Title: Distributed phase reference schemes for QKD: Explicit attacks and security considerations

Date: Jun 02, 2007 02:30 PM

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Abstract: <div id="Cleaner">"Distributed phase reference schemes are a new class of protocols for Quantum Key Distribution, in which the quantum signals have overall phase-relationships to each other. This is expected to protect against some loss-related attacks. However, proving the full security of these schemes is a new challenge for theorists, as one can no longer identify individual signals (such as qubits in BB84, for instance), and so the security proof techniques do not apply directly.<div id="Cleaner"><div id="Cleaner">In this talk I will present two such protocols (the Differential Phase Shift and the Coherent One Way protocols). Their ""unconditionnal security"" has not been proven yet, but I will present some specific attacks on these schemes, which give us upper bounds for the security, as well as a ""feeling"" on how these schemes should perform."

Distributed Phase Reference schemes for QKD :

Explicit attacks and security considerations

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Outline

- Distributed Phase Reference schemes
  2 examples : DPS & COW protocols

- Can we directly apply the standard methods to prove the security of these schemes ?

- Examples of explicit attacks :
  – Beam Splitting attack
  – New non-zero-error attacks

[Inoue, Waks, Yamamoto, PRA 68, 022317 (2003)]

Differential Phase Shift protocol

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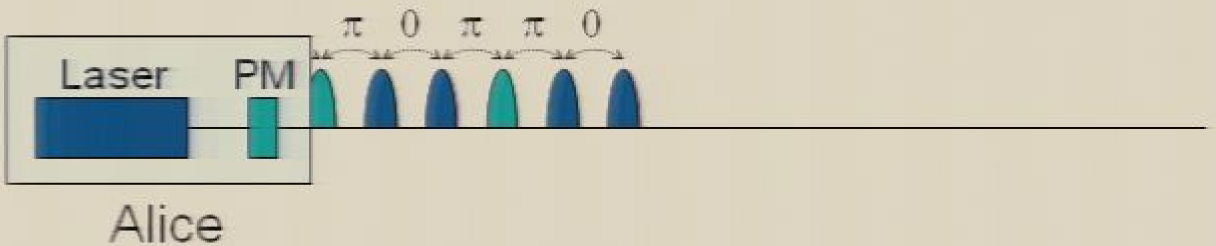

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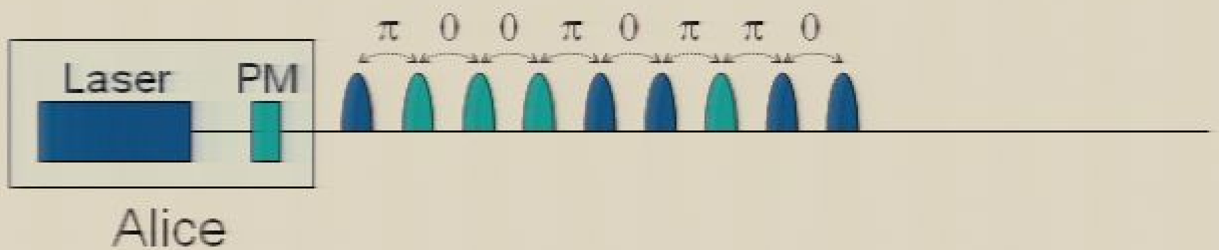

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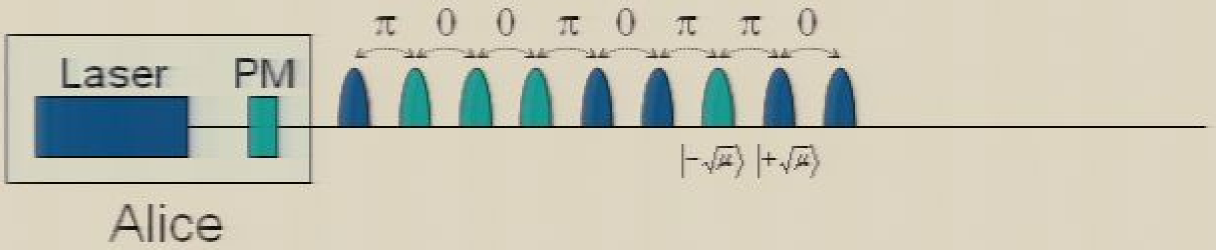

- Alice sends a train of *weak coherent pulses* $\left|\pm\sqrt{\mu}\right\rangle$.
- The pulses have *overall phase relationships* (mode-locked laser); the *classical bits* are encoded in the *relative phase* between two successive coherent pulses.

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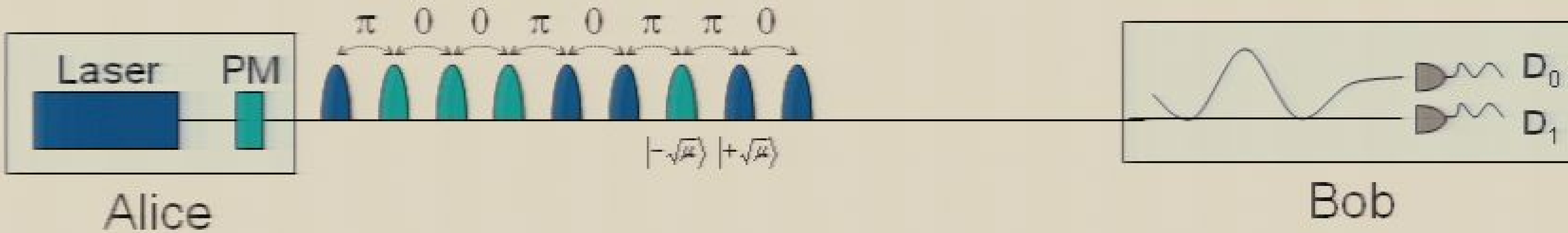

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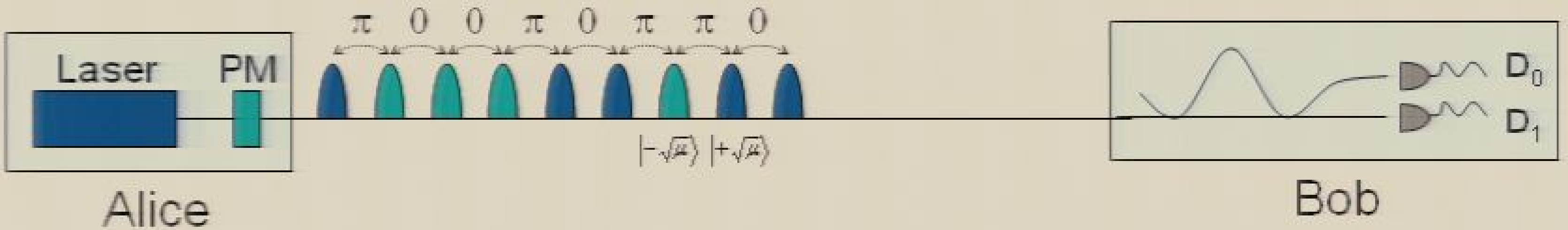

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  - Security parameters : $Q,V \quad \left(Q=\frac{1-V}{2}\right)$

