Title: Perfect Cluster States from Imperfect Entanglement in Optical Lattices

Date: Jun 15, 2007 12:35 PM

URL: http://pirsa.org/07060055

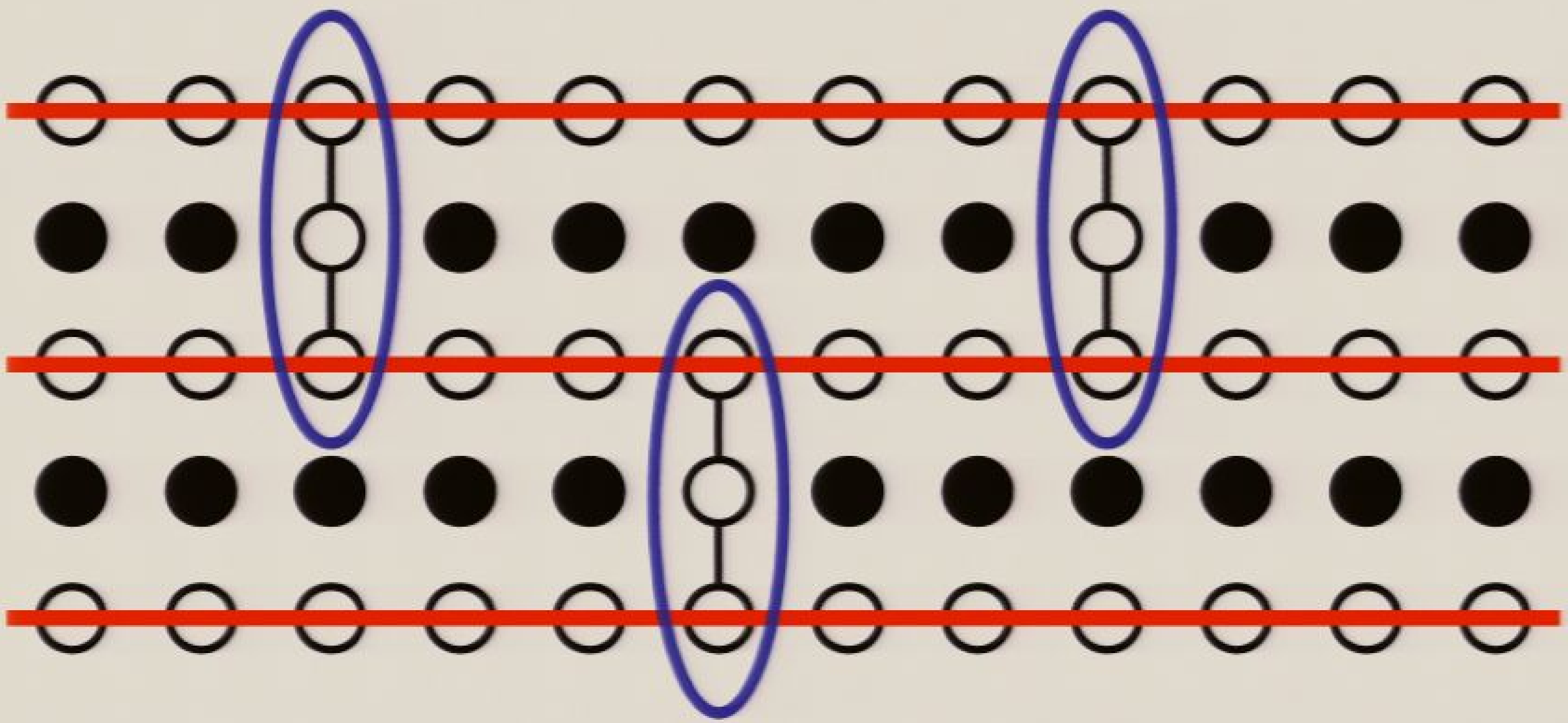
Abstract:

What's One-Way Quantum Computing?

1) Remove unwanted qubits: Z-basis measurements

→ *real-space quantum circuit*

2) Computation via XY measurements



horizontal chains = quantum wires & 1-qubit gates

vertical links = 2-qubit gates

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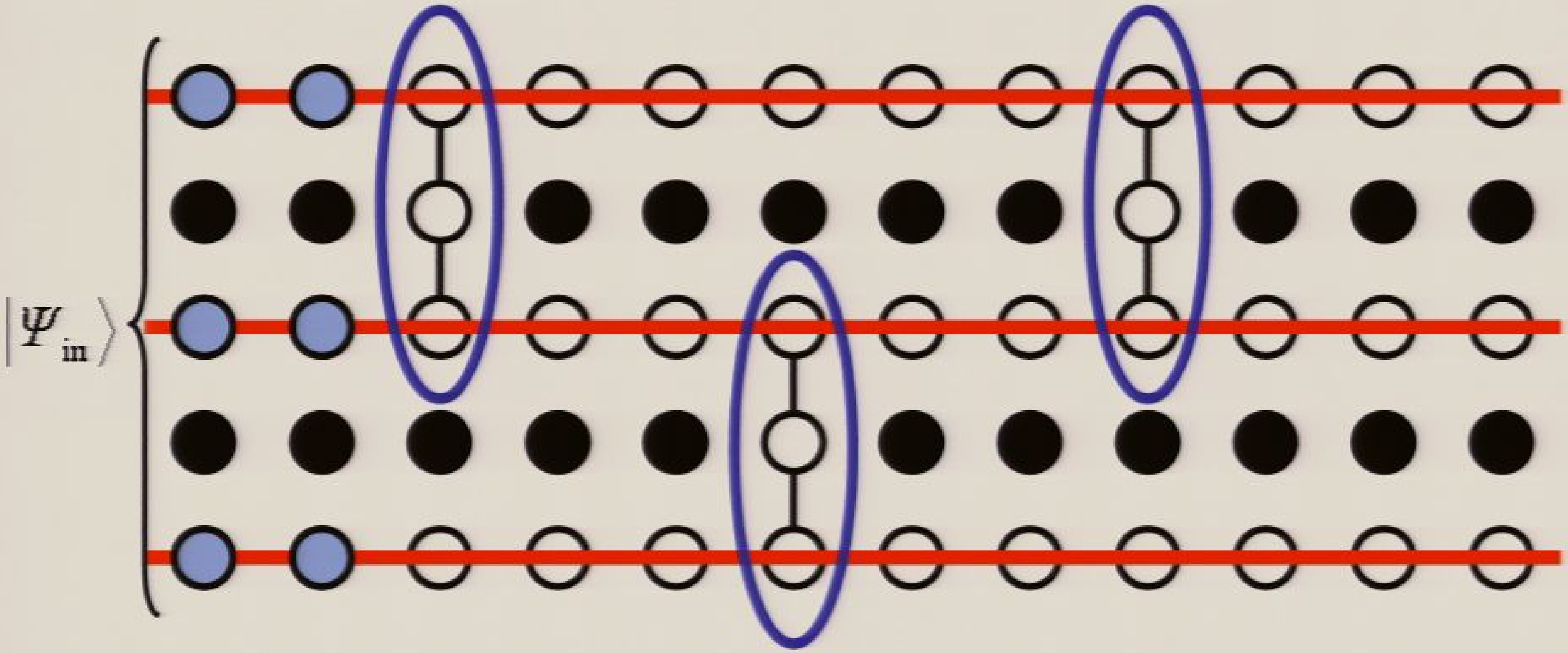

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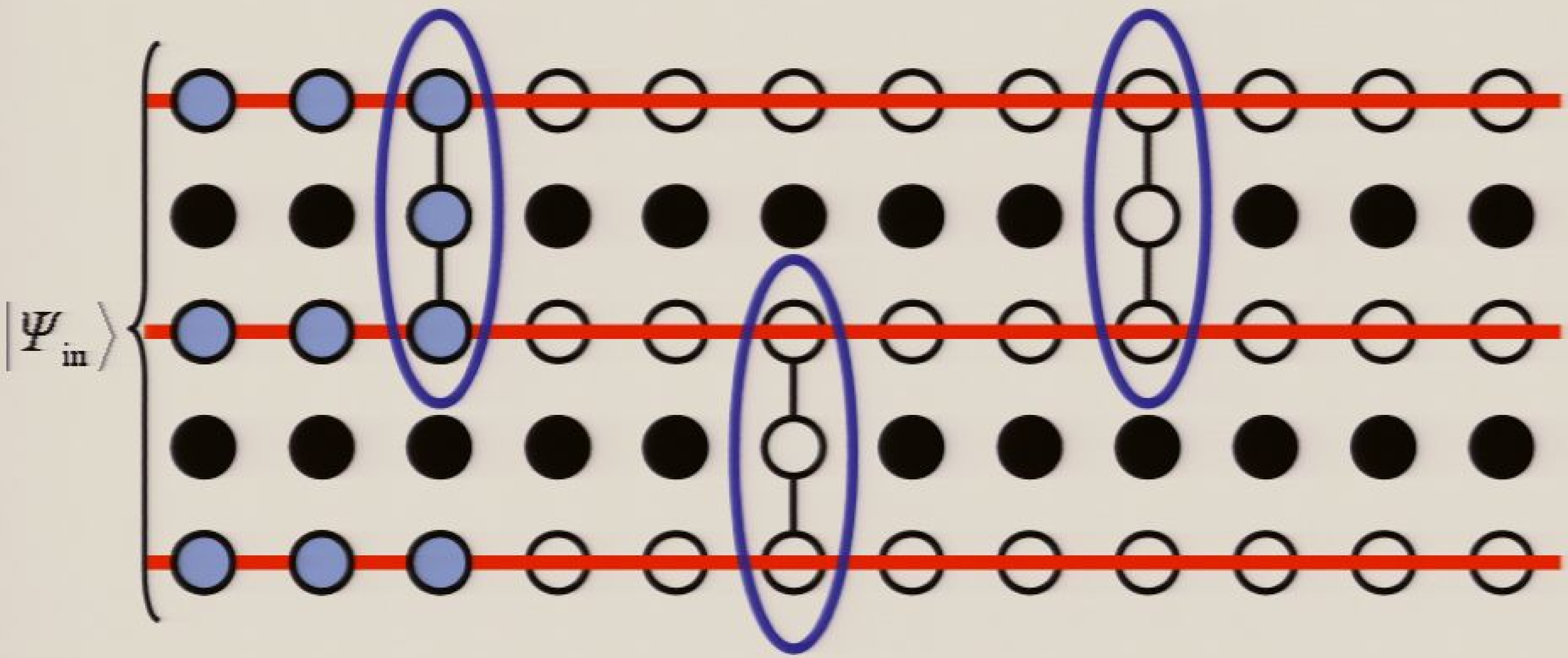

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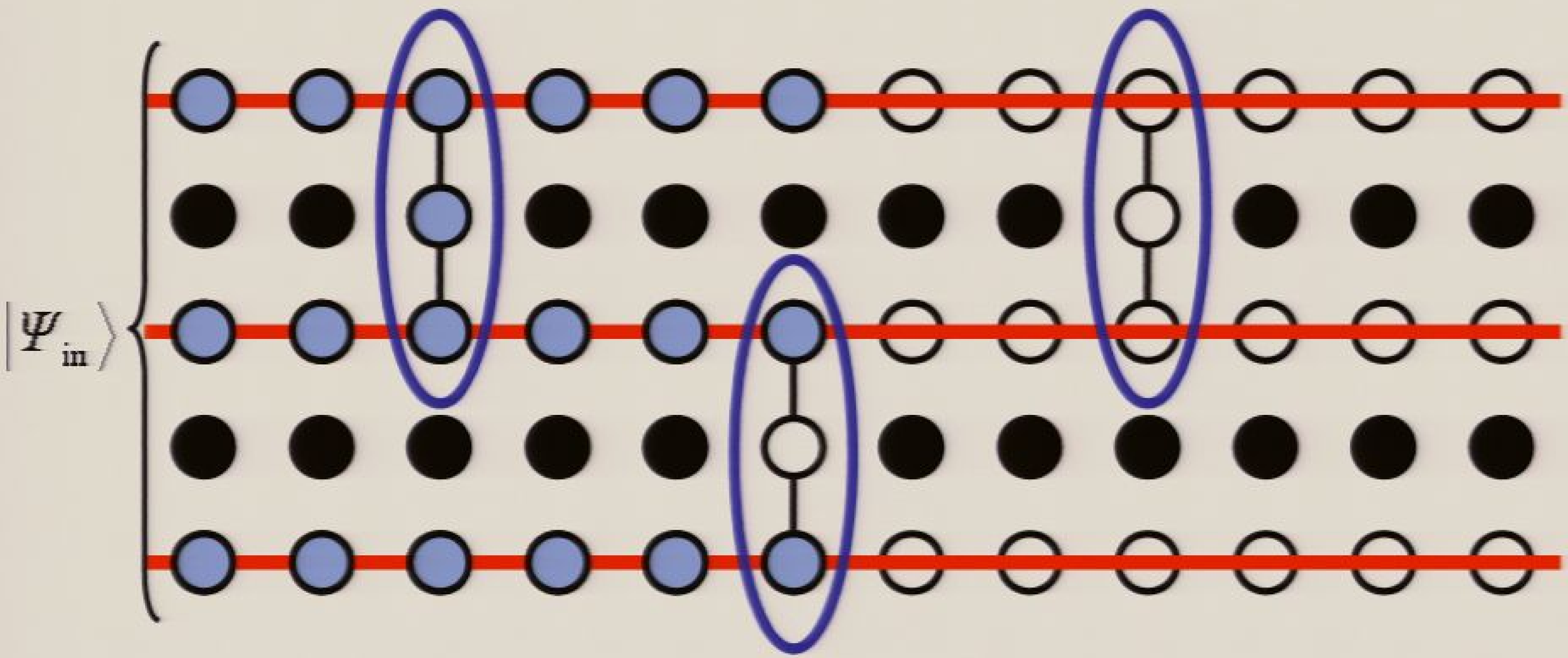

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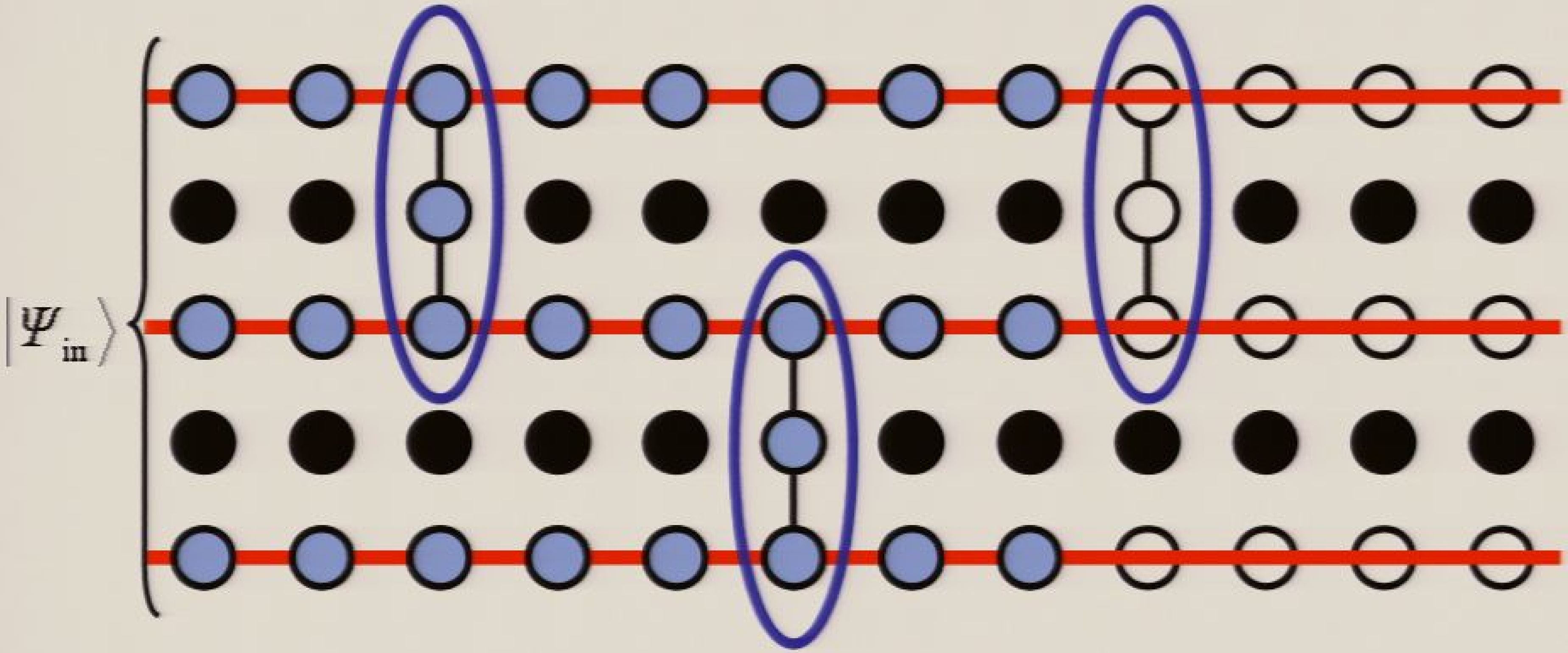

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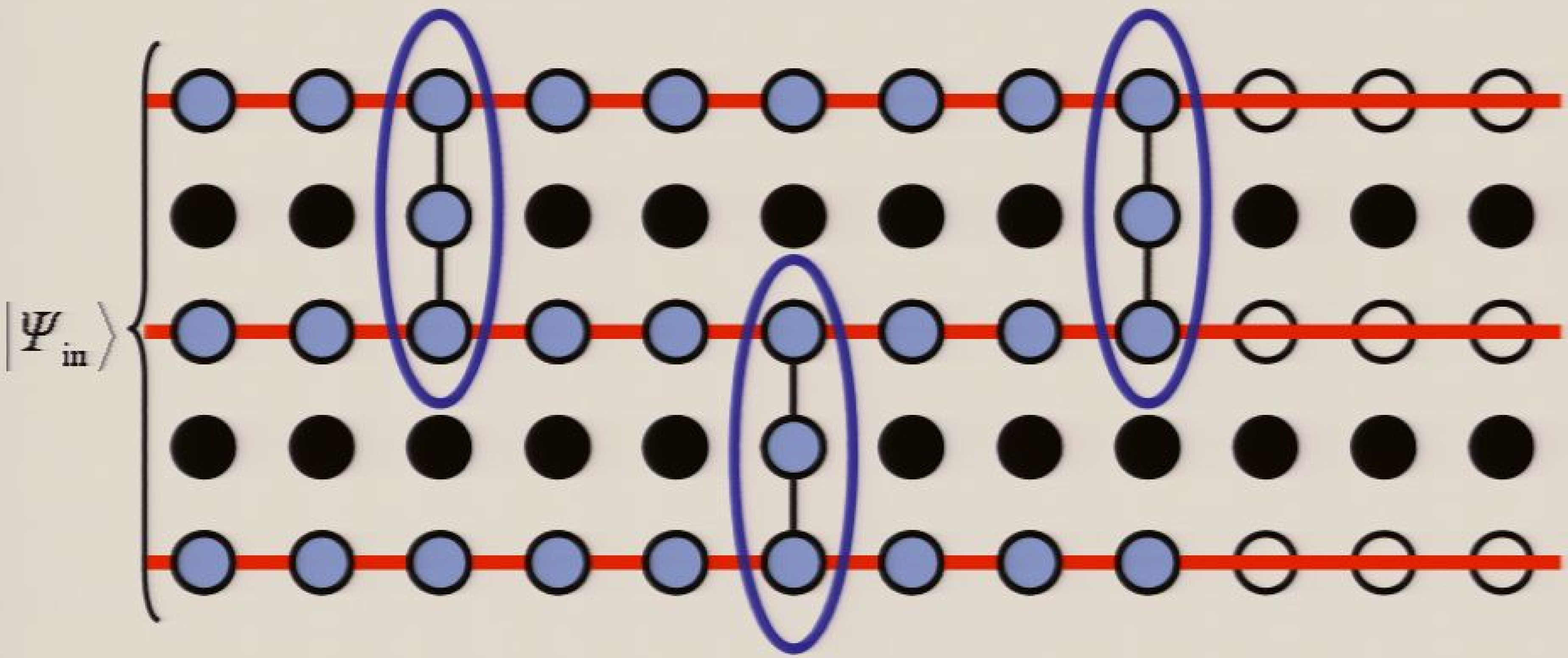

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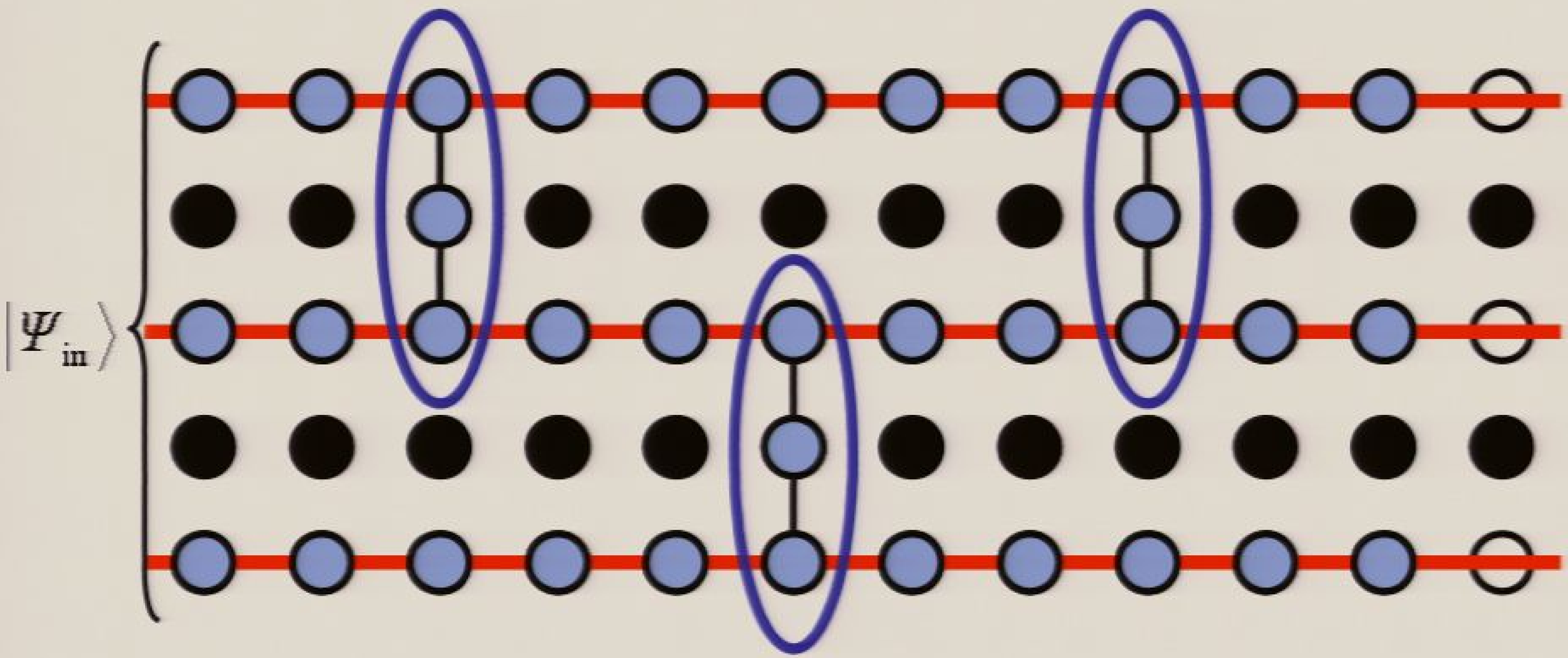

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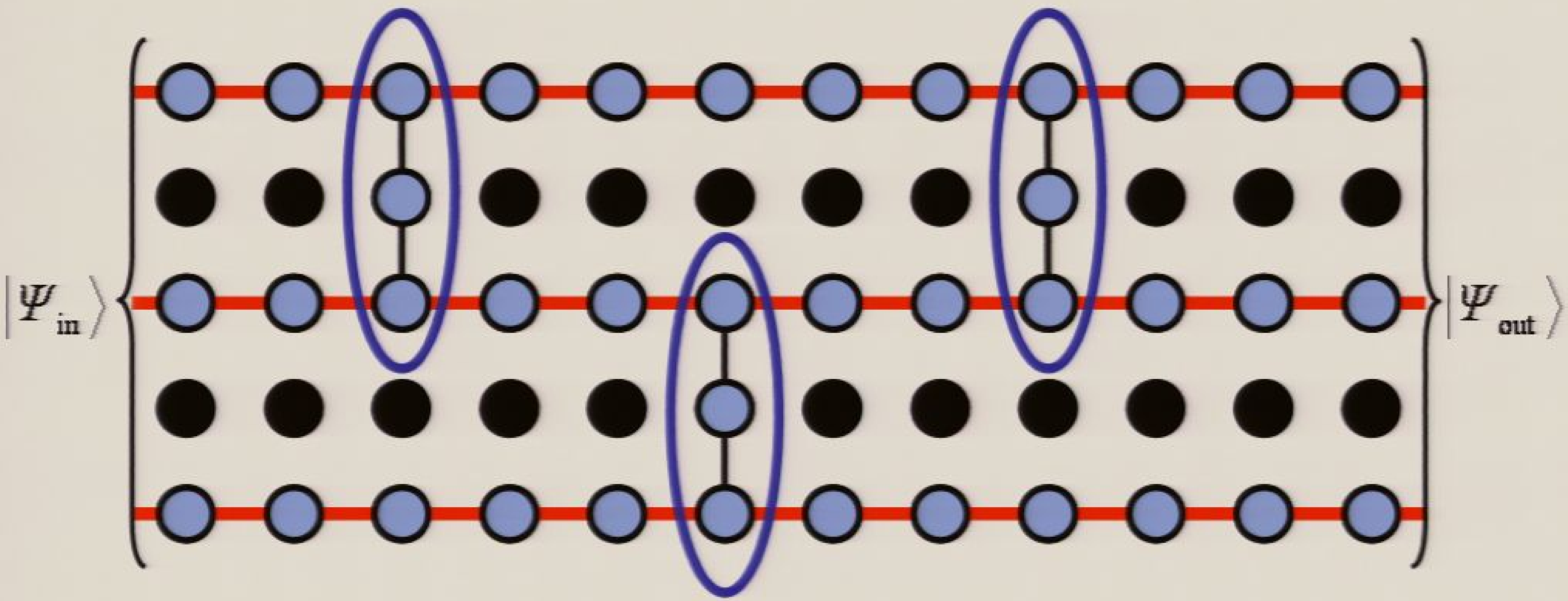

