Title: Quantum Mechanics with real clocks and rods

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Abstract:

Quantum mechanics with real clocks and rods.

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Environment-induced decoherence plays an important role in the solution of the measurement problem of quantum mechanics. It allows us to understand how the interaction with the environment induces a local suppression of interference between a set of preferred states, associated with the ``pointer basis''.

More precisely, a) decoherence induces a fast suppression of the interference terms of the reduced matrix describing the system coupled to the measurement device, and b) it selects a preferred set of states that are robust in spite of their interaction with the environment.

These facts are a direct consequence of the standard unitary time evolution of the total system-environment composition and therefore the global phase coherence is not destroyed but simply transferred from the system to the environment.

We have recently noticed that quantum mechanics also leads to other kinds of loss of coherence due to the quantum effects of real clocks

R. G, R. Porto, J. Pullin, New J. Phys. 6, 45 (2004) and Phys. Rev. Lett. 93, 240401 (2004)

As ordinarily formulated, quantum mechanics involves an idealization. That is, the use of a perfect classical clock to measure times.

The equations of quantum mechanics, when cast in terms of the variable that is really measured by a clock in the laboratory, differ from the traditional Schroedinger description and induce a loss of coherence.

Contrary to what happens with the environment-induced decoherence this new type of decoherence is associated to a non-unitary evolution in the physical time. The origin of the lack of unitarity is the fact that in quantum mechanics the determination of the state of a system is only possible by repeating an experiment.
If one uses a real clock, which has thermal and quantum fluctuations, each experimental run will correspond to a different value of the evolution parameter.

Due to the extreme accuracy that real clocks can reach this effect is very small, as we shall see.
The aim of this talk is to present a discussion of the interplay between these two types of decoherence and their consequences for the problem of measurements in quantum mechanics. We will also discuss the consequences of the use of real measuring rods in the determination of positions on the same problem.

OUTLINE

1) The evolution equation in terms of a real clock variable

2) Limitations to how good a clock or a measuring rod can be

3) Loss of unitarity and loss of entanglement

4) Consequences for the problem of measurement in Quantum Mechanics

The evolution equation in terms of a real clock variable

Given a physical situation of interest described by a (multi-dimensional) phase space *q, p* we start by choosing a ``clock". By this we mean a physical quantity (more precisely a set of quantities, like when one chooses a clock and a calendar to monitor periods of more than a day) that we will use to keep track of the passage of *time*.

An example of such a variable could be the angular position of the hand of an analog watch. Let us denote it by $T(q,p)$. We then identify some physical variables that we wish to study as a function of time. We shall call them generically $O(q,p)$.

We then quantize the system and work in the Heisenberg picture
Notice that we are not in any way modifying quantum mechanics. We assume that the system has a Hamiltonian evolution in terms of an external parameter t, which is a classical variable.

$$P_{T_0}(t) = \int_{T_0-\Delta T}^{T_0+\Delta T} dT \sum_k |T,k,t><T,k,t|$$

$$P_{O_0}(t) = \int_{O_0-\Delta O}^{O_0+\Delta O} dO \sum_j |O,j,t><O,j,t|$$

What is the probability that the observable *O* take a given value O_0 given that the clock indicates a certain time T_0

$$\mathcal{P}\left(O \in [O_0 - \Delta O, O_0 + \Delta O] | T \in [T_0 - \Delta T, T_0 + \Delta T]\right) = \lim_{\tau\to\infty} \frac{\int_{-\tau}^{\tau} dt \operatorname{Tr}\left(P_{O_0}(t) P_{T_0}(t) \rho P_{T_0}(t)\right)}{\int_{-\tau}^{\tau} dt \operatorname{Tr}\left(P_{T_0}(t)\rho\right)}$$

The reason for the integrals is that we do not know for what value of the external ideal time *t* the clock will take the value T_0

$$\rho = \rho_{\rm cl} \otimes \rho_{\rm sys}$$

$$U = U_{\rm cl} \otimes U_{\rm sys}.$$

Up to now we have considered the quantum states as described by a density matrix at a time *t*. Since the latter is unobservable, we would like to shift to a description where we have density matrices as functions of the observable time *T*.

