Title: Symmetric extendibility of quantum states

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Abstract: Imagine that Alice and Bob share a quantum state, from which they want to distill something useful like entanglement or secret key. For this they need to communicate classically and they want to do this by one way communication from Alice to Bob. For some states, it might happen that the state is a part of a tripartite state shared with Charlie, which is invariant if Bob's and Charlie's systems are switched. Such a state is called a symmetric extension, and if it exists Alice and Bob have no chance of distilling key or entanglement by one way communication. I will present some results characterizing which quantum states have symmetric extension.

Symmetric extendibility of quantum states

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Properties of bipartite quantum states

Property	Task	(true/false)
Distillable entanglement?	Teleport	(yes/no)
Pure entanglement?	Distill entanglement	(yes/maybe)
NPPT entanglement?	Distill entanglement	(maybe/no)
Entanglement?	Secret key	(maybe/no)
Symmetric extension?	1-way secret key	(no/maybe)

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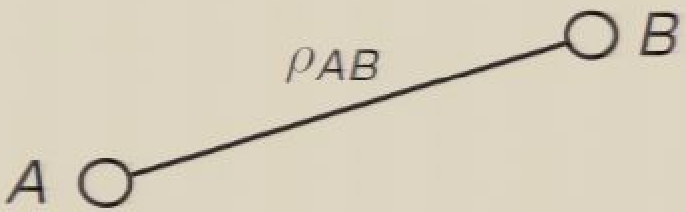
Symmetric extension

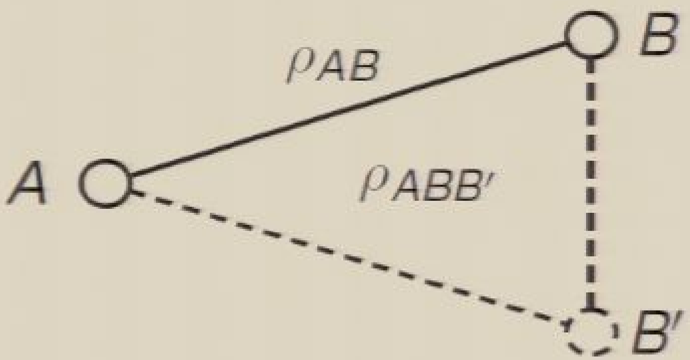


$\exists \rho_{ABB'}$ such that

- $\mathrm{tr}_{B'} \rho_{ABB'} = \rho_{AB}$

Any classical/public information from A is received by both B and B'.

Symmetric extension



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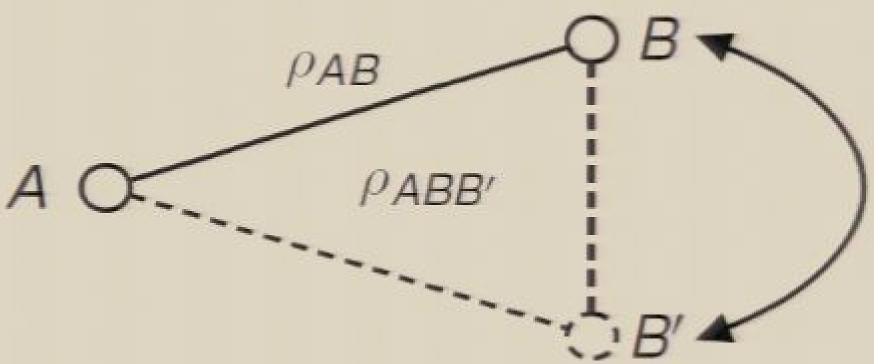

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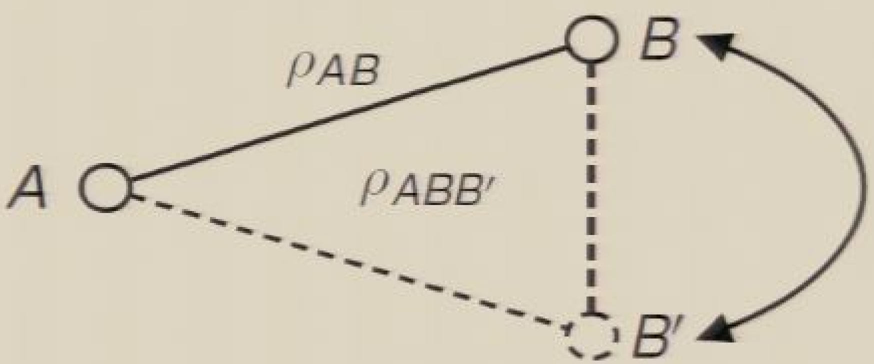

