

Title: Quantum Cellular Automata Applications

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Abstract:

Applications of Quantum Cellular Automata

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Canadian Quantum Information Student's Conference,
2007

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Applications we will discuss are...

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- Single spin Measurement (described in [P., Mosca, Cappellaro, Cory. PRL 97, 100501 (2006)]).
- Large (macroscopic) Schrödinger Cat State creation and verification (work in progress).



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- In NMR, as in other settings, it would be desirable to be able to measure a single spin.



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- This would allow, among other things, NMR quantum computers that are not reliant on large ensembles.
- An NMR spectrometer can only detect the magnetic footprint of a large enough ensemble, of roughly size 10^6 spins.



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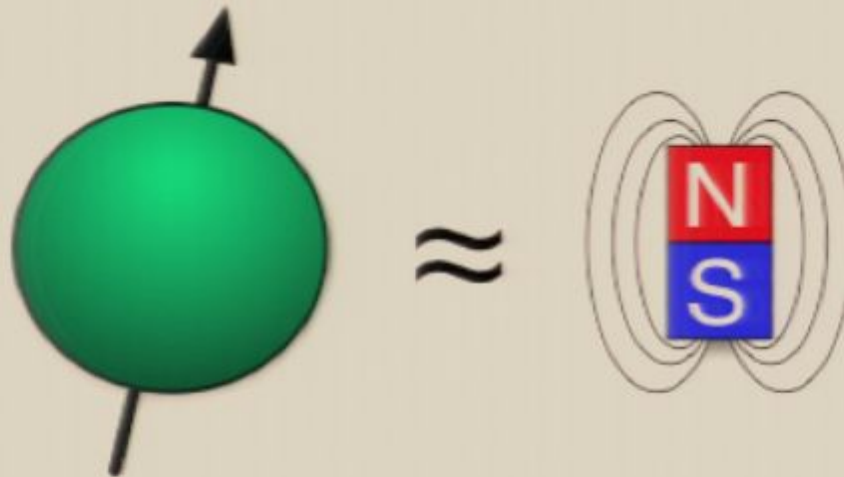
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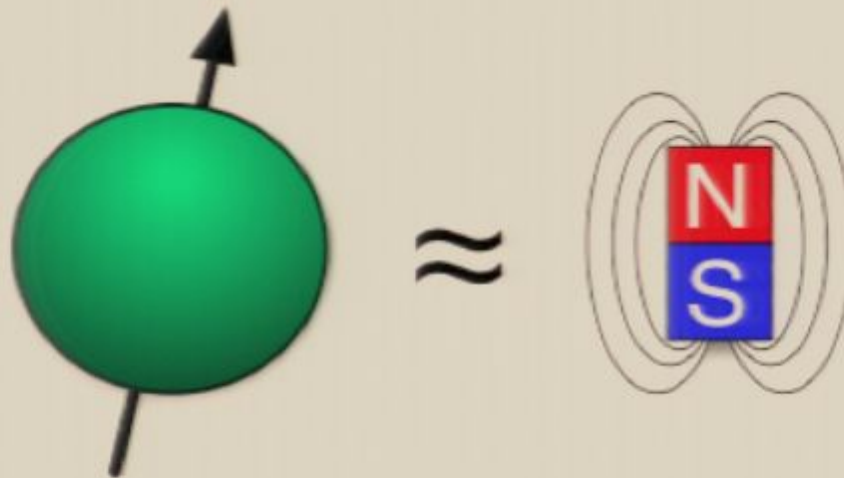
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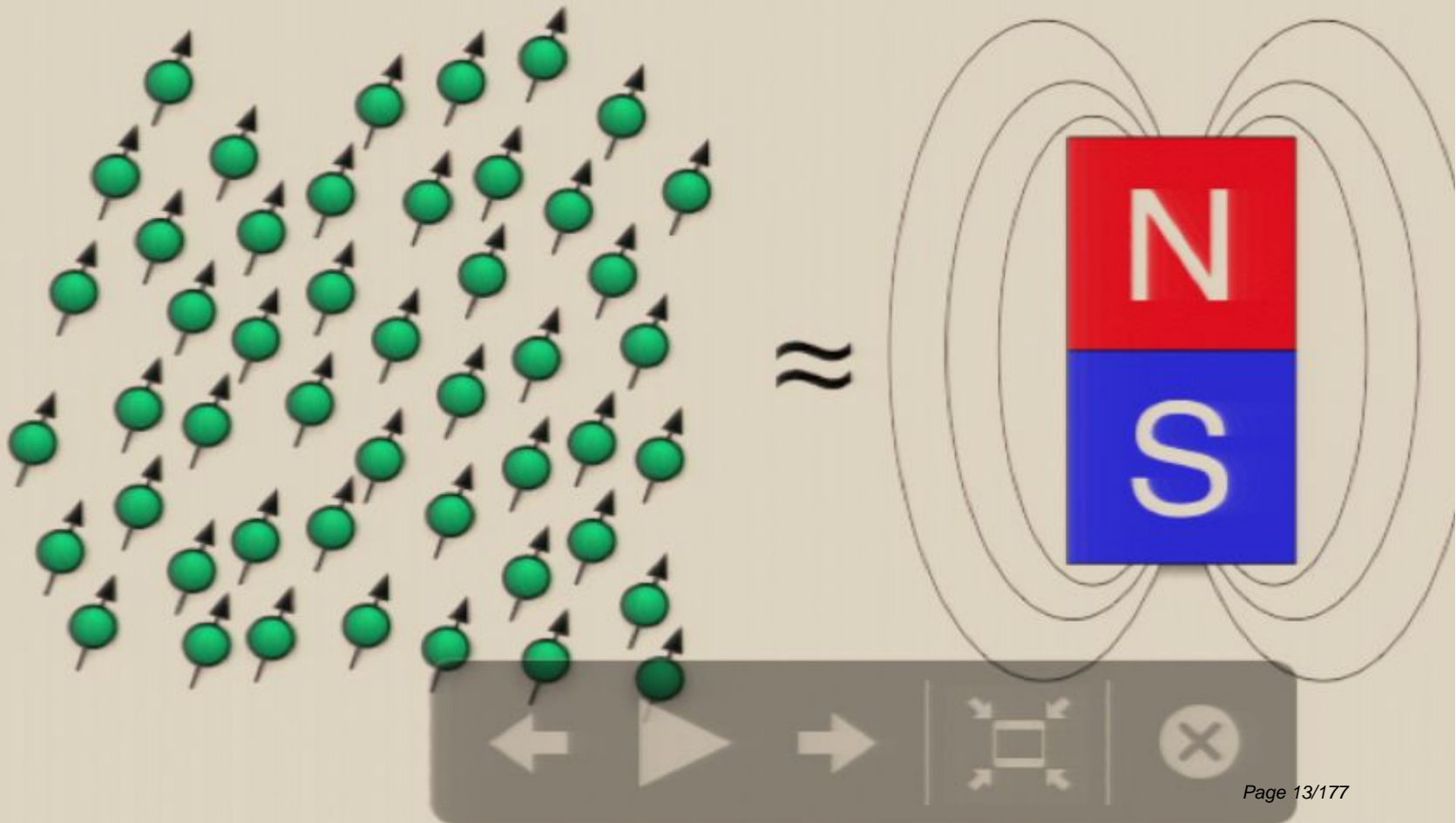
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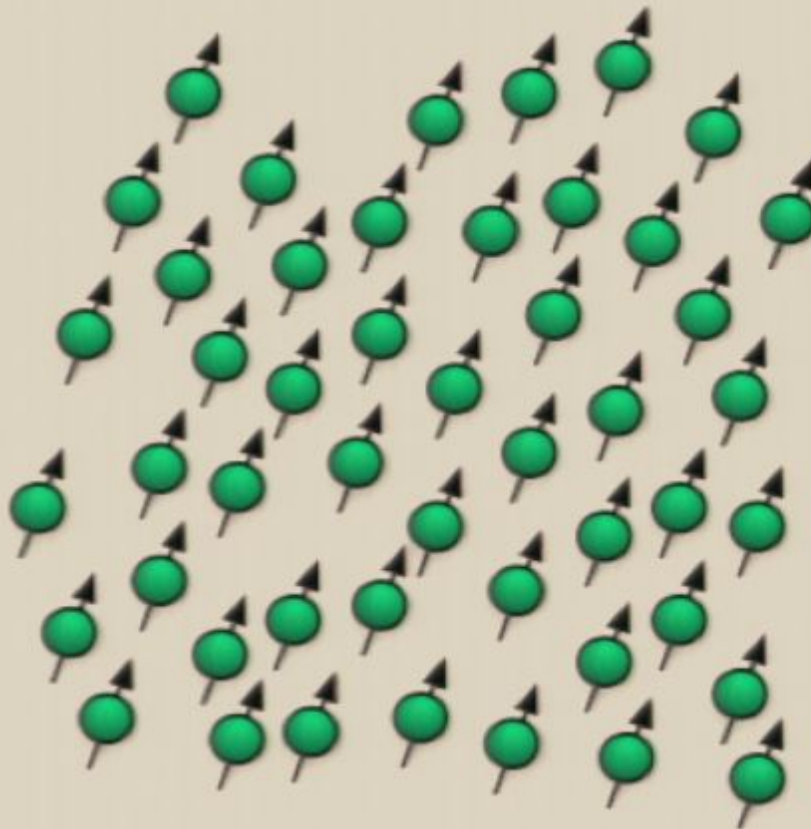
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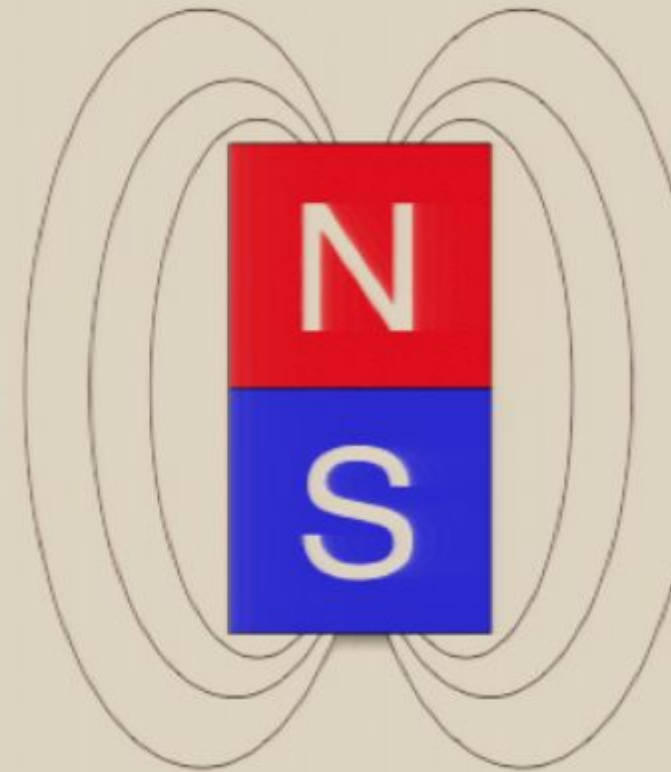
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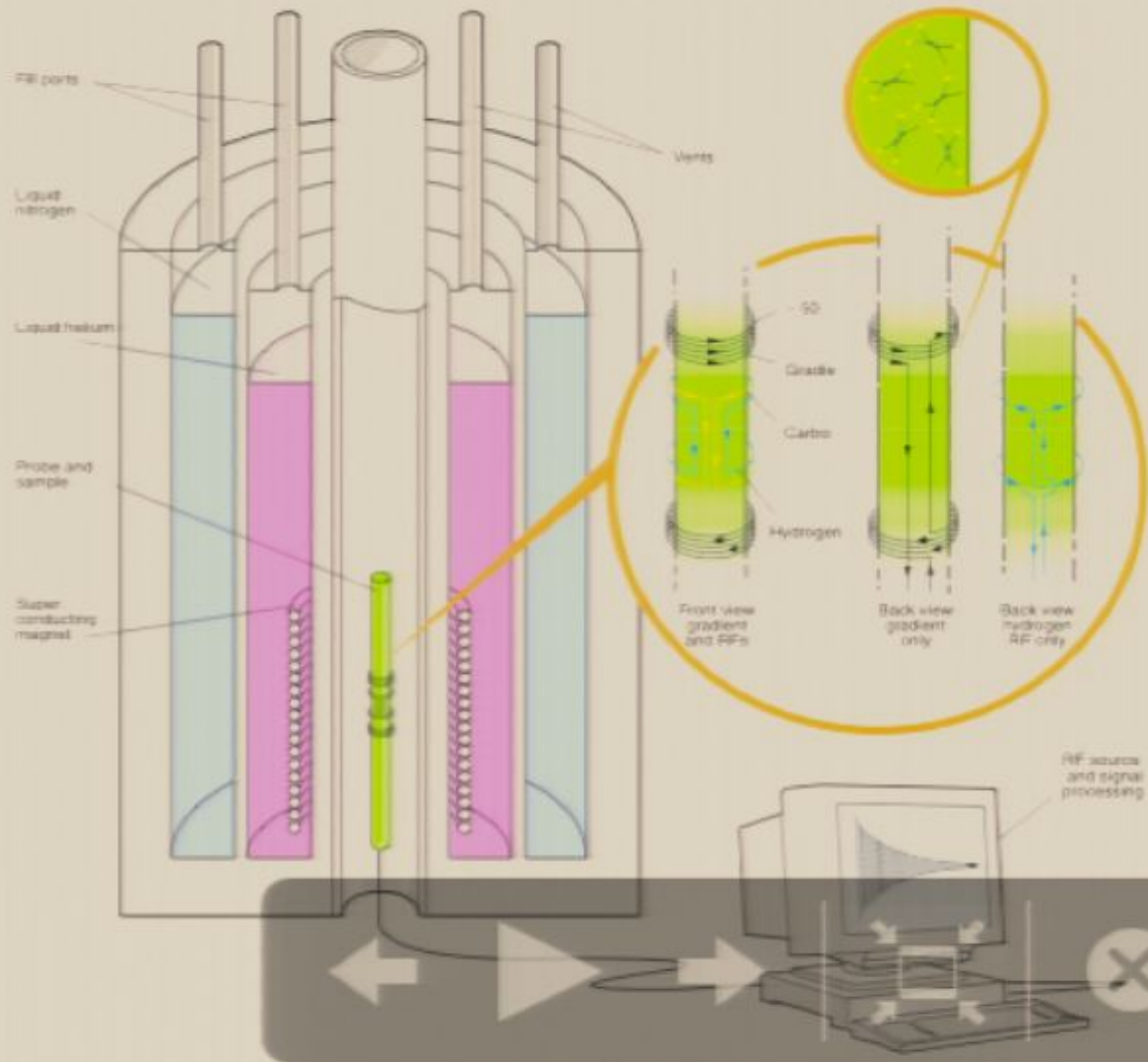
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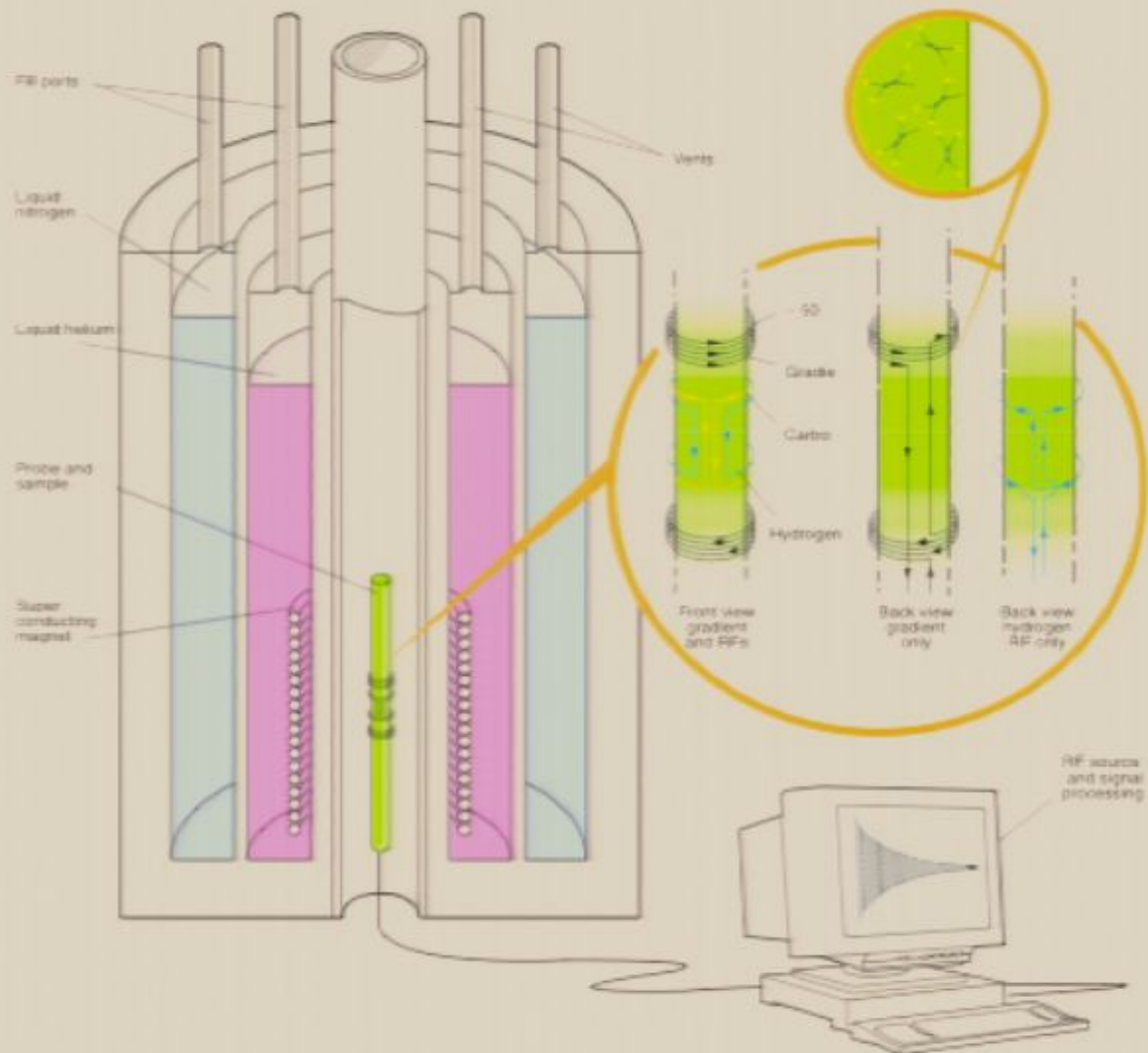
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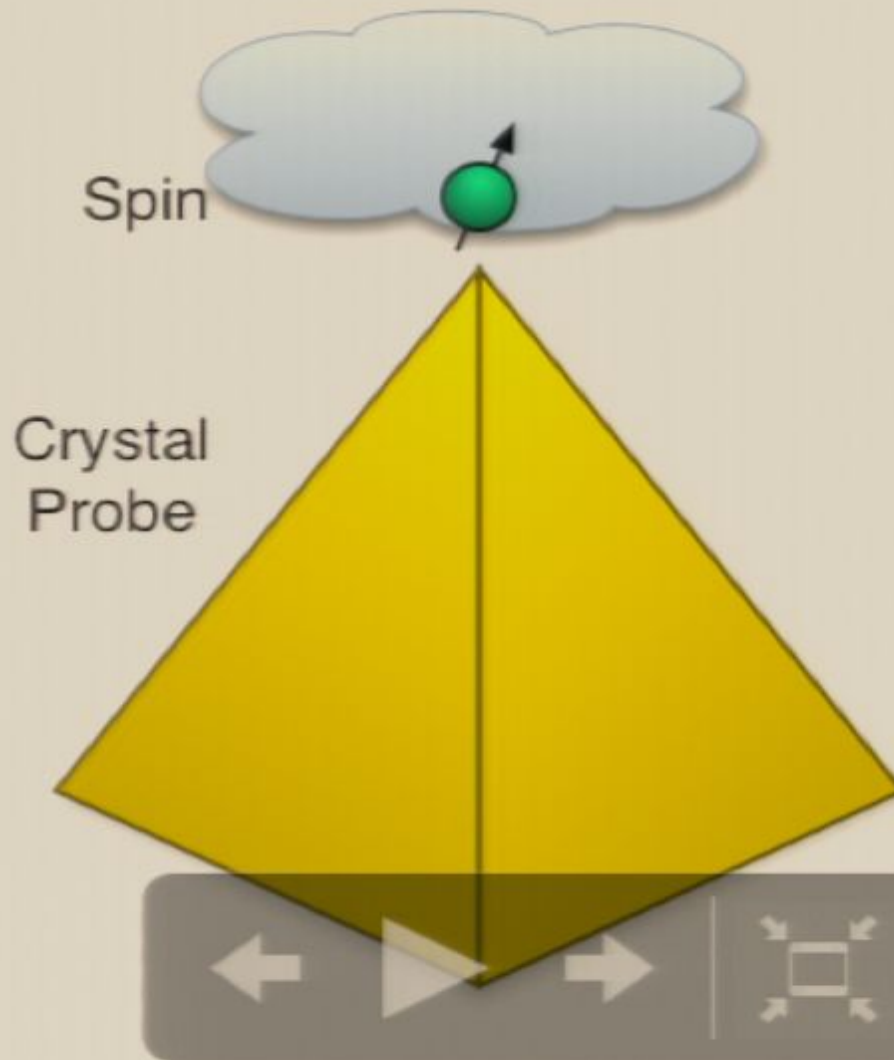
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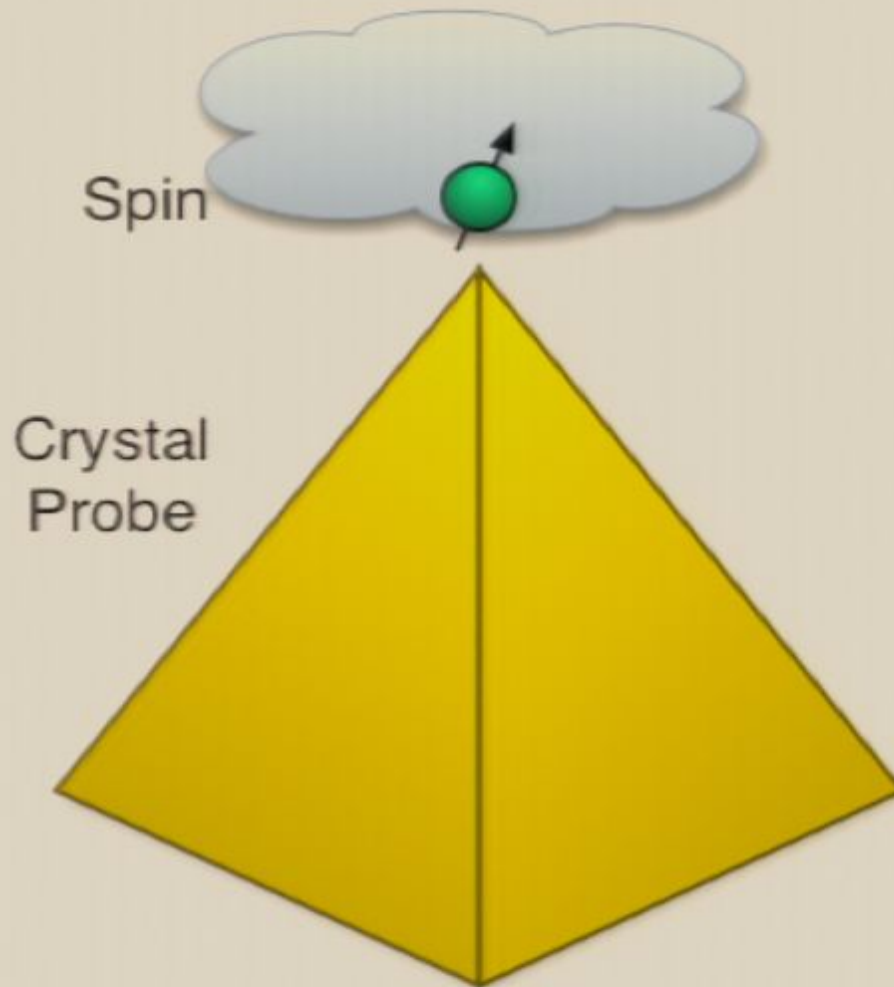
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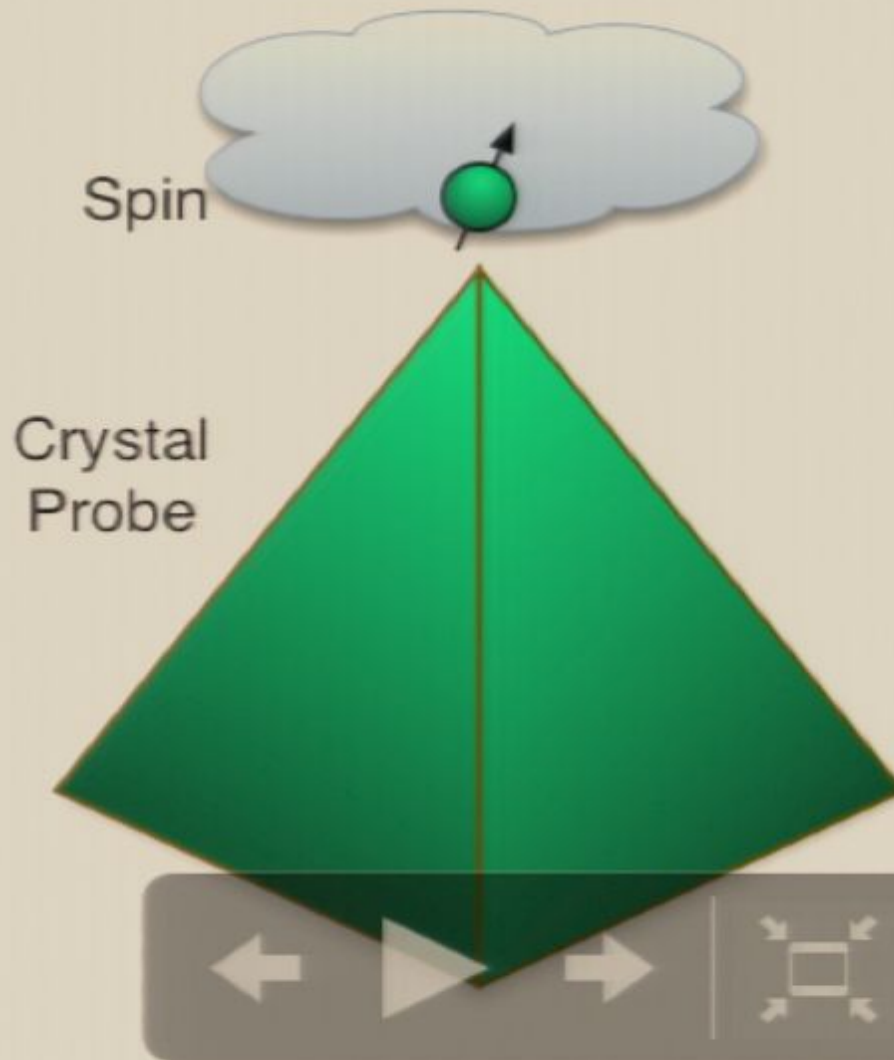
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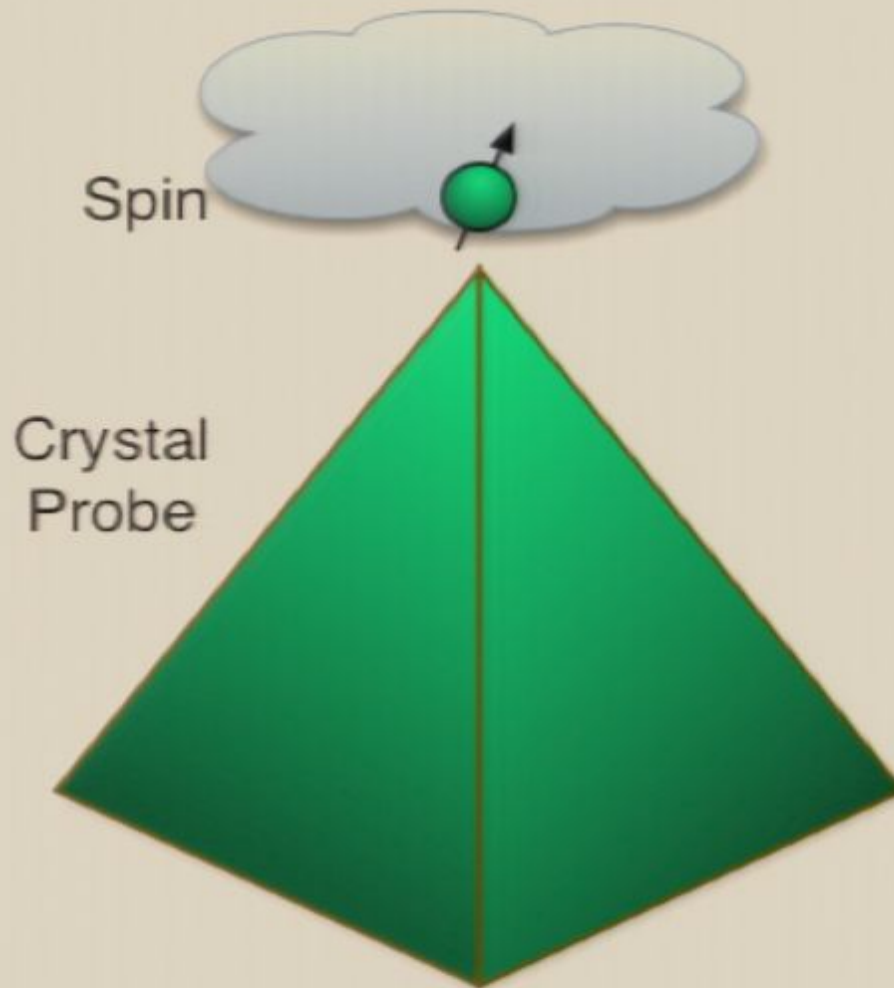
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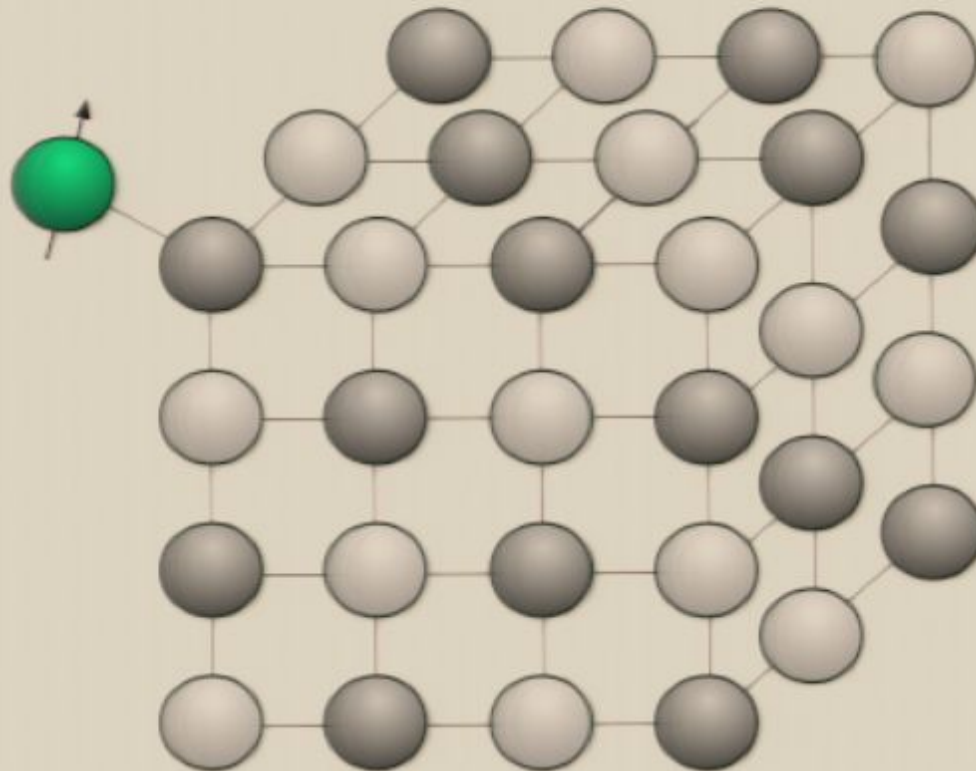
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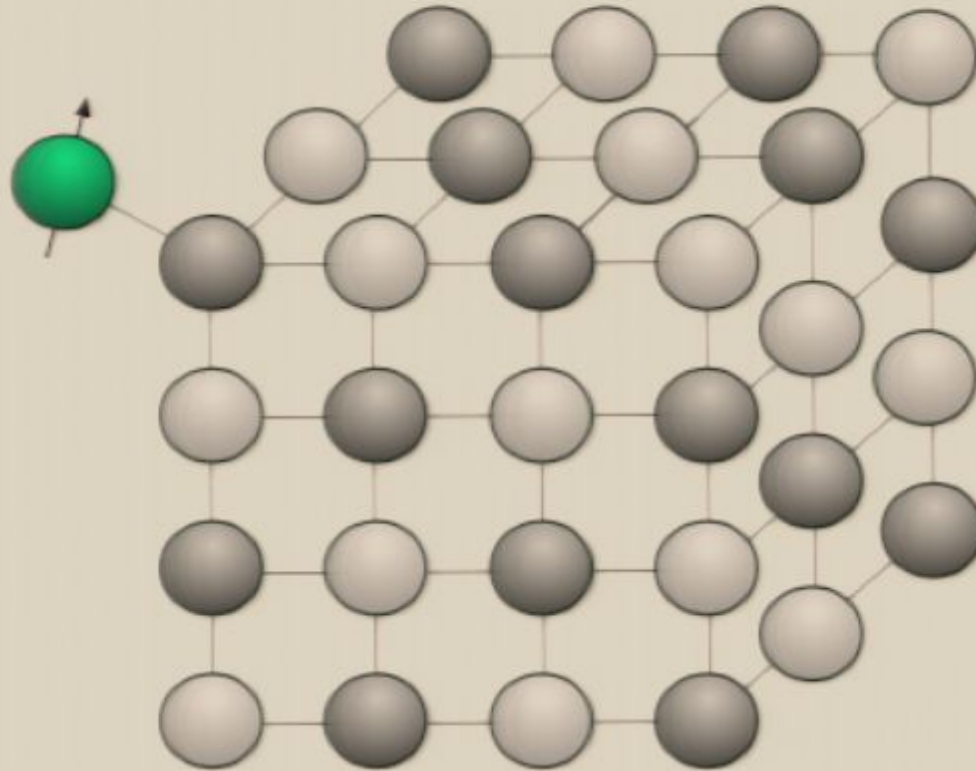
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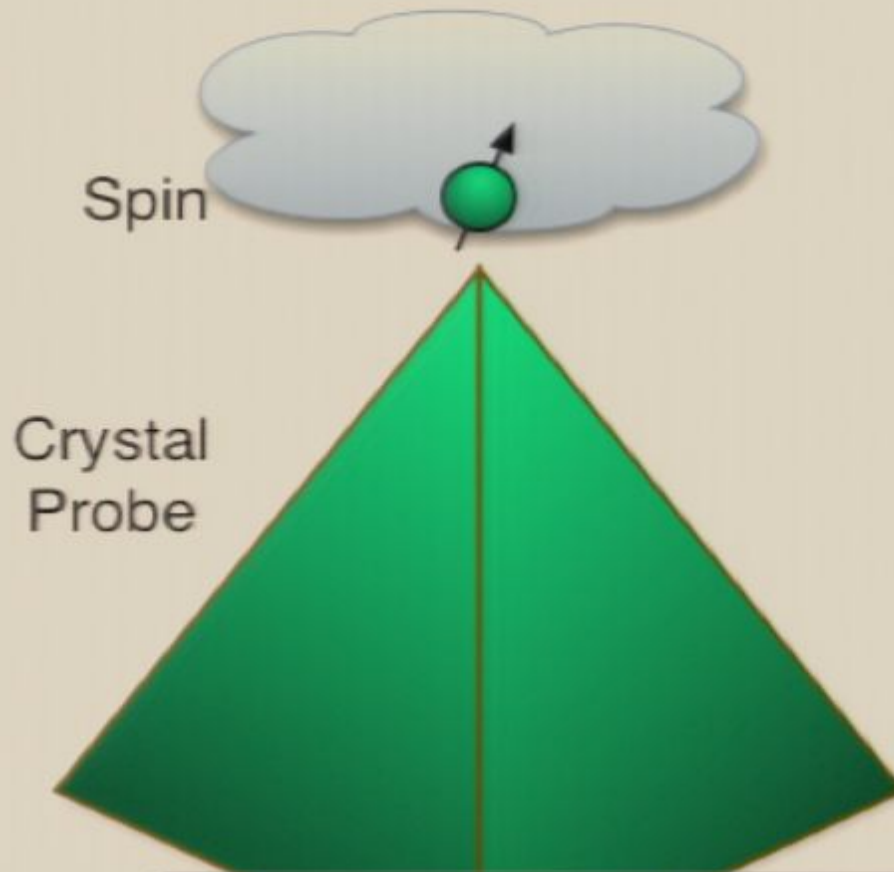
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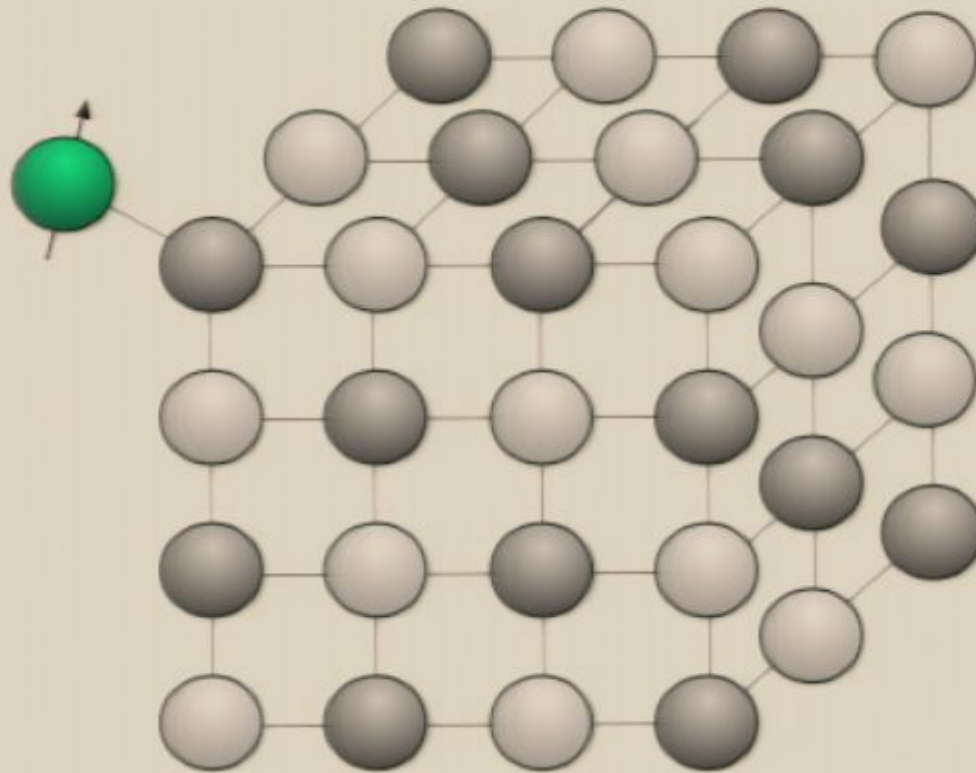
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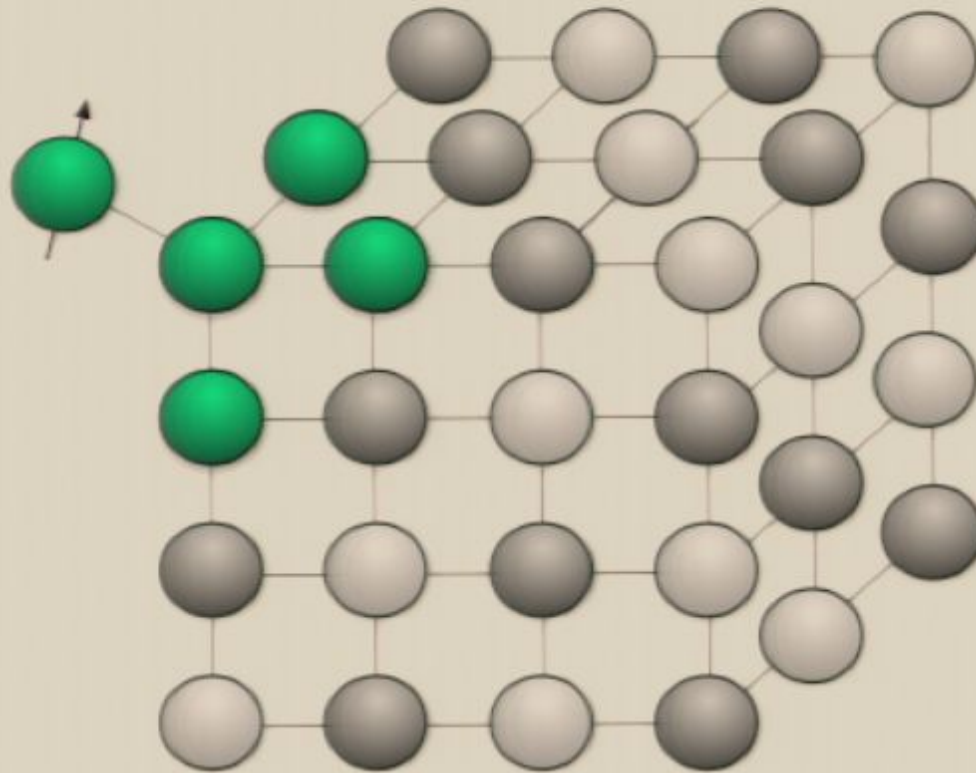
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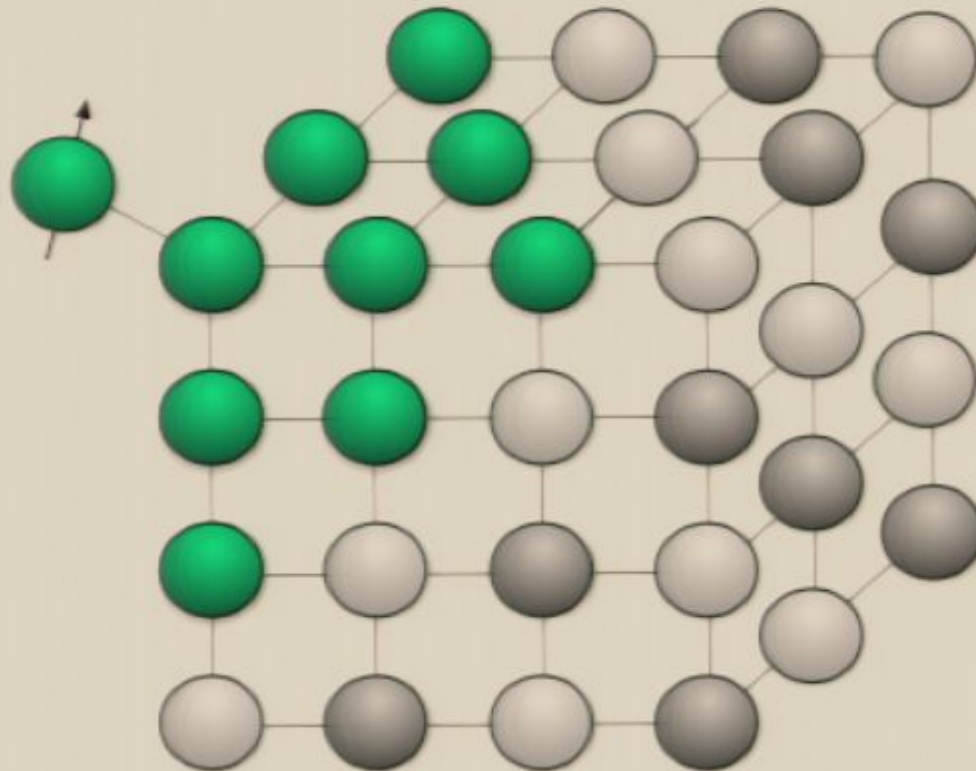
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- A *Cellular Automata* consists of a lattice structure, where each cell is in one of a finite number of given states