Recall:

$$\mathcal{P}\left(O|t\right) \equiv \frac{\mathrm{Tr}\left(P_O(0)\rho(t)\right)}{\mathrm{Tr}\left(\rho(t)\right)}$$

$$\mathcal{P}\left(O \in [O_0 \pm \Delta O] | T \in [T_0 \pm \Delta T]\right) =$$

$$\lim_{\tau\to\infty} \frac{\int_{-\tau}^{\tau} dt\, \mathrm{Tr}\left(U_{\mathrm{sys}}(t)^\dagger P_O(0) U_{\mathrm{sys}}(t) U_{\mathrm{cl}}(t)^\dagger P_T(0) U_{\mathrm{cl}}(t) \rho_{\mathrm{sys}} \otimes \rho_{\mathrm{cl}}\right)}{\int_{-\tau}^{\tau} dt\, \mathrm{Tr}\left(P_T(t)\rho_{\mathrm{cl}}\right) \mathrm{Tr}\left(\rho_{\mathrm{sys}}\right)}$$

$$= \lim_{\tau\to\infty} \frac{\int_{-\tau}^{\tau} dt\, \mathrm{Tr}\left(U_{\mathrm{sys}}(t)^\dagger P_O(0) U_{\mathrm{sys}}(t) \rho_{\mathrm{sys}}\right) \mathrm{Tr}\left(U_{\mathrm{cl}}(t)^\dagger P_T(0) U_{\mathrm{cl}}(t) \rho_{\mathrm{cl}}\right)}{\int_{-\tau}^{\tau} dt\, \mathrm{Tr}\left(P_T(t)\rho_{\mathrm{cl}}\right) \mathrm{Tr}\left(\rho_{\mathrm{sys}}\right)}.$$

Now we may define the probability density that the resulting measurement of the clock variable takes the value T when the ideal time takes the value t :

$$\mathcal{P}_t(T) \equiv \frac{\mathrm{Tr}\left(P_T(0) U_{\mathrm{cl}}(t) \rho_{\mathrm{cl}} U_{\mathrm{cl}}(t)^\dagger\right)}{\int_{-\infty}^{\infty} dt\, \mathrm{Tr}\left(P_T(t)\rho_{\mathrm{cl}}\right)},$$

$$\int_{-\infty}^{\infty} dt \mathcal{P}_t(T) = 1.$$

$$\rho(T) \equiv \int_{-\infty}^{\infty} dt U_{\rm sys}(t) \rho_{\rm sys} U_{\rm sys}(t)^{\dagger} \mathcal{P}_t(T)$$

We have therefore ended with the standard probability expression with an "effective" density matrix in the Schrödinger picture given by $\rho(T)$

Unitarity is lost since one ends up with a density matrix that is a superposition of density matrices associated with different values of *t*

Now that we have identified what will play the role of a density matrix in terms of a "real clock" evolution, we would like to see what happens if we assume the "real clock" is behaving semi-classically.
To do this we assume that

$$\mathcal{P}_t(T) = f(T-t)$$

where *f* is a function that decays very rapidly for values of *T* far from the maximum of the probability distribution

$$f(T-t) = \delta(T-t) + a(T)\delta'(T-t) + b(T)\delta''(T-t) + \ldots$$

$$\frac{\partial\rho(T)}{\partial T} = i[\rho(T), H] + \sigma(T)[H,[H,\rho(T)]].$$

$$\sigma(T) = \partial b(T)/\partial T.$$

there would be terms involving further commutators if we had kept more terms in the expansion of *f*

Conserved quantities are automatically preserved by the modified evolution. This equation was first obtained in a different context by:
G. J. Milburn, Phys. Rev *A44*, 5401 (1991).