[Gisin *et al*, quant-ph/0411022 (2004);
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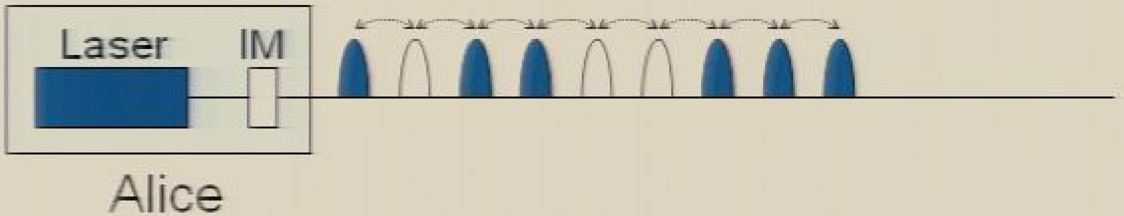

- Alice sends a train of *weak coherent pulses* $|\sqrt{\mu}\rangle$ or *empty pulses* $|0\rangle$.
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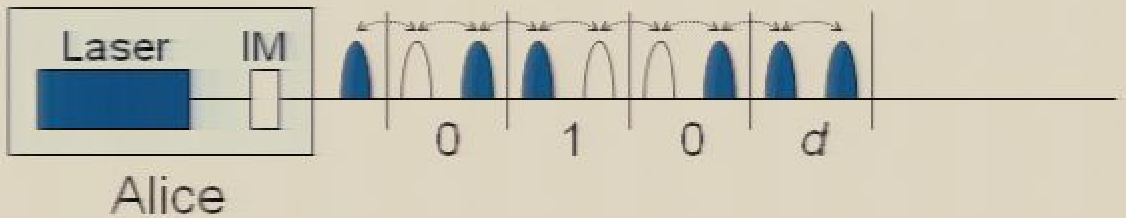

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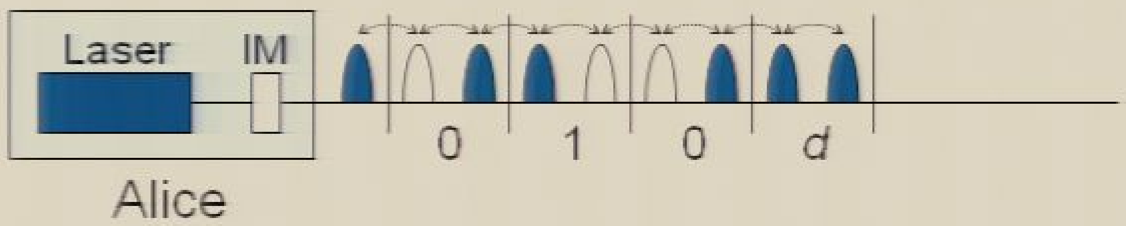

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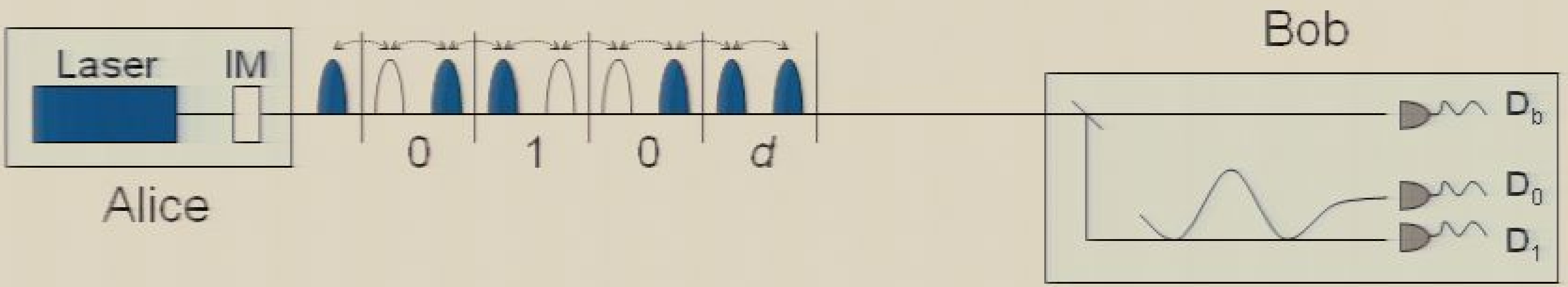

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- On a first line, Bob measures the time of arrival of the pulses to get the bits.
- Bob also takes a fraction of the incoming signal and inputs it into an unbalanced interferometer to check for the coherence between two successive non-empty pulses.

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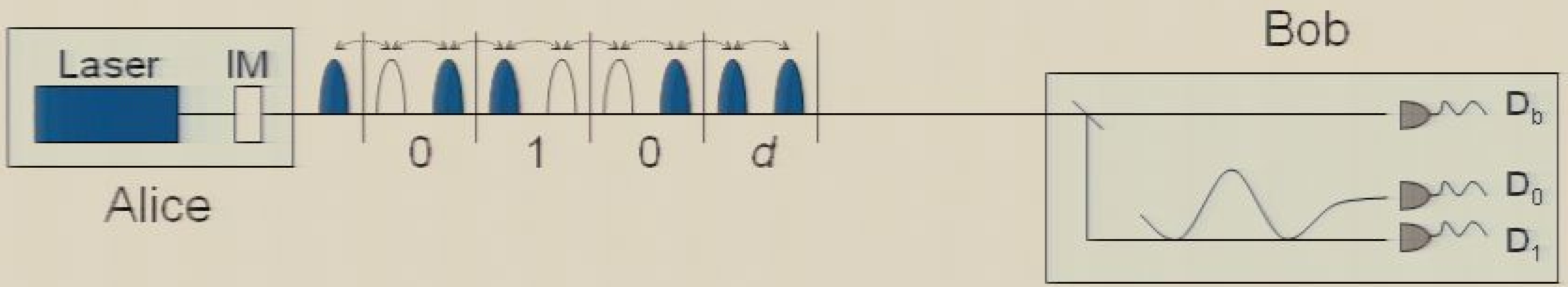

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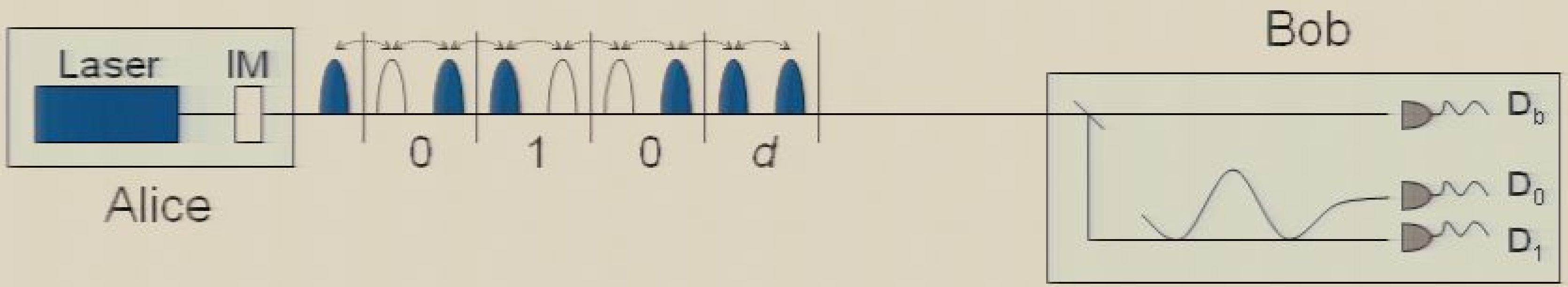

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- Problem : for those distributed phase reference schemes, we cannot define such qubits…

  The standard proofs do not apply directly!
- Before we try to develop new techniques, we want to get a feeling of how secure these schemes should be, of their limitations…

  In order to do so, we study specific attacks.

