Title: Macroscopic observables and quantization

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Abstract:

Macroscopic observables and quantization

Klaas Landsman

Radboud Universiteit Nijmegen
Institute for Mathematics, Astrophysics and Particle Physics

Operational Quantum Physics and the Quantum-Classical Contrast, Perimeter Institute, June 7, 2007

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Copenhagen

These uncertainties are simply a consequence of the fact that we describe the experiment in terms of classical physics (1958)

However far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms (1949)

Conceptual goals

- Ultimate goal: *make sense of the Copenhagen Interpretation*
  - Identification of classical contexts in suitable limits
  - Explanation of Born rule for probabilities of events
  - Proof of randomness of measurement outcomes
- Not discussed today but essential for complete picture: complementarity of different classical contexts through
  - Coexistence (modern Many Worlds à la Wallace)
  - Localization (topos framework of Isham et al)

Technical goals

- Construct classical contexts as commutative C*-algebras 'extracted' from noncommutative C*-algebra
  - 'Classical limit' $\lim_{\hbar \to 0}$ is well understood through notion of C*-deformation quantization
  - Macroscopic limit $\lim_{N \to \infty}$ typically uses heavy machinery (von Neumann algebras, $\infty\bigotimes$)
  - Both limits can be brought under same heading (namely continuous fields of C*-algebras)
- Transparent proof of Born rule and of randomness

Pedigree

- **Copenhagen:** Scheibe (1973), Camilleri (2005)
- **Macroscopic observables**: Bogoliubov (1958), Haag (1962), Hepp (1972), Werner&Duffield, Raggio (1990)
- **Quantization**: Berezin (1974), Rieffel (1989)
- **Born rule**: Finkelstein (1963), Hartle (1968), ...
- **Randomness**: Kolmogorov (1965), Chaitin (1976), Coleman & Lesniewski (u), Gutmann (1995)

Deformation quantization

- **Idea**: classical system is boundary at $\hbar = 0$ of fibre bundle of quantum theories defined for $\hbar > 0$
- **Execution**: do not glue phase space to Hilbert spaces but function algebra to operator algebras
- **Technically**: classical & quantum algebras of observables form **continuous field of C*-algebras**
  - Key difference with vector bundles: fibres need not be isomorphic (and may be ∞-dim)

Bundle of C*-algebras

- **Base space** $I \subset \mathbb{R}$ containing 0 as accumulation point
  - Quantization: ħ takes values in I, e.g. $I = \mathbb{R}$
  - Large systems: $I = 0 \cup \{1/N, N \in \mathbb{N}\}$
- **Bundle of C*-algebras** A **is disjoint union** $\coprod_{x \in I} A_x$
  - Quantization: A_0 commutative (classical)
  - Large systems: two possible continuous fields
    - ***macroscopic*** observables: A_0 commutative
    - ***quasilocal*** observables: A_0 highly noncommutative

Continuous field of C*-algebras

- **Topology on bundle of C*algebras** $\coprod_{x \in I} A_x$ is added by specifying all continuous sections $\sigma : x \in I \mapsto \sigma(x) \in A_x$
  - function $x \mapsto \|\sigma(x)\|$ from $I \to \mathbb{R}^+$ is continuous
  - all sections form C*-algebra A that is $C_0(I)$ module
- **Upshot:** can study 'transition' to A_0 by (norm) continuity:
  - For $\hbar \in I = \mathbb{R}$ this is the 'usual' classical limit $\hbar \to 0$
  - For $I = 0 \cup \{1/N, N \in \mathbb{N}\}$ this is the limit $N \to \infty$
- One *approximates* a quantum system by a classical one and a finite system by an infinite one in a (norm) controlled way

Example of quantization

- Classical particle moving on $\mathbb{R}^n$: $A_0 = C_0(\mathbb{R}^{2n})$
- Quantum algebra of observables: $A_\hbar = K(L^2(\mathbb{R}^n)) \,\forall \hbar \neq 0$
- Topology: each function $f \in C_0(\mathbb{R}^{2n})$ defines section