For instance, Rule 30 from S. Wolfram's "A New Kind of Science" is:



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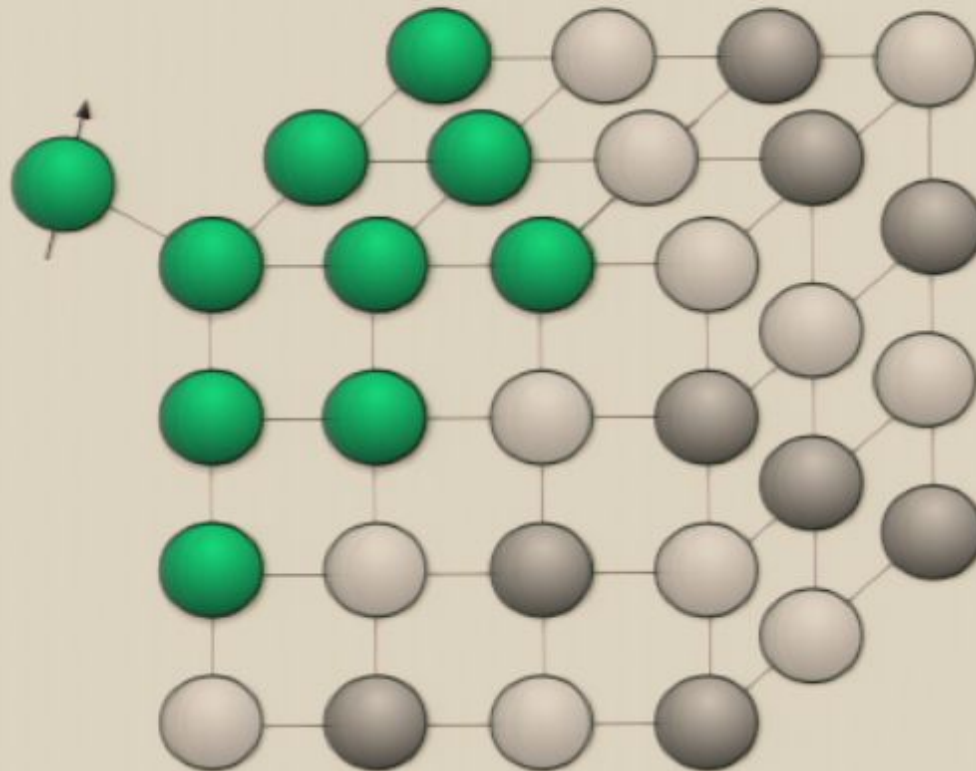
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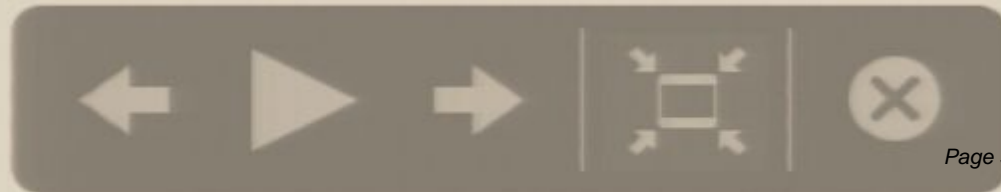
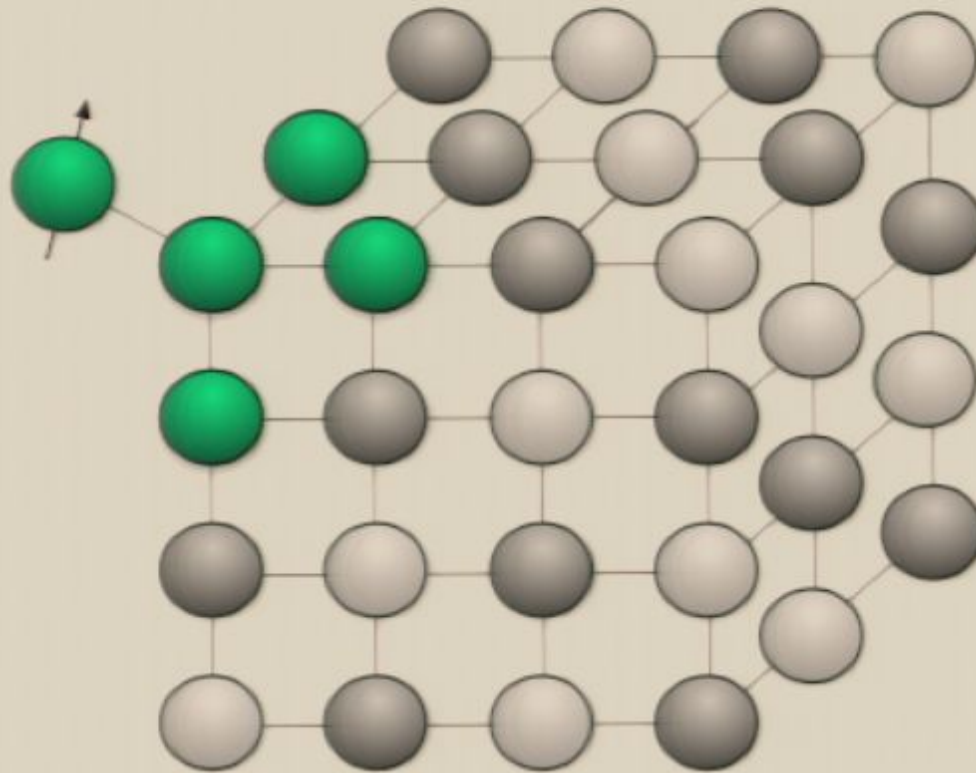
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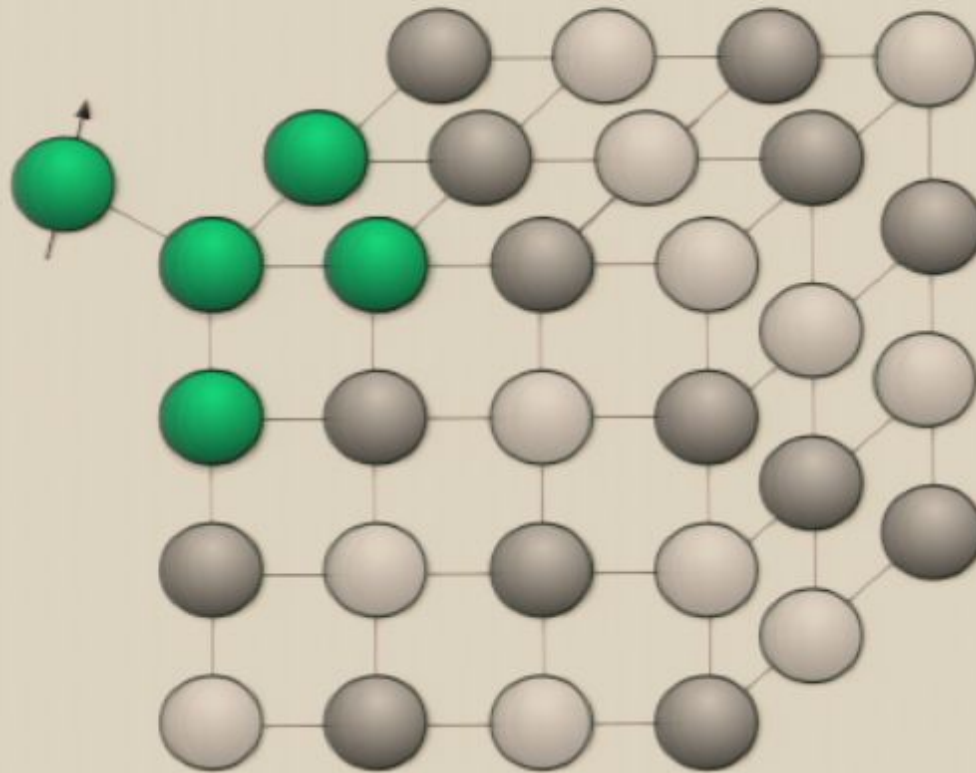
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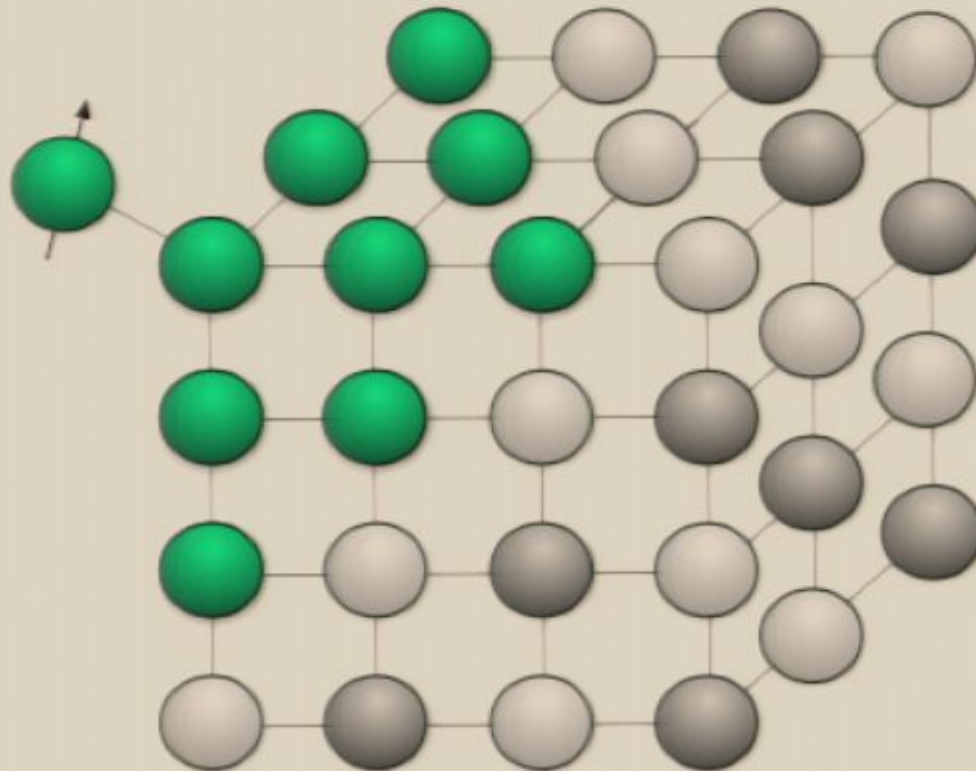
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- A *Cellular Automata* consists of a lattice structure, where each cell is in one of a finite number of given states
- At each discrete time-step, every cell is updated, in parallel, according to a local, spatially uniform rule.