Beam-Splitting attack on DPS

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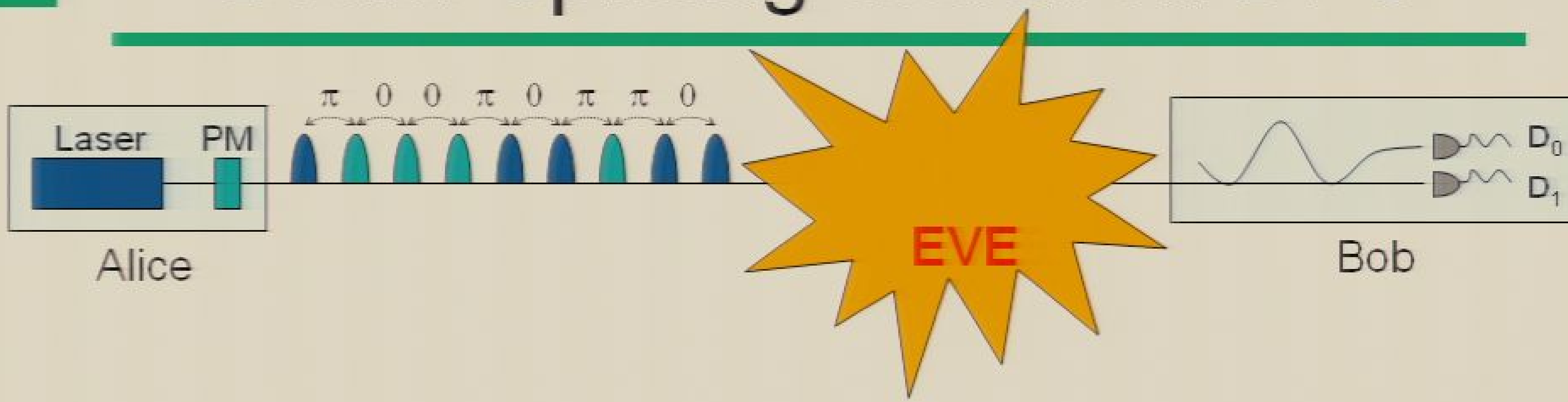

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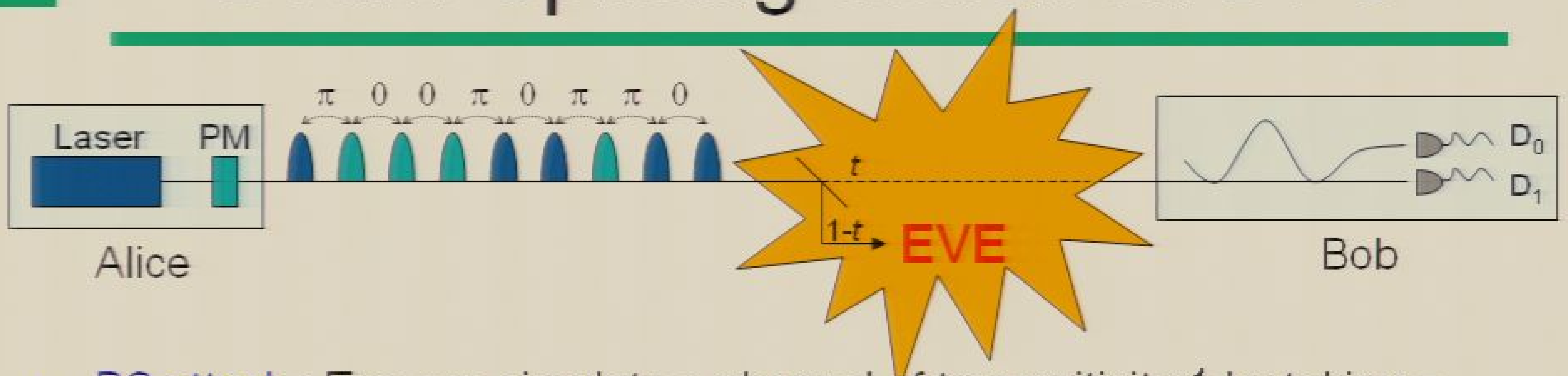

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  For each bit detected by Bob, one can compute her states $\rho_E^{A=0/1}$ and ρ_E .
  Eve's information is upper-bounded by Holevo's quantity

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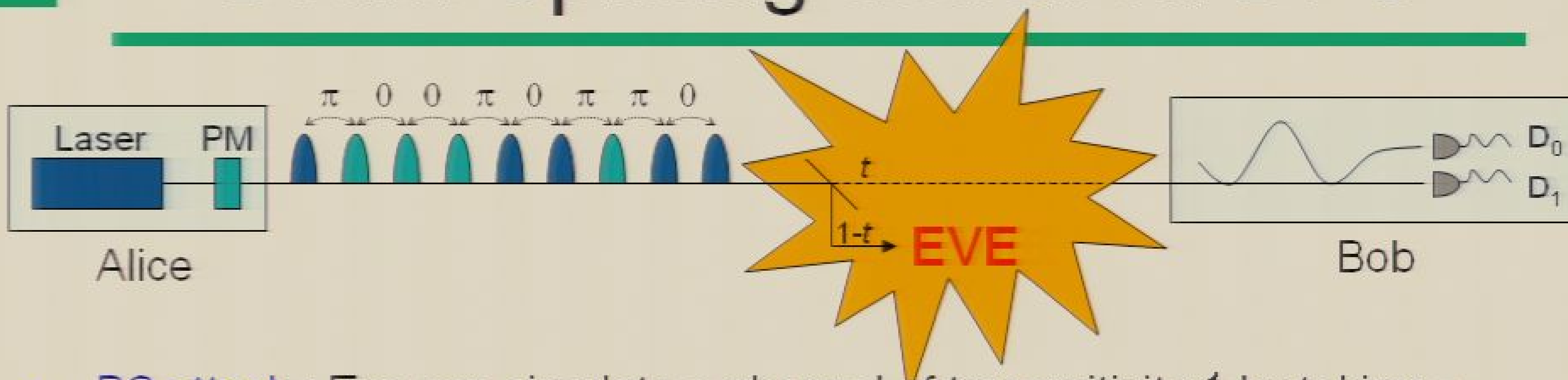

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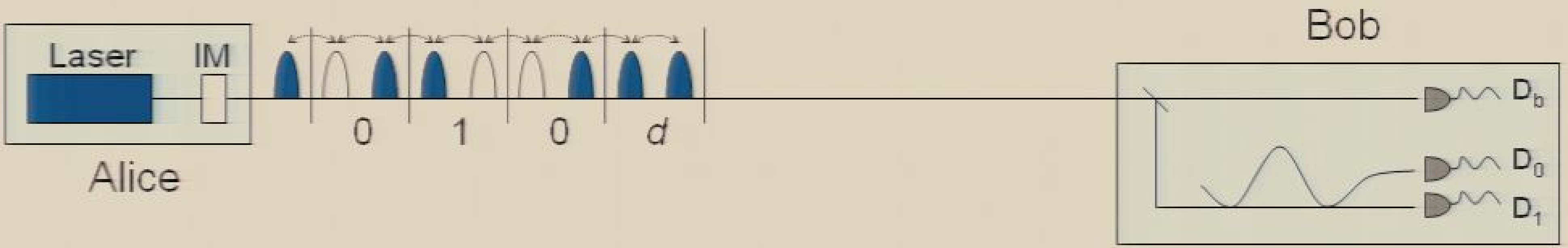

