Title: Toward a 5-qubit solid state quantum computer

Date: Jun 04, 2007 03:50 PM

URL: http://pirsa.org/07060025

Abstract:

~~Toward a 5-qubit solid state quantum computer~~

Quantum feedback control and continuous error correction

Martin Laforest

Institute for Quantum Computing

University of Waterloo

Canadian Quantum Information Students Conference 2007

Outline

- Motivation
- Classical feedback control
- Quantum feedback control
- Discrete quantum error correction
- Continuous quantum error correction

Motivation

- For feedback control
  - Increase stability of systems
  - Increase the control
  - Suppress unwanted evolutions
- For continuous error correction
  - Generalize to unknown state
  - Incorporate less restrictive system requirements then discrete QEC.

Classical and quantum control

- Any system comes with a certain internal dynamics
- We want to:
  - Drive an initial (potentially partially unknown) state to a specific state
  - Control the dynamics of the system to effectively implement a different dynamics
- Open loop control
  - "Go blind" from start to beginning
- Closed loop control
  - Start the control
  - Acquire some information about the system
  - Modify the controller
  - And so on

Classical feedback

- Given a stochastic (Ito, Wiener) process

$$d\mathbf{x} = \mathbf{F}(\mathbf{x}, \mathbf{u})dt + \mathcal{G}(\mathbf{x}, \mathbf{u}) \cdot \mathbf{dW}$$

- The observation process is

$$d\mathbf{y} = \mathbf{H}(\mathbf{x}, t)dt + \mathcal{R}(t) \cdot \mathbf{dW}$$

- The randomness arise from Wiener increment

$$dW \in \mathcal{N}(0, dt)$$

$$\langle dW(t)dW(t')\rangle = \delta(t - t')dt$$

Classical feedback

- Choose control parameters **u** as a function of the entire history of d**y**
- The wanted behavior of the system is absorb into a cost function

$$\mathcal{C}(P(\mathbf{x}), \mathbf{u}, \mathbf{x}_f)$$

$$dP = -\sum_{i=1}^{n} \frac{\partial}{\partial x_i}(F_i P)dt + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2}{\partial x_i \partial x_j}([\mathcal{G}\mathcal{G}^T]_{ij}P)dt$$
$$+[\mathbf{H}(\mathbf{x},t) - \langle \mathbf{H}(\mathbf{x},t)\rangle]^T(\mathcal{R}\mathcal{R}^T)[\mathbf{dy} - \langle \mathbf{H}(\mathbf{x},t)\rangle dt]P$$

Kushner-Stratonovich eq. for *a posteriori* probability density

Classical feedback

- Optimal control:
  - Minimizing C
  - Very sensitive to dynamics variation
  - Need precise knowledge of the system
  - Need good noise model
- Sub optimal robust control:
  - Minimizing C over a distribution of $\mathbf{F}(\mathbf{x},\mathbf{u})$
  - Not as efficient

Quantum feedback

- Measurement will disturb the system
- But we can use weak measurement
- Can treat measurement and action of the environment on the same footing

$$\dot{\rho} = -i[H,\rho] + \mathcal{D}[A]\rho + \mathcal{D}[Q]\rho$$

$$\mathcal{D}[\mathcal{O}]\rho = \mathcal{O}\rho\mathcal{O}^\dagger - \frac{1}{2}\{\mathcal{O}^\dagger\mathcal{O},\rho\}$$

- But, we have access to the measurement record

$$d\rho_c = -i[H,\rho_c]dt + \mathcal{D}(A)\rho_c dt + \mathcal{D}(Q)\rho_c dt + \mathcal{H}[Q]\rho_c dW$$

$$\mathcal{H}[\mathcal{O}]\rho_c = \mathcal{O}\rho_c + \rho_c\mathcal{O}^\dagger - Tr[(\mathcal{O}+\mathcal{O}^\dagger)\rho_c]\rho_c$$

- Measurement process

$$dy = Tr[(Q+Q^\dagger)\rho]dt + dW$$

Quantum feedback

- Cost function

$$\mathcal{C} = \left\langle \int_0^T f(\rho_c(t), u(t), t)dt + f_f(\rho_c(T), T) \right\rangle$$

- Quantum Bellman equation

$$\mathcal{C}(t_i)^* = min \Bigg[f(\rho_c(t_i), u(t_i), t_i)\Delta t$$

$$+ \int \mathcal{C}^*(t_{i+1}) P_c(\rho(t_{i+1}) | \rho_c(t_i), u(t_i)) d\rho(t_{i+1}) \Bigg]$$

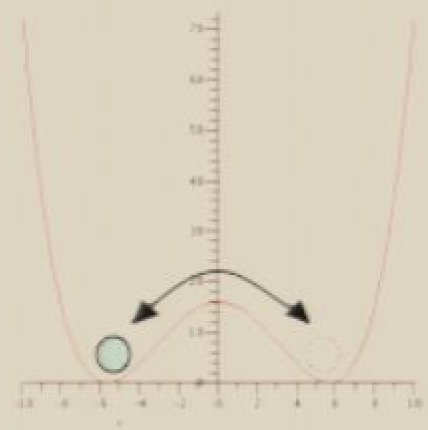
Quantum feedback

- Example with double well potential:

$$H = H_0 + H_{fb}$$

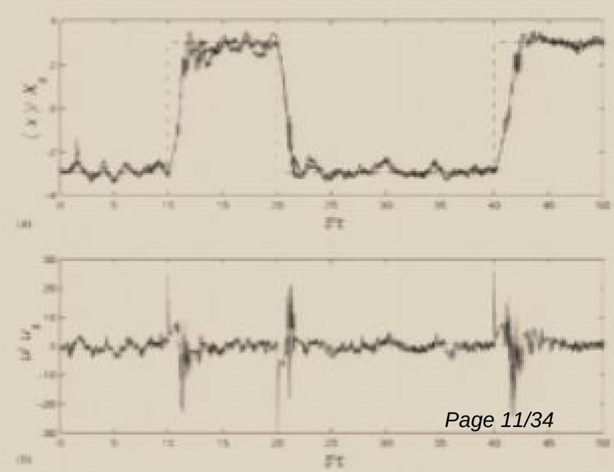
$$H_{fb} = u\hat{x},\ Q = \hat{x}$$

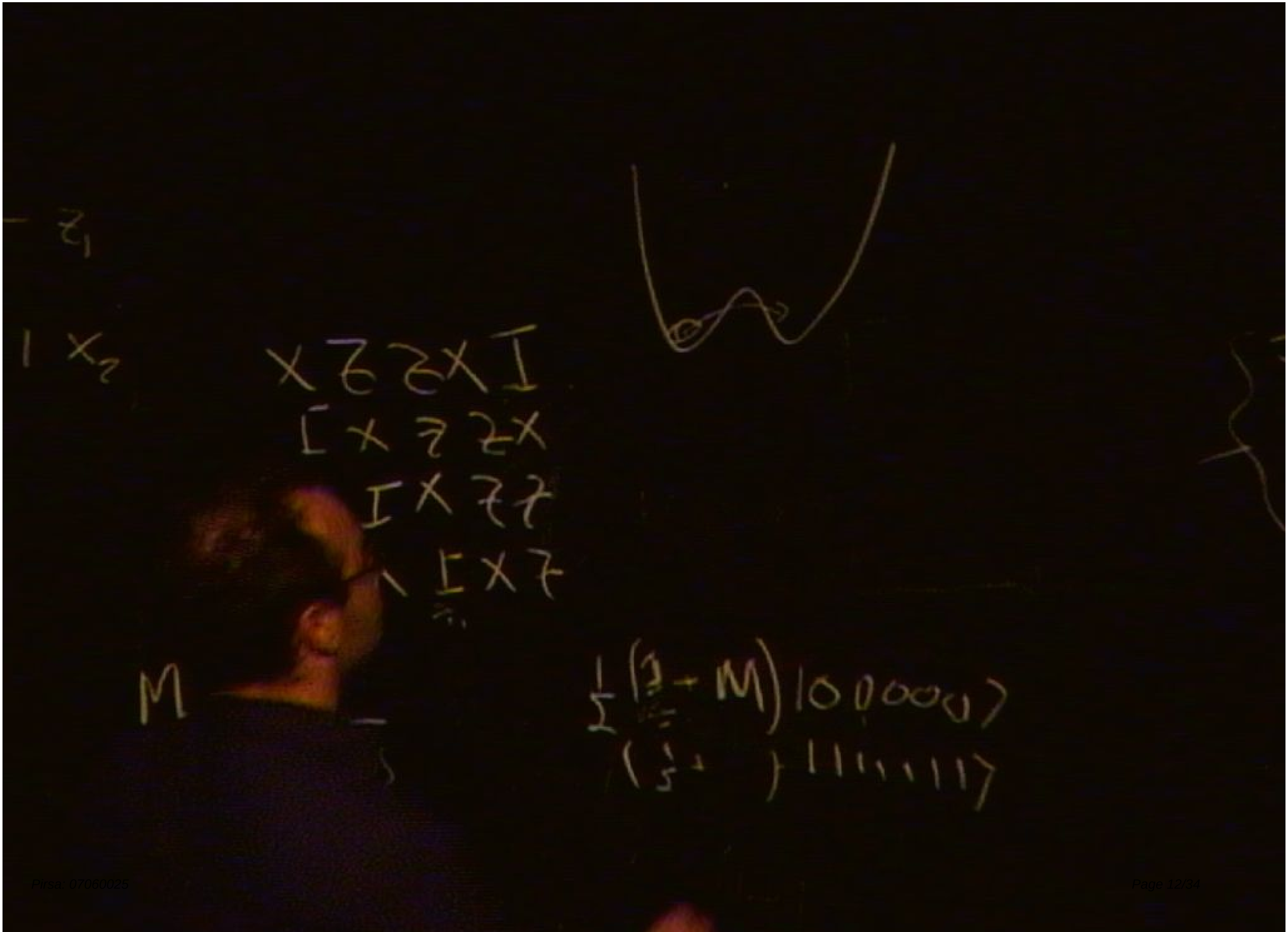
$$H_0 = \frac{1}{2}\hat{p}^2 - A\hat{x}^2 + B\hat{x}^4$$