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“Rule 30” Evolution

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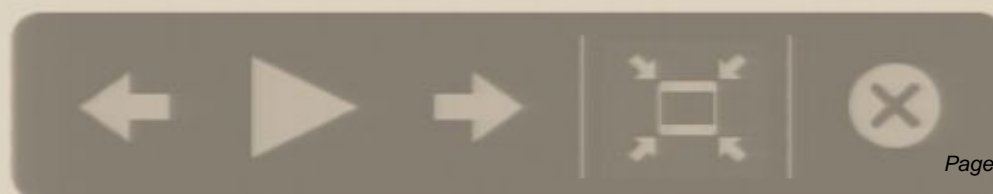
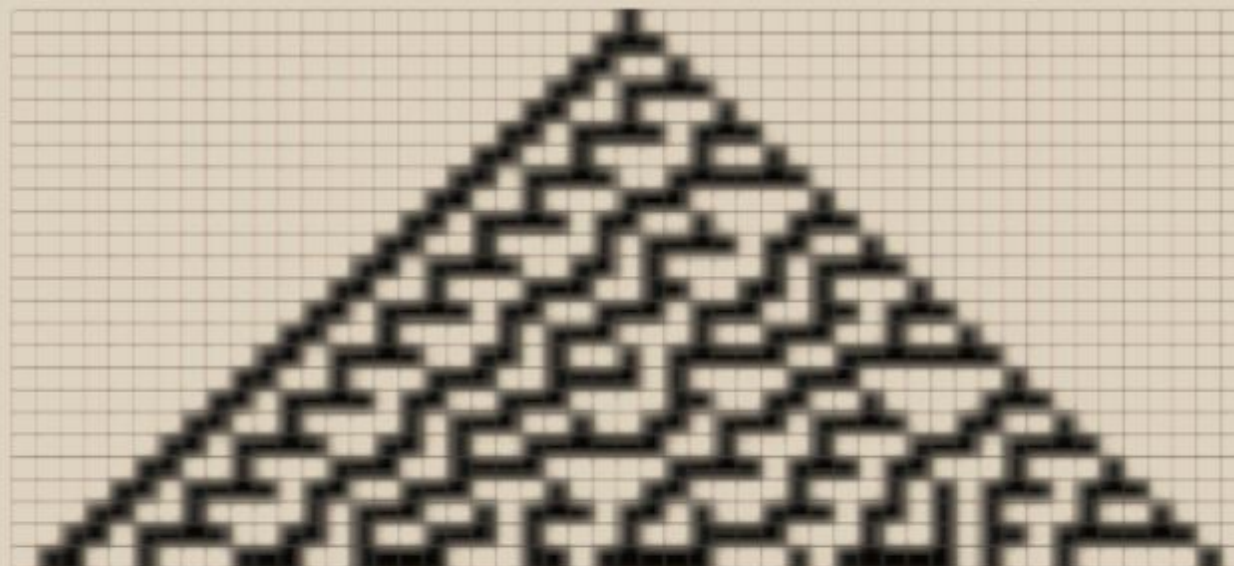
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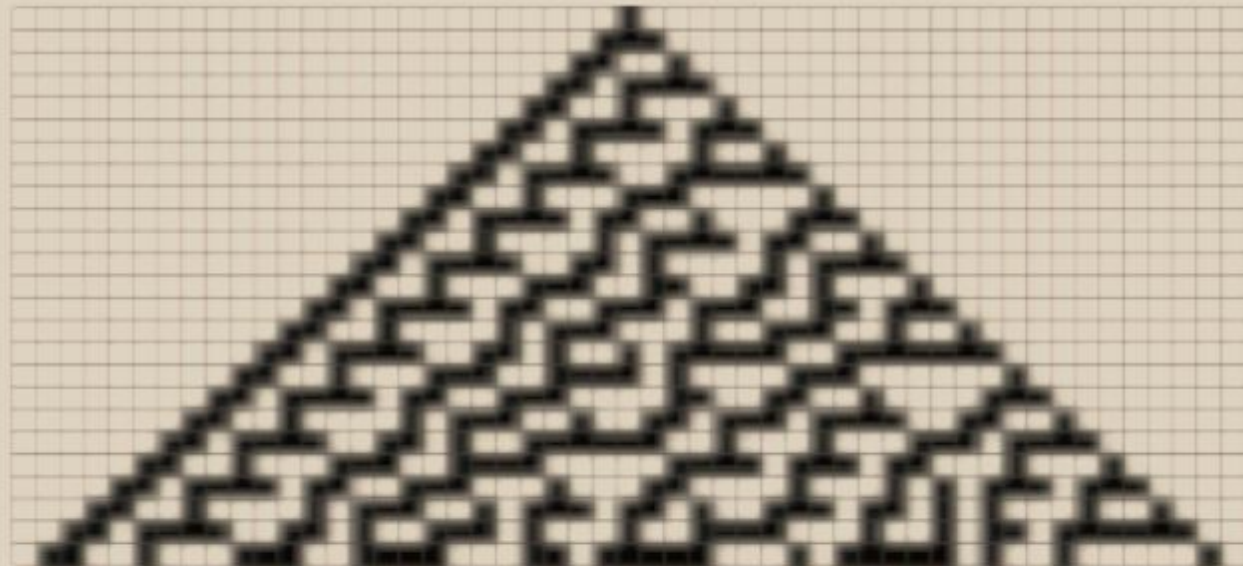
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- There are many ways of making a reversible classical CA, most famous is the Margolus CA.
- These can be extended to make quantum CA, or QCA
- We shall study another QCA type, the Coloured QCA, or CQCA.



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Coloured QCA.

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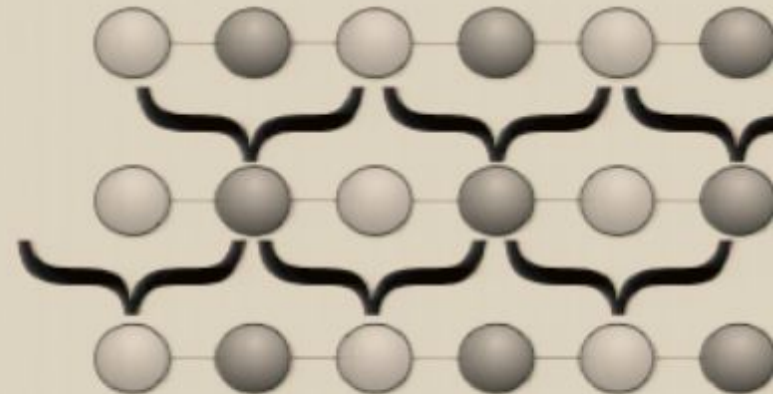
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- In a CQCA each lattice is assigned a colour in a checkerboard fashion.



Coloured QCA.

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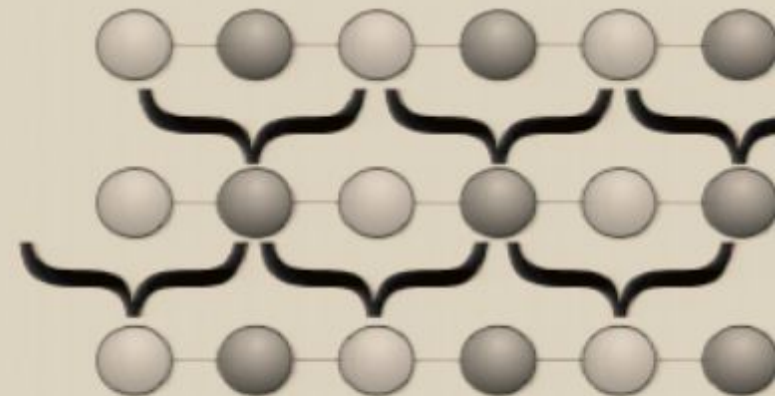
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- In a CQCA each lattice is assigned a colour in a checkerboard fashion.
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Coloured QCA.

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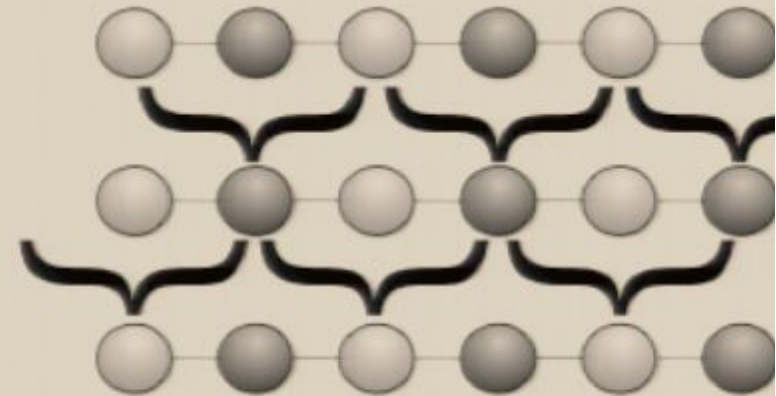
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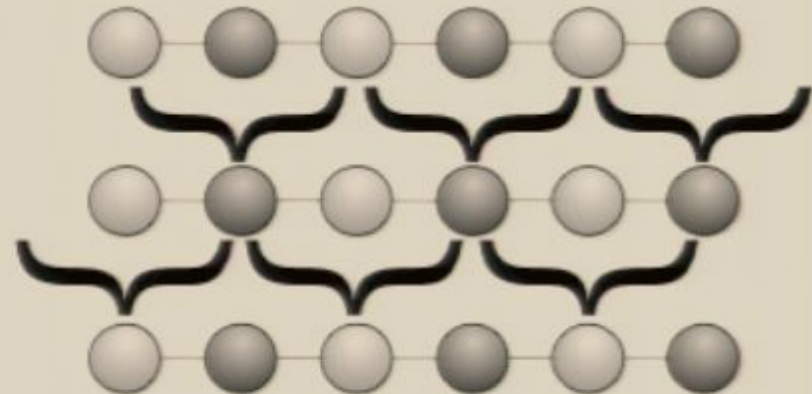
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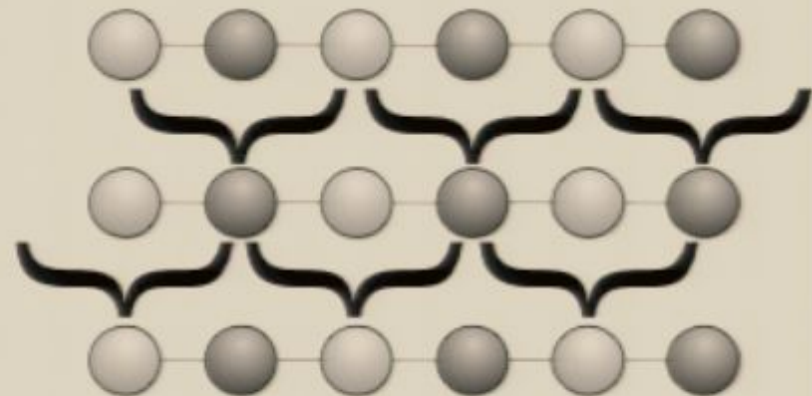
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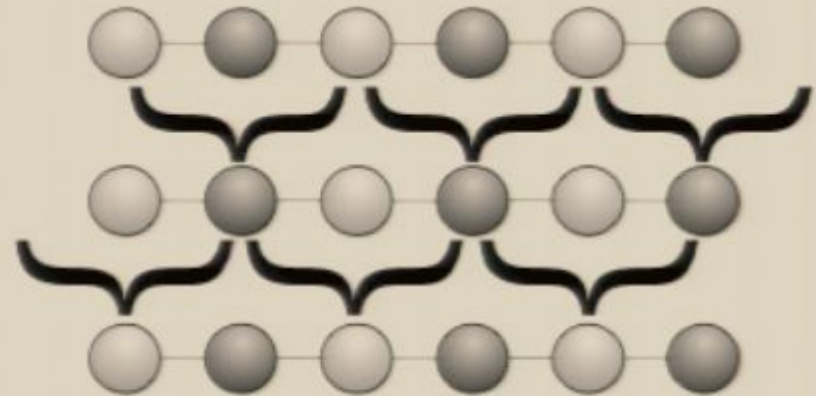
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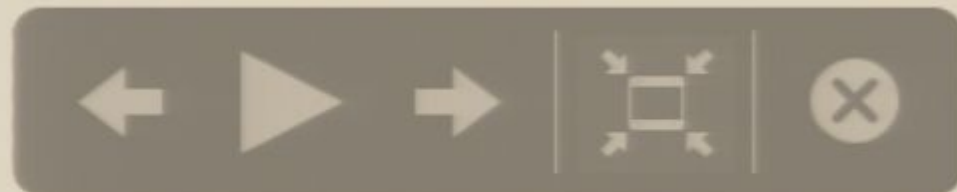
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- At each time step only points of a certain colour are updated with a unitary dependant on their neighbours' values.
- Neighbours which are of the same colour are **not** distinguishable.



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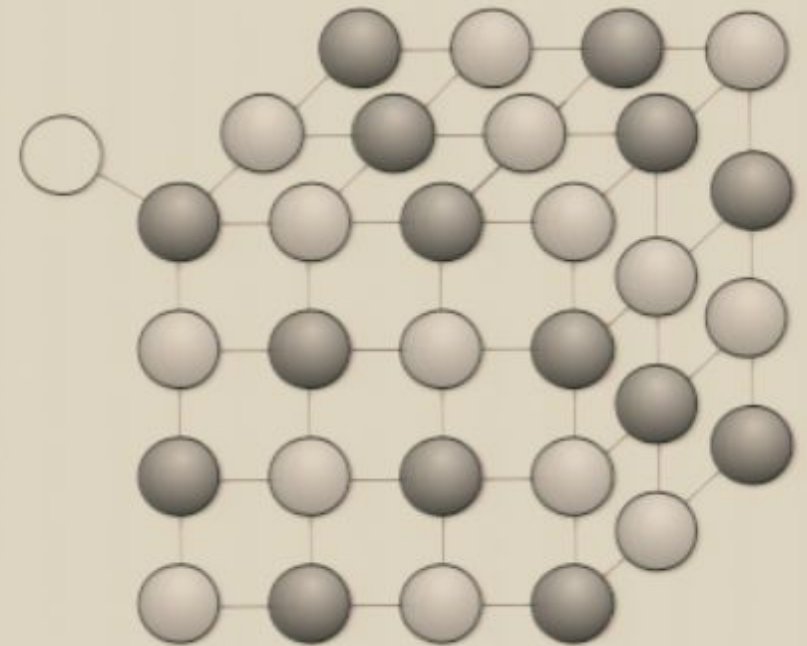
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- Suppose we have a three dimensional 2 colour finite QCA.



