Title: Time of occurence and spacetime localization of events as observables in quantum theory

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Abstract:

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Klaus Fredenhagen
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Introduction

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Analogous problem in quantum mechanics:

Time as a quantum mechanical observable

Time of occurence observable of a given event

Time of an event in classical mechanics:
F function on phase space representing the event.
Times of occurence

$$\{t \in \mathbb{R}, F(q(t), p(t)) = 0\}$$

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Example: 1d free motion, with the event "passing through the origin"

$$x(t) = x + \frac{p}{m} t$$

i.e. associated classical time observable

$$T = -m\frac{x}{p}.$$

Quantum mechanics: Aharanov's time operator

$$T = -\frac{m}{2}(p^{-1}x + xp^{-1})$$

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with domain $\{\phi \in \mathcal{D}(\mathbb{R}), 0 \notin \text{supp}\phi\}$.
The adjoint of T has eigenfunctions

$$\psi(p) = |p|^{-\frac{1}{2}} e^{\mu|p|}(a\Theta(p) + b\Theta(-p))$$

with eigenvalues $\lambda = \frac{im}{2}\mu$. ψ is normalizable iff $\Re\mu < 0$, hence T^{**} is maximally symmetric with deficiency indices $(2, 0)$, and T has no selfadjoint extension.

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Proof:
Let $P(I)$ be a spectral projection of T for a finite interval I. Then $P(I)e^{iHt}P(I) = 0$ for large $|t|$. The Fouriertransform is analytic and vanishes outside of the spectrum of H. Thus $P(I) = 0$ or $\mathrm{sp}H = \mathbb{R}$.

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New construction: "Time of occurence" (Brunetti-Fredenhagen 2002):

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Interpretation: $B(\mathbb{R})$ measures the duration of the event

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$\frac{d}{dE}$ derivative operator with Dirichlet boundary conditions on $\partial\mathrm{spec}(H)$.

Example: Particle moving freely in 1 dimension.
Event: Particle stays in a neighbourhood of the origin.
Event represented by the projection

$$A_a\Phi(x) = \begin{cases} \Phi(x) & , \quad |x| \leq a/2 \\ 0 & , \quad \text{else} \end{cases} \tag{1}$$

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$$P(G) = \int_G d^4x \, P(x)$$

with values in the positive contractions on $\mathcal{H}_f$.

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(as representations of the translation group)
$H^+_{>2m} = \{p \in \mathbb{M}^*, p^2 > 4m^2, p_0 > 0\}$ 2 particle momentum spectrum

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("Töplitz quantization" of Minkowski space)

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NC Minkowski space: noncommuting coordinates

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with testfunctions f and Weyl operators e^{ikq} satisfying the Weyl relations

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The operators $\hat{q}^\mu$ satisfy the same commutation relations as the coordinates q^μ on a dense domain, but are not selfadjoint and cannot be exponentiated to yield the Weyl relations .

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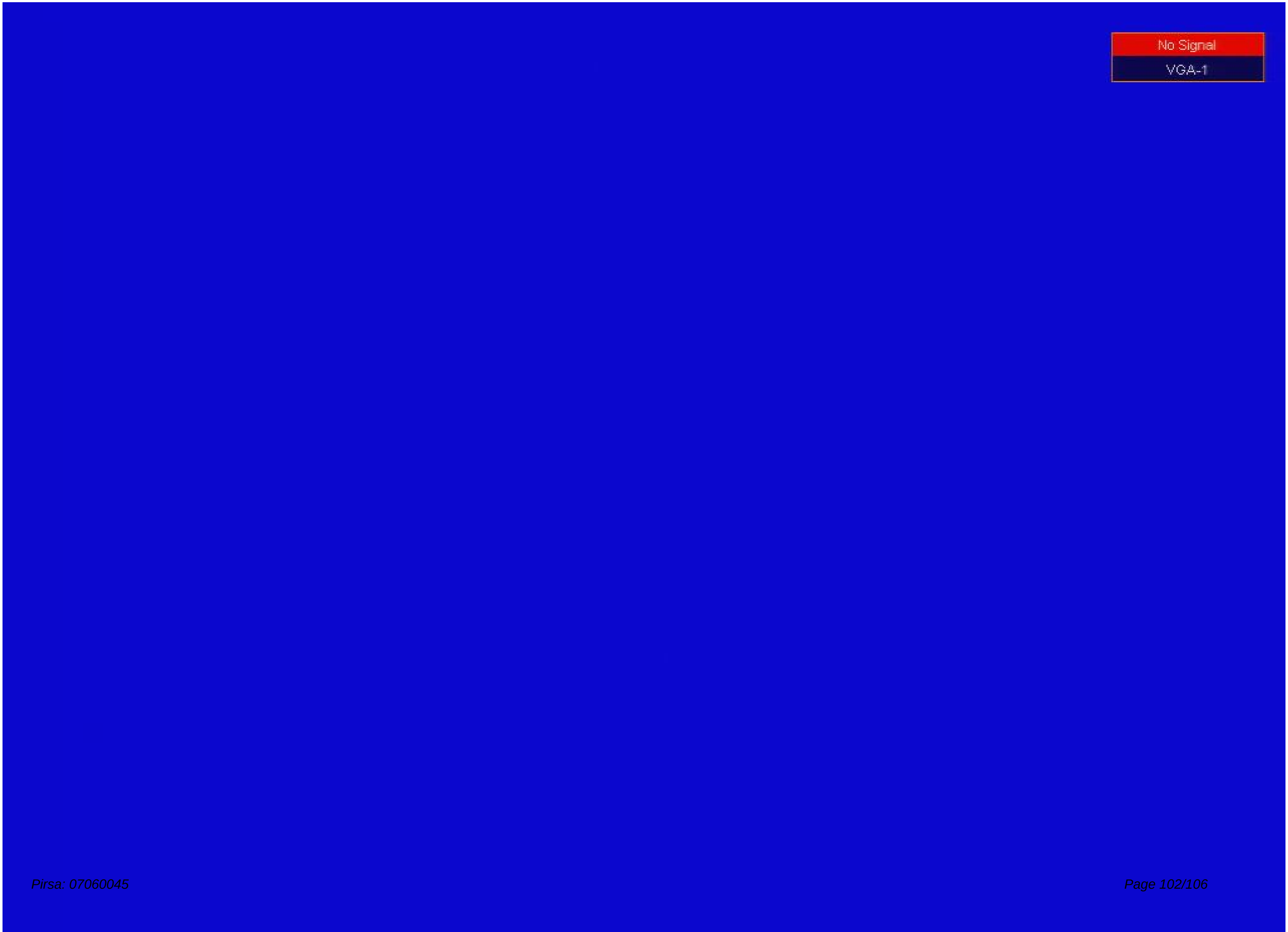
No Signal
VGA-1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- Starting from a noncommutative spacetime one obtains a deformation of the given spacetime by a completely positive map.
- In analogy to renormalization theory one may interpret parametric spacetime as bare spacetime and the observable spacetime as the physical spacetime.