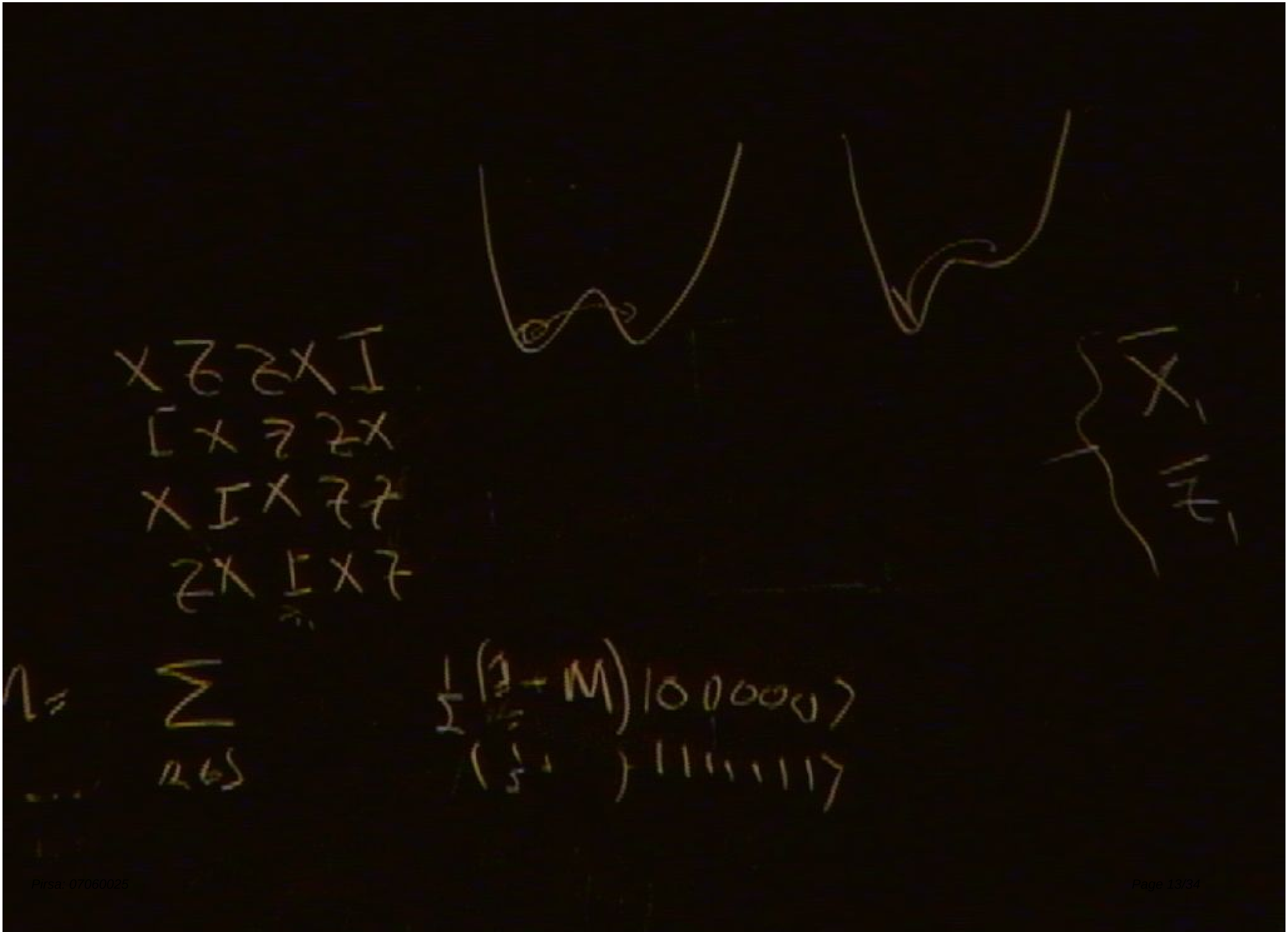


$$d\rho_c = -i[H_o + H_{fb}, \rho_c]dt + 2\beta\mathcal{D}(\hat{x})\rho_c dt +$$

$$2k\mathcal{D}(\hat{x})\rho_c dt + \sqrt{2k}\mathcal{H}[\hat{x}]\rho_c dW$$



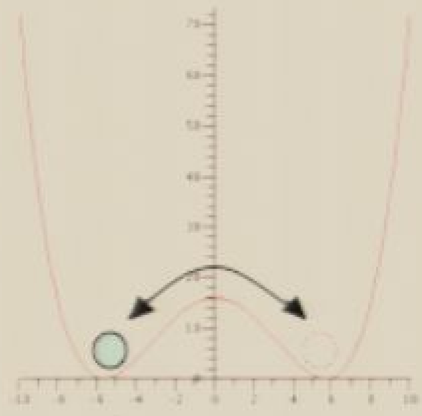

XZZXI
IXZZX
IXZZ
IXZ
M



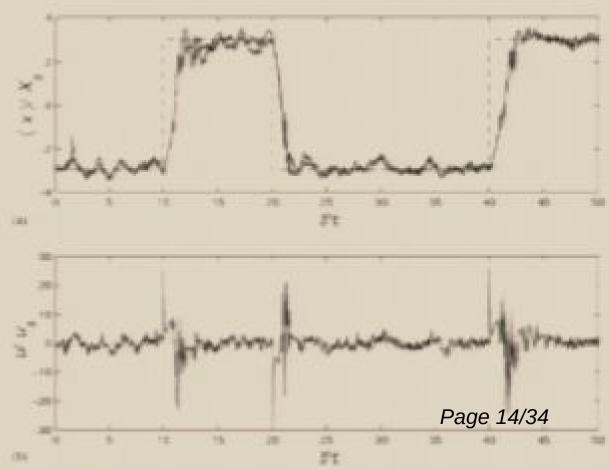
Quantum feedback

- Example with double well potential:

$$H = H_0 + H_{fb}$$
$$H_{fb} = u\hat{x},\ Q = \hat{x}$$
$$H_0 = \frac{1}{2}\hat{p}^2 - A\hat{x}^2 + B\hat{x}^4$$



$$d\rho_c = -i[H_o + H_{fb}, \rho_c]dt + 2\beta\mathcal{D}(\hat{x})\rho_c dt +$$
$$2k\mathcal{D}(\hat{x})\rho_c dt + \sqrt{2k}\mathcal{H}[\hat{x}]\rho_c dW$$



Quantum feedback

- Measurement will disturb the system
- But we can use weak measurement
- Can treat measurement and action of the environment on the same footing

$$\dot{\rho} = -i[H, \rho] + \mathcal{D}[A]\rho + \mathcal{D}[Q]\rho$$

$$\mathcal{D}[\mathcal{O}]\rho = \mathcal{O}\rho\mathcal{O}^\dagger - \frac{1}{2}\{\mathcal{O}^\dagger\mathcal{O}, \rho\}$$

- But, we have access to the measurement record

$$d\rho_c = -i[H, \rho_c]dt + \mathcal{D}(A)\rho_c dt + \mathcal{D}(Q)\rho_c dt + \mathcal{H}[Q]\rho_c dW$$

$$\mathcal{H}[\mathcal{O}]\rho_c = \mathcal{O}\rho_c + \rho_c\mathcal{O}^\dagger - Tr[(\mathcal{O} + \mathcal{O}^\dagger)\rho_c]\rho_c$$

- Measurement process

$$dy = Tr[(Q + Q^\dagger)\rho]dt + dW$$

Quantum feedback

- Measurement will disturb the system
- But we can use weak measurement
- Can treat measurement and action of the environment on the same footing

$$\dot{\rho} = -i[H,\rho] + \mathcal{D}[A]\rho + \mathcal{D}[Q]\rho$$

$$\mathcal{D}[\mathcal{O}]\rho = \mathcal{O}\rho\mathcal{O}^\dagger - \frac{1}{2}\{\mathcal{O}^\dagger\mathcal{O},\rho\}$$

- But, we have access to the measurement record

$$d\rho_c = -i[H,\rho_c]dt + \mathcal{D}(A)\rho_c dt + \mathcal{D}(Q)\rho_c dt + \mathcal{H}[Q]\rho_c dW$$

$$\mathcal{H}[\mathcal{O}]\rho_c = \mathcal{O}\rho_c + \rho_c\mathcal{O}^\dagger - Tr[(\mathcal{O}+\mathcal{O}^\dagger)\rho_c]\rho_c$$

- Measurement process

$$dy = Tr[(Q+Q^\dagger)\rho]dt + dW$$

Quantum feedback

- Cost function

$$\mathcal{C} = \left\langle \int_0^T f(\rho_c(t), u(t), t)dt + f_f(\rho_c(T), T) \right\rangle$$

- Quantum Bellman equation

$$\mathcal{C}(t_i)^* = min \left[f(\rho_c(t_i), u(t_i), t_i)\Delta t \right.$$

$$\left. + \int \mathcal{C}^*(t_{i+1}) P_c(\rho(t_{i+1}) | \rho_c(t_i), u(t_i)) d\rho(t_{i+1}) \right]$$