  $\hbar \mapsto Q_\hbar(f) \in A_\hbar$ where $Q_\hbar$ is any decent quantization map, e.g. $Q_\hbar(f) = \int_{\mathbb{R}^{2n}} \frac{d^n p d^n q}{(2\pi\hbar)^n} f(p,q)|p,q\rangle\langle p,q|$

  Asymptotic commutativity: $\lim_{\hbar\to 0} \|Q_\hbar(f)Q_\hbar(g) - Q_\hbar(fg)\| = 0$

  Poisson bracket: $\lim_{\hbar\to 0} \left\| \frac{i}{\hbar}[Q_\hbar(f), Q_\hbar(g)] - Q_\hbar(\{f,g\}) \right\| = 0$

Large systems

- N copies of system with C*-algebra B, e.g $B = M_2(\mathbb{C})$
- Two kinds of limiting observables for large N:
  - ***Quasilocal*** observables are limits of ***local*** observables $a_N^{loc} = b_M \otimes 1 \otimes \cdots \otimes 1,\ b_M \in B^{\otimes M}$
  - ***Macroscopic*** observables are limits of ***averages***

$$a_N^{av} = \mathrm{symm}(b_M \otimes 1 \otimes \cdots \otimes 1),\ b_M \in B^{\otimes M}$$

Example: $a_N^{av} \;=\; \frac{1}{N}(b_1 \otimes 1 \cdots \otimes 1 + \cdots 1 \otimes \cdots 1 \otimes b_1)$

- Macroscopic observables *commute*: $\lim_{N\to\infty}[a_N, a'_N] = 0$

(with each other *and* with all quasilocal observables)

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Two continuous fields

- These different limiting observables give rise to two different bundles of C*-algebras over $I = 0 \cup \{1/N, N \in \mathbb{N}\}$:

$$A^{(q)}_{1/N} = A^{(c)}_{1/N} = B^{\otimes N}$$

$$\begin{aligned} A^{(q)}_0 &= \underline{\lim}_{N\to\infty} B^{\otimes N} \\ A^{(c)}_0 &= C(S(B)) \end{aligned}$$