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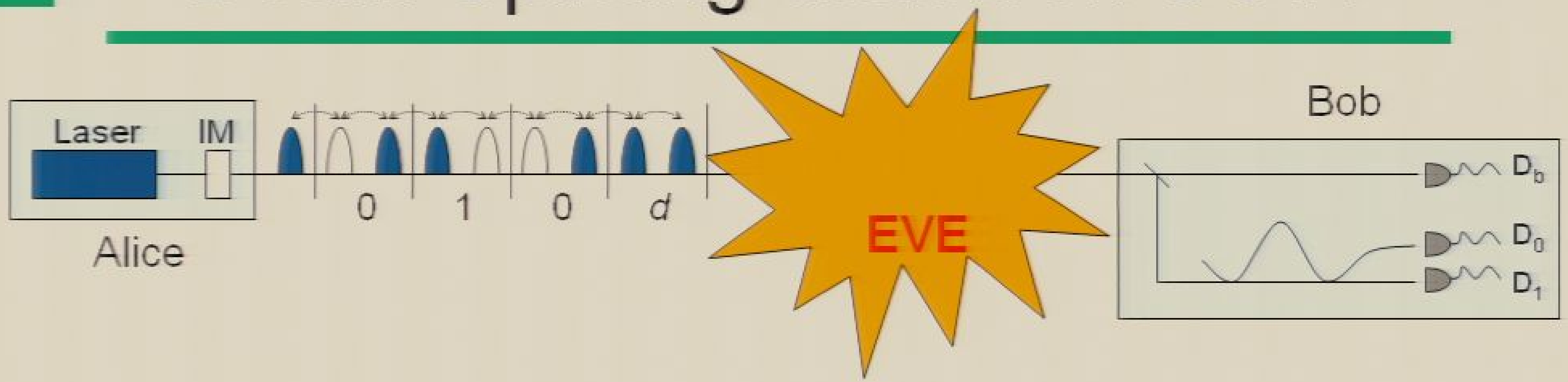

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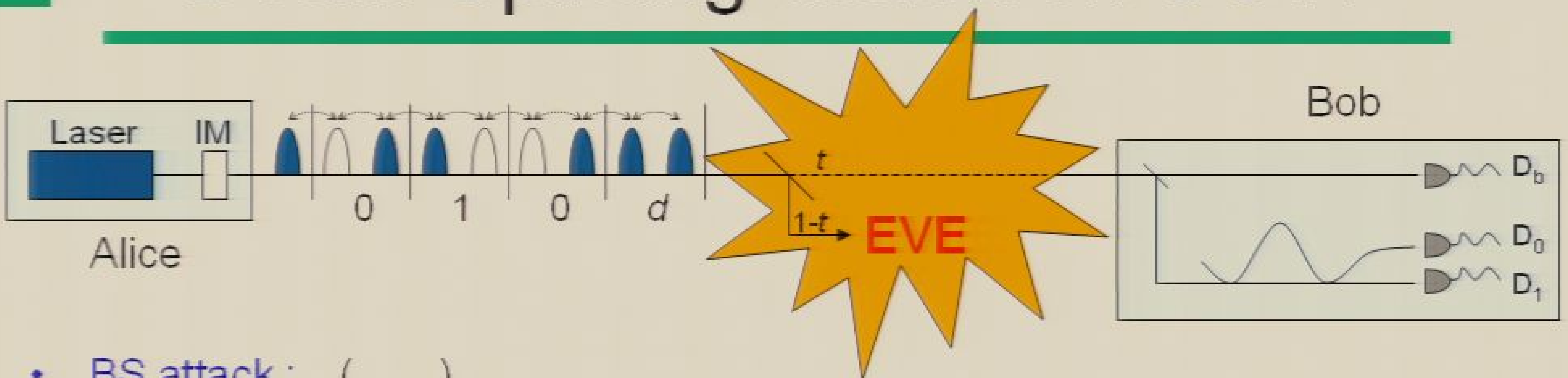

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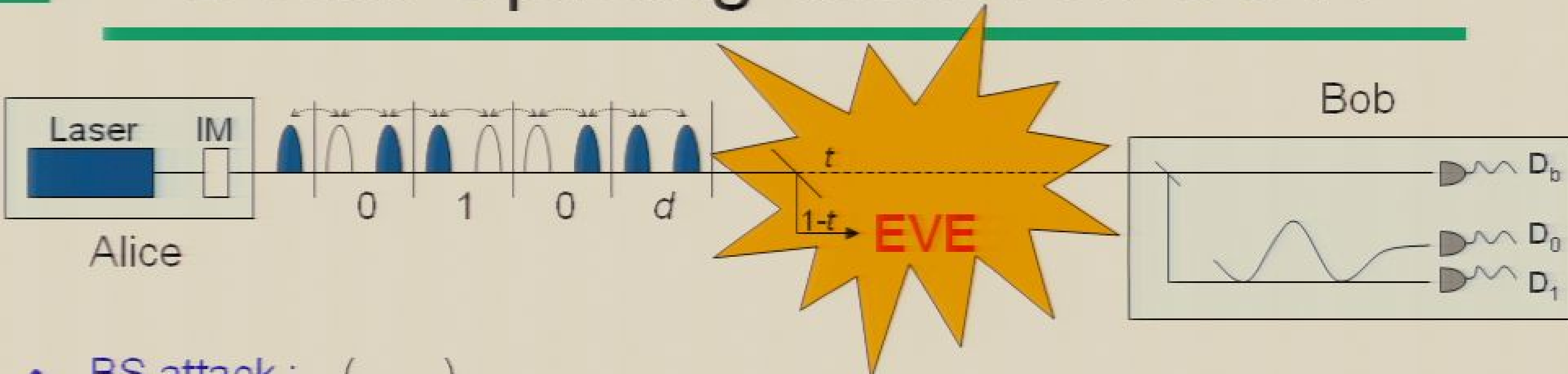

