Title: Quantum Key Distribution Networks

Date: Jun 02, 2007 03:30 PM

URL: http://pirsa.org/07060013

Abstract: Current physical implementations of quantum key distribution (QKD) require communicating parties to be close together. We will explore methods for allowing parties separated by long distances to communicate by combining many QKD links in a network and discuss the resulting security properties.

University of Waterloo

IQC Institute for Quantum Computing

Quantum steganography

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Saturday, June 2, 2007

[1] Supported in part by Canada's NSERC and Sun Microsystems.

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- No active or passive adversary should be able to detect that you're doing something out of the ordinary.
- Steganography has been used in espionage, as a form of encryption, and in watermarking.

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- This induces a 6-bit (2 red, 2 blue, 2 green) hidden image.

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- In cryptography, we usually have greater trust in schemes where the entire algorithm is publicly known, and only a secret key is shared between the sender and receiver.

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An information-theoretic framework

An information-theoretic framework for steganography[2]

- Model things using probability distributions.

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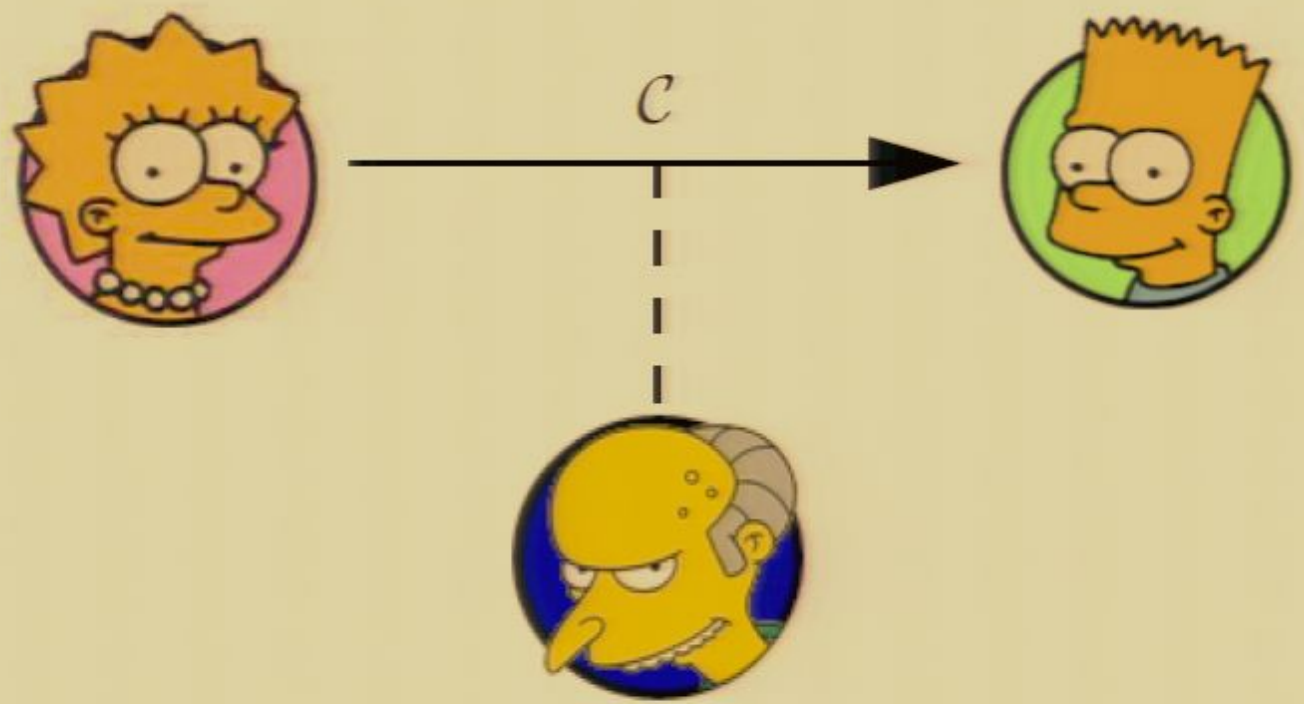