A-B Cube

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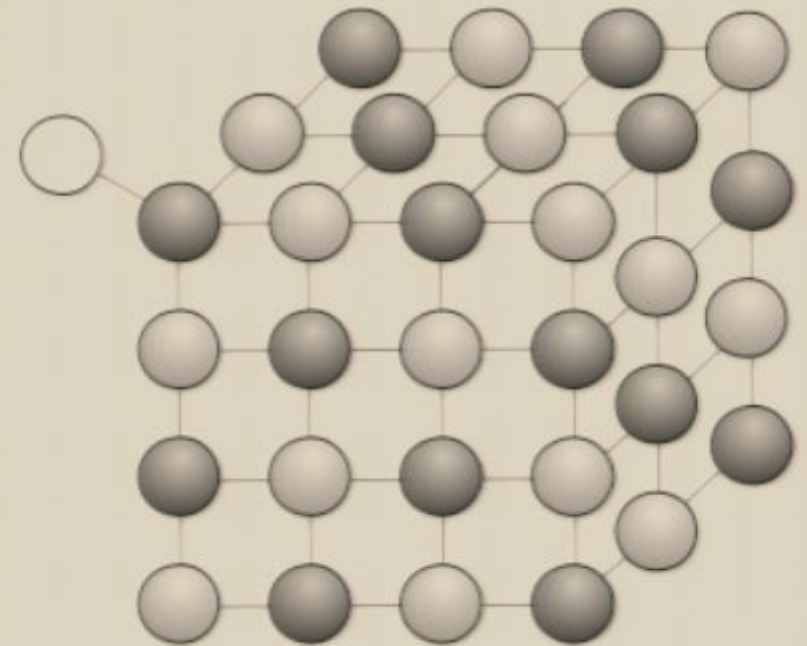
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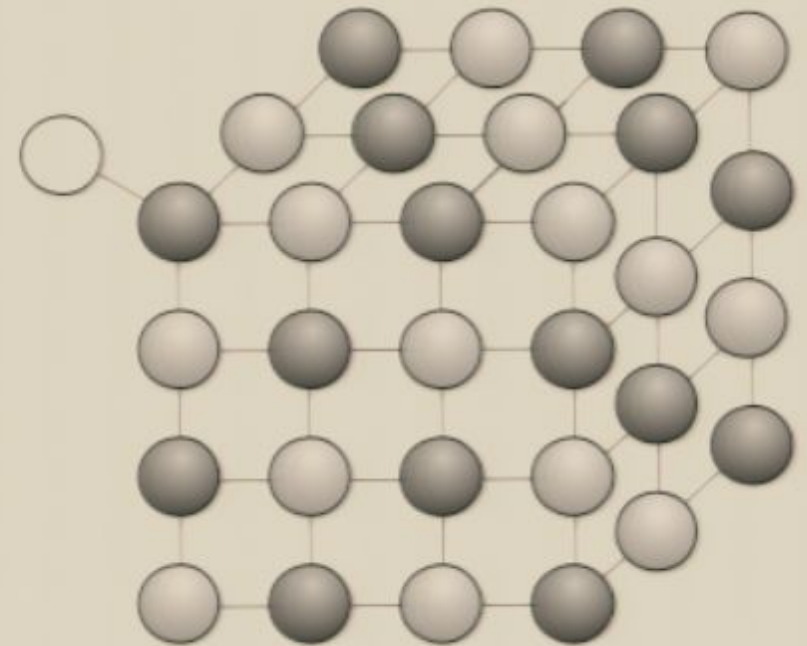
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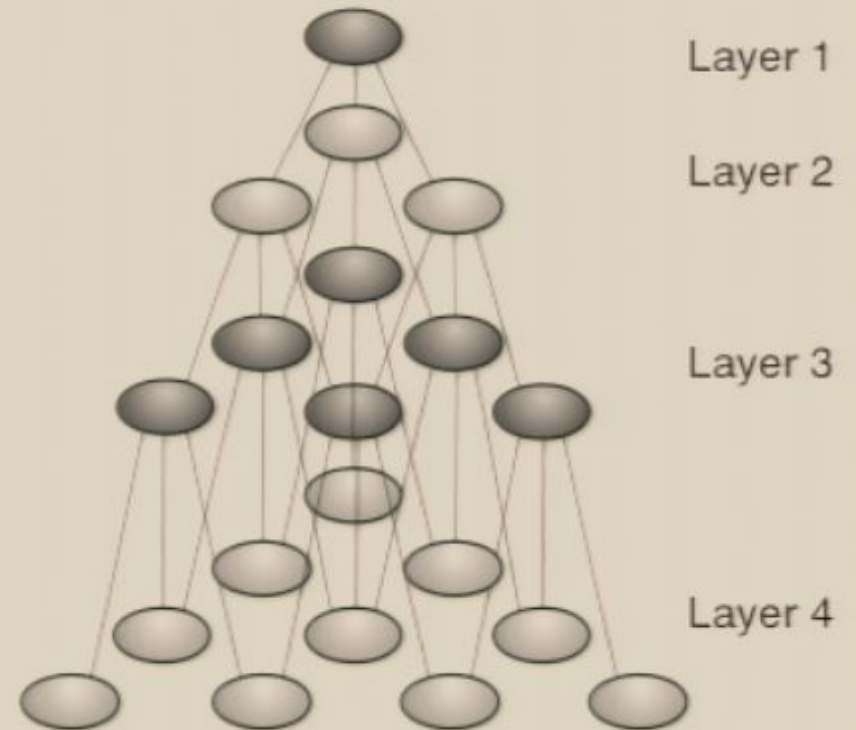
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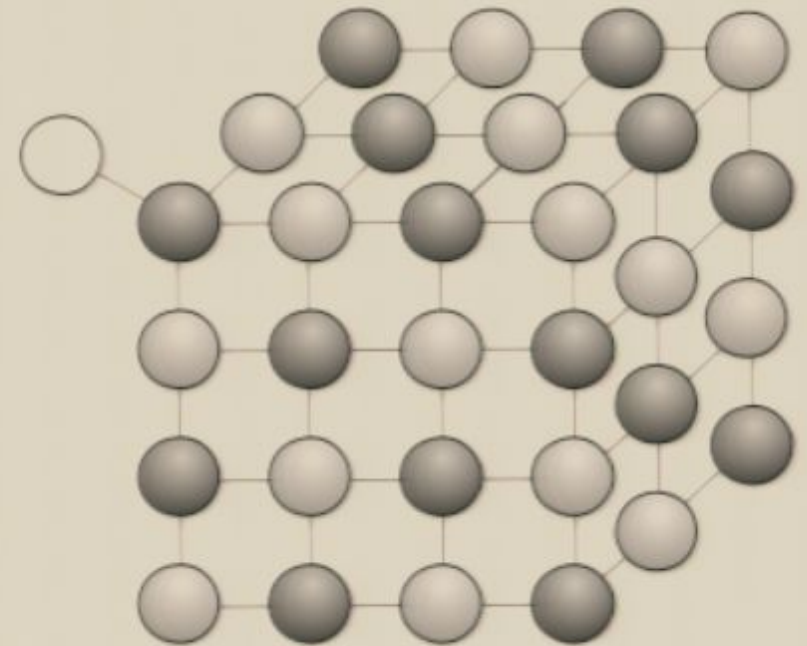
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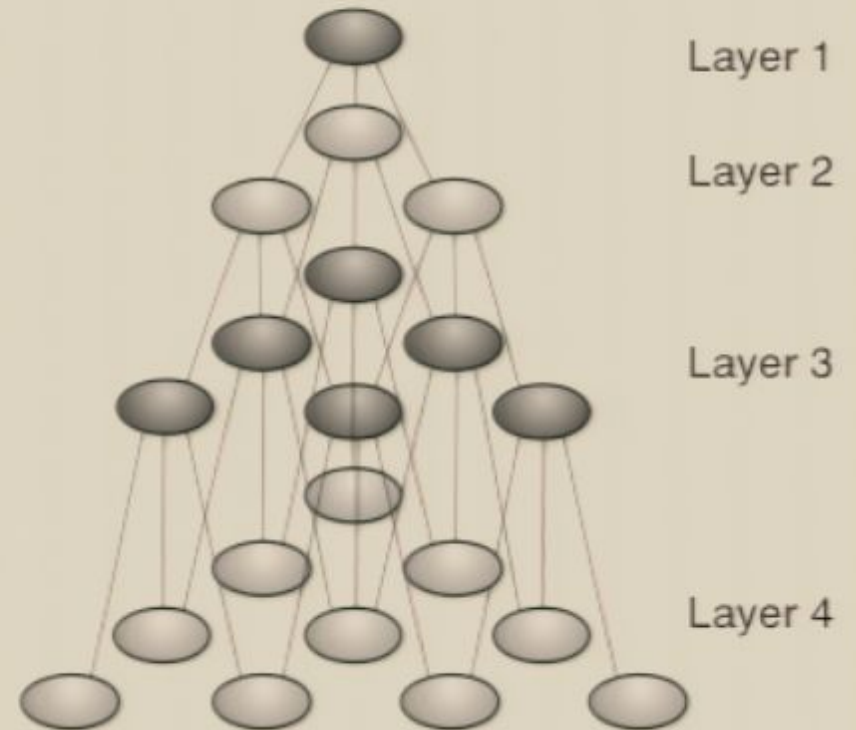
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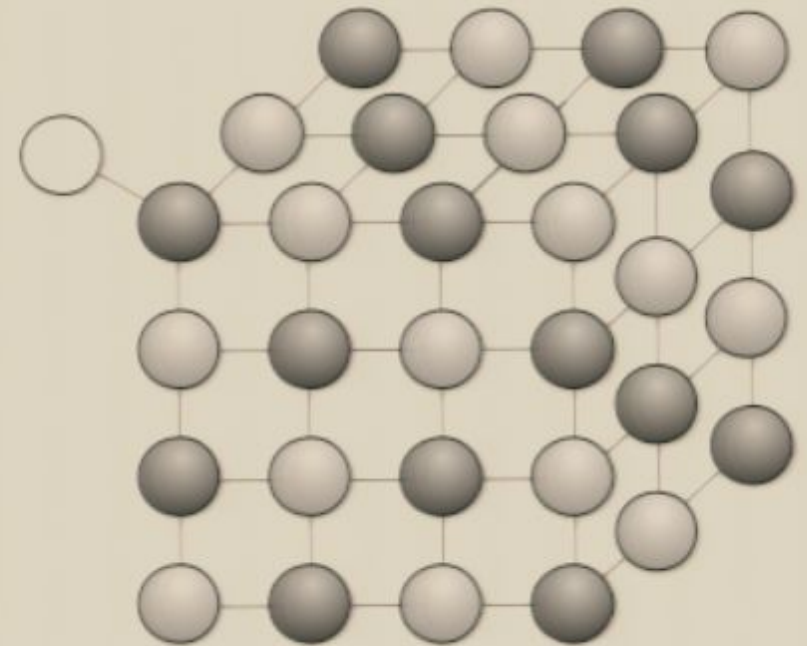
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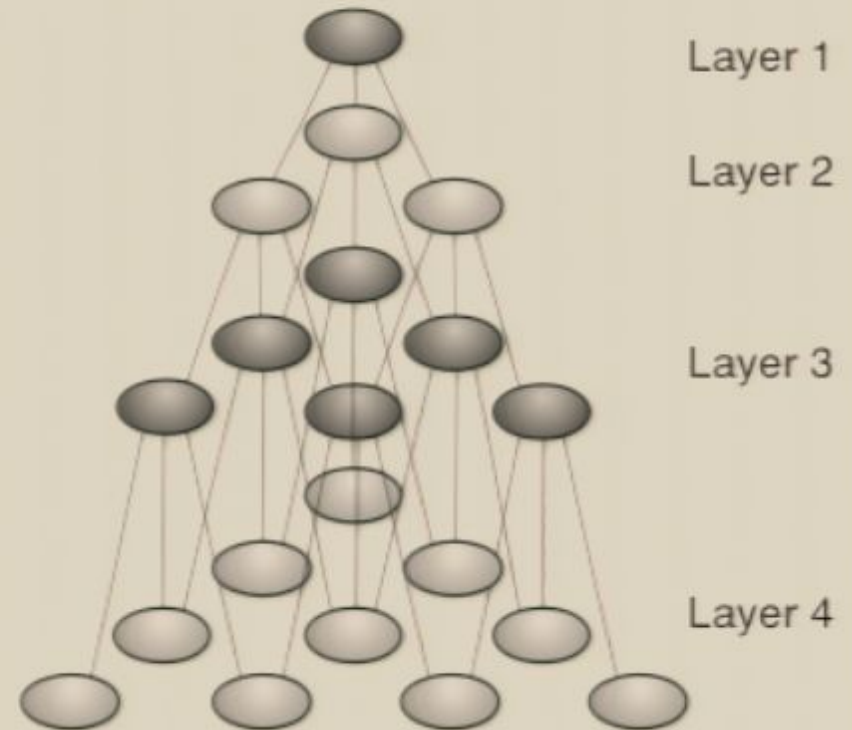
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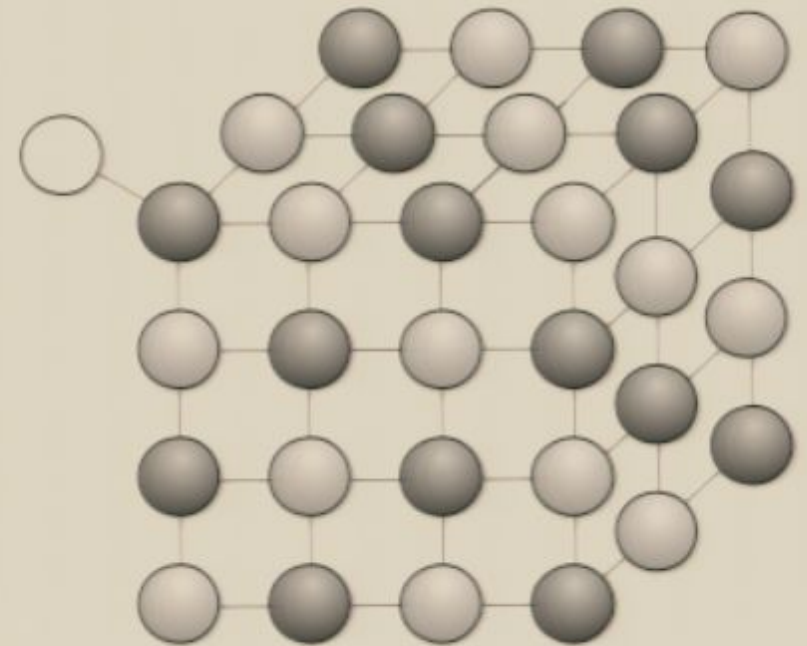
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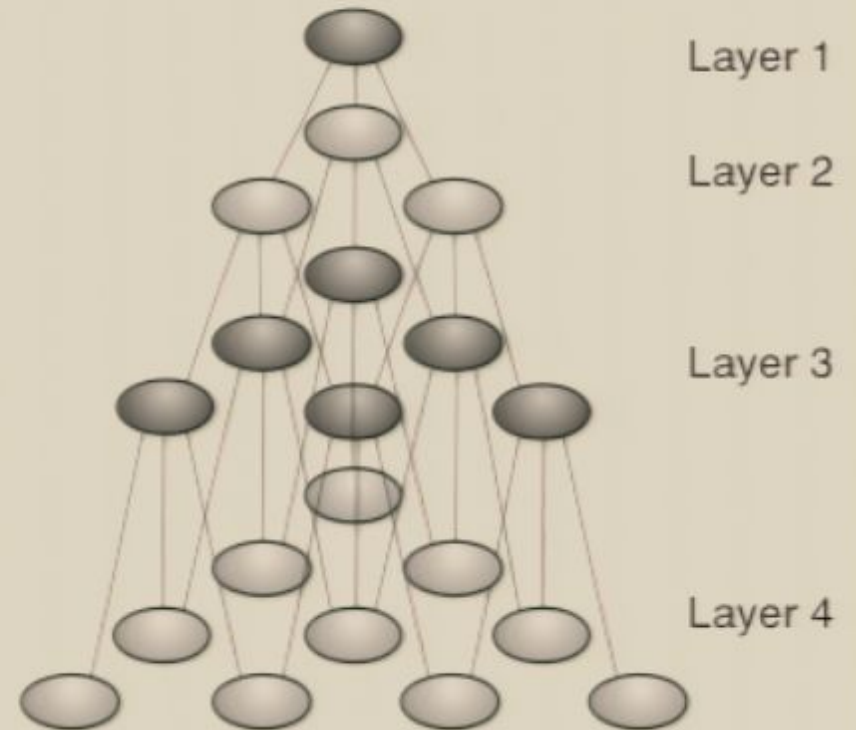
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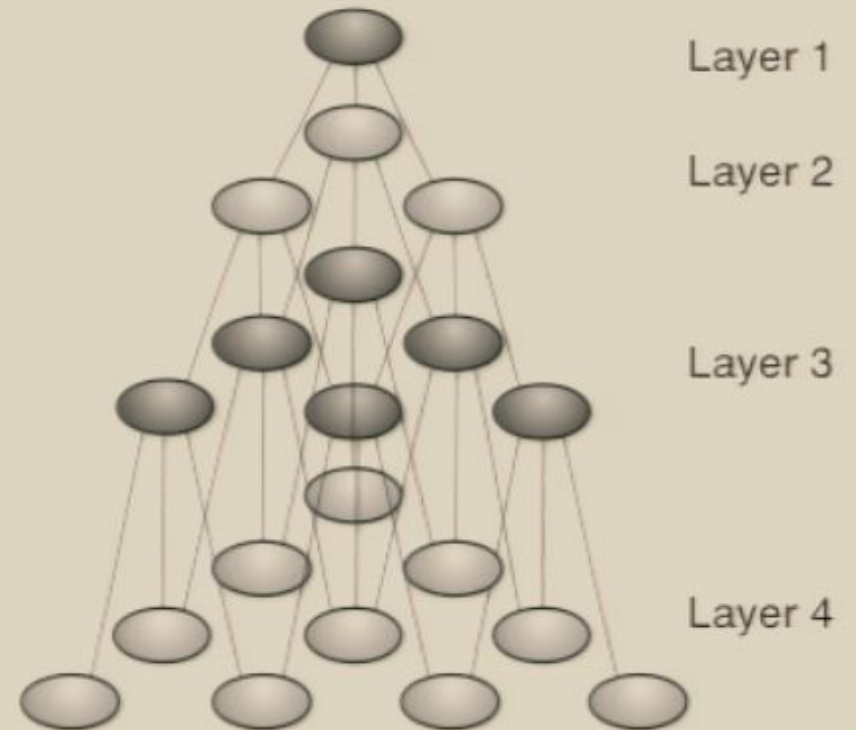
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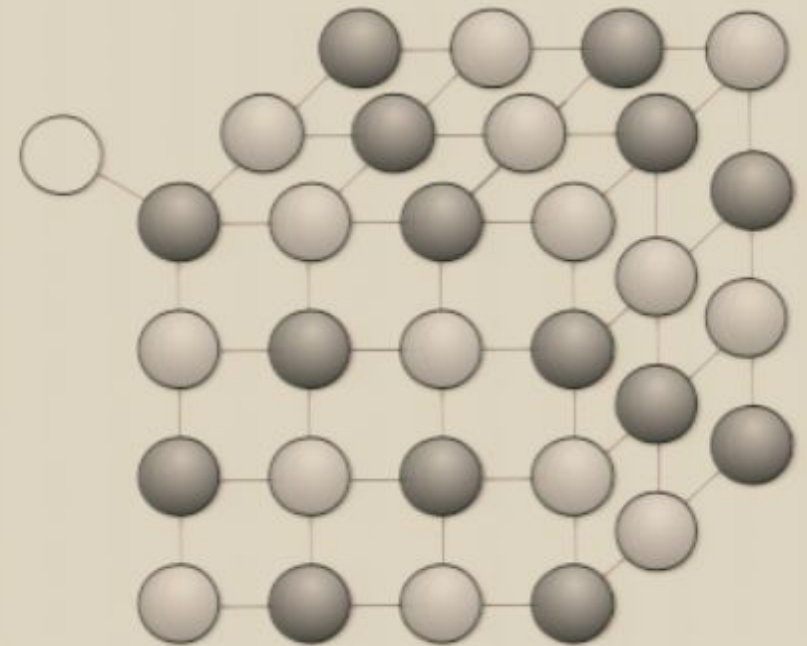
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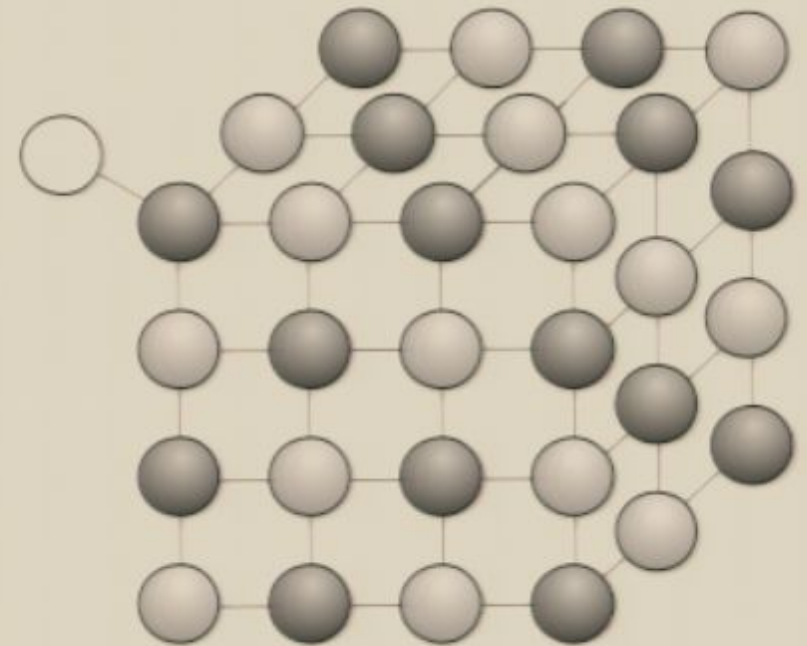
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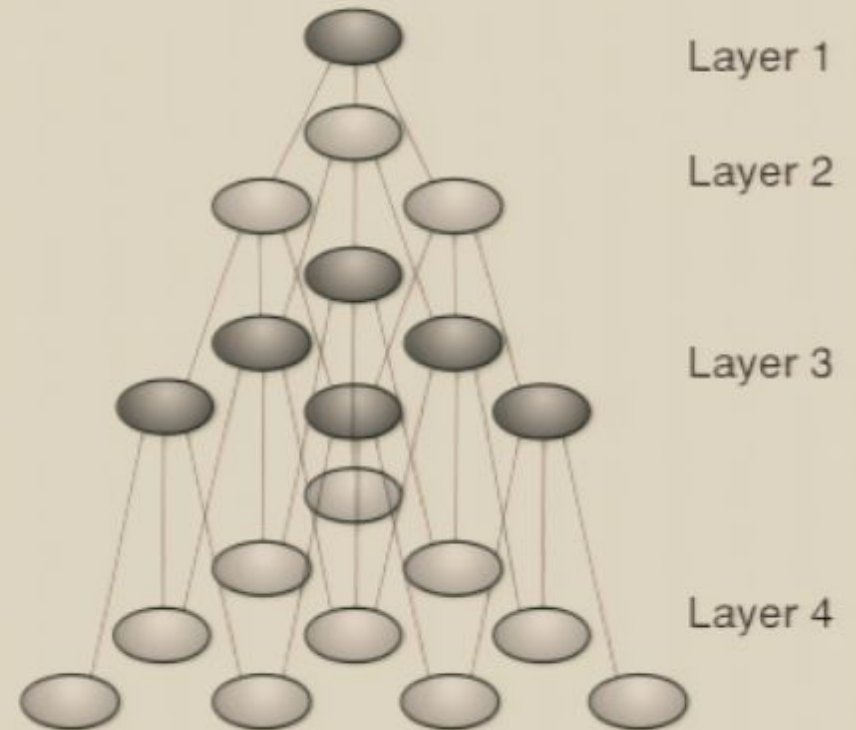
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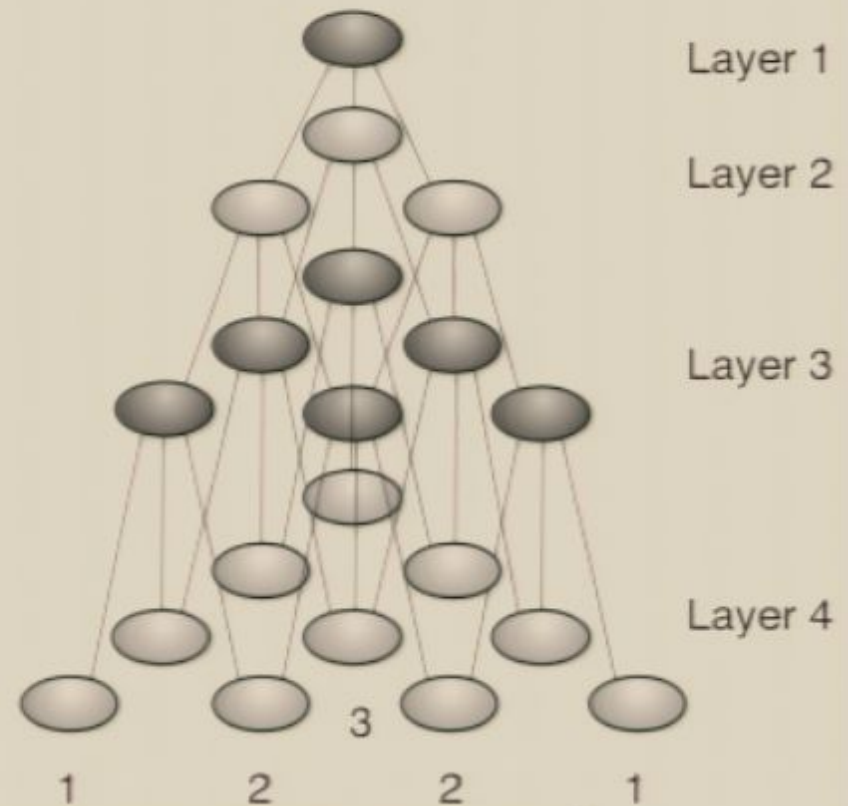
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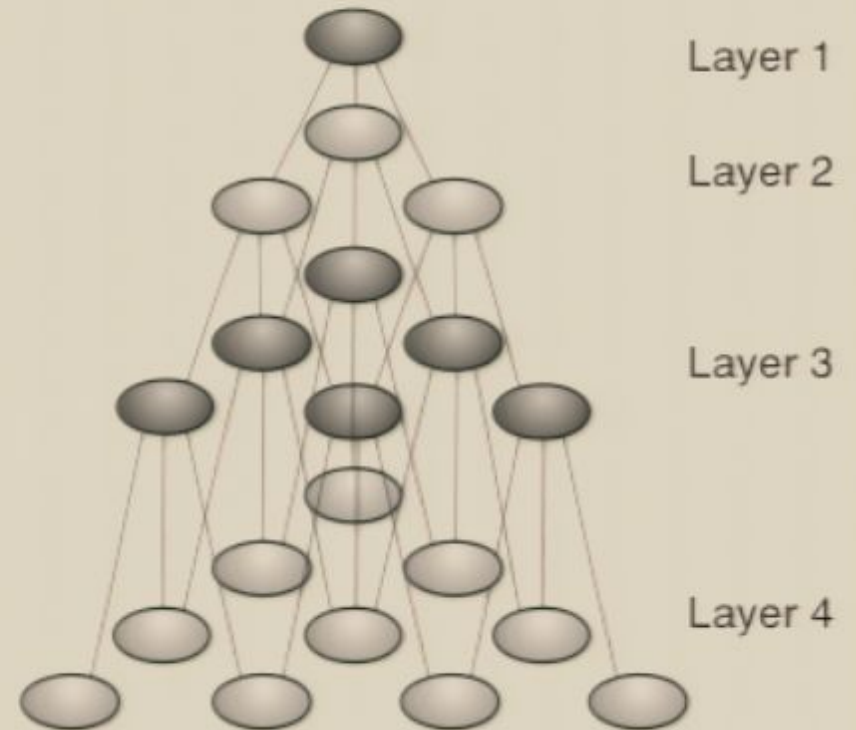
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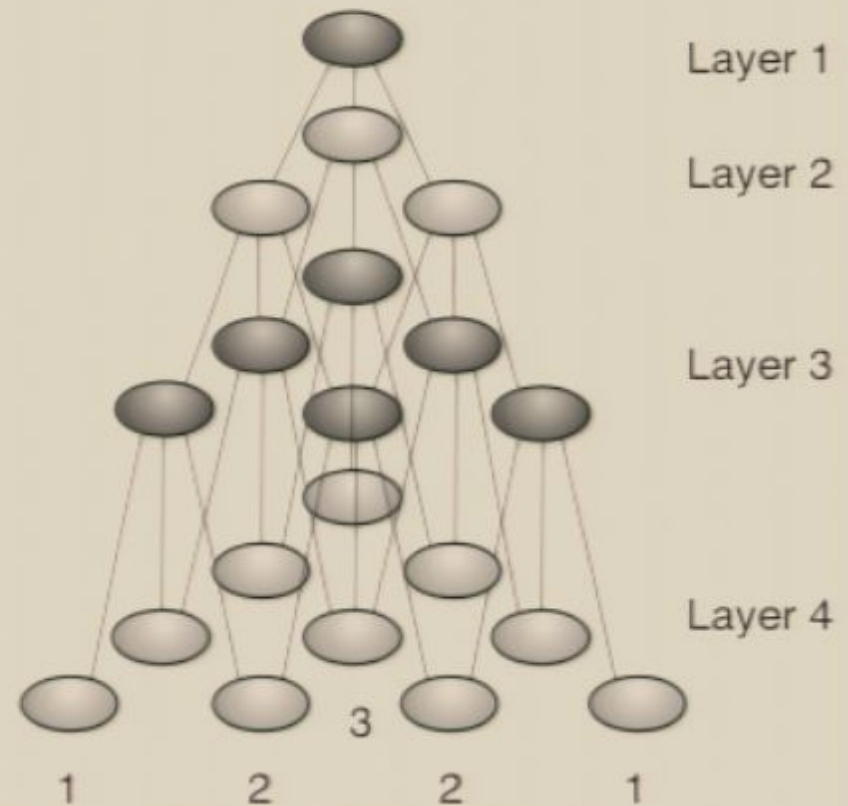
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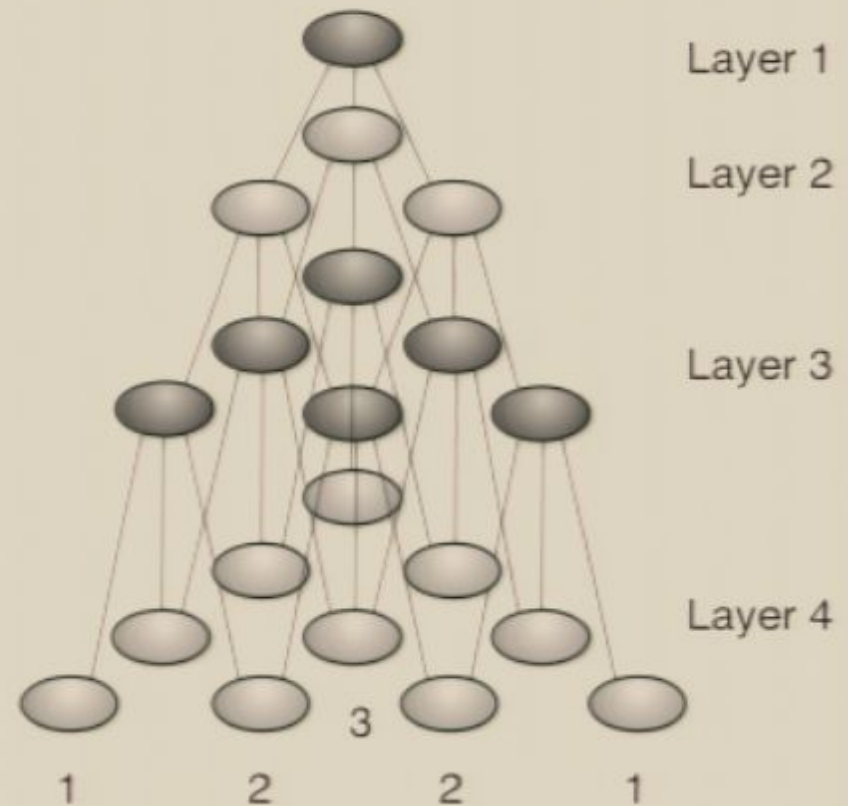
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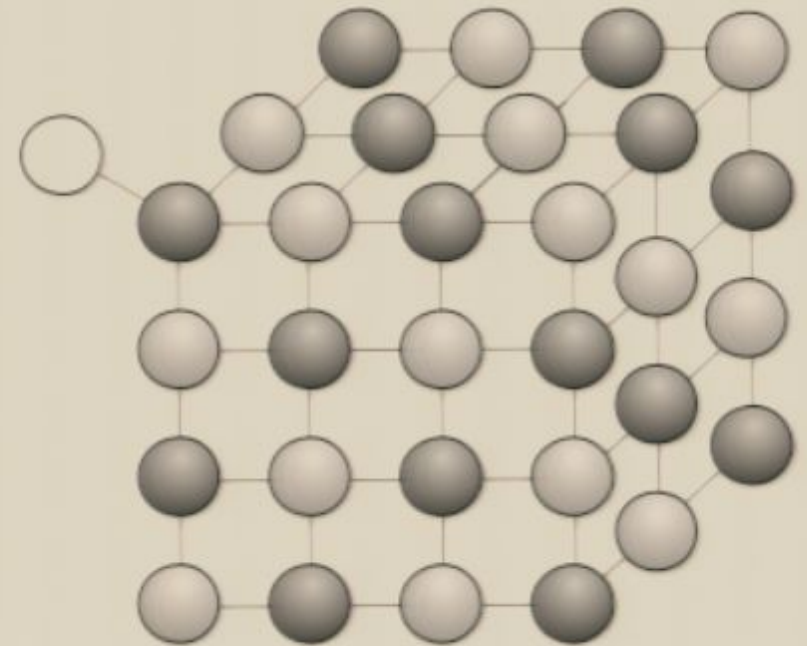
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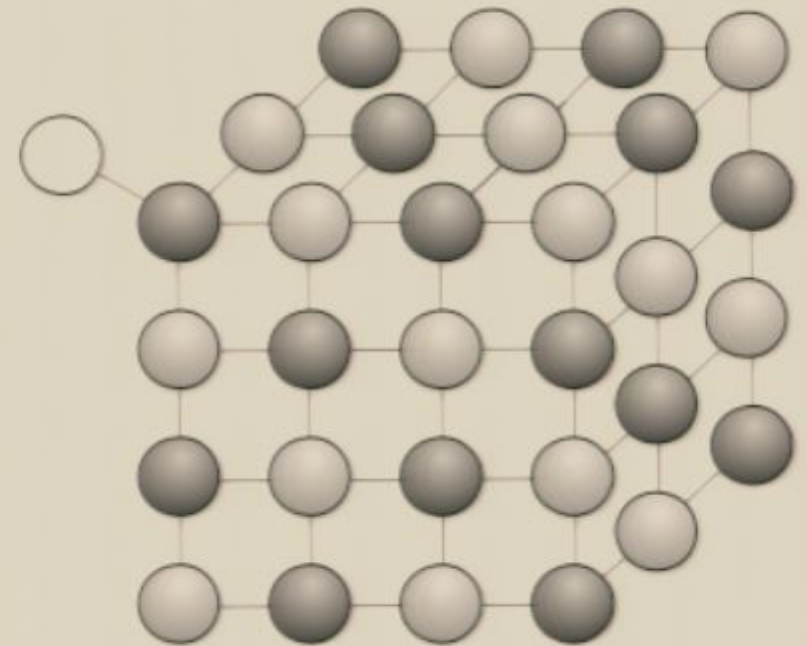
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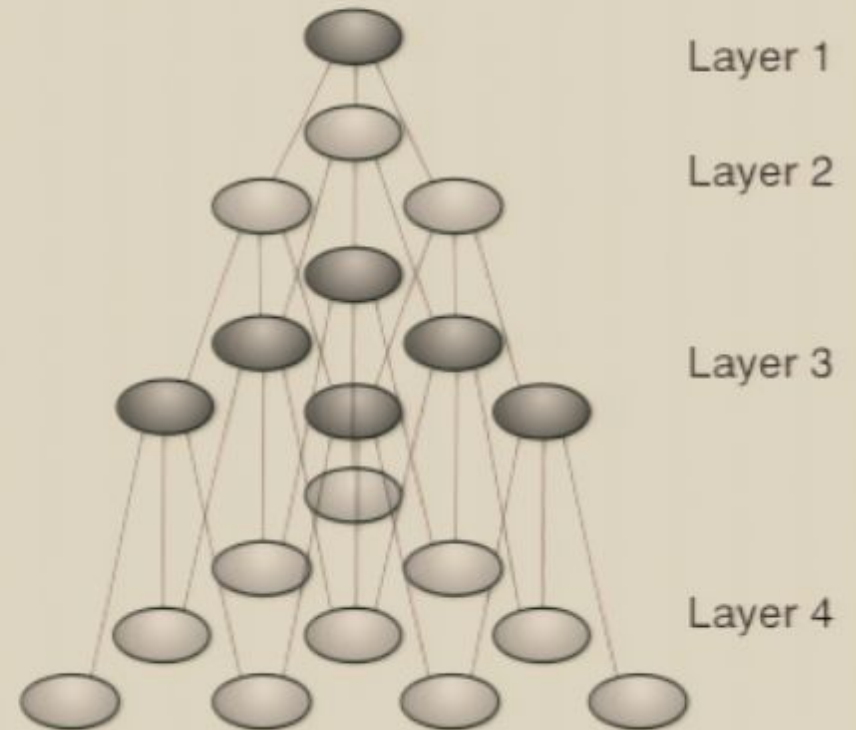
