Title: Quantum Mechanics in Phase Space

Date: Jun 02, 2007 10:00 AM

URL: http://pirsa.org/07060008

Abstract: Many authors have proposed what are known as "phase-space" or "classical" representations of quantum mechanics. A unifying framework is given which illustrates the relationship among these various theories. Examples relevant to quantum computing will be given.

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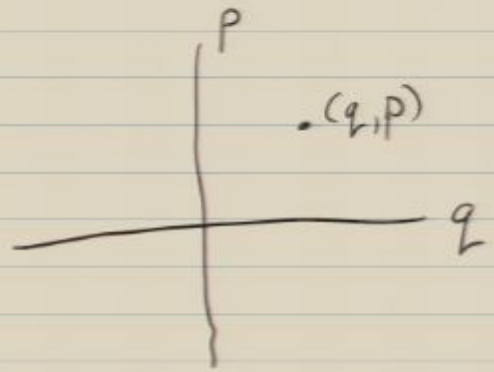
Classical Phase Space



- each point (q,p) represents a state of a classical system
- as system evolves, a path is traced out
- ignorance of system $\rightarrow$ system is represented as a probability distribution $\rho(q,p)$

Louiville equation:

$$\frac{d}{dt}\rho = \{H, \rho\}$$

$$\{H, \rho\} = H\left(\overleftarrow{\partial}_q \overrightarrow{\partial}_p - \overleftarrow{\partial}_p \overrightarrow{\partial}_q\right)\rho$$

Quantization:

$$q \longmapsto \hat{q}, \quad p \longmapsto \hat{p}$$

$$[\hat{q}, \hat{p}] = i\hbar$$

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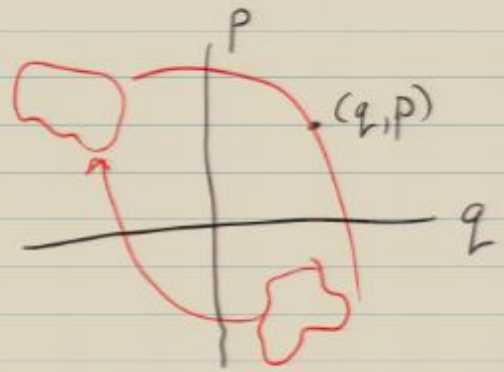

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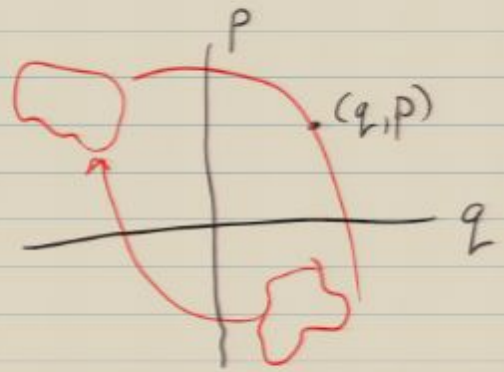

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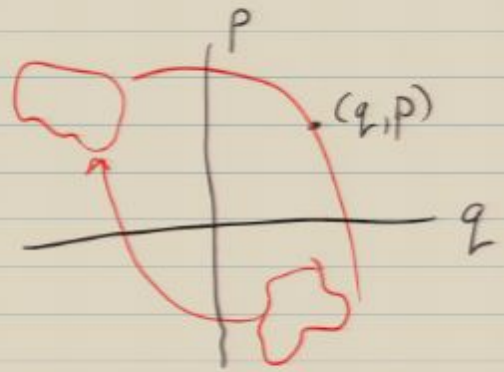

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ordering problem: $qp \longmapsto \hat{q}\hat{p}$ or $\hat{p}\hat{q}$?

1929 - Weyl: average over all possible orderings

$$\hat{W}(qp) = \frac{1}{2}(\hat{q}\hat{p} + \hat{p}\hat{q})$$

One can show: $\hat{W}(e^{\sigma q + \tau p}) = e^{\sigma \hat{q} + \tau \hat{p}}$

$$f(q,p) = \int_{\mathbb{R}^2} d\sigma d\tau \; F(\sigma,\tau)\, e^{i\sigma q + i\tau p}$$

$$F(\sigma,\tau) = \frac{1}{(2\pi)^2} \int_{\mathbb{R}^2} dq\, dp \; f(q,p)\, e^{-i\tau p - i\sigma q}$$

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$$f(q,p) = \hbar \int du\, e^{-ipu} \left\langle q + \frac{\hbar u}{2} \middle| \hat{W}(f) \middle| q - \frac{\hbar u}{2} \right\rangle$$

$$\rho(q,p) = \frac{W(\hat{\rho})}{2\pi\hbar}$$

<u>Problem</u>: ρ is negative! (For all but Guassian states)

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Wigner function

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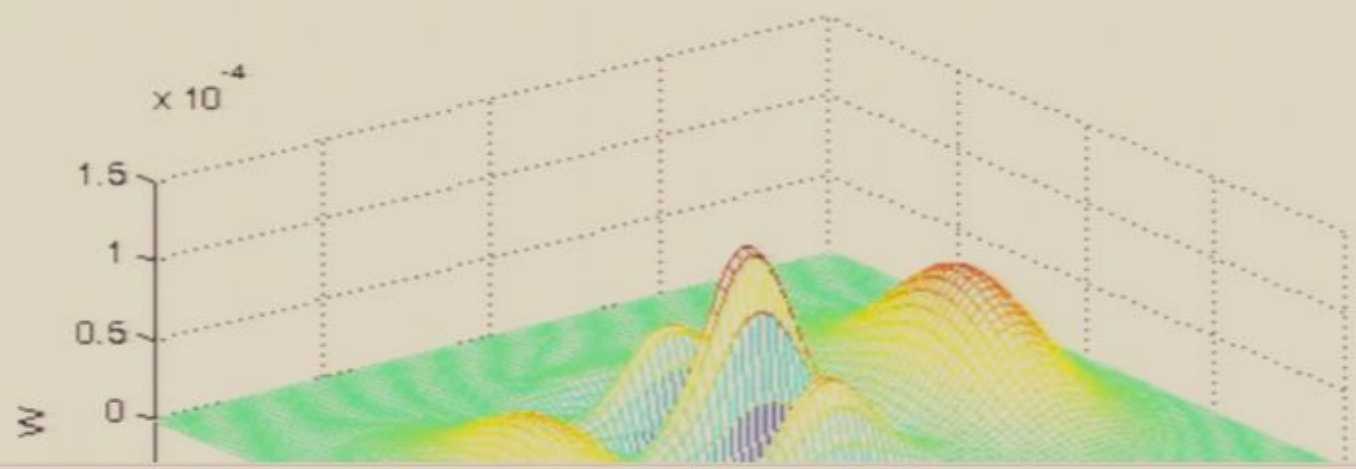

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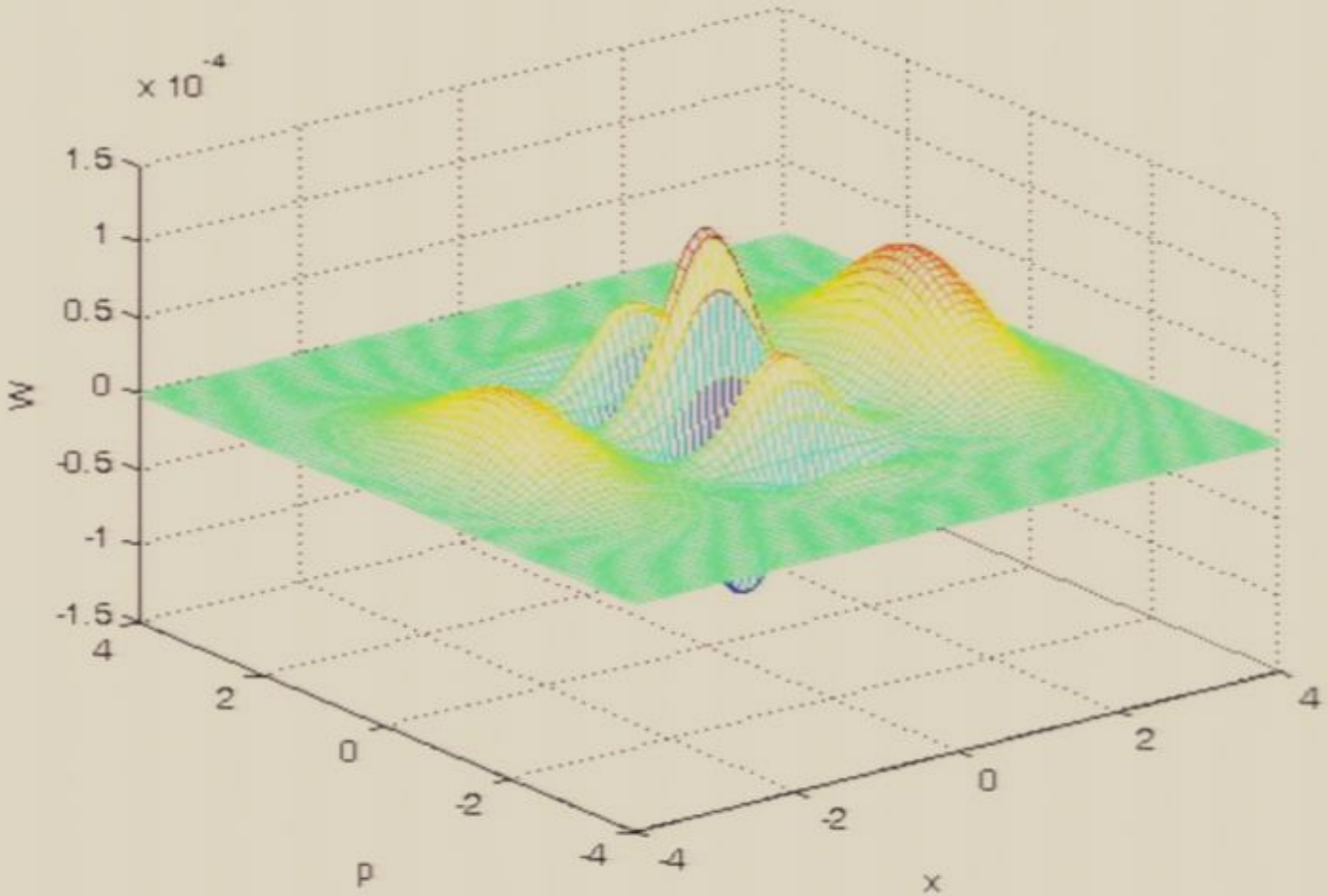

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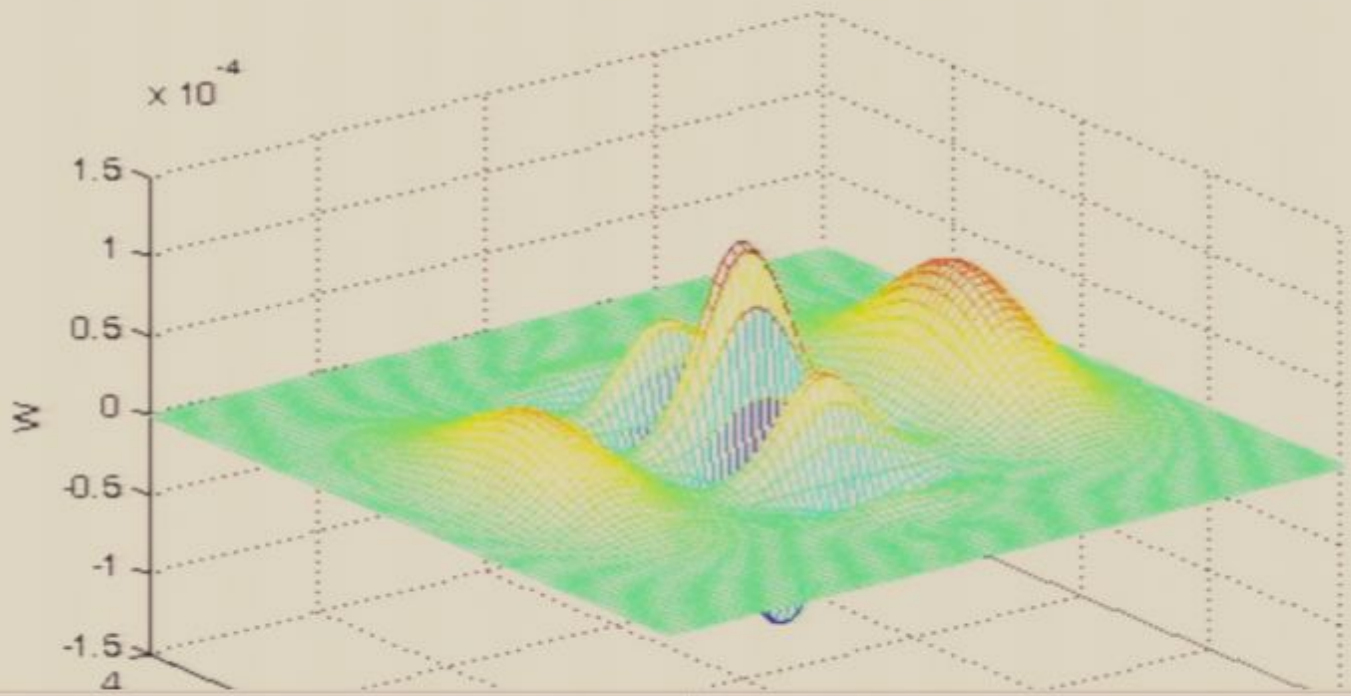

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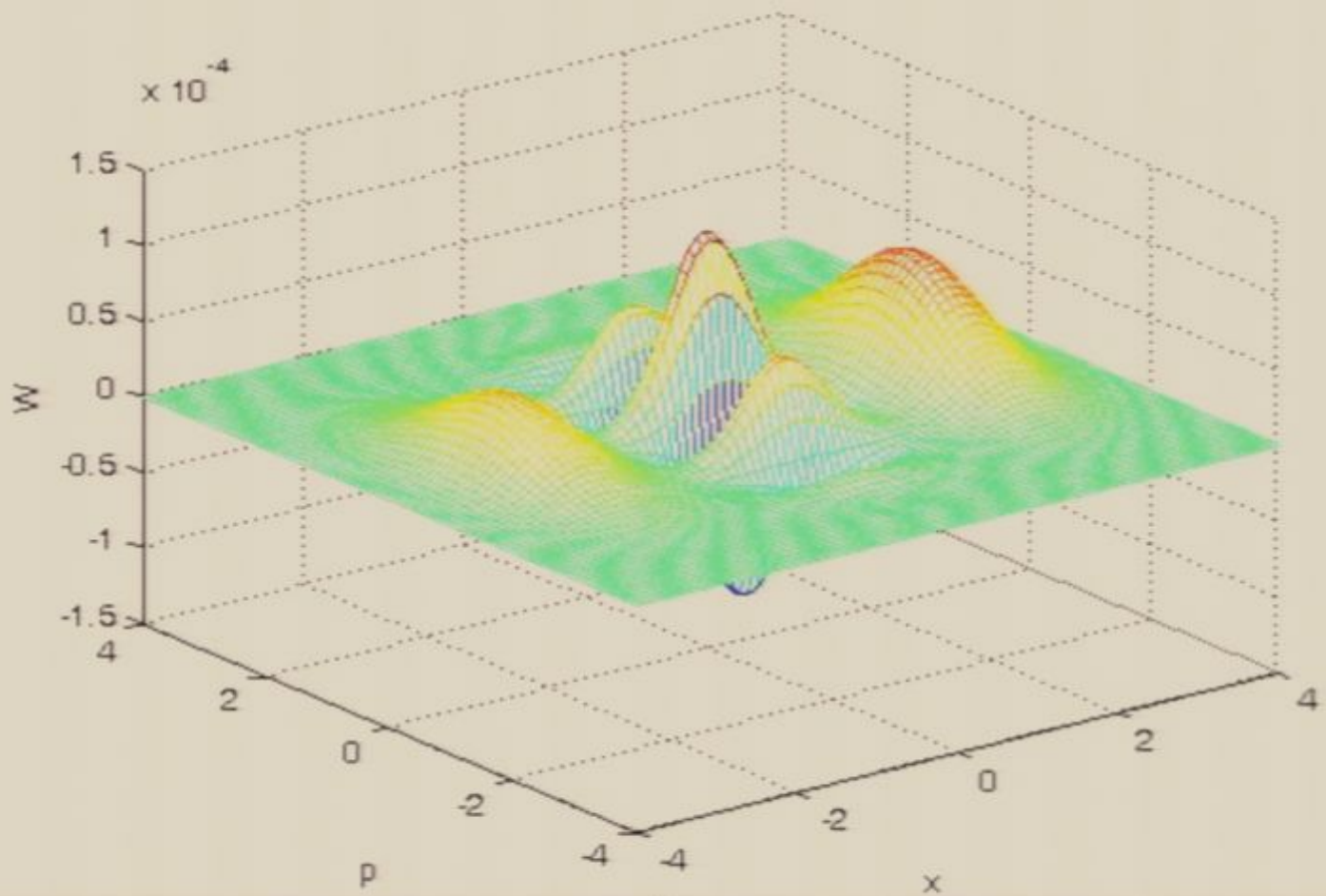