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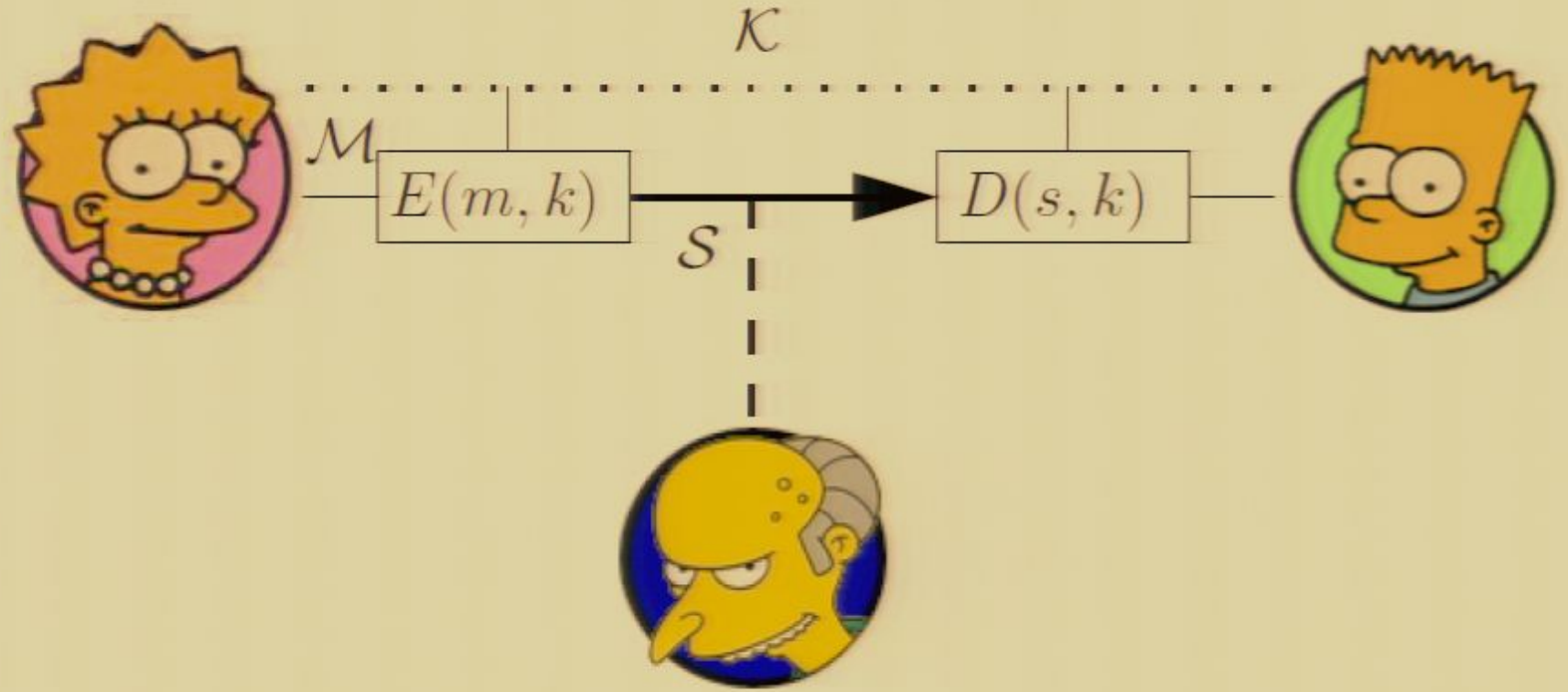

