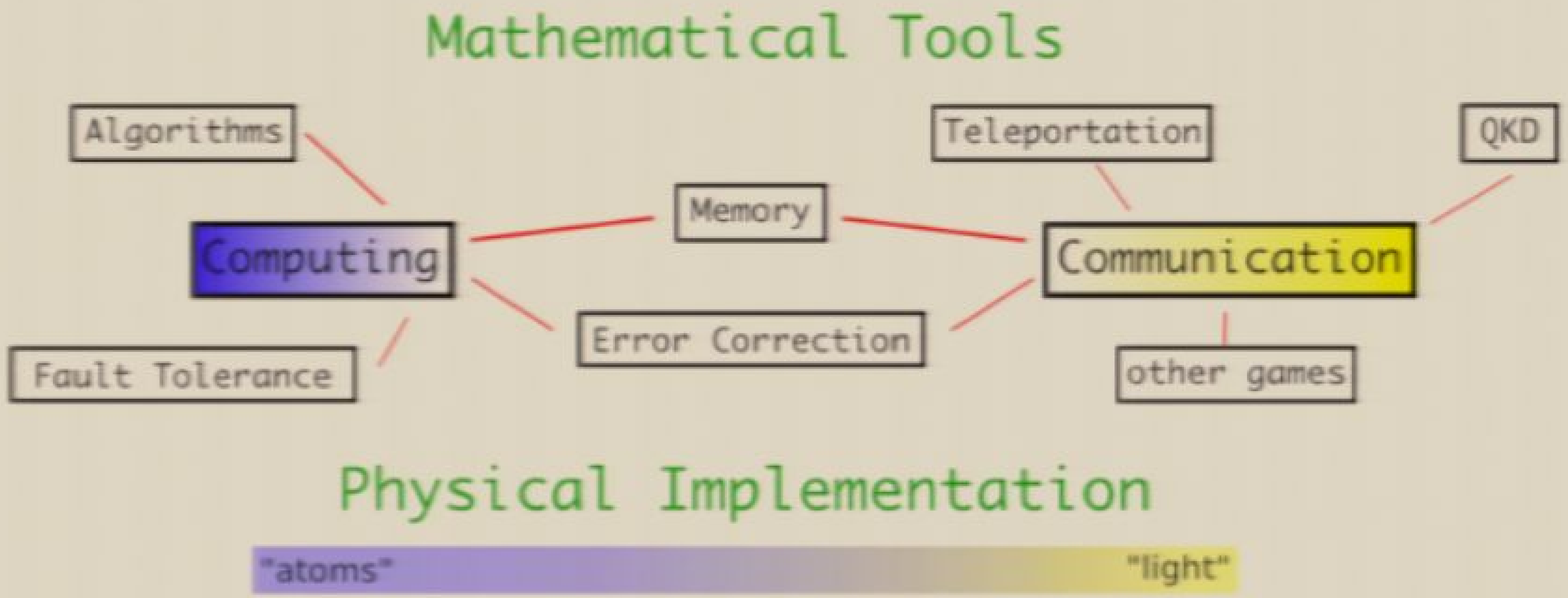
Title: Verifying Entanglement in Quantum Optical Systems