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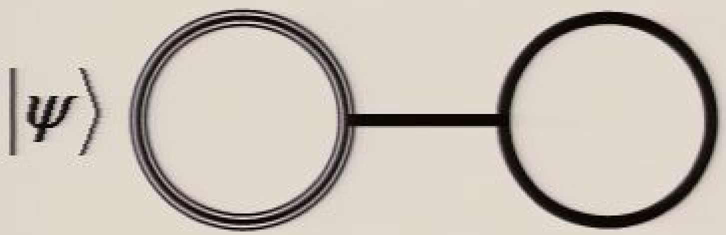

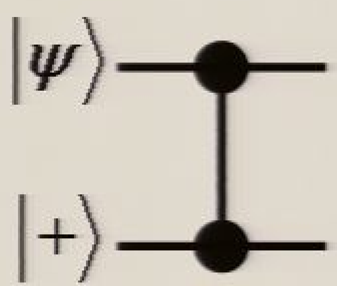

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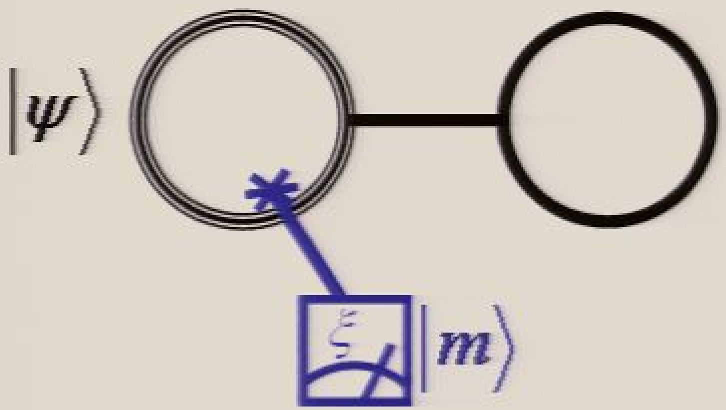

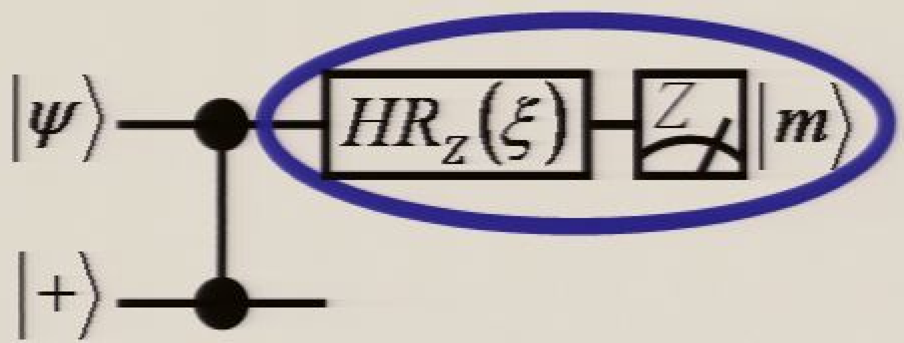

ξ-basis measurement:

$$\frac{1}{\sqrt{2}}\left\{|0\rangle \pm e^{i\xi}|1\rangle\right\}$$

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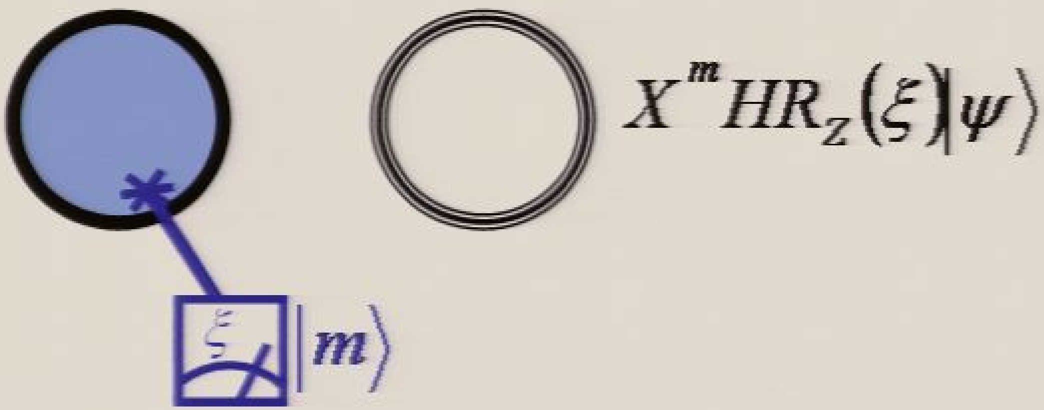

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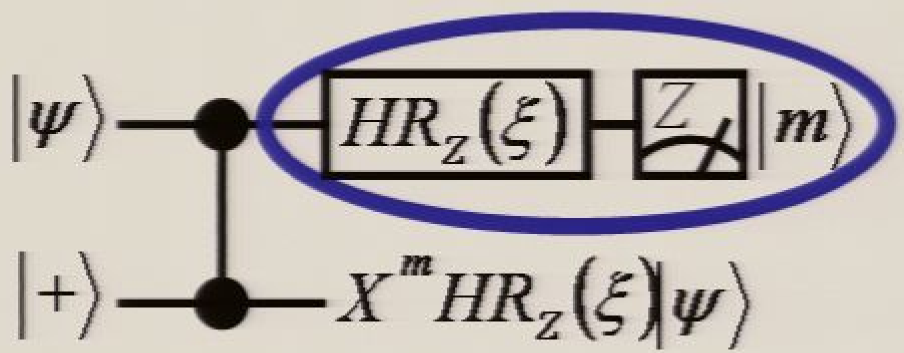

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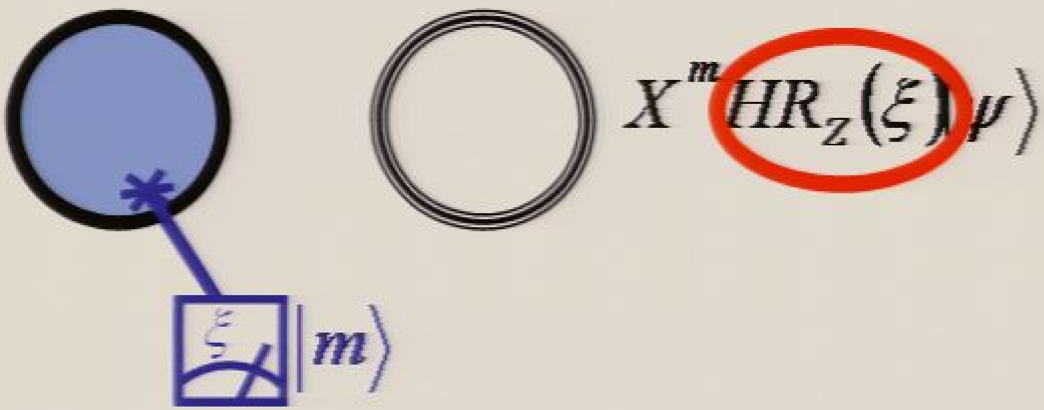

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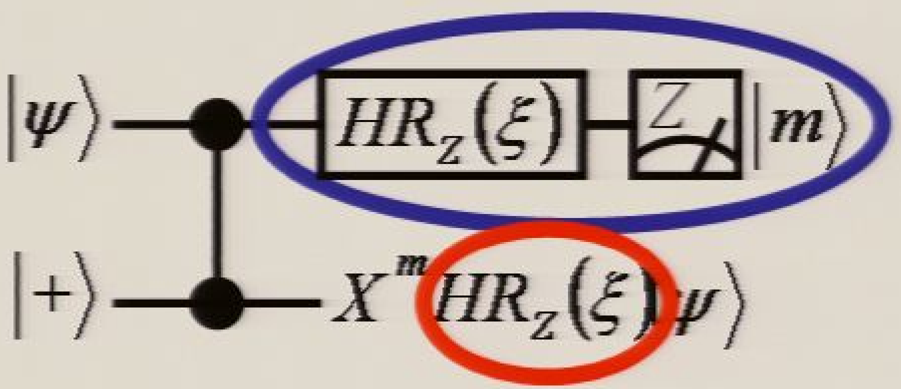

Universal for single-qubit unitaries

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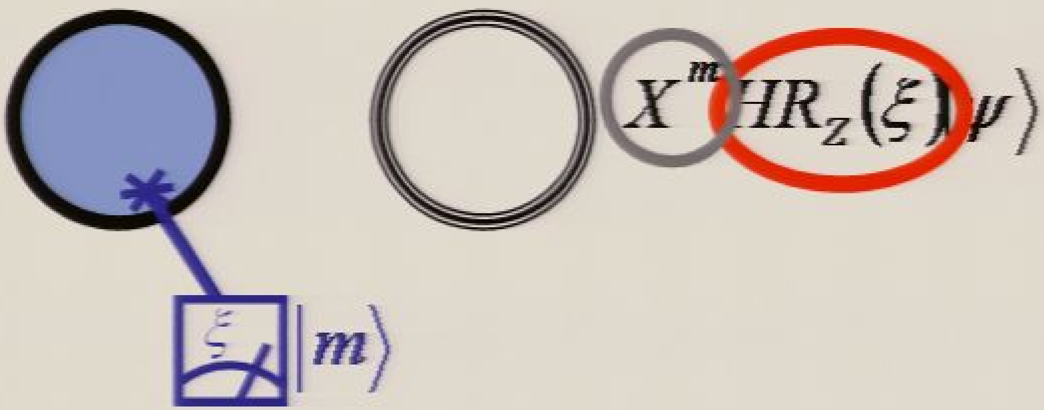

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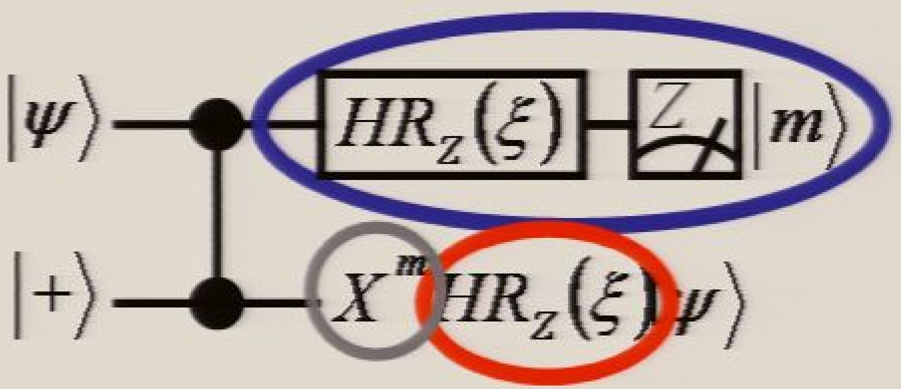

Universal for single-qubit unitaries

X by-products handled by feedforward

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$|\psi\rangle$

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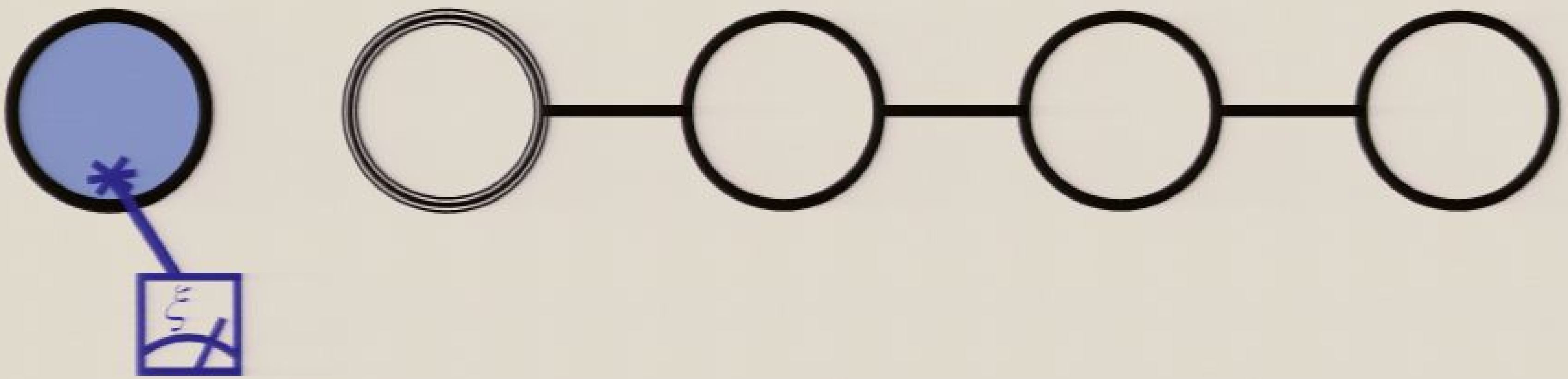

$$HR_Z(\xi)|\psi\rangle$$

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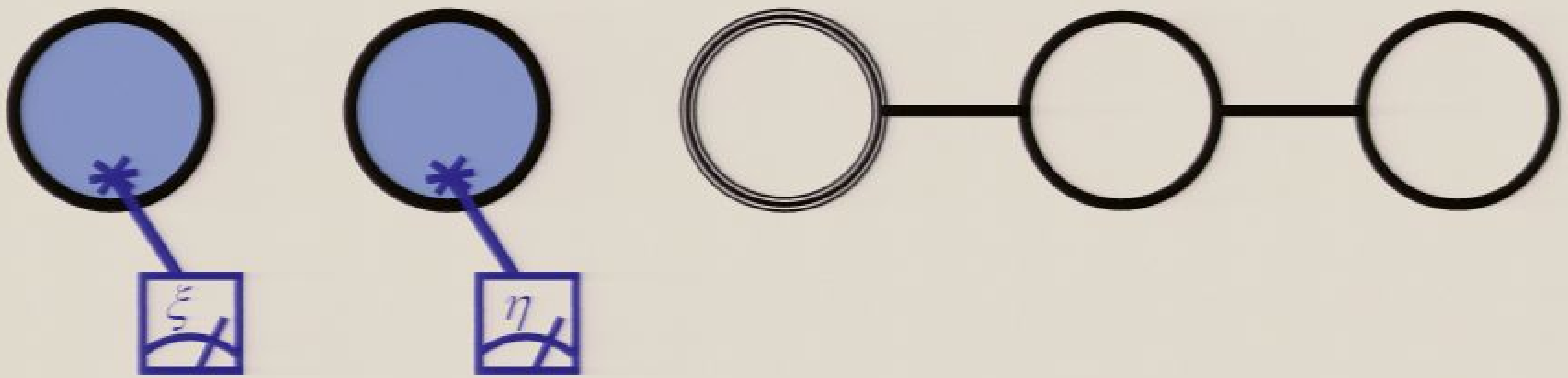

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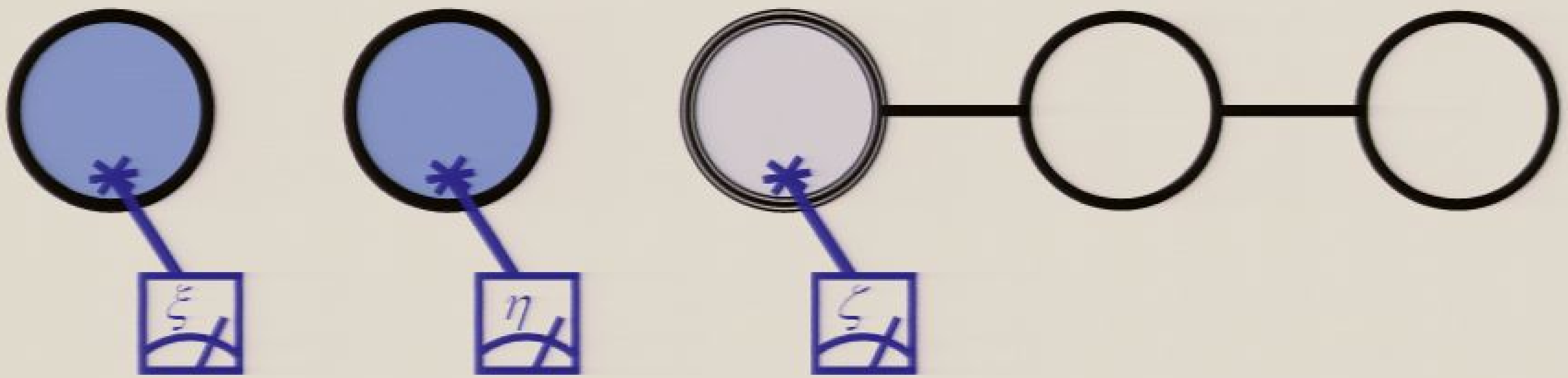

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$$H\, HR_Z(\zeta)\, HR_Z(\eta) HR_Z(\xi)|\psi\rangle$$

$$R_Z(\zeta)\, R_X(\eta) R_Z(\xi)|\psi\rangle$$

$$R(\xi, \eta, \zeta)|\psi\rangle$$

Efficient Generation of Cluster States

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Ising interaction

(q. dots, SC circuits, optical lattices)

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$$\hat{U}(t) = \prod_j \exp\left[i\phi \left(\frac{\hat{\sigma}_Z^{(j)} - 1}{2} \right) \left(\frac{\hat{\sigma}_Z^{(j+1)} - 1}{2} \right) \right] \qquad \phi \equiv gt$$

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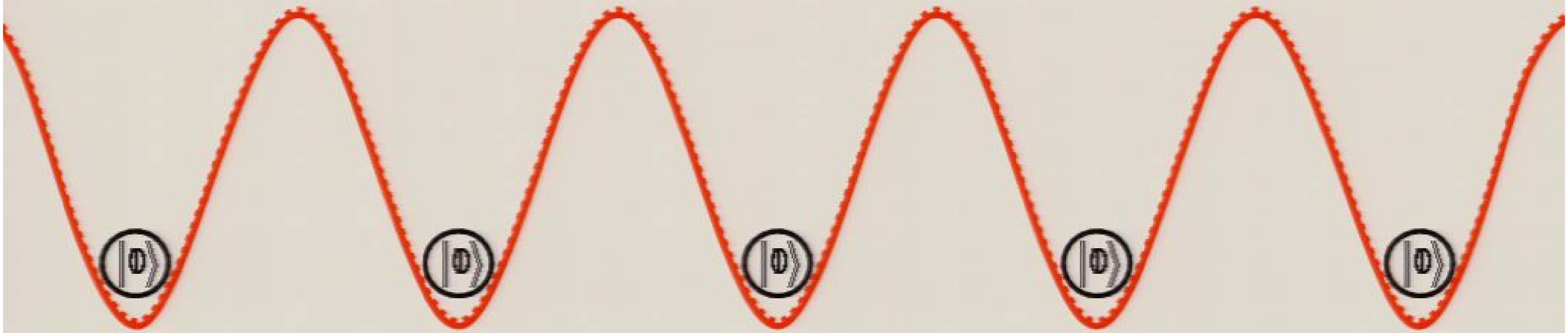
$$\phi = \pi$$

$$\boxed{\hat{U}(t) = \prod_j CZ^{(j,j+1)}}$$

Efficient Generation of Cluster States

Cold collisions in optical lattices



$$|0\rangle_j |0\rangle_{j+1}$$

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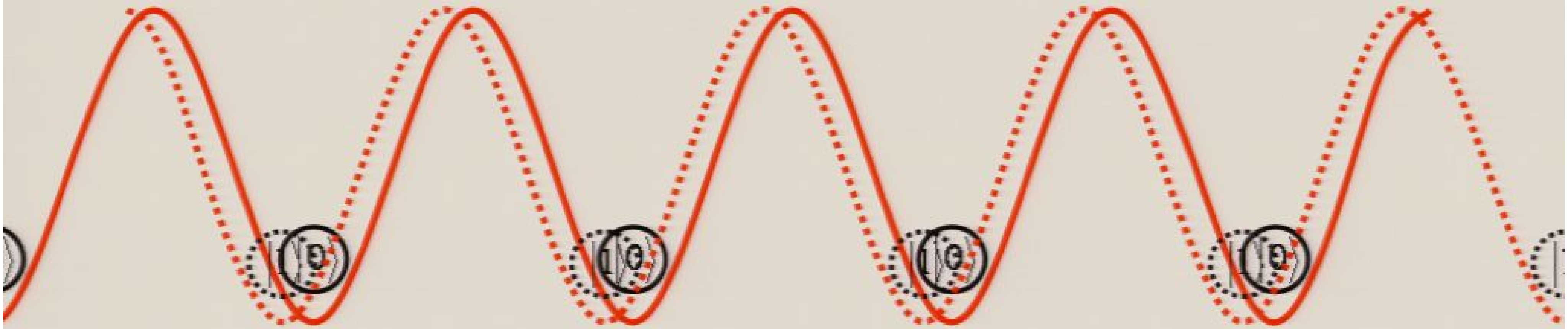

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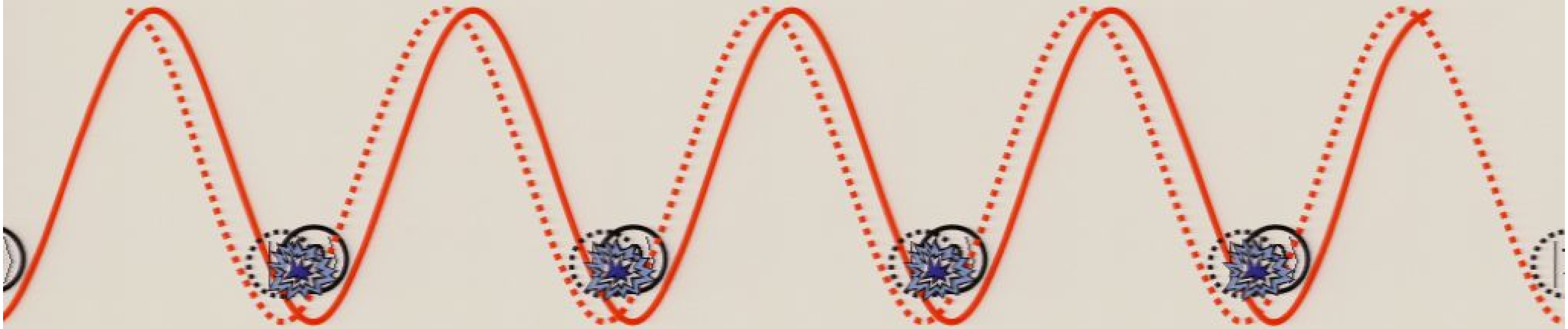

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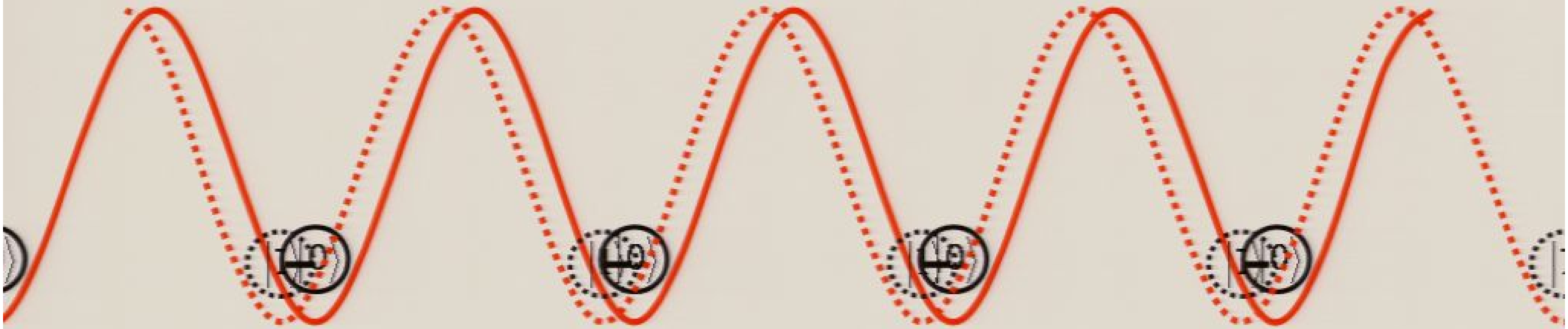

$e^{i\phi}$ $e^{i\phi}$ $e^{i\phi}$ $e^{i\phi}$

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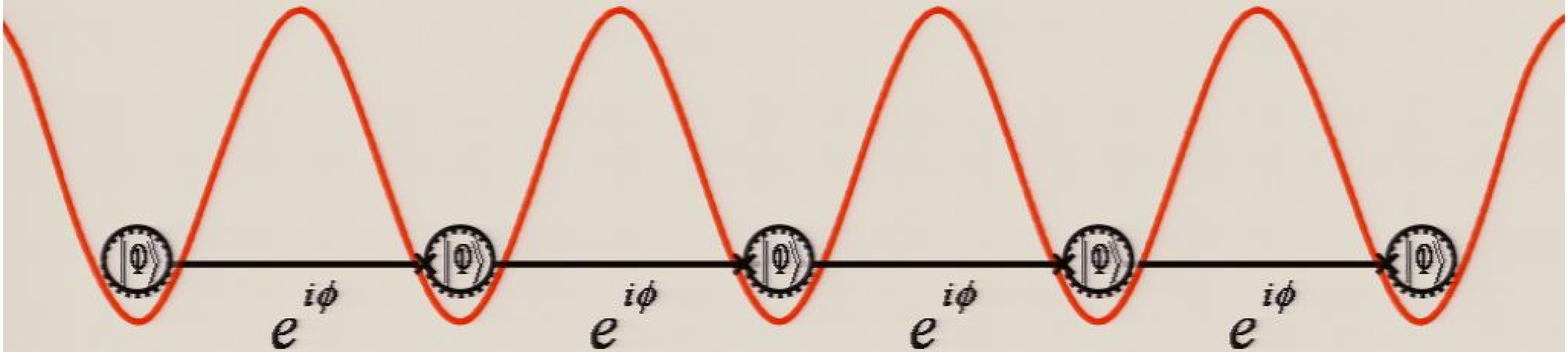

