Title: Information Flow in the Heisenberg Picture

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Abstract:

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Cédric Bény

Perimeter Institute, June 2, 2007

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- They represent information which has been preserved by the channel

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The set $\mathcal{E}^\dagger(\Delta)$ characterizes the observables preserved by the channel $\mathcal{E}$

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There is a channel $\mathcal{R}$ such that

$$(\mathcal{R} \circ \mathcal{E})^\dagger(P) = P$$

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Correctable observables

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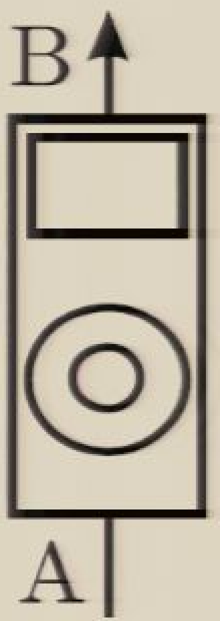
Complementary channel



- Consider the iPod^{TM}

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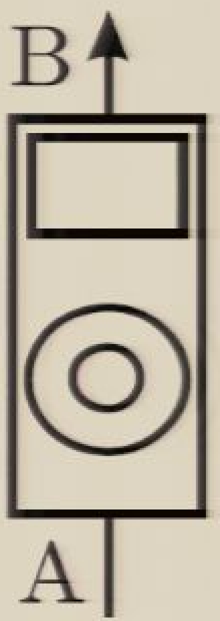

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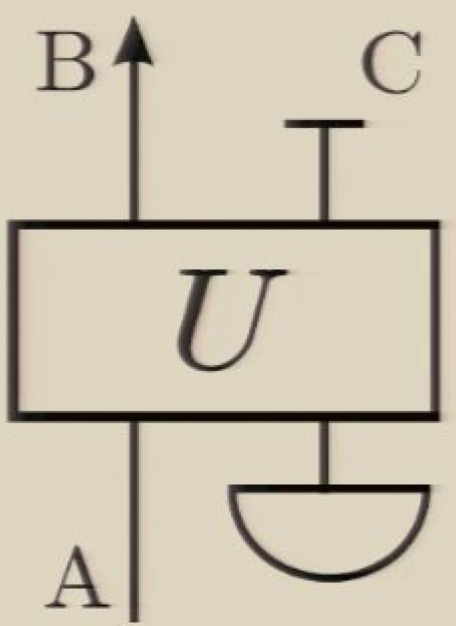

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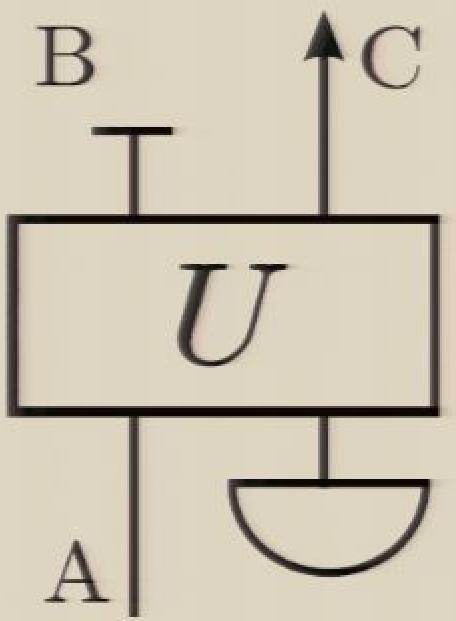

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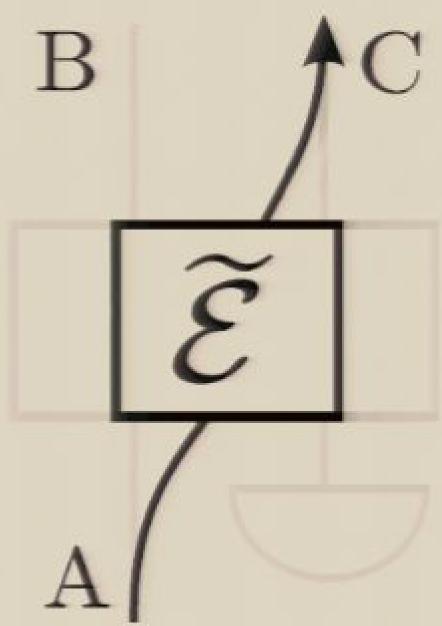

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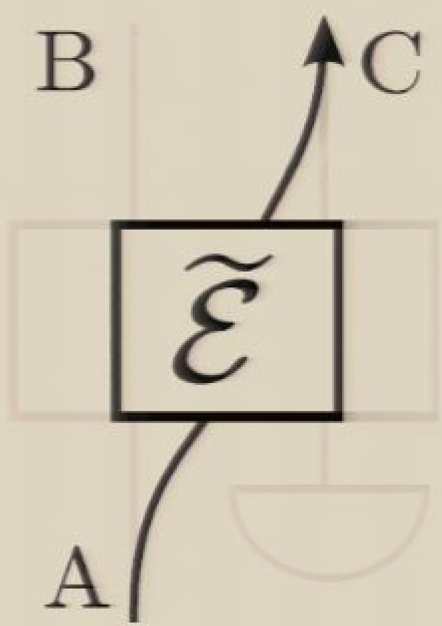

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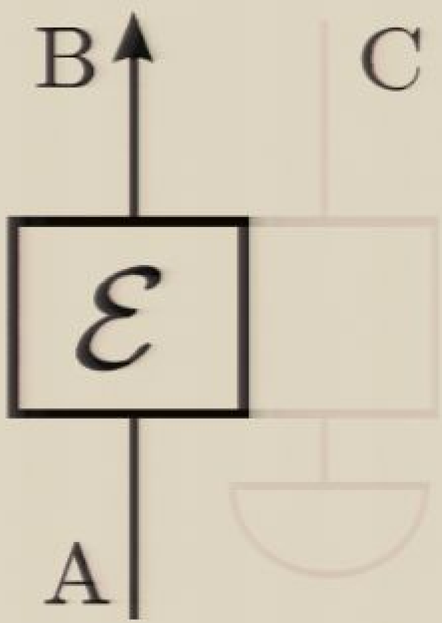
- $\widetilde{\mathcal{E}}$ characterizes the information which escapes into the "environment"

Predictive measurements

- $\mathcal{E}^\dagger(\Delta)$ is the information about A accessible in B



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- or, it is the information obtained about A which can serve for prediction on the state of B

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- The set $\mathcal{I}$ generalizes the notion of pointer states selected by the interaction with the environment

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- This is a perfect measurement of the observable Z

Example: coherent states

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- Then attempt to infer properties of the field by measuring the electrons
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- The effective channel on the field will correspond to a coarse-grained phase-space measurement

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- This describes the emergence of an approximately commuting phase space

Thank you for your attention!

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