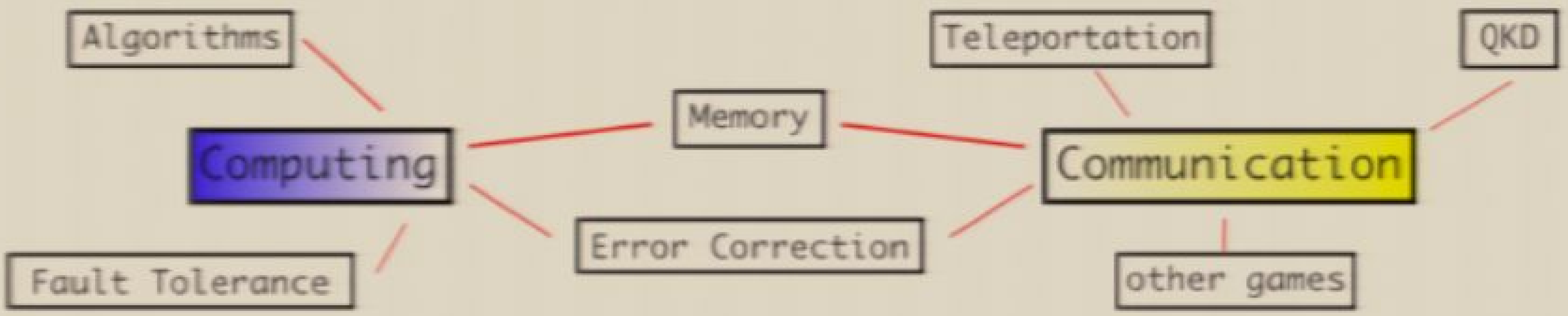
Date: Jun 05, 2007 10:50 AM

URL: http://pirsa.org/07060030

Abstract: We present an entanglement verification method for systems with underlying qubit-mode structure, which does not require full knowledge of the bi-partite density matrix. It is applied to a quantum key distribution experiment with coherent signal states and one of two different detection schemes: For heterodyne detection, it is possible to detect entanglement even in the presence of loss and noise whereas for Stokes operator measurements, entanglement verification fails.


Mathematical Tools
Algorithms
Teleportation
QKD
Memory
Computing
Communication
Fault Tolerance
Error Correction
other games
Physical Implementation
"atoms"
"light"

Mathematical Tools



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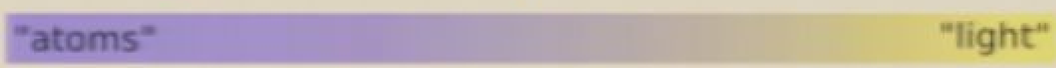

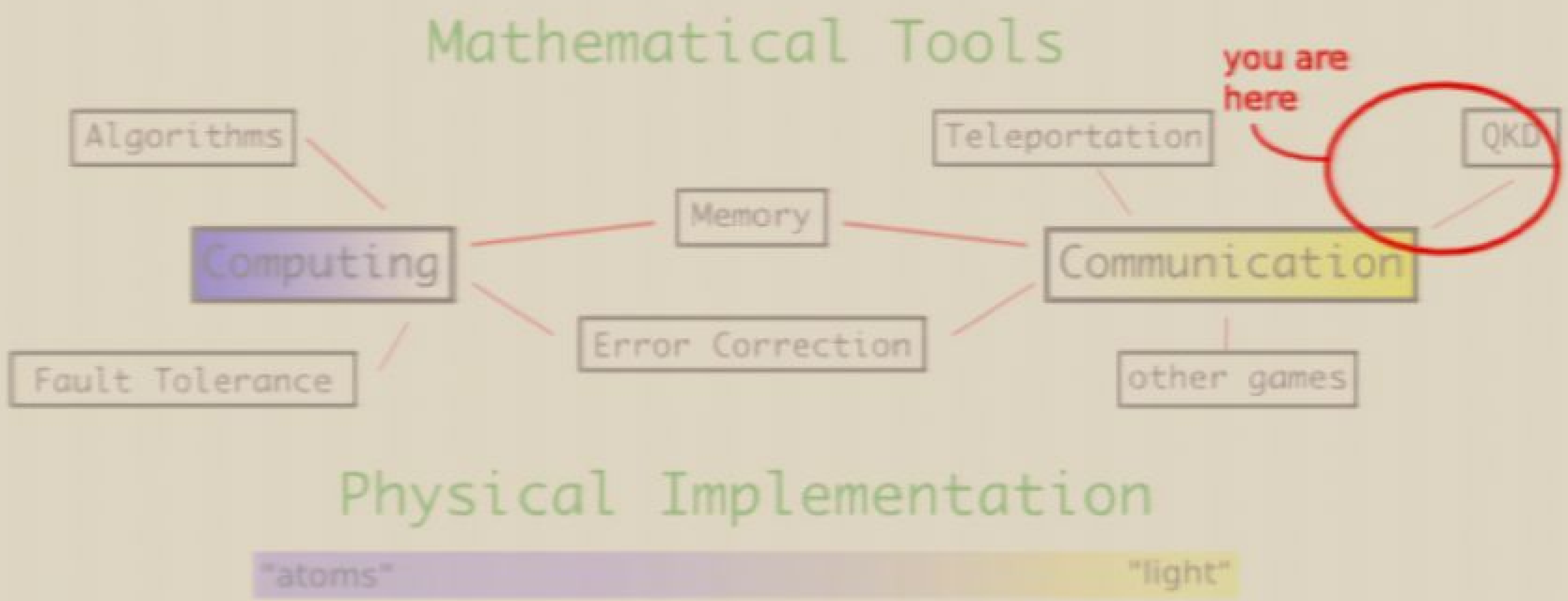
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Zoom In

Communication

Single Photons

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Continuous Variables

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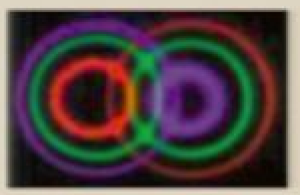
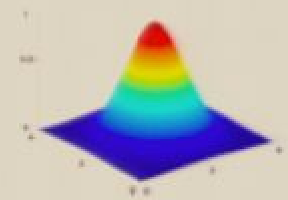
Single Photons | "light" | Continuous Variables