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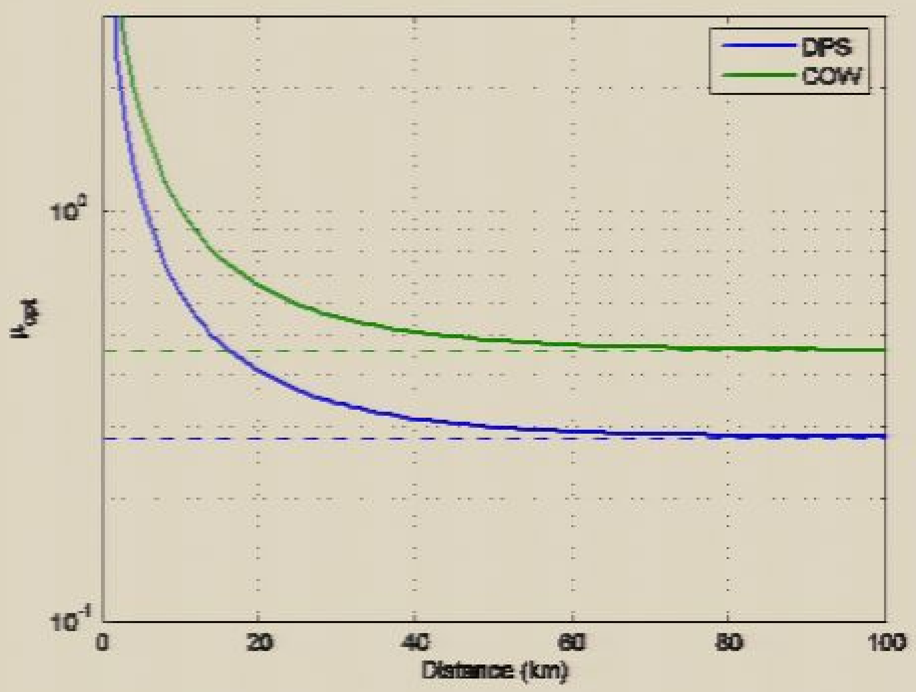
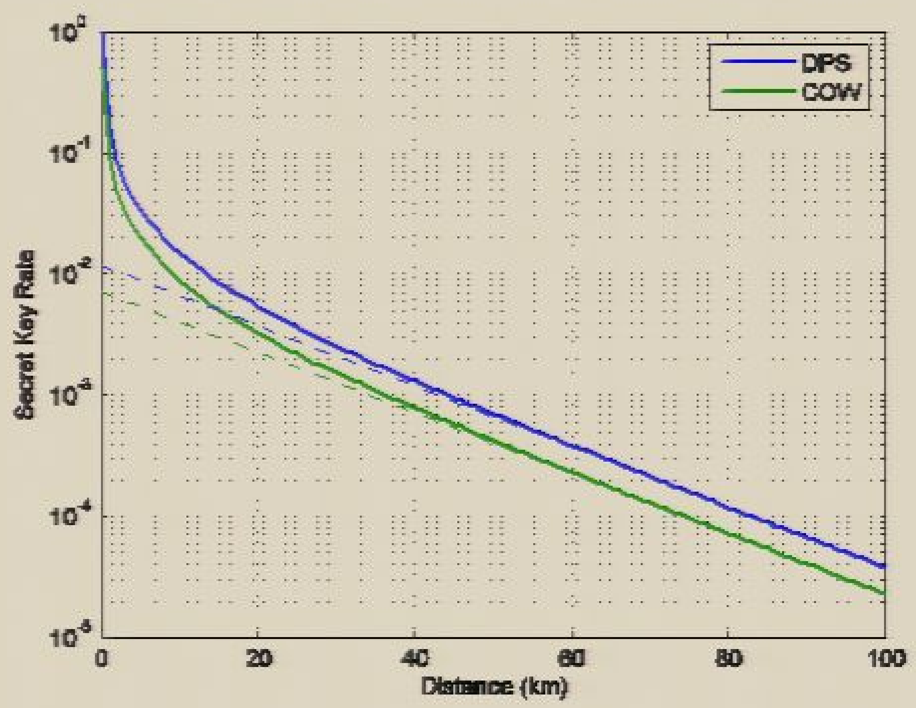
BS attack : DPS vs COW





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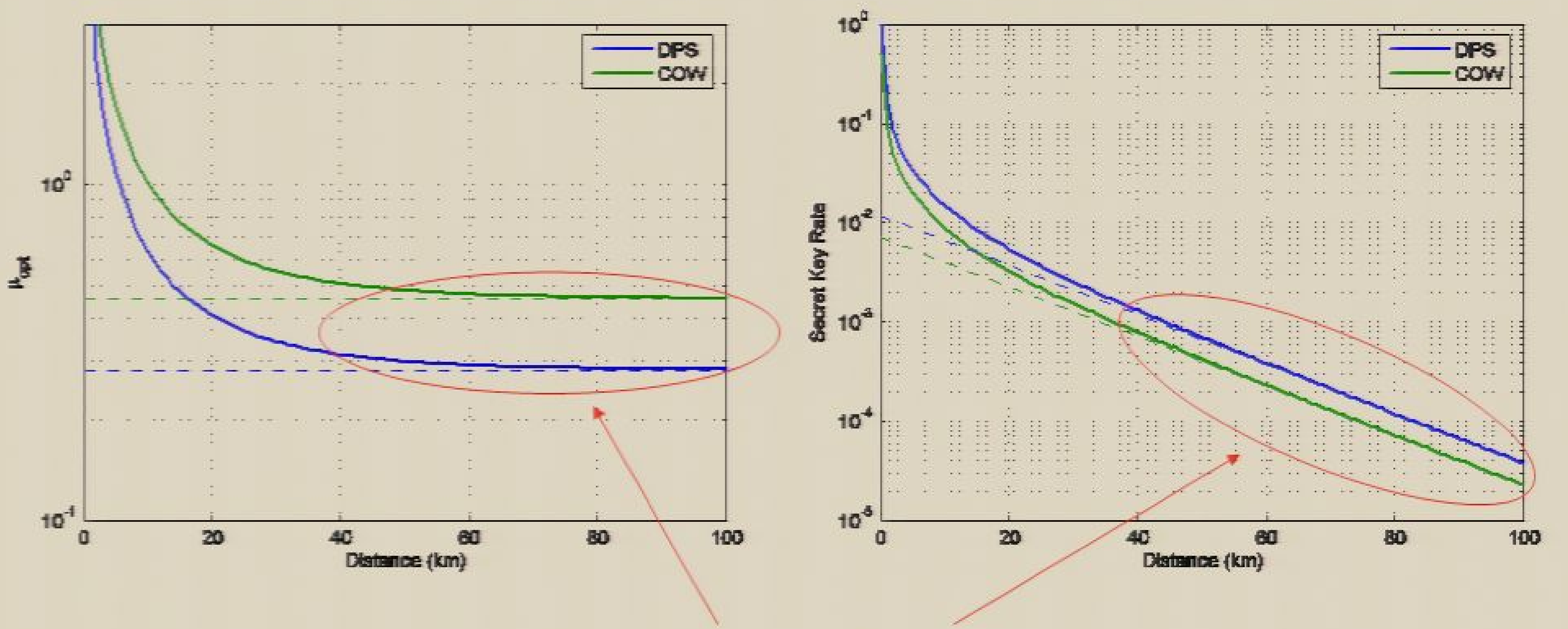

Linear regime :

for $\mu t << 1, \quad r = r_0 t \eta, \quad r_0 = r_0(\mu)$

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For $\mu t << 1$, $\left|\sqrt{\mu}\right\rangle_A \rightarrow |0\rangle_B |\varphi\rangle_E + \sqrt{\mu t}|1\rangle_B |\psi\rangle_E$

The price to pay will be the introduction of errors…

[Ex: For the above attack on a sequence $\left|\sqrt{\mu}\sqrt{\mu}\right\rangle$, the interference will show a visibility $V = \left|\langle\varphi|\psi\rangle\right|^2$]

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Loss of visibility, QBER :

$$V_{\sigma\omega} = \mathrm{Re}\left\langle\psi^{01}_{\sigma\omega}\middle|\psi^{10}_{\sigma\omega}\right\rangle, \quad V_{\sigma\omega,\sigma'\omega'} = \mathrm{Re}\left\langle\varphi_{\sigma\omega}\middle|\psi^{01}_{\sigma\omega}\right\rangle\left\langle\psi^{10}_{\sigma'\omega'}\middle|\varphi_{\sigma'\omega'}\right\rangle, \quad QBER = \frac{1-V}{2}$$