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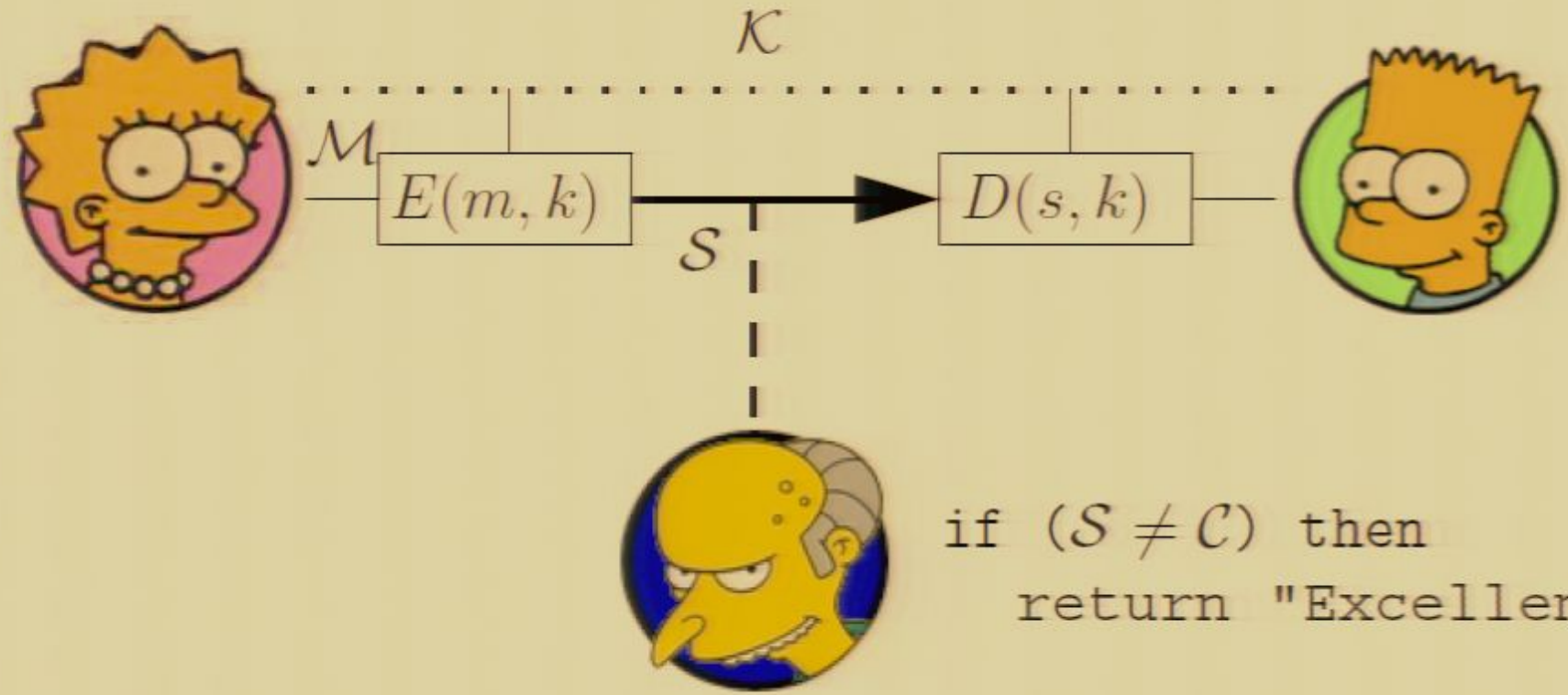

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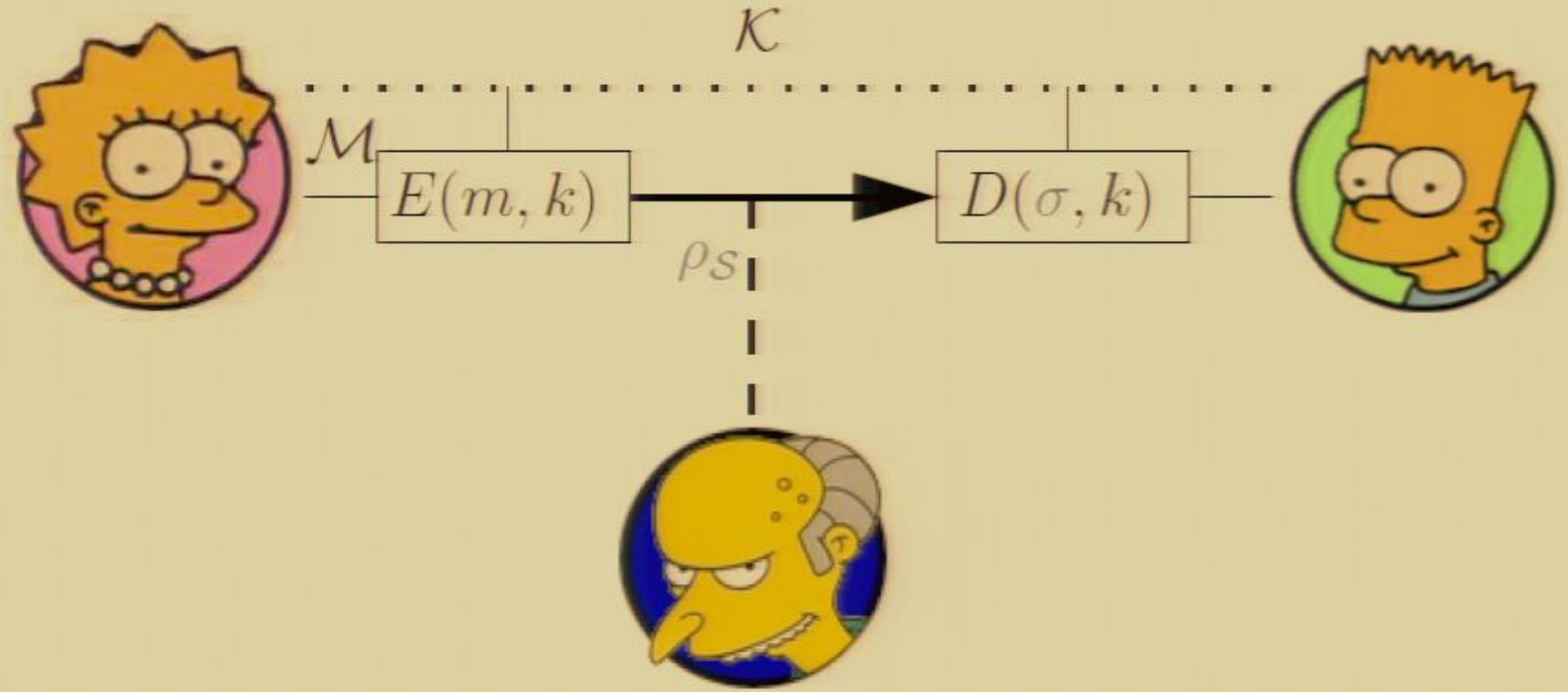