$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$

e.g. from downconversion
encode information in H or V polarisation
$\Rightarrow$ dim($\mathcal{H}$)=2

encode information in α
(amplitude & phase)

measure $\hat{x}$: Re(α)
$\hat{p}$: Im(α)

Zoom In

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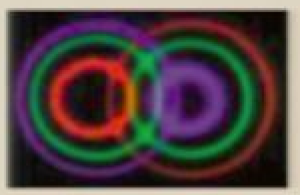

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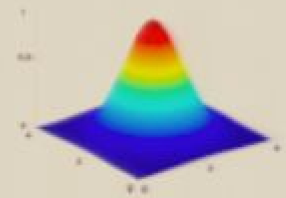

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high fidelity operations
with low probability

deterministic operations
with low fidelity

Mode-Mode and Qubit-Mode Entanglement

Hauke Häseler, Tobias Moroder, Norbert Lütkenhaus

University of Erlangen, Germany
Institute for Quantum Computing, Canada

June 5, 2007

Entanglement

The definition of entanglement is simple:

$$\text{If } \rho_{AB} = \rho_A \otimes \rho_B \rightarrow \text{separable.}$$
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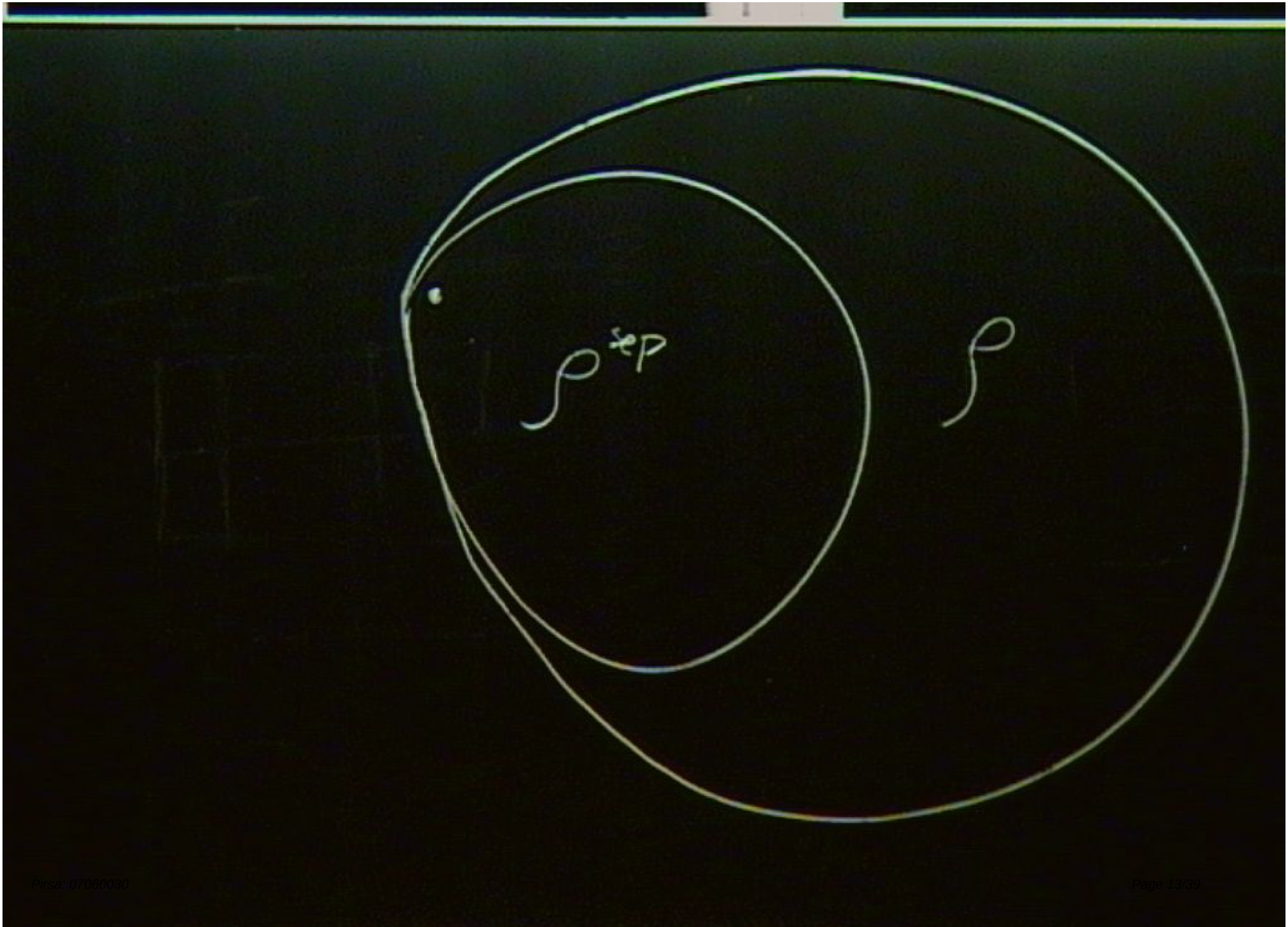
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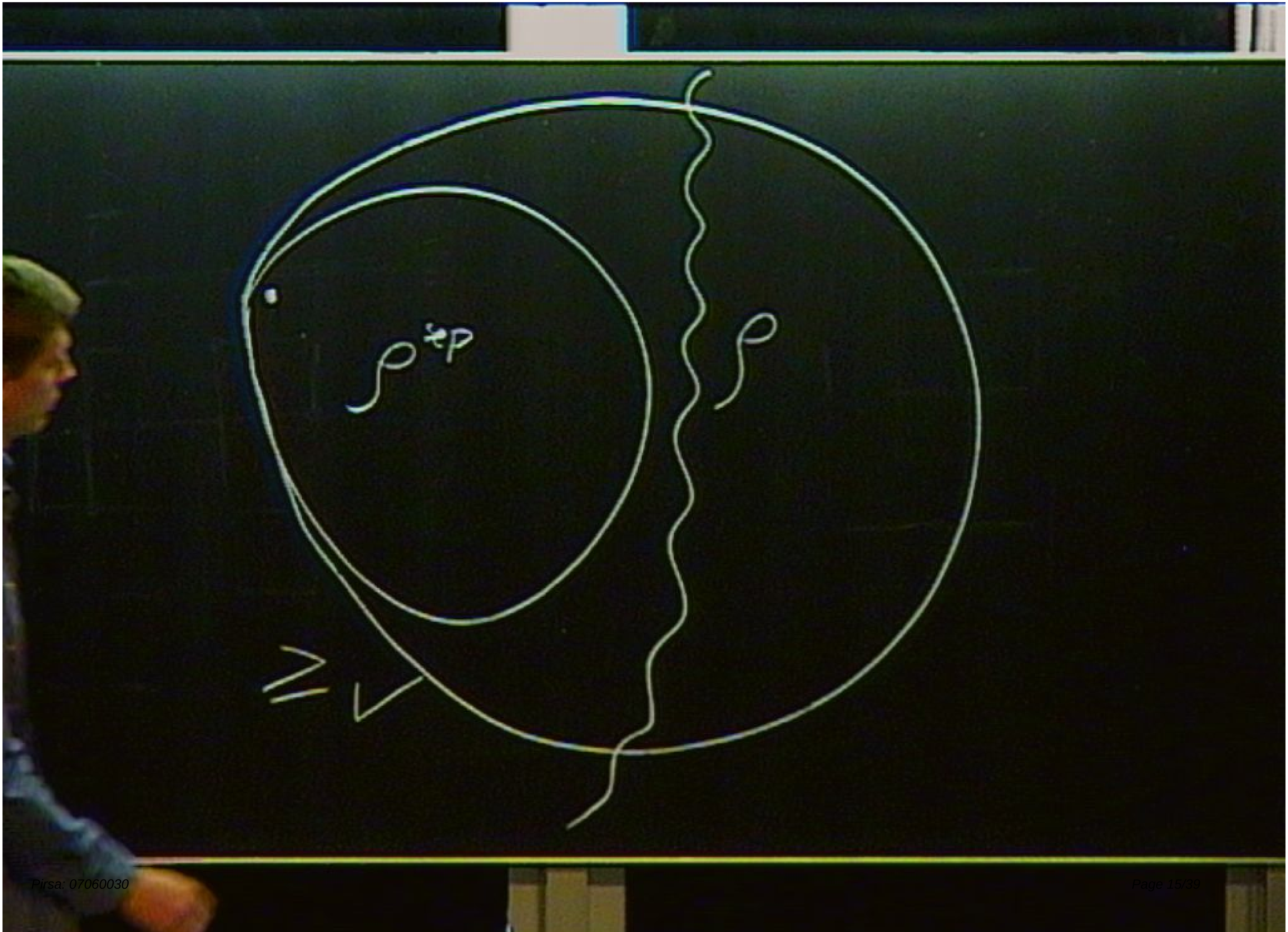
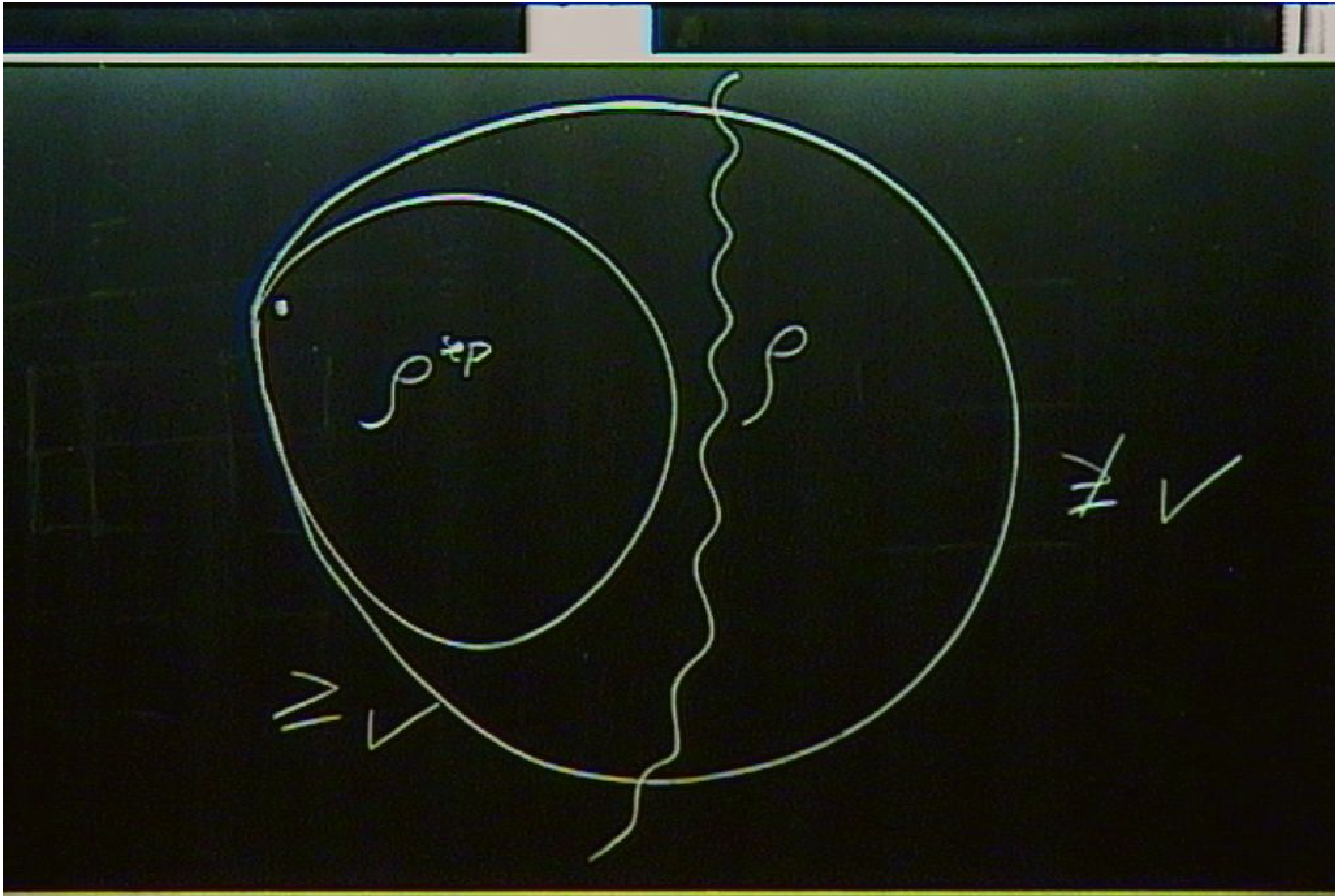
Usual technique: Suppose we measure on $\rho^{sep} \rightarrow$ find some inequality that is true $\forall$ separable states.