- sequence $(a_0, a_1, \ldots, a_N, \ldots)$, $a_N \in B^{\otimes N}$ is continuous section of
  - $A^{(q)} : \exists\, (a_N^{loc}),\ \lim_{N\to\infty} \|a_N - a_N^{loc}\| = 0$ and $a_0 = \lim_N a_N$
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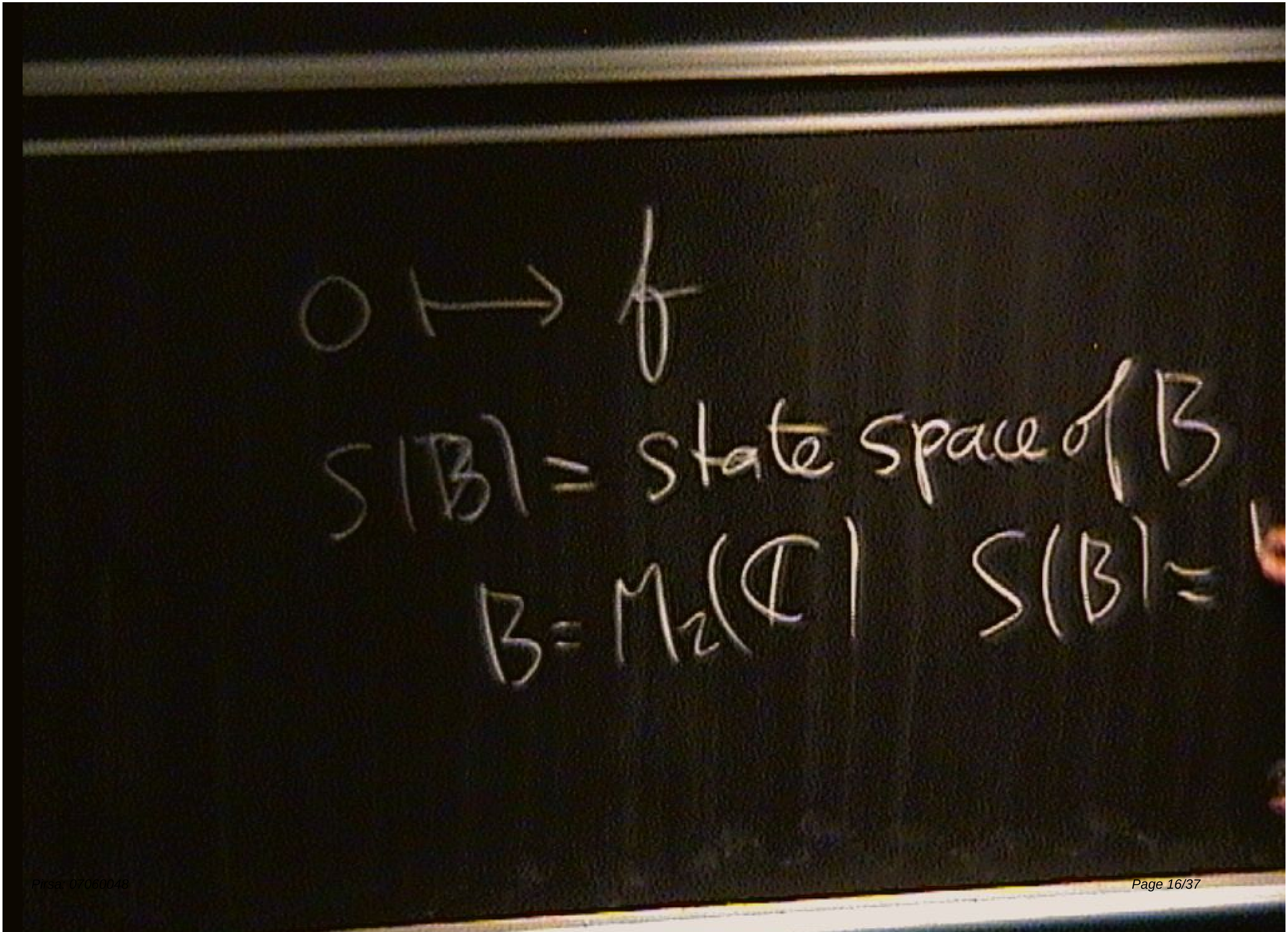
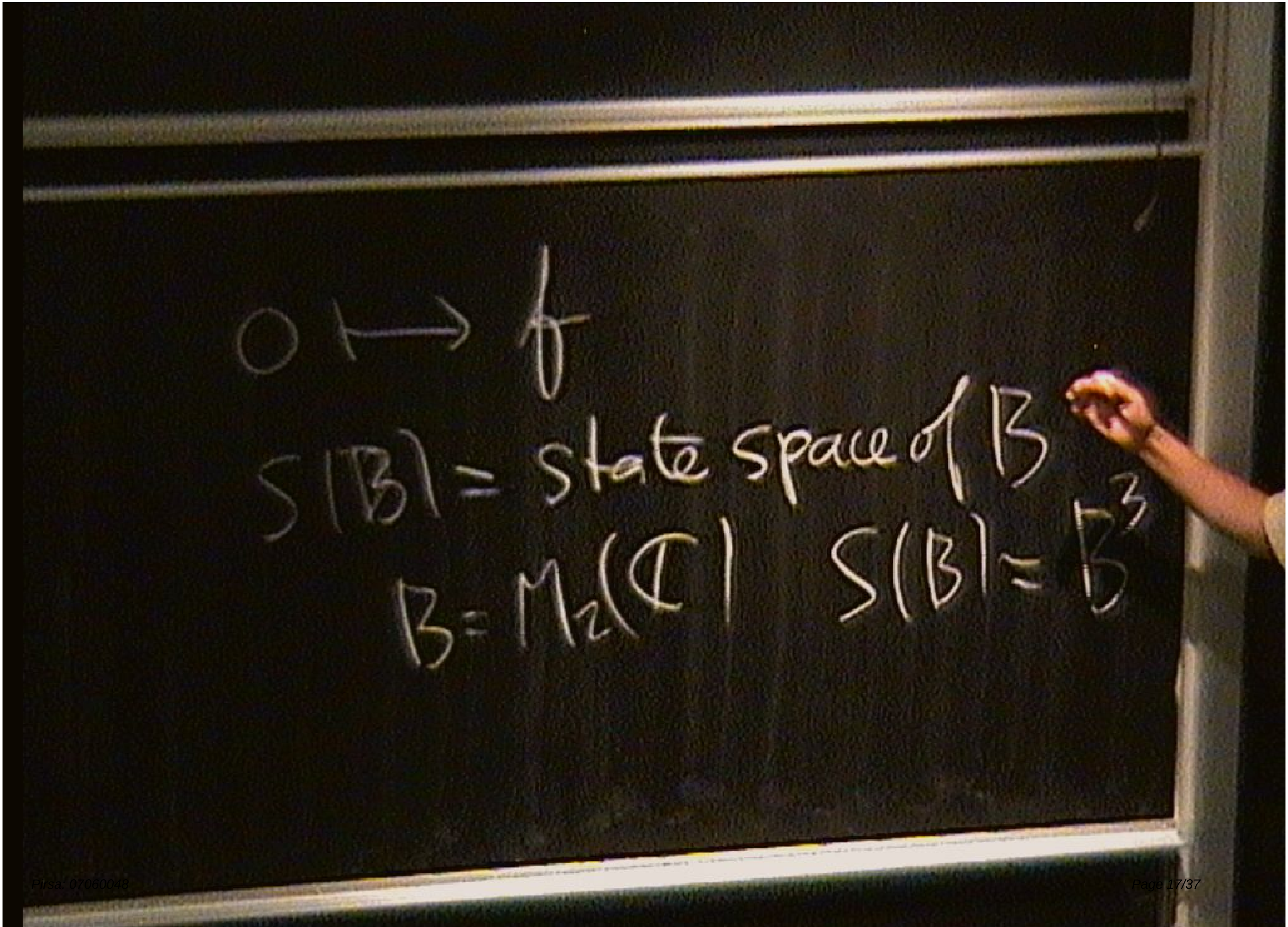