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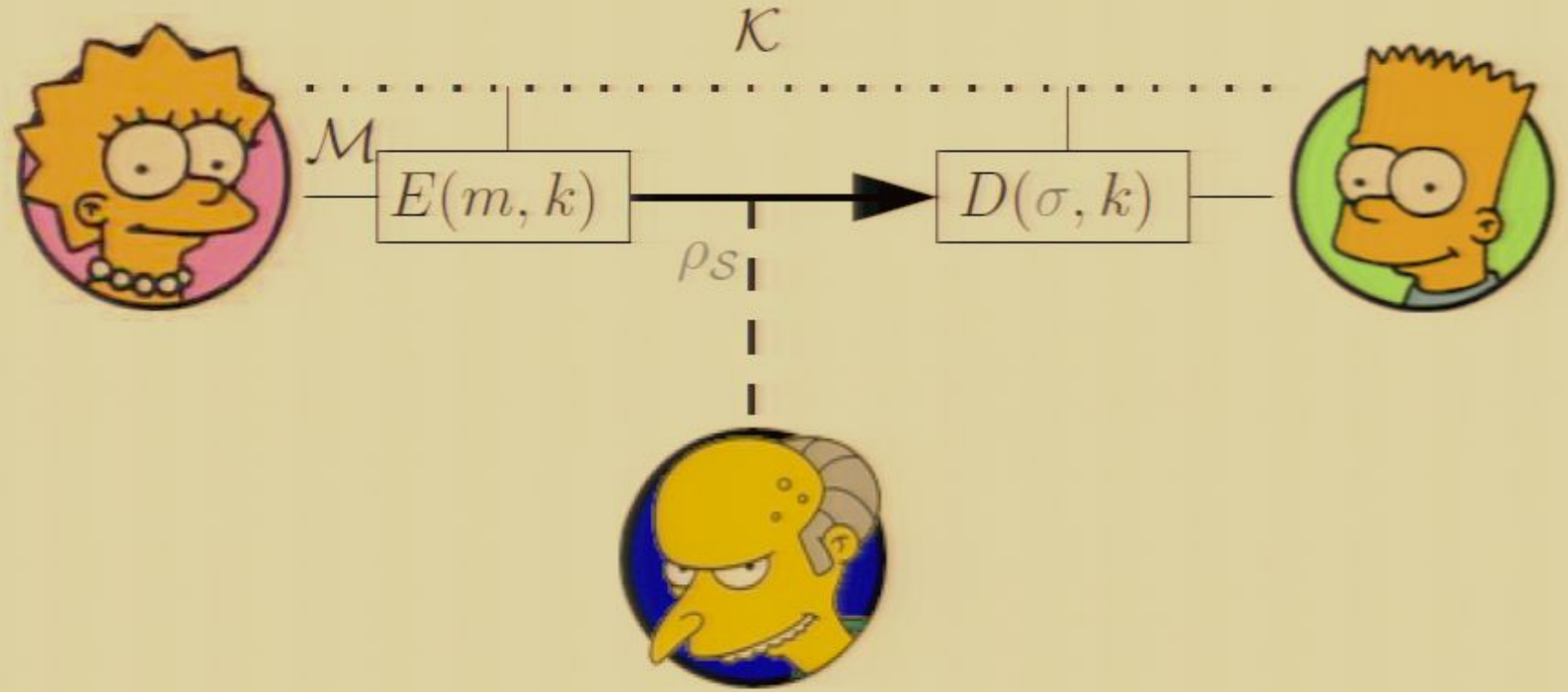

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- We can use the same definition of information rate as in the classical case.