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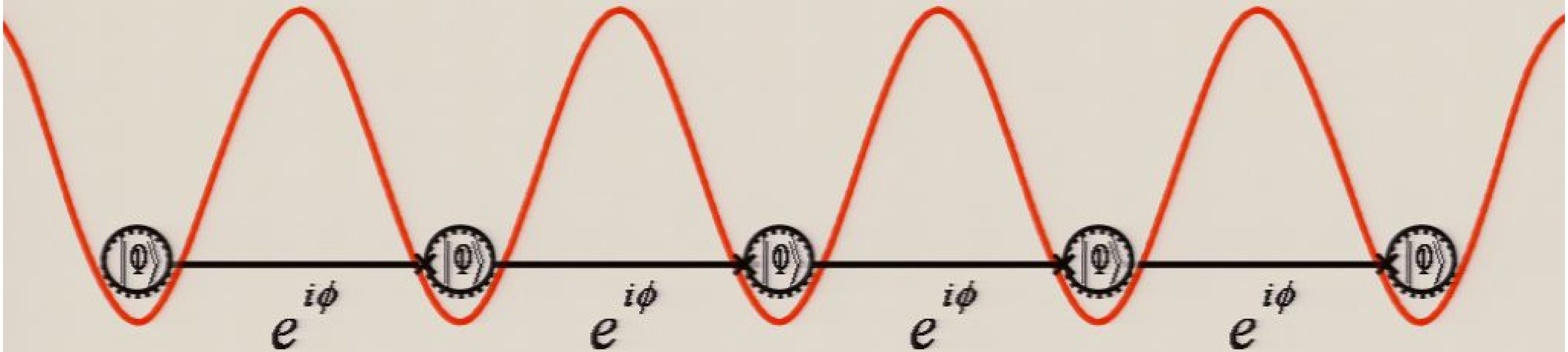

$$|0\rangle_j|0\rangle_{j+1} \quad\quad |0\rangle_j|0\rangle_{j+1}$$

$$|0\rangle_j|1\rangle_{j+1} \quad\quad |0\rangle_j|1\rangle_{j+1}$$

$$|1\rangle_j|0\rangle_{j+1} \quad \Rightarrow \quad e^{i\phi}|1\rangle_j|0\rangle_{j+1}$$

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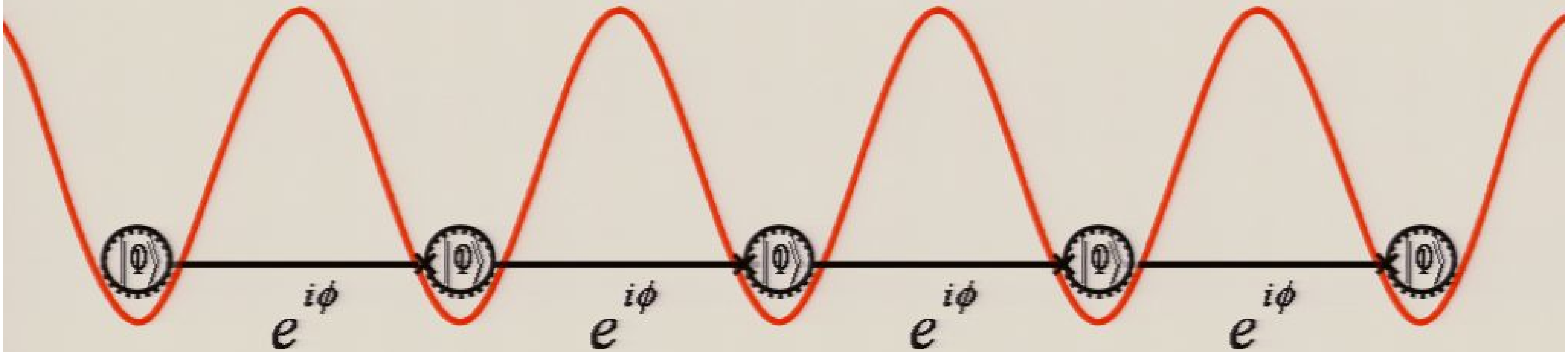

$$\hat{U}(t) = \prod_j \left(I^{(j)} \otimes X^{(j+1)}\right) CZ^{(j,j+1)} \left(I^{(j)} \otimes X^{(j+1)}\right)$$

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$\phi = \pi$

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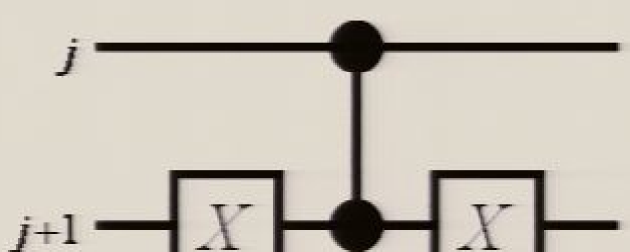

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Same θ for all entanglement links

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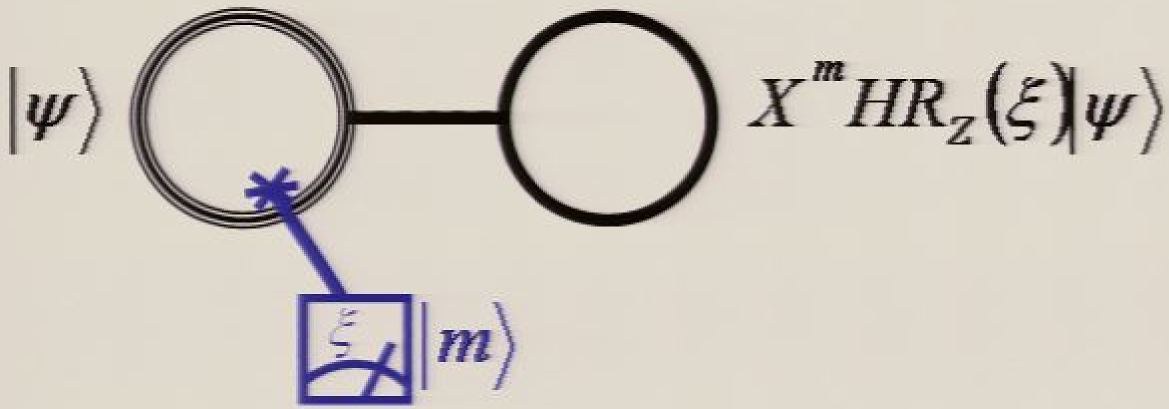

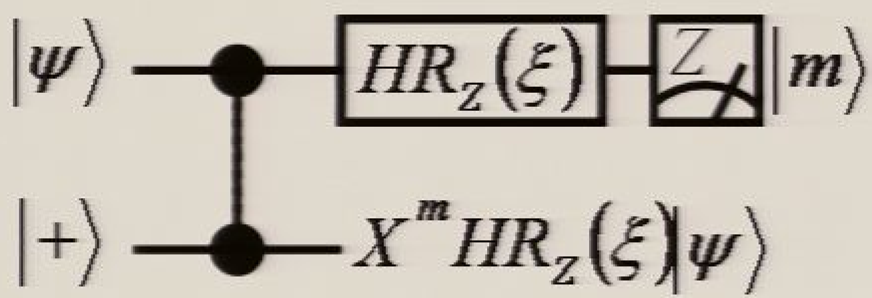

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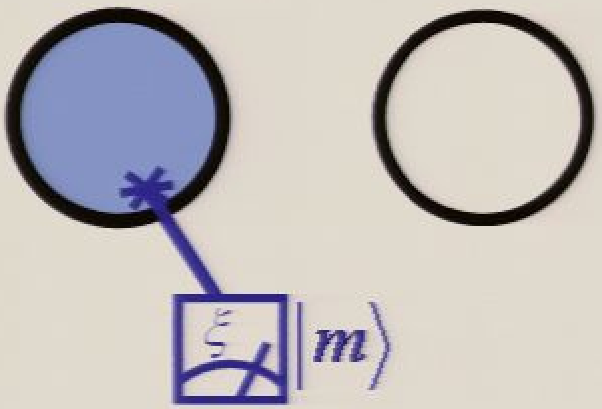

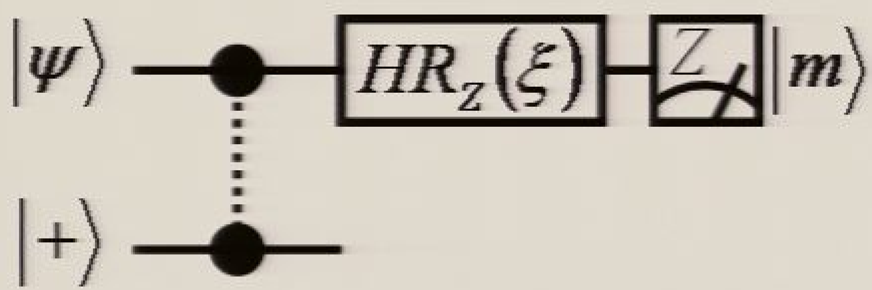

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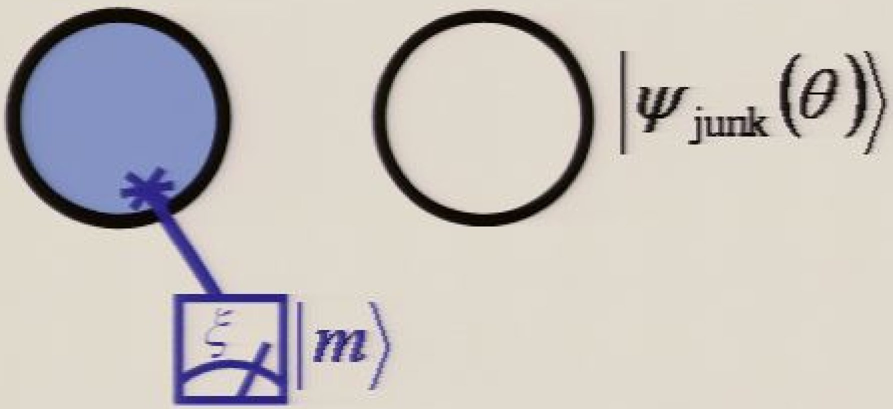

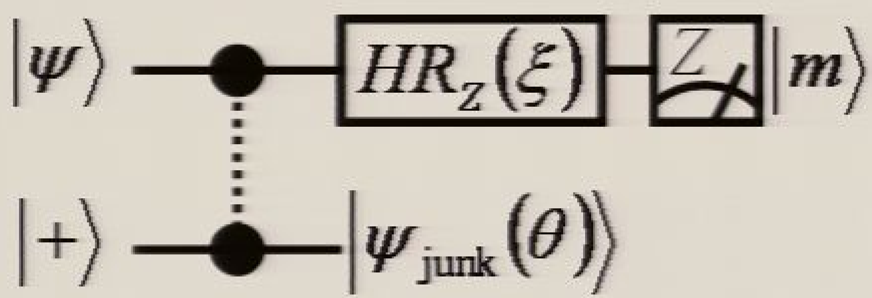

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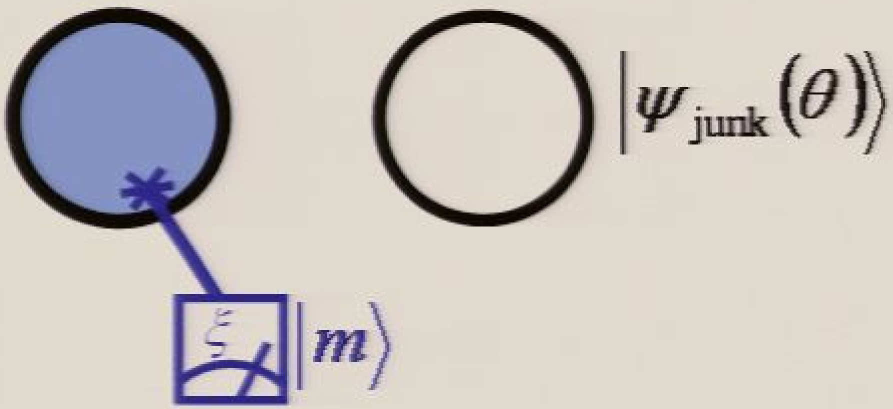

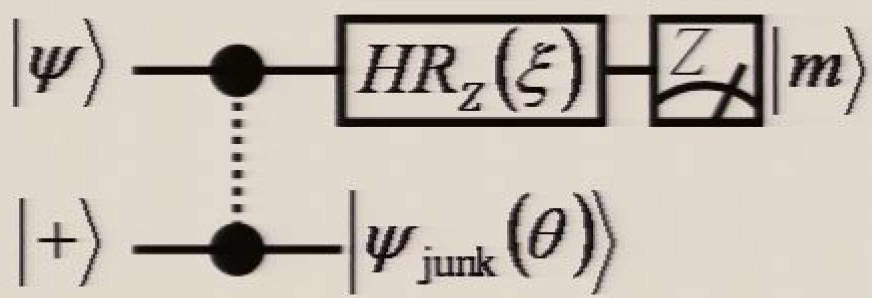

Fidelity loss is small if
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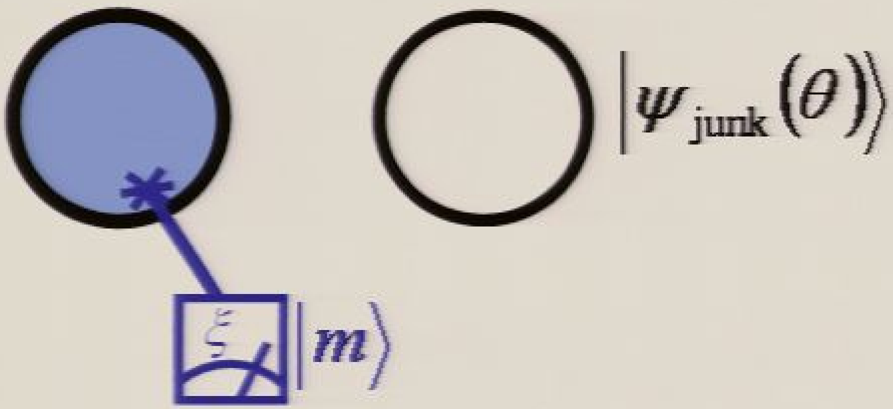

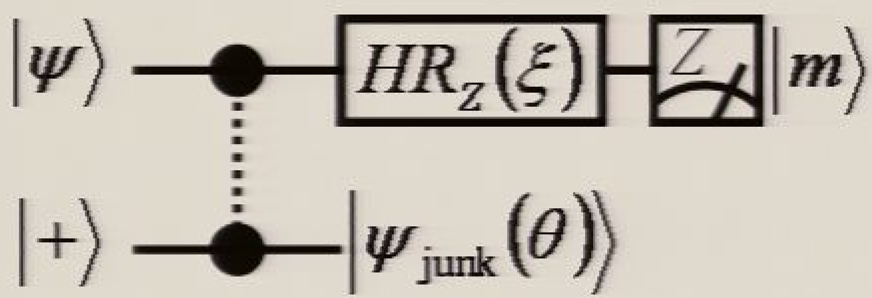

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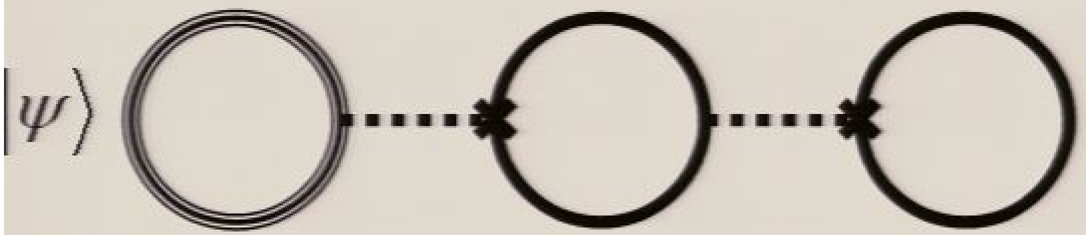
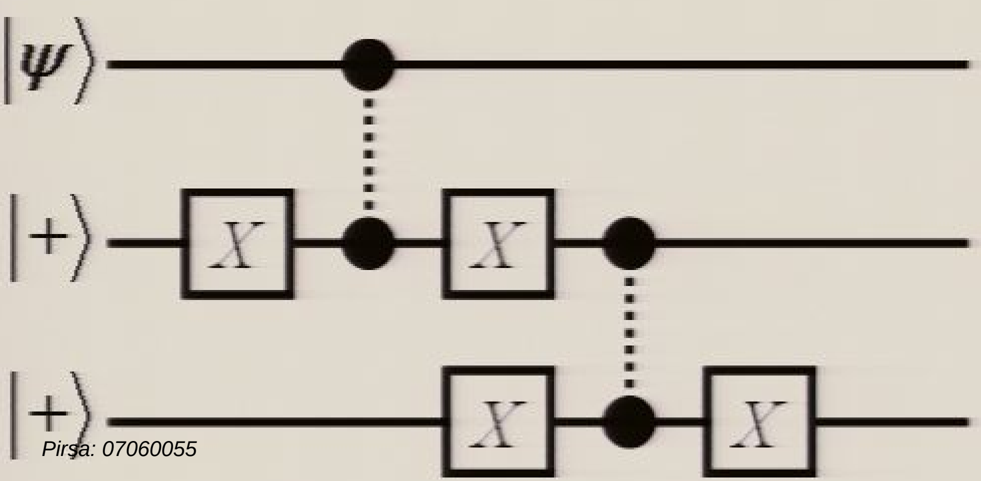
Fidelity losses add up

M. S. Tame, *et al.*, *PRA* **72**, 012319 (2005).

SOLUTION: Combine two imperfect links





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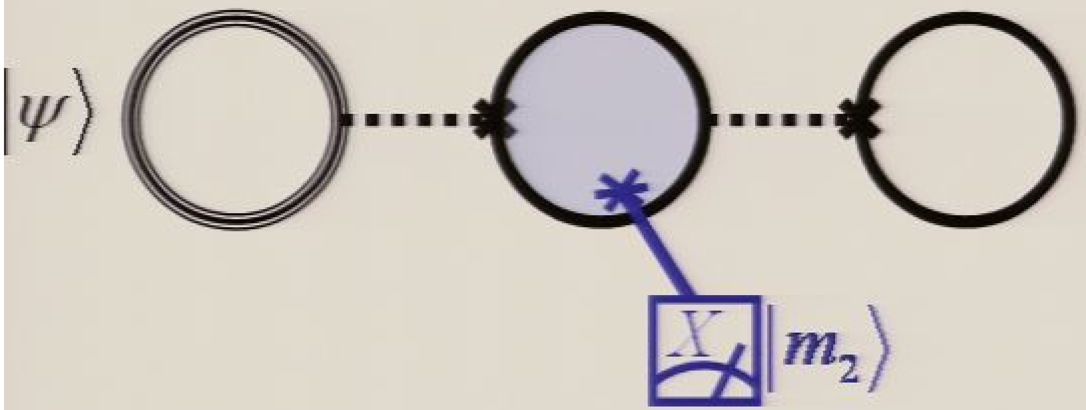

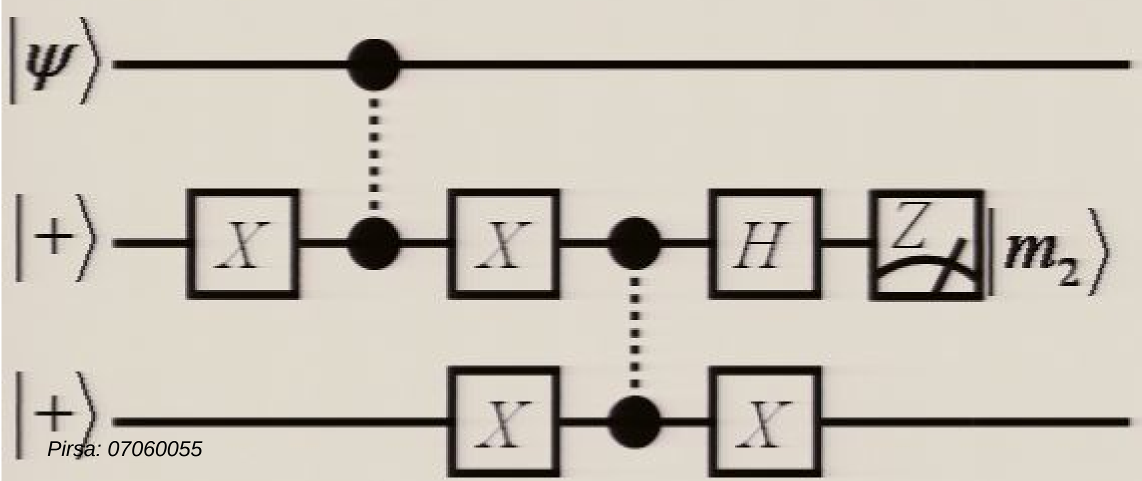

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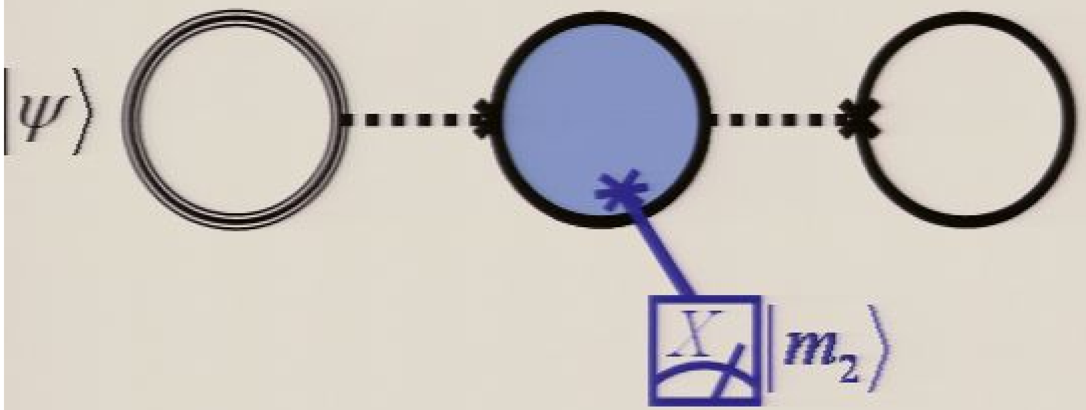

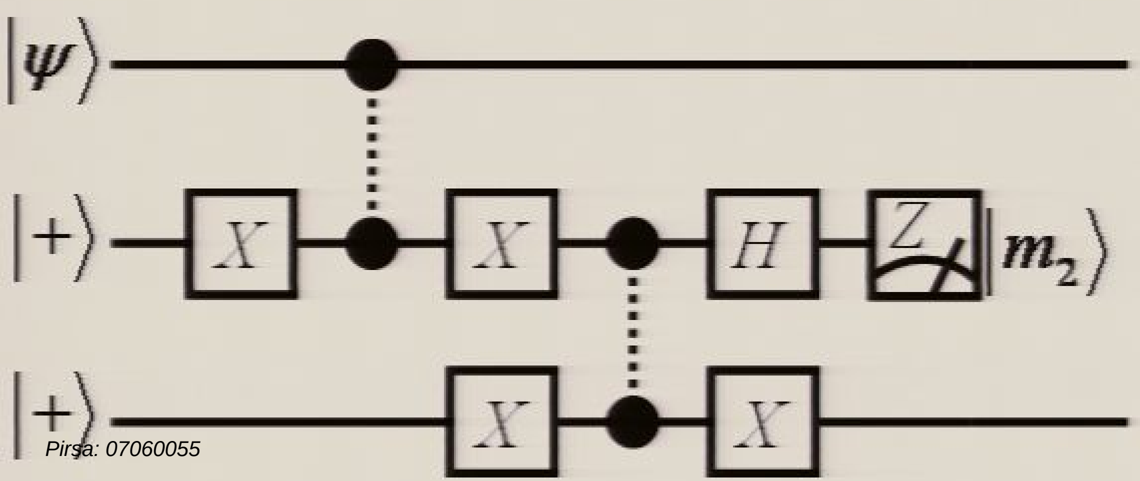

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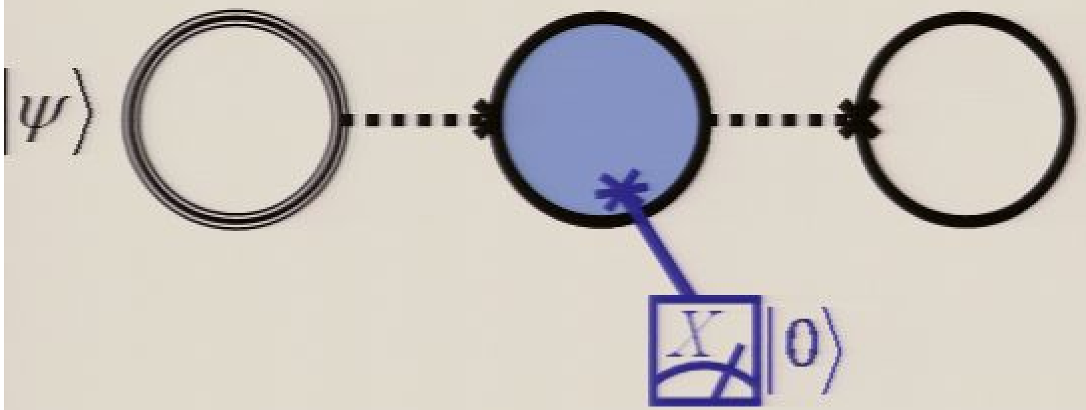

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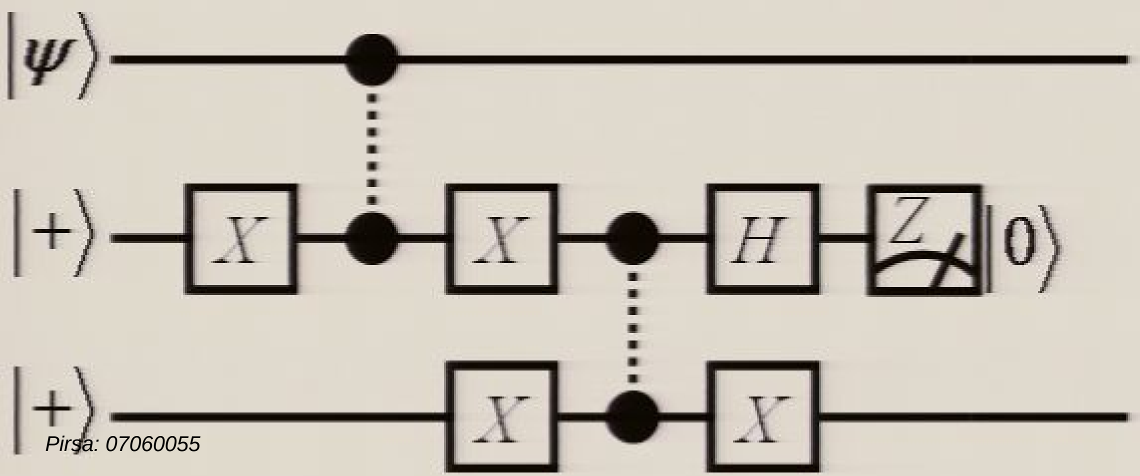