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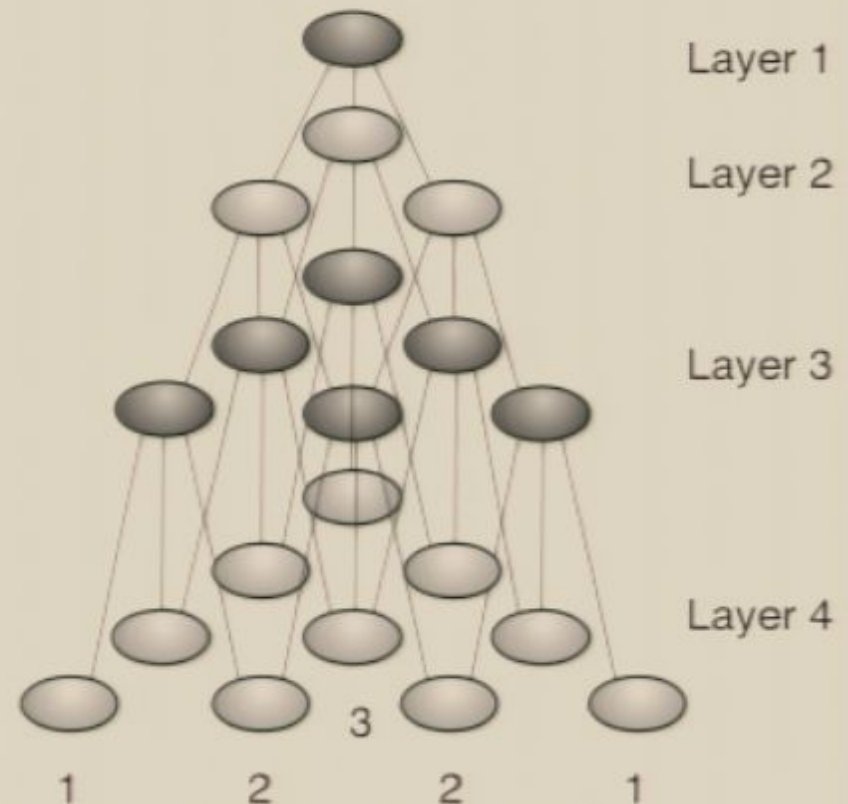
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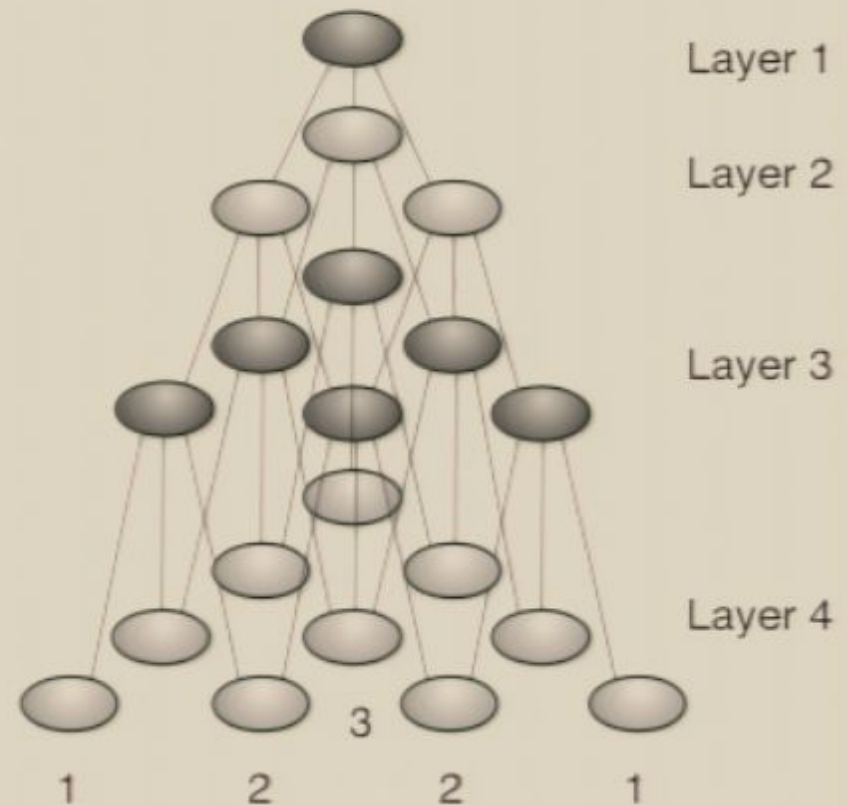
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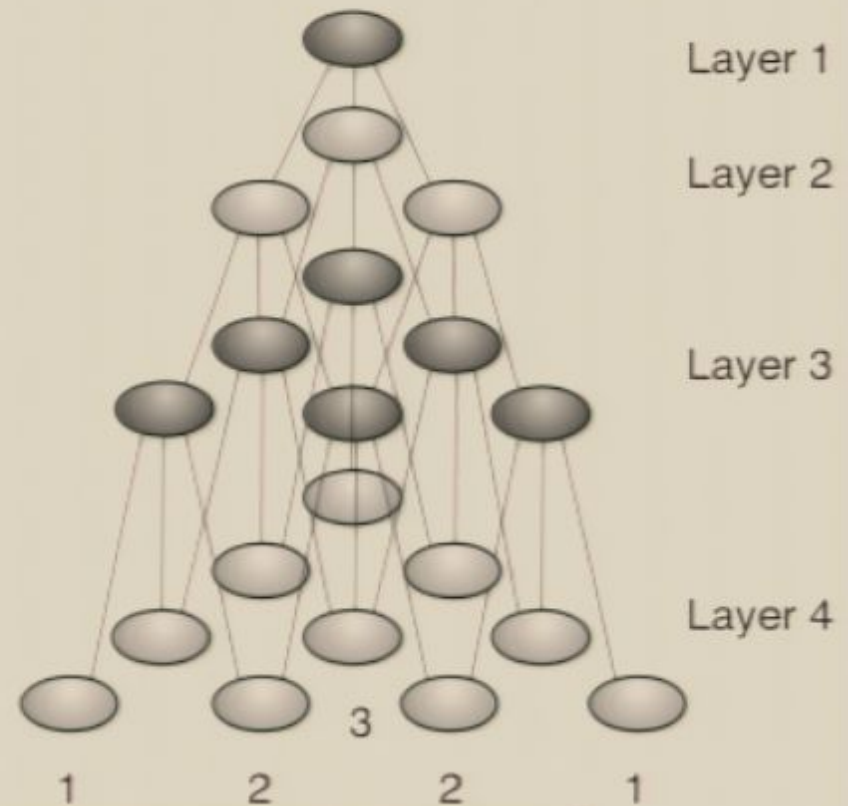
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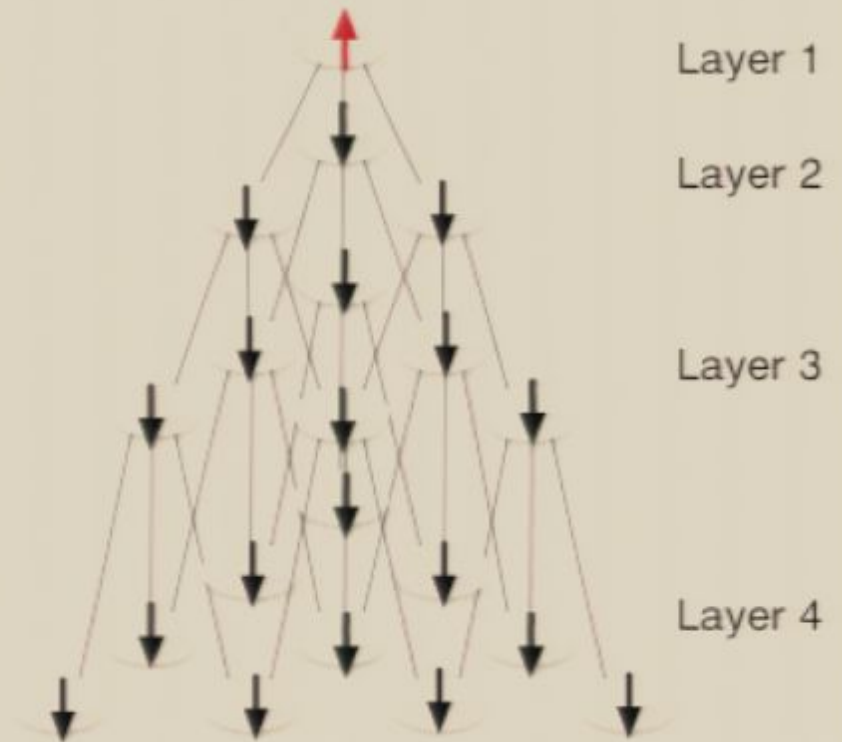
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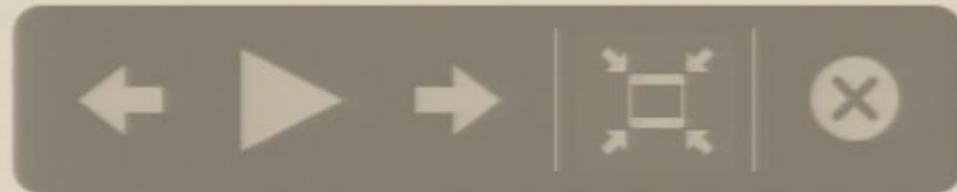
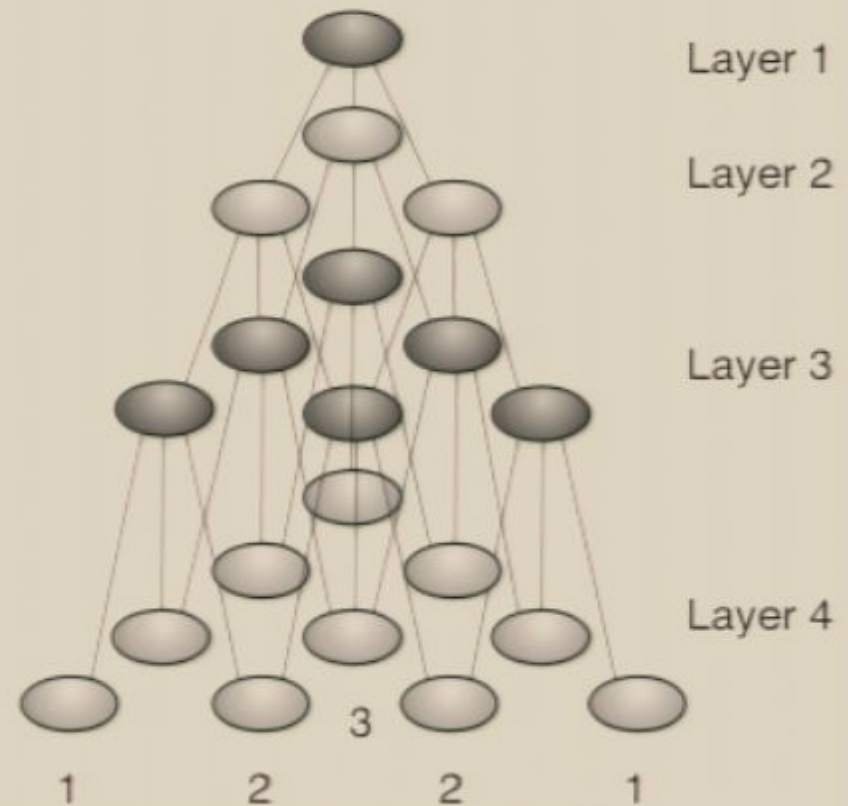
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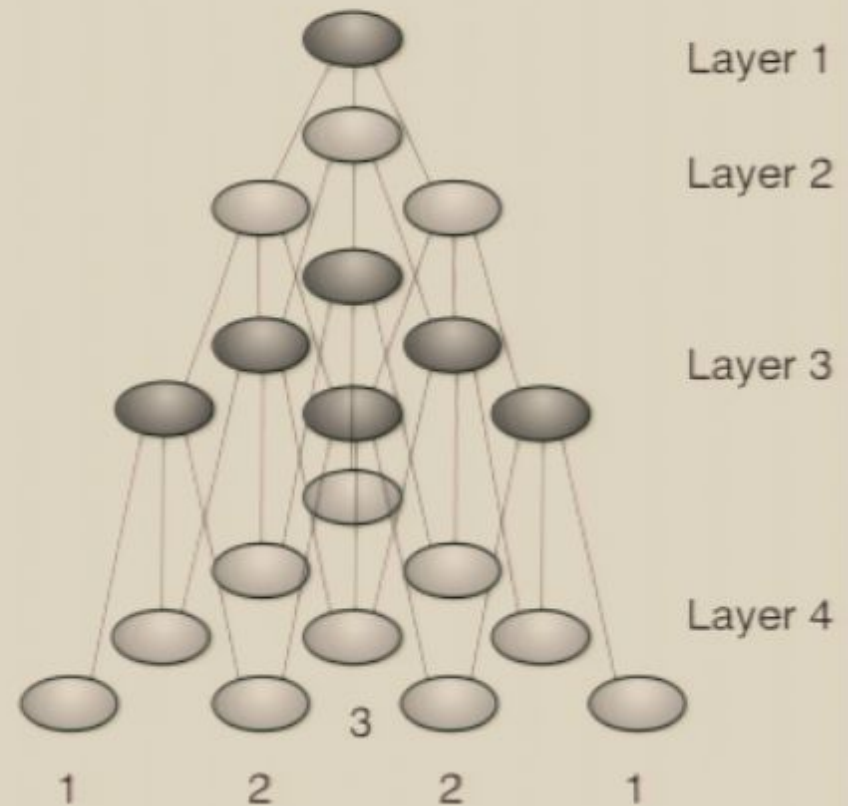
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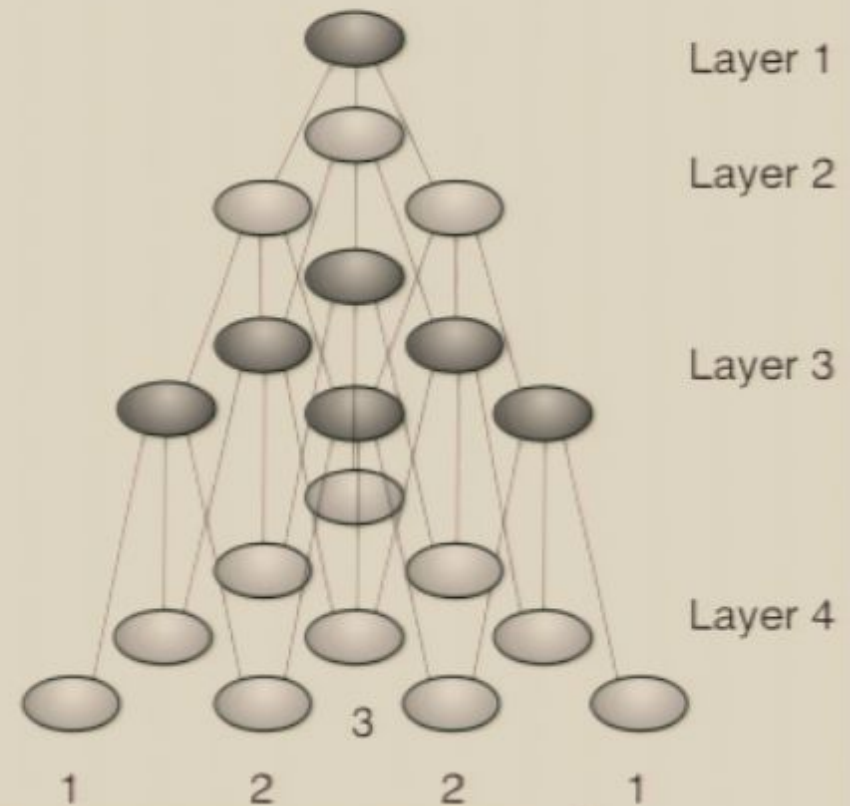
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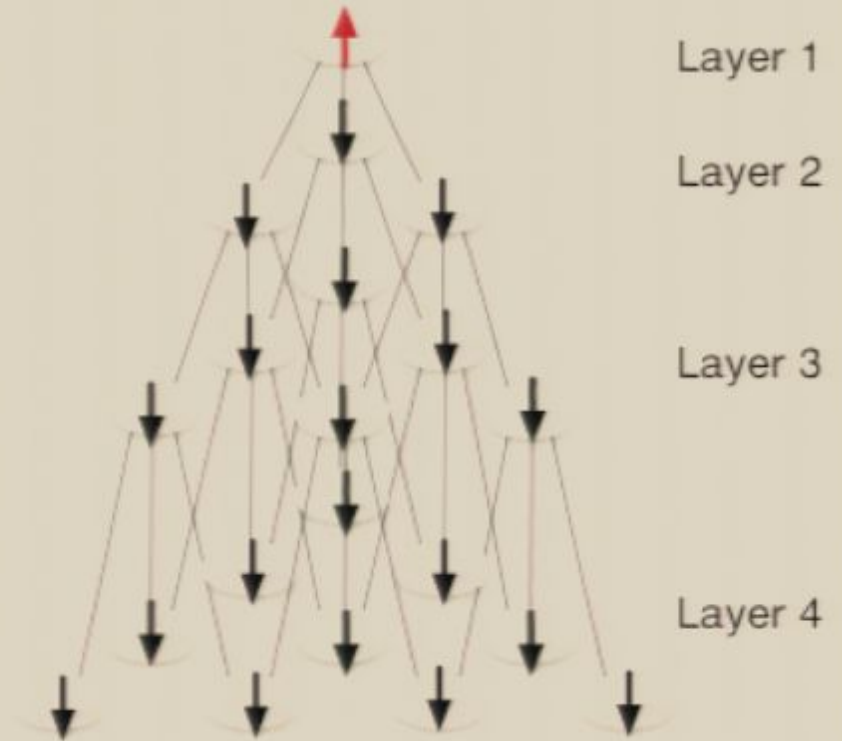
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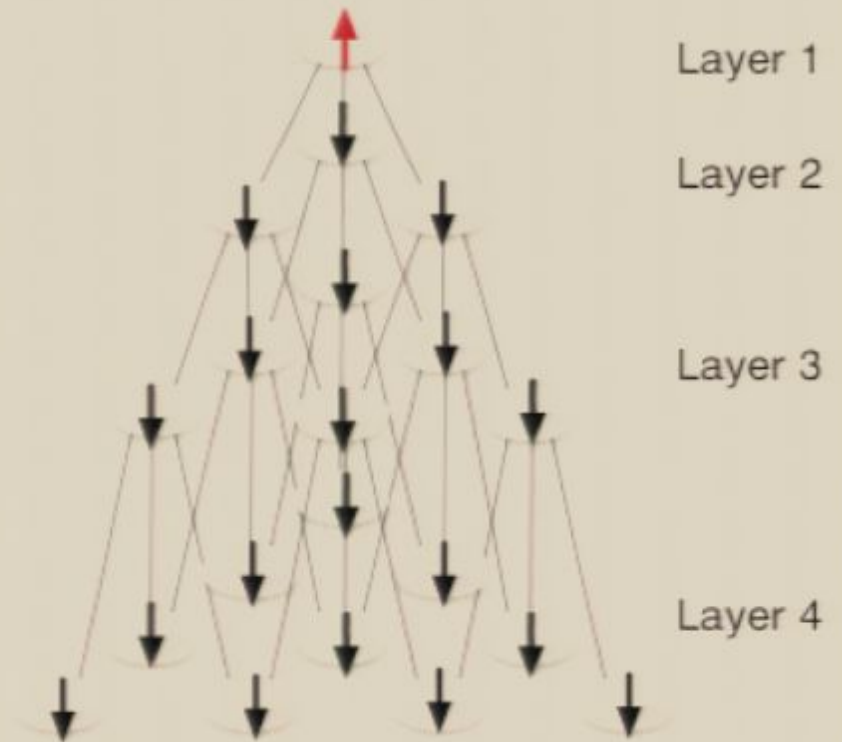
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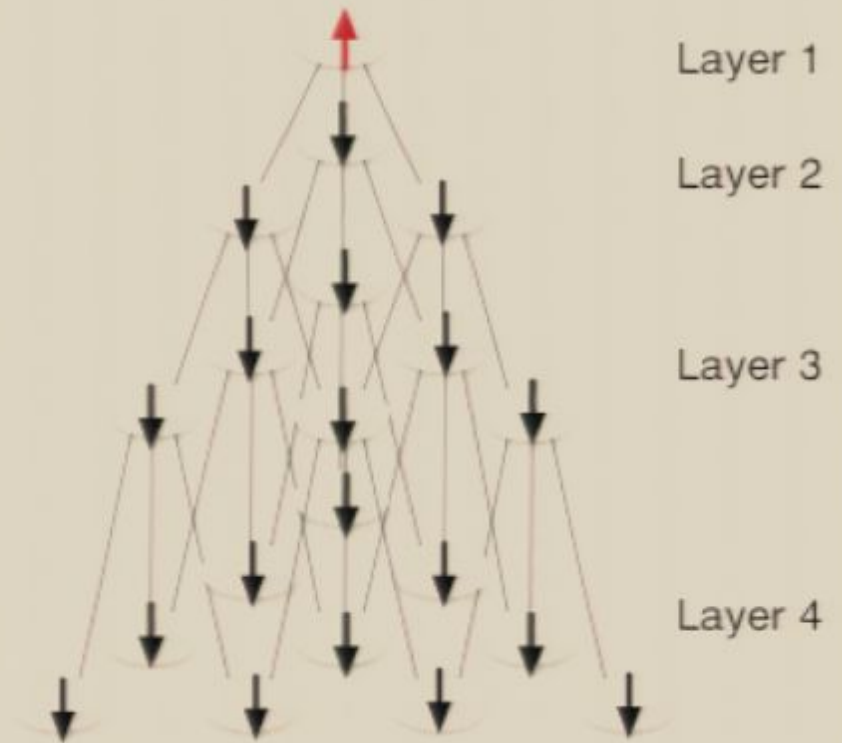
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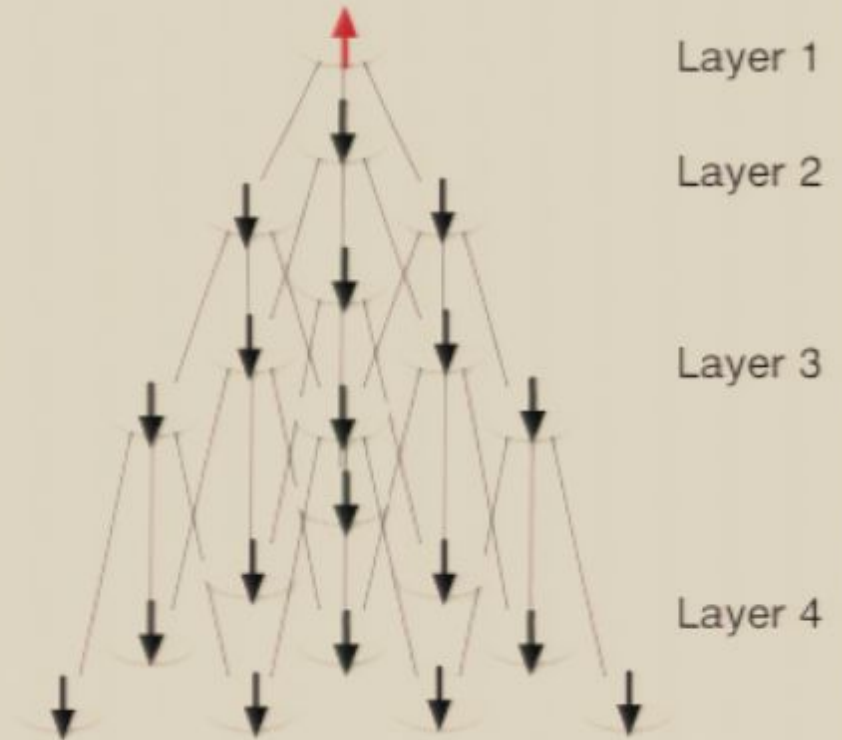
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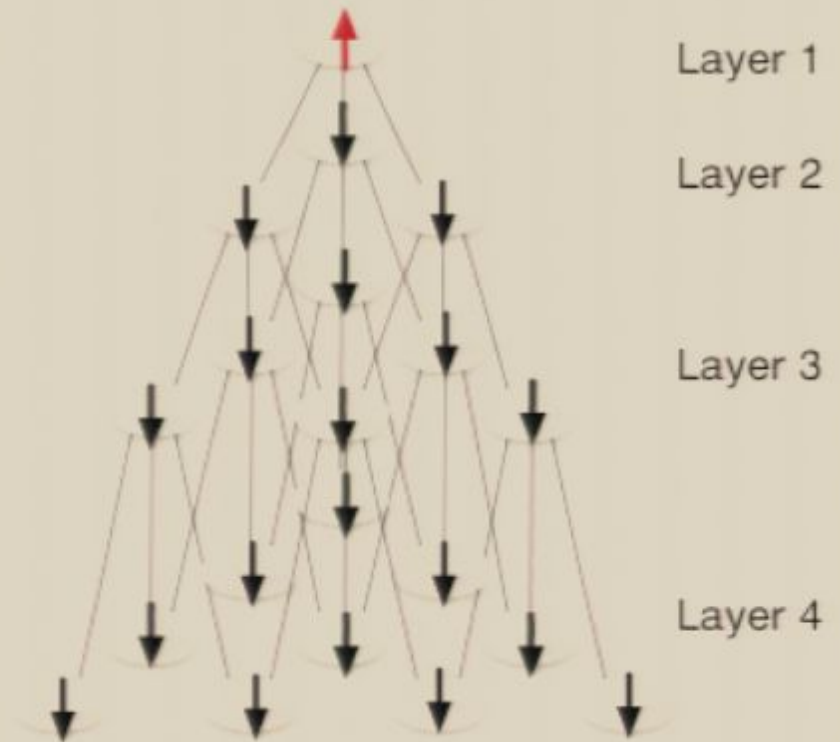
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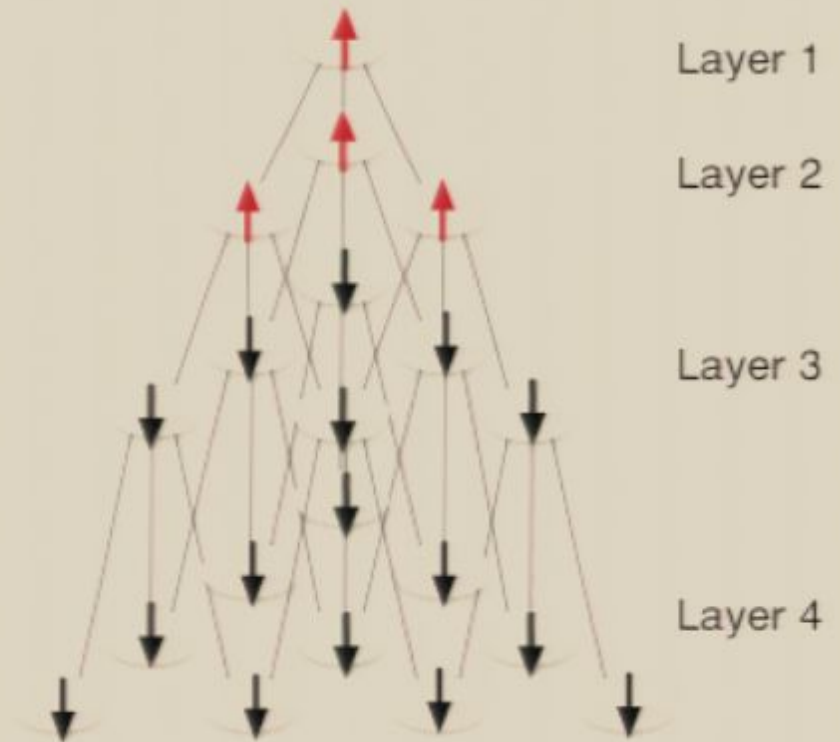
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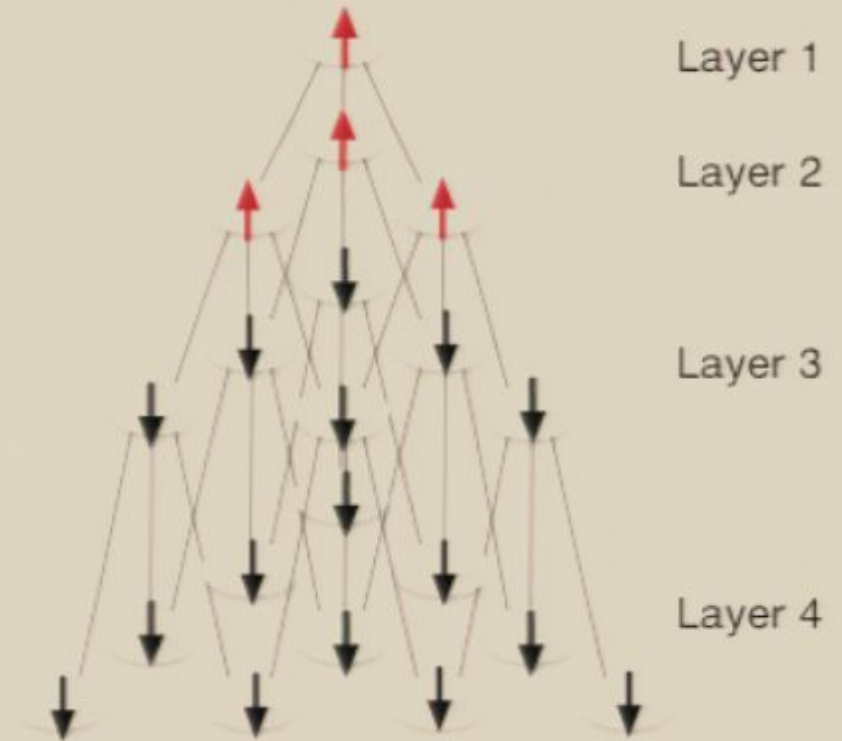
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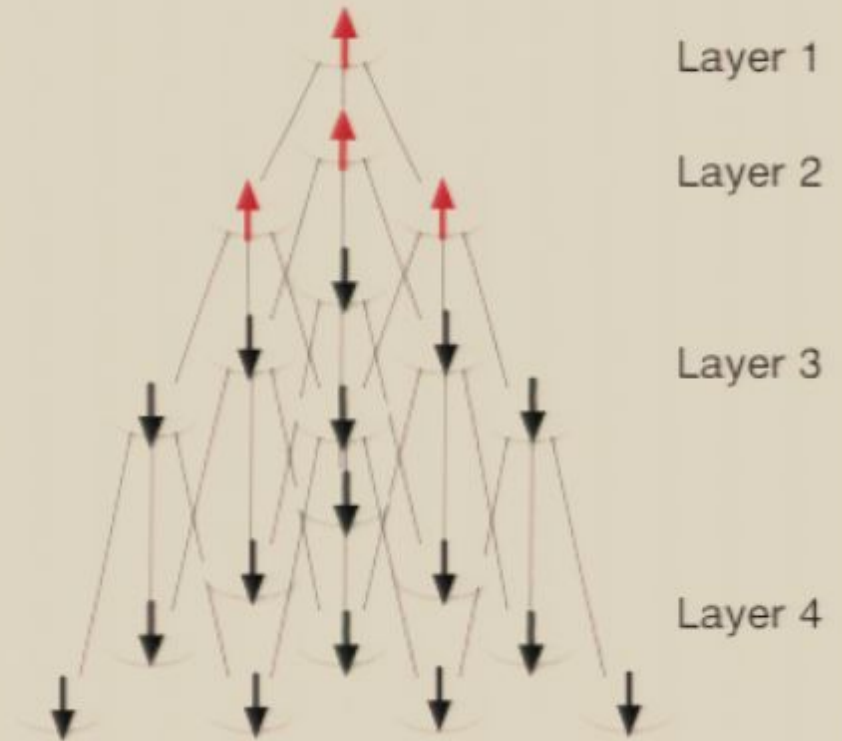
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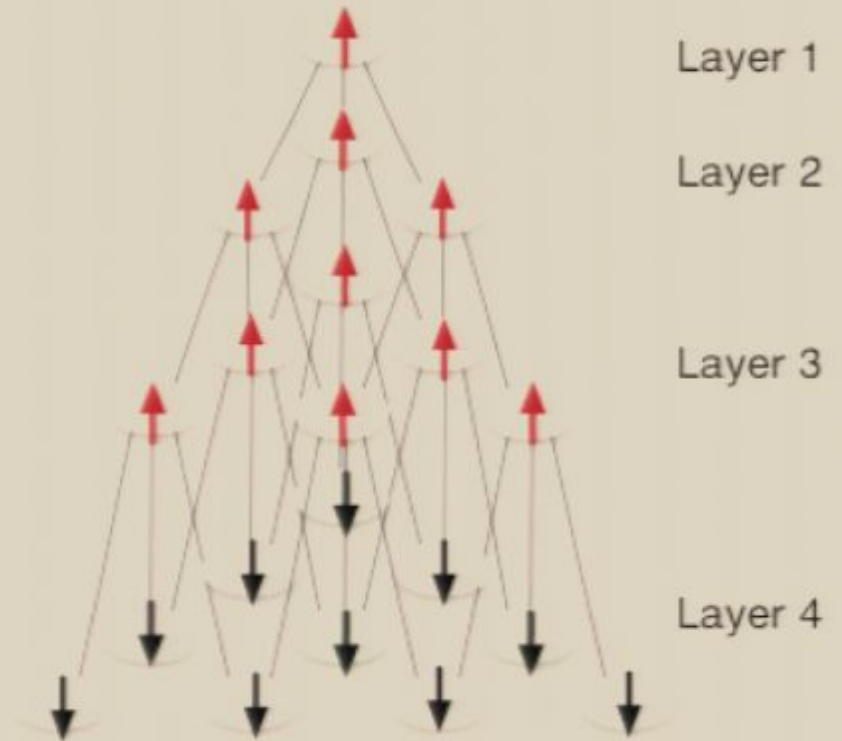
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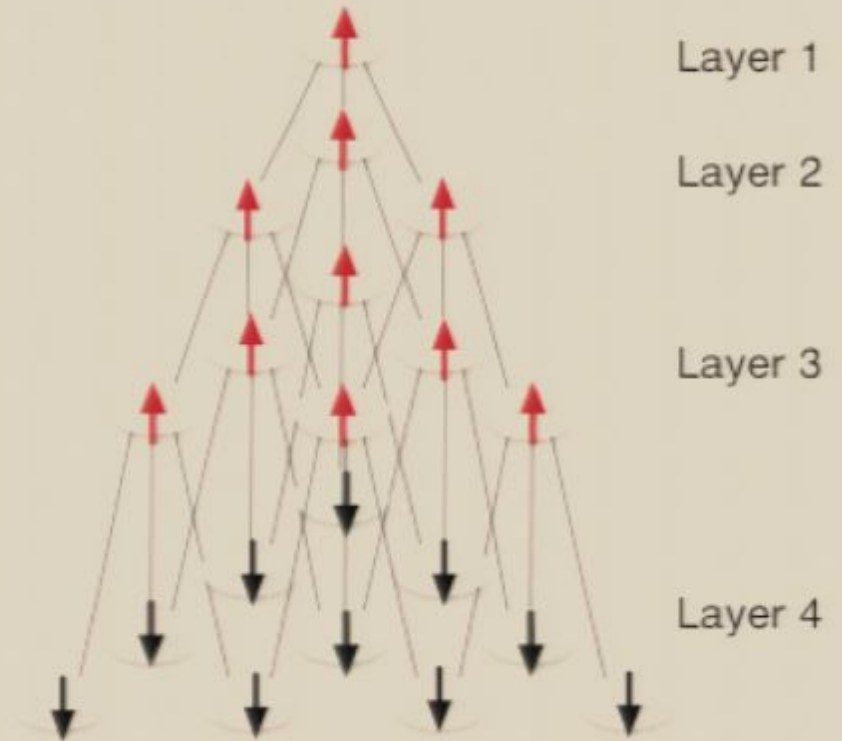
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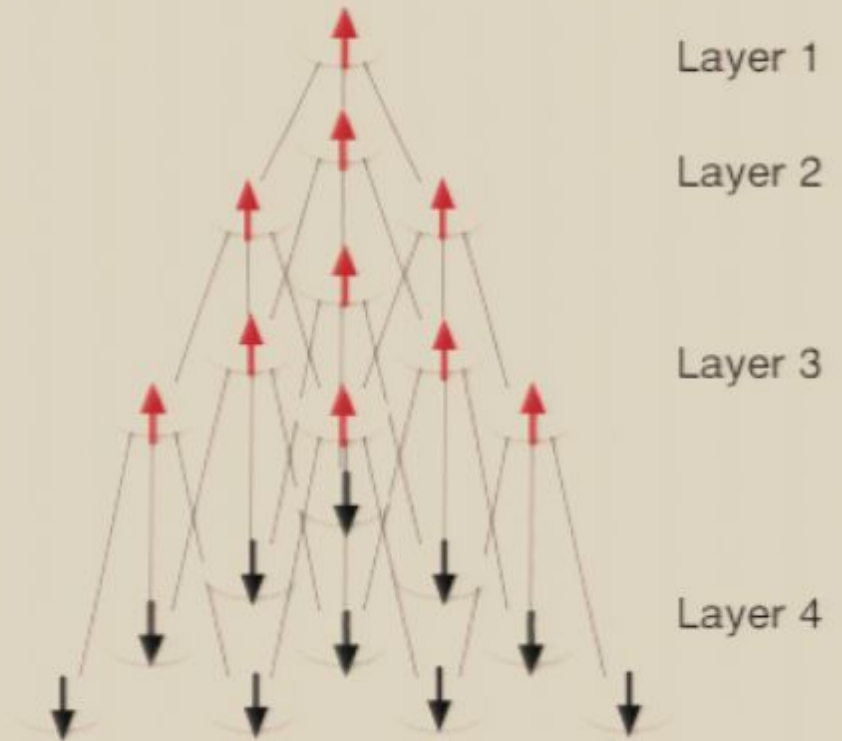
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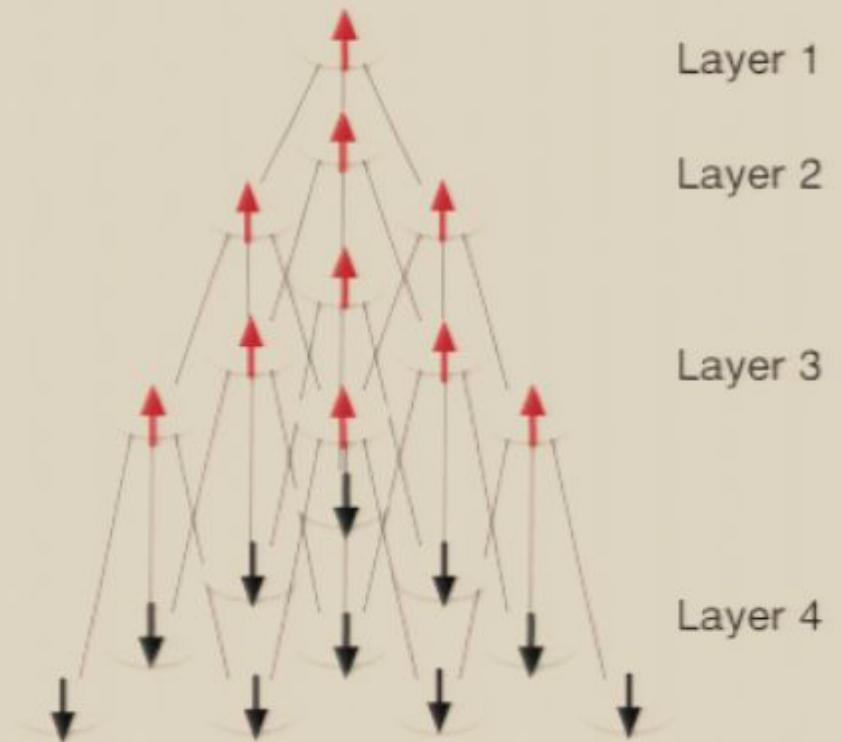
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In subsequent steps we need to apply NOT gates to spins with neighbour fields -2 , -1 , and 0 .