If violated by actual outcomes, we have shown entanglement


ρ^{rep}
ρ
$\geq$



ρ^{sep}
ρ
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$\ngeq V$

Example 1 [1]

Alice & Bob both measure $\hat{x}$ & $\hat{p}$
→ $\hat{x}_A, \hat{x}_B$ & $\hat{p}_A, \hat{p}_B$ with the usual uncertainty relation

$$\mathrm{Var}(\hat{x}_A)\mathrm{Var}(\hat{p}_A) \geq \frac{1}{4}$$

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Construct new operators

$$\hat{X} = a\hat{x}_A + \frac{1}{a}\hat{x}_B \qquad \hat{P} = a\hat{p}_A - \frac{1}{a}\hat{p}_B$$

Then calculate the sum of uncertainties:

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In Words:

Simultaneous eigenstate of $\hat{X}$ and $\hat{P} \Rightarrow$ from $\langle \hat{P}_A \rangle$, we can infer the value of $\langle \hat{P}_B \rangle$ to greater precision than the uncertainty principle allows

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Example 2 [2]

Partial Transposition

Again, start with $\rho^{sep} = \sum_i p_i \, \rho_A^i \otimes \rho_B^i \geq 0$

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- infinite size
- consider sub-determinants

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Your engraving
goes here.
iPod
(PRODUCT)
4GB

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- consider sub-determinants
- sufficient for entanglement

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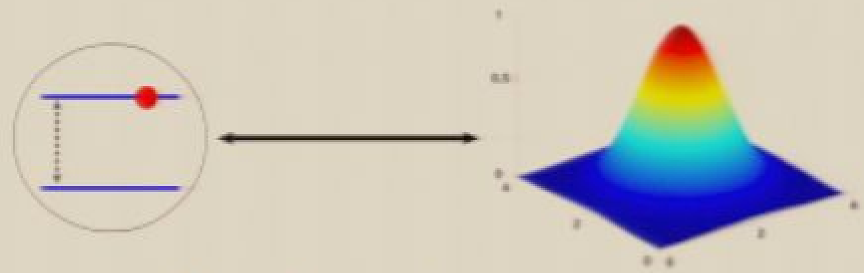
Qubit and Mode [3]



Intuition: QUBITS: correlated or anti-correlated results
MODES: Increased precision from distant measurements on non-commuting observables

[3] J. Rigas *et al.*, PRA **73** 012341 (2006)

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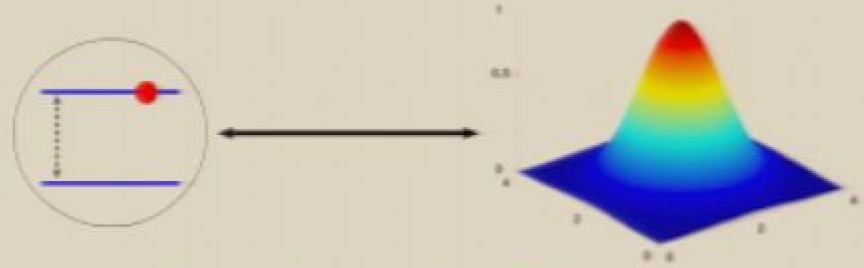

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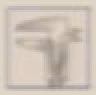
ρ_{AB} acts on $\mathcal{H}_A \otimes \mathcal{H}_B$



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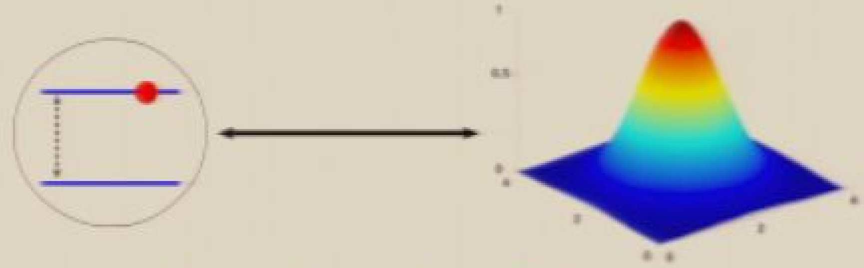

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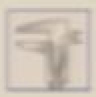

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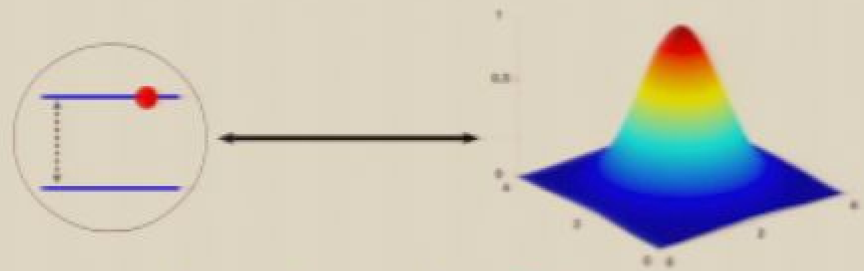

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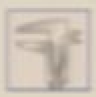