$O \longmapsto f$
$S(B) =$ state space of B
$B = M_2(\mathbb{C})$ $S(B) =$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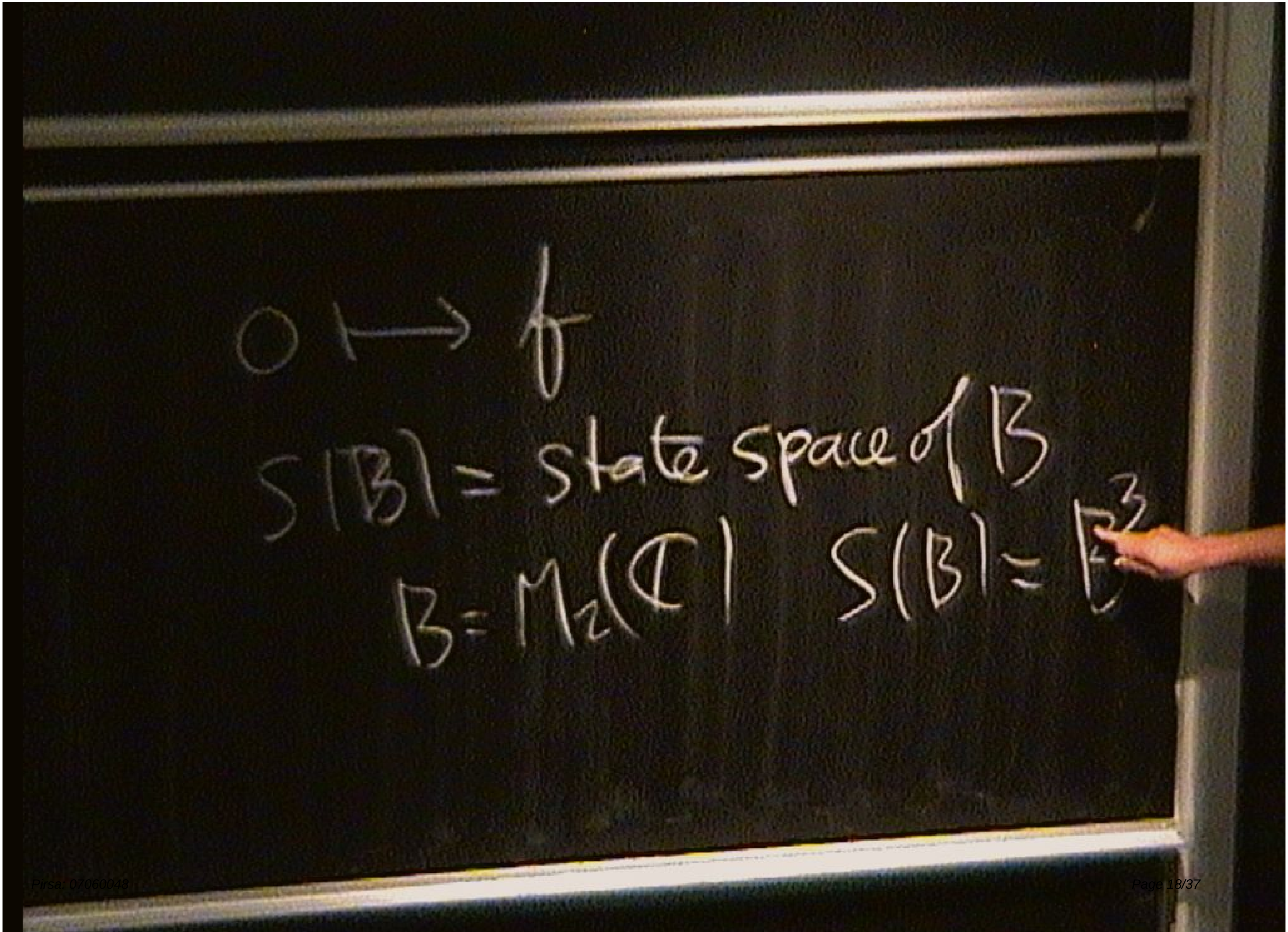