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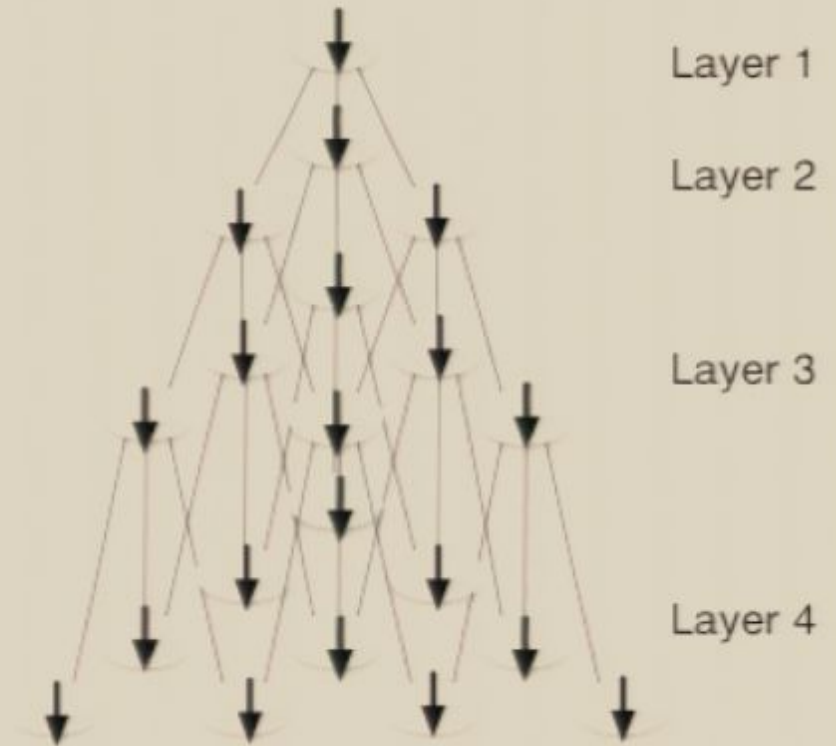
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Notice that if the top spin is initially in the state $|\downarrow\rangle$, then the procedure leaves the lattice invariant.