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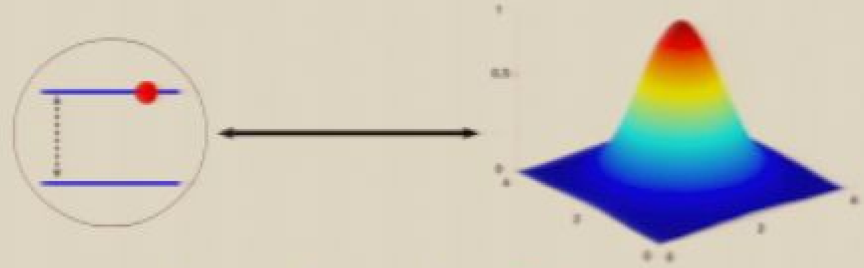

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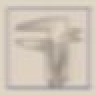

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simple, sufficient entanglement criterion

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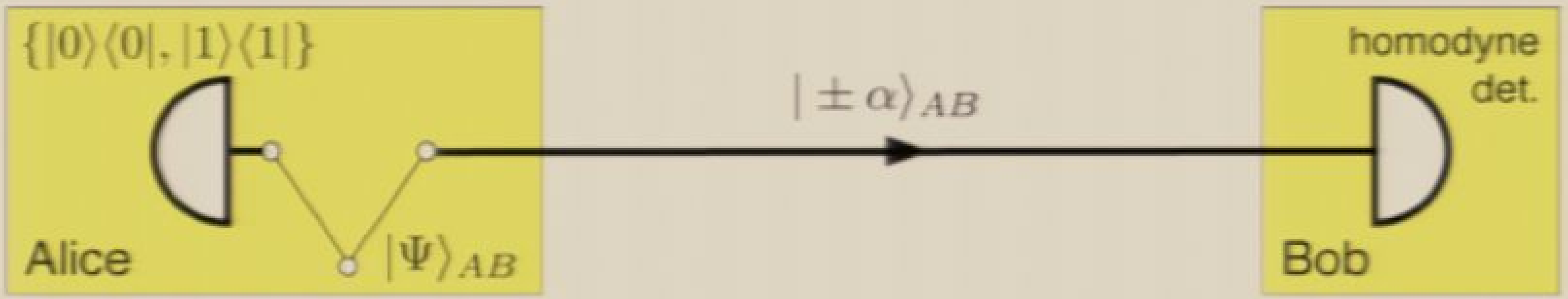
Reiterate

- Non-local measurement operators on a "qubit-mode" state
- Arrange measurement outcomes in a special matrix χ
- Rearrange according to partial transposition
- Check the positivity $\Rightarrow$ sufficient ent. criterion

Applications



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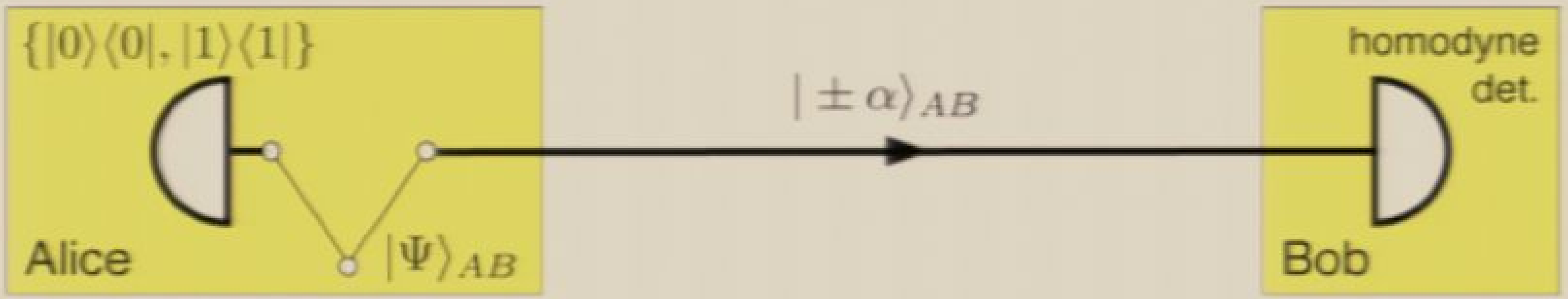