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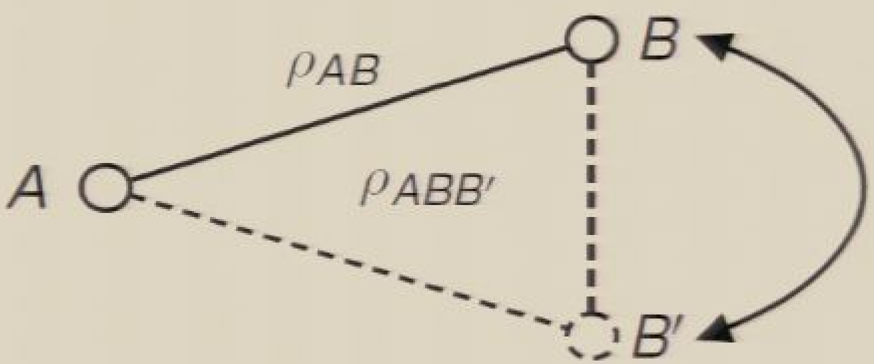

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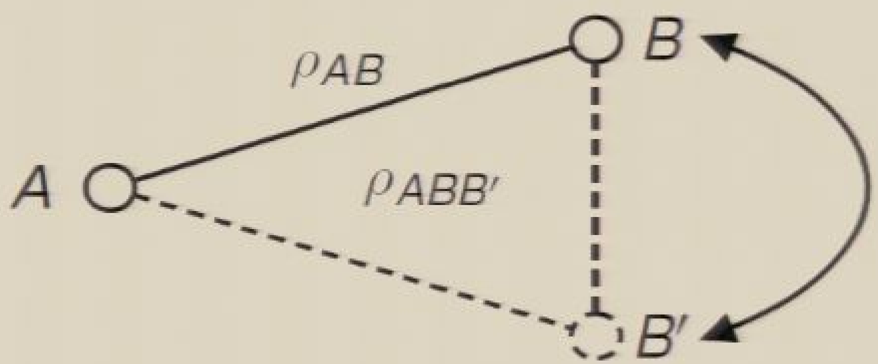

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Structure of symmetric extendible states

Decomposition into pure-extendible states:

$$\rho_{AB} = \sum_j p_j \rho^j_{AB},$$

where the ρ^j_{AB} are extendible to pure states.

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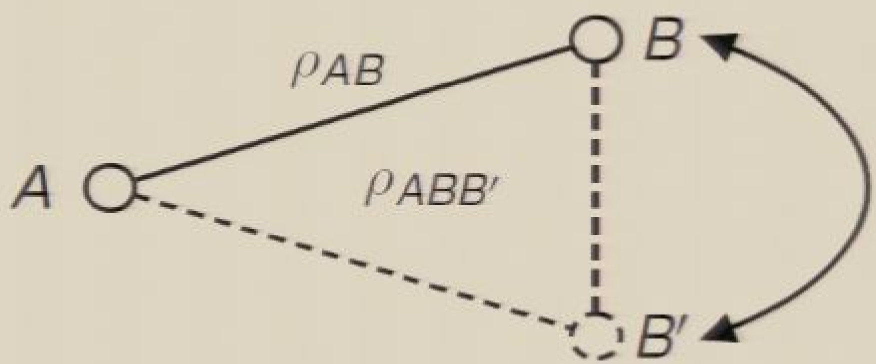

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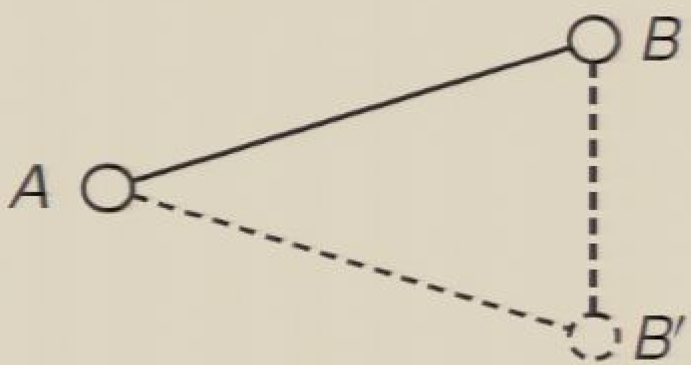