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- For COW :

$$\left|0,\sqrt{\mu}\right\rangle_A \to |00\rangle_B|\varphi_0\rangle_E + \sqrt{(1-Q)\mu t}|01\rangle_B\left|\psi_0^{01}\right\rangle_E + \sqrt{Q\mu t}|10\rangle_B\left|\psi_0^{10}\right\rangle_E$$

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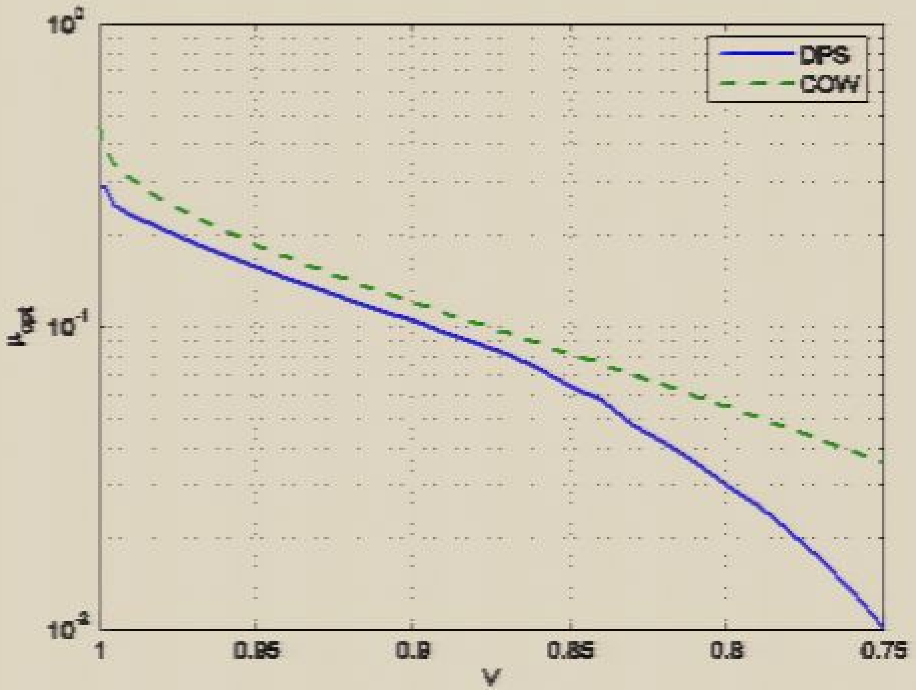
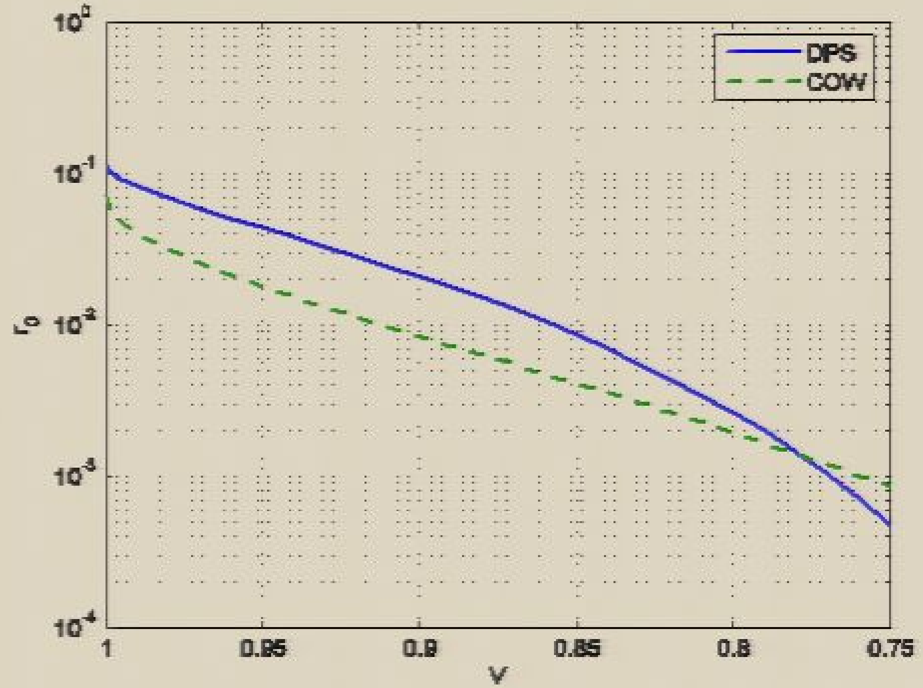
- The two rounds of optimization were performed numerically.

Security bounds for our attacks





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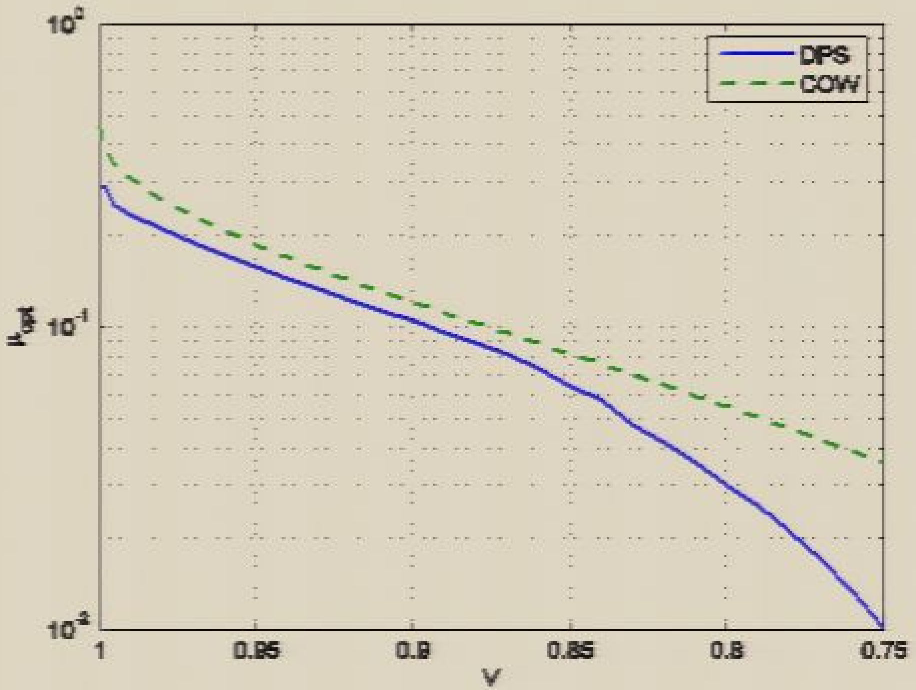