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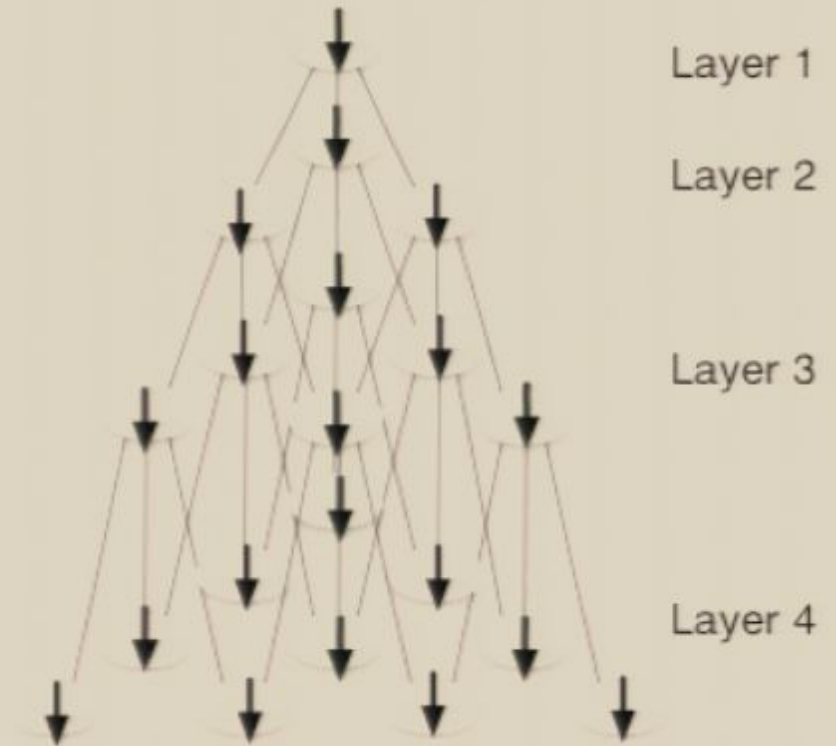
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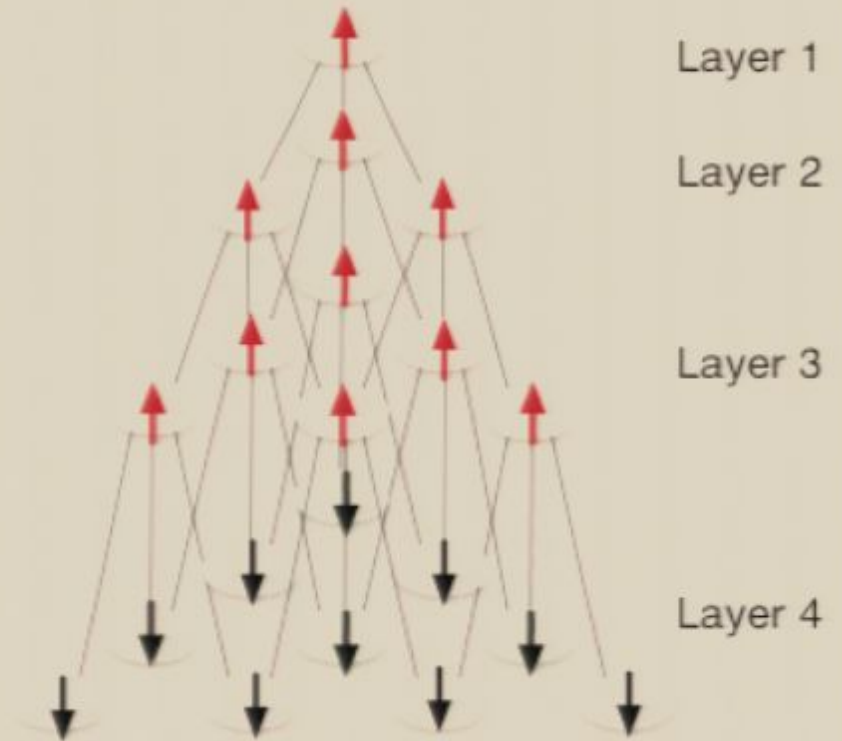
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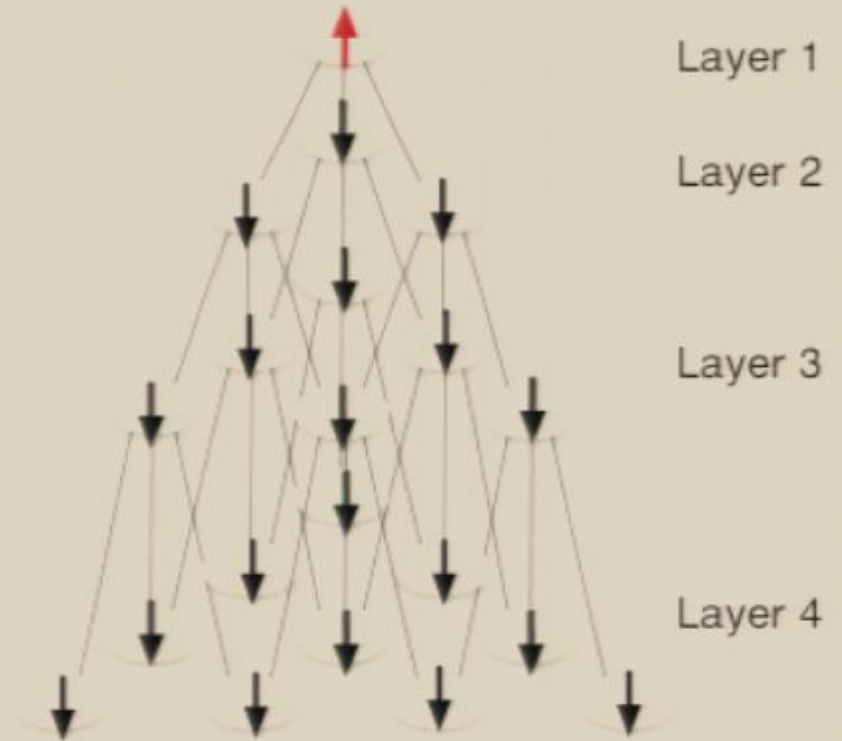
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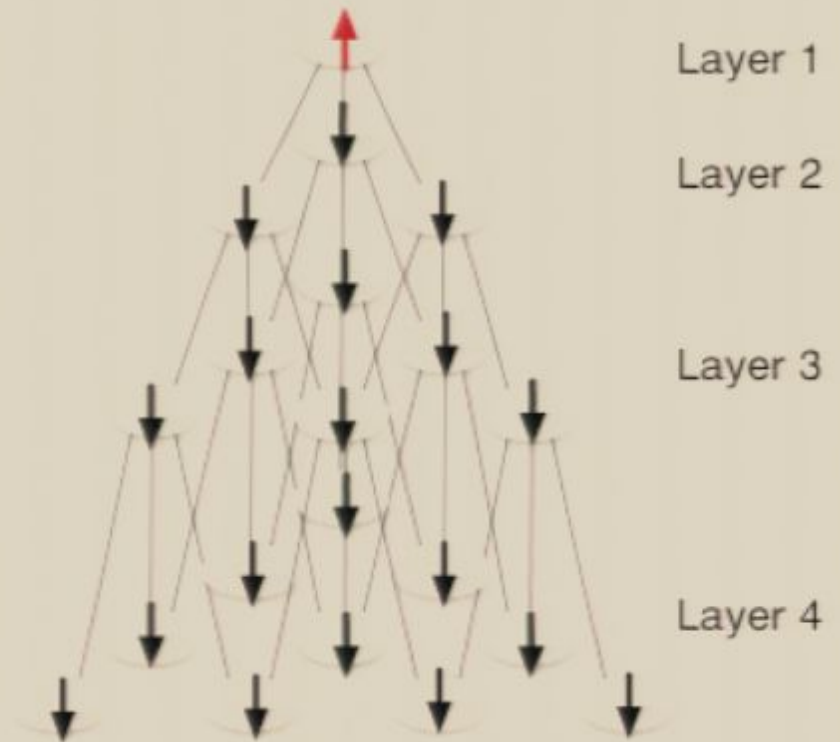
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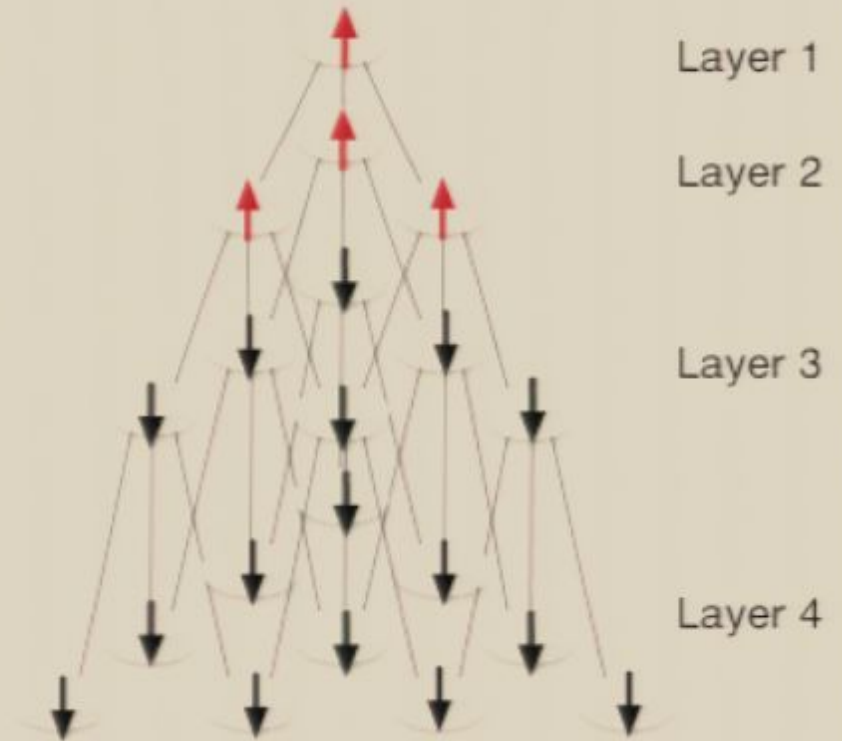
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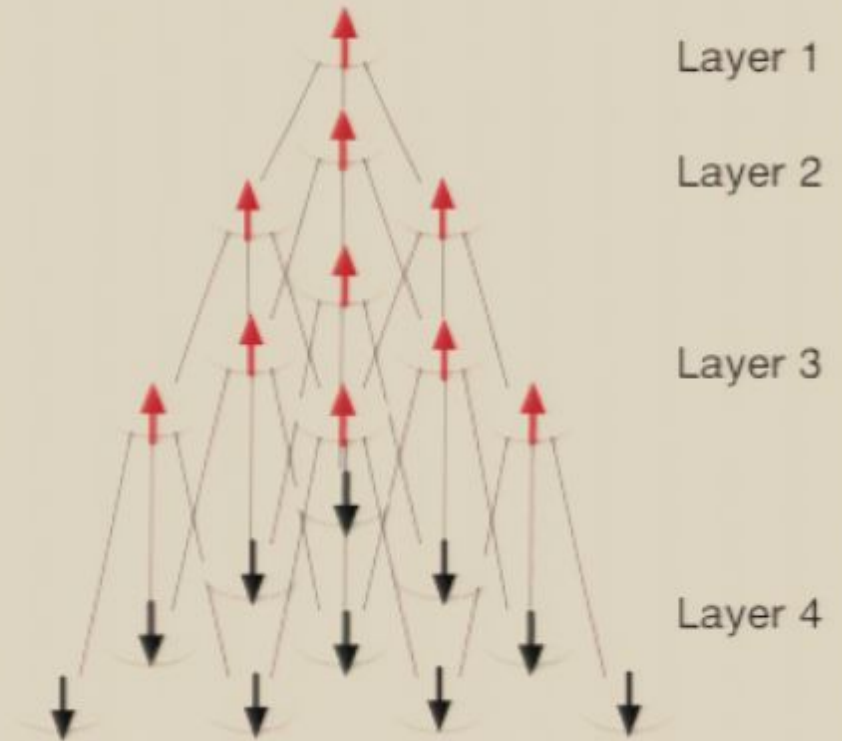
- Suppose the top spin is initially in the $|\uparrow\rangle$ state.
- If we were to apply a NOT gate to B spins with neighbour field -2 , all second layer spins would be flipped to the $|\uparrow\rangle$ state,
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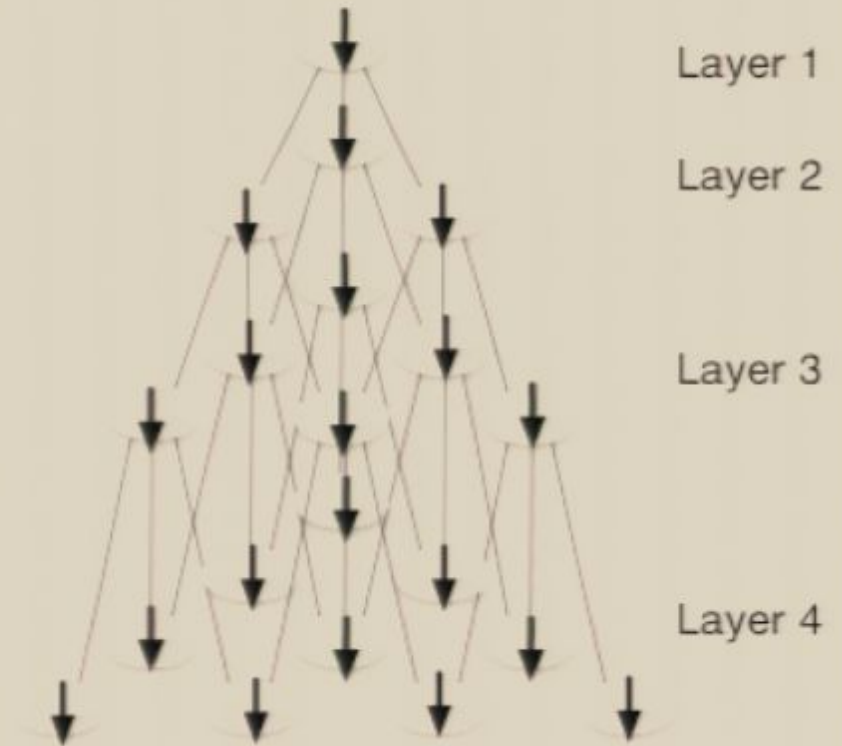
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Notice that if the top spin is initially in the state $|\downarrow\rangle$, then the procedure leaves the lattice invariant.



The QCA rule can be summarized as

$$NOT(A, \{-2, -1, 0\})NOT(B, \{-2, -1, 0\})$$

The algorithm simply repeats these two steps until enough spins have been properly polarized. It take $O(\sqrt[3]{N})$ steps to polarize N spins.

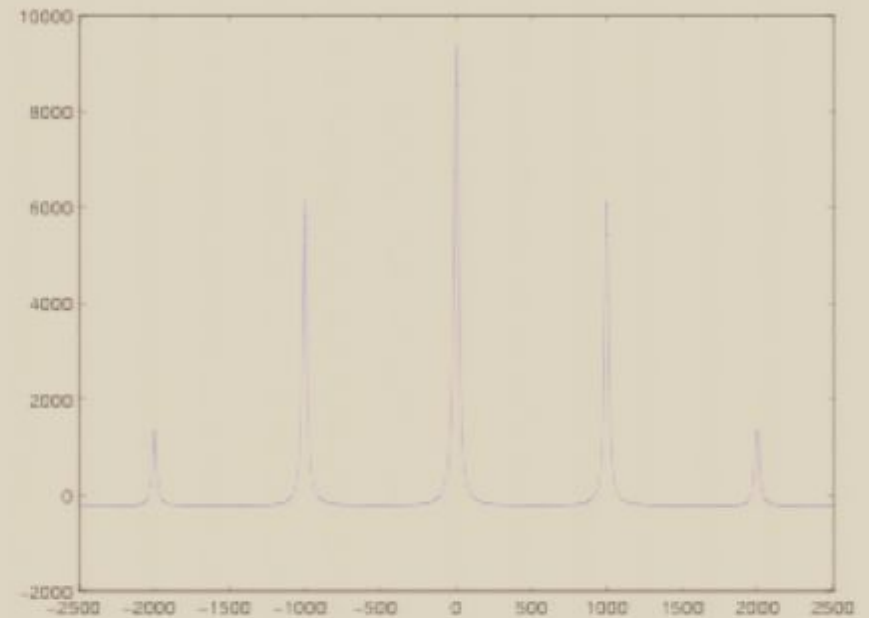


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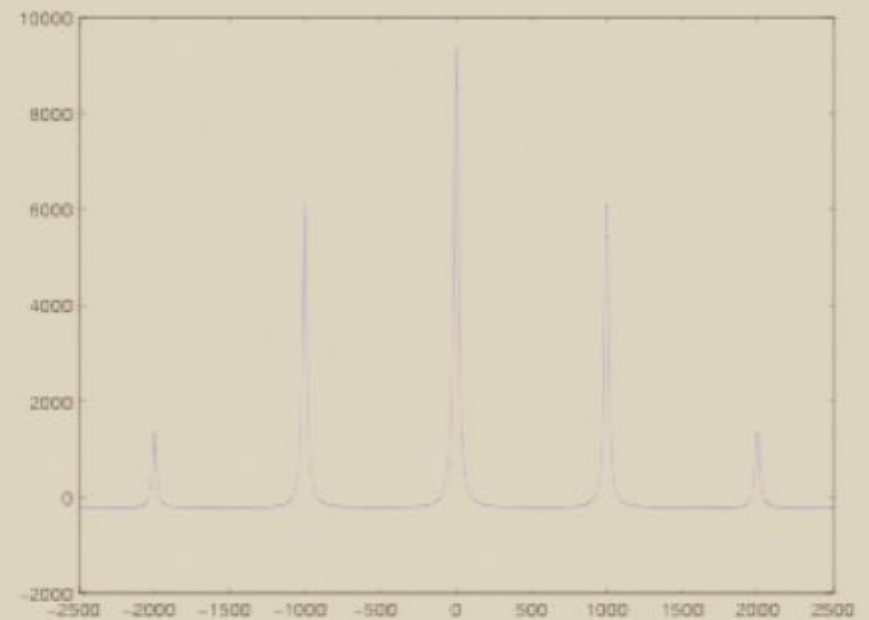
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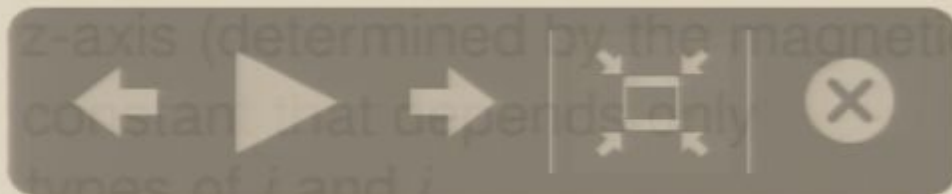


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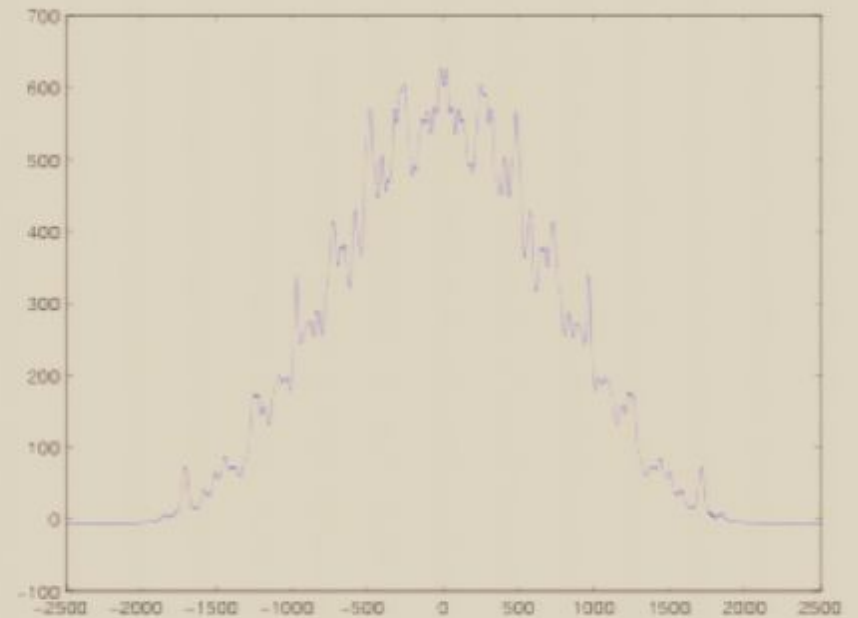
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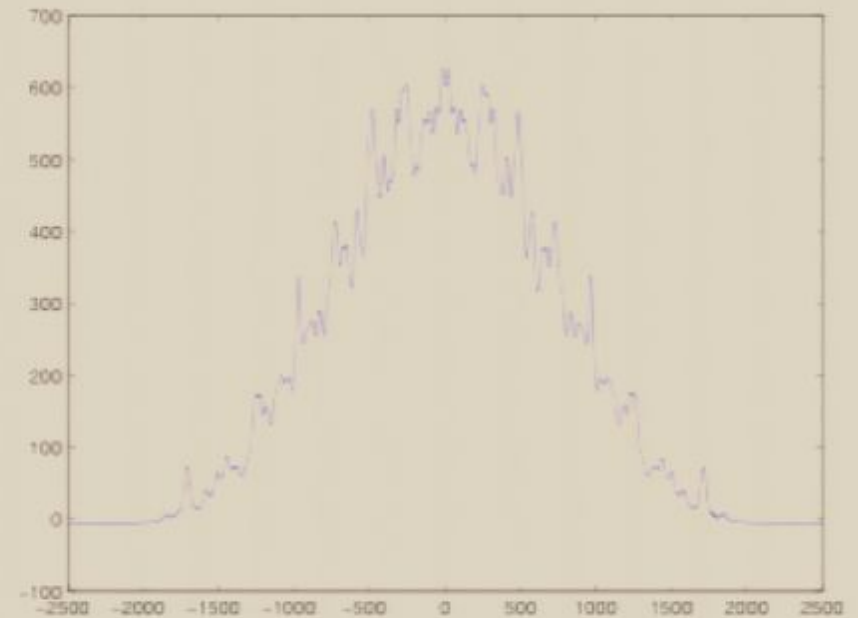
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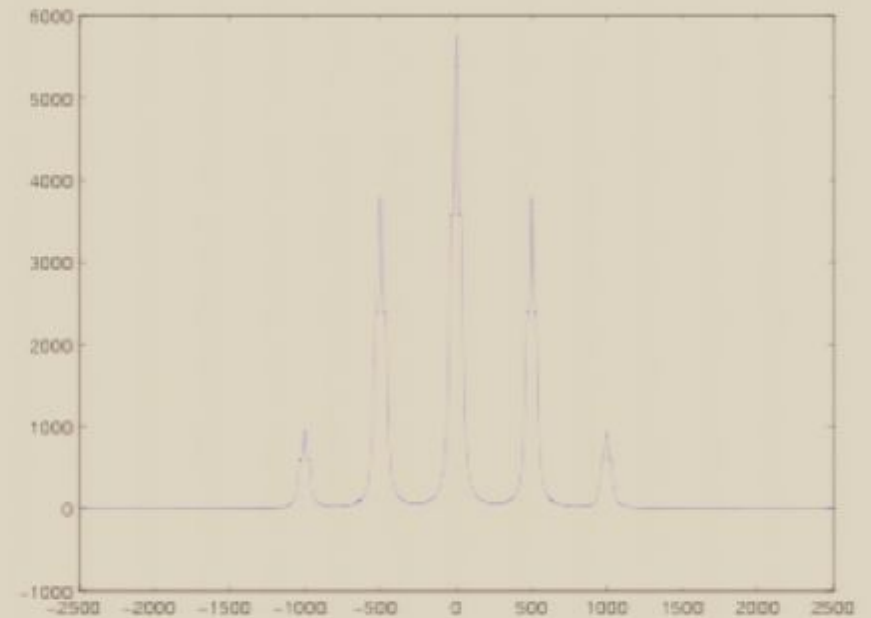


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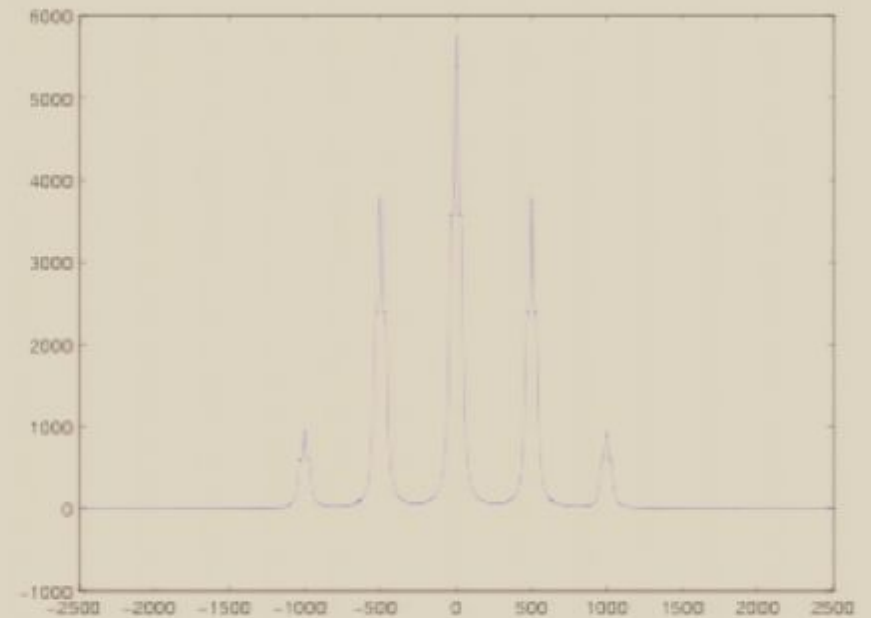