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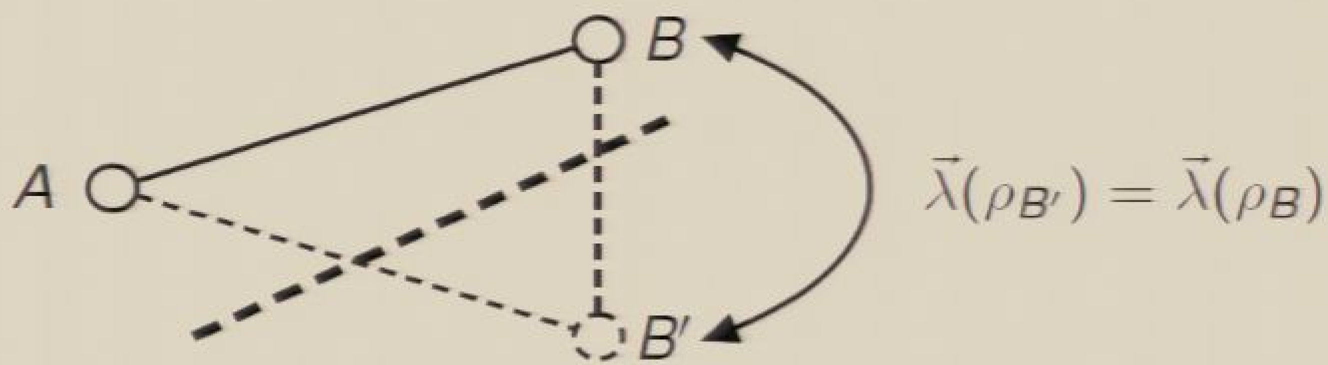

Two qubits

Necessary *and sufficient* conditions

Pure-extendible states

$$\vec{\lambda}(\rho_{AB}) = \vec{\lambda}(\rho_B)$$

Rank-2 states

$$\lambda_{\max}(\rho_{AB}) \leq \lambda_{\max}(\rho_B)$$

Conjecture for general *2-qubit* states

Symmetric extension only depends on $\vec{\lambda}(\rho_{AB})$ and $\vec{\lambda}(\rho_B)$ (numerically verified).

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Two qubits – Bell diagonal states

$$\rho_{AB} = p_I|\Phi^+\rangle\langle\Phi^+| + p_x|\Psi^+\rangle\langle\Psi^+| + p_y|\Psi^-\rangle\langle\Psi^-| + p_z|\Phi^-\rangle\langle\Phi^-|,$$

where

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$
$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

can also be written as

$$\rho_{AB} = \frac{1}{4}I - \frac{1}{2}(\gamma_x X \otimes X + \gamma_y Y \otimes Y + \gamma_z Z \otimes Z)$$

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Theorem [Renes, Doherty]

A bell-diagonal state has symmetric extension iff one of

$$\gamma_x\gamma_y\gamma_z - \gamma_x^2\gamma_y^2 - \gamma_x^2\gamma_z^2 - \gamma_y^2\gamma_z^2 \geq 0$$

$$\min(\gamma_i\gamma_j \pm \gamma_i\gamma_k \pm \gamma_j\gamma_k, -(\gamma_k \pm \gamma_j \pm \gamma_i)) \geq 0$$

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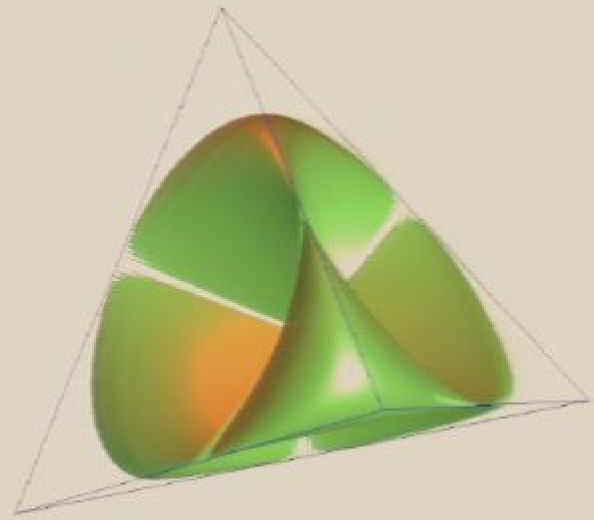