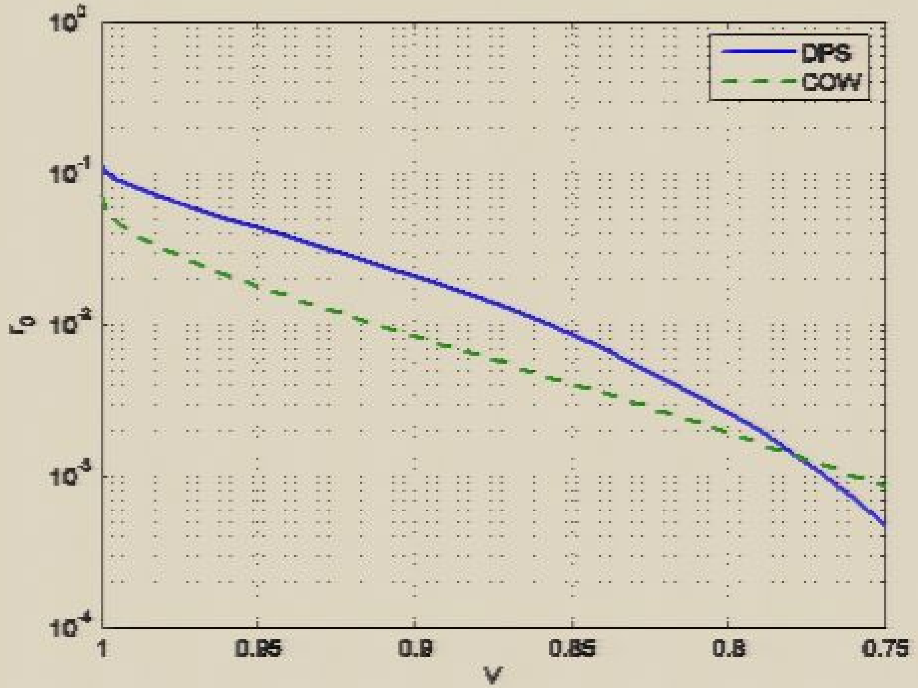

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  However, we hope our attacks are not that far from the optimal ones, and that our upper bounds are not that bad !

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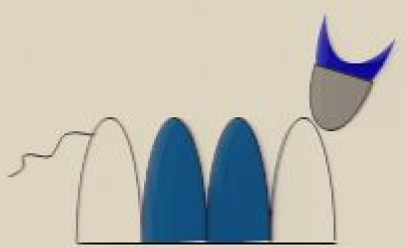
for your attention !

(and by the way, I also love you all very much ;-) !)



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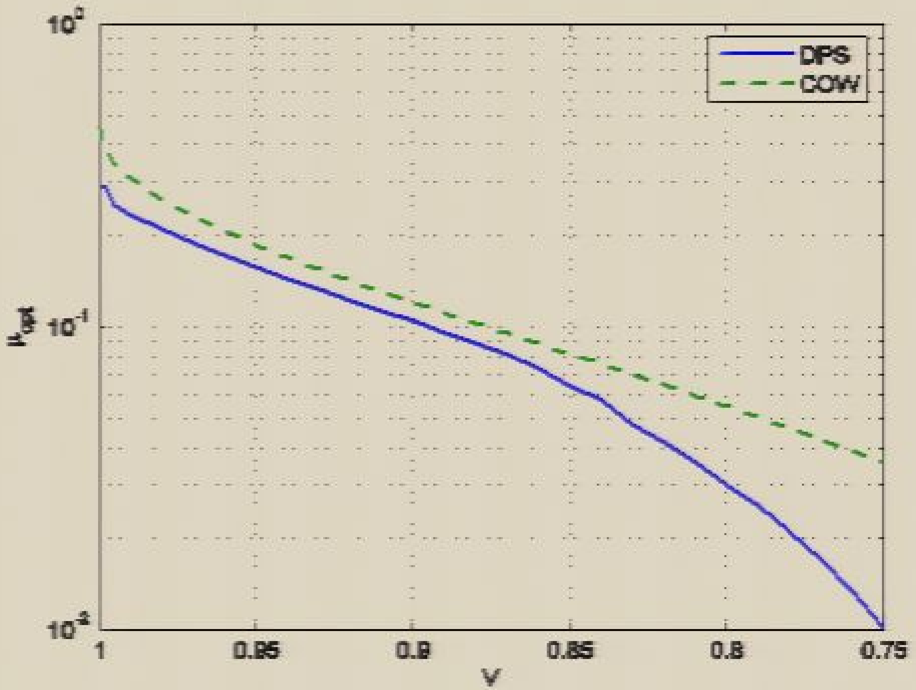
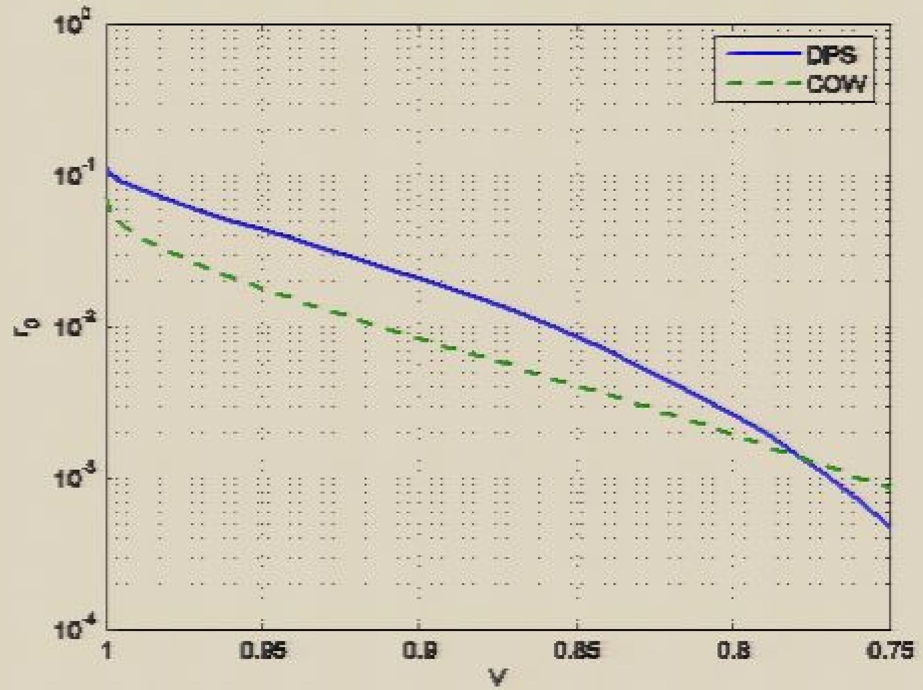
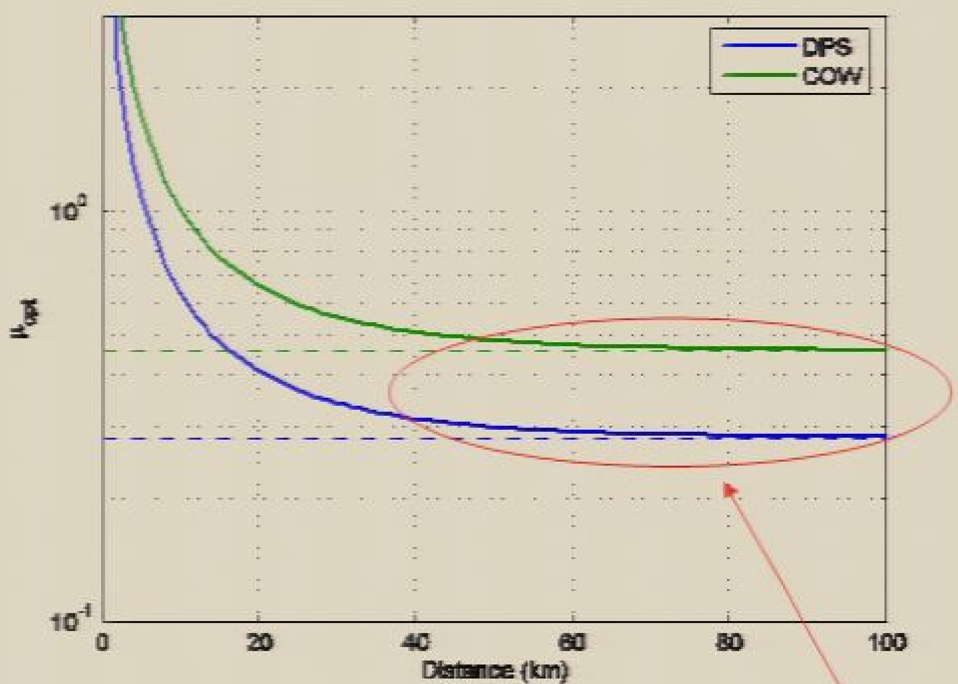
Security bounds for our attacks