The extra term induces loss of coherence. In fact, if $\sigma(T)$ is a constant.

$$\rho(T)_{nm} = \rho_{nm}(0)e^{-i\omega_{nm}T}e^{-\sigma\omega_{nm}^2 T}$$

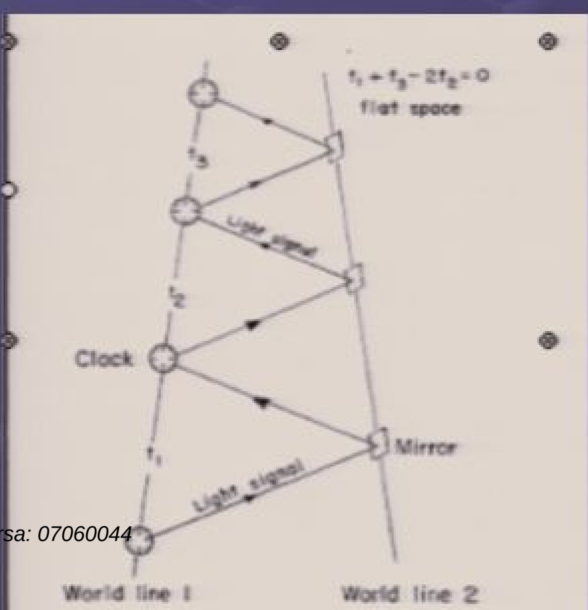
We therefore see that the off-diagonal elements of the density matrix go to zero exponentially at a rate governed by σ i.e. by how badly the clock's wavefunction spreads.

Limitations to how good a clock or a rod can be

We have established that when we study quantum mechanics with a physical clock, unitarity is lost, and pure states evolve into mixed states. The effects are more pronounced the worse the clock is.

Which raises the question: is there a fundamental limitation to how good a clock can be?
This is a contentious point: I will give three independent arguments leading to an estimate of such a limitation:

A) Salecker and Wigner (1957) and Ng and van Dam (1995)



They consider a clock consisting of two mirrors between which a light ray bounces back and forth. Every bounce is a "tick" of the clock.

They note that by the time the light bounced off a mirror and returns, the original mirror's wave-function would have spread. The width of the spread limits the accuracy of the clock

$$\delta t \approx \sqrt{\frac{t}{M}} \qquad (\hbar = c = 1)$$

So this tells us that one can build an arbitrarily accurate clock just by increasing its mass.

However, Ng and van Dam pointed out that there is a limit to this. Basically, if one piles up enough mass in a concentrated region of space one ends up with a black hole.

$$\delta T = T^{1/3} t_p^{2/3}$$

There is also a corresponding uncertainty for the measurements of lengths.

B) S.Lloyd and J. Ng Scientific American 291 52 (2004)
Giovannetti, Lloyd and Maccone Science 306 1330 (2004)

In order to map out the geometry of spacetime they fill space with clocks exchanging signals with the other clocks and measuring their time of arrival, like the GPS. We can think of this procedure as a special kind of computation. The total number of elementary measurement events of the clocks is bounded by the Margolus-Levitin theorem.

To perform an elementary logical operation in time requires an average amount of energy $E \geqslant \pi\hbar/2\Delta t.$

Therefore the maximum number of measurements is $n_{MAX} \approx \frac{Mc^2T}{\hbar}$

Again, to prevent black hole formation the total mass is bounded: within a volume of radius *l* $M \leq \frac{lc^2}{2G}$

Thus, the total number of operations or cells of space-time is bounded by $\frac{cTl}{l_P^2} \approx \frac{l^2}{l_P^2}$

And the volume of each cell is $\frac{l^3}{l^2/l_P^2} = l l_P^2$

And therefore the cells are separated by an average distance $\delta l = l^{1/3} l_P^{2/3}$

Notice that this limitation for the measurement of length and times is also related with the holographic bound if one assumes an entropy per cell of order one. This limits were obtained from heuristic considerations. Is there a derivation from first principles of these bounds? We don't have a complete theory of quantum Gravity but…

We have recently established that the kinematical structure of loop quantum gravity in spherical symmetry implies the holographic principle irrespective of the details of the dynamics. It stems from the fact that the elementary volume that any dynamical operator may involve goes as $r l_P^2$