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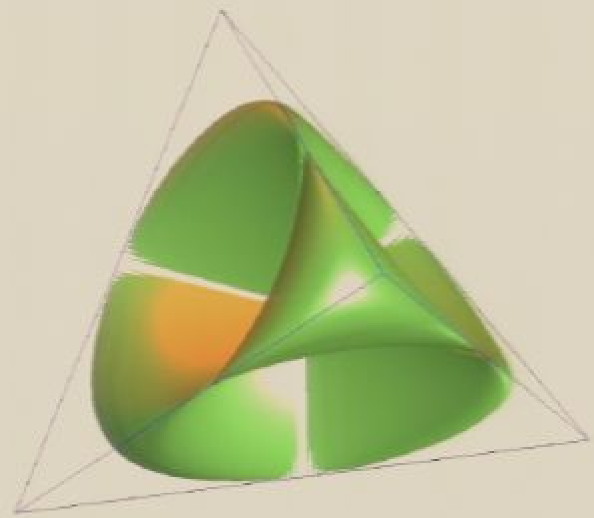

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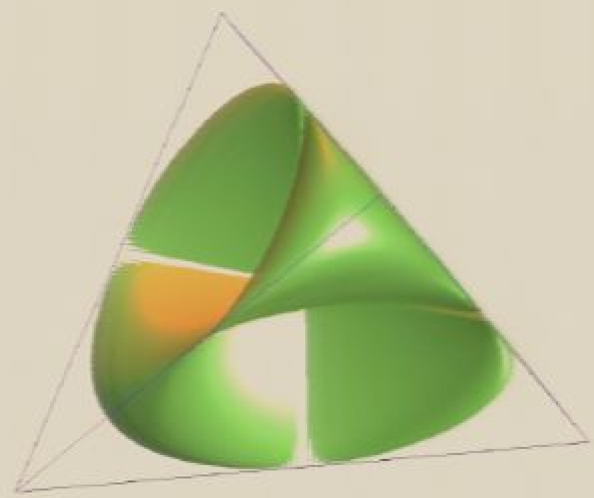

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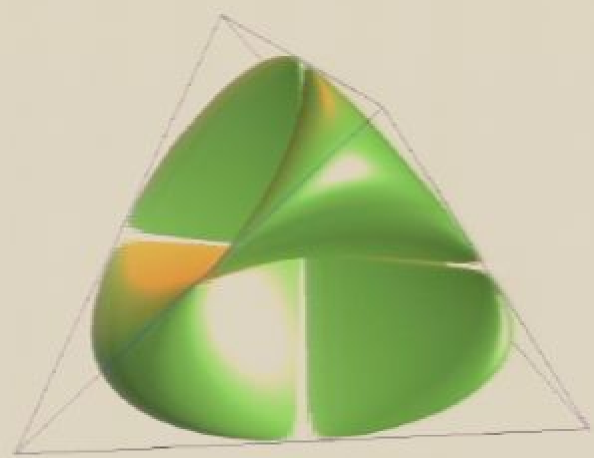

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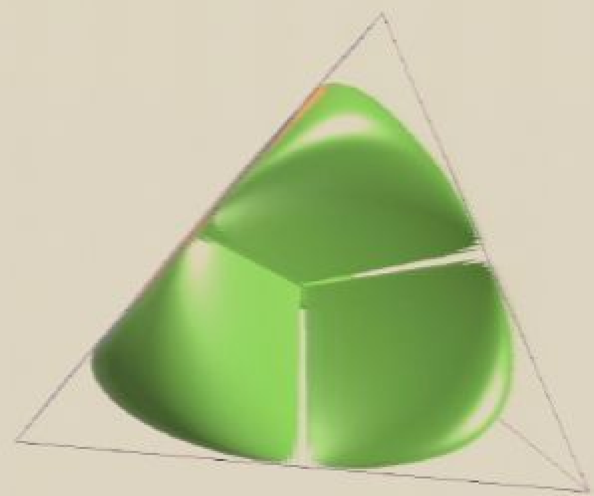


Higher dimensional symmetrized states

- Can symmetrize by random unitary on A and corresponding on B (twirling)
- Only requires one-way communication
  $\Rightarrow$ conserves symmetric extension
- $\rho \to \int (U \otimes U)\rho(U \otimes U)^\dagger \mathrm{d}U$ give Werner states
- $\rho \to \int (U \otimes U)\rho(U^* \otimes U^*)^\dagger \mathrm{d}U$ give isotropic states