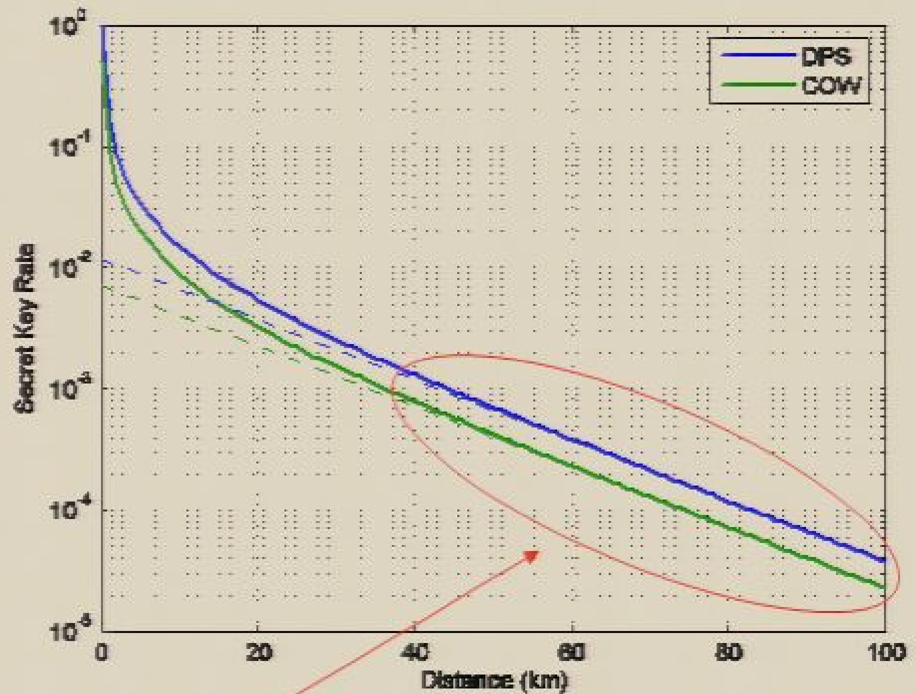




Secret key rates : $r = r_0 t \eta$

BS attack : DPS vs COW

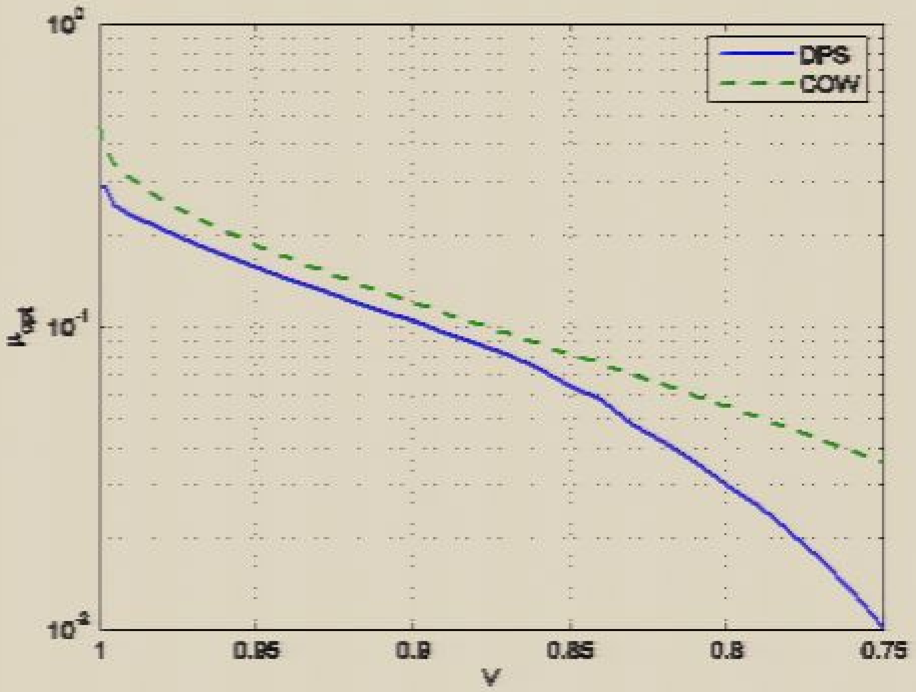


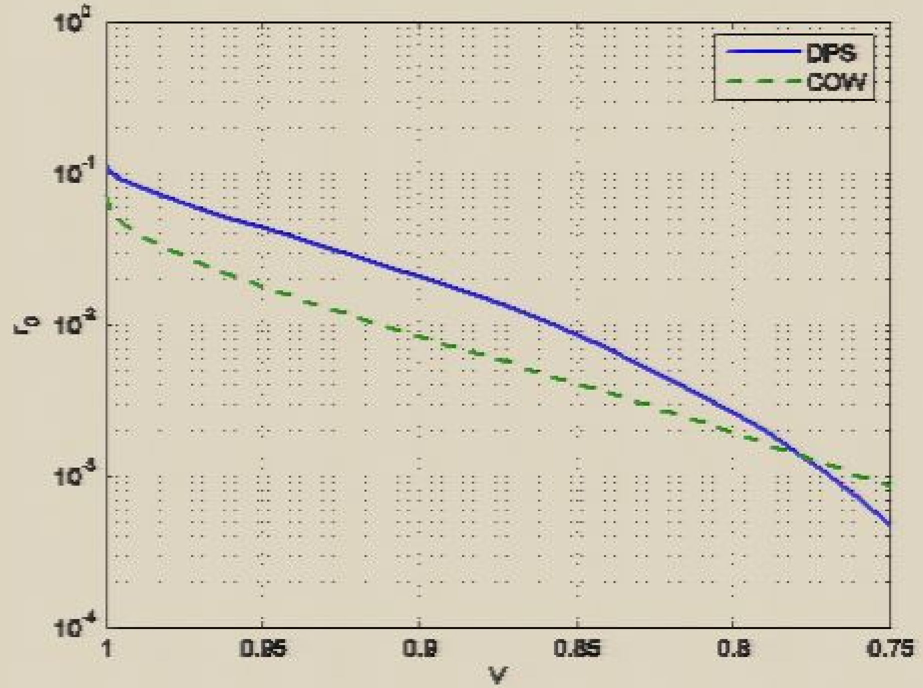


Linear regime :

for $\mu t << 1, \quad r = r_0 t \eta, \quad r_0 = r_0(\mu)$

Security bounds for our attacks





Secret key rates : $r = r_0 t \eta$