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- Each of the following states will be constructed with probability $\frac{1}{4}$:

$\|0\rangle$	$\|1\rangle$
$\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle) = \|+\rangle$	$\frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle) = \|-\rangle$

BB84 quantum key distribution

- In the BB84 quantum key distribution protocol, the quantum states sent across the wire are generated as follows:
- Choose a, b uniformly at random from $\{0, 1\}$.
- Construct the state $|\psi\rangle = H^a|b\rangle$, where H is the Hadamard matrix.
- Each of the following states will be constructed with probability $\frac{1}{4}$:

$\|0\rangle$	$\|1\rangle$
$\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle) = \|+\rangle$	$\frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle) = \|-\rangle$

- The mixed state for the probability distribution of states generated in this manner is the totally mixed state:

$$\rho_{\mathrm{BB84}} = \frac{1}{2}I = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$$

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- Encoding $E(m, k) = H^k|m\rangle$
- Decoding $D(\sigma, k) =$ measure $H^k \sigma H^{k\dagger}$ in the computational basis

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- In either case, this scheme won't be perfectly secure.

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- Encoding $E(m, k) = H^m|k\rangle$
- Decoding $D(\sigma, k)$ (more complicated: recipient uses knowledge of key to help determine if he got the right basis or not)

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- However, we now have some decoding errors because the receiver doesn't always know what basis to measure in.
- We have a decoding scheme that gives a probability of error in decoding of $\frac{1}{4}$ for all message distributions $\mathcal{M}$.

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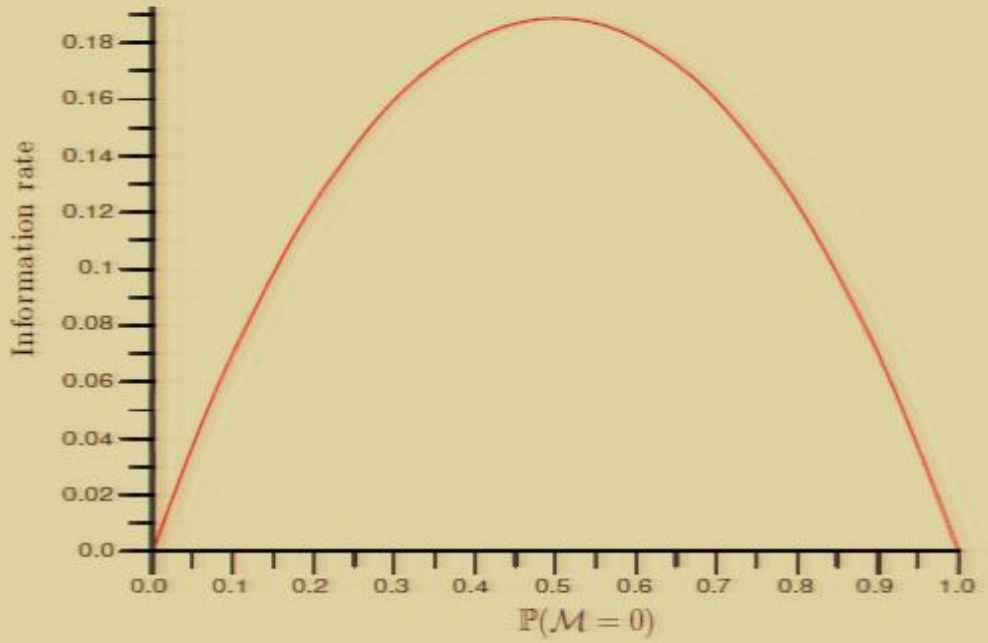
- The information rate for this scheme is

$$h\left(\frac{1}{4}+\frac{1}{2}\mathbb{P}(\mathcal{M}=0)\right)-h\left(\frac{3}{4}\right)$$

where $h(x)=-x\log_2(x)+(1-x)\log_2(1-x)$.



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Classical one-time pad	any	perfect	1
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Future work

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- Find quantum steganography schemes for other covertext distributions, e.g., the six-state protocol, the 3-state protocol, or other quantum communication scenarios.