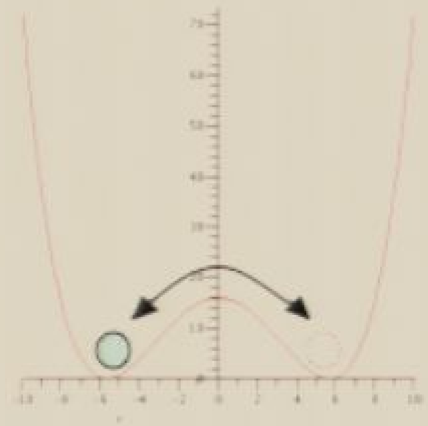
Quantum feedback

- Example with double well potential:

$$H = H_0 + H_{fb}$$

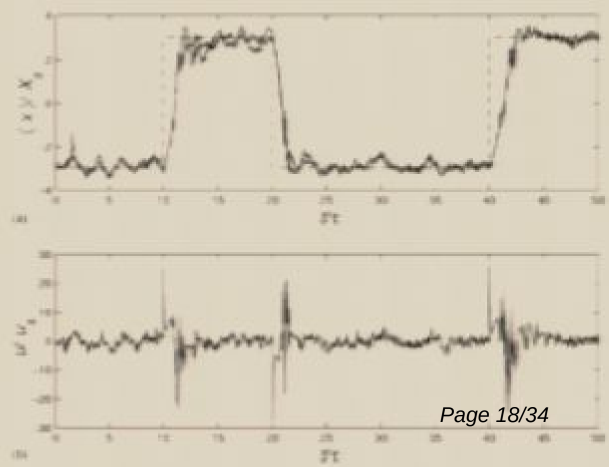
$$H_{fb} = u\hat{x},\ Q = \hat{x}$$

$$H_0 = \frac{1}{2}\hat{p}^2 - A\hat{x}^2 + B\hat{x}^4$$



$$d\rho_c = -i[H_o + H_{fb}, \rho_c]dt + 2\beta\mathcal{D}(\hat{x})\rho_c dt +$$

$$2k\mathcal{D}(\hat{x})\rho_c dt + \sqrt{2k}\mathcal{H}[\hat{x}]\rho_c dW$$



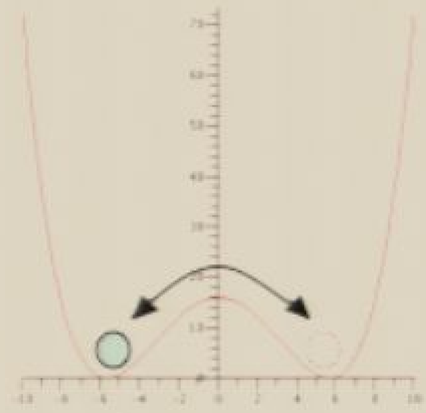
Quantum error correction

- Software solution to combat decoherence and errors
- Encode state of a system in a bigger system
- Measure only information about the error
  - Do not need to know the state of the system
- Recover encoded state through conditional unitary correction

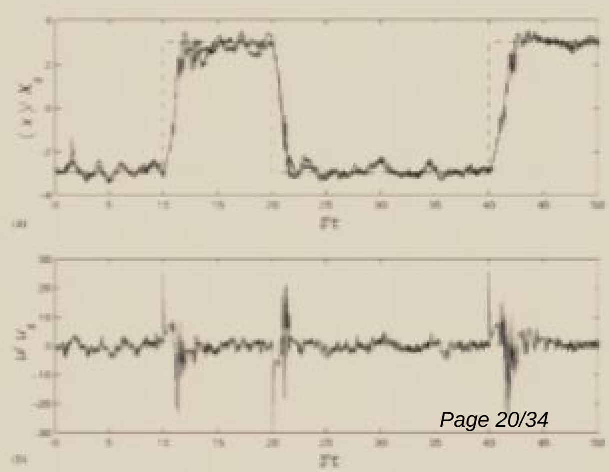
Quantum feedback

- Example with double well potential:

$$H = H_0 + H_{fb}$$
$$H_{fb} = u\hat{x},\ Q = \hat{x}$$
$$H_0 = \frac{1}{2}\hat{p}^2 - A\hat{x}^2 + B\hat{x}^4$$



$$d\rho_c = -i[H_o + H_{fb}, \rho_c]dt + 2\beta\mathcal{D}(\hat{x})\rho_c dt +$$
$$2k\mathcal{D}(\hat{x})\rho_c dt + \sqrt{2k}\mathcal{H}[\hat{x}]\rho_c dW$$



Quantum error correction

- Software solution to combat decoherence and errors
- Encode state of a system in a bigger system
- Measure only information about the error
  - Do not need to know the state of the system
- Recover encoded state through conditional unitary correction

Quantum error correction

- Let's use the qubit language (2-level system)
- Can correct 1 qubit using 3 against bit flip error X, e.g.

$$|0\rangle \leftrightarrow |1\rangle$$

- Use the redundancy code:

$$|\bar{0}\rangle \rightarrow |000\rangle,\ |\bar{1}\rangle \rightarrow |111\rangle$$

- The error X_1, X_2, X_3 can affect the state
- Do two two qubit measurements Z_1Z_2 and Z_2Z_3

$$(1,1) \rightarrow \mathbb{1},\ (-1,1) \rightarrow X_1$$

$$(1,-1) \rightarrow X_3,\ (-1,-1) \rightarrow X_2$$

Quantum error correction

- Require strong, error free projective measurement
- Assume fast, discrete, error-free recovery operation

- We will relax those two assumptions
  - Weak noisy measurement only
  - Slow, continuous recovery

$$
\begin{aligned}
d\rho_c = &\gamma(\mathcal{D}[X_1] + \mathcal{D}[X_2] + \mathcal{D}[X_3])\rho_c dt \\
&+\kappa(\mathcal{D}[Z_1 Z_2] + \mathcal{D}[Z_2 Z_3] + \mathcal{D}[Z_1 Z_3])\rho_c dt \\
&+\sqrt{\kappa}(\mathcal{H}[Z_1 Z_2]dW_1 + \mathcal{H}[Z_2 Z_3]dW_2 + \mathcal{H}[Z_1 Z_3]dW_3)\rho_c \\
&-i[u_1 X_1 + u_2 X_2 + u_3 X_3, \rho_c]dt
\end{aligned}
$$

$$dQ_1 = 2\kappa\langle Z_1 Z_2\rangle_c dt + \sqrt{\kappa}dW_1 \qquad u_1 = \frac{u}{8}(1 - \langle Z_1 Z_2\rangle_c)(1 - \langle Z_2 Z_3\rangle_c)(1 - \langle Z_1 Z_3\rangle_c)$$

$$dQ_2 = 2\kappa\langle Z_2 Z_3\rangle_c dt + \sqrt{\kappa}dW_2 \qquad u_2 = \frac{u}{8}(1 - \langle Z_1 Z_2\rangle_c)(1 - \langle Z_2 Z_3\rangle_c)(1 + \langle Z_1 Z_3\rangle_c)$$

$$dQ_3 = 2\kappa\langle Z_1 Z_3\rangle_c dt + \sqrt{\kappa}dW_3 \qquad u_3 = \frac{u}{8}(1 + \langle Z_1 Z_2\rangle_c)(1 - \langle Z_2 Z_3\rangle_c)(1 - \langle Z_1 Z_3\rangle_c)$$