Loss of unitarity and loss of entanglement

If the best accuracy one can get with a clock is given by

$$\delta T = T^{1/3} t_P^{2/3}$$

$$\sigma(T) = \left(\frac{t_P}{T_{\max} - T}\right)^{1/3} t_P$$

$$\rho(T)_{nm} = \rho_{nm}(0)\, e^{-i\omega_{nm} T}\, e^{-\omega_{nm}^2 T_{\rm Planck}^{4/3} T^{2/3}}.$$

So we conclude that *any* physical system that we study in the lab will suffer loss of quantum coherence at least at the rate given by the formula above. This is a fundamental inescapable limit. A pure state inevitably will become a mixed state due to the impossibility of having a perfect classical clock in nature.

The prospects for detecting the fundamental decoherence we propose are quite weak. Bose-Einstein condensates, which can have 1 million atoms in coherent states can have energy differences for which the fundamental decoherence exponents become of order unity only in times larger than the age of the universe. However LISA would be able to detect these length uncertainties.

A point that could be raised is that atomic clocks currently have an accuracy that is less than ten orders of magnitude worse than the absolute limit we derived in the previous section. Couldn't improvements in atomic clock technology actually get better than our supposed absolute limit?
This seems unlikely. When one studies in detail the most recent proposals to improve atomic clocks, they require the use of entangled states that have to remain coherent. Our effect would actually prevent the improvement of atomic clocks beyond the absolute limit!

See for instance A. Andre, A. Sorensen, M. Lukin, Phys. Rev. Lett. **92**, 230801 (2004)

Real rods and entanglement loss:

In field theory both time and spatial coordinates are ideal elements. Let us consider non relativistic electrons

$$\hat{\sigma}^z(v_1) = \int_{v_1} \hat{\psi}_a^{+}(u^i)\sigma^z_{ab}\hat{\psi}^z(u^i)\prod_i du^i$$

Measures the spin in the volume v_1 centered in a point with fiducial coordinates x_1^i

$$\hat{X}^i(x^j)$$

Quantum rods:
Assign Euclidean coordinates identifying the position of the detector

We may define conditional probabilities as in the case of real clocks:
For instance:

$$\hat{P}^z_{x_1^i}(\varepsilon_a) = \int_{v_1} du\, |u,\varepsilon_a><u,\varepsilon_a|$$

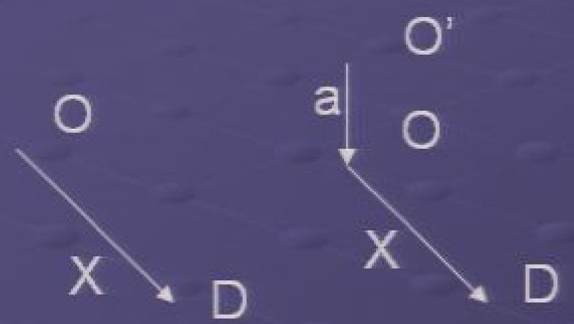
$$P(\varepsilon_a | X^j) = \int \prod_i dx_1^i \mathcal{P}_{x_1}(X^j)\mathrm{Tr}(\hat{P}^z_{x_1}(\varepsilon_a)\rho) \qquad \int \prod_i dx_1^i \mathcal{P}_{x_1}(X^j) = 1$$

Where $\mathcal{P}$ is the probability density that the measurement X^j occurs at x_1^i

Due to the limitations in the measurements of lengths $\mathcal{P}$ is *not* a Dirac delta.