Werner states

- Werner states are of the form

$$\rho = p\frac{P_{\text{as}}}{d^2 - d} + (1-p)\frac{P_{\text{sym}}}{d^2+1}$$

- Any state of this form has symmetric extension for $d \geq 3$

Isotropic states

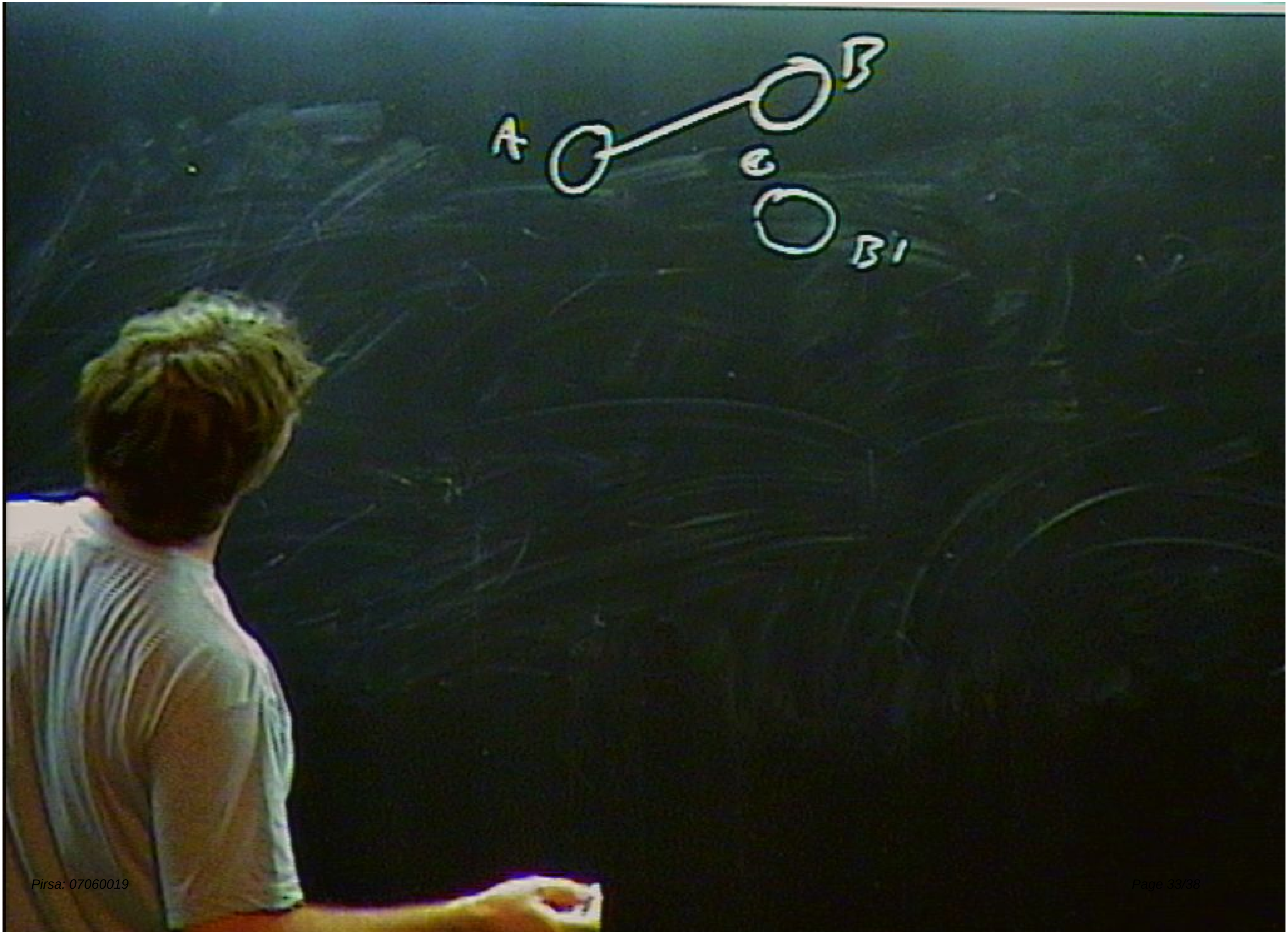
- Isotropic states are of the form

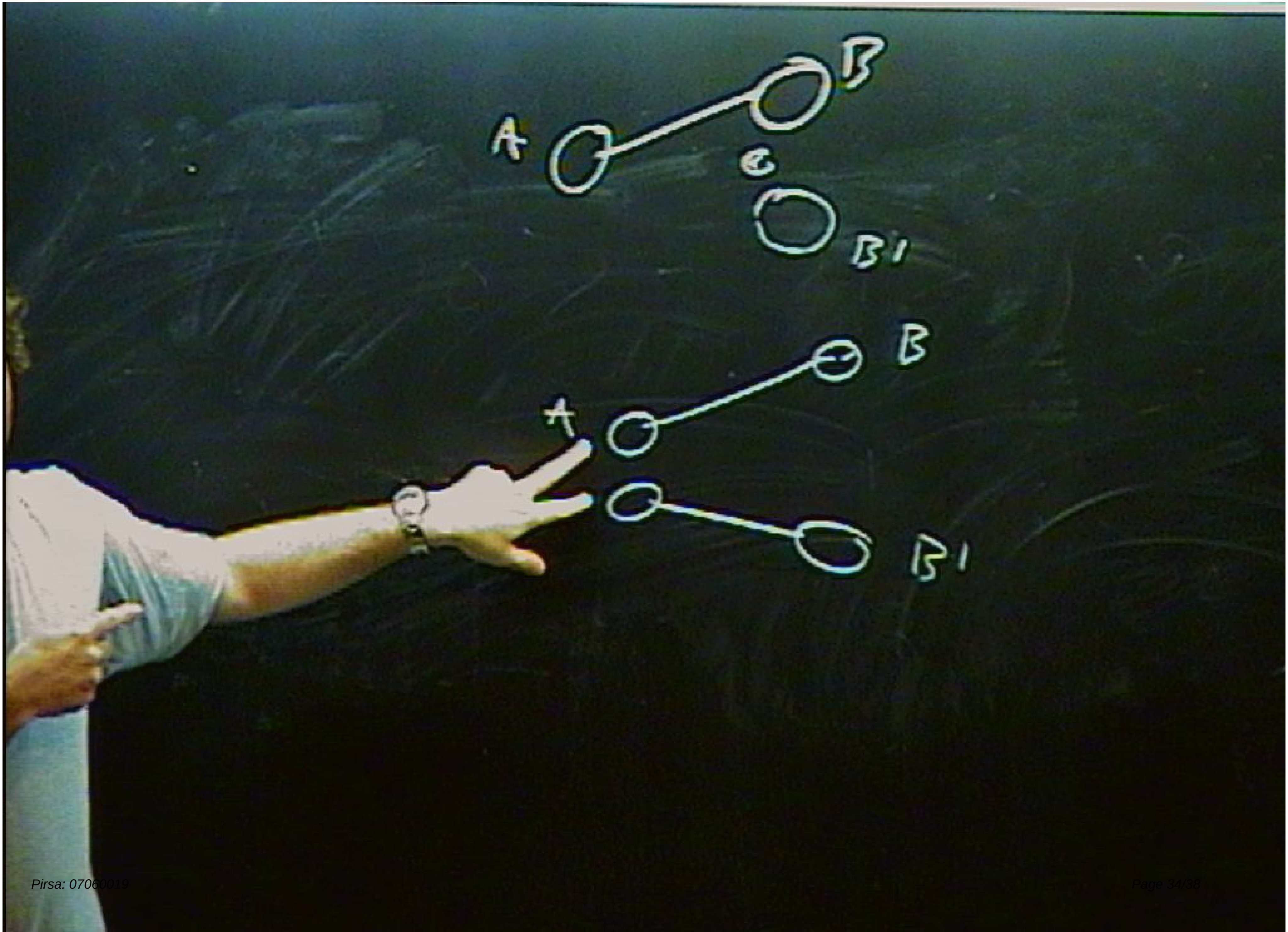
$$\rho = \alpha|\phi^+\rangle\langle\phi^+| + (1-\alpha)I/d^2$$

- The twirling conserves fidelity to $|\phi^+\rangle$
- The states have symmetric extension iff $\langle\phi^+|\,\rho\,|\phi^+\rangle \leq \frac{d+1}{2d}$

Summary

- Symmetric extension limits one-way quantum tasks
- Extendible states can be decomposed to pure-extendible
- Criteria simplify for two qubits


A
B
C
B'