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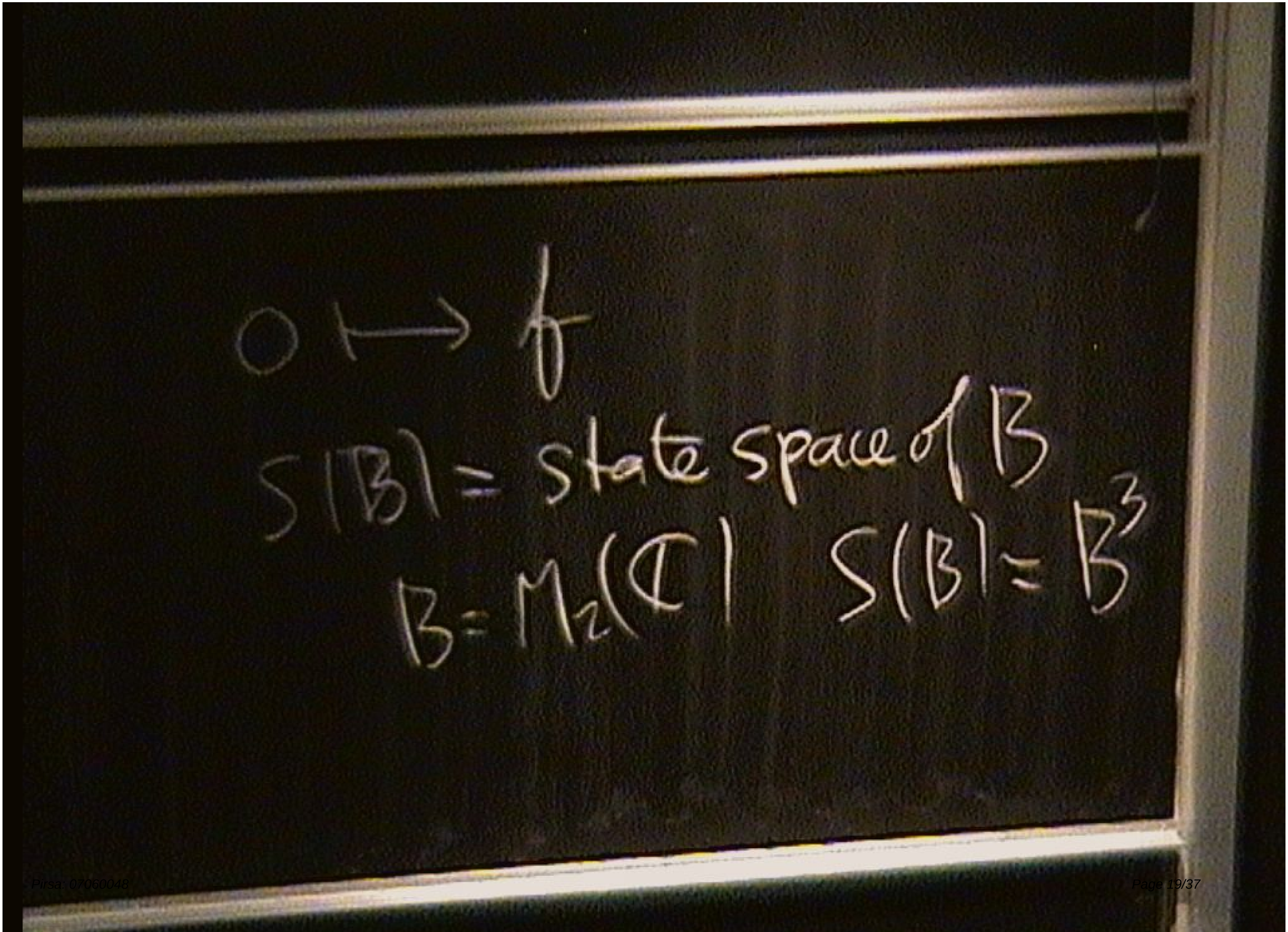
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Born rule