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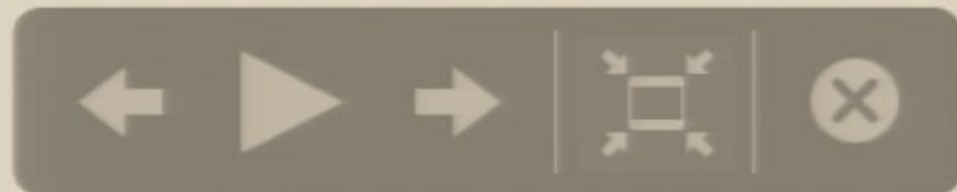
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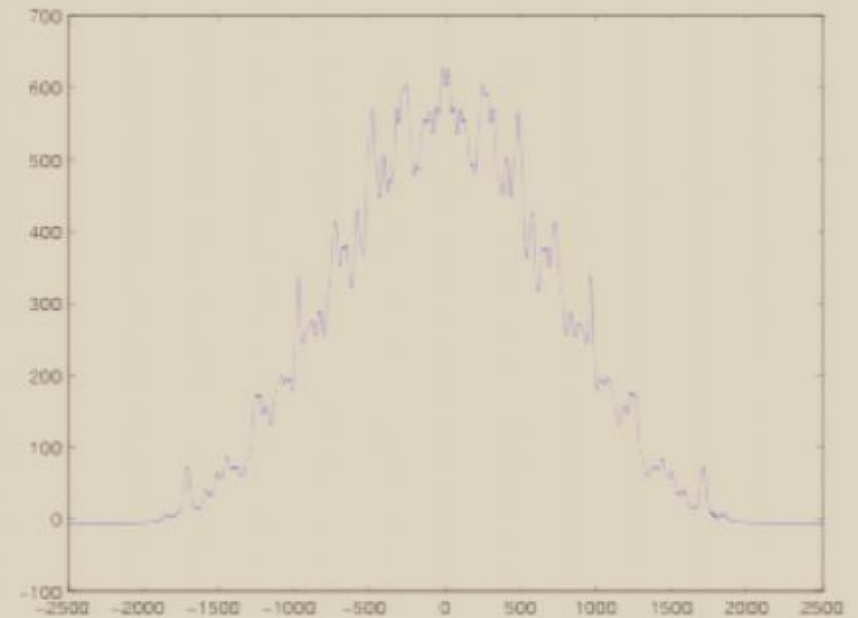
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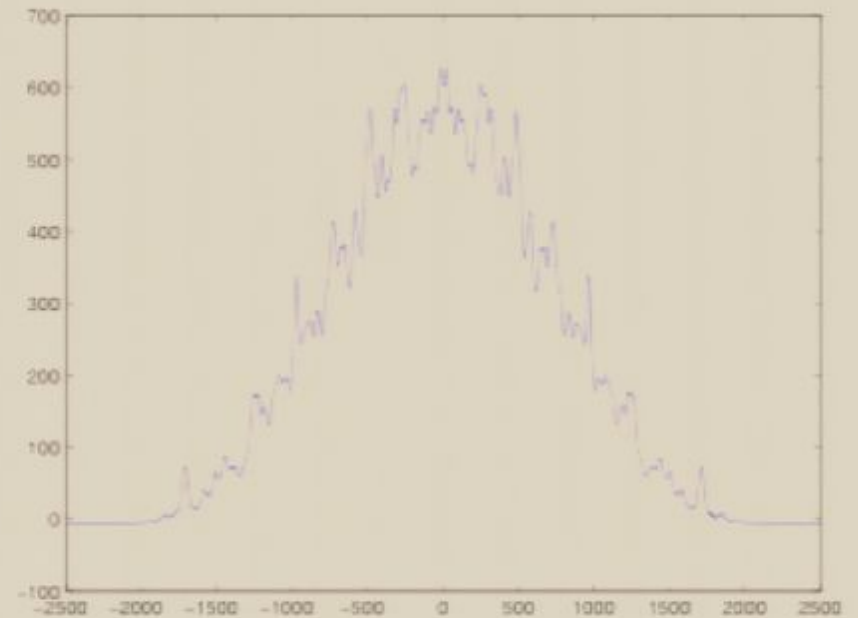
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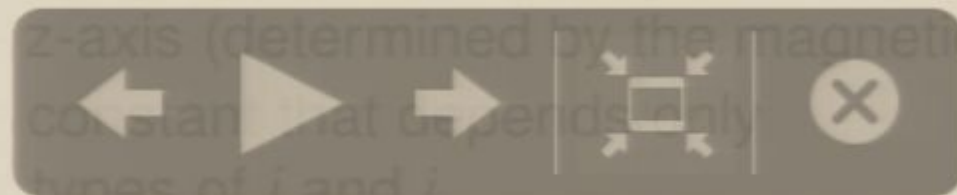
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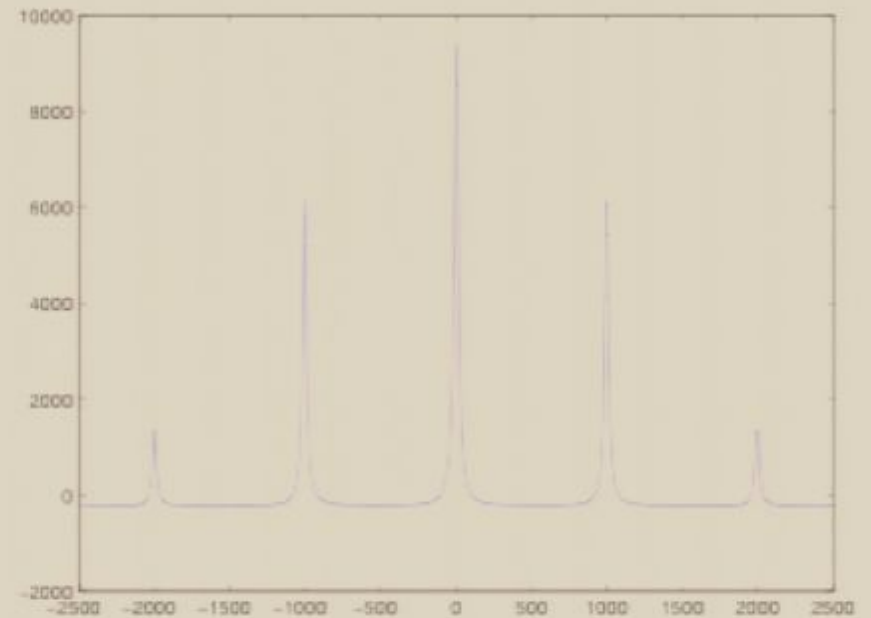


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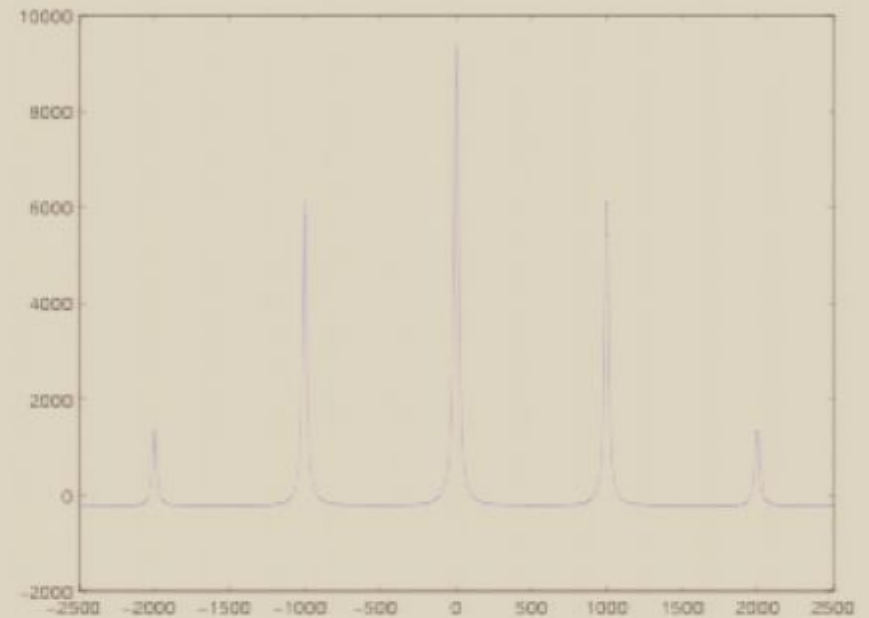
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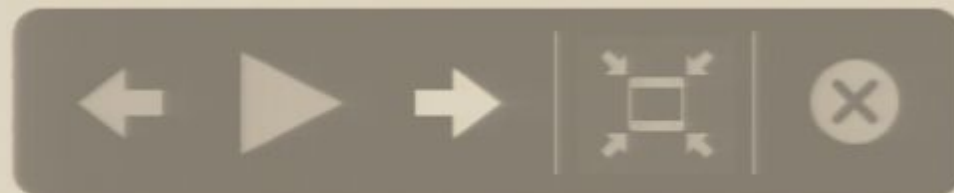
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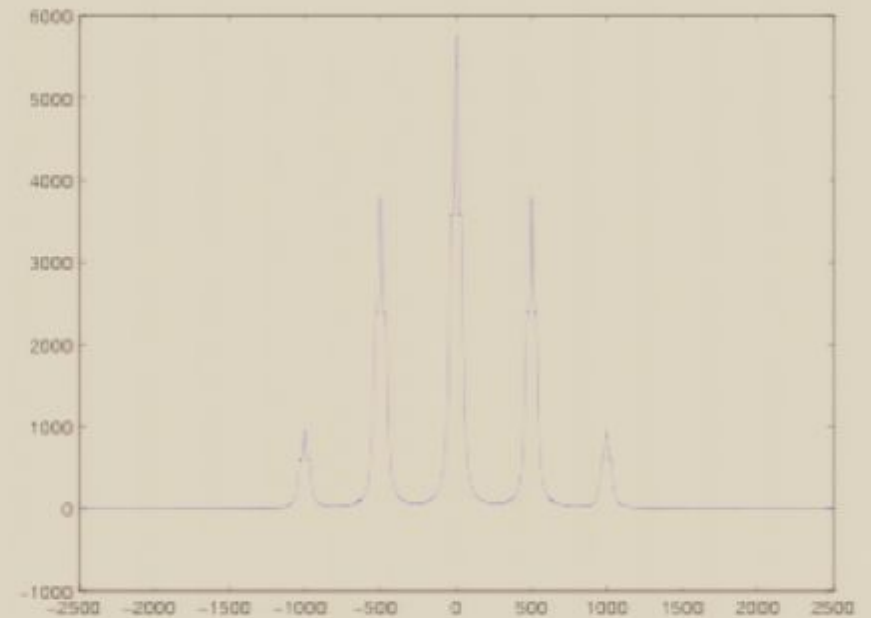
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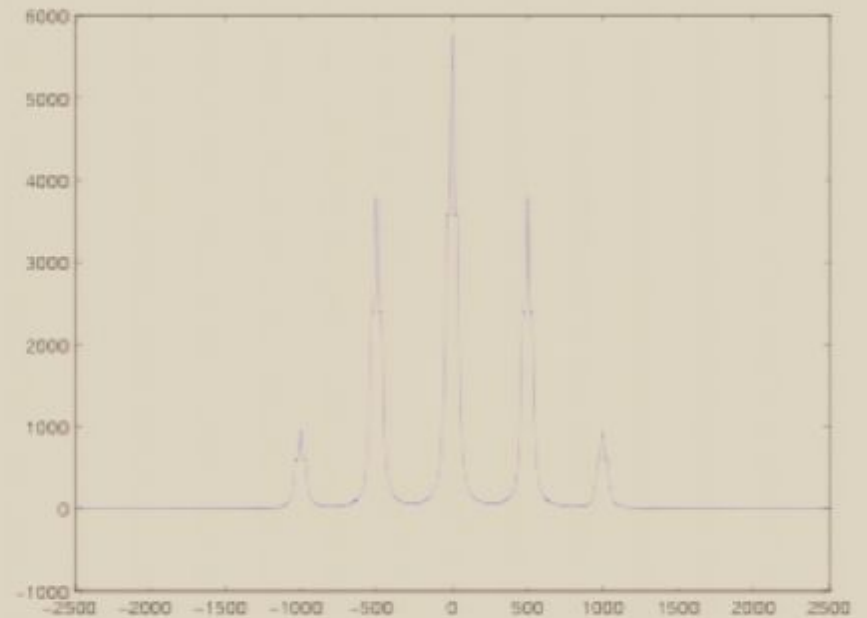
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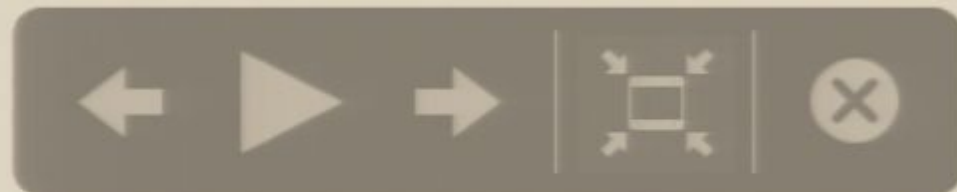
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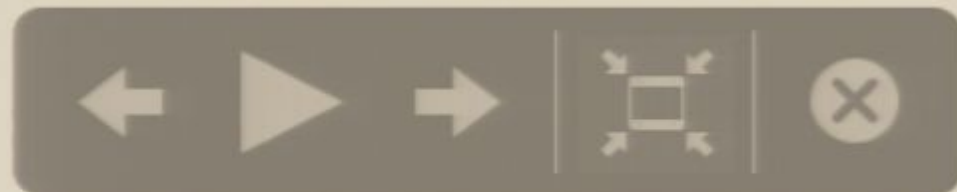
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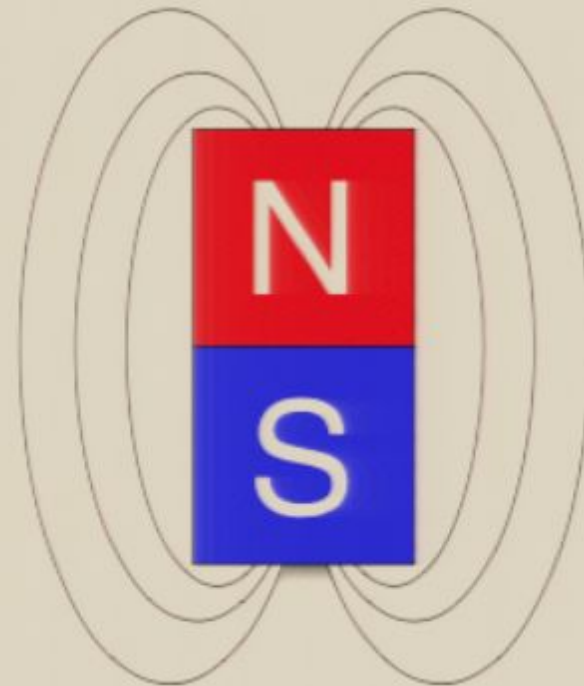
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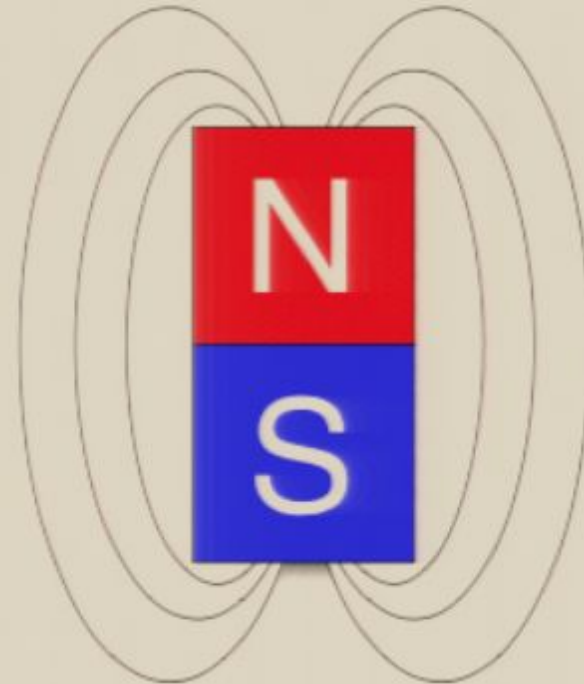
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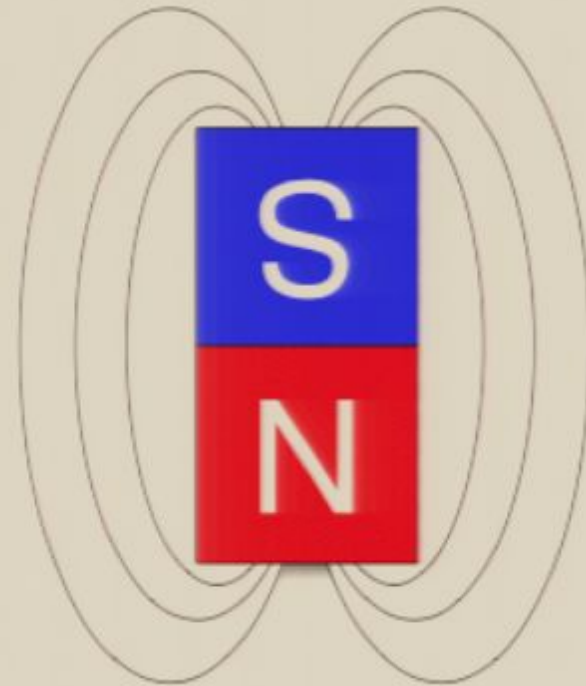
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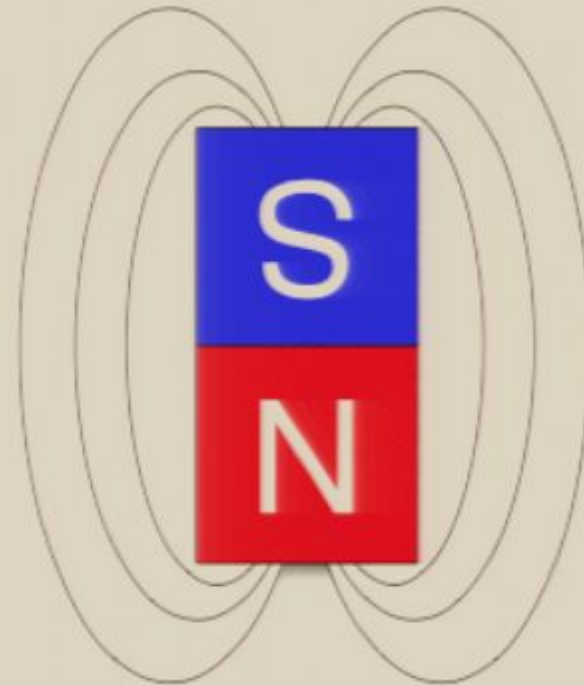
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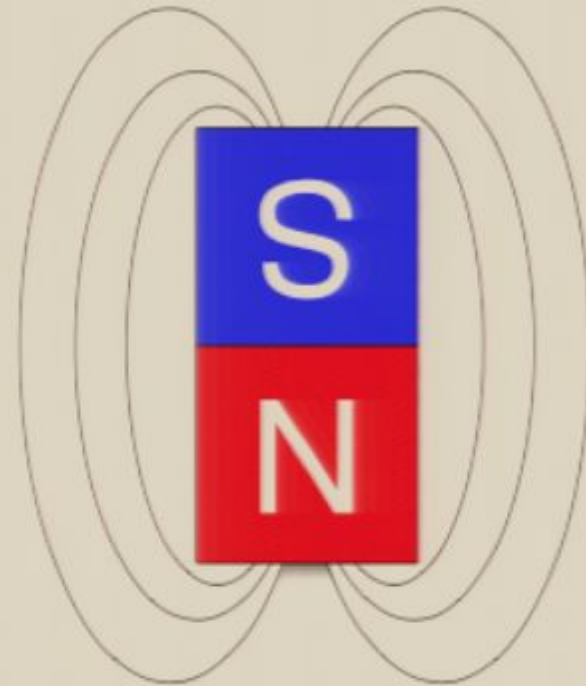
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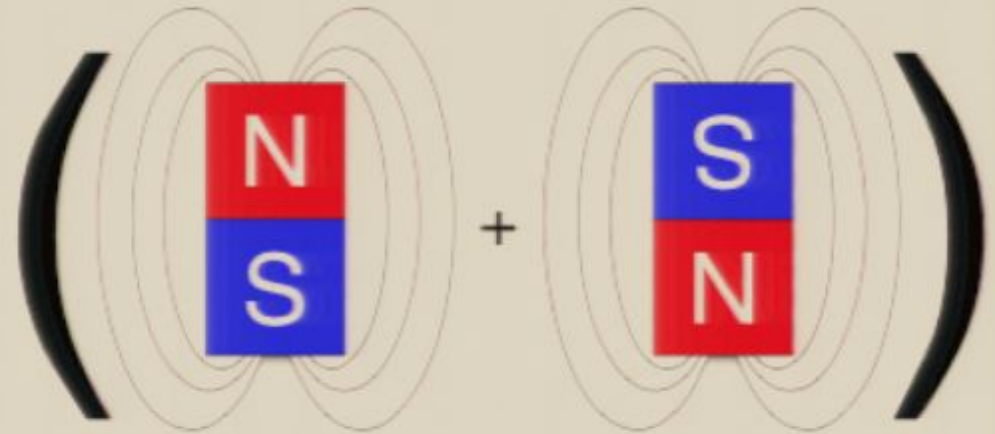
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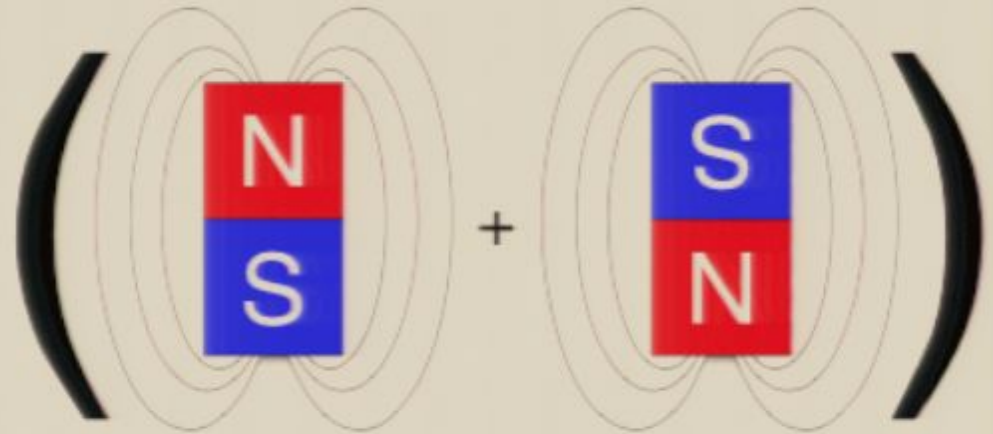
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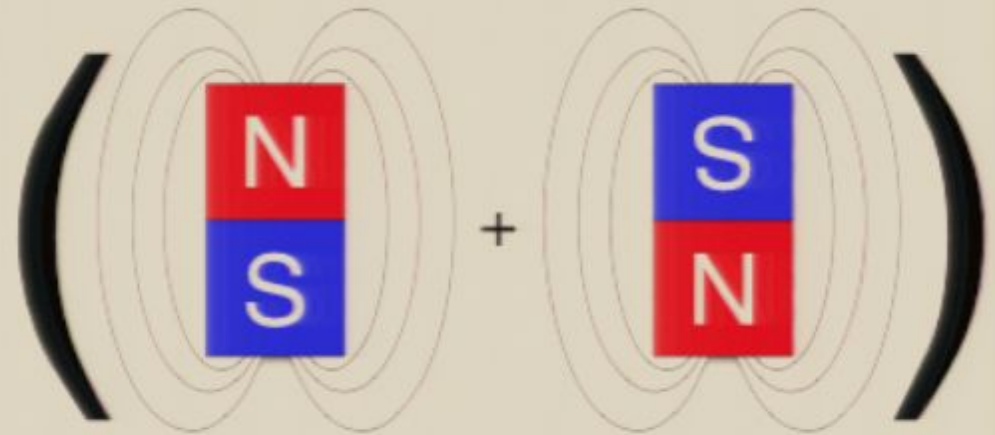
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- In order to verify that we succeeded in creating a cat state it suffices to show that:

$$H(\text{tr}_{\text{ancilla}}(\Delta^\dagger(\Delta(H|0\rangle \otimes \rho_{\text{ancilla}})))) = |0\rangle$$



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- Technically, in the absence of perfect polarization the state we create is not a superposition of two basis states, but rather the superposition of two mixtures of basis states ρ_0 and ρ_1 , *i.e.* the state can be written as $\frac{1}{\sqrt{2}} (\rho_0 + \rho_1)$



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What's next...

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- An experiment of this sort requires a high degree of quantum control in solid state NMR. It will not be implemented any time soon.



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Thank you for waking up!



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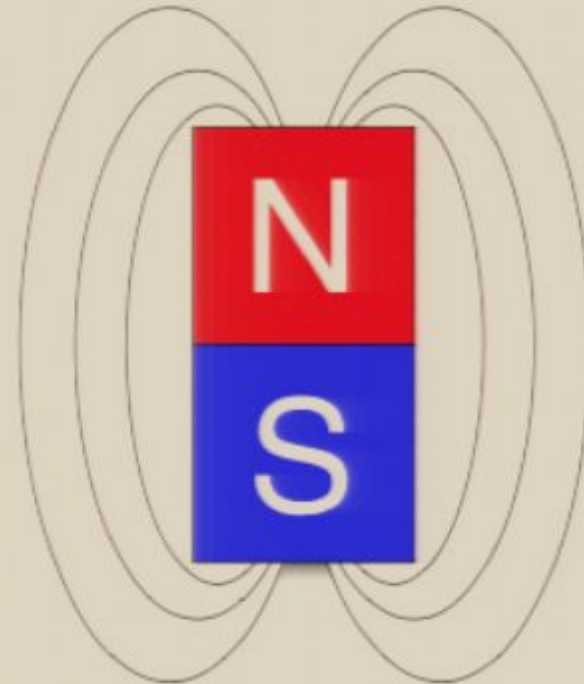
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Lets call our spin-amplification process Δ . Then

$$\Delta (|0\rangle \otimes \rho_{\text{ancilla}}) =$$



$$\Delta \left[\left(\frac{1}{\sqrt{2}} |0\rangle + |1\rangle \right) \otimes \rho \right]$$



$$\Delta \left[\left(\frac{1}{\sqrt{2}} |10\rangle + |11\rangle \right) \otimes \mathcal{P} \right]$$

$$\frac{1}{\sqrt{2}} \Delta(|10\rangle \otimes \mathcal{P}) + \frac{1}{\sqrt{2}} \Delta(|11\rangle \otimes \mathcal{P})$$

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$$\frac{1}{\sqrt{2}} \Delta(|10\rangle \otimes \mathcal{P}) + \frac{1}{\sqrt{2}} \Delta(|11\rangle \otimes \mathcal{P})$$

$$\Delta \left(\frac{1}{\sqrt{2}} 10 \right) \ll$$



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