One may consider that $\mathcal{P}$ is a Gaussian whose spread grows with the distance between the origin of the rods and the detector.

$$\mathcal{P}_x(X) = \left(\frac{1}{\pi D(X)}\right)^{3/2} \exp - \frac{(X-x)^2}{D(X)} \qquad \text{with } D(X) = l_p^{4/3}\, l(X)^{2/3}$$



$$P(\varepsilon_a \mid X^j) \neq P(\varepsilon_a \mid X^j + a^j)$$

In order to study the effects on entangled systems we compare Bell's Inequality violations for a given position of the detectors as measured from O and O'
Let us consider a two particle entangled state

$$|\psi_{12}> = \frac{1}{\sqrt{2 v_1 v_2}}\left(|v_1+, v_2-> + |v_1-, v_2+>\right)$$

One can check that for any set of Bell operators $\hat{Q}, \hat{R}, \hat{S}, \hat{T}.$ such that

$${}_0<\psi|\hat{Q}_x \hat{S}_y + \hat{R}_x \hat{S}_y + \hat{R}_x T_y - \hat{Q}_x \hat{T}_y|\psi>_0 \geq 2$$

There exists O' sufficiently far from O such that:

$$<\psi|\hat{Q}_{X+a}\hat{S}_{Y+a}+\hat{R}_{X+a}\hat{S}_{Y+a}+\hat{R}_{X+a}\hat{T}_{Y+a}-\hat{Q}_{X+a}\hat{T}_{Y+a}|\psi>_{O'}\leq 2$$

for

$$a^{2/3}l_p^{4/3}\approx|\vec{X}-\vec{Y}|^2$$

This immediately suggests that many entangled systems will present entanglement loss for any set of local observers.

Implications for the measurement problem of quantum mechanics.

The measurement problem in quantum mechanics is related to the fact that in ordinary quantum mechanics the measurement apparatus is assumed to be always in an eigenstate after a measurement has been performed.
The usual explanation for this is that there exists interaction with the environment. This selects a preferred basis, i.e., a particular set of quasi-classical states often referred to as ``pointer states'' that are robust, in the sense of retaining correlations over time in spite of their immersion in the environment.

Decoherence then quickly damps superpositions between the localized preferred states when only the system is considered. This is taken as an explanation of the appearance to a local observer of a ``classical'' world of determinate, ``objective'' (robust) properties.

The main problem with such a point of view is how is one to interpret the local suppression of interference in spite of the fact that the total state describing the system-environment combination retains full coherence. One may raise the question whether retention of the full coherence could ever lead to empirical conflicts with the ascription of definite values to macroscopic systems.

The usual point of view is that it would be very difficult to reconstruct the off diagonal elements of the density matrix in practical circumstances. However, at least as a matter of principle, one could indeed reconstruct such terms
The evolution of the whole system remains unitary and the coherence of the measurement device will eventually reappear (revivals) .

The fundamental decoherence induced by real clocks suppresses exponentially the off diagonal terms.

Revivals of these terms cannot occur no matter how long one waits.

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W. Zurek, Phys. Rev. **D26**, 1862 (1982).

S+A: $\{|+>, |->\}.$ E: N two level "atoms" $\{|+>_k, |->_k\}.$

The total Hamiltonian of S+A+E is

$$H_{\text{int}} = \hbar \sum_k \left(g_k \sigma_z \otimes \sigma_z^k \otimes \prod_{j \neq k} I_j \right).$$

The initial state:

$$|\Psi(0)> = (a|+> + b|->) \prod_{k=1}^{N} \otimes [\alpha_k |+>_k + \beta_k |->_k],$$

evolves into:

$$\begin{aligned} |\Psi(t)> \; &= \; a|+> \prod_{k=1}^{N} \otimes [\alpha_k \exp(i g_k t) \, |+>_k + \beta_k \exp(-i g_k t) \, |->] \\ &+ b|-> \prod_{k=1}^{N} \otimes [\alpha_k \exp(-i g_k t) \, |+>_k + \beta_k \exp(i g_k t) \, |->]. \end{aligned}$$

Taking the trace of $\rho(t) = |\Psi(t)><\Psi(t)|,$ over the environment degrees of freedom

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Taking the trace of $\rho(t) = |\Psi(t)><\Psi(t)|$, over the environment degrees of freedom

$$\rho_c(t) = |a|^2|+><+| + |b|^2|-><-| + z(t)ab^*|+><-| + z^*(t)a^*b|-><+|$$

$$z(t) = \prod_{k=1}^{N} \left[\cos\left(2g_k t\right) + i\left(|\alpha_k|^2 - |\beta_k|^2\right)\sin\left(2g_k t\right)\right].$$

If *z(t)* vanishes the reduced density matrix is a "proper mixture" representing several outcomes with its corresponding probabilities.