- Example of example: $B = M_2(\mathbb{C})$, $b_1 = |1\rangle\langle 1|$
- Then $a_N^{av} \equiv f_N$ is ***frequency operator*** on $\bigotimes^N \mathbb{C}^2$: its eigenstates are $|x_1\rangle \cdots |x_N\rangle$, $x_i = 0, 1$ with eigenvalues $(\sum_{i=1}^N x_i)/N$ (i.e. frequency of 1)
- Unique *continuous* section $(f_0, f_1, \ldots)$ of $A^{(c)}$ has

  limit $f_0(\psi) := \lim_{N\to\infty} {}^{\otimes N}\langle\psi|f_N|\psi\rangle^{\otimes N} = |\langle\psi|1\rangle|^2$
- **Interpretation**: $\lim_{N\to\infty} f_N$ is classical (dispersion-free) with (sharp) value given by Born probability

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Poisson structure

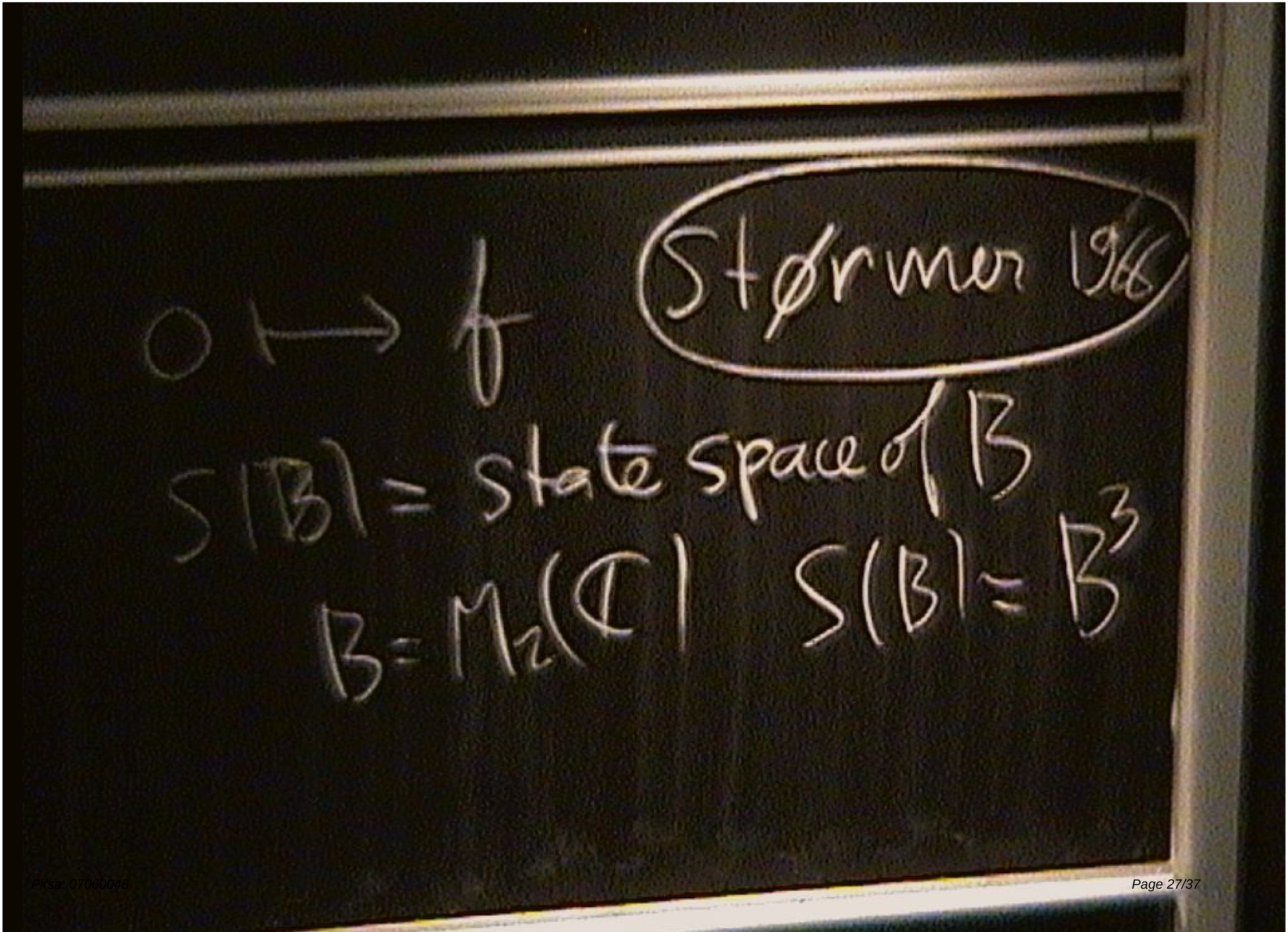
- Classical algebra $A_0^{(c)} = C(S(B))$ of macroscopic observables has dense subalgebra $C^\infty(S(B))$ with natural ***Poisson bracket.*** On affine functions this is given by $\{\hat{b}, \hat{c}\} = i\widehat{[b,c]}$, $\hat{b} : \omega \mapsto \omega(b),\, b \in B$
- Theorem: For any two continuous sections $(a_N), (b_N)$ of the field $A^{(c)}$ the section $(\{a_0, b_0\}, \ldots, iN[a_N, b_N], \ldots)$ is continuous, so that $\lim_{N\to\infty} iN[a_N, b_N] = \{a_0, b_0\}$
- Corollary: if (bounded) N-particle Hamiltonians h_N converge to classical Hamiltonian h_0 in the sense that section (h_N) is continuous, then quantum dynamics of h_N converges (in norm) to classical dynamics of h_0