Quantum error correction

- No optimal strategy have been used
- If we don't do the measurement, there is still feedback
- We can introduce a cost function that minimize the component of the system outside the encoded subspace

$$\mathcal{C} = 1 - Tr(\Pi_C \rho_c)$$

$$\Pi_C = |\bar{0}\rangle\langle\bar{0}| + |\bar{1}\rangle\langle\bar{1}| = \frac{1}{4}(\mathbb{1} + Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3)$$

- We can use the SME to derive

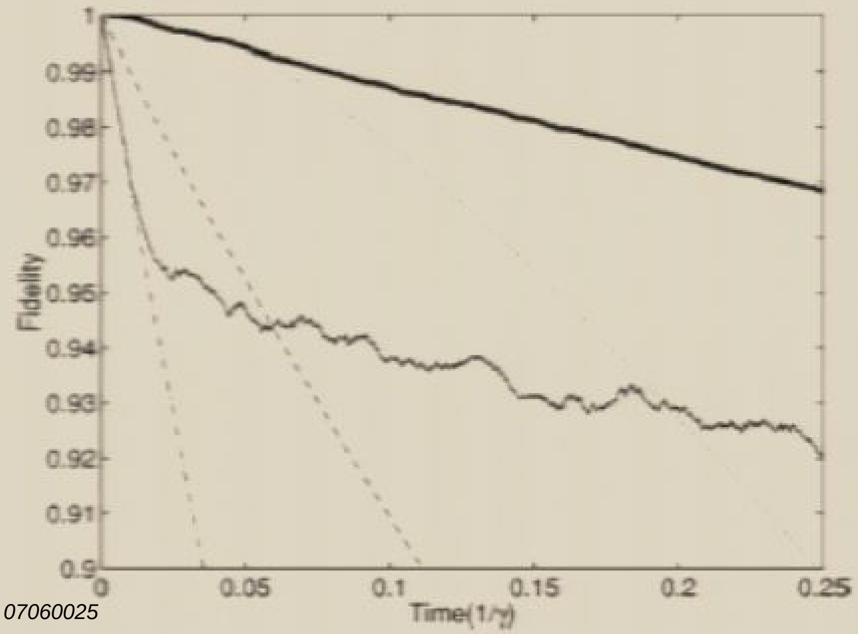
$$\frac{d}{dt}\mathcal{C} = 1 - 2u_1\langle Y_1 Z_2 + Y_1 Z_3\rangle_c + 2u_2\langle Z_1 Y_2 + Y_2 Z_3\rangle_c + 2u_3\langle Z_1 Y_3 + Z_2 Y_3\rangle_c$$

- This is minimized by

$$u_1 = u\, sgn\langle Y_1 Z_2 + Y_1 Z_3\rangle_c$$
$$u_2 = u\, sgn\langle Z_1 Y_2 + Y_2 Z_3\rangle_c$$
$$u_3 = u\, sgn\langle Z_1 Y_3 + Z_2 Y_3\rangle$$

Quantum error correction

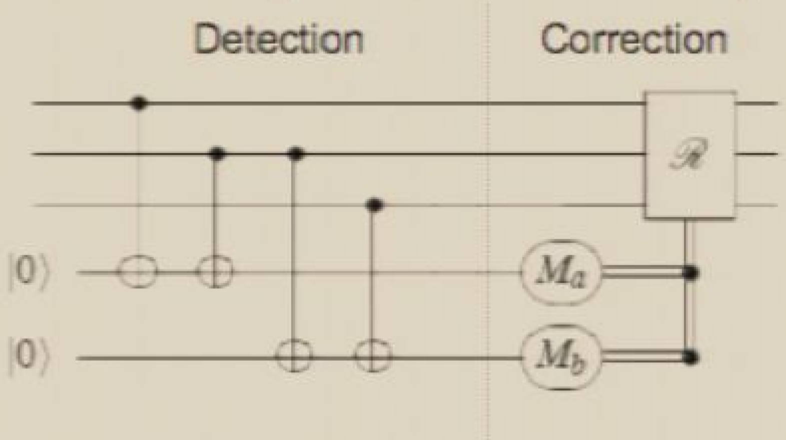
- We need to integrate the SME to determine u_i
- So we need $\rho_c(0)$, which is unknown
- Can be shown that the feedback solution in independent of initial state
- We can use $\rho_c(0)=|000\rangle\langle 000| + |111\rangle\langle 111|$

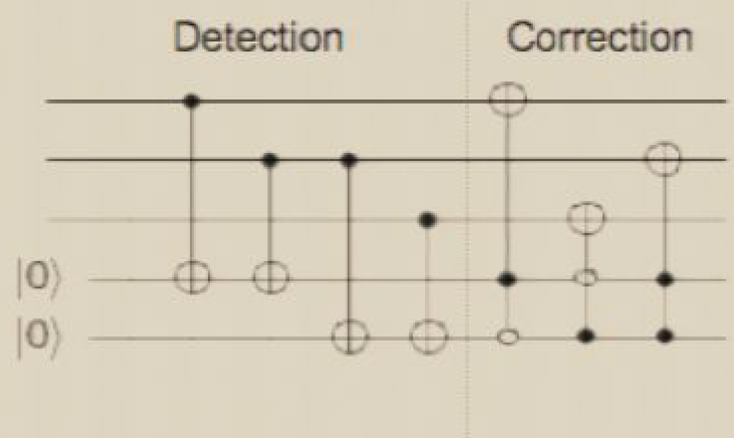


Results looks a little nicer then they really are!

Quantum error correction

- So far, we use classical information to classically tune our dynamics.
- Can we do it using quantum information and coherent feedback?
- Let's modify the measure step