But *z(t)* is a multiperiodic function that will retake the initial value for sufficiently large times. (Poincare Recurrence)

Although this time is usually large, perhaps exceeding the age of the universe, at least in principle it implies that the measurement process does not correspond to a change from a pure to a mixed state in a fundamental way.

If one redoes the derivation using the effective equation we derived for quantum mechanics with real clocks one gets:

$$z'(t) = z(t)\prod_k \exp(-(2g_k)^2 t_P^{4/3} t^{2/3})$$

If one includes real clocks in quantum mechanics revivals are avoided and the pure states resulting from environment decoherence appear to be experimentally

Are there other kinds of correlations that allow us to distinguish between the reduced matrix and a "proper mixture" after the measurement?

D'Espagnat has proposed certain observables that are preserved by the unitary evolution that involve the system and the environment. Such observables would change drastically in value if the reduced density matrix were to turn into an proper mixture. In the model we are considering:

$$\hat{M} = \hat{S}_x \otimes \prod_k \otimes \hat{s}_x^k$$

If the evolution is unitary and the initial state is $|\psi(0)>$

$$<M> = (a^* b + b^* a) \prod_k (\alpha_k^* \beta_k + \beta_k^* \alpha_k)$$

While if the state is in a proper mixture $<M> = 0$

Conserved quantities are preserved by the evolution with real clocks but if positions are taken into account, the measurement of these kinds of operators will probably be impossible because of the previously mentioned fundamental limitations in the measurement of positions.

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Conserved quantities are preserved by the evolution with real clocks but if positions are taken into account, the measurement of these kinds of operators will probably be impossible because of the previously mentioned fundamental limitations in the measurement of positions.

Summarizing, when real rods and clocks are taken into account the transition from the pure states resulting from environment decoherence to mixed states seem to be totally unobservable, not only "for all practical purposes" as is usually claimed but because of reasons of principle related with the fundamental structure of spacetime.

Of course, even if the measuring device is after the measurement in a "proper mixture" , problems still persist with the interpretation of quantum mechanics.
The fact that the reduced density matrix at the end is in a diagonal form is not necessarily a completely satisfactory solution to the measurement problem.
This is known as the "and-or" problem. As Bell put it

"If one were not actually on the lookout for probabilities, ...
the obvious interpretation of even ρ' (the diagonal density matrix)
would be that the system is in a state in which various states"

$$|\Psi_1 >< \Psi_1| \quad \text{and} \quad |\Psi_2 >< \Psi_2| \quad \text{and} \quad \ldots$$

coexist.
This is not at all a *probability* interpretation, in which the different terms are seen not as *coexisting* but as *alternatives*

It is not obvious how our contribution to the problem changes anything in the discussion of this point.

Conclusions and final remarks.

1) Local observers will probably see loss of unitarity and entanglement due to the use of real clocks and rods.

2) It could be strictly impossible to distinguish between the reduced density matrix resulting from enviromental decoherence and proper mixtures.

3) If this is the case, Everett's relative state ("many worlds") interpretation, loses its compelling nature.

4) We need to study more realistic models…

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the obvious interpretation of even ρ' (the diagonal density matrix)
would be that the system is in a state in which various states"

$$|\Psi_1><\Psi_1| \quad \text{and} \quad |\Psi_2><\Psi_2| \quad \text{and} \quad \ldots$$

coexist.
This is not at all a *probability* interpretation, in which the different terms are seen not as *coexisting* but as *alternatives*

It is not obvious how our contribution to the problem changes anything in the discussion of this point.

Conclusions and final remarks.

1) Local observers will probably see loss of unitarity and entanglement due to the use of real clocks and rods.

2) It could be strictly impossible to distinguish between the reduced density matrix resulting from enviromental decoherence and proper mixtures.

3) If this is the case, Everett's relative state (“many worlds") interpretation, loses its compelling nature.

4) We need to study more realistic models…