Quantum De Finetti

- Quasilocal and macroscopic observables are related through ***global fields of states***: these are families of states $(\omega_N \in S(B^{\otimes N}))_{N=1,2,\ldots}$ with limits to $A_0^{(c)}$ ***and*** $A_0^{(q)}$: $\lim_{N\to\infty} \omega_N(a_N^{(c,q)}) = \omega_0^{(c,q)}(a_0^{(c,q)})$ for some $\omega_0^{(c,q)} \in S(A_0^{(c,q)})$ and all continuous sections $(a_N^{(c,q)})$ of $A_0^{(c,q)}$

- Example: ***exchangeable*** states $\omega_0^{(q)}$ on $A_0^{(q)}$, $\omega_N := \omega_0^{(q)}|B^{\otimes N}$ is permutation invariant: $\sigma_N^* \omega_N = \omega_N \, \forall N, \sigma_N \in \mathfrak{S}_N$

$$\Rightarrow \omega_0^{(q)} = \int_{S(B)} d\mu(\rho)\, \rho^{\otimes\infty} \text{ and } \omega_0^{(c)}(f) = \int_{S(B)} d\mu(\rho)\, f(\rho)$$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Randomness of QM

- Randomness in classical physics is always caused by either uncertainty in the initial state (Poincaré) or by averaging over time (Boltzmann, Einstein)
- Randomness in quantum physics is believed to be "irreducible" (having fixed a classical context)
- Can test for randomness by asking if sequence of measurement outcomes is random
- What is a good definition of random sequences?

What is a random sequence?

- Do spin-z measurements in state $|\psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ form a random sequence ? $(+\hbar/2 \equiv 1, \ -\hbar/2 \equiv 0)$
- Is the decimal expansion of π a random sequence?
- Why is a coin toss yielding 0000000000000000000000 considered suspicious (and hence non-random) whereas 1100101011000011010010 looks random? Aren't both sequences equally (im)probable?
- Many attempts from 1900 (Hilbert's 6th Problem) onwards to define probability and random sequences (Borel, von Mises); solution in 1960s in ***Algorithmic Complexity Theory*** (Kolmogorov, Chaitin, Solomonoff, Martin-Löf)

Definition of a random sequence

- ***Algorithmic complexity*** of finite sequence x is length of shortest computer program (on UTM) producing x

$$K_U(x|\ell(x)) = \min_{p:U(p,\ell(x))=x} \ell(p)$$

- Finite sequence x is ***random*** if $K_U(x|n) \geq n$
- Infinite sequence x is random if $\lim_{n\to\infty} \frac{K_U(x_1 \cdots x_n|n)}{n} = 1$
- If x is random, then it passes all possible statistical tests for randomness (but not vice versa, cf. π)
- Most random numbers cannot be proved to be random

Proof of randomness (T.B.C.)

- ***Basic idea***: Coleman & Lesniewski, Gutmann (1995) define projection $P_N : \otimes^N \mathbb{C}^2 \to \otimes^N \mathbb{C}^2$ onto span of all qubits $|x_1\rangle \cdots |x_N\rangle$ for which string $x_1 \cdots x_N$ is random and hope that "$P_\infty |\psi\rangle^{\otimes\infty} = |\psi\rangle^{\otimes\infty}$" for $|\psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$

- ***Technical execution***: sequence $(P_N)_{N=1,2,\ldots}$ has limit function $P_0 \in C(B^3)$ in continuous field $A^{(c)}$ built from C*-algebra $B = M_2(\mathbb{C})$, $P_0(\psi) := \lim_{N\to\infty} {}^{\otimes N}\langle\psi|P_N|\psi\rangle^{\otimes N} = 1$

- ***Interpretation***: $\lim_{N\to\infty} P_N$ is classical (dispersion-free) with sharp value 1 (="yes") in state $|\psi\rangle^{\otimes\infty} \equiv \psi$

Conclusion

- In large systems it is essential to control the approximation of a finite system by an infinite one: this can be done using continuous fields of C*-algebras
- Large systems have two interesting types of limiting observables (each giving rise to its own continuous field)
  - quasilocal ones (limits of local observables)
  - macroscopic ones (limits of averages), which form a commutative algebra (called for by Copenhagen)
- Formalism can be applied to repeated measurements and gives easy proofs of Born rule and randomness of QM

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