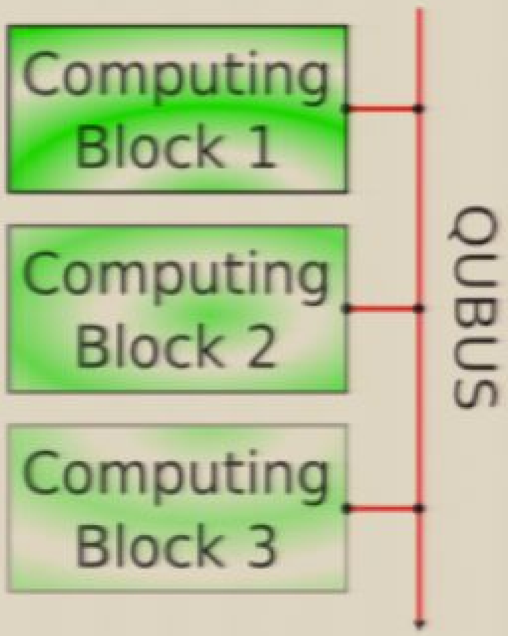

Thank you

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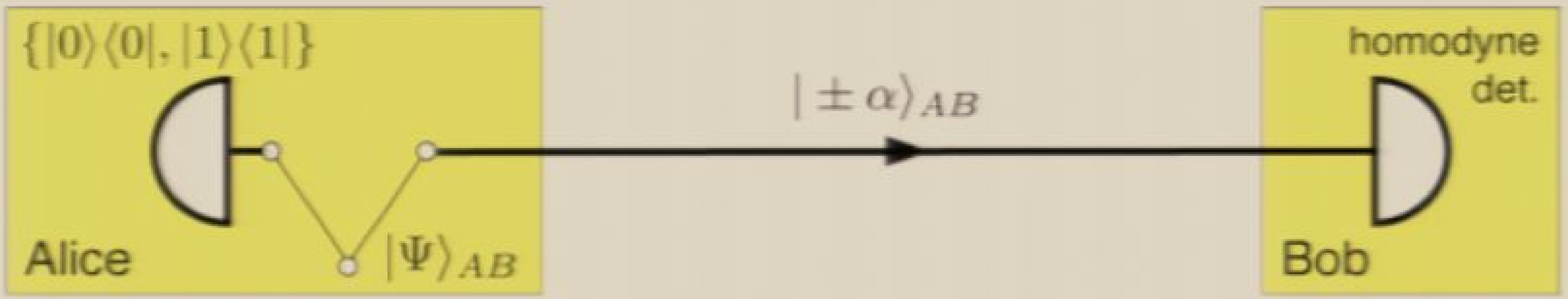

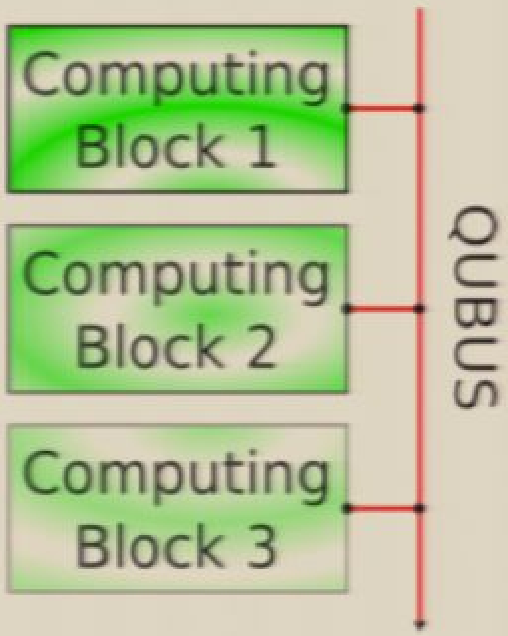

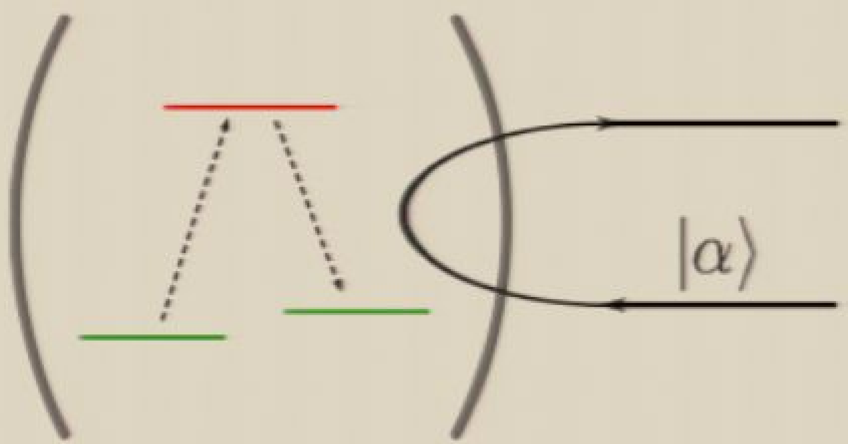

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