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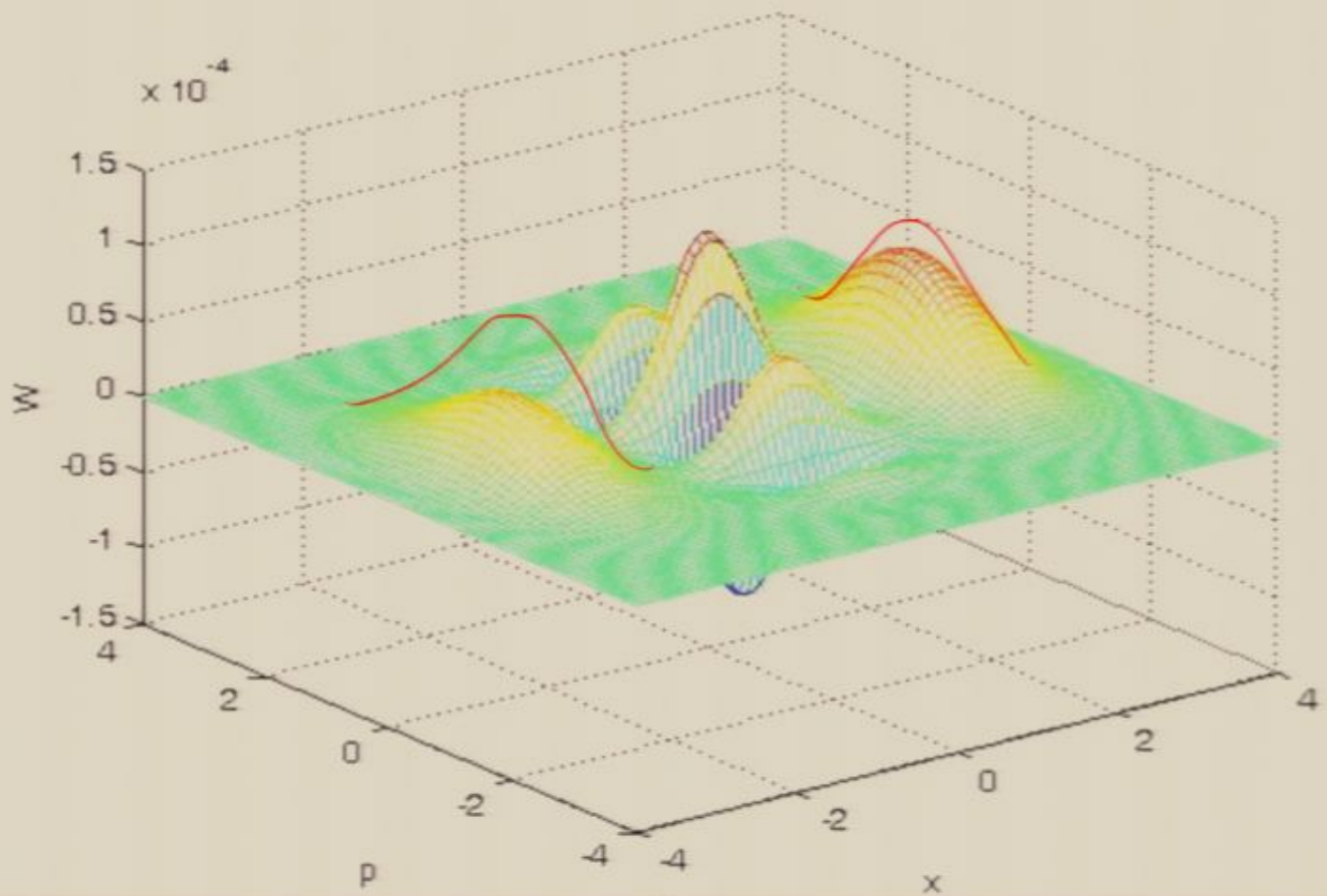

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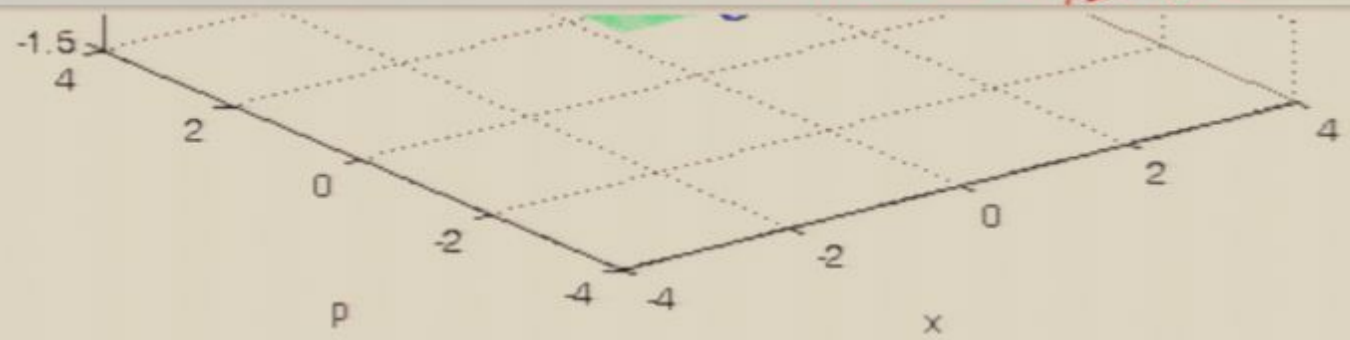

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Evolution of Wigner function is classical + quantum correction!

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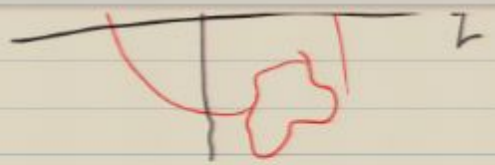

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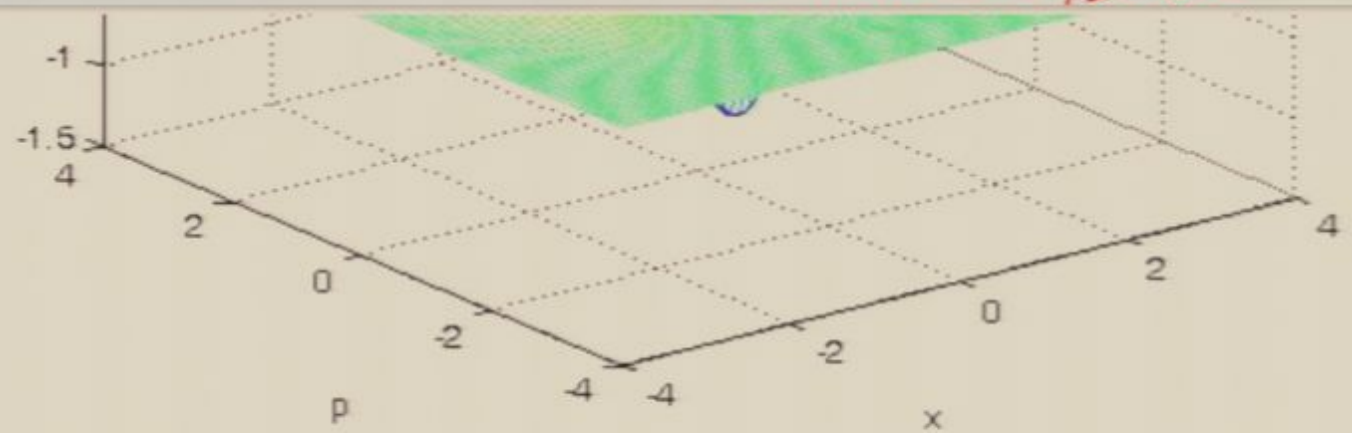

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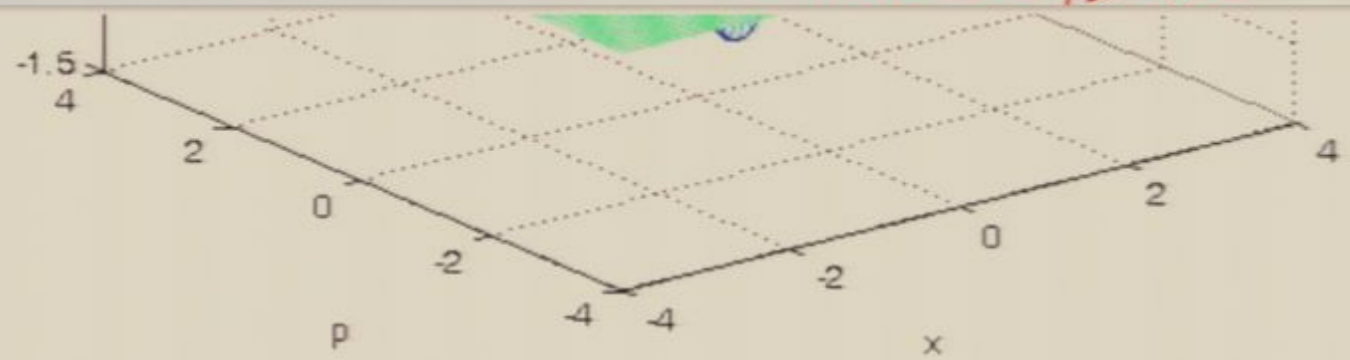

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Evolution of Wigner function is classical

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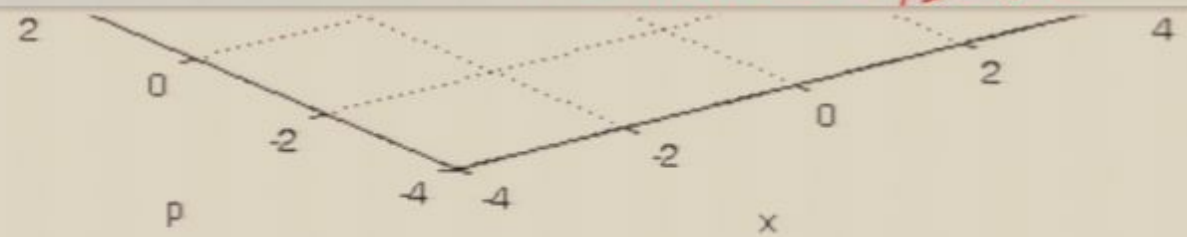

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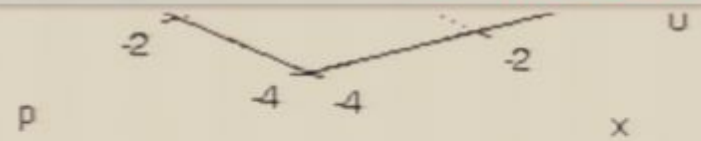

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Question: what about finite systems??

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Existing research

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87 – Wootters

Discrete phase space as $n \times n$ lattice of n prime. If n is composite, discrete phase space is Cartesian product of prime phase spaces.

99 – Weigert, et al.

Phase space as both a continuum and discrete set of points on a sphere for a spin $n = 2j + 1$.

01 – Hardy

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02 – Bianucci, et al.

- Discrete phase space as $2n \times 2n$ lattice.
- Analyze Grover's algorithm, etc

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- Discrete phase space as $2n \times 2n$ lattice.
- Analyze Grover's algorithm, etc.
- Valid for any n but contains 4-fold redundancy

04 – Vourdas

Discrete phase space as $n \times n$ lattice valid for

Existing research - Microsoft Office OneNote 2003

Existing research

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So on and so forth...

Existing research - Microsoft Office OneNote 2003

Vector representations of linear operators

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So on and so forth...

Vector representations of linear operators - Microsoft Office OneNote 2003

Vector representations of linear operators

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Abstract complex Hilbert space $\mathcal{H}$, $\dim(\mathcal{H}) =: n$

Algebra of linear operator $L(\mathcal{H})$ with inner product $\langle \hat{A}, \hat{B} \rangle := \mathrm{Tr}(\hat{A}^\dagger \hat{B})$

Let $\{\hat{E}_\alpha : \alpha \in \Sigma\}$ be a basis of Hermetian operators

There exists a dual basis $\{\hat{E}^\alpha : \alpha \in \Sigma\} \subset L(\mathcal{H})$ s.t.

$$\langle \hat{E}^\alpha, \hat{E}_\beta \rangle = \delta^\alpha{}_\beta$$

The two bases transform via Gram matrix

$$\hat{E}_\alpha = G_{\alpha\beta} \hat{E}^\beta$$

$$G_{\alpha\beta} := \langle \hat{E}_\alpha, \hat{E}_\beta \rangle$$

$$\Rightarrow (G^{-1})_{\alpha\beta} = \langle \hat{E}^\alpha, \hat{E}^\beta \rangle =: G^{\alpha\beta}$$

$\forall \hat{A} \in L(\mathcal{H})$ the transformation

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Vector representations of linear operators

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$$\hat{A} = A_\alpha \hat{E}^\alpha \quad , \quad A_\alpha = \langle \hat{E}_\alpha, \hat{A} \rangle \qquad (*)$$

is an isomorphism between $L(\mathcal{H})$ and $\mathbb{C}^\Sigma$

Call $\{A_\alpha : \alpha \in \Sigma\} =: A \subset \mathbb{C}^\Sigma$ the vector representation of $\hat{A}$

The inner product induced on $\mathbb{C}^\Sigma$ by $(*)$ is

$$\langle \hat{A}, \hat{B} \rangle = A_\alpha B_\beta G^{\alpha\beta} =: \langle A, B \rangle$$

The algebra induced on $\mathbb{C}^\Sigma$ is given by

$$(A+B)_\alpha = A_\alpha + B_\alpha$$

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Vector representations of linear operators

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Vector representations of linear operators

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Vector representations of linear operators

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Vector representations of linear operators

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<u>Open Question</u>: What properties on Γ are sufficient to imply $(**)$?

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Quantum Mechanics on $\mathbb{C}^{\Sigma}$

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<u>Axiom 1</u>: a physical system with n distinguishable preparations is represented by a state $\rho \in \mathbb{C}^{\Sigma}$

<u>Axiom 2</u>: Extraction of classical information from a system in state ρ is described by a measurement $\{P_k : k \in \Delta\}$ where the probability of outcome k being observed is

$$Pr(k) = \langle P_k, \rho \rangle$$

<u>Axiom 3</u>: The state representing a system evolves according to

$$\frac{d}{dt}\rho = \{\{H, \rho\}\} = \frac{1}{i\hbar}\left(H \star \rho - \rho \star H\right),$$

where $H \in \mathbb{C}^{\Sigma}$ is any real function.

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Quantum Mechanics on $\mathbb{C}^\Sigma$

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Def: The *identity* is the unique $\mathbb{1} \in \mathbb{C}^\Sigma$ s.t.

$$\mathbb{1} \star A = A \star \mathbb{1} = A, \quad \forall A \in \mathbb{C}^\Sigma$$

Def: a *pure state* is any $\rho \in \mathbb{C}^\Sigma$ s.t.

$$\rho \star \rho = \rho.$$

Def: a *state* is any $\rho \in \mathbb{C}^\Sigma$ s.t.

$$\langle \rho, \sigma \rangle \geq 0, \quad \forall \sigma \in \mathbb{C}^\Sigma \text{ pure}$$

and

$$\langle \rho, \mathbb{1} \rangle = 1.$$

Def: a *measurement* is any set $\{P_k : k \in \Delta\} \subset \mathbb{C}^\Sigma$ s.t.

$$\langle P_k, \sigma \rangle \geq 0 \quad \forall \sigma \in \mathbb{C}^\Sigma \text{ pure}, \ \forall k \in \Delta$$

and

$$\sum_{k \in \Delta} P_k = \mathbb{1}.$$

Axiom 1: a closed quantum system with a distinguishable

Quantum Mechanics on GE - Microsoft Office OneNote 2003

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state is real

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Quantum Mechanics on $\mathbb{C}^{\Sigma}$

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discrete star product

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where $H \in \mathbb{C}^{\Sigma}$ is any real function.

Example 1 - Microsoft Office OneNote 2003

Example 1

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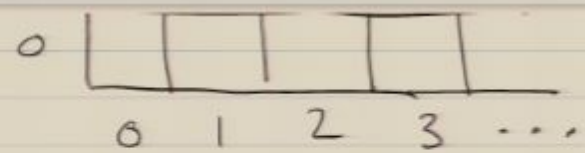

The inner product on $\mathbb{C}^{\Sigma}$ is

$$\langle A, B \rangle = \sum_{\alpha} A_{\alpha} B_{\alpha}$$

an multiplication is

$$(A \star B)_{\alpha} = \sum_{\beta\gamma} A_{\beta} B_{\gamma} \Gamma_{\alpha\beta\gamma}$$

$$\Gamma_{\alpha\beta\gamma} = e^{\frac{4\pi i}{n}\left[p_1(q_3-q_2)+p_2(q_1-q_3)+p_3(q_2-q_2)\right]}$$

$\alpha = (q_1, p_1)$
$\beta = (q_2, p_2)$
$\gamma = (q_3, p_3)$

Is this space isomorphic to $L(\mathcal{H})$? i.e. does there exist a basis such that $G_{\alpha\beta} = \delta_{\alpha\beta} = \langle \hat{E}_{\alpha}, \hat{E}_{\beta} \rangle$ and $\Gamma_{\alpha\beta\gamma} = \langle \hat{E}_{\alpha}, \hat{E}_{\beta} \hat{E}_{\gamma} \rangle$?

YES!

Example 1 - Microsoft Office OneNote 2003

Example 1

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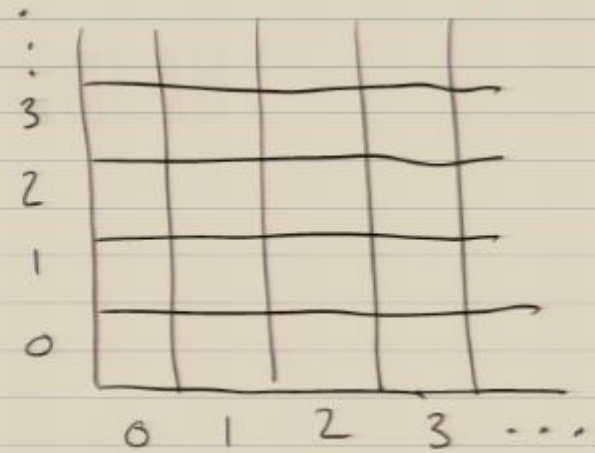
Wootters (87)

$\Sigma = \mathbb{Z}_n \times \mathbb{Z}_n$ ← discrete phase space

n prime



Each $\alpha \in \Sigma$

$\alpha = (q, p)$

The inner product on $\mathbb{C}^{\Sigma}$ is

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Example 1 - Microsoft Office OneNote 2003

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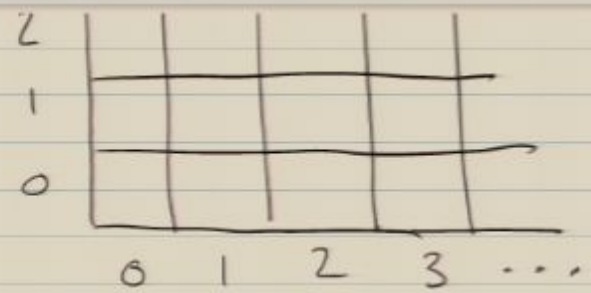

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Example 1 - Microsoft Office OneNote 2003

Example 1

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<u>YES!</u>

Consider $\hat{z}$ with $\mathrm{spec}(\hat{z}) = \{\omega^k : k \in \mathbb{Z}_n\}$, $\omega = e^{i\frac{2\pi}{n}}$

Eigenvectors of $\hat{z}$ form a basis for $\mathcal{H}$

$$\{|k\rangle : k \in \mathbb{Z}_n\}$$

Consider also $\hat{X}$ defined by

Example 1 - Microsoft Office OneNote 2003

Example 1

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exist a basis such that $c_{\alpha\beta} = \delta_{\alpha\beta} = \langle L_\alpha, L_\beta \rangle$

and $\Gamma_{\alpha\beta\gamma} = \langle \hat{E}_\alpha, \hat{E}_\beta \hat{E}_\gamma \rangle$?

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Consider $\hat{Z}$ with $\mathrm{spec}(\hat{Z}) = \{\omega^k : k \in \mathbb{Z}_n\}$, $\omega = e^{i\frac{2\pi}{n}}$

Eigenvectors of $\hat{Z}$ form a basis for $\mathcal{H}$

$$\{|k\rangle : k \in \mathbb{Z}_n\}$$

Consider also $\hat{X}$ defined by

$$\hat{X}|k\rangle = |k+1\rangle$$

$\hat{Z}, \hat{X}$ are called generalized Pauli operators

<u>$n=2$</u> $\mathrm{spec}(\hat{Z}) = \{1, -1\} \Rightarrow$ eigenbasis $\{|0\rangle, |1\rangle\}$

$$\left.\begin{array}{l} \hat{Z}|0\rangle = |0\rangle, \quad \hat{Z}|1\rangle = -|1\rangle \\ \hat{X}|0\rangle = |1\rangle, \quad \hat{X}|1\rangle = |0\rangle \end{array}\right\} \text{usual Pauli operators}$$

Example 1 - Microsoft Office OneNote 2003

Example 1

May 31, 2007
10:51 AM

<u>YES!</u>

Consider $\hat{Z}$ with $\mathrm{spec}(\hat{Z}) = \{\omega^k : k \in \mathbb{Z}_n\}$, $\omega = e^{i\frac{2\pi}{n}}$

Eigenvectors of $\hat{Z}$ form a basis for $\mathcal{H}$

$$\{|k\rangle : k \in \mathbb{Z}_n\}$$

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<u>$n=2$</u> $\quad \mathrm{spec}(\hat{Z}) = \{1, -1\} \Rightarrow$ eigenbasis $\{|0\rangle, |1\rangle\}$

$$\left.\begin{array}{l} \hat{Z}|0\rangle = |0\rangle, \quad \hat{Z}|1\rangle = -|1\rangle \\ \hat{X}|0\rangle = |1\rangle, \quad \hat{X}|1\rangle = |0\rangle \end{array}\right\} \text{usual Pauli operators}$$

$$\left\{ \frac{1}{n} \sum_{k=0}^{n-1} \omega^{pk - qj + \frac{kj}{2}} \hat{X}^k \hat{Z}^j : (q,p) \in \mathbb{Z}_n \times \mathbb{Z}_n \right\}$$

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$\mathcal{H}=\mathbb{C}^2$ spec(Z) = $\{1,-1\}$ → eigenbasis $\{|0\rangle,|1\rangle\}$

$$\left.\begin{array}{l}\hat{Z}|0\rangle=|0\rangle\ ,\ \hat{Z}|1\rangle=-|1\rangle\\ \hat{X}|0\rangle=|1\rangle\ ,\ \hat{X}|1\rangle=|0\rangle\end{array}\right\}\text{usual Pauli operators}$$

$$\left\{\frac{1}{n}\sum_{k,j=0}^{n-1}\omega^{pk-qj+\frac{kj}{2}}\hat{X}^k\hat{Z}^j\ :\ (q,p)\in\mathbb{Z}_n\times\mathbb{Z}_n\right\}$$

is a basis for $L(\mathcal{H})$ with the above properties.

$|0\rangle$

$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$

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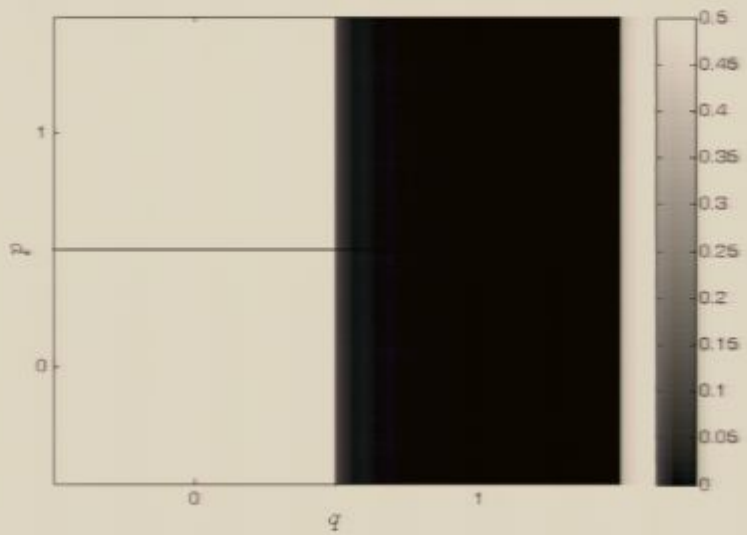
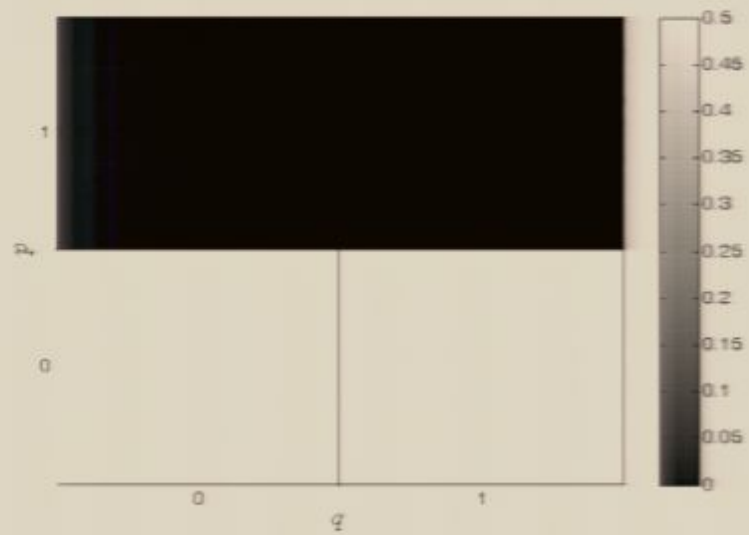
$|0\rangle$



$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$



trine measurement

$|1\rangle$

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Example 1

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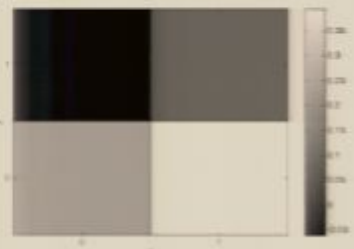
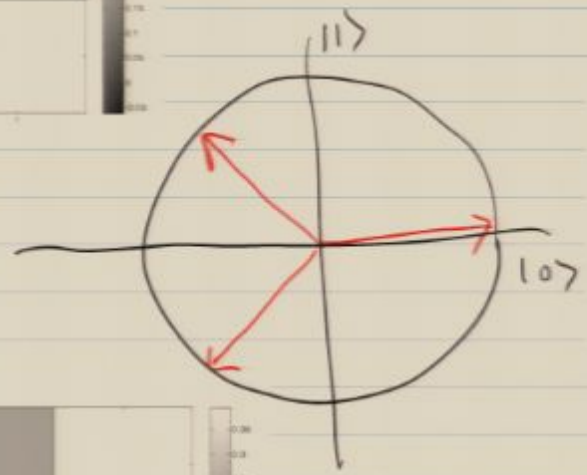
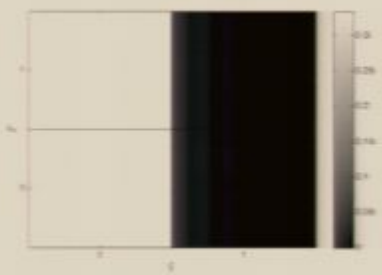
trine measurement







GHZ state — $\frac{1}{\sqrt{n}}(|00\cdots\rangle + |11\cdots\rangle)$

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Example 1

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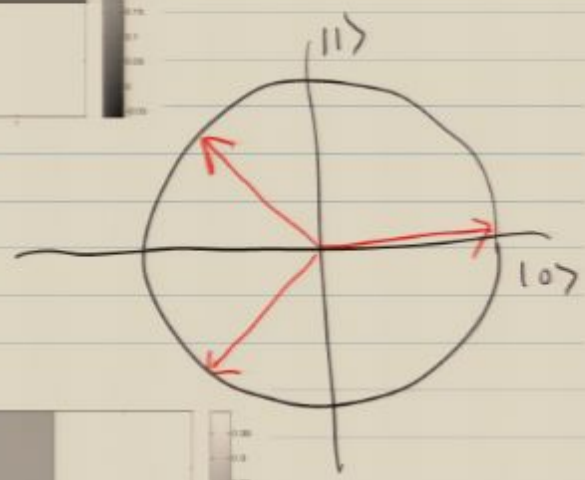
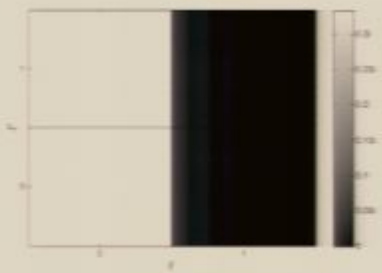
trine measurement





GHZ state — $\frac{1}{\sqrt{n}}\left(|00\cdots\rangle + |11\cdots\rangle\right)$

$n=4$ $n=8$ $n=16$

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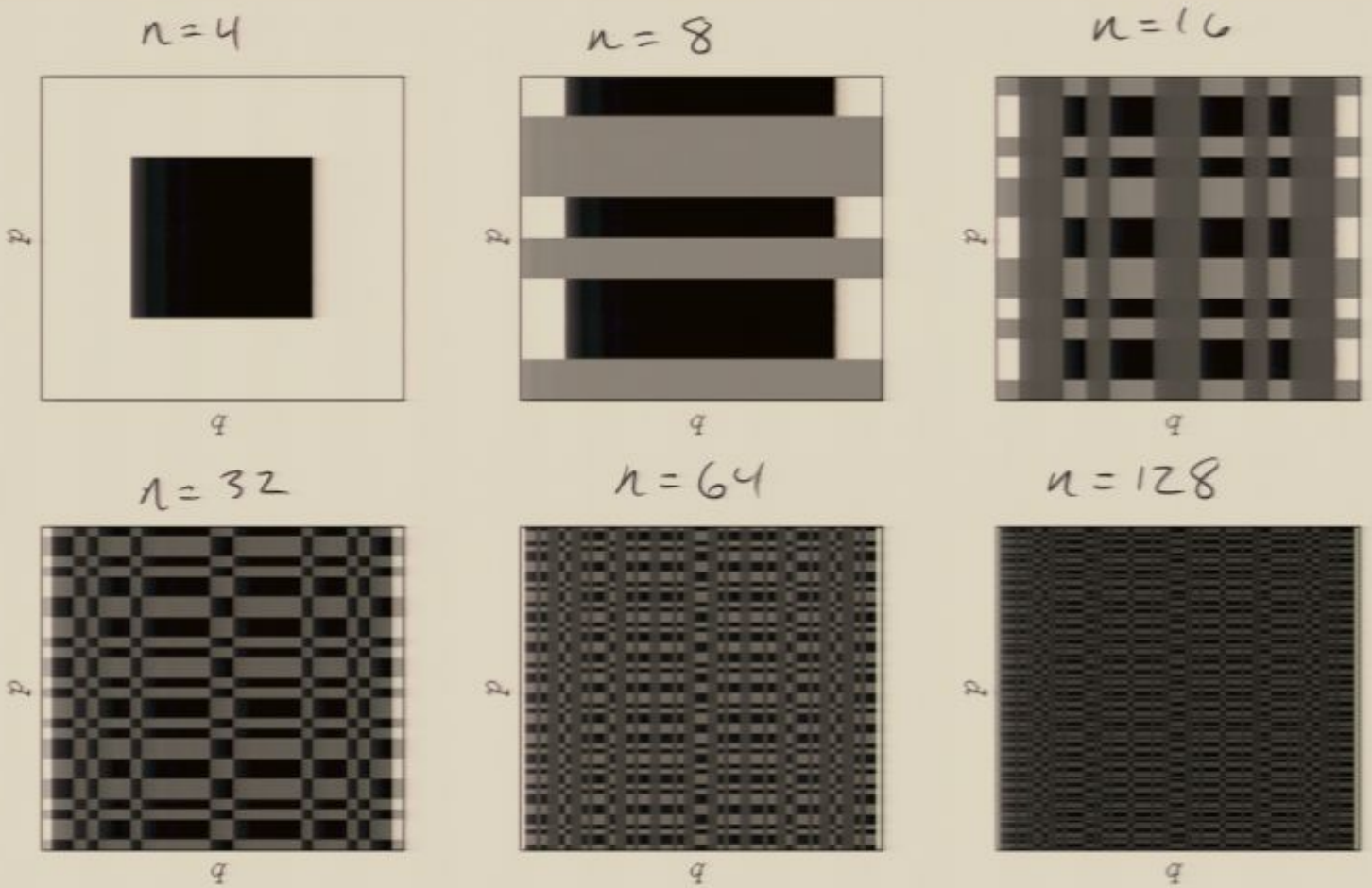
GHZ state — $\frac{1}{\sqrt{n}}\left(|00\cdots\rangle + |11\cdots\rangle\right)$

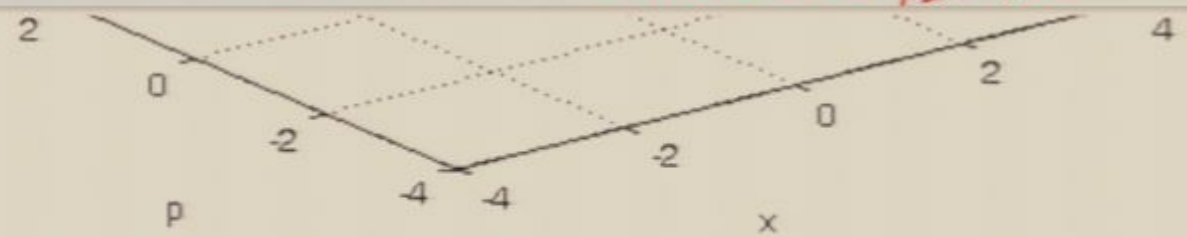


Introduction - Microsoft Office OneNote 2003

Introduction

CHRIS FERRIE, IQC

June 1, 2007
12:22 PM



<u>1949</u> – <u>Moyal</u>: $\hat{W}(f)\hat{W}(g) = \hat{W}(f \star g)$ star product

$$f \star g = f(q,p)\, e^{\frac{i\hbar}{2}\left(\overleftarrow{\partial_q}\overrightarrow{\partial_p} - \overleftarrow{\partial_p}\overrightarrow{\partial_q}\right)}\, g(q,p)$$

$$\frac{d}{dt}\hat{\ell} = \frac{1}{i\hbar}[\hat{H}, \hat{\ell}] \longmapsto \frac{d}{dt}\ell = \{\{H, \ell\}\}$$ Moyal bracket

$$= \{H, \ell\} + \text{"}O(\hbar)\text{"}$$

Evolution of Wigner function is classical + quantum correction!

Question: What about finite systems??

Introduction - Microsoft Office OneNote 2003

Introduction

CHRIS FERRIE, IQC

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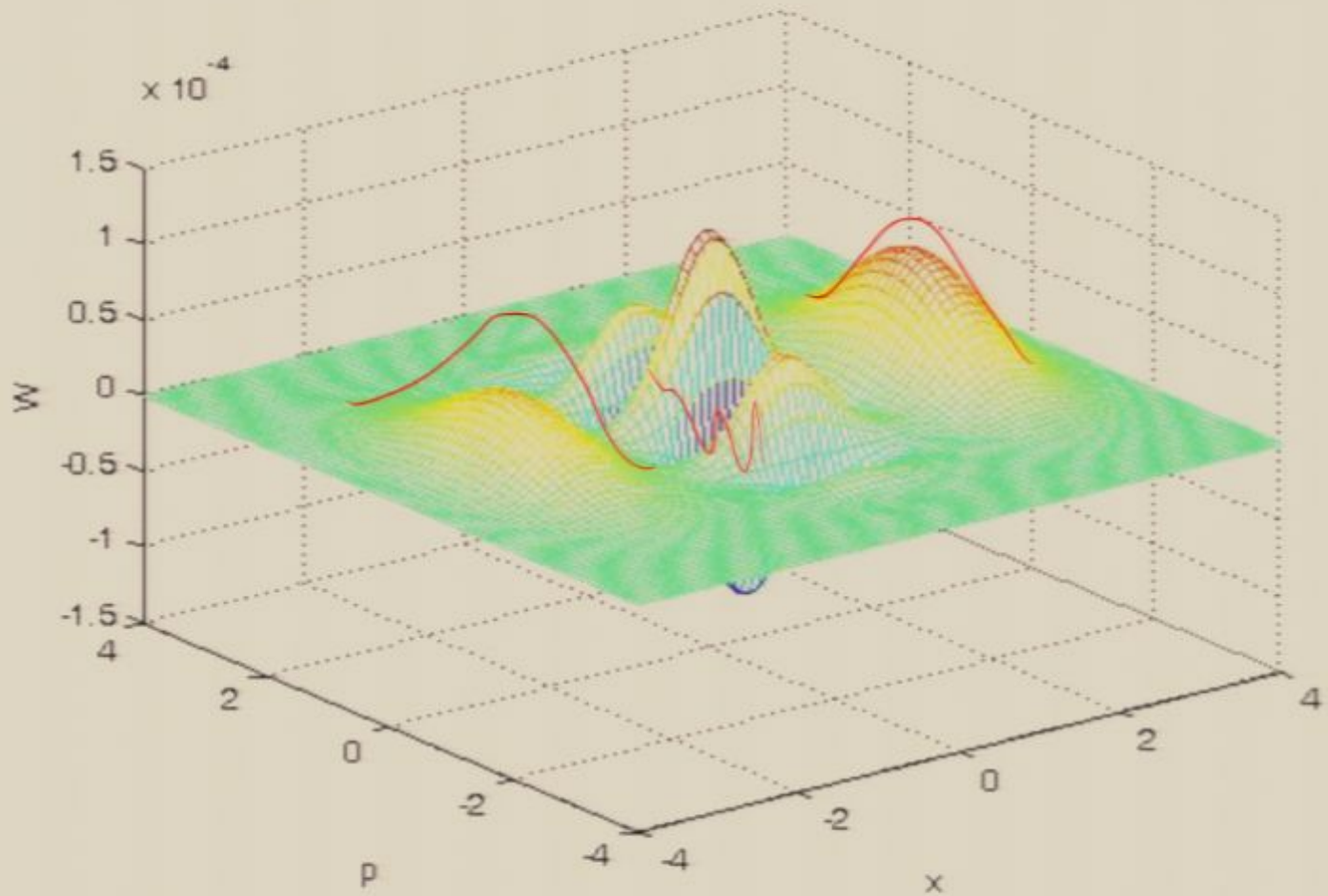

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Example 1

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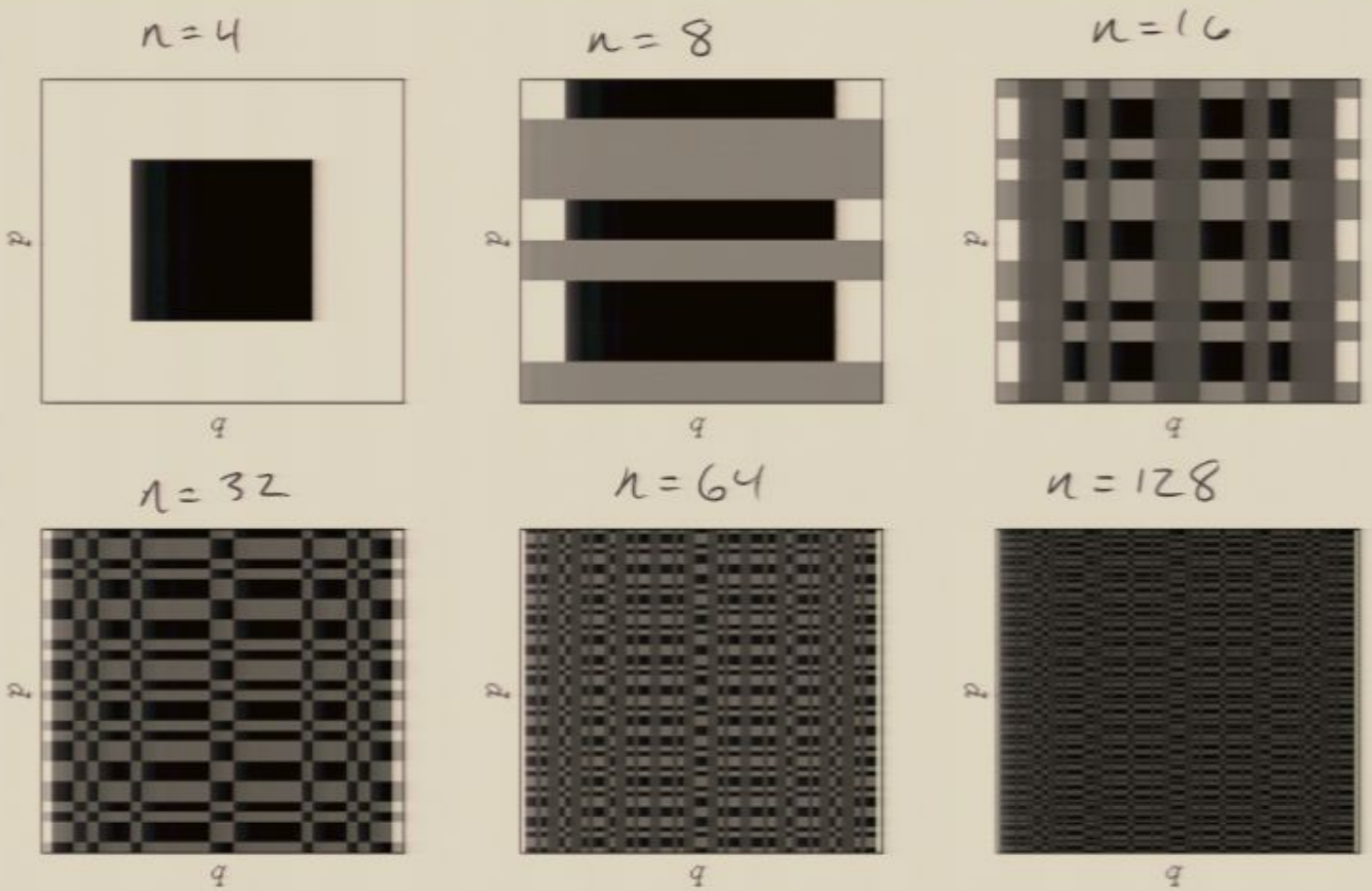
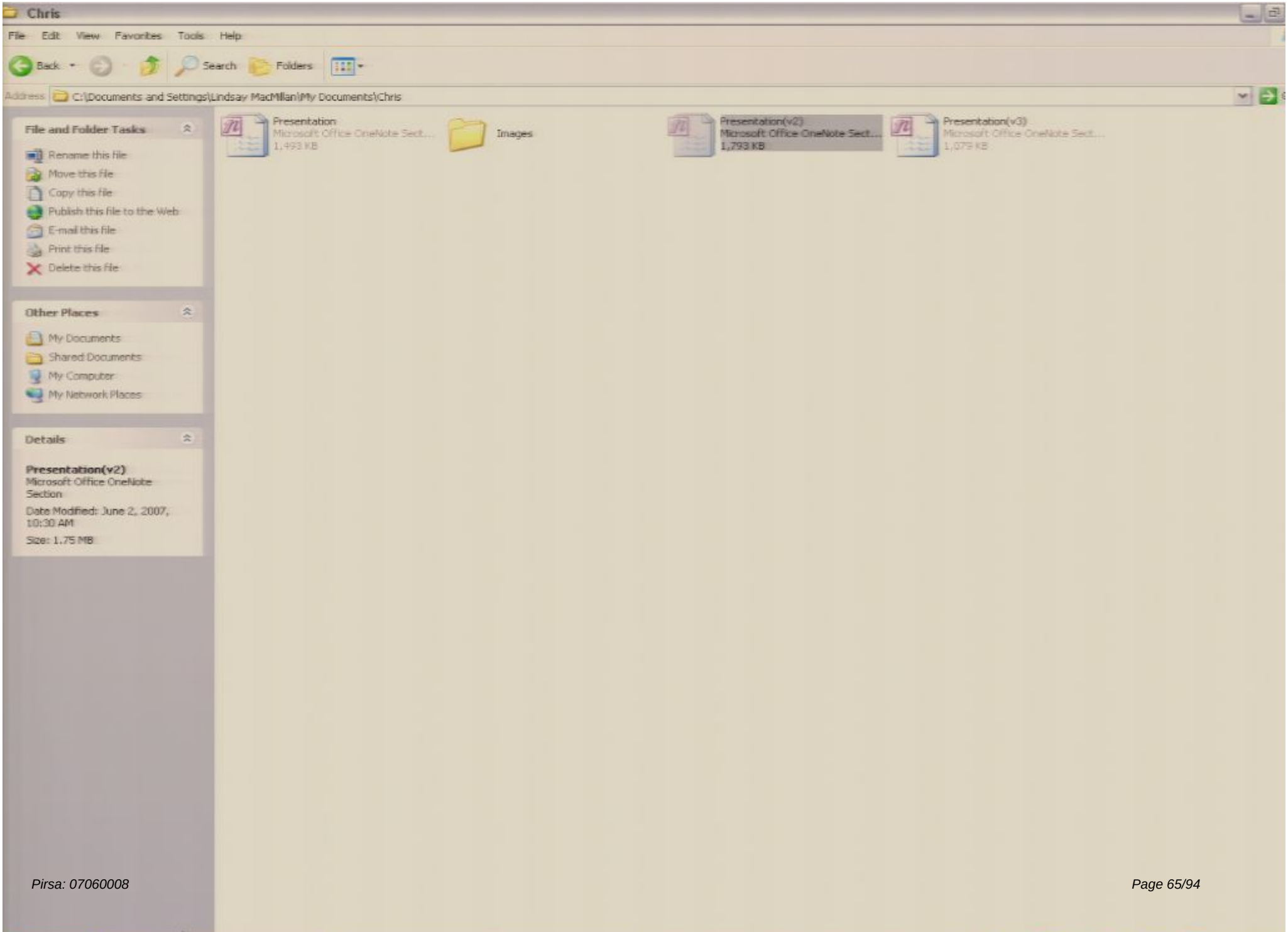
GHZ state — $\frac{1}{\sqrt{n}}\left(|00\cdots\rangle + |11\cdots\rangle\right)$

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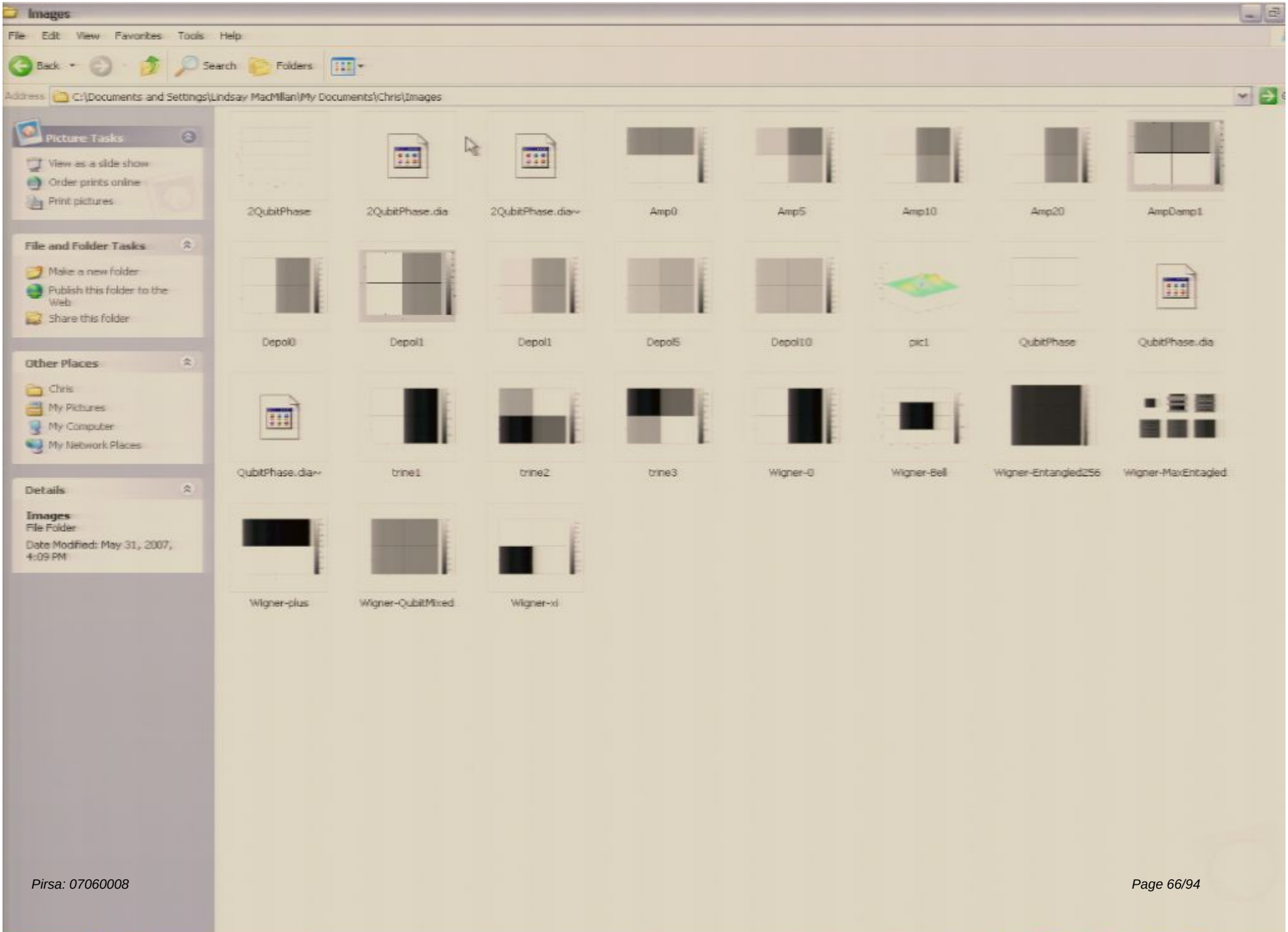
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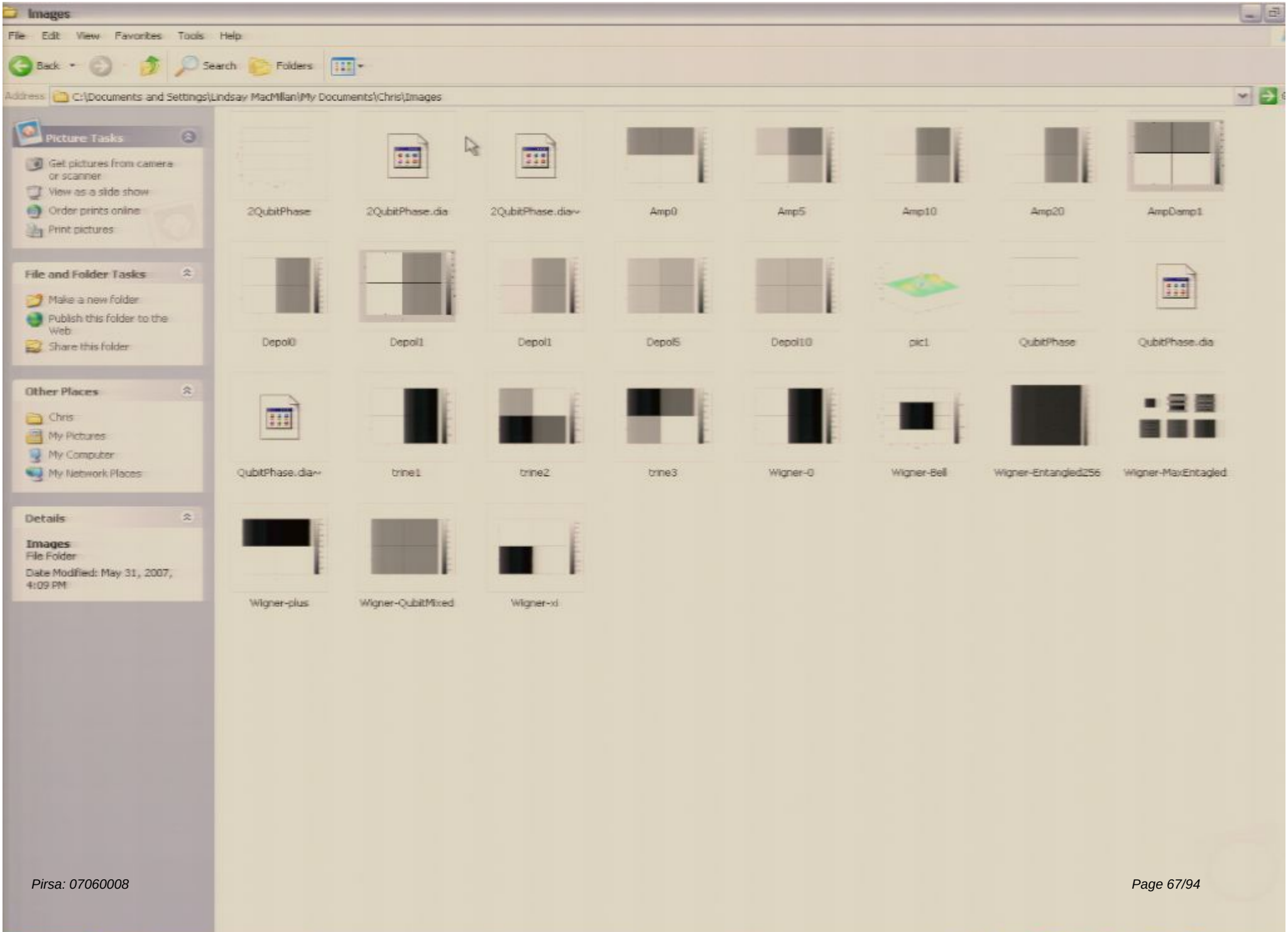
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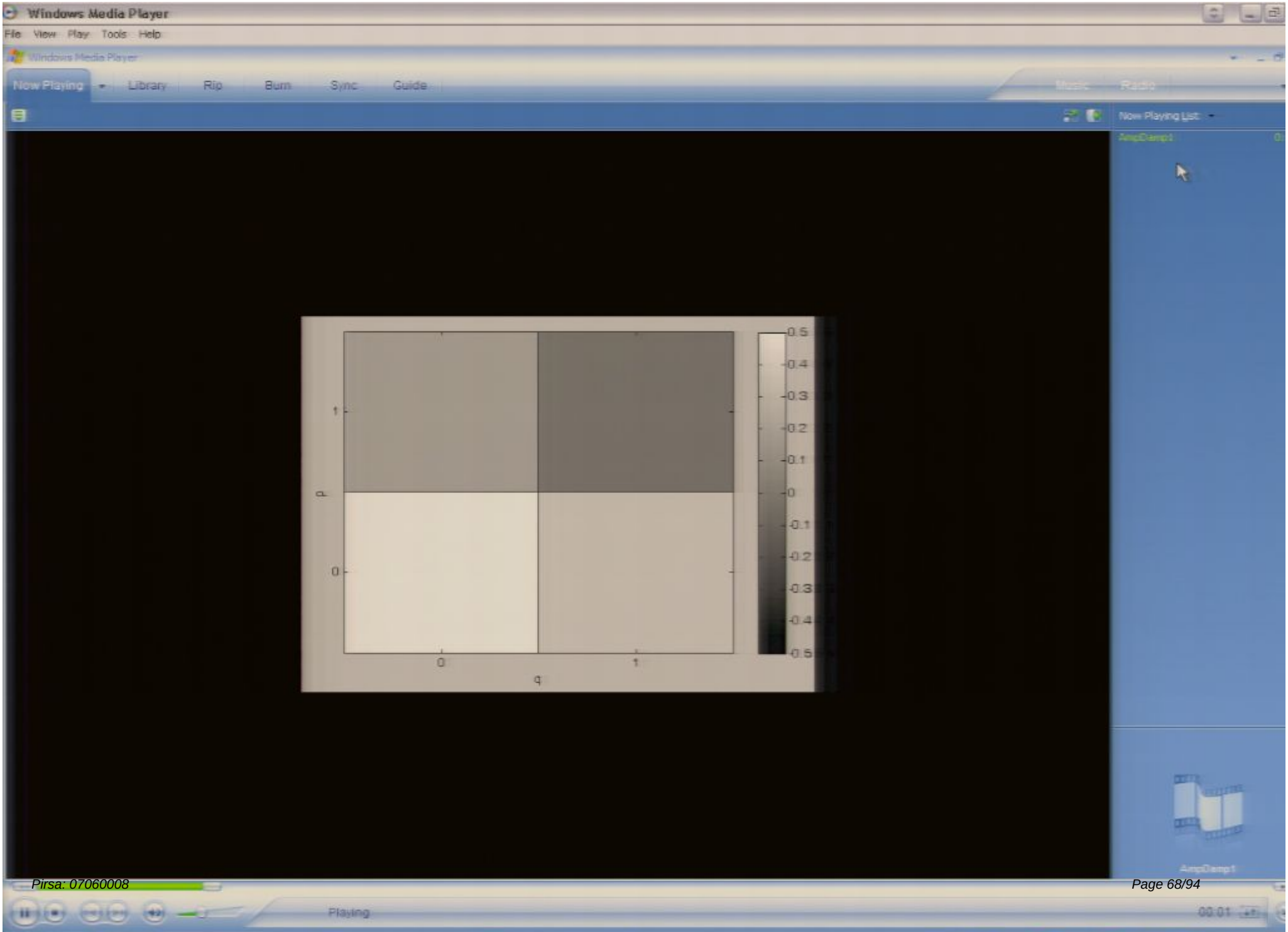
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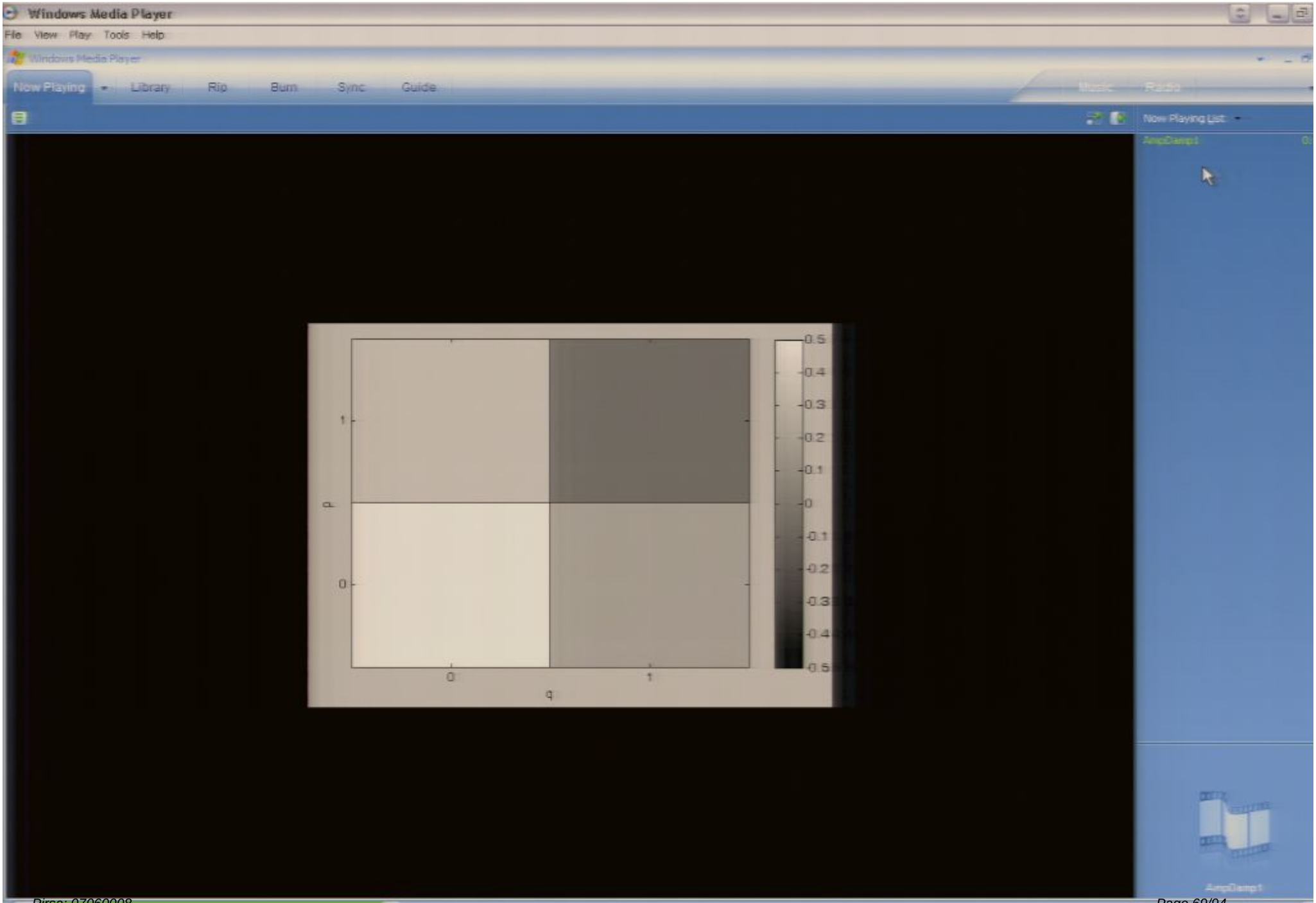
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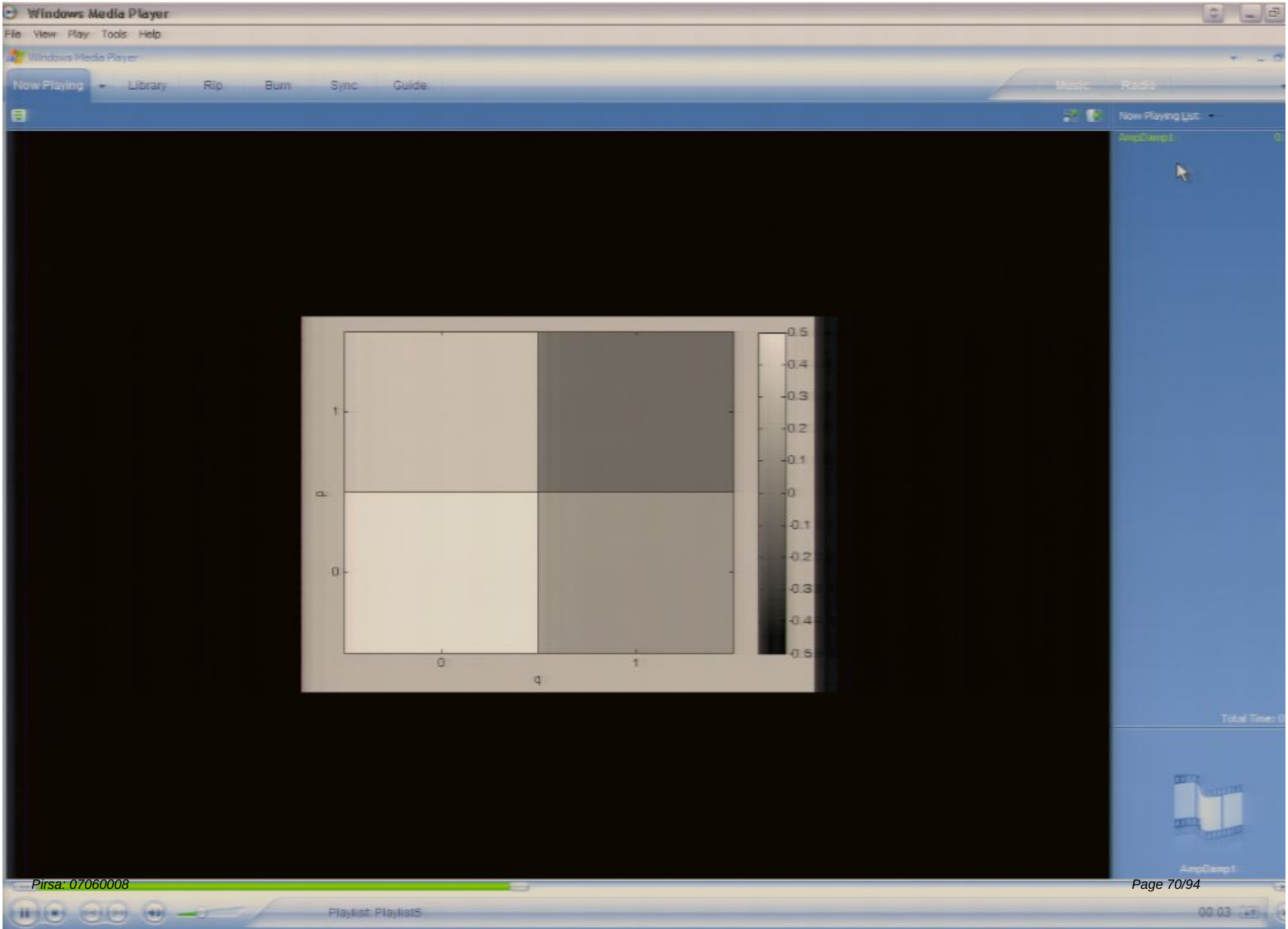
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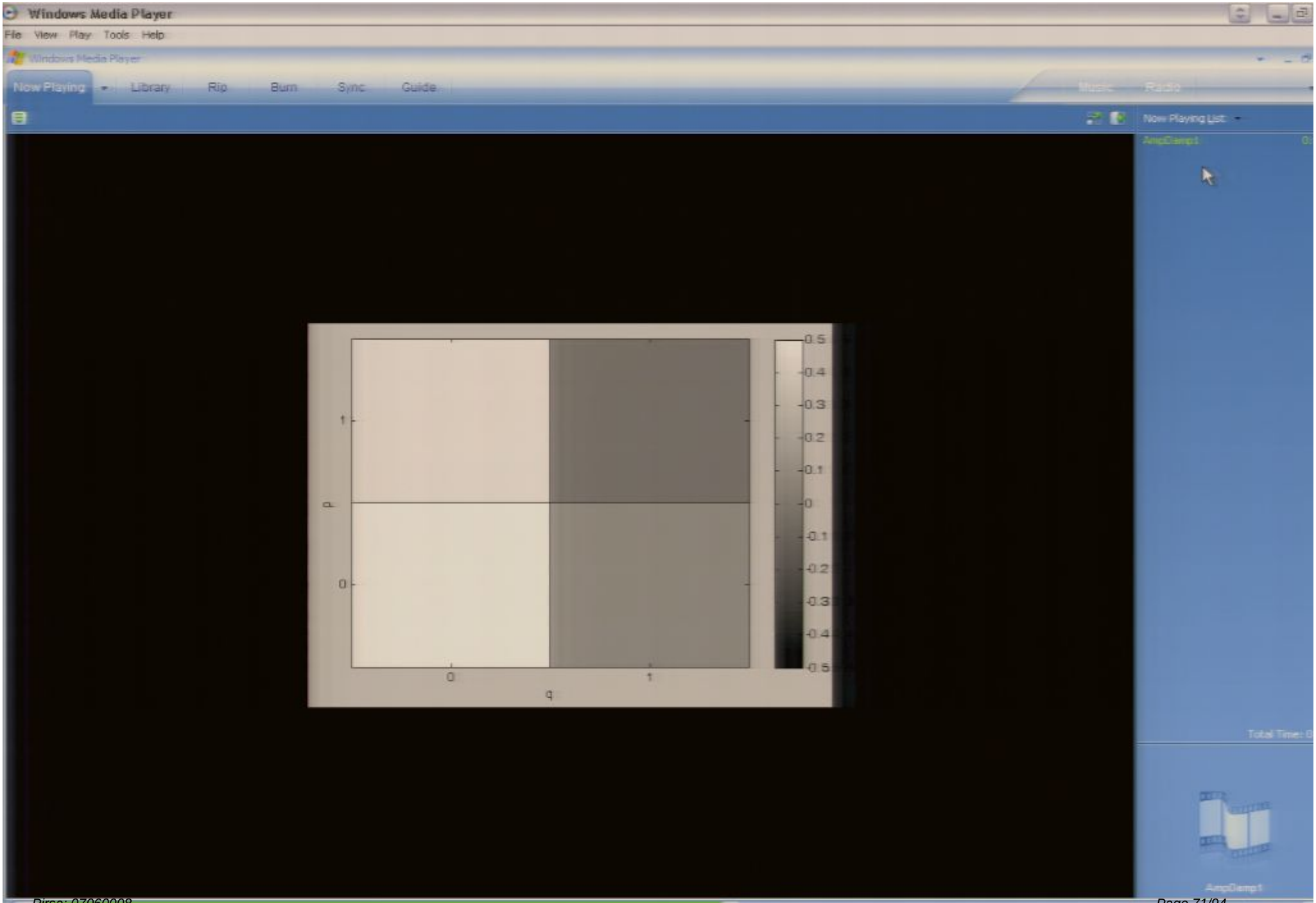
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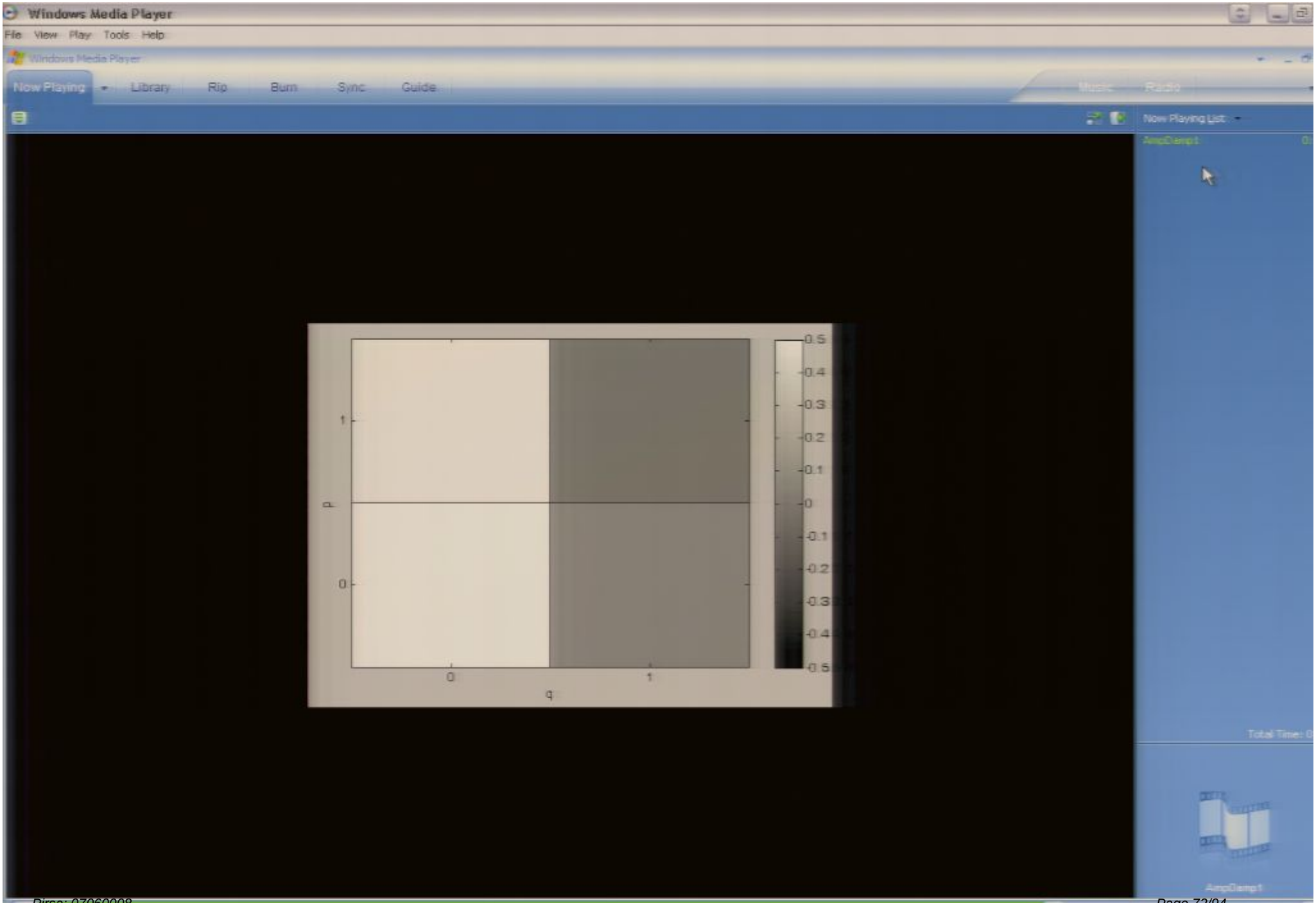
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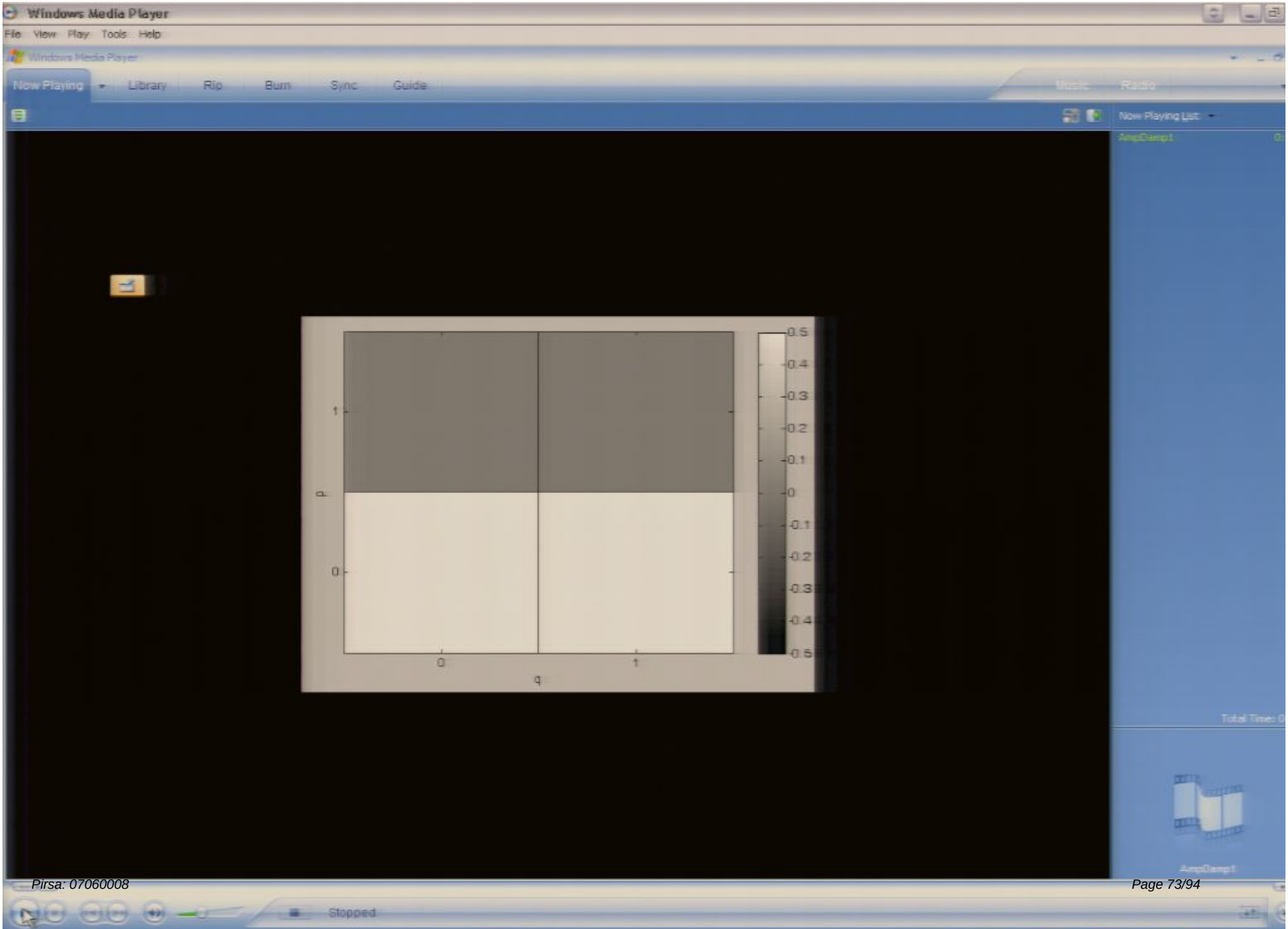
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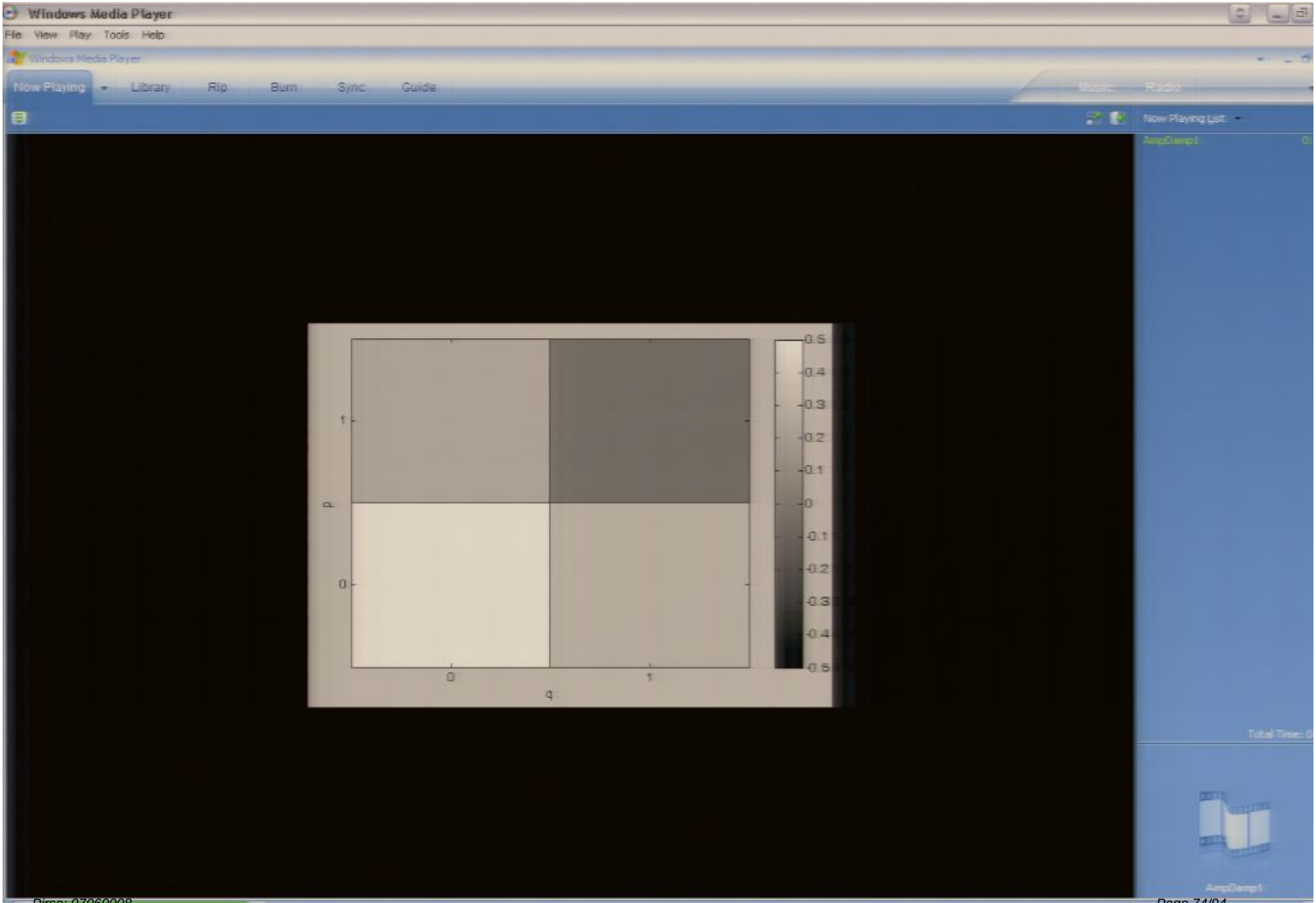
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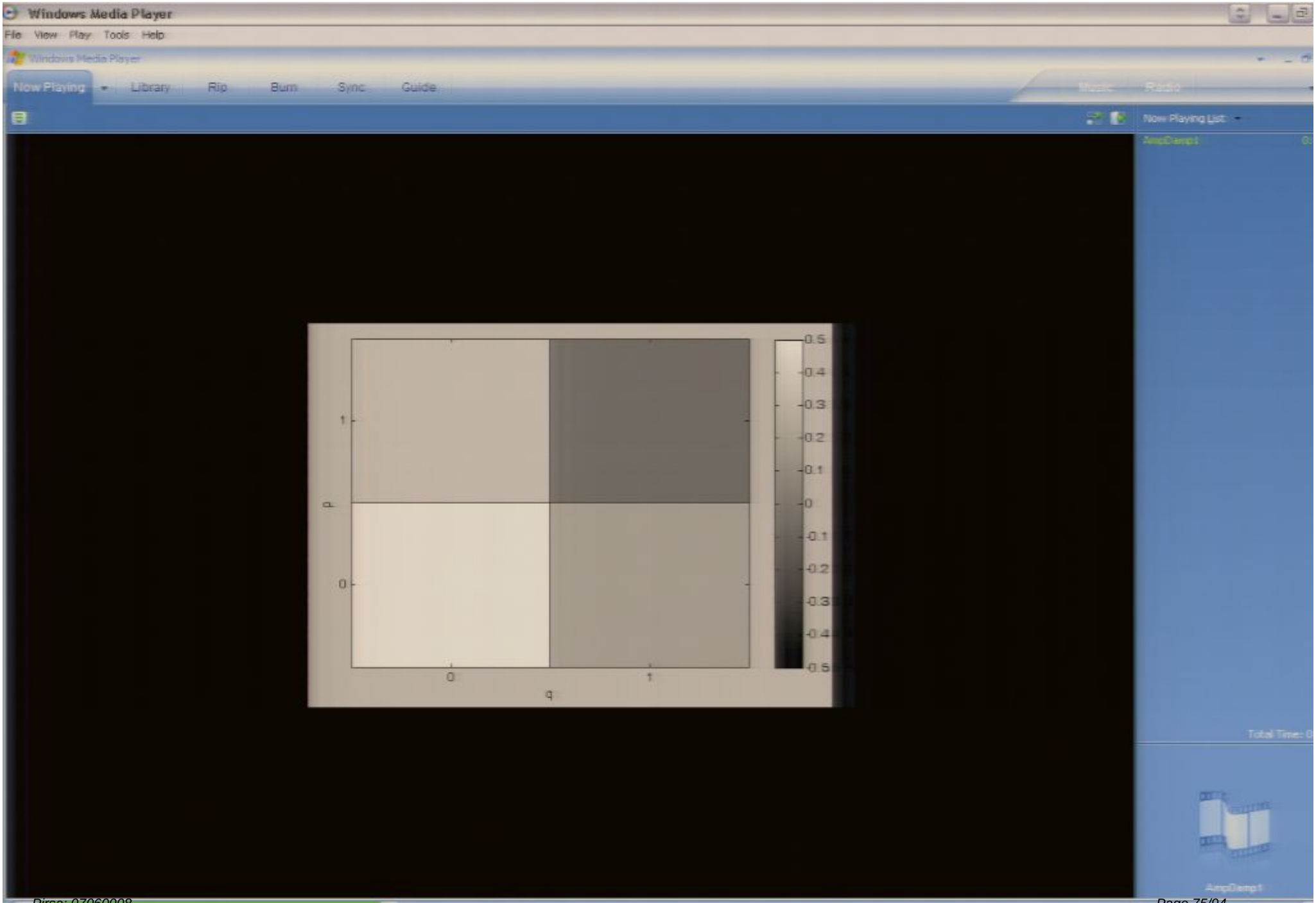
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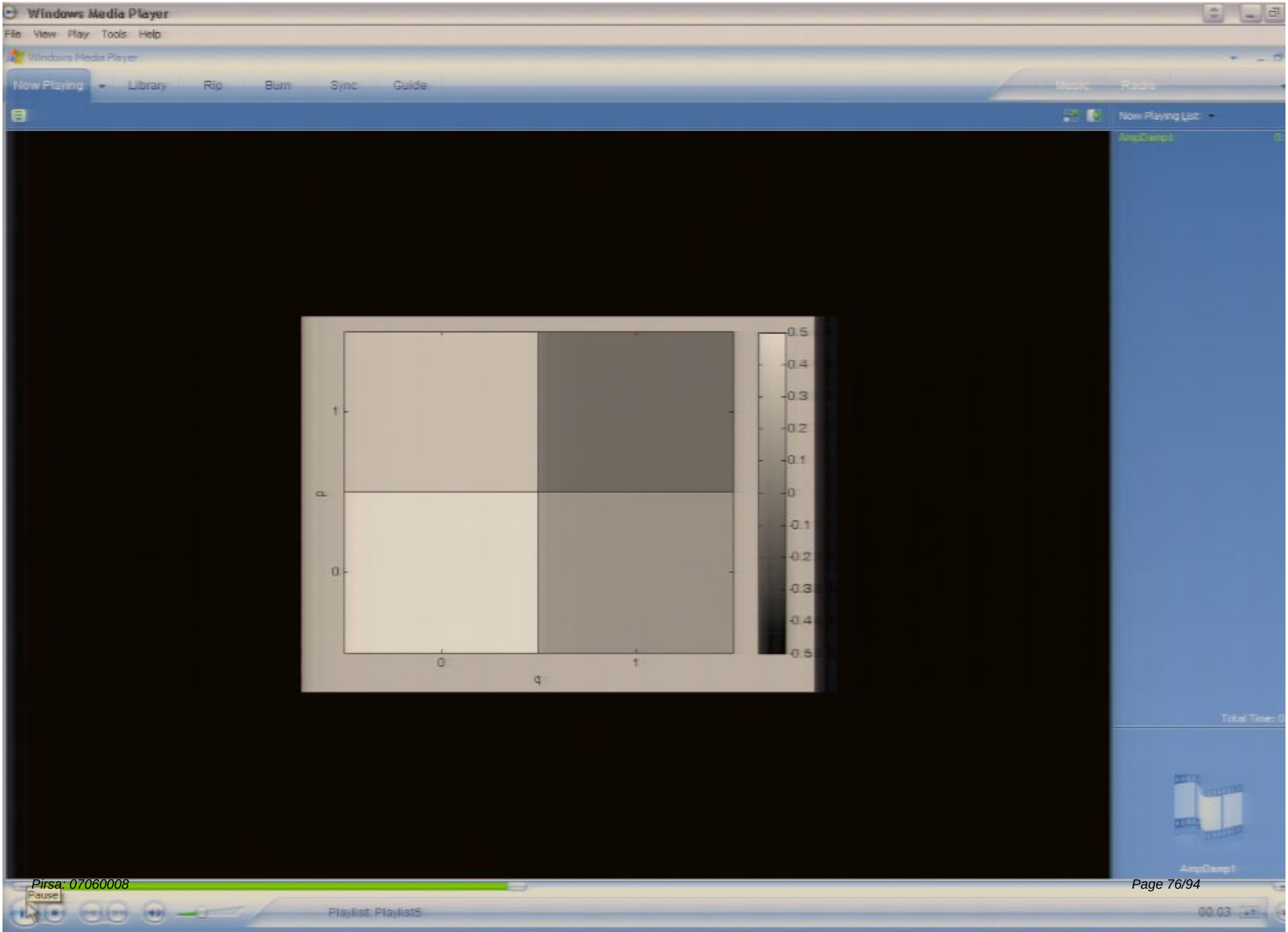
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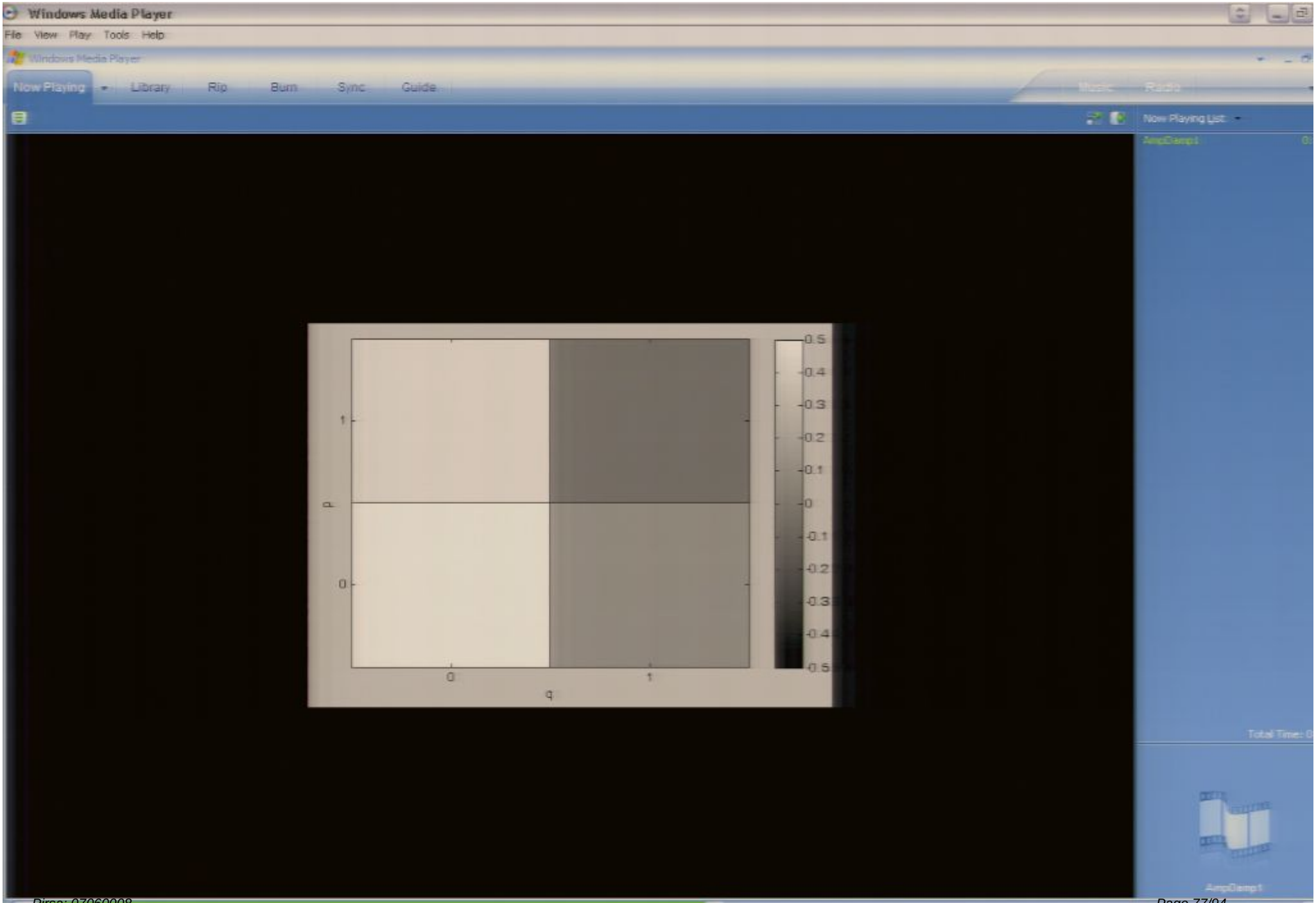
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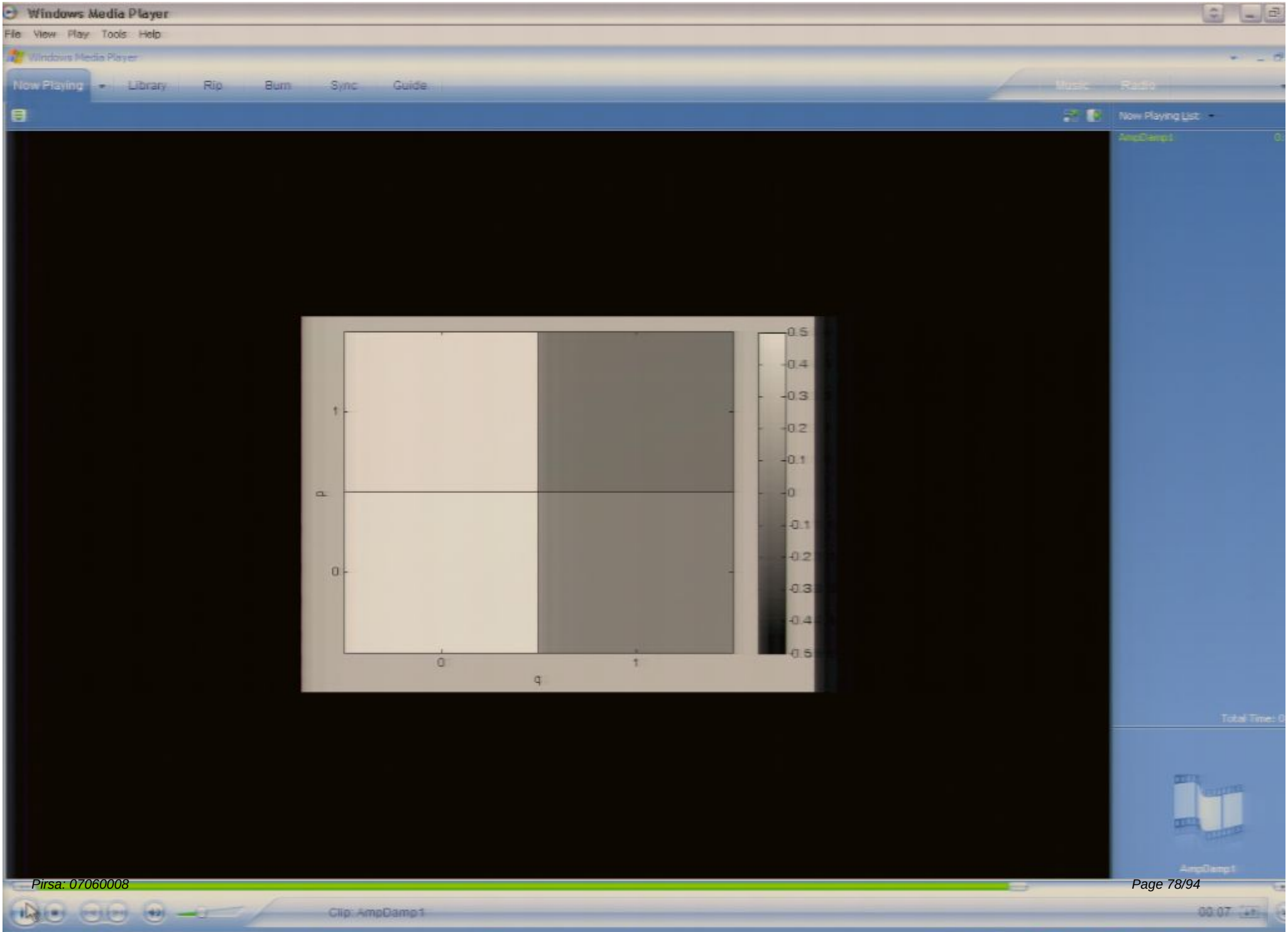
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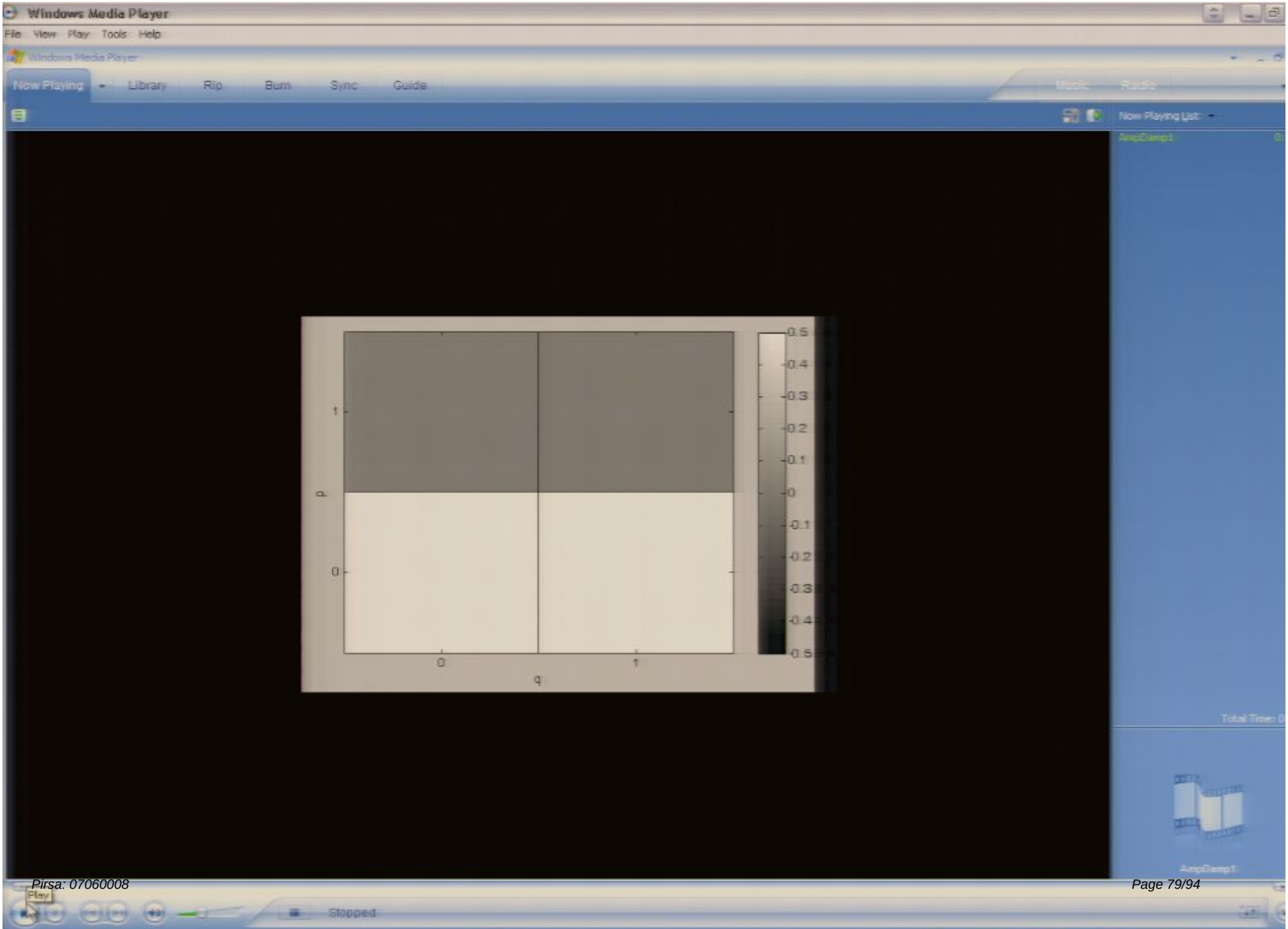
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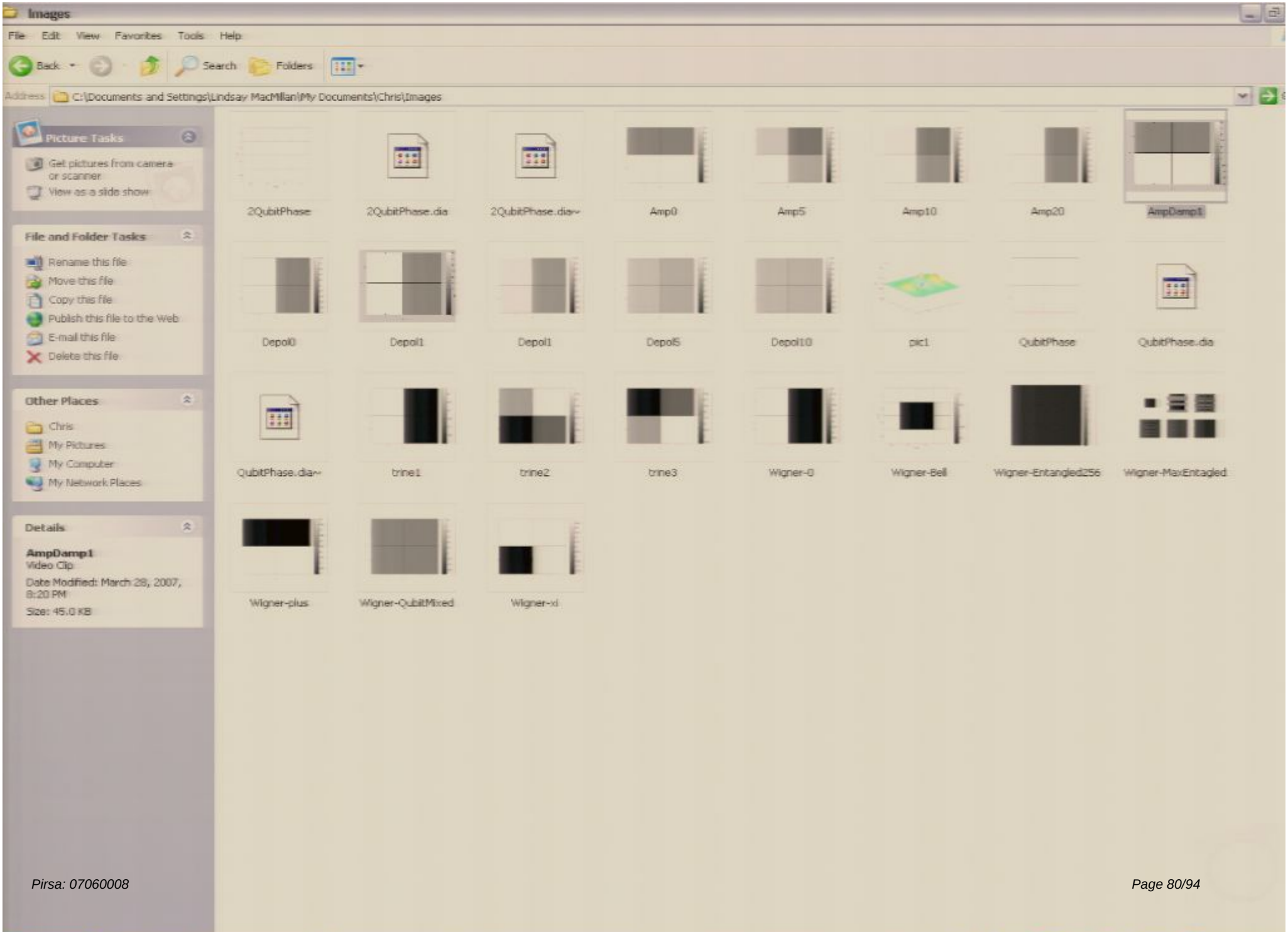
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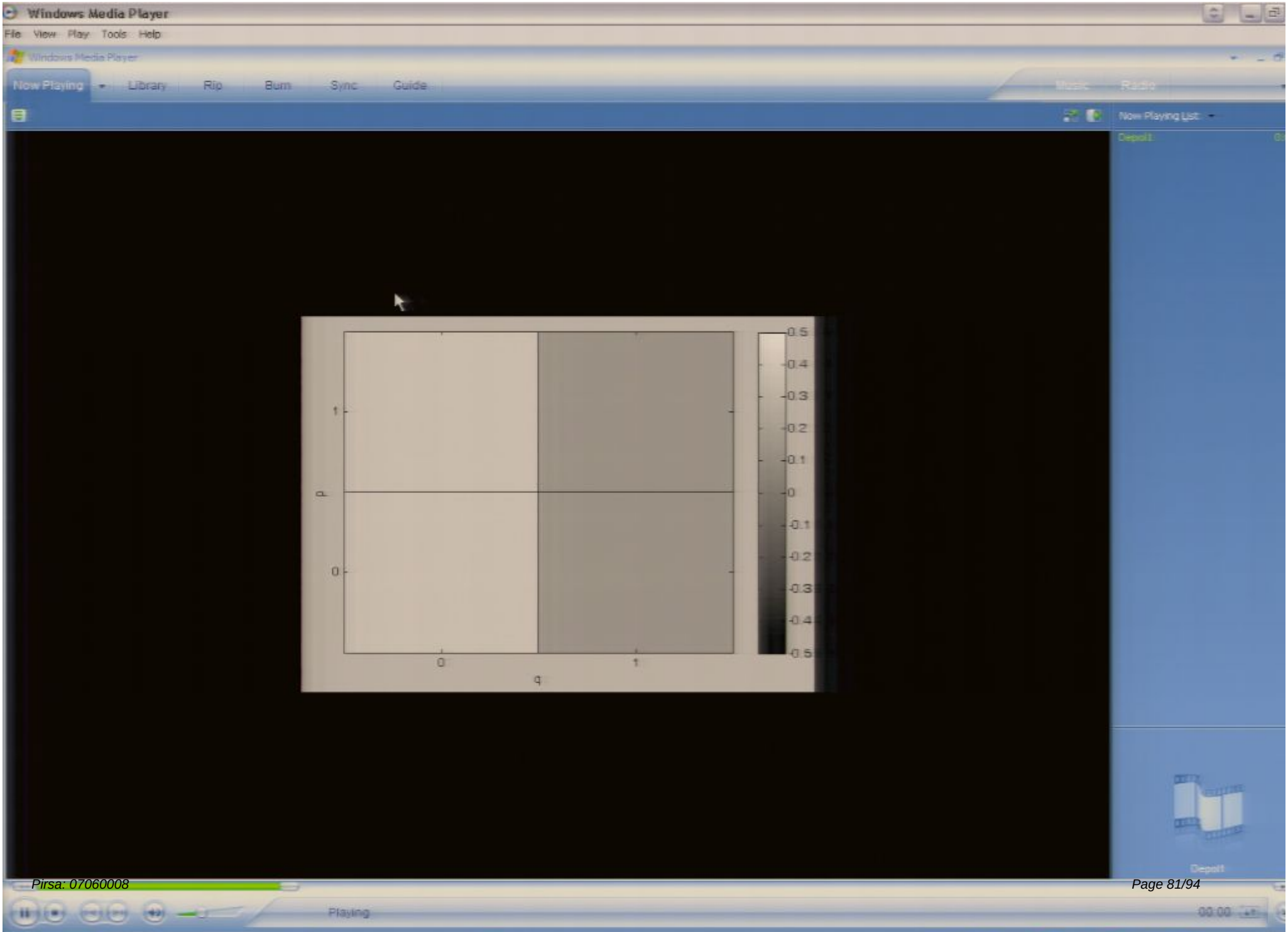
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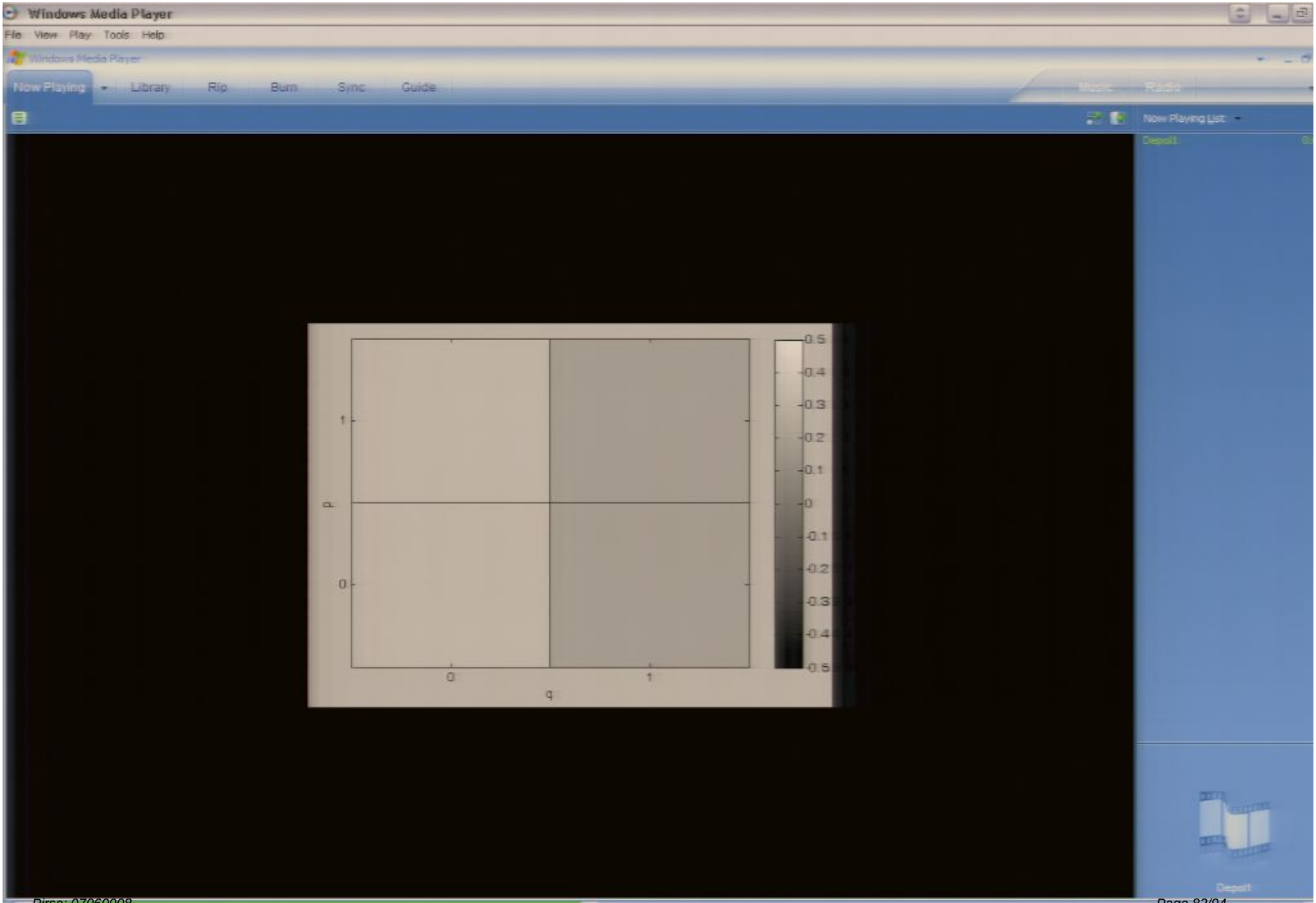