A
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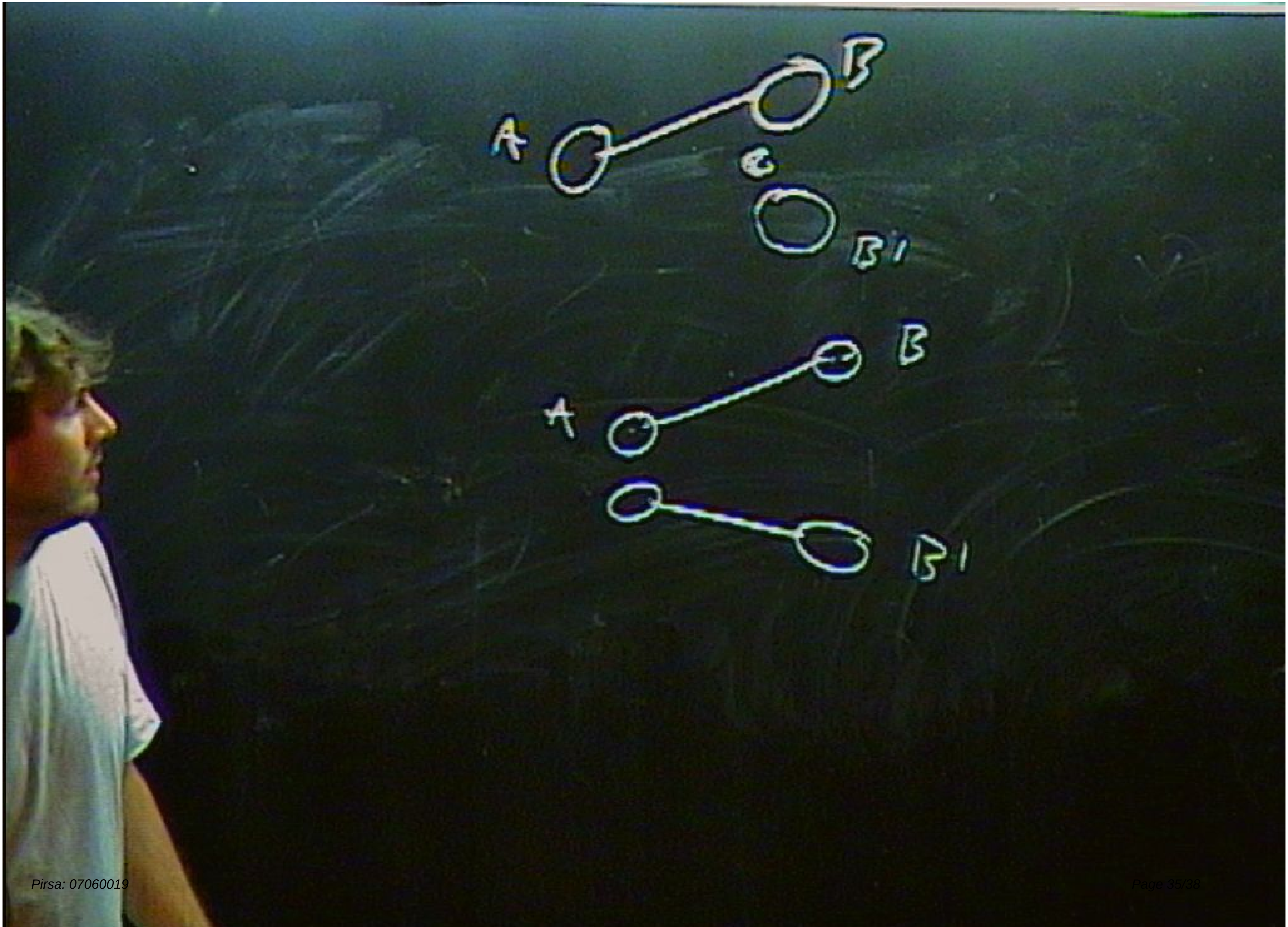
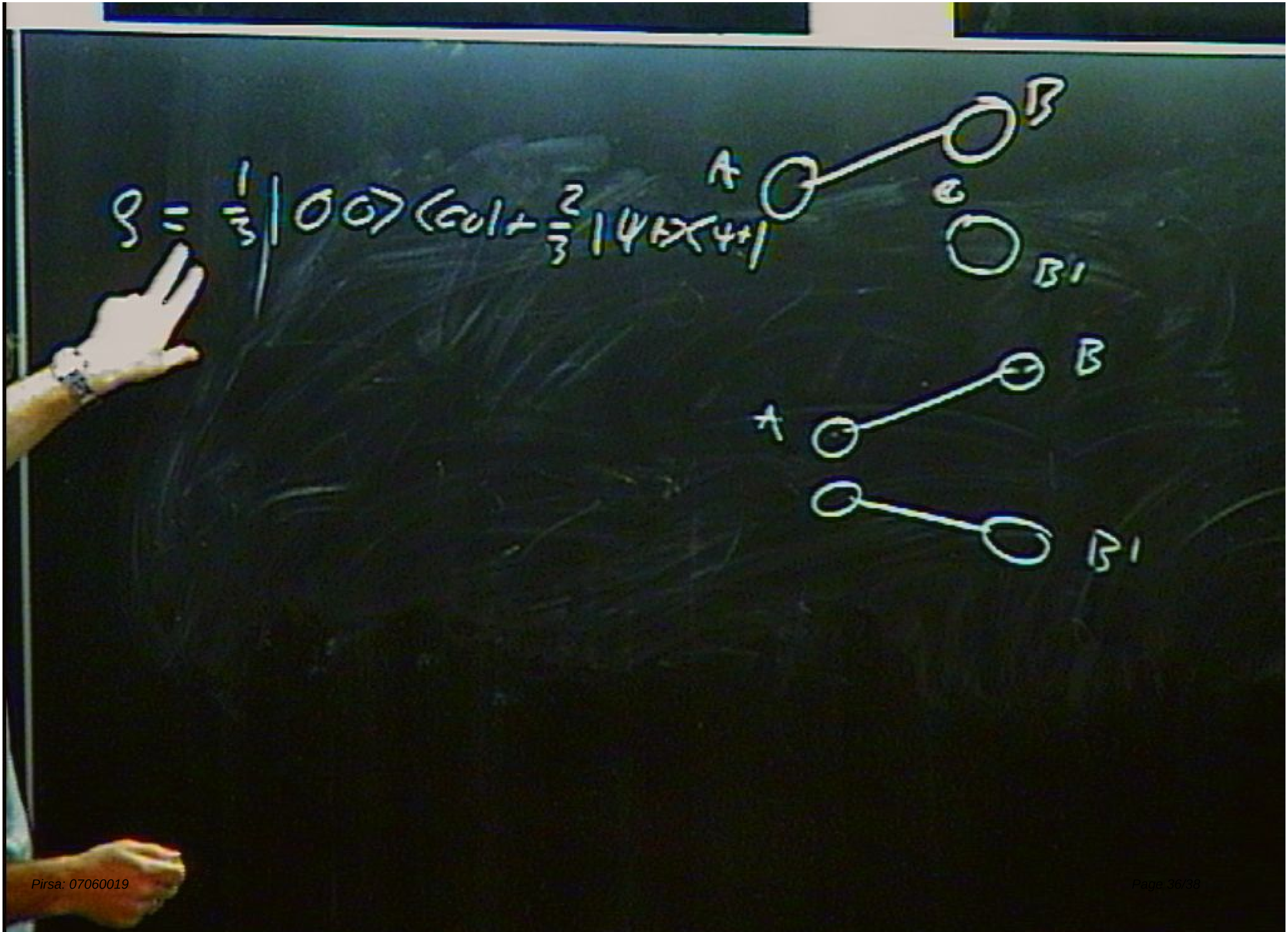
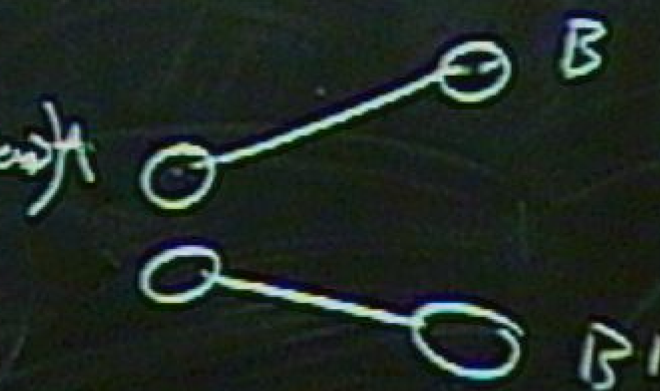

A
B
C
B'
A
B
B'


ρ = 1/3 |00⟩⟨00| + 2/3 |ψ+⟩⟨ψ+|
A
B
B'
A
B
B'

$$\rho = \frac{1}{3}|00\rangle\langle 00| + \frac{2}{3}|\psi^+\rangle\langle\psi^+|$$

$$|W\rangle = \frac{1}{\sqrt{3}}\left(|001\rangle + |010\rangle + |100\rangle\right)$$



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