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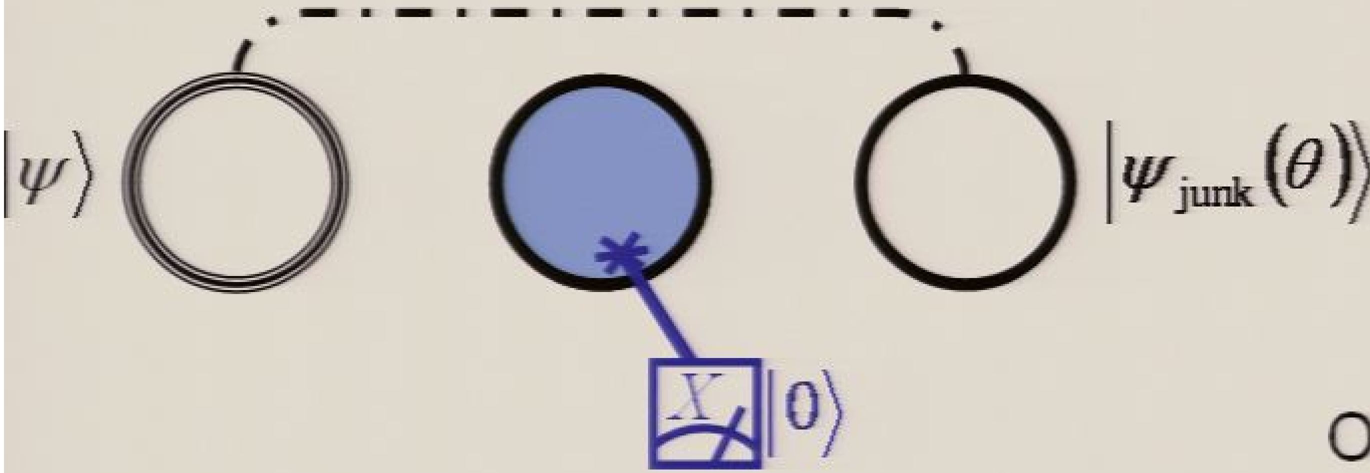

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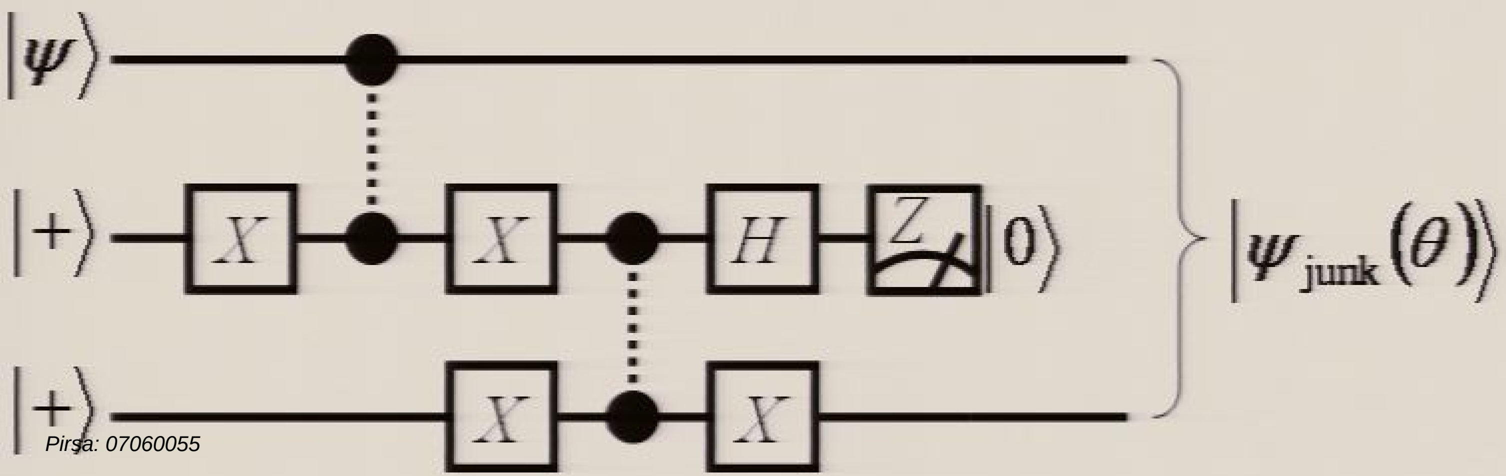

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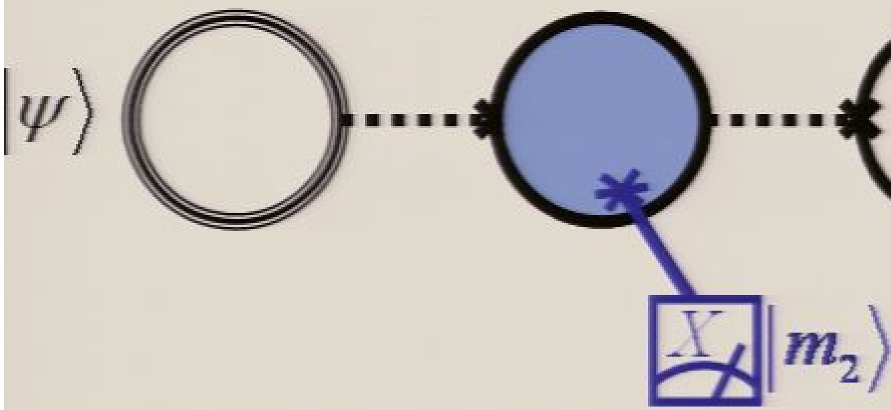

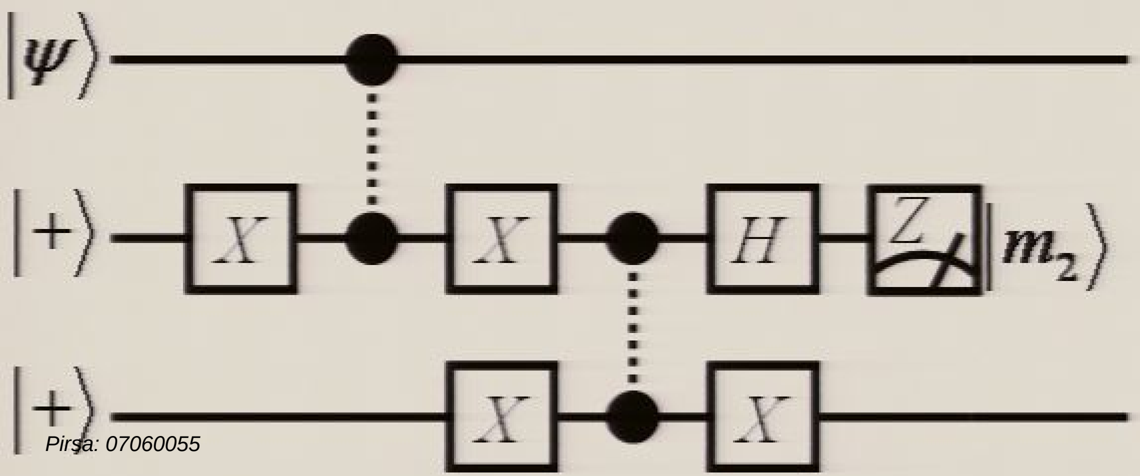

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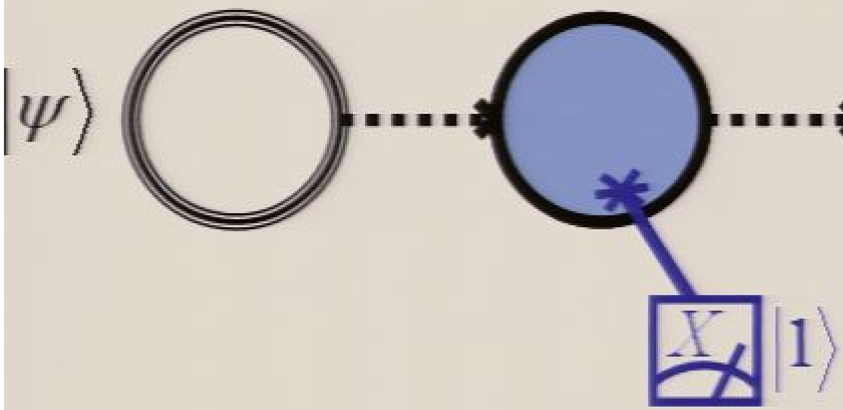
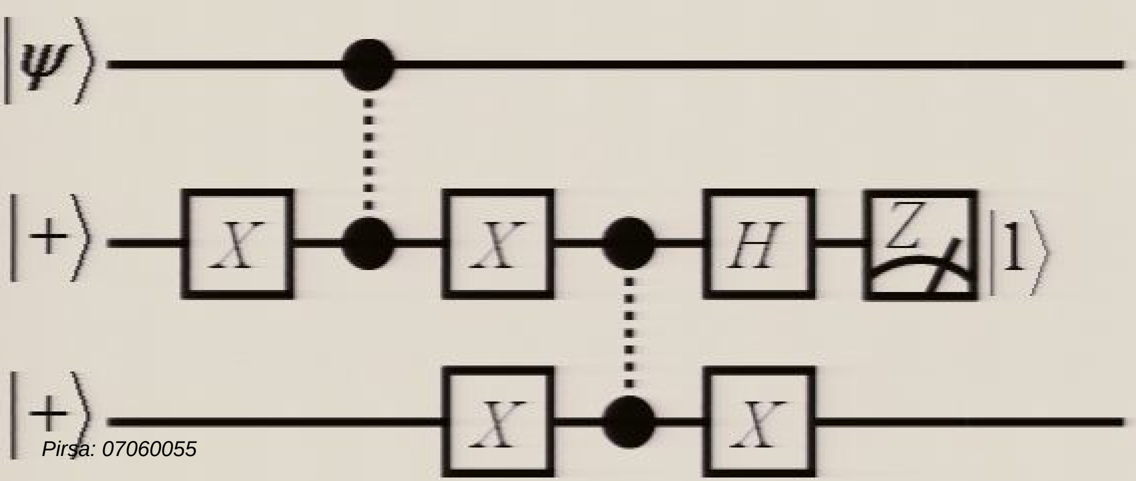


Outcome: $\boxed{m_2 = 1}$



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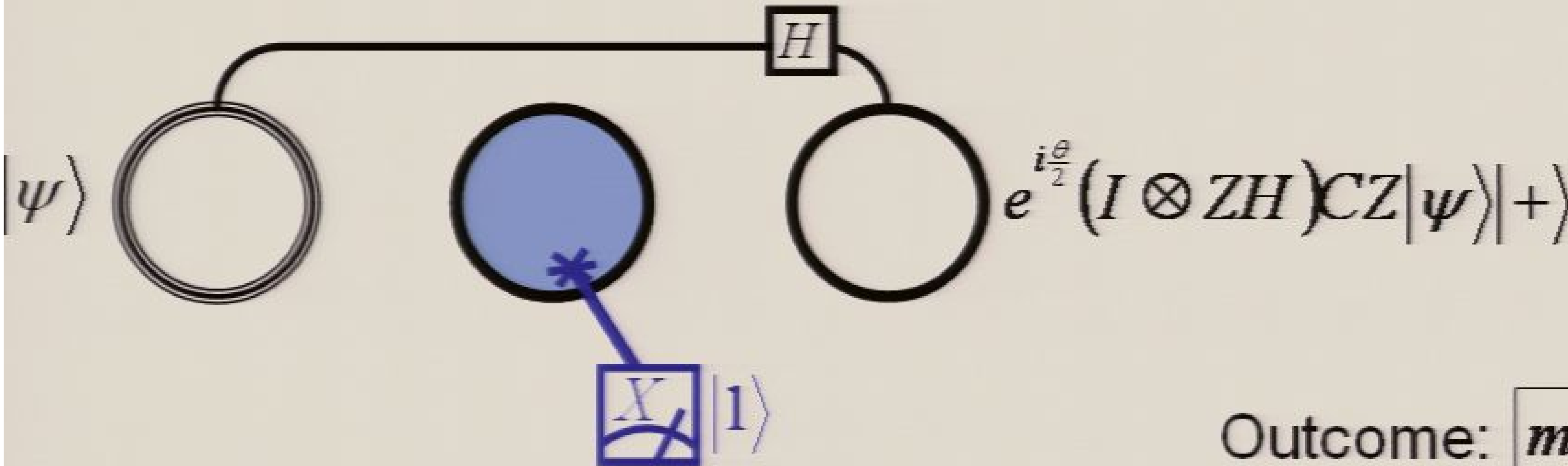

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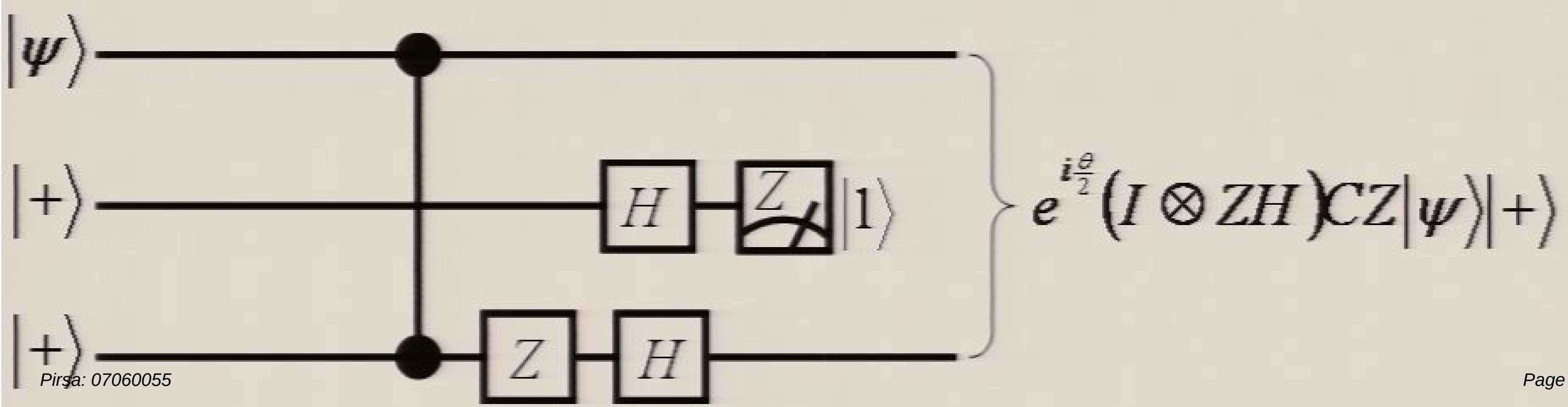

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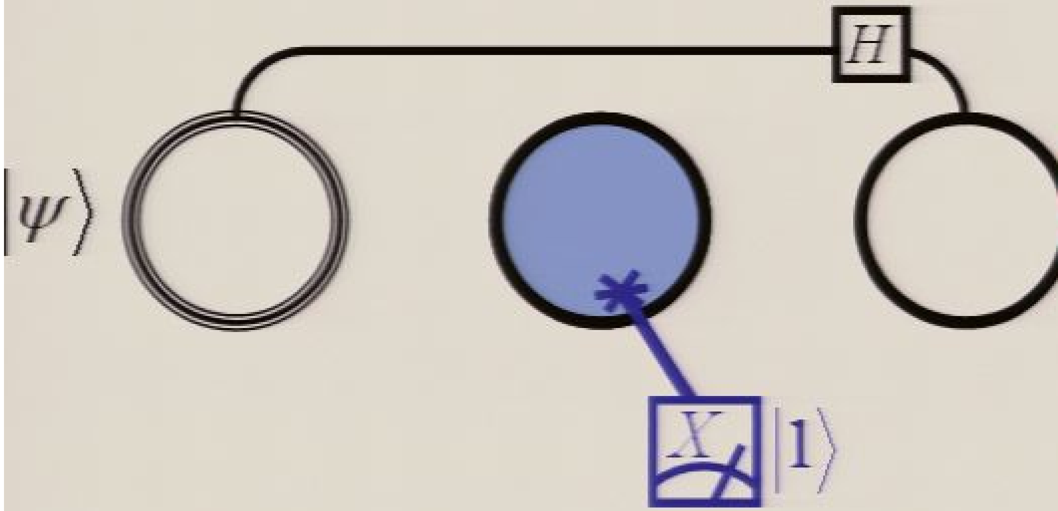

$$e^{i\frac{\theta}{2}}\left(I\otimes ZH\right)CZ|\psi\rangle|+\rangle$$

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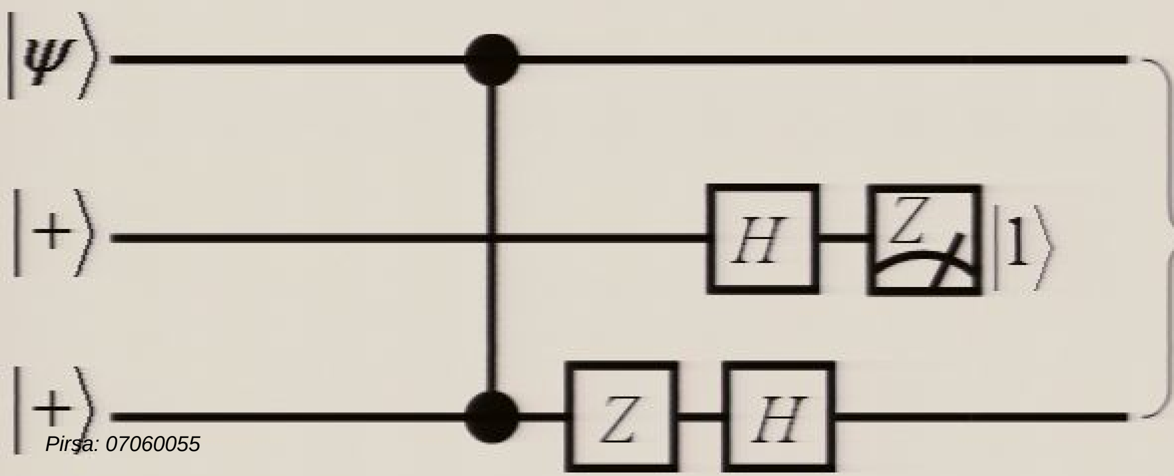

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Harmless global phase

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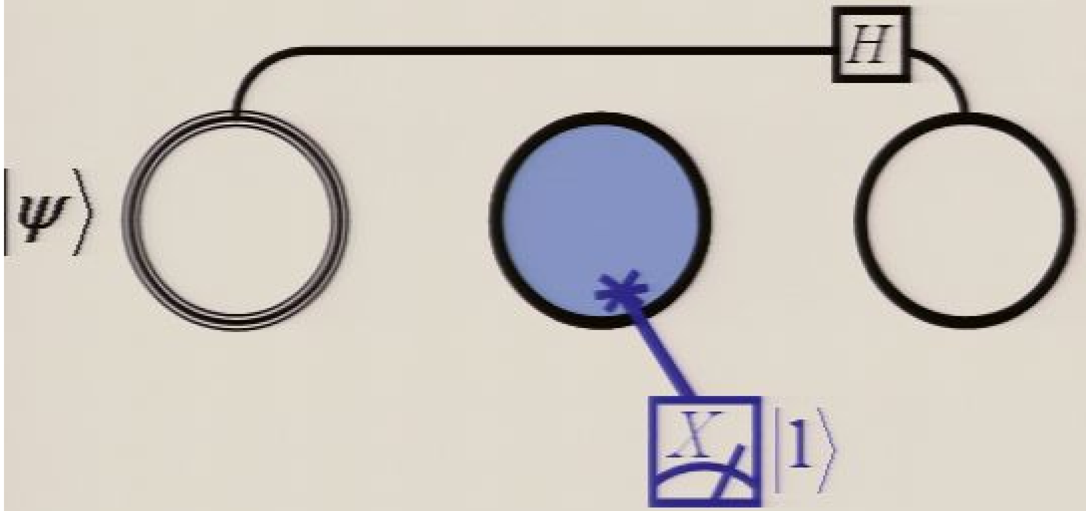

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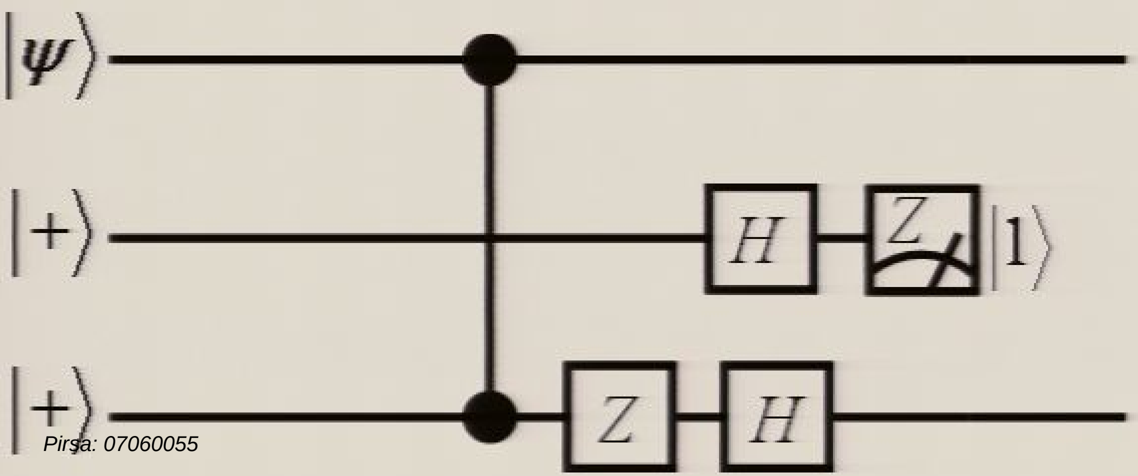
SUCCESS!



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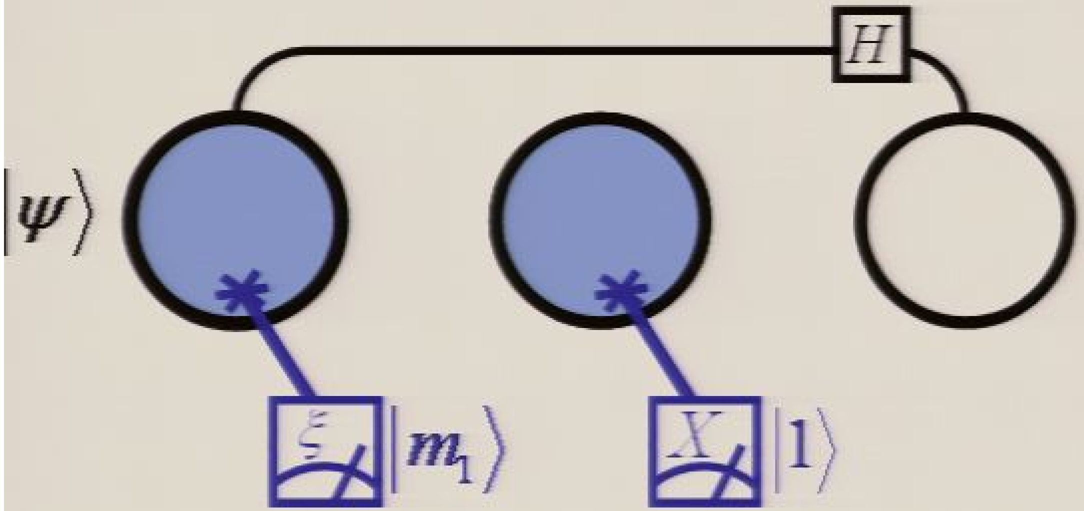

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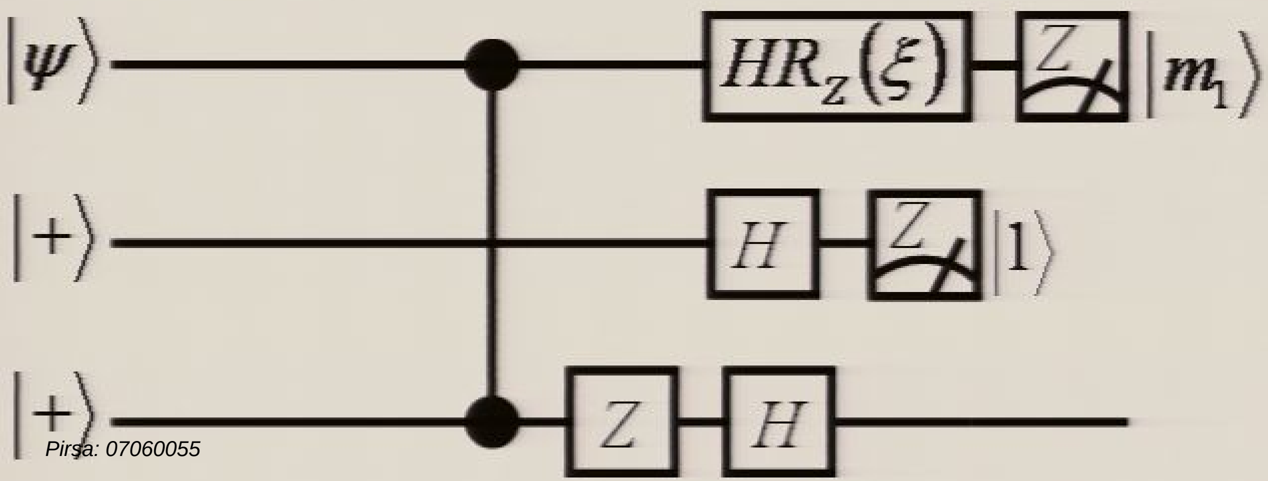

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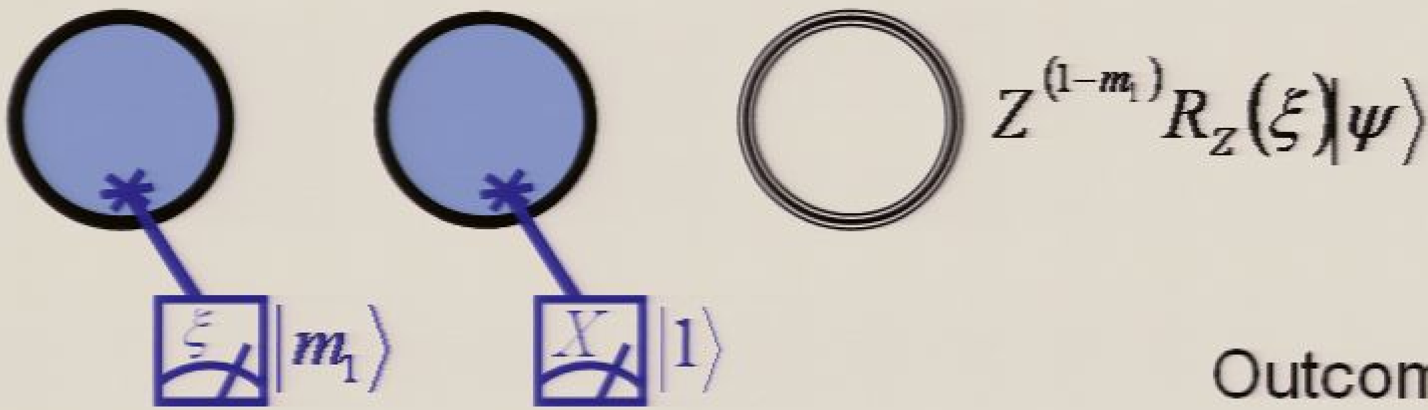

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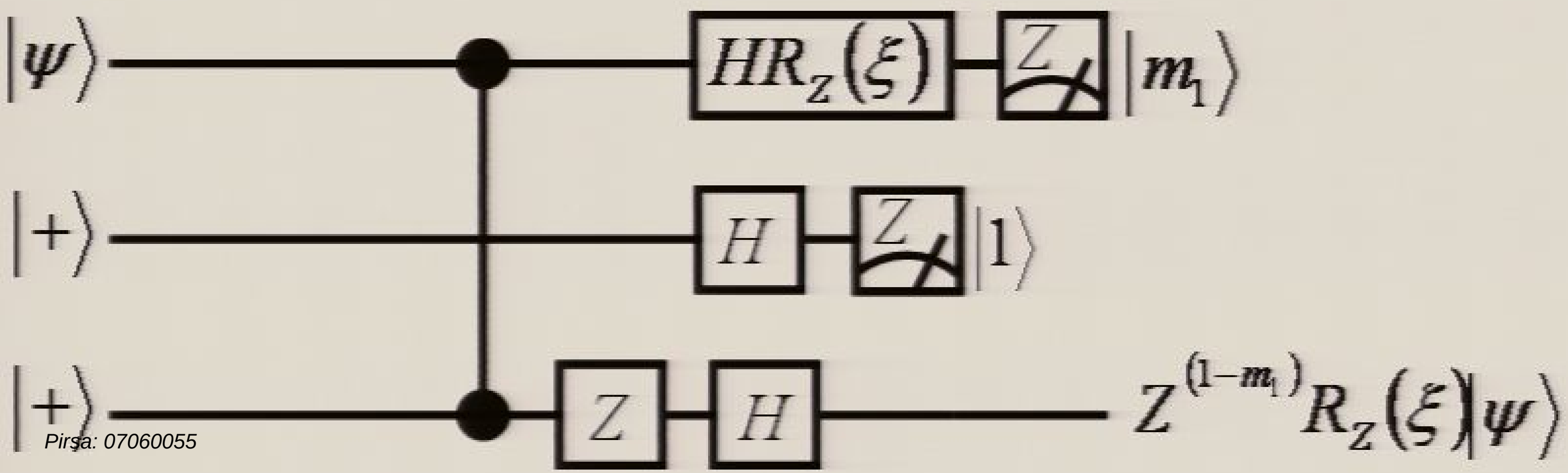

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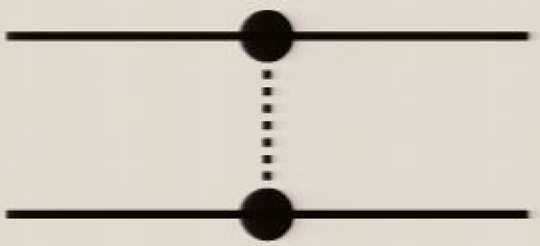
Finite probability even for weak entanglement!

SOLUTION: Combine two imperfect links

$CS(\pi+\theta)$ won't work!

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -e^{i\theta} \end{pmatrix}$$



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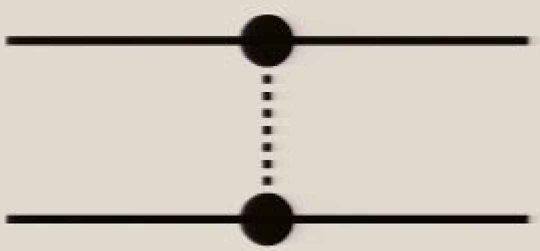

Need $[I\otimes X]CS(\pi+\theta)[I\otimes X]$

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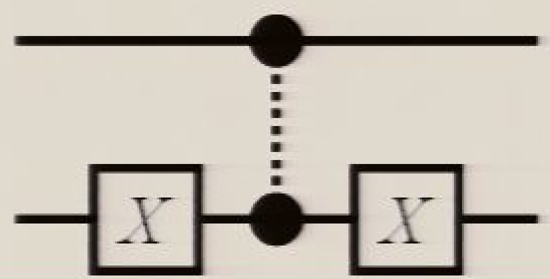

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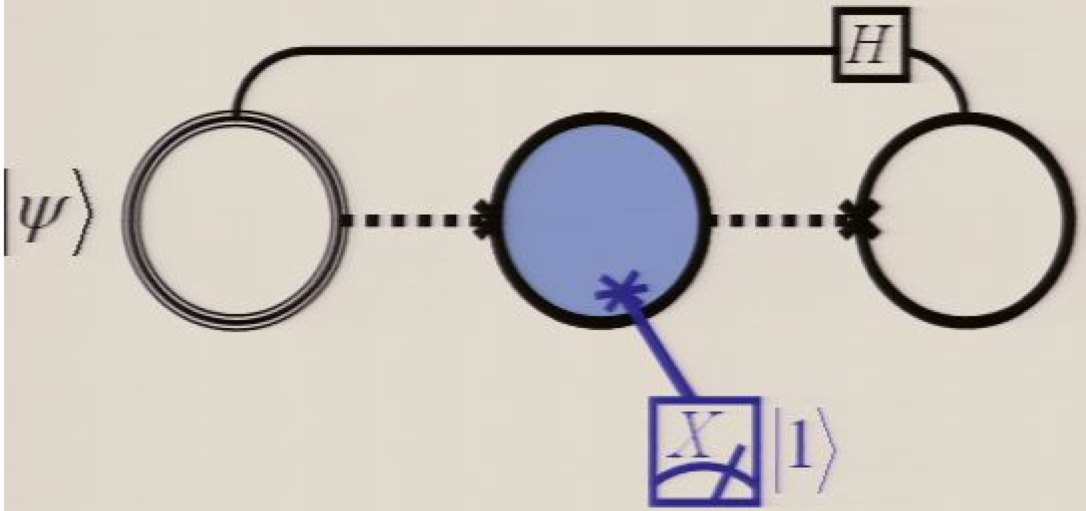

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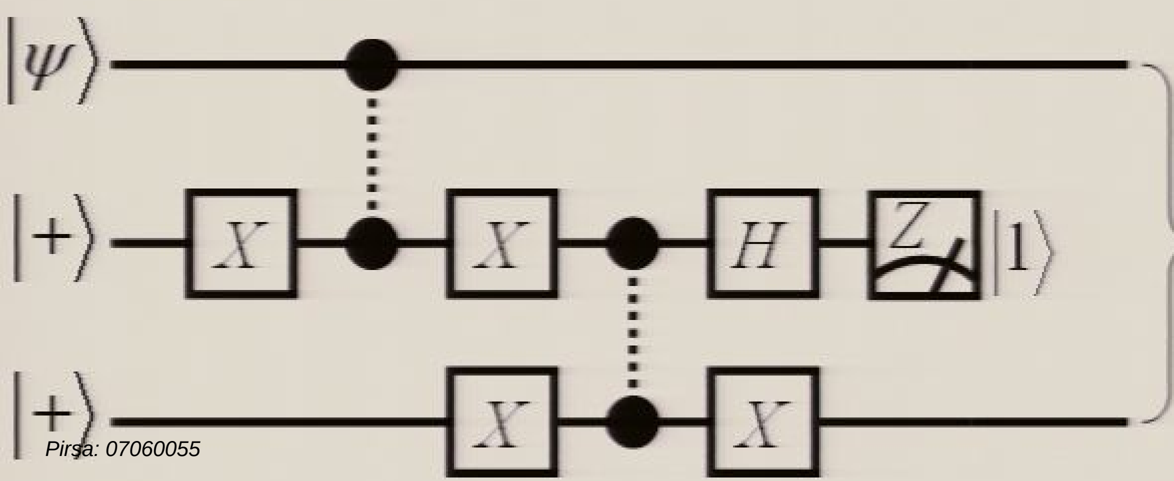

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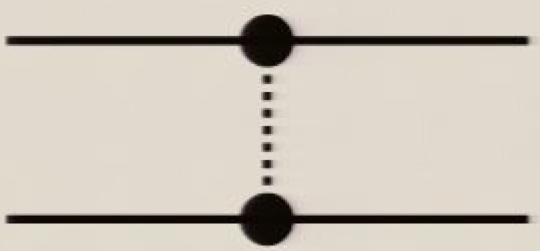

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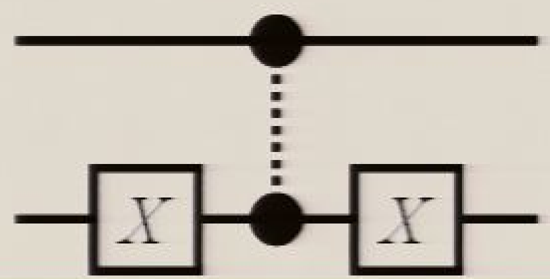

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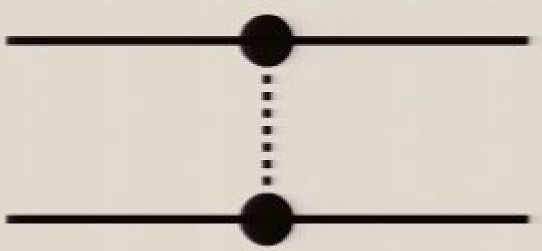

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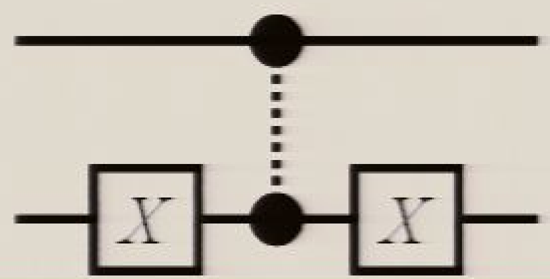
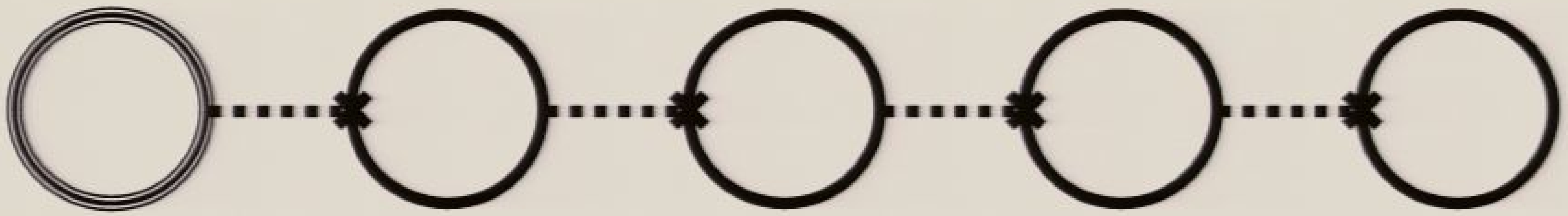


Natural with cold collisions. Also possible with Ising:

$$\hat{H} = \hbar g \sum_j \left(\frac{\hat{\sigma}_Z^{(j)} - 1}{2} \right) \left(\frac{\hat{\sigma}_Z^{(j+1)} + 1}{2} \right)$$

SOLUTION: Combine two imperfect links

…or four imperfect links



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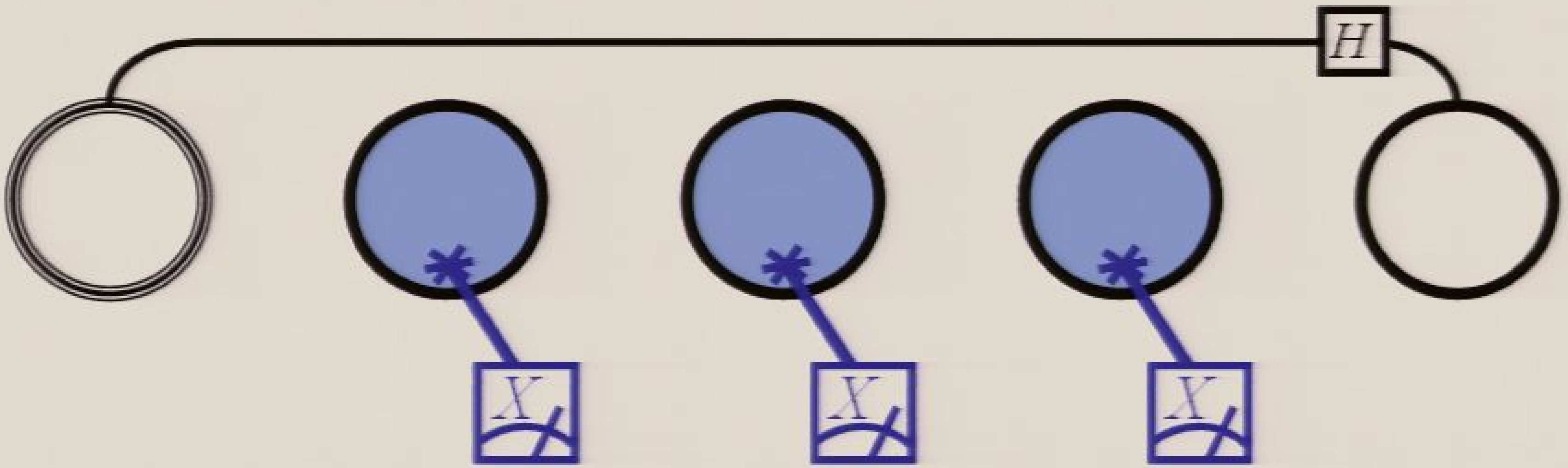

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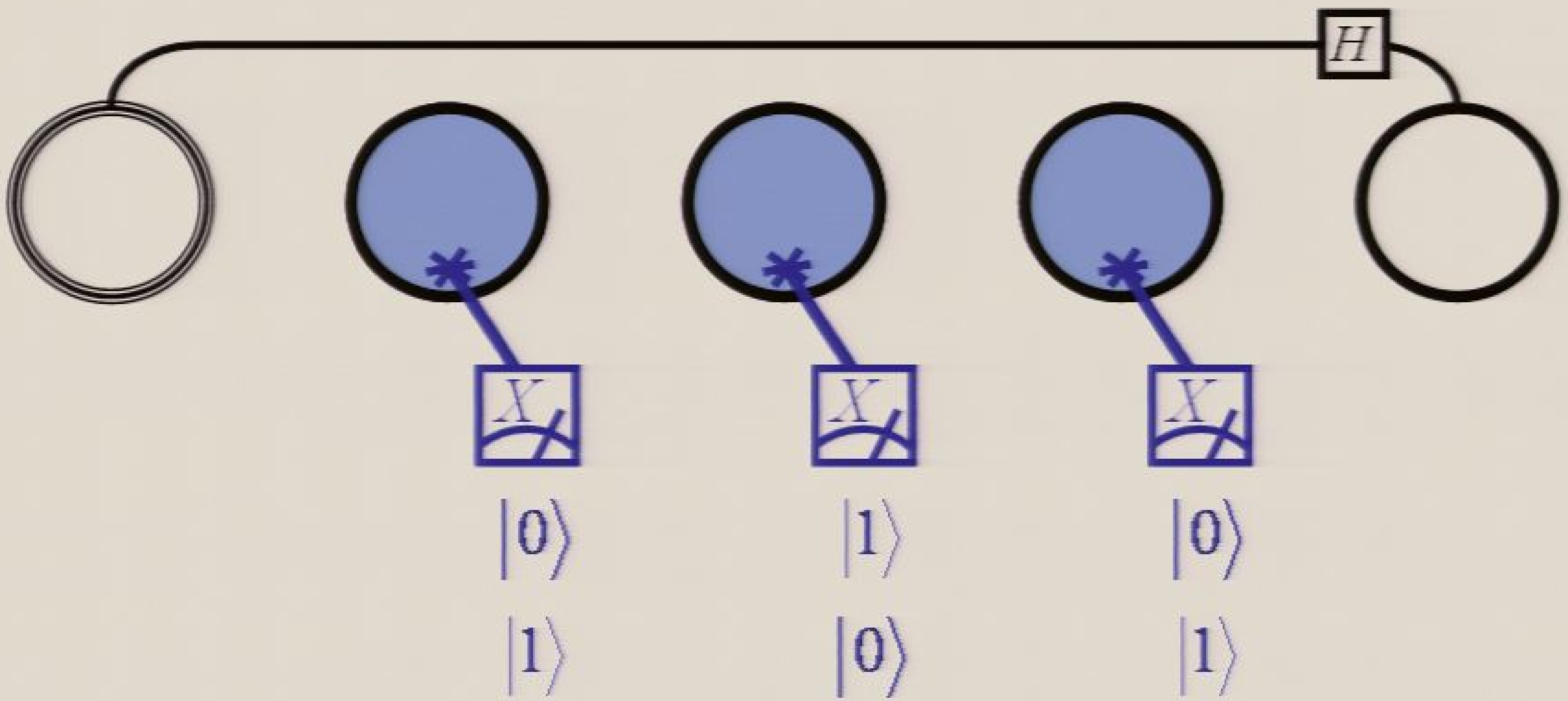

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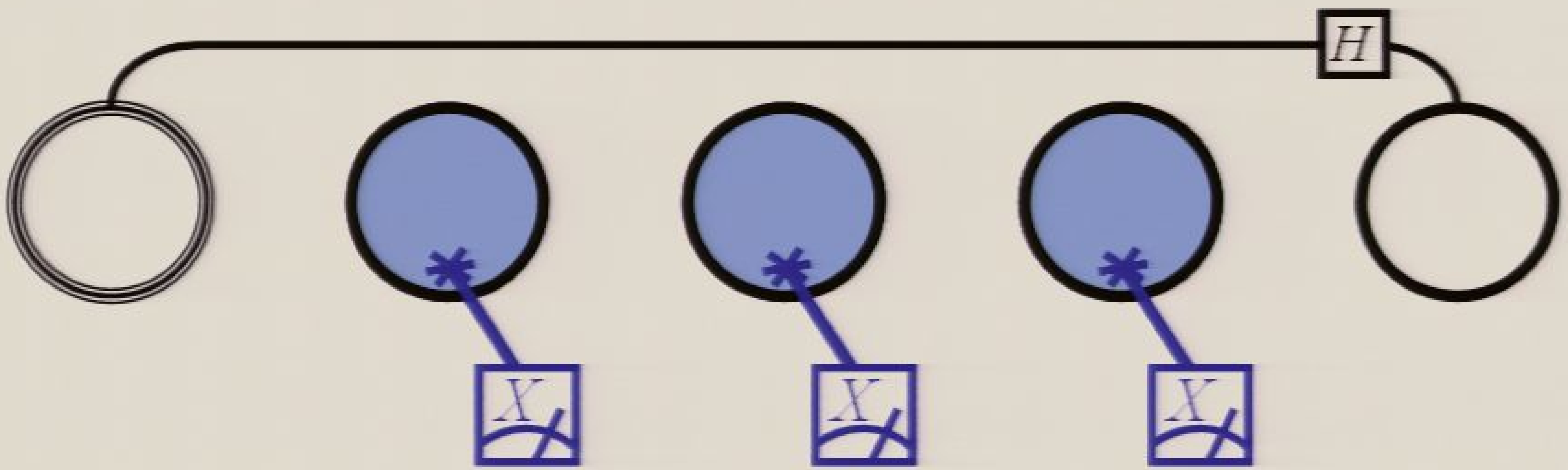

$\lvert 0\rangle$	$\lvert 1\rangle$	$\lvert 0\rangle$
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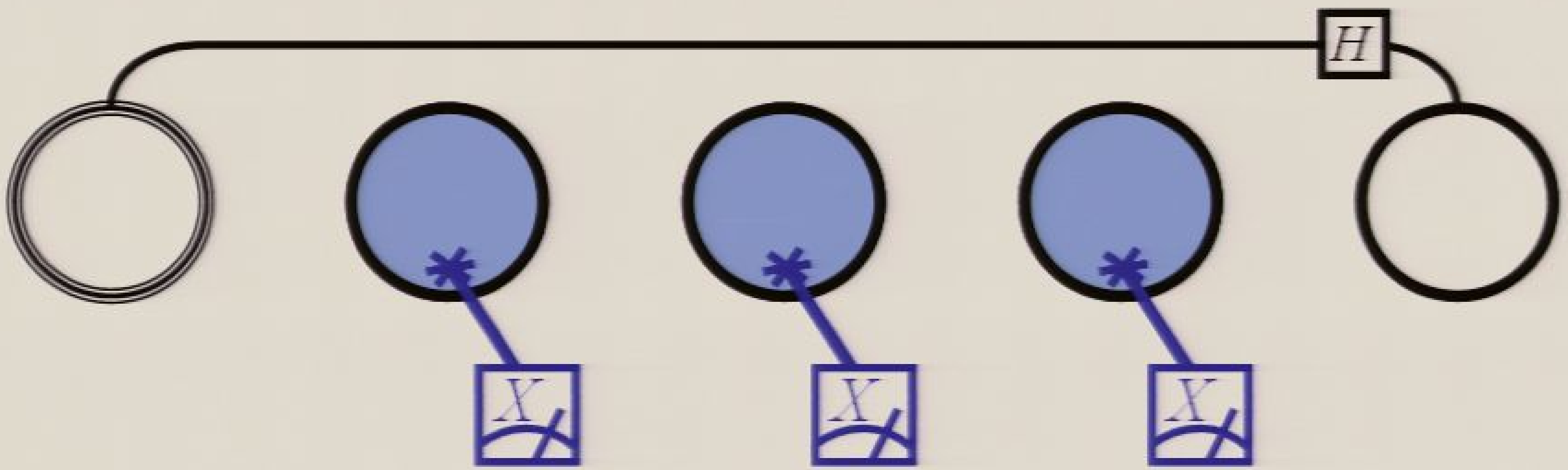

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$|010\rangle, |111\rangle: \; e^{i\frac{\theta}{2}}(I \otimes ZH)CZ|\psi\rangle|+\rangle$

$|101\rangle: \; e^{i\frac{\theta}{2}}(I \otimes H)CZ|\psi\rangle|+\rangle$

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…or $N+1$ imperfect links (odd N measured qubits)

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Larger sequences constructed from smaller sequences:

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Composition rule: $|010\rangle|x\rangle|1\rangle \rightarrow |01001\rangle$, $|01011\rangle$

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$R_{Z}(\xi)$ is not!

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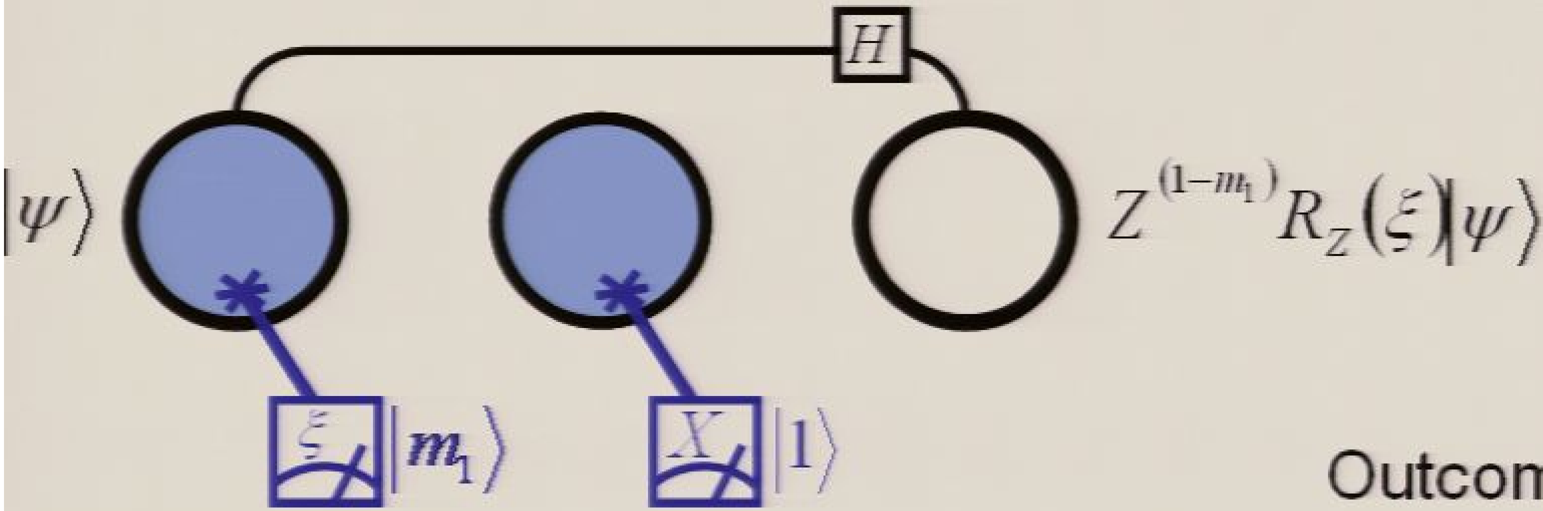
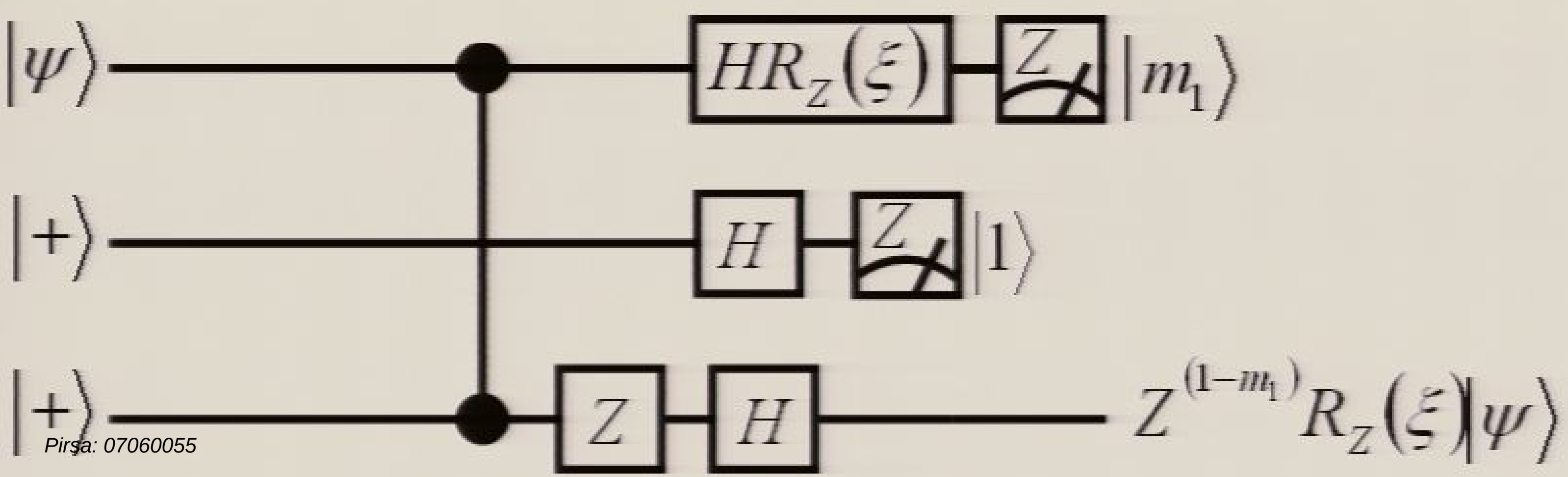


Outcome: $\boxed{m_2 = 1}$

SUCCESS!



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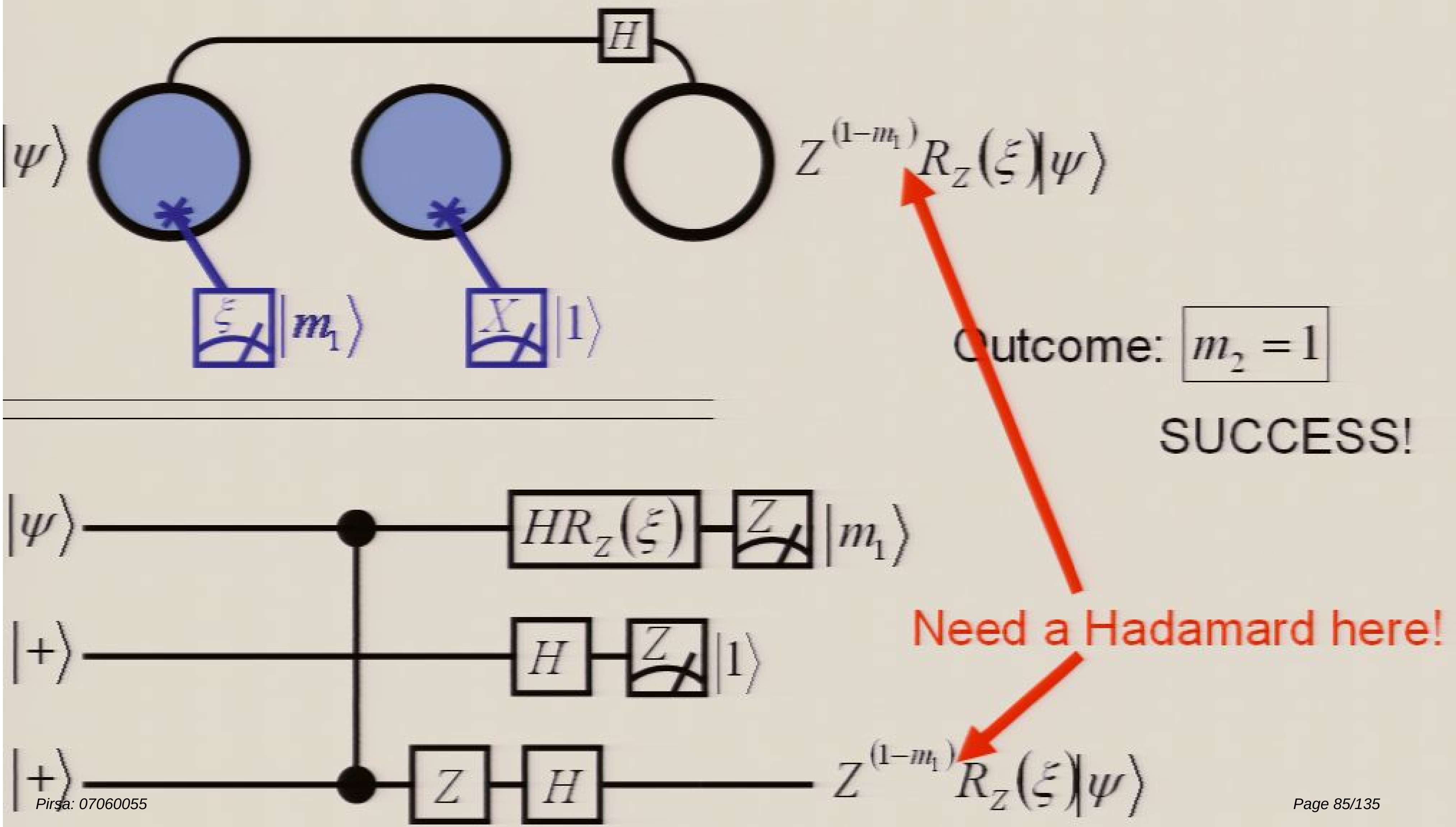

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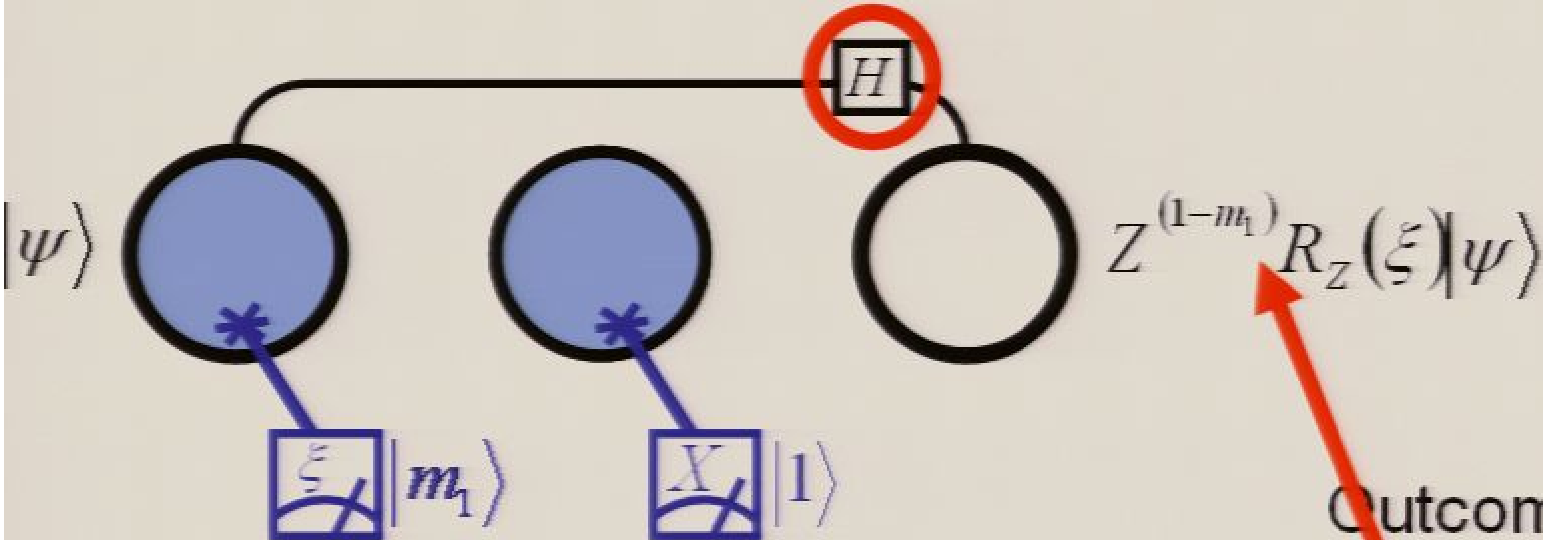

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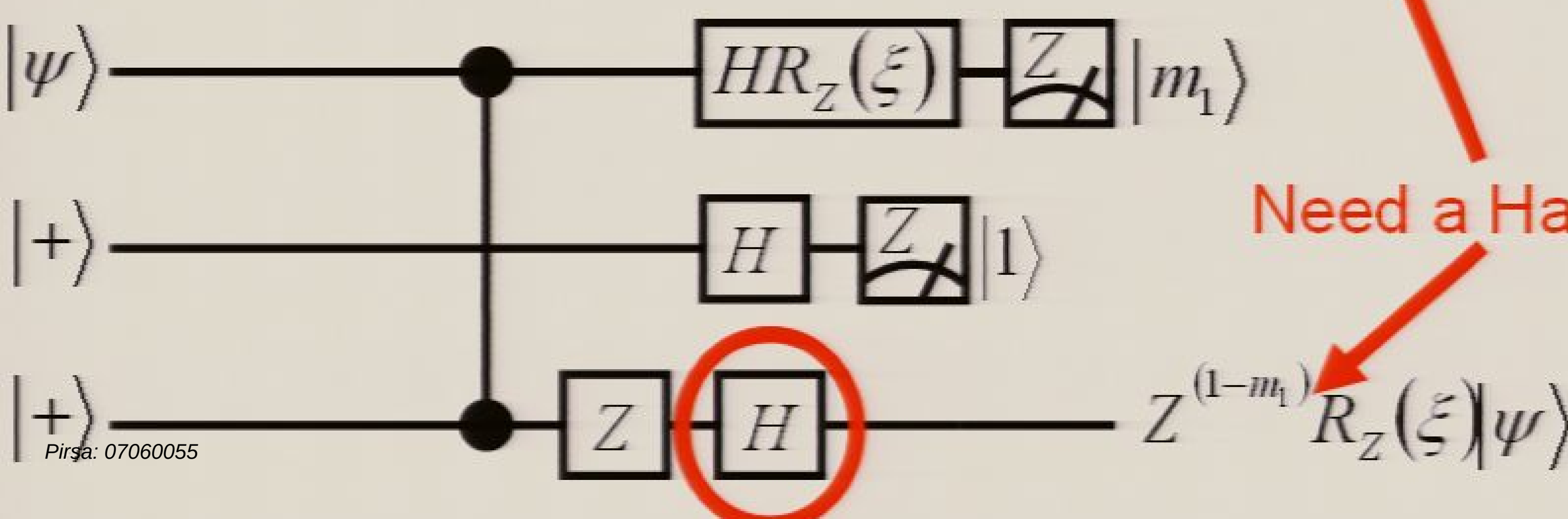

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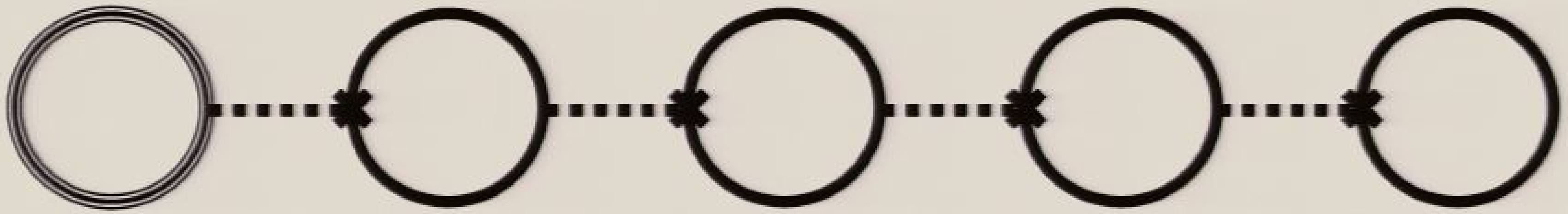

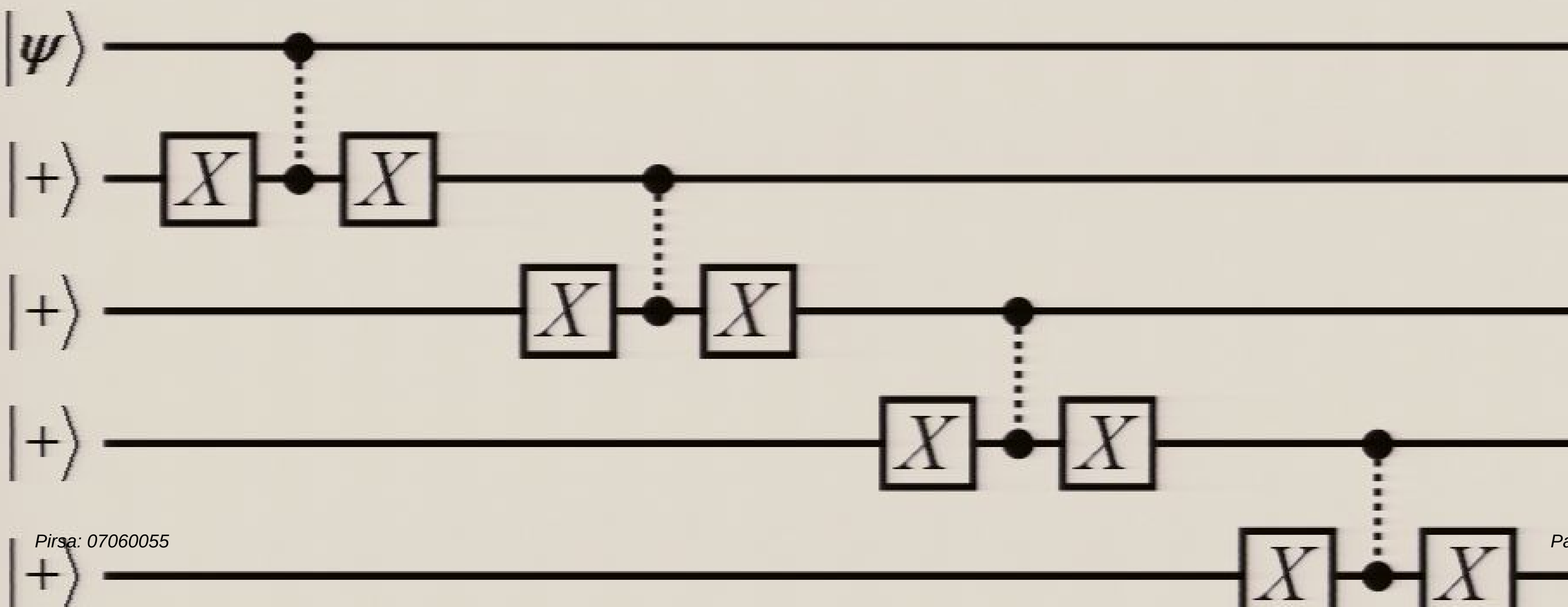

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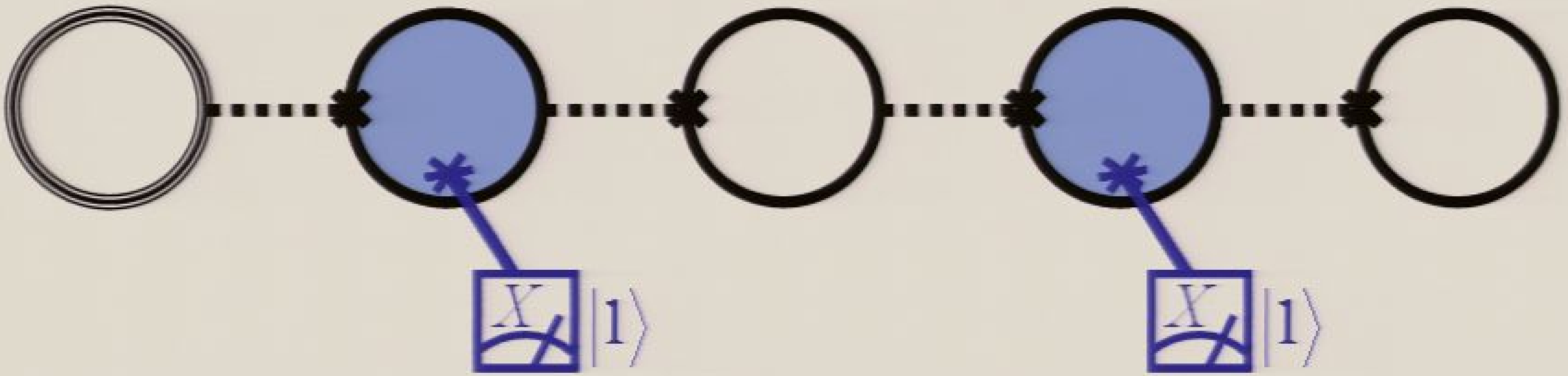

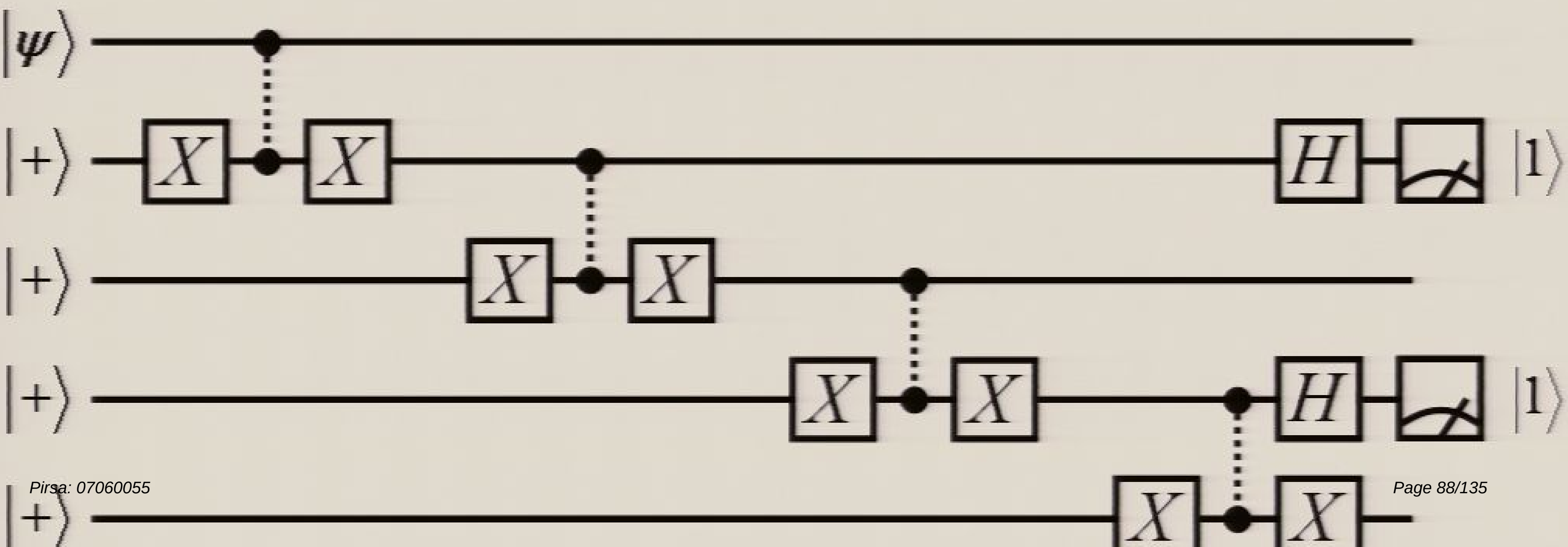

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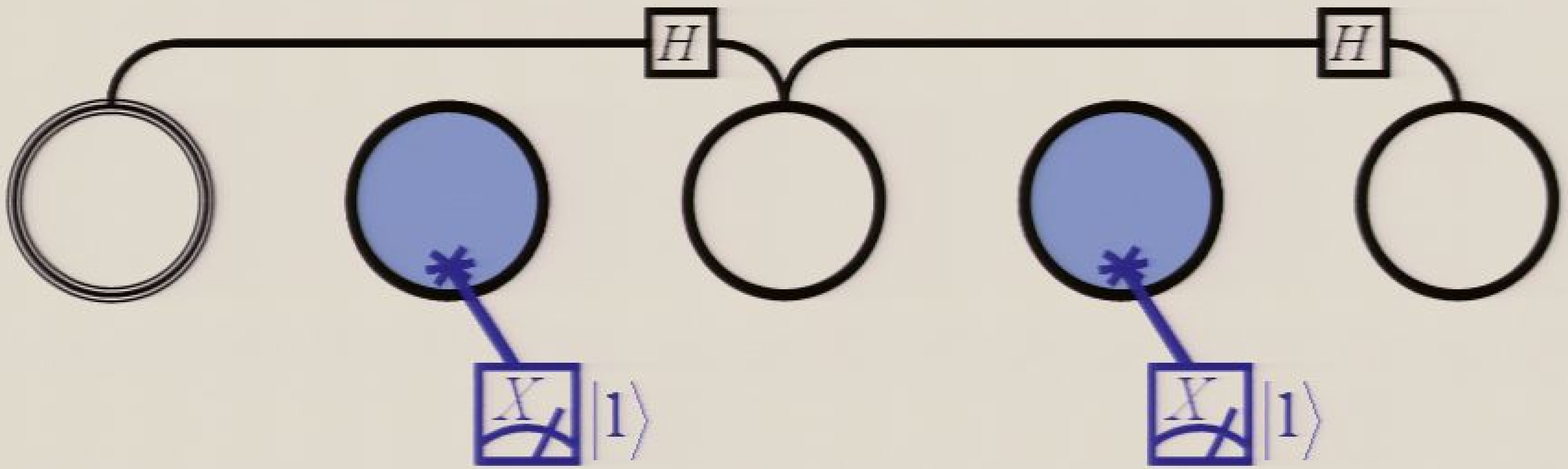

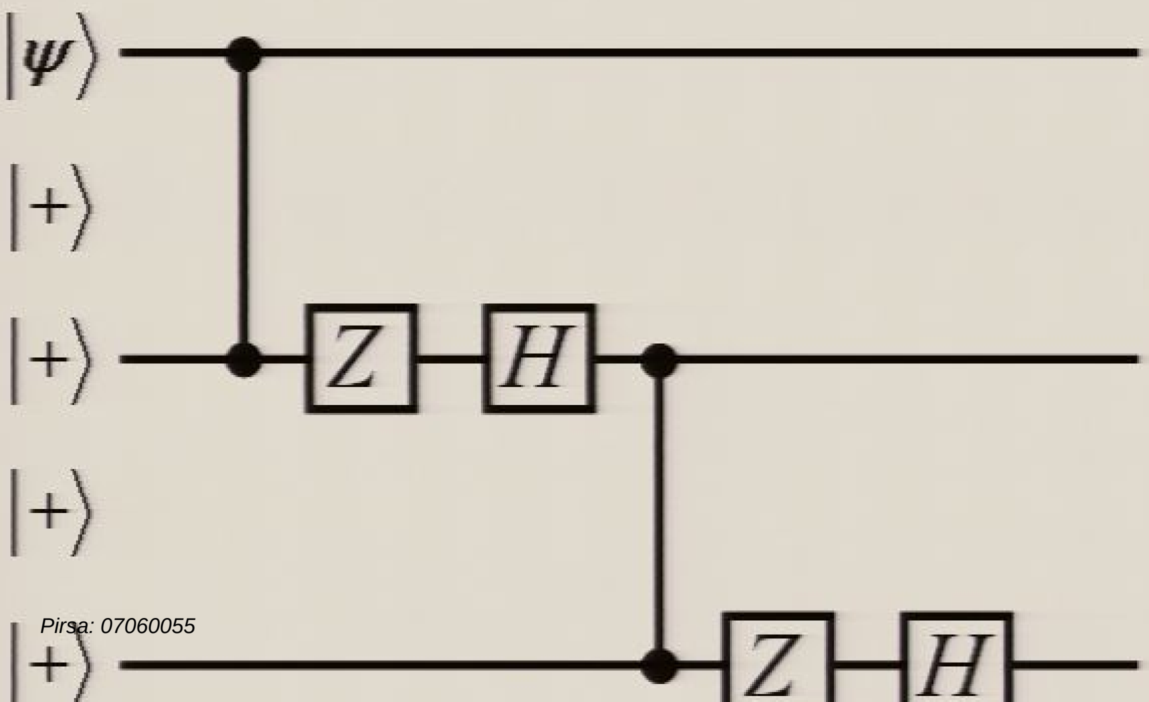

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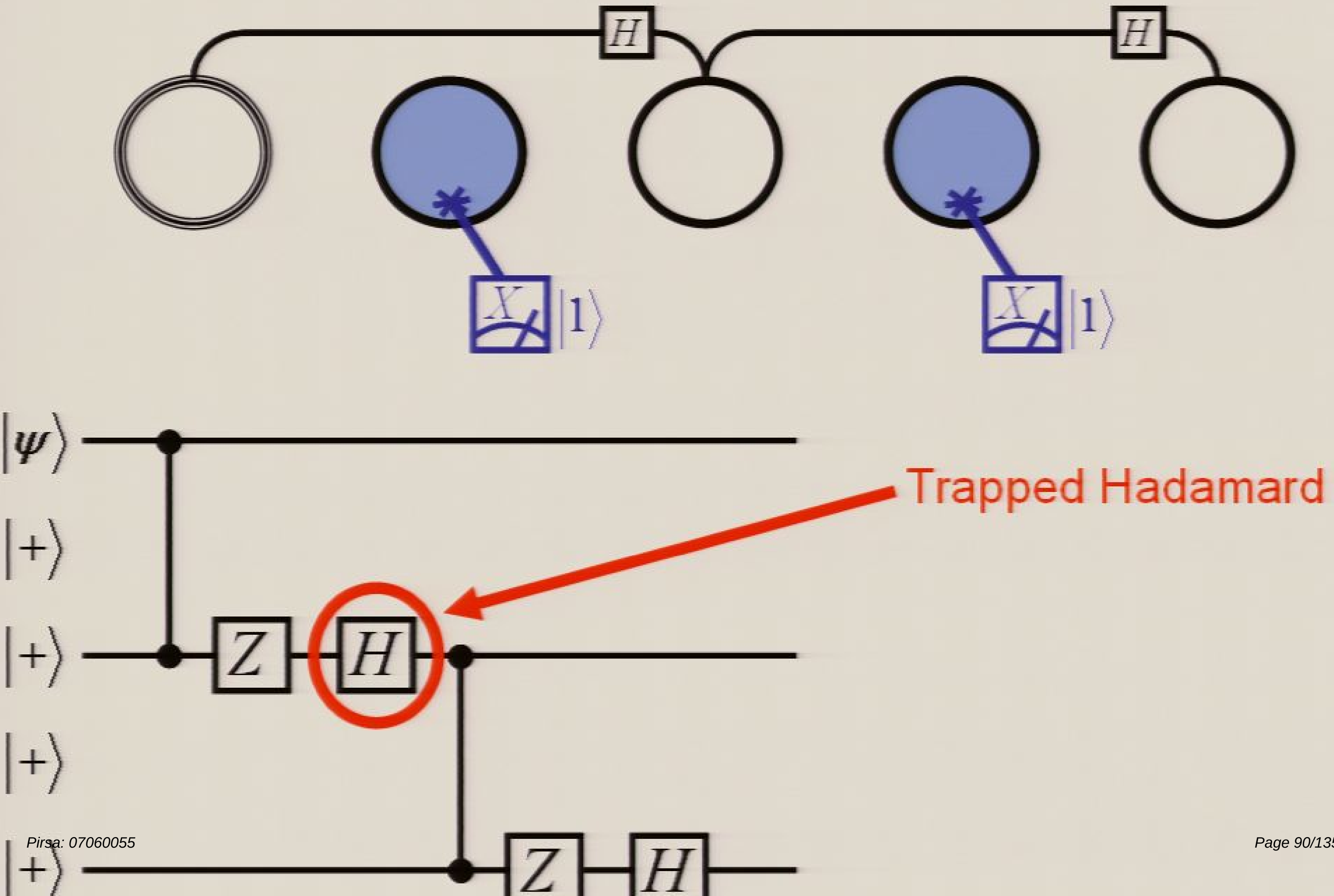

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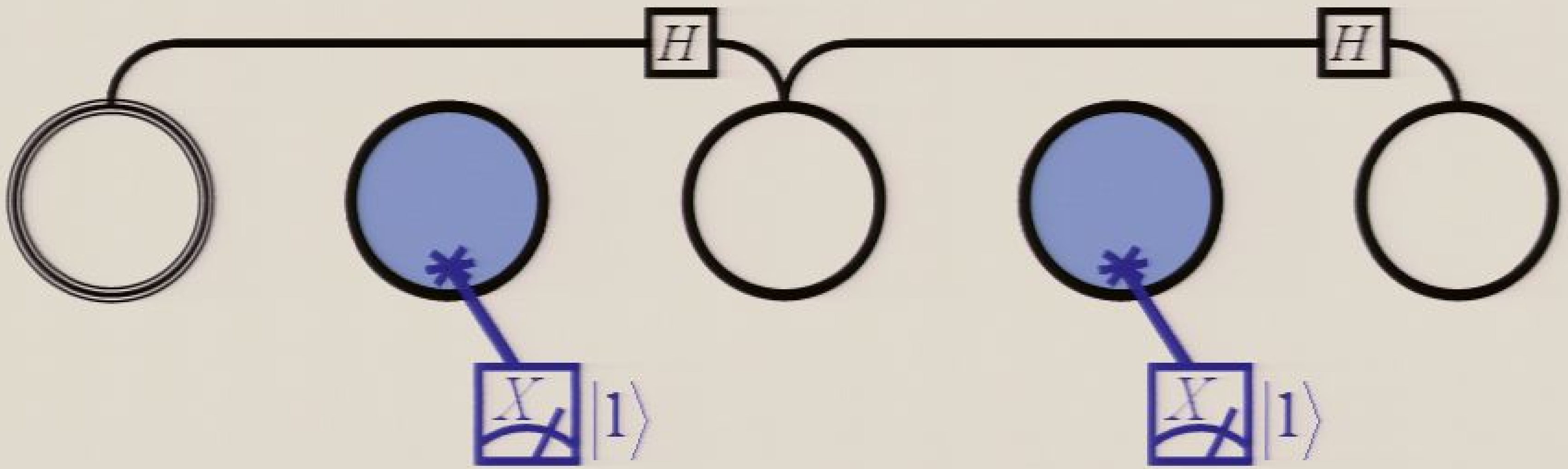

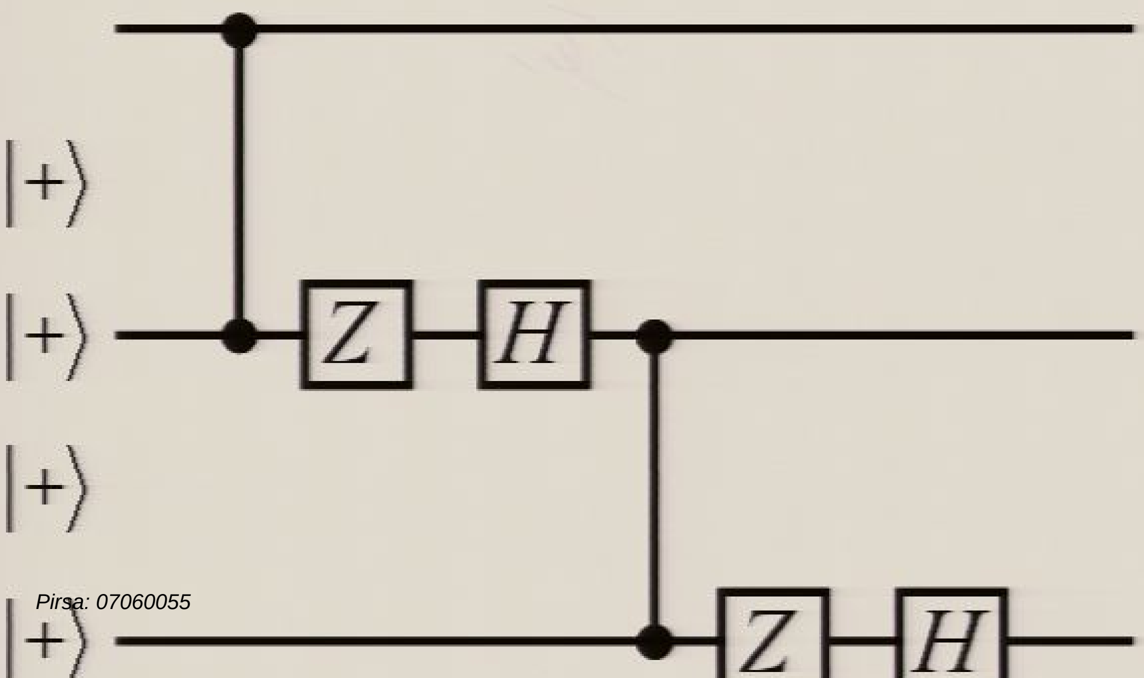

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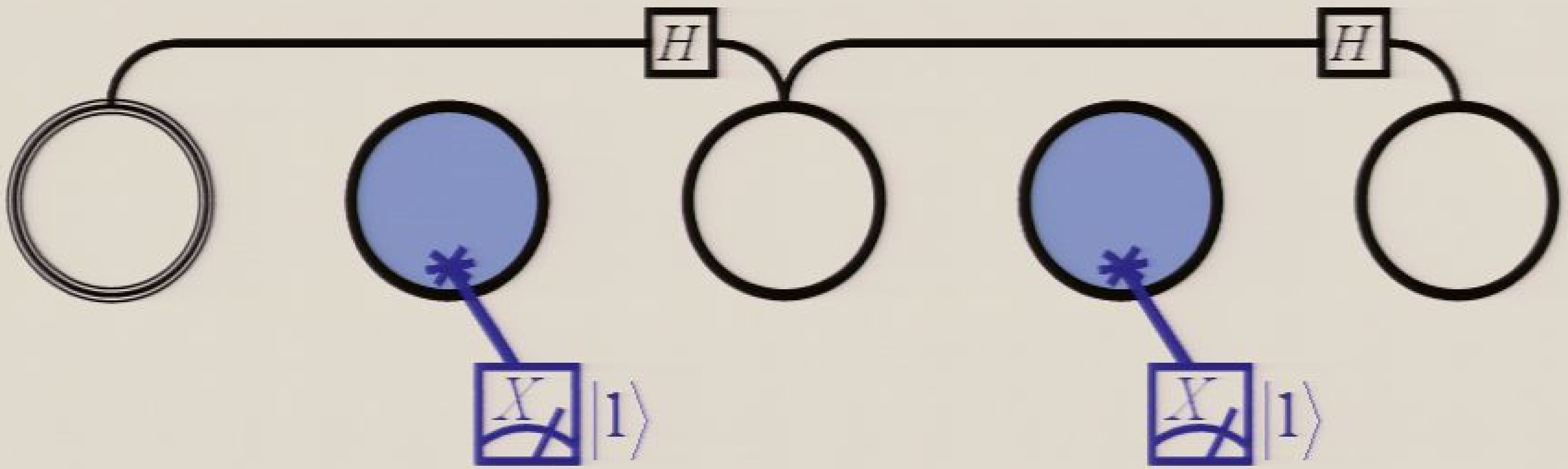

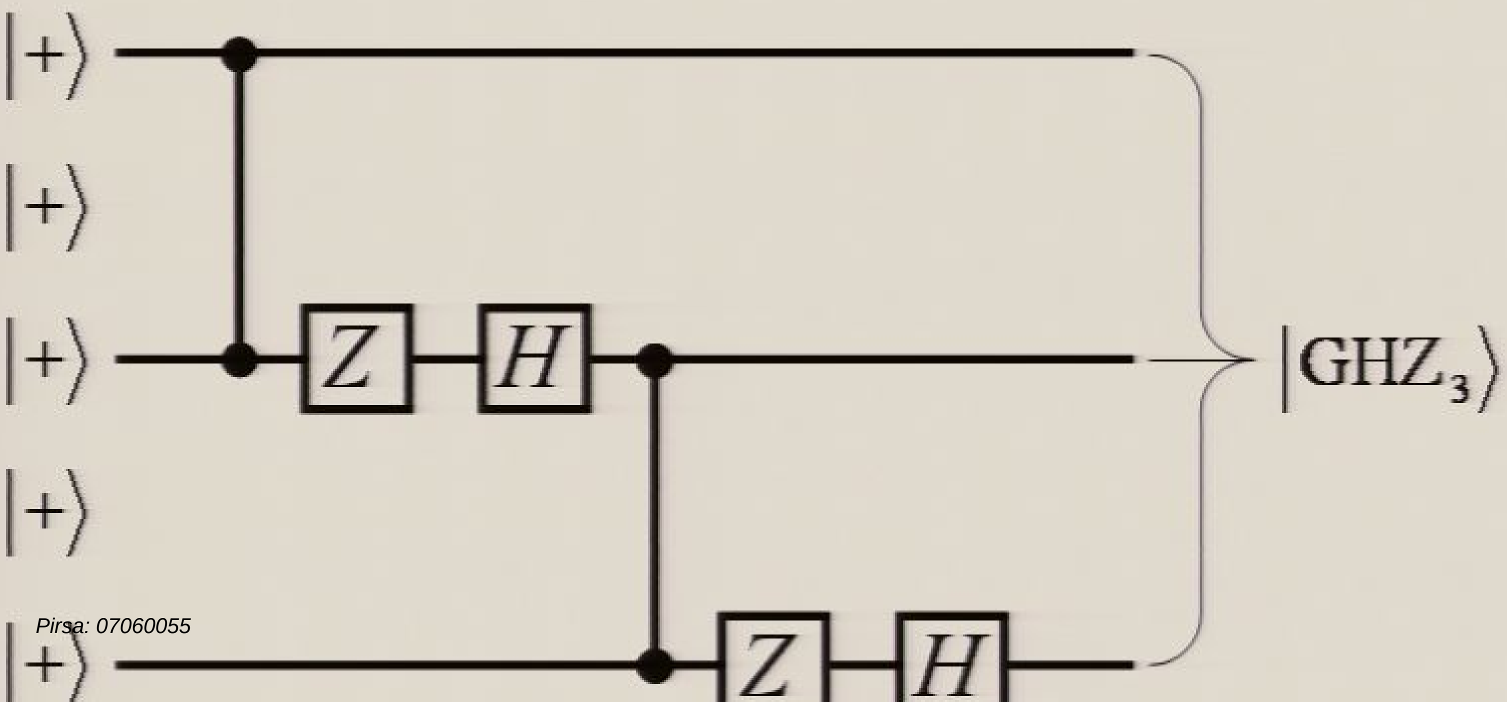

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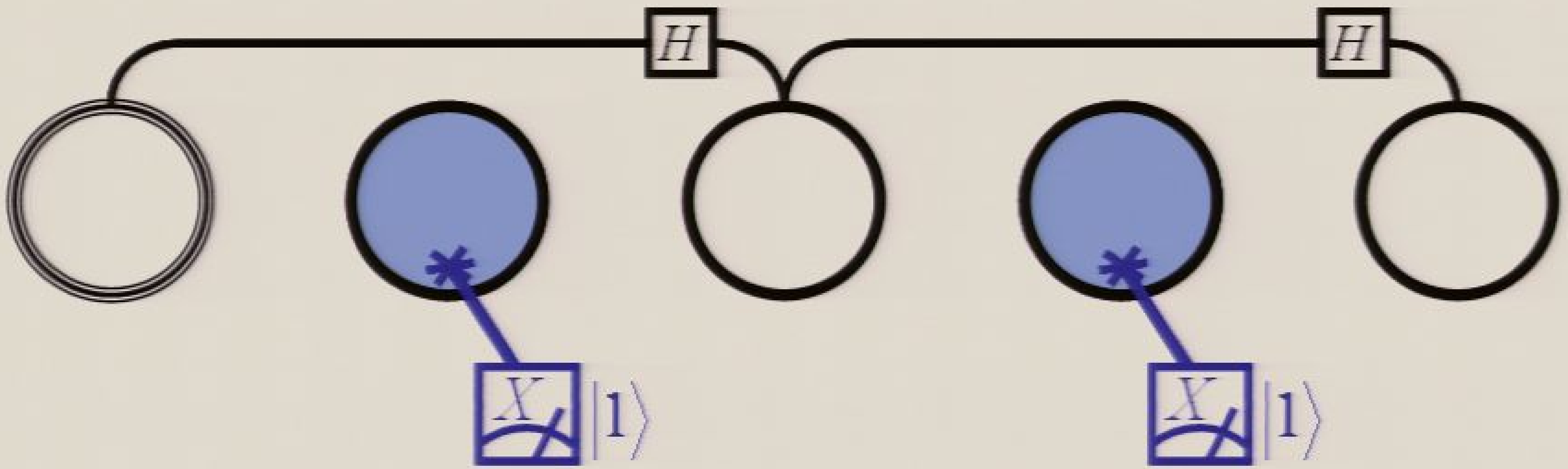
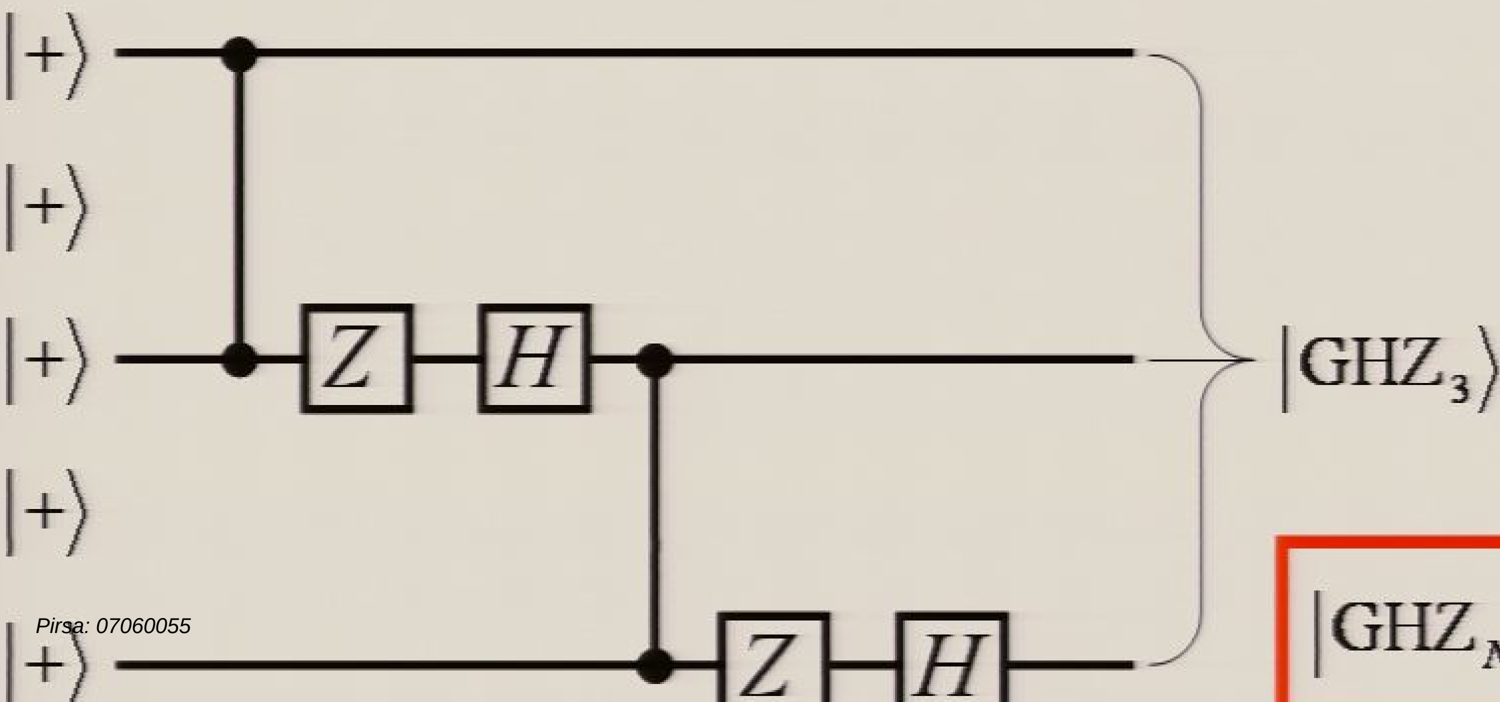
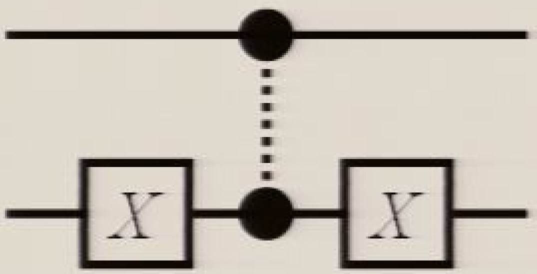




$$|\mathrm{GHZ}_N\rangle \overset{\text{LOCC}}{\neq} |\mathrm{cluster}_N\rangle$$

Beyond LOCC: Selective Entanglement

$$[I \otimes X]\, CS(\pi + \theta)\, [I \otimes X]$$



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -e^{i\theta} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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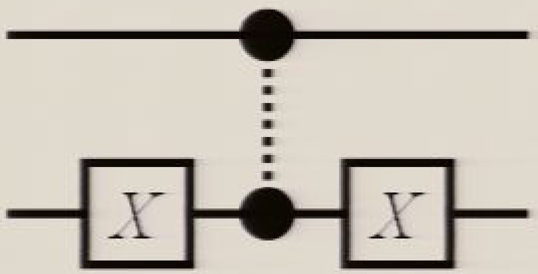

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Eigenstates won't become entangled!

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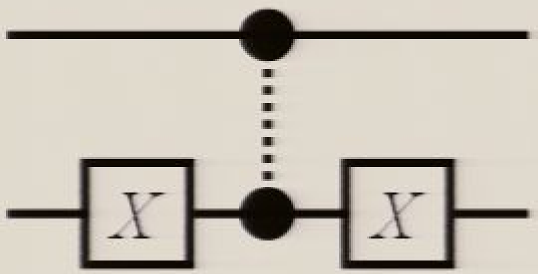


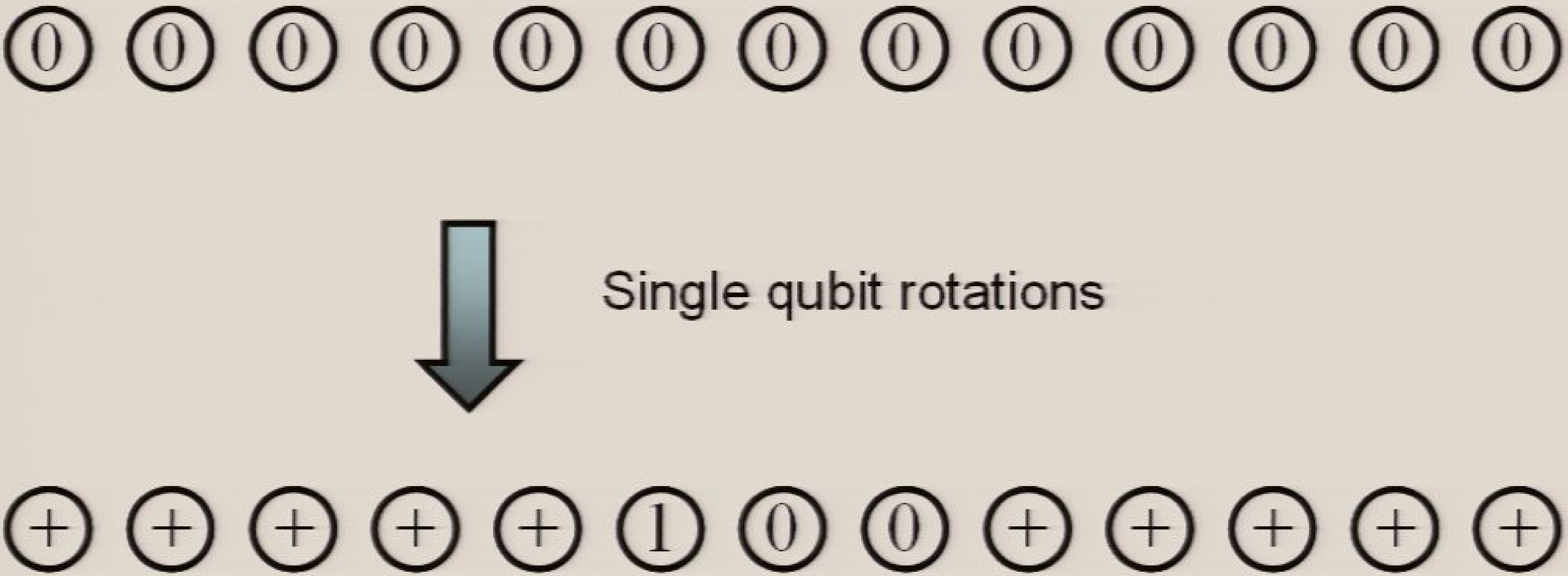
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -e^{i\theta} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Eigenstates won't become entangled!

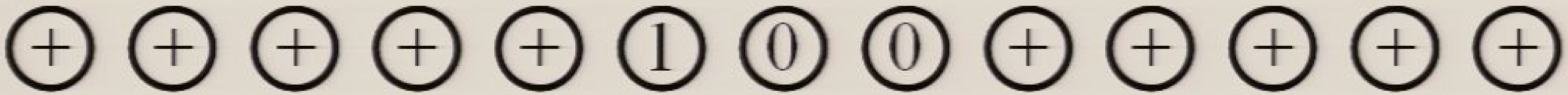
$$|0\rangle|\chi\rangle = \begin{pmatrix} \alpha \\ \beta \\ 0 \\ 0 \end{pmatrix} \qquad |\chi\rangle|1\rangle = \begin{pmatrix} 0 \\ \alpha \\ 0 \\ \beta \end{pmatrix} \qquad |1\rangle|0\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Beyond LOCC: Selective Entanglement

Beyond LOCC: Selective Entanglement



Beyond LOCC: Selective Entanglement



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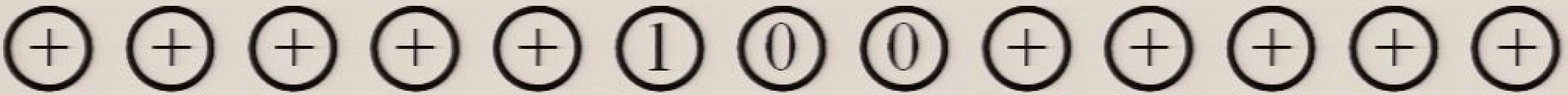
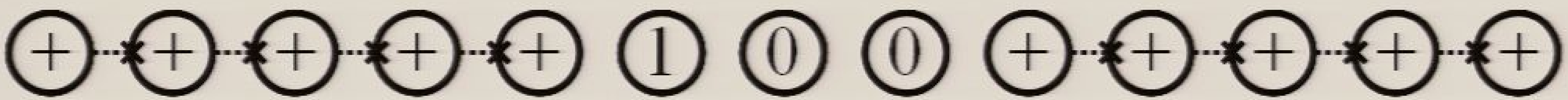


Imperfect entanglement
(eigenstates $|+\rangle|1\rangle$, $|0\rangle|+\rangle$, $|1\rangle|0\rangle$,
and $|0\rangle|0\rangle$ are unaffected)



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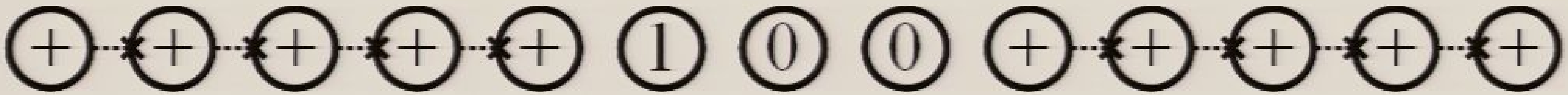
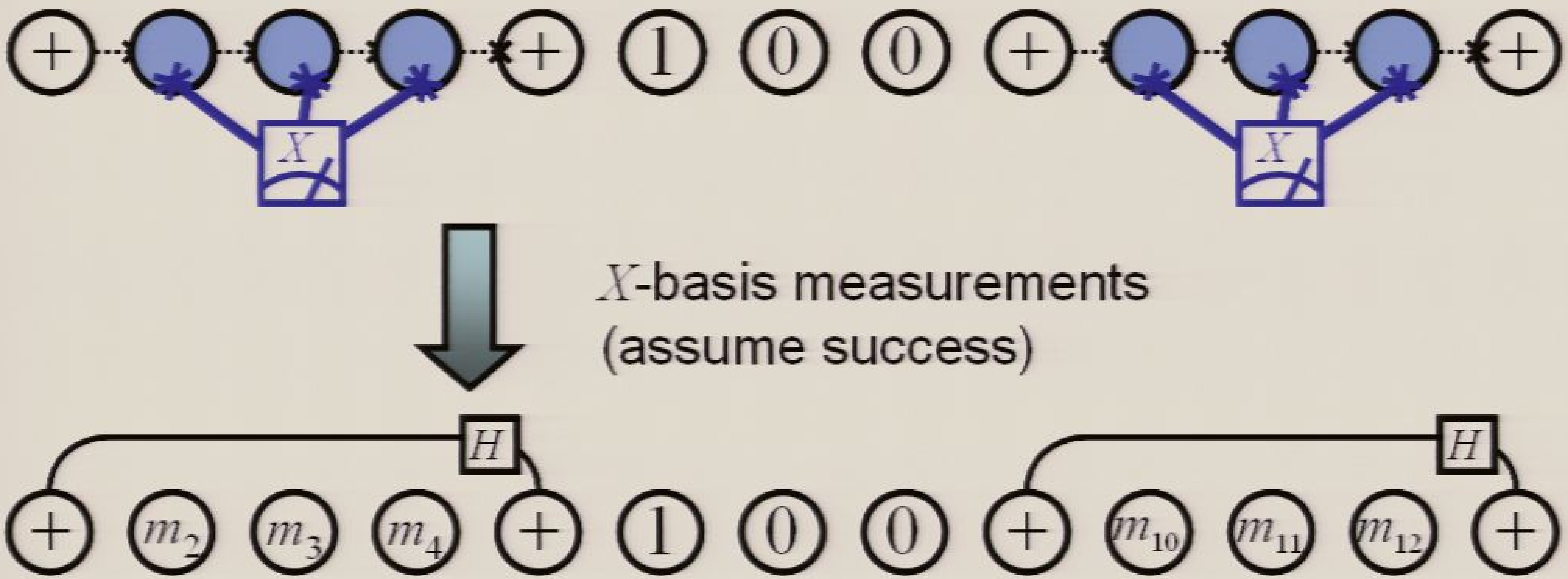


Beyond LOCC: Selective Entanglement



Untrapped Hadamards!

Beyond LOCC: Selective Entanglement

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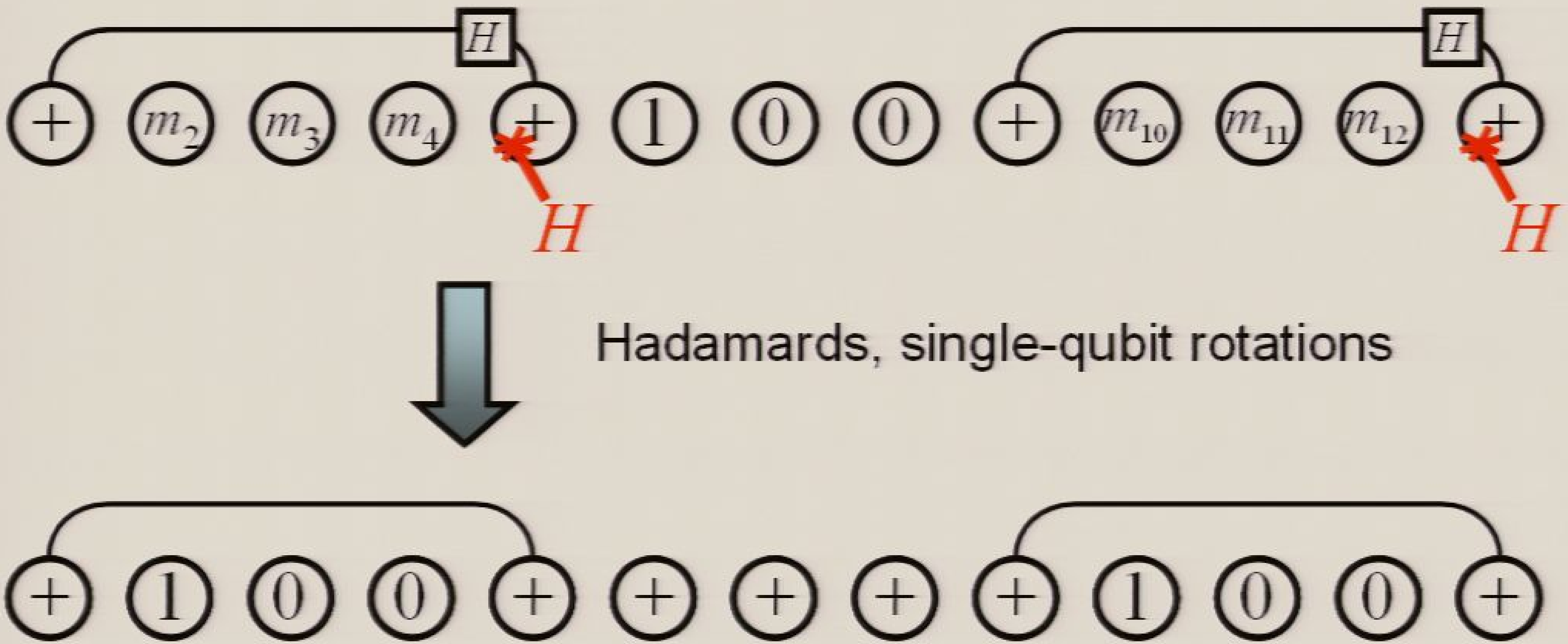

2-qubit cluster states!

Beyond LOCC: Selective Entanglement

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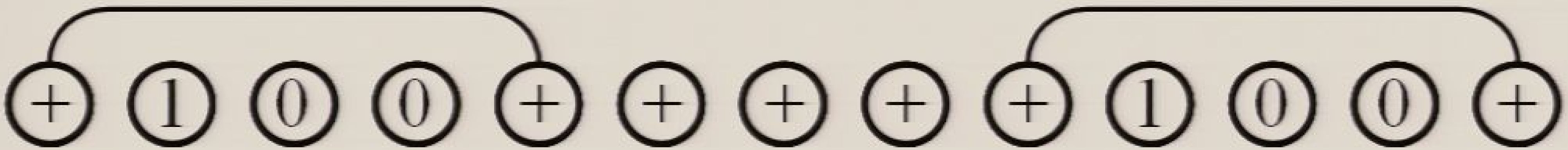

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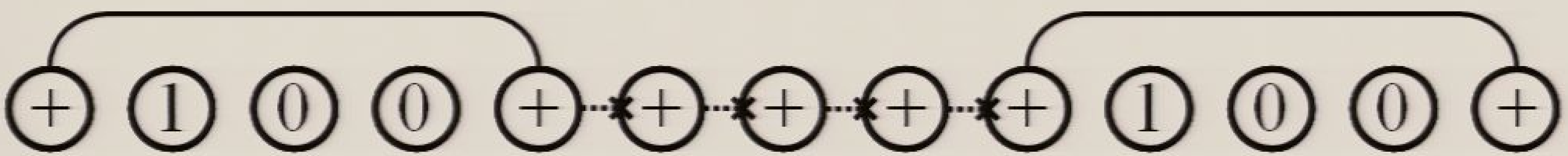

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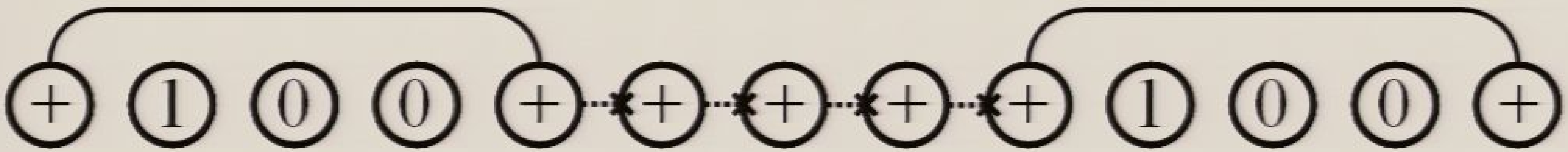

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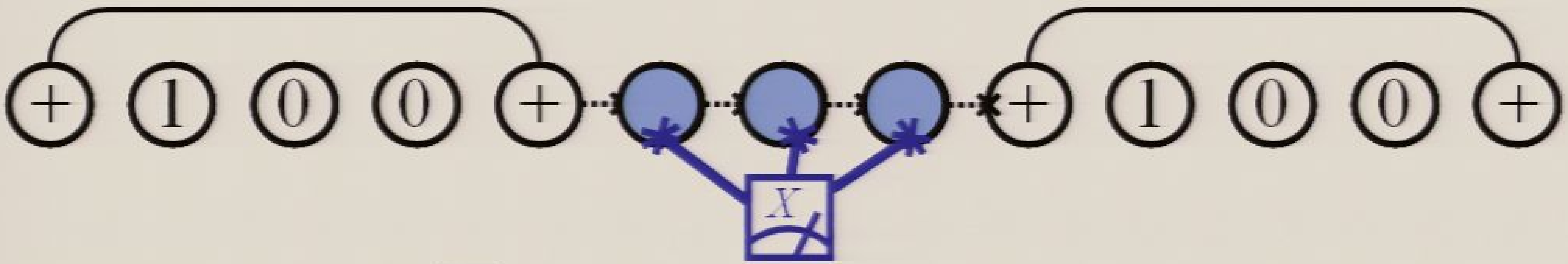
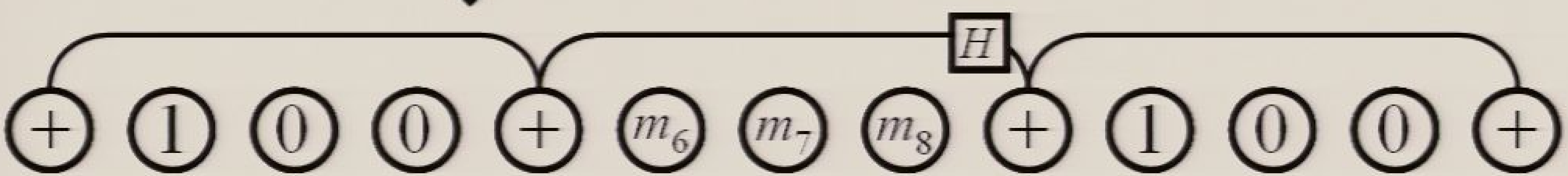


X-basis measurements
(assume success)



Trapped Hadamard!

Beyond LOCC: Selective Entanglement

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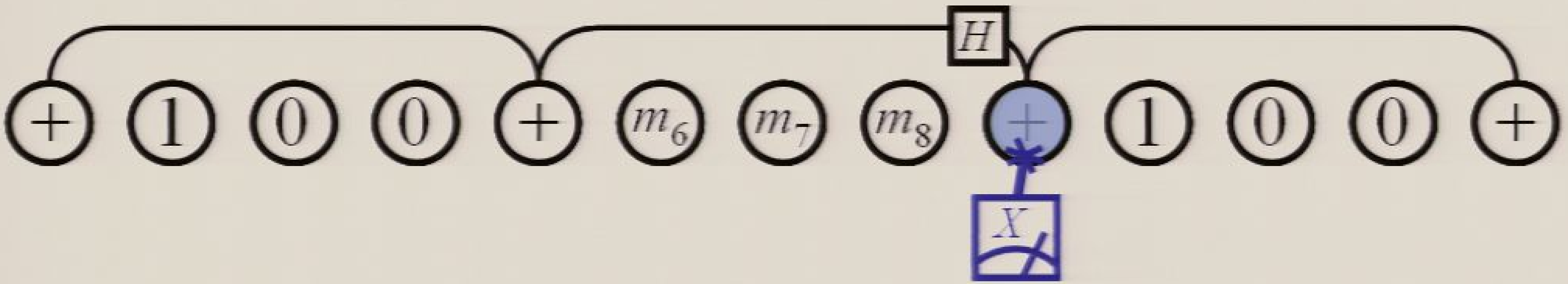
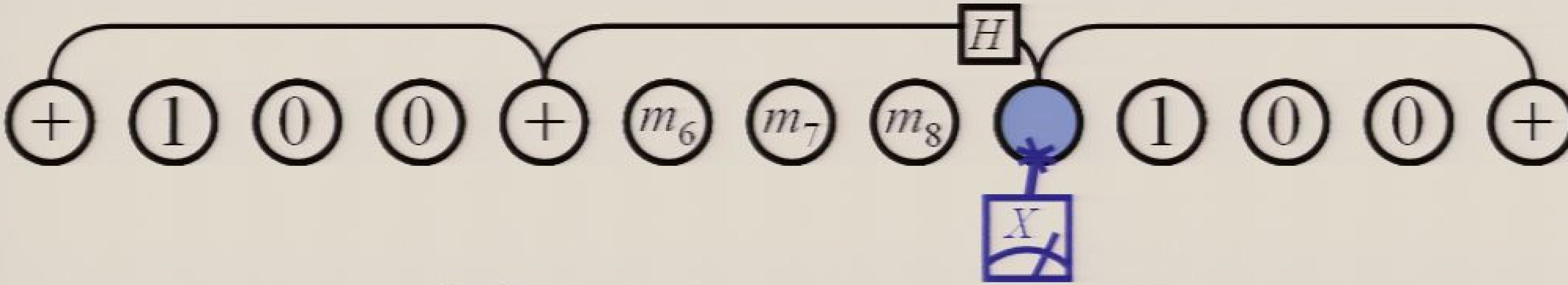
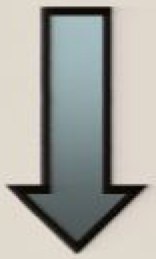


Beyond LOCC: Selective Entanglement





X-basis measurement
(arbitrary outcome)

3-qubit cluster state!

(generalizes to N qubits)

Beyond LOCC: Selective Entanglement

"If at first you don't succeed, try, try again"

Beyond LOCC: Selective Entanglement

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Beyond LOCC: Selective Entanglement

"If at first you don't succeed, try, try again"

If $|m_6 m_7 m_8\rangle \neq \{|010\rangle, |101\rangle, |111\rangle\}$, then re-initialize,

Beyond LOCC: Selective Entanglement

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If $\left|m_6 m_7 m_8\right\rangle \neq \left\{|010\rangle, |101\rangle, |111\rangle\right\}$, then re-initialize,

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Beyond LOCC: Selective Entanglement

“If at first you don’t succeed, try, try again”

If $\left|m_6 m_7 m_8\right\rangle \neq \left\{|010\rangle, |101\rangle, |111\rangle\right\}$, then re-initialize,

re-entangle,

and re-measure. Repeat until successful!

Beyond LOCC: Selective Entanglement

"If at first you don't succeed, try, try again"

Beyond LOCC: Selective Entanglement

"If at first you don't succeed, try, try again"

X-basis measurements can be performed simultaneously (constant depth)

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Average fraction of successful outcomes per step:

$$F_{\text{succ}} = P_{\text{succ}}^{(3)} = \frac{3}{8}\cos^4\left(\frac{\theta}{2}\right)$$

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Beyond LOCC: Selective Entanglement

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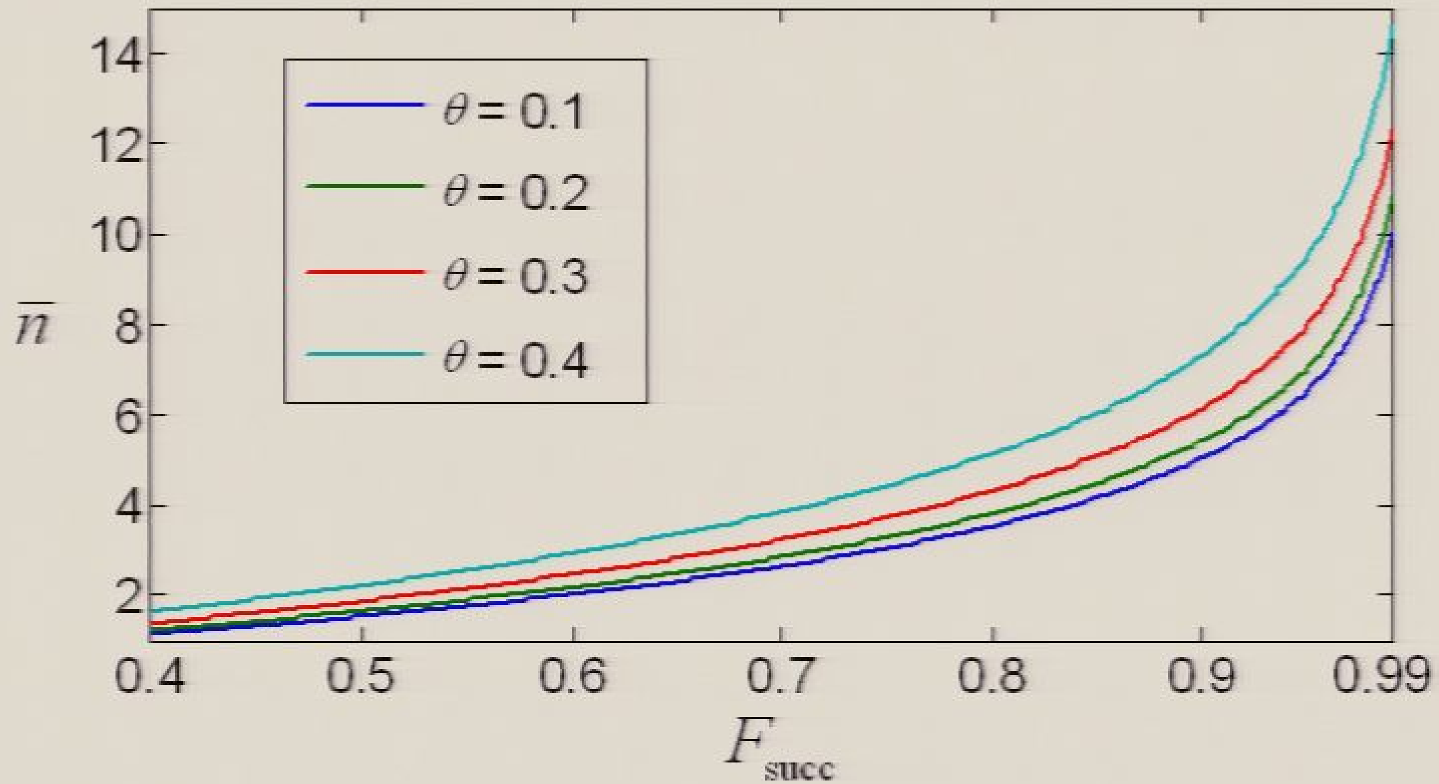

Average number of steps to attain desired success fraction:

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Beyond LOCC: Selective Entanglement

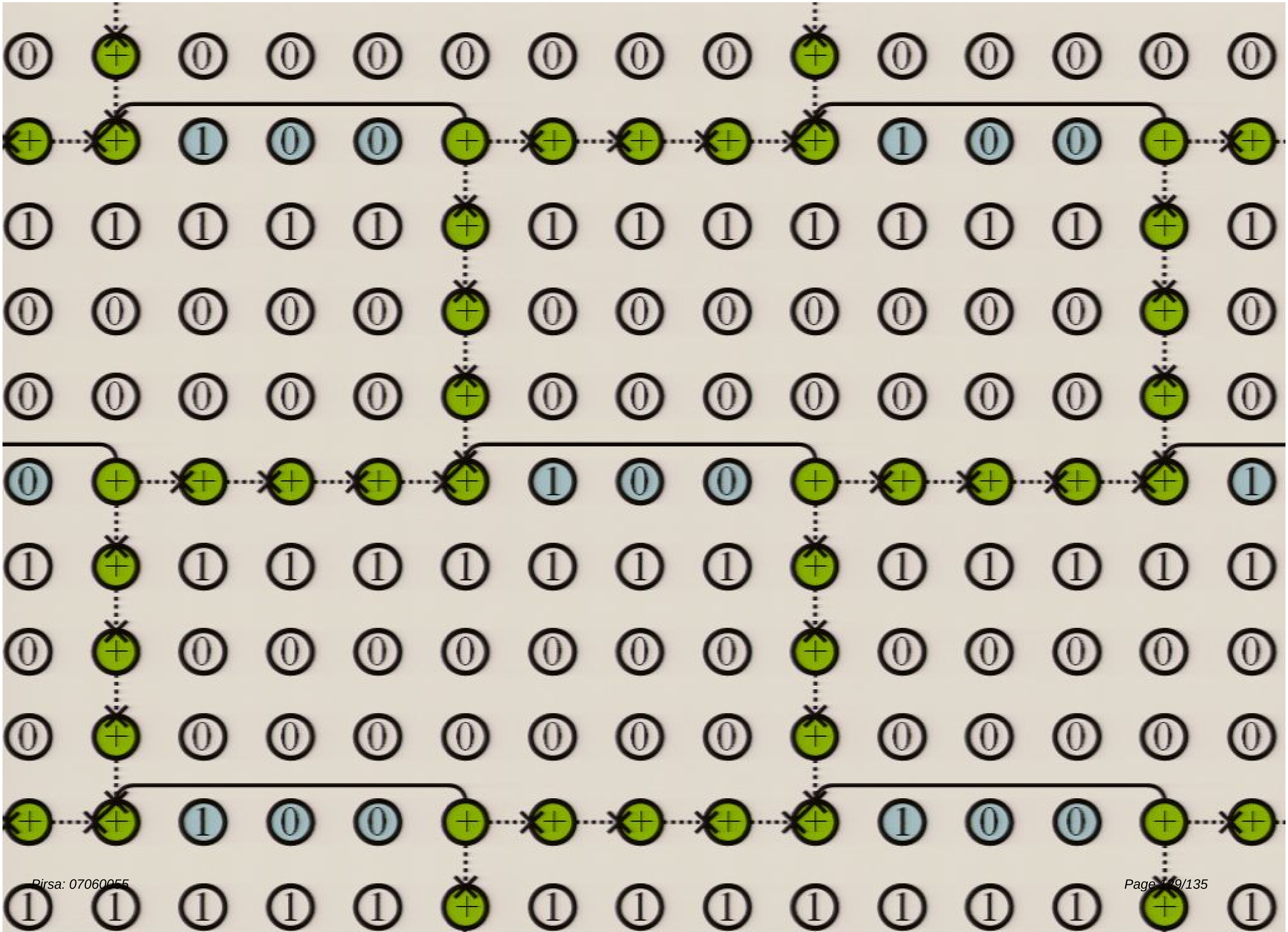
Generalizes to 2D & 3D

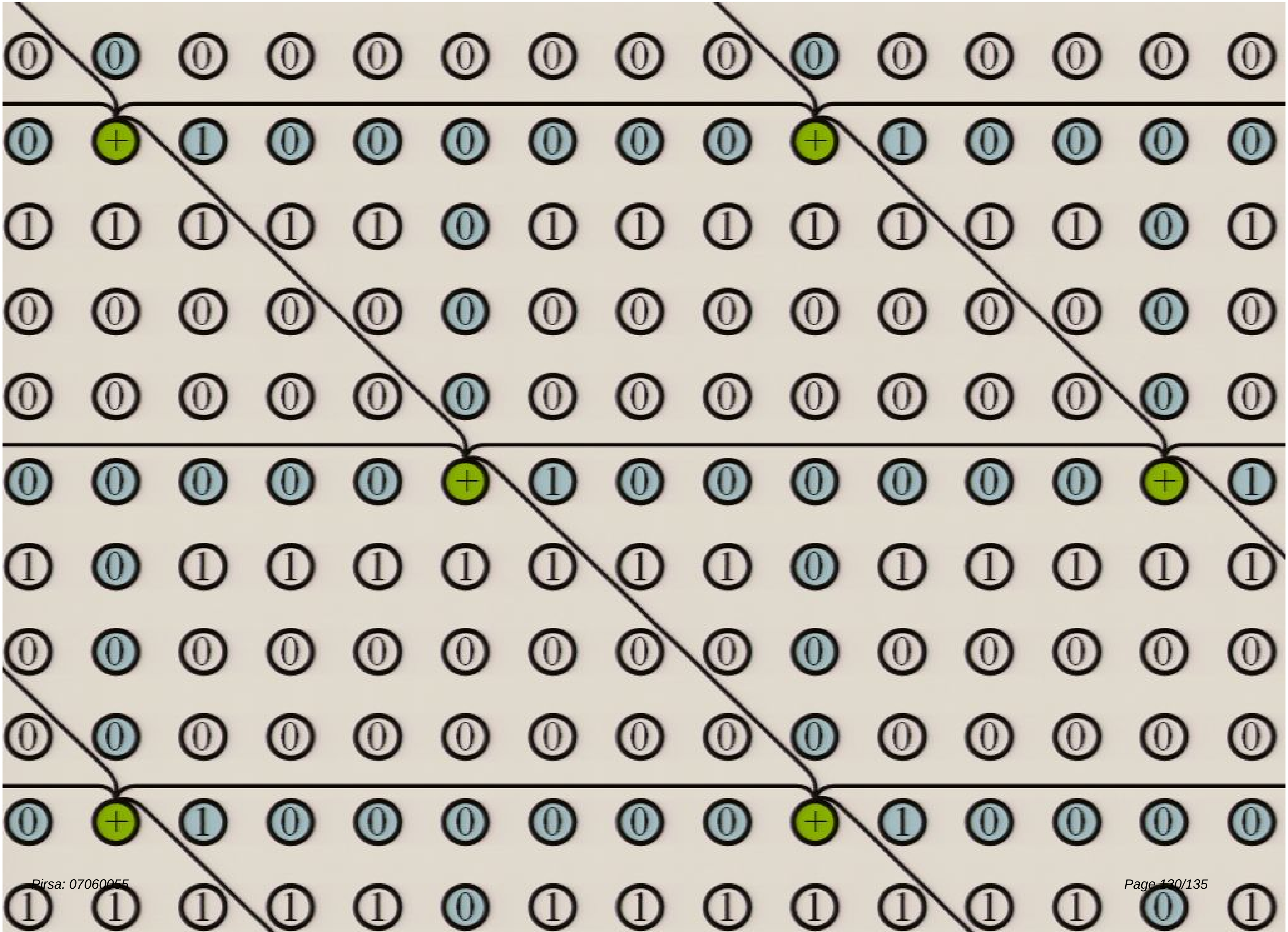
1) Entangle checkerboard pattern of 5-qubit chains

2) Convert desired fraction into 2-qubit cluster states

3) Entangle chains linking 2-qubit cluster states

4) Convert desired fraction into perfect links





Bonus Features

Constant overhead (32 physical qubits * N)

Direct construction of perfect real-space quantum circuits for one-way quantum computing (without Z-basis measurements)

Direct construction of various other perfect graph states

CONCLUSION

PROBLEM: imperfect entanglement / systematic phase errors

SOLUTION (part 1): combine imperfect links via measurement

SOLUTION (part 2): selective entangling via state preparation

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SOLUTION (part 1): combine imperfect links via measurement

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FUTURE WORK

Construction of encoded cluster states (in 3D?)

Use of multi-qubit addressing

A better understanding of why this works!?!?

THANK YOU!

David Feder (PhD supervisor)

End of slide show, click to exit.