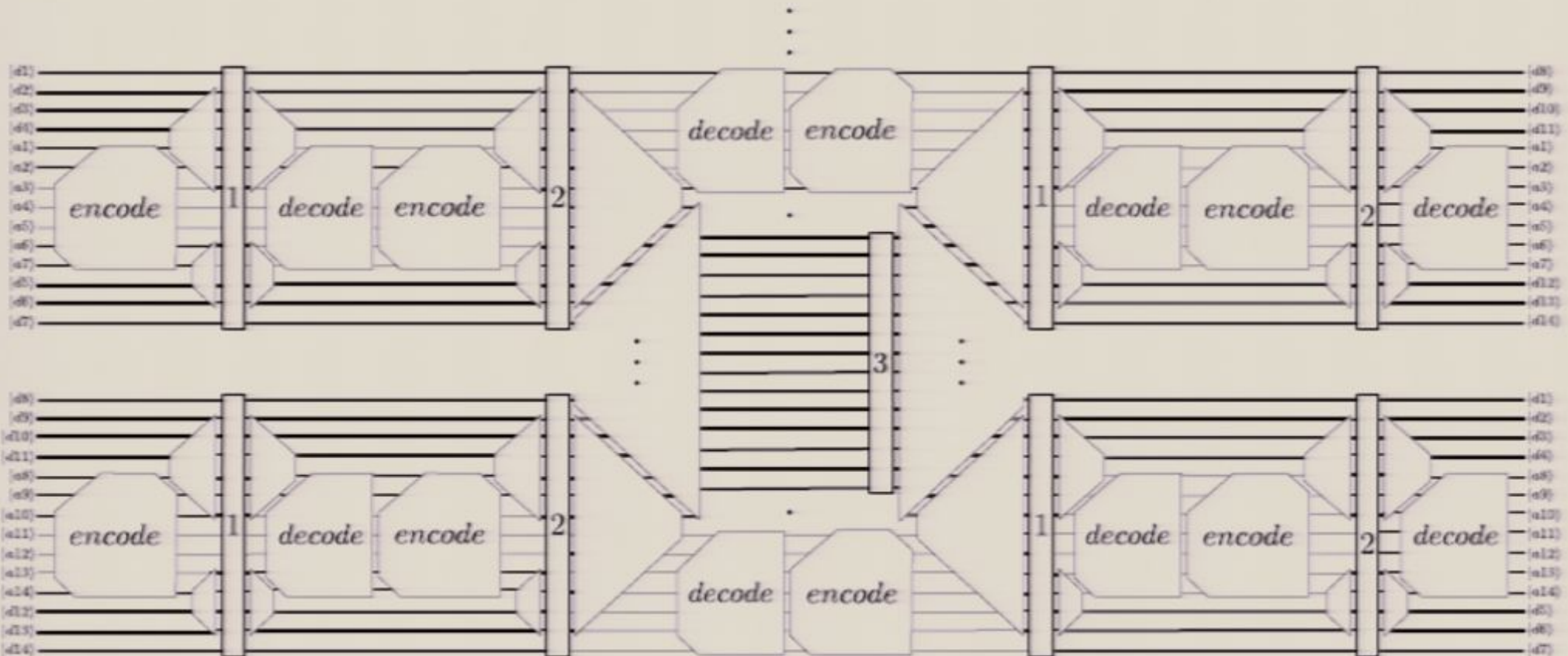


5 mm

- 1 wire per 40  $\mu\text{m}$

# Level 3 circuitry

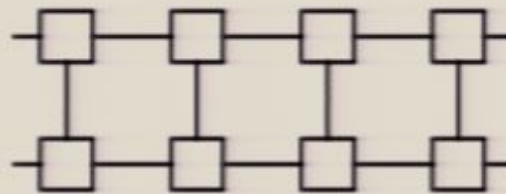
- Linear nearest neighbor with fault-tolerant swap



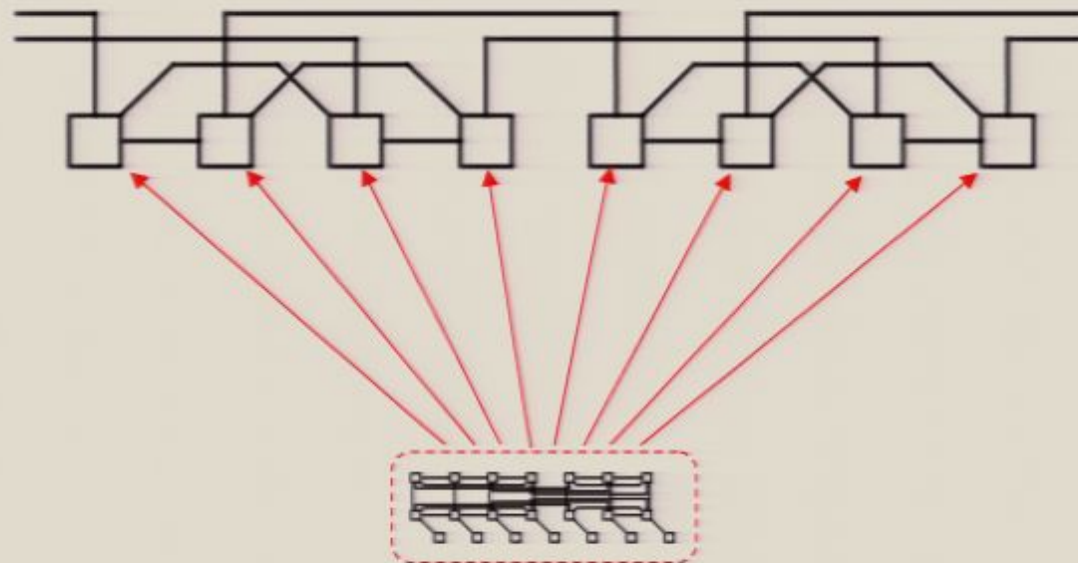
- Each horizontal line represents  $2 \times 21^2$  qubits

# Level 2 circuitry

- Can't do logical bilinear directly

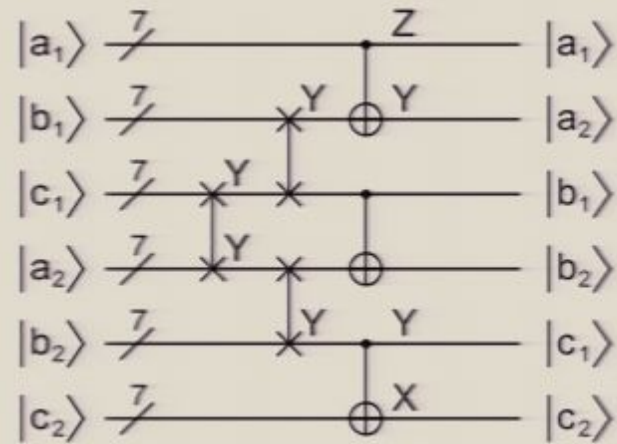


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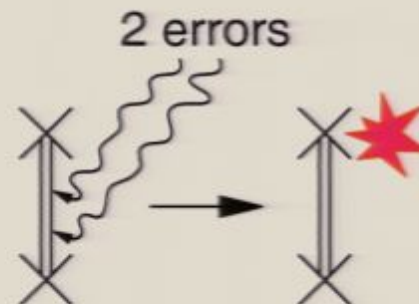
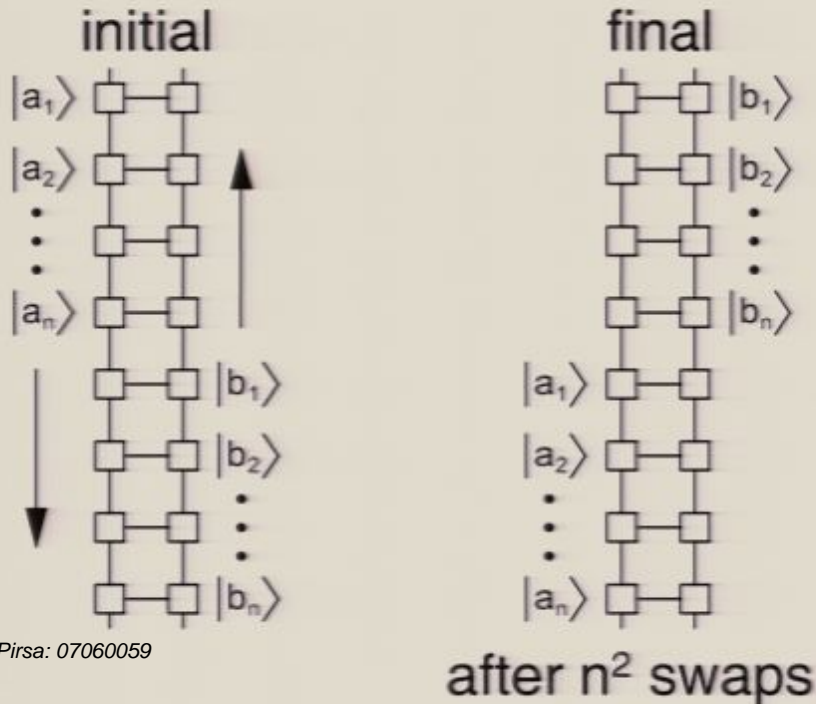


# Level 2 circuitry

- Must still avoid linear nearest neighbor

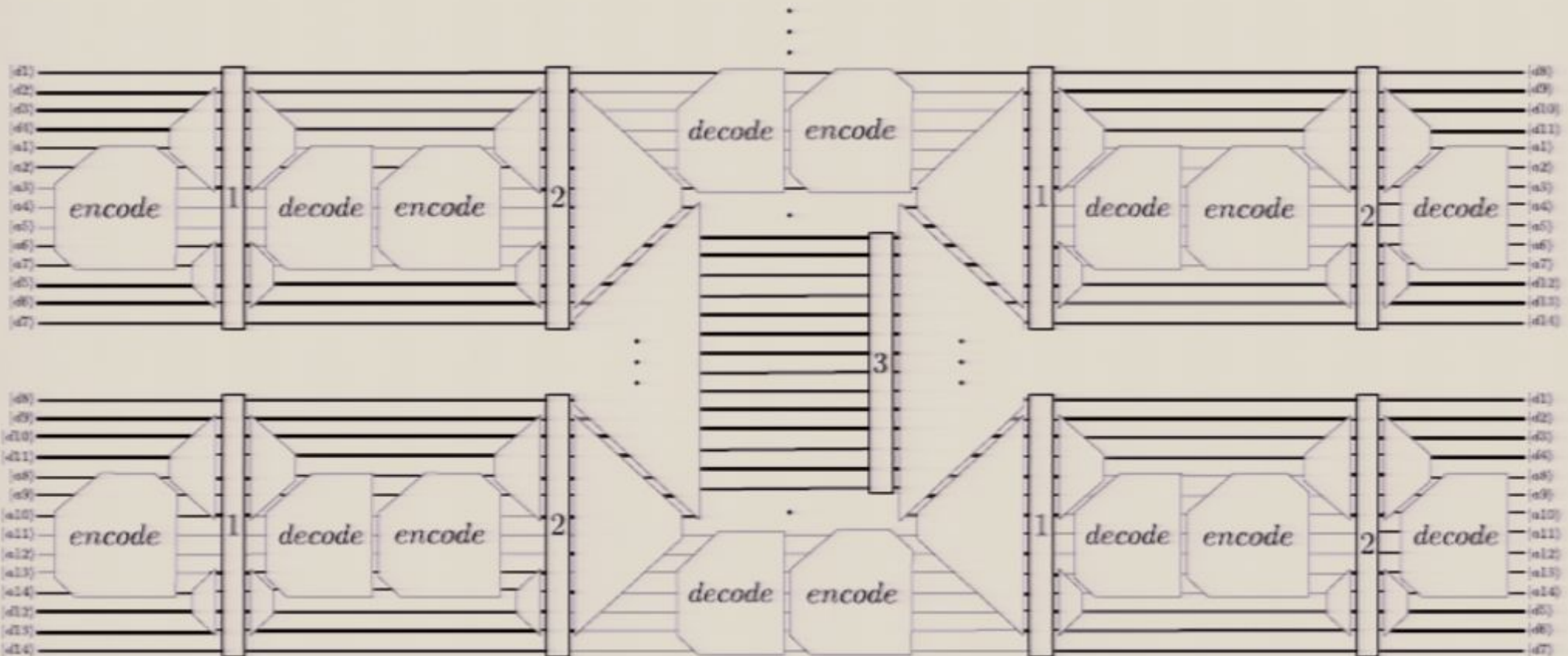


- Need logical bilinear network
- Permits fault-tolerant swap



# Level 3 circuitry

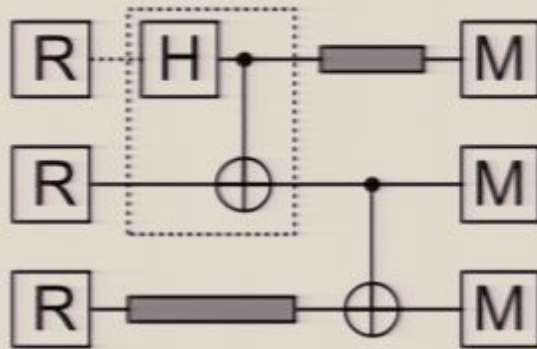
- Linear nearest neighbor with fault-tolerant swap



- Each horizontal line represents  $2 \times 21^2$  qubits

# Calculating thresholds

- Random example pretending to cope with one error



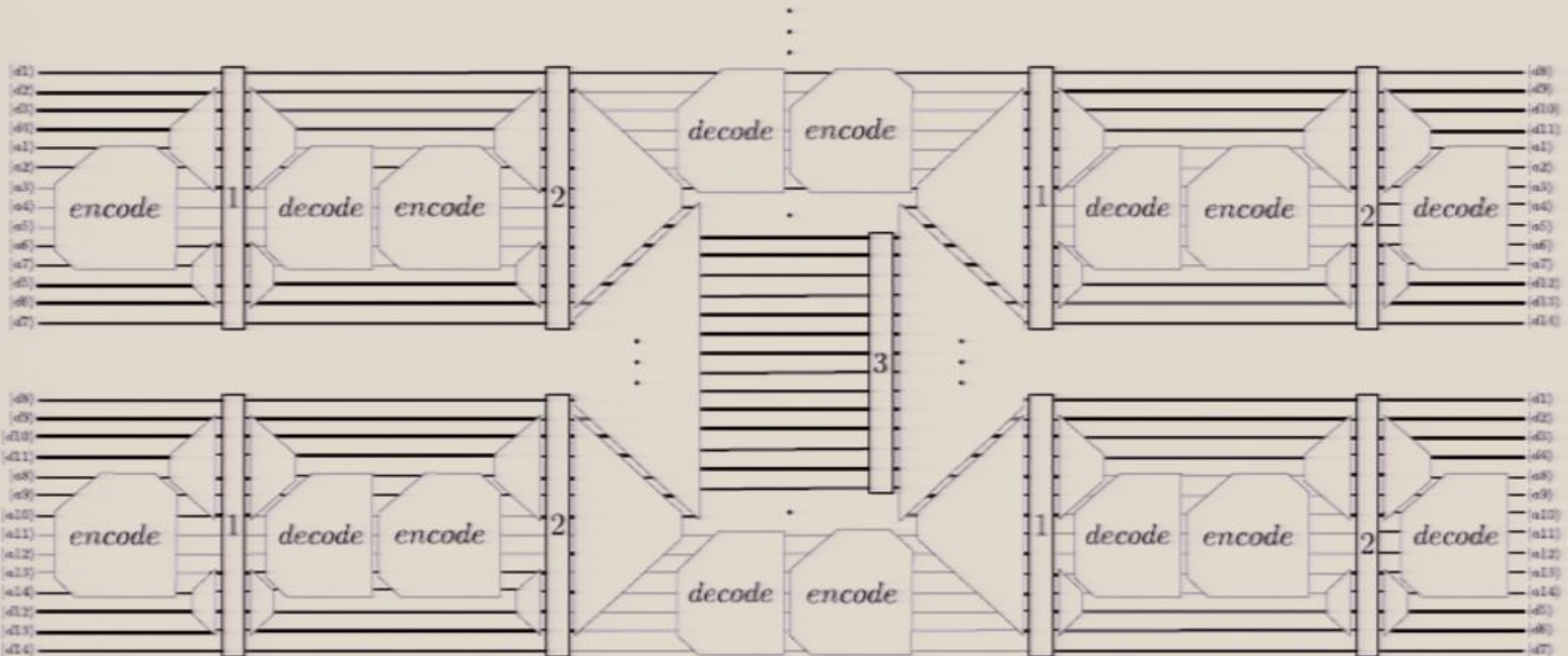
- Resets: 3
- Gates: 2
- Waits: 2
- Measurements: 3

$$P_{fail} = 1 - P_{success}$$

$$\begin{aligned}
 &= 1 - (1 - P_{reset})^3 (1 - P_{gate})^2 (1 - P_{wait})^2 (1 - P_{meas})^3 \\
 &\quad - P_{reset} (1 - P_{reset})^2 (1 - P_{gate})^2 (1 - P_{wait})^2 (1 - P_{meas})^3 \\
 &\quad - P_{gate} (1 - P_{reset})^3 (1 - P_{gate}) (1 - P_{wait})^2 (1 - P_{meas})^3 \\
 &\quad - P_{wait} (1 - P_{reset})^3 (1 - P_{gate})^2 (1 - P_{wait}) (1 - P_{meas})^3 \\
 &\quad - P_{meas} (1 - P_{reset})^3 (1 - P_{gate})^2 (1 - P_{wait})^2 (1 - P_{meas})^2
 \end{aligned}$$

# Level 3 circuitry

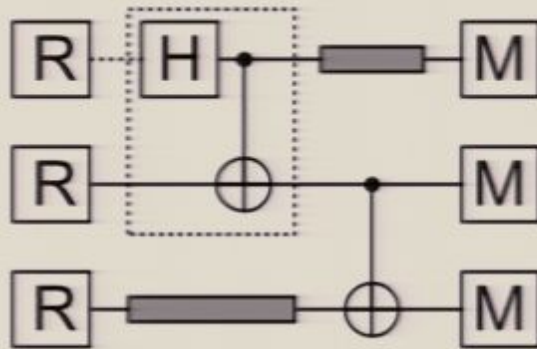
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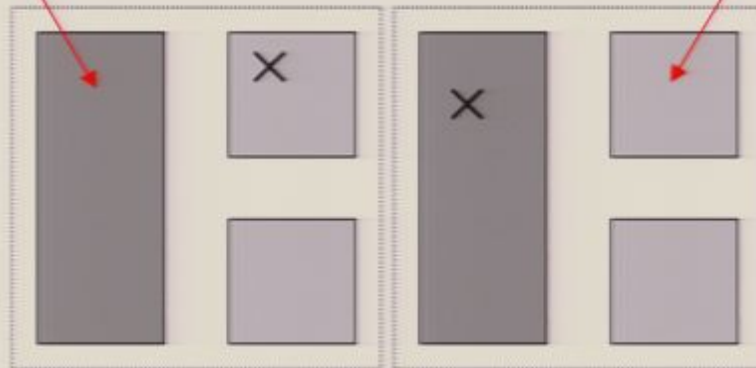


# Extended rectangles

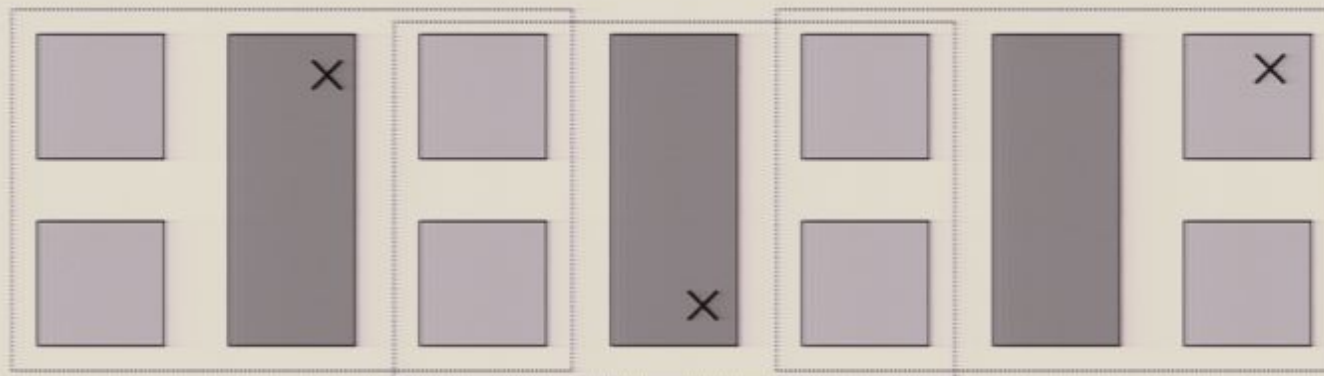
- How much circuitry needs to be included?

Logical interaction

Error correction



X



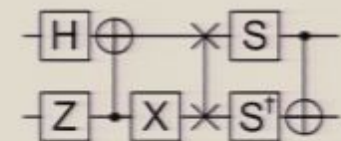
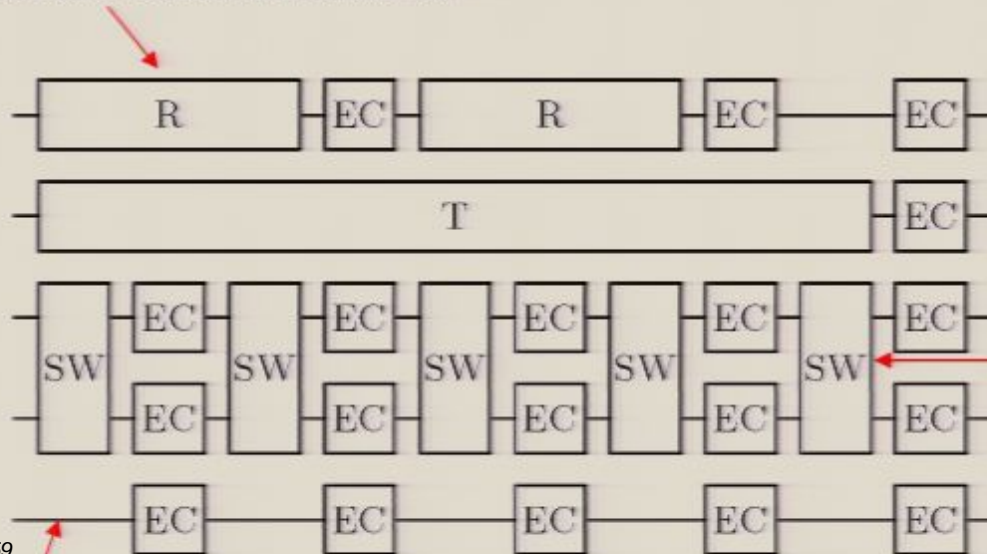
# Which threshold?

- Must calculate threshold of most complex gate
- Universal gate set:
  - H, X, Z, S, S<sup>†</sup>, and all combinations (23)
  - CNOT, SWAP
  - Measure, initialize, wait
  - T-gate ( $\pi/8$ -gate)

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

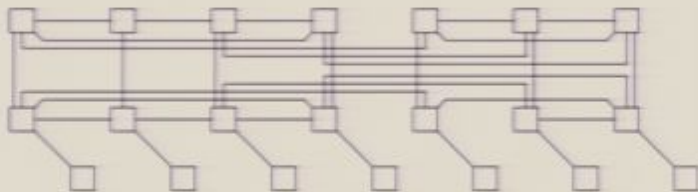
measure and initialize



Zhang et. al.,  
PRA 67 042313 (2003)

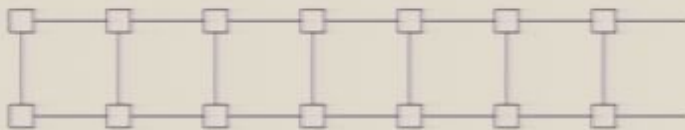
# Three universal gate sets

- Non-local network



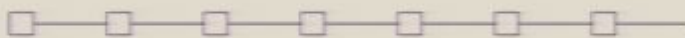
	$i = m$	$i = S$	$i = r$	depth
$j = m$	$206 + 28t_r$	100	28	$15 + 2t_r$
$j = S$	$398 + 56t_r$	207	56	$15 + 2t_r$
$j = T$	$1067 + 133t_r$	289	98	$75 + 10t_r$
$j = r$	$366 + 42t_r$	125	42	$30 + 4t_r$

- Bilinear network

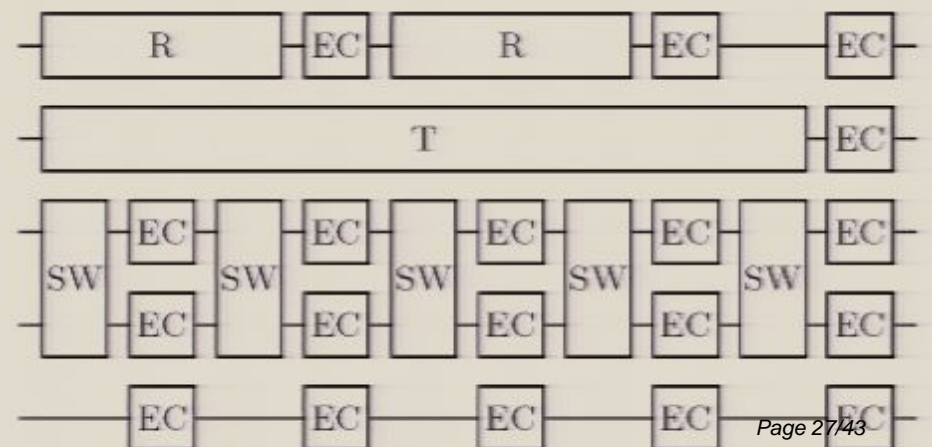


	$i = m$	$i = S$	$i = T$	$i = r$	depth
$j = m$	710	408	0	40	45
$j = S$	1114	1122	0	80	45
$j = T$	3171	1330	28	137	225
$j = r$	1129	510	0	57	90

- Linear nearest neighbor



	$i = m$	$i = S$	$i = T$	$i = r$	depth
$j = m$	558	204	0	28	38
$j = S$	824	603	0	56	38
$j = T$	2496	670	28	98	190
$j = r$	974	255	0	42	76



# Thresholds for the architecture

- Four variables:  $p_{\text{swap}}$ ,  $p_{\text{memory}}$ ,  $p_{\text{readout}}$ ,  $t_{\text{readout}}$
- Set:  $p_{\text{memory}} = 0.1 p_{\text{swap}}$ ,  $p_{\text{readout}} = p_{\text{swap}}$ ,  $t_{\text{readout}} = 10$

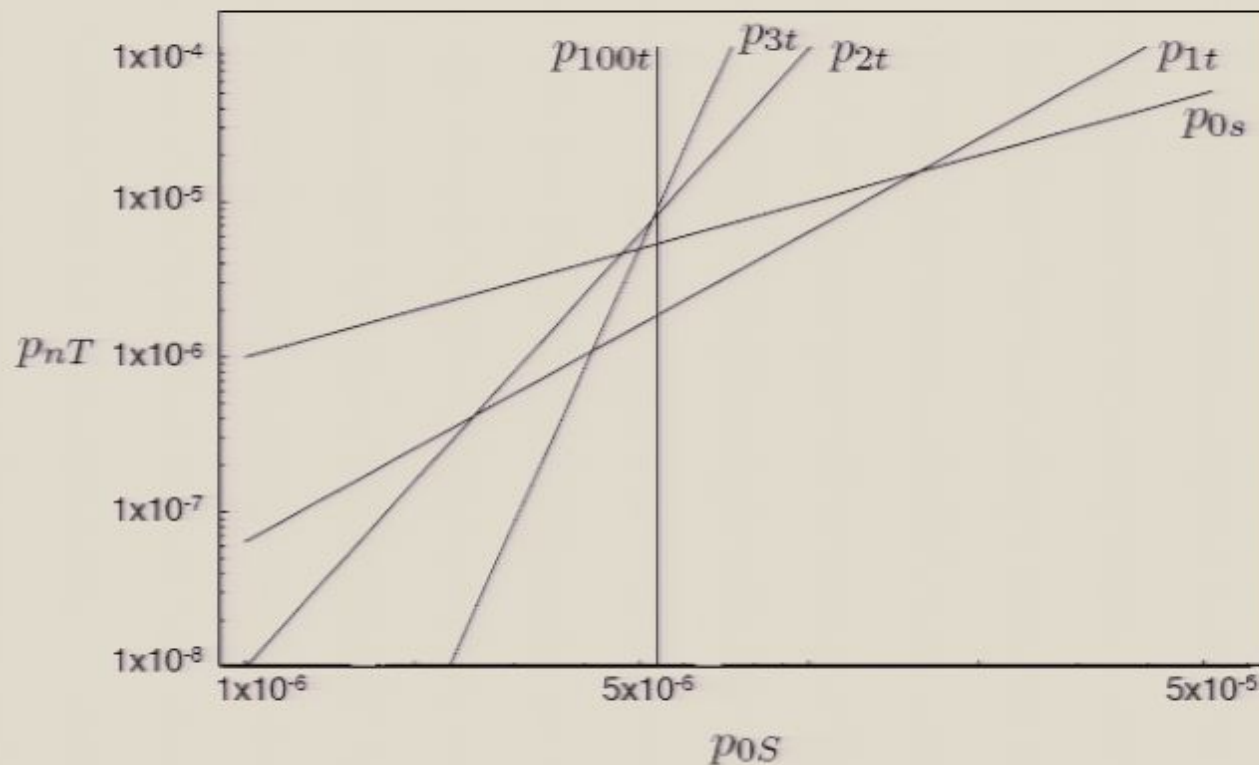
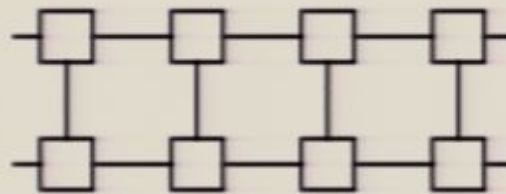


FIG. 1:  $p_0S$  and  $p_nT(p_0S, 0.1, 1.0, 10.0)$  for  $n = \{1, 2, 3, 100\}$ .  
The lower bound to the 100  $T$  threshold is  $5.36 \times 10^{-6}$ .

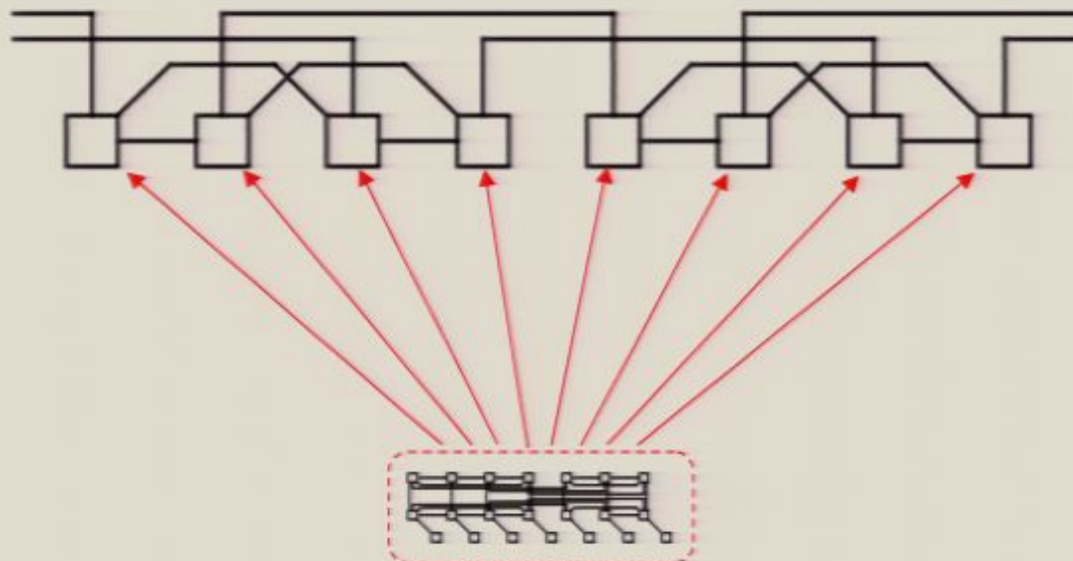
- Infinite level threshold:  $\sim 5 \times 10^{-6}$ , level-1 threshold  $\sim 10^{-5}$

# Level 2 circuitry

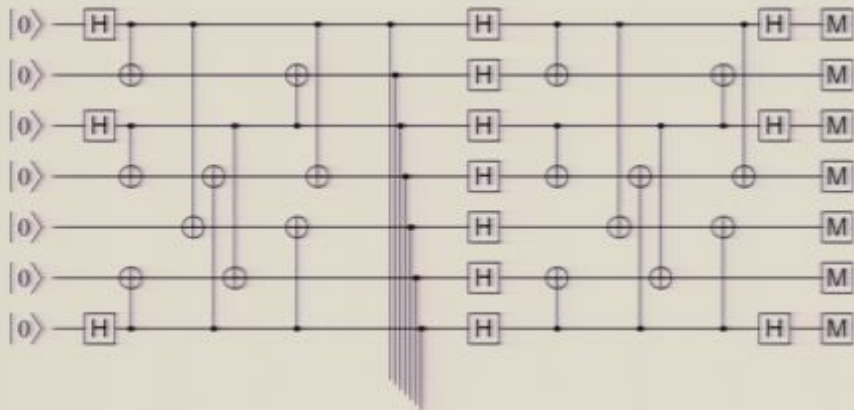
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- Need to stretch the design

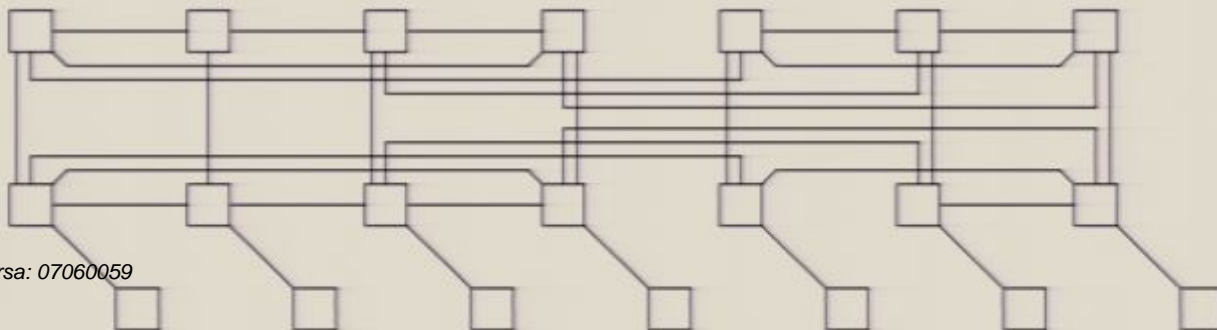
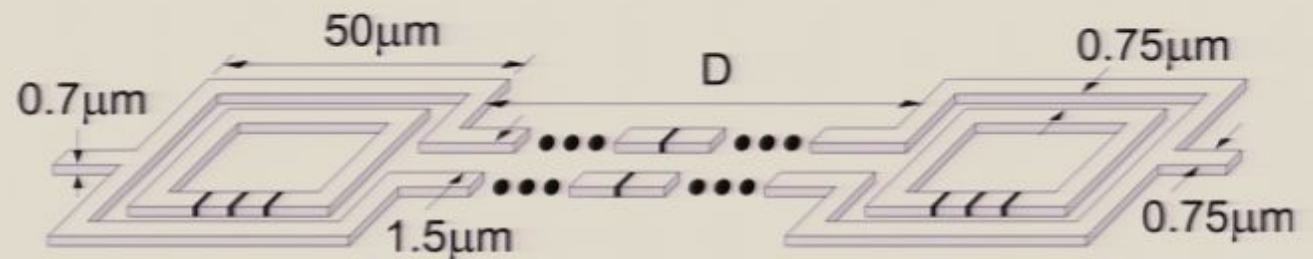


# Laying out the circuitry



- Error correction

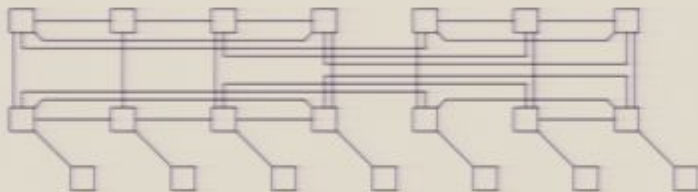
- Coupler



- Network

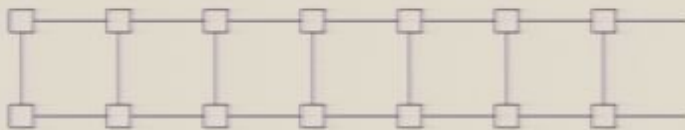
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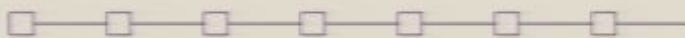
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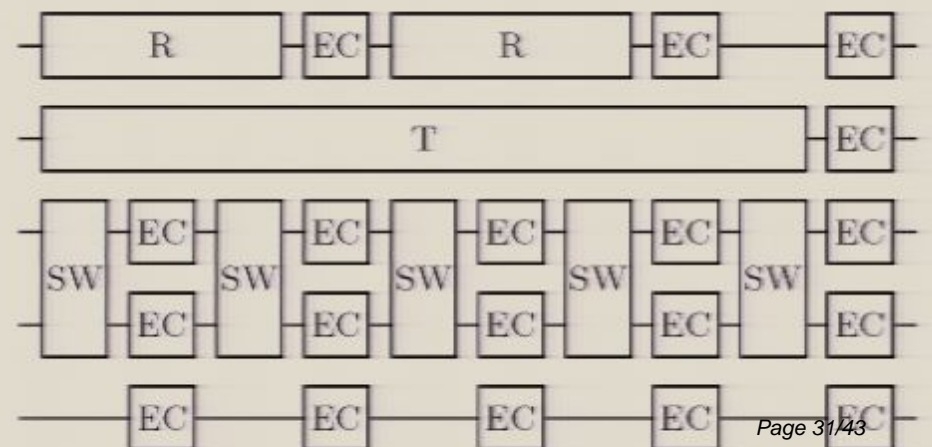


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$j = S$	824	603	0	56	38
$j = T$	2496	670	28	98	190
$j = r$	974	255	0	42	76



# Thresholds for the architecture

- Four variables:  $p_{\text{swap}}$ ,  $p_{\text{memory}}$ ,  $p_{\text{readout}}$ ,  $t_{\text{readout}}$
- Set:  $p_{\text{memory}} = 0.1 p_{\text{swap}}$ ,  $p_{\text{readout}} = p_{\text{swap}}$ ,  $t_{\text{readout}} = 10$

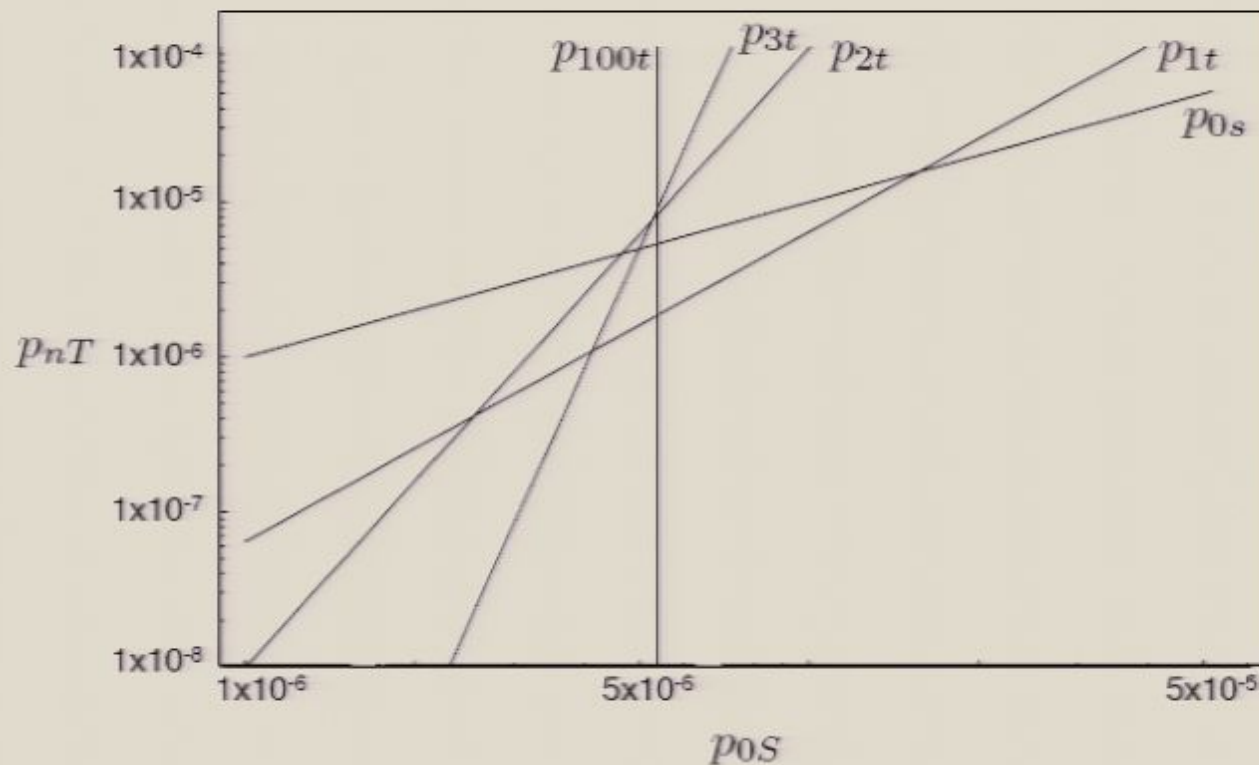


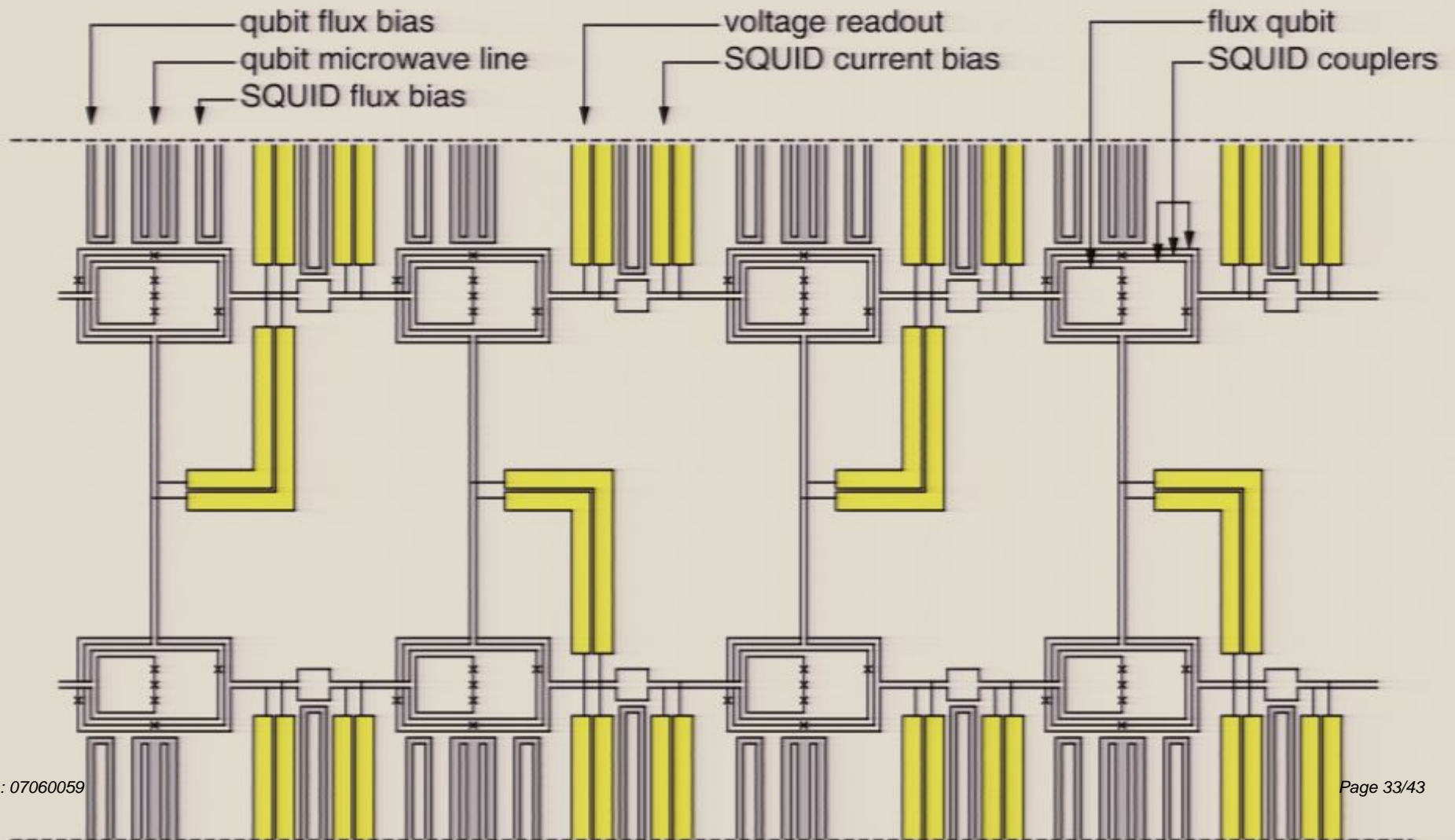
FIG. 1:  $p_{0S}$  and  $p_{nT}(p_{0S}, 0.1, 1.0, 10.0)$  for  $n = \{1, 2, 3, 100\}$ .  
The lower bound to the 100  $T$  threshold is  $5.36 \times 10^{-6}$ .

- Infinite level threshold:  $\sim 5 \times 10^{-6}$ , level-1 threshold  $\sim 10^{-5}$



# Bilinear details

- Comparable threshold  $2 \times 10^{-6}$  (Stephens, quant-ph/0702201)
- Simpler construction likely to outweigh lower threshold



# Thresholds for the architecture

- Four variables:  $p_{\text{swap}}$ ,  $p_{\text{memory}}$ ,  $p_{\text{readout}}$ ,  $t_{\text{readout}}$
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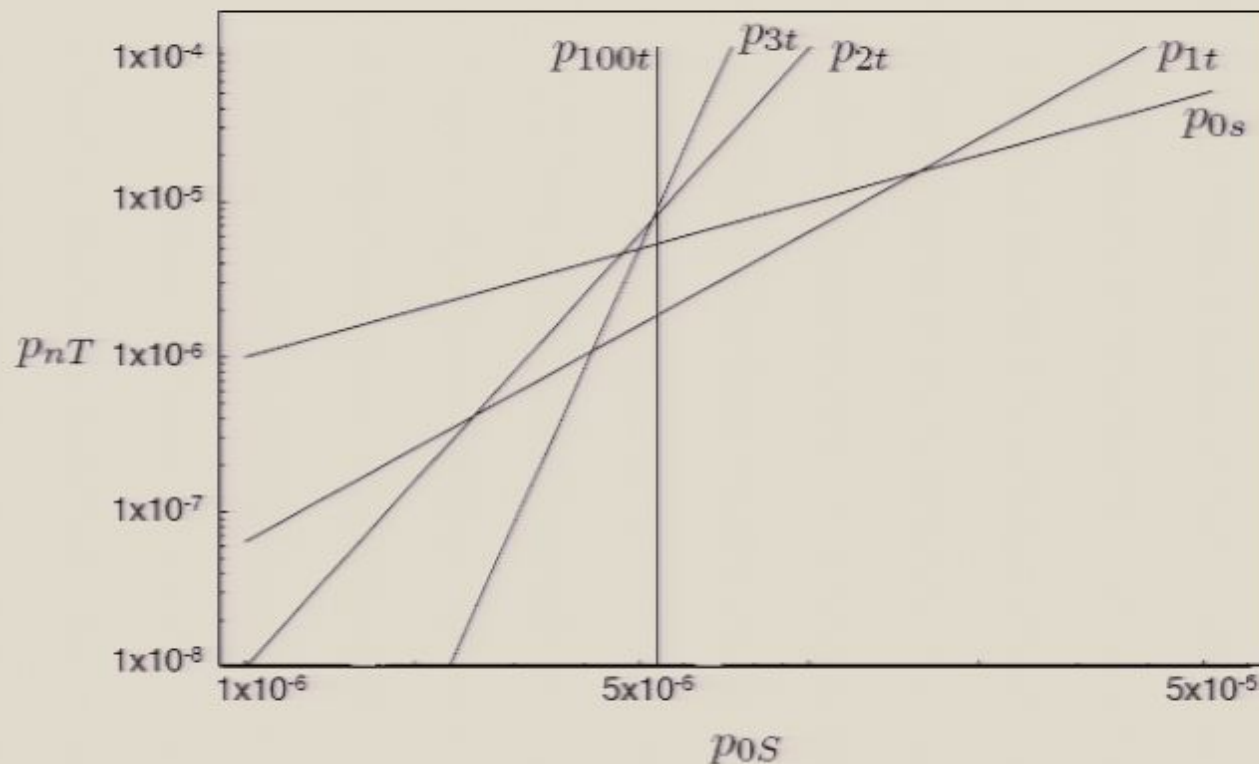
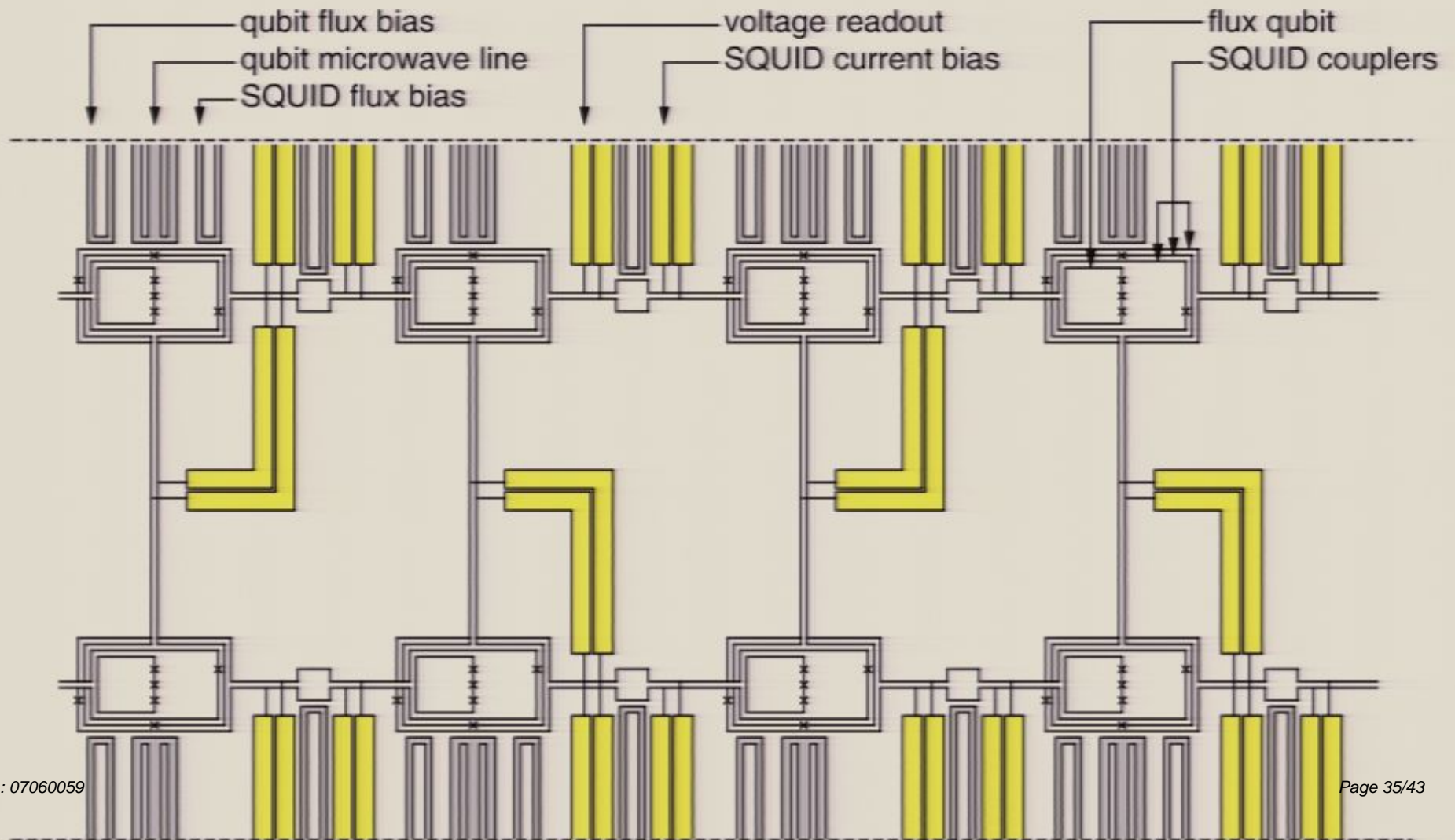


FIG. 1:  $p_{0S}$  and  $p_n T(p_{0S}, 0.1, 1.0, 10.0)$  for  $n = \{1, 2, 3, 100\}$ .  
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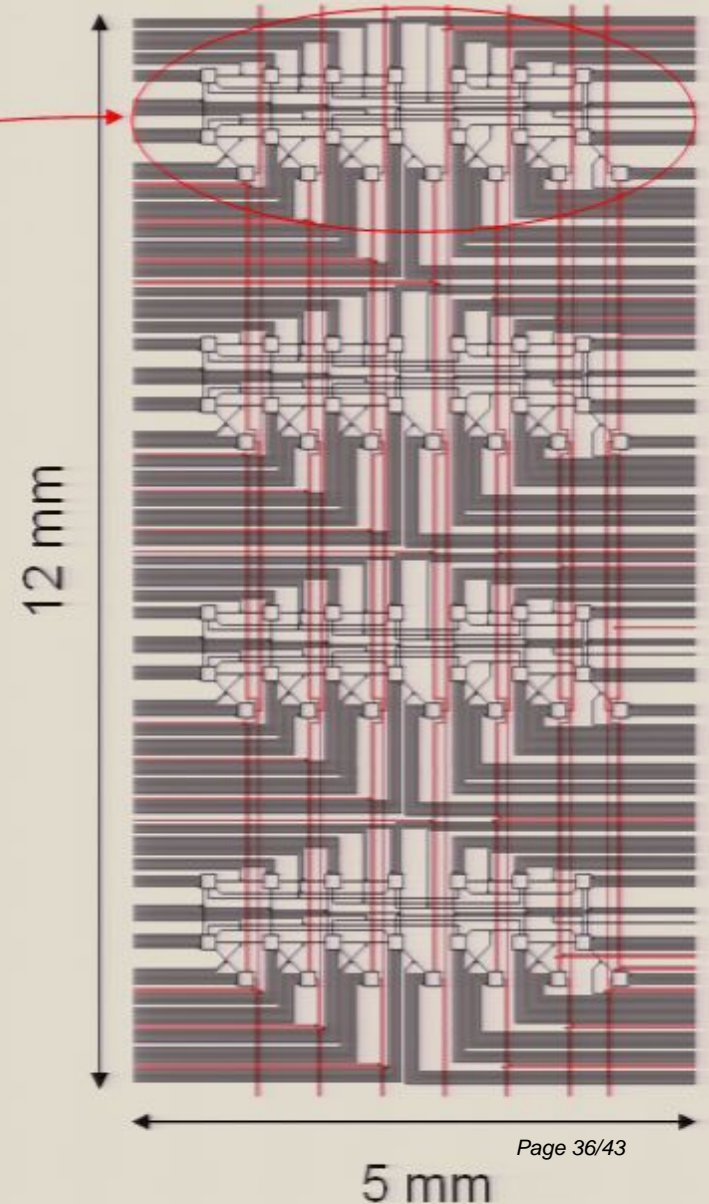
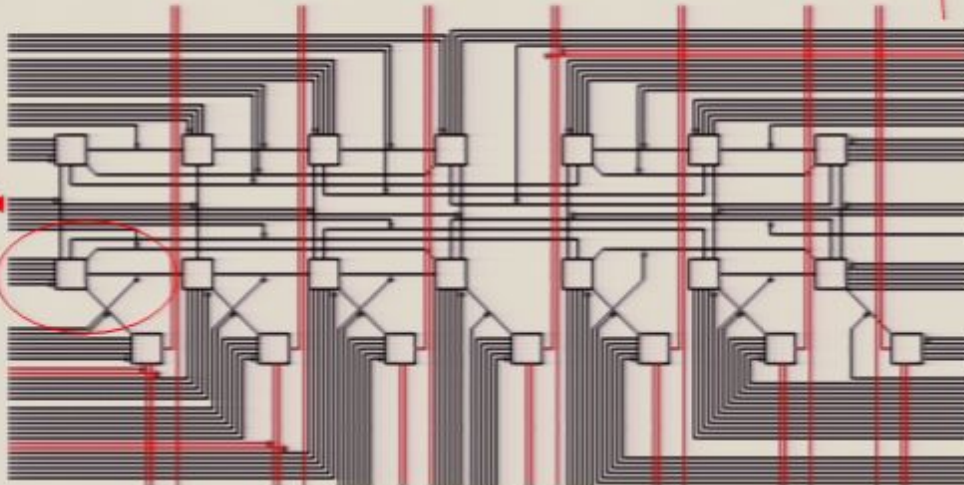
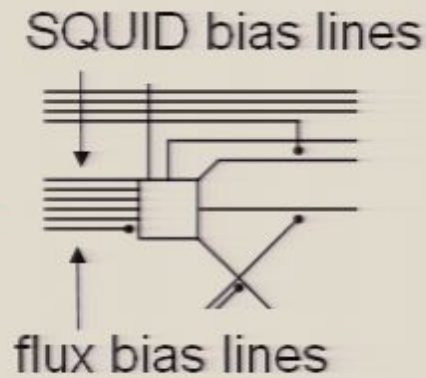
# Bilinear details

- Comparable threshold  $2 \times 10^{-6}$  (Stephens, quant-ph/0702201)
- Simpler construction likely to outweigh lower threshold



# Level 2 circuitry

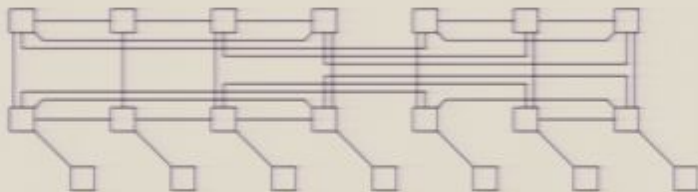
- Must control each qubit



- 1 wire per  $40 \mu\text{m}$

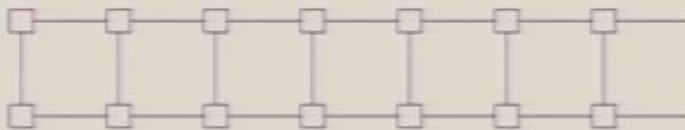
# Three universal gate sets

- Non-local network



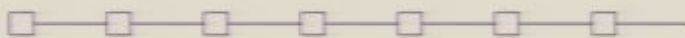
	$i = m$	$i = S$	$i = r$	depth
$j = m$	$206 + 28t_r$	100	28	$15 + 2t_r$
$j = S$	$398 + 56t_r$	207	56	$15 + 2t_r$
$j = T$	$1067 + 133t_r$	289	98	$75 + 10t_r$
$j = r$	$366 + 42t_r$	125	42	$30 + 4t_r$

- Bilinear network

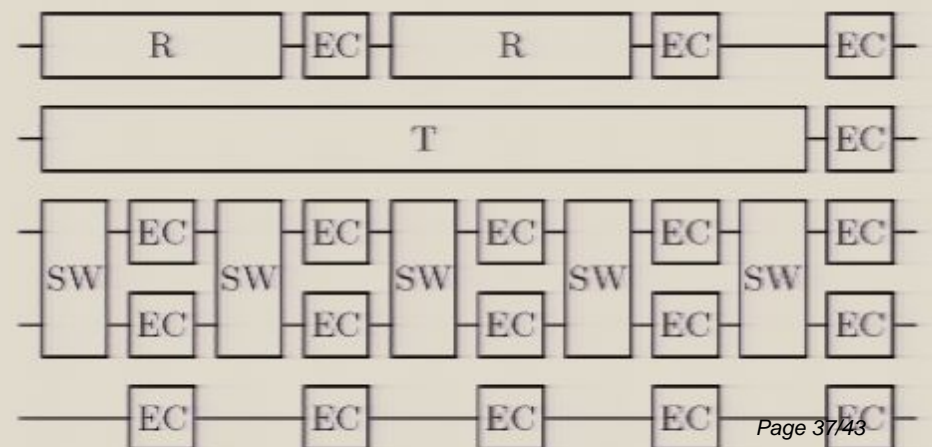


	$i = m$	$i = S$	$i = T$	$i = r$	depth
$j = m$	710	408	0	40	45
$j = S$	1114	1122	0	80	45
$j = T$	3171	1330	28	137	225
$j = r$	1129	510	0	57	90

- Linear nearest neighbor

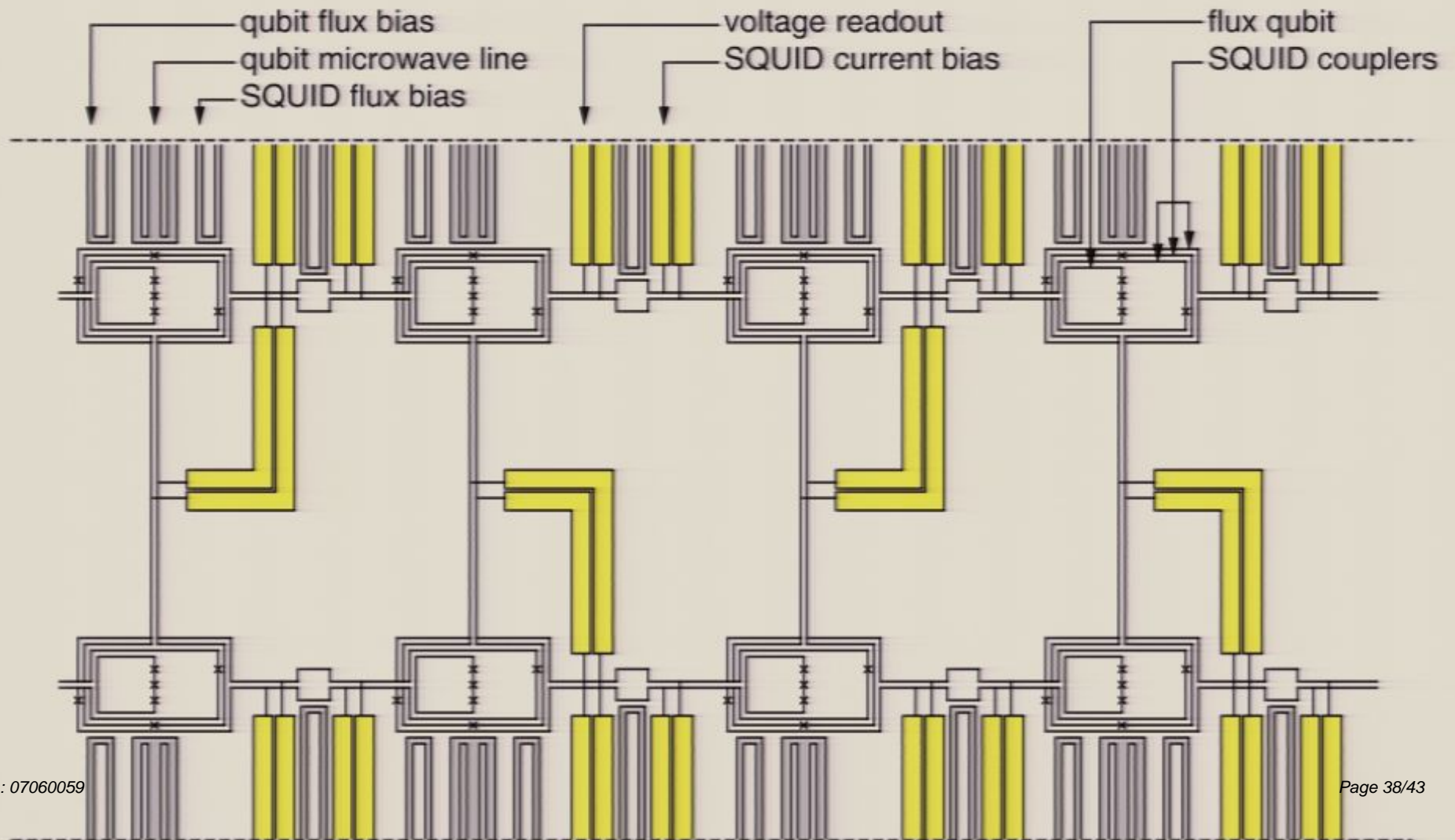


	$i = m$	$i = S$	$i = T$	$i = r$	depth
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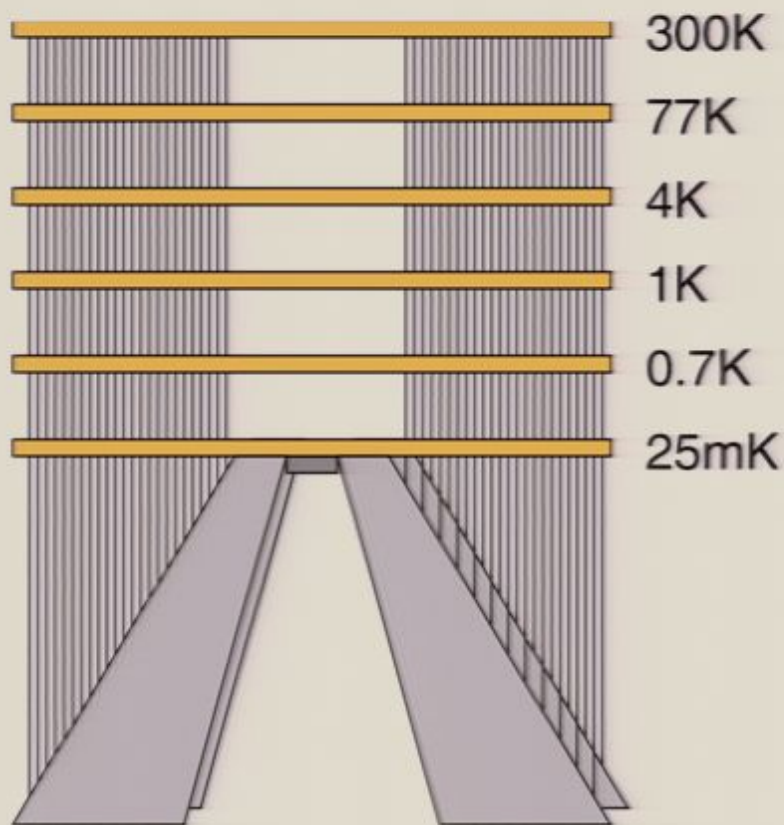


# Bilinear details

- Comparable threshold  $2 \times 10^{-6}$  (Stephens, quant-ph/0702201)
- Simpler construction likely to outweigh lower threshold



# Further work



- Cooling – details
- Wire density – serious limitation
- Crosstalk and shielding
- Interchip spanning schemes
- Algorithms without error correction

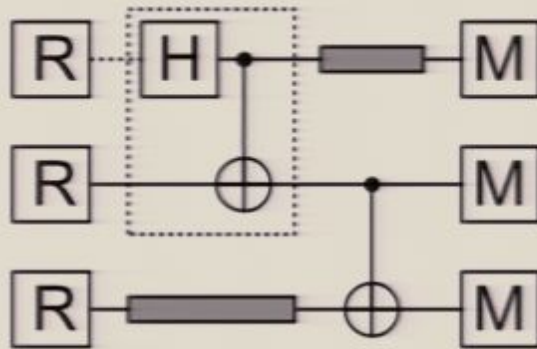
# Conclusion

- There is no universal threshold of  $10^{-4}$
- Without long-range interactions, threshold much lower
- For flux qubit architecture, threshold  $\sim 5 \times 10^{-6}$
- Implies gate error rates of  $\sim 10^{-7}$  needed
- May need to compute without error correction



# Calculating thresholds

- Random example pretending to cope with one error



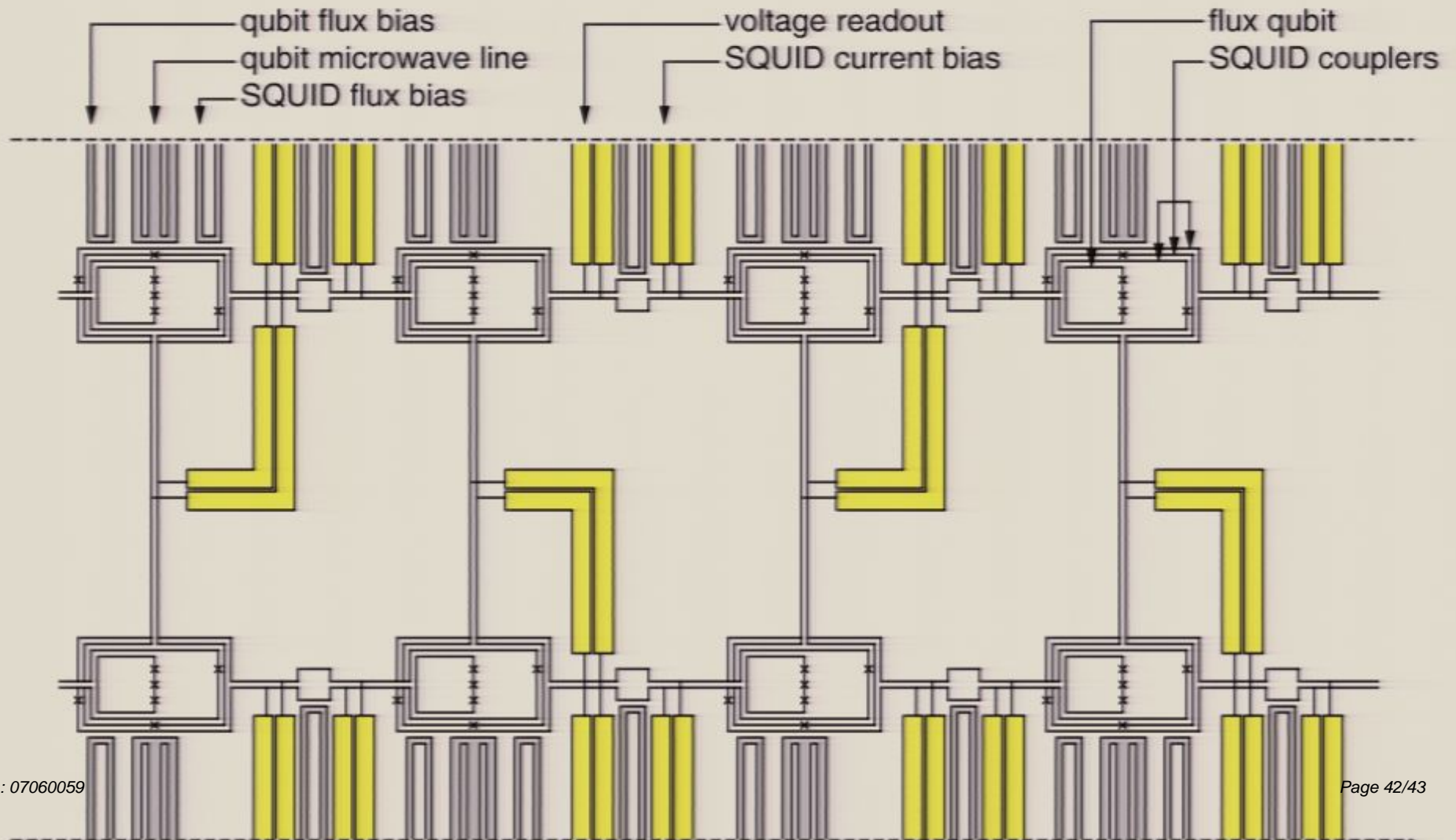
- Resets: 3
- Gates: 2
- Waits: 2
- Measurements: 3

$$P_{fail} = 1 - P_{success}$$

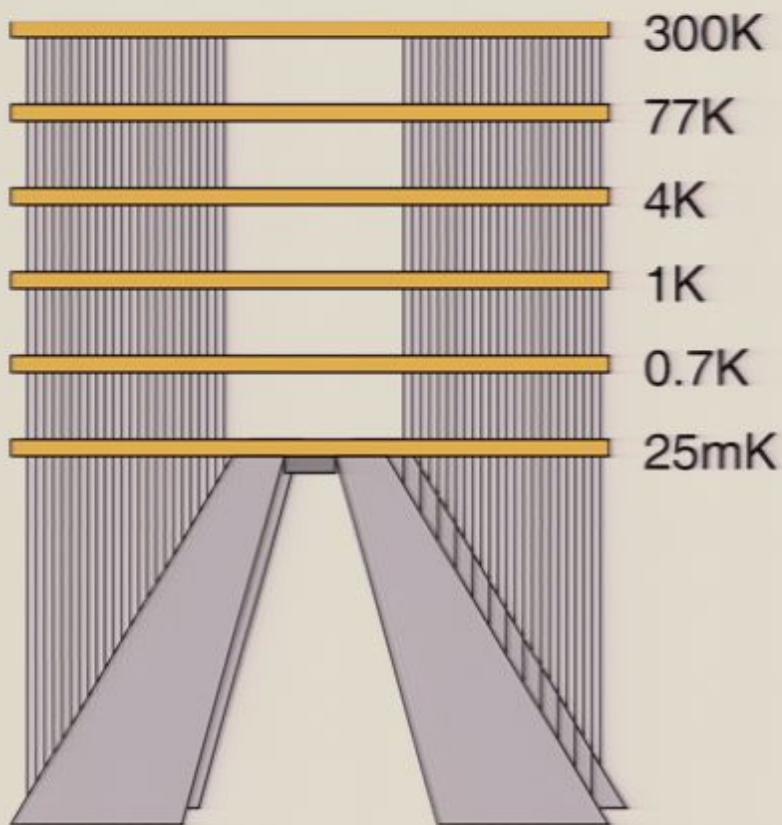
$$\begin{aligned}
 &= 1 - (1 - P_{reset})^3 (1 - P_{gate})^2 (1 - P_{wait})^2 (1 - P_{meas})^3 \\
 &\quad - P_{reset} (1 - P_{reset})^2 (1 - P_{gate})^2 (1 - P_{wait})^2 (1 - P_{meas})^3 \\
 &\quad - P_{gate} (1 - P_{reset})^3 (1 - P_{gate}) (1 - P_{wait})^2 (1 - P_{meas})^3 \\
 &\quad - P_{wait} (1 - P_{reset})^3 (1 - P_{gate})^2 (1 - P_{wait}) (1 - P_{meas})^3 \\
 &\quad - P_{meas} (1 - P_{reset})^3 (1 - P_{gate})^2 (1 - P_{wait})^2 (1 - P_{meas})^2
 \end{aligned}$$

# Bilinear details

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# Further work



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