- The aim is to do both *at the same time*

$$H = H_C + H_D + i[H_D, H_C]$$

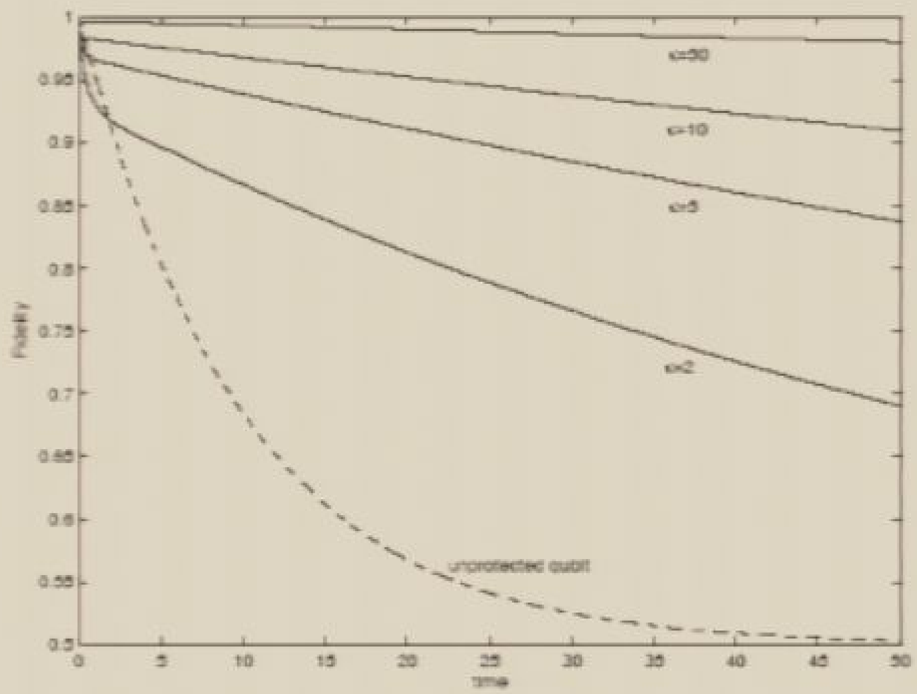
$$H_C = |00101\rangle\langle 00100| + |11001\rangle\langle 11000| + |10010\rangle\langle 100000|$$
$$+|01110\rangle\langle 01100| + |01011\rangle\langle 01000| + |10111\rangle\langle 10100| + h.c.$$

$$H_D = |00001\rangle\langle 00101| + |11101\rangle\langle 11001| + |00010\rangle\langle 10010|$$
$$+|11110\rangle\langle 01110| + |00011\rangle\langle 01011| + |11111\rangle\langle 10111| + h.c.$$

Quantum error correction

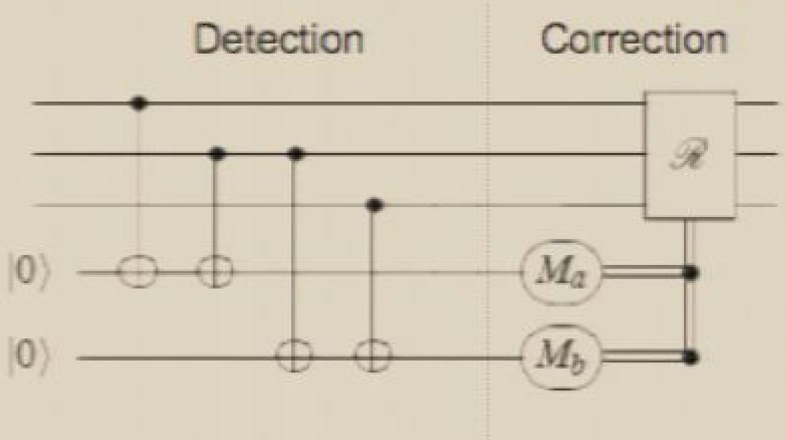
- We know have the desired dynamic

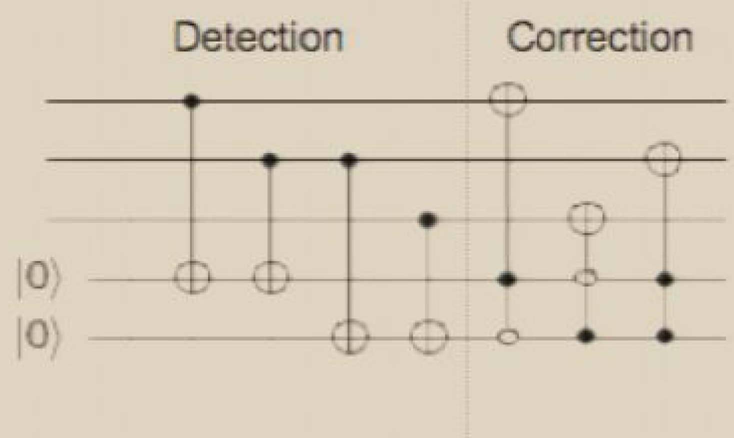
$$\frac{d\rho}{dt} = \gamma(\mathcal{D}[X_1] + \mathcal{D}[X_2] + \mathcal{D}[X_3])\rho$$
$$+\lambda(\mathcal{D}[I_4^-] + \mathcal{D}[I_5^-])\rho$$
$$-i\kappa[H, \rho]$$



Quantum error correction

- So far, we use classical information to classically tune our dynamics.
- Can we do it using quantum information and coherent feedback?
- Let's modify the measure step





- The aim is to do both *at the same time*

$$H = H_C + H_D + i[H_D, H_C]$$

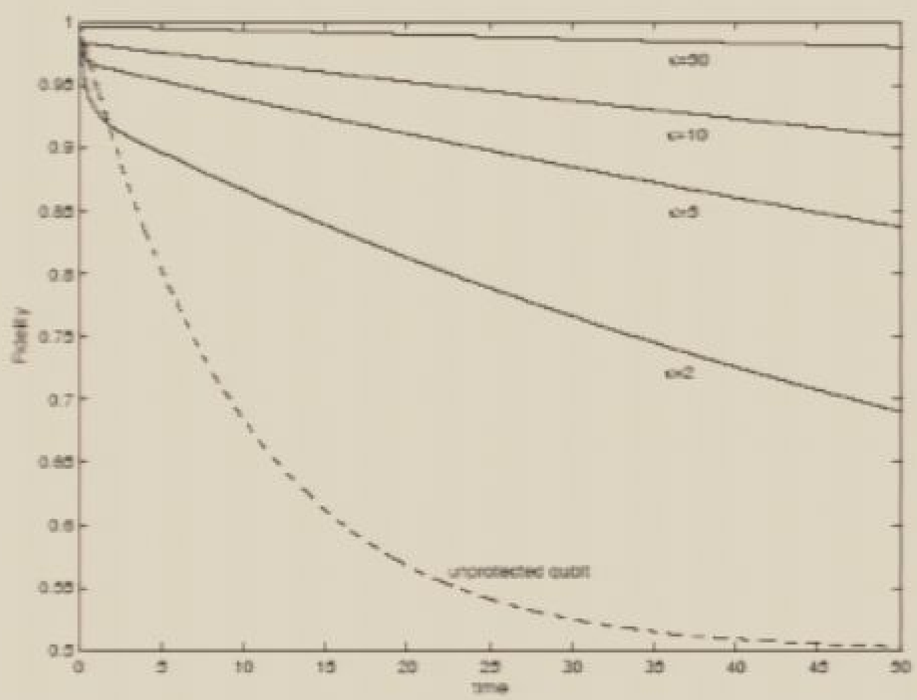
$$H_C = |00101\rangle\langle 00100| + |11001\rangle\langle 11000| + |10010\rangle\langle 100000| + |01110\rangle\langle 01100| + |01011\rangle\langle 01000| + |10111\rangle\langle 10100| + h.c.$$

$$H_D = |00001\rangle\langle 00101| + |11101\rangle\langle 11001| + |00010\rangle\langle 10010| + |11110\rangle\langle 01110| + |00011\rangle\langle 01011| + |11111\rangle\langle 10111| + h.c.$$

Quantum error correction

- We know have the desired dynamic

$$\frac{d\rho}{dt} = \gamma(\mathcal{D}[X_1] + \mathcal{D}[X_2] + \mathcal{D}[X_3])\rho$$
$$+\lambda(\mathcal{D}[I_4^-] + \mathcal{D}[I_5^-])\rho$$
$$-i\kappa[H, \rho]$$



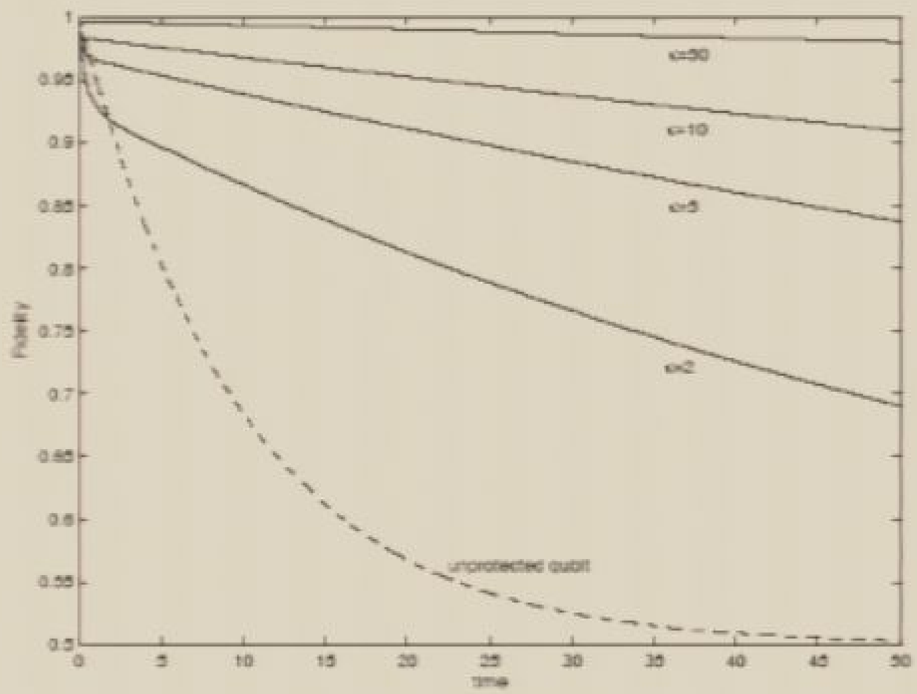
Conclusion

- Quantum feedback actively fight the dynamics of a system to bring a known initial state to a final state
- Quantum error correction fight erros on an unknown state in a known subspace
- Both ideas are complementary and can be used together
- What is the complexity overhead for bigger codes

Quantum error correction

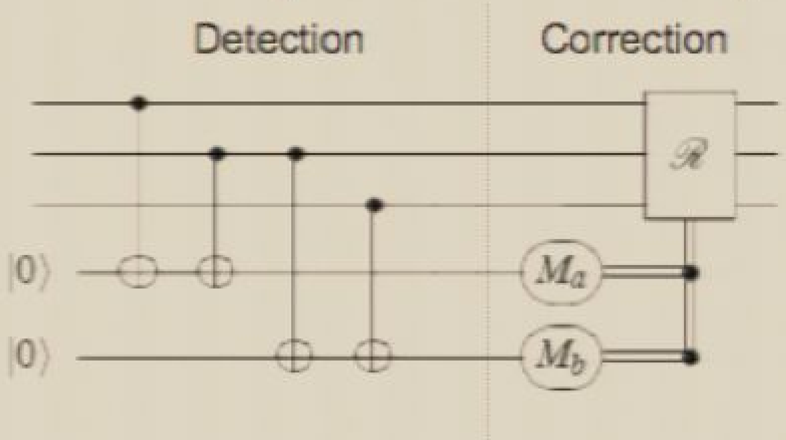
- We know have the desired dynamic

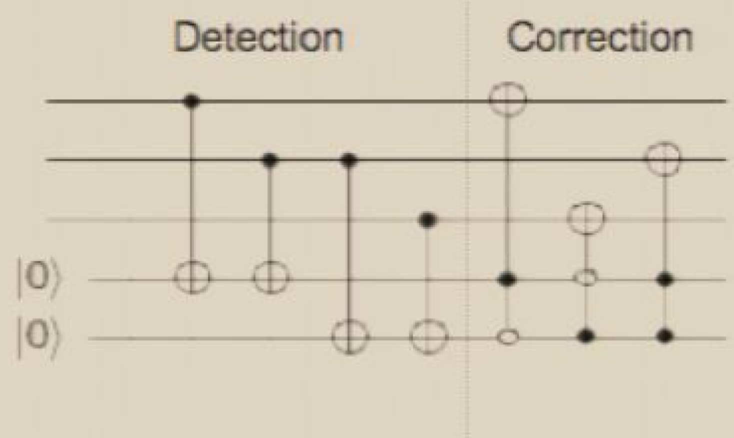
$$\frac{d\rho}{dt} = \gamma(\mathcal{D}[X_1] + \mathcal{D}[X_2] + \mathcal{D}[X_3])\rho$$
$$+\lambda(\mathcal{D}[I_4^-] + \mathcal{D}[I_5^-])\rho$$
$$-i\kappa[H, \rho]$$



Quantum error correction

- So far, we use classical information to classically tune our dynamics.
- Can we do it using quantum information and coherent feedback?
- Let's modify the measure step



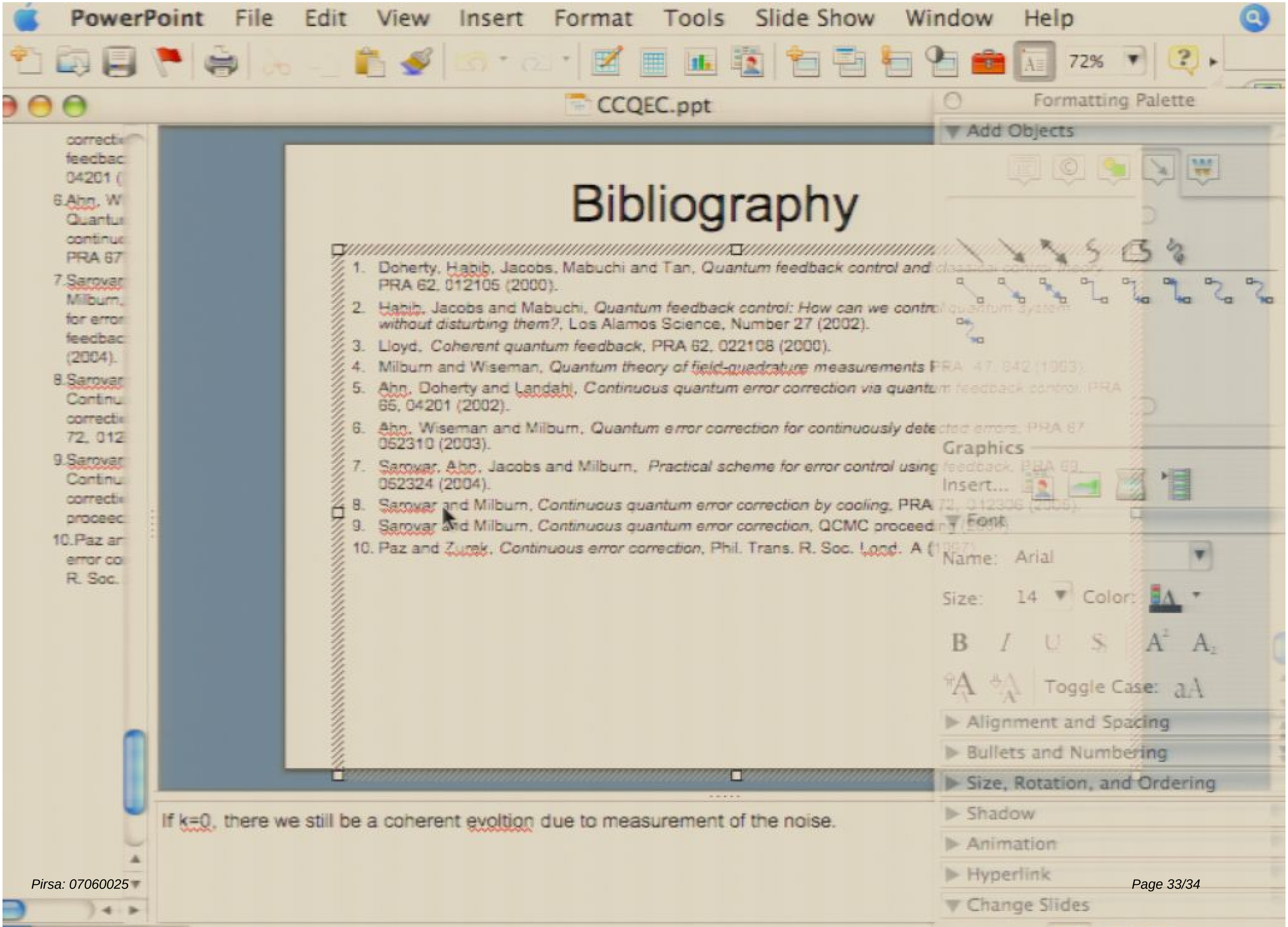


- The aim is to do both *at the same time*

$$H = H_C + H_D + i[H_D, H_C]$$

$$H_C = |00101\rangle\langle 00100| + |11001\rangle\langle 11000| + |10010\rangle\langle 100000|$$
$$+|01110\rangle\langle 01100| + |01011\rangle\langle 01000| + |10111\rangle\langle 10100| + h.c.$$

$$H_D = |00001\rangle\langle 00101| + |11101\rangle\langle 11001| + |00010\rangle\langle 10010|$$
$$+|11110\rangle\langle 01110| + |00011\rangle\langle 01011| + |11111\rangle\langle 10111| + h.c.$$


PowerPoint File Edit View Insert Format Tools Slide Show Window Help
72%
CCQEC.ppt
Formatting Palette
Add Objects
Bibliography
1. Doherty, Habib, Jacobs, Mabuchi and Tan, Quantum feedback control and
PRA 62, 012105 (2000).
2. Habib, Jacobs and Mabuchi, Quantum feedback control: How can we contro
without disturbing them?, Los Alamos Science, Number 27 (2002).
3. Lloyd, Coherent quantum feedback, PRA 62, 022108 (2000).
4. Milburn and Wiseman, Quantum theory of field-quadrature measurements
5. Ahn, Doherty and Landahl, Continuous quantum error correction via quantu
65, 04201 (2002).
6. Ahn, Wiseman and Milburn, Quantum error correction for continuously dete
062310 (2003).
7. Sarovar, Ahn, Jacobs and Milburn, Practical scheme for error control using
052324 (2004).
8. Sarovar and Milburn, Continuous quantum error correction by cooling, PRA
9. Sarovar and Milburn, Continuous quantum error correction, QCMC proceed
10. Paz and Zurek, Continuous error correction, Phil. Trans. R. Soc. Lond. A (
Graphics
Insert...
Font
Name: Arial
Size: 14
Color:
Toggle Case:
Alignment and Spacing
Bullets and Numbering
Size, Rotation, and Ordering
Shadow
Animation
Hyperlink
Change Slides
If k=0, there we still be a coherent evoltion due to measurement of the noise.

Bibliography

1. Doherty, Habib, Jacobs, Mabuchi and Tan, *Quantum feedback control and classical control theory* , PRA 62, 012105 (2000).
2. Habib, Jacobs and Mabuchi, *Quantum feedback control: How can we control quantum system without disturbing them?*, Los Alamos Science, Number 27 (2002).
3. Lloyd, *Coherent quantum feedback*, PRA 62, 022108 (2000).
4. Milburn and Wiseman, *Quantum theory of field-quadrature measurements* PRA 47, 642 (1993).
5. Ahn, Doherty and Landahl, *Continuous quantum error correction via quantum feedback control*, PRA 65, 04201 (2002).
6. Ahn, Wiseman and Milburn, *Quantum error correction for continuously detected errors*, PRA 67, 052310 (2003).
7. Sarovar, Ahn, Jacobs and Milburn, *Practical scheme for error control using feedback*, PRA 69, 052324 (2004).
8. Sarovar and Milburn, *Continuous quantum error correction by cooling*, PRA 72, 012306 (2005).
9. Sarovar and Milburn, *Continuous quantum error correction*, QCMC proceeding (2004).
10. Paz and Zurek, *Continuous error correction*, Phil. Trans. R. Soc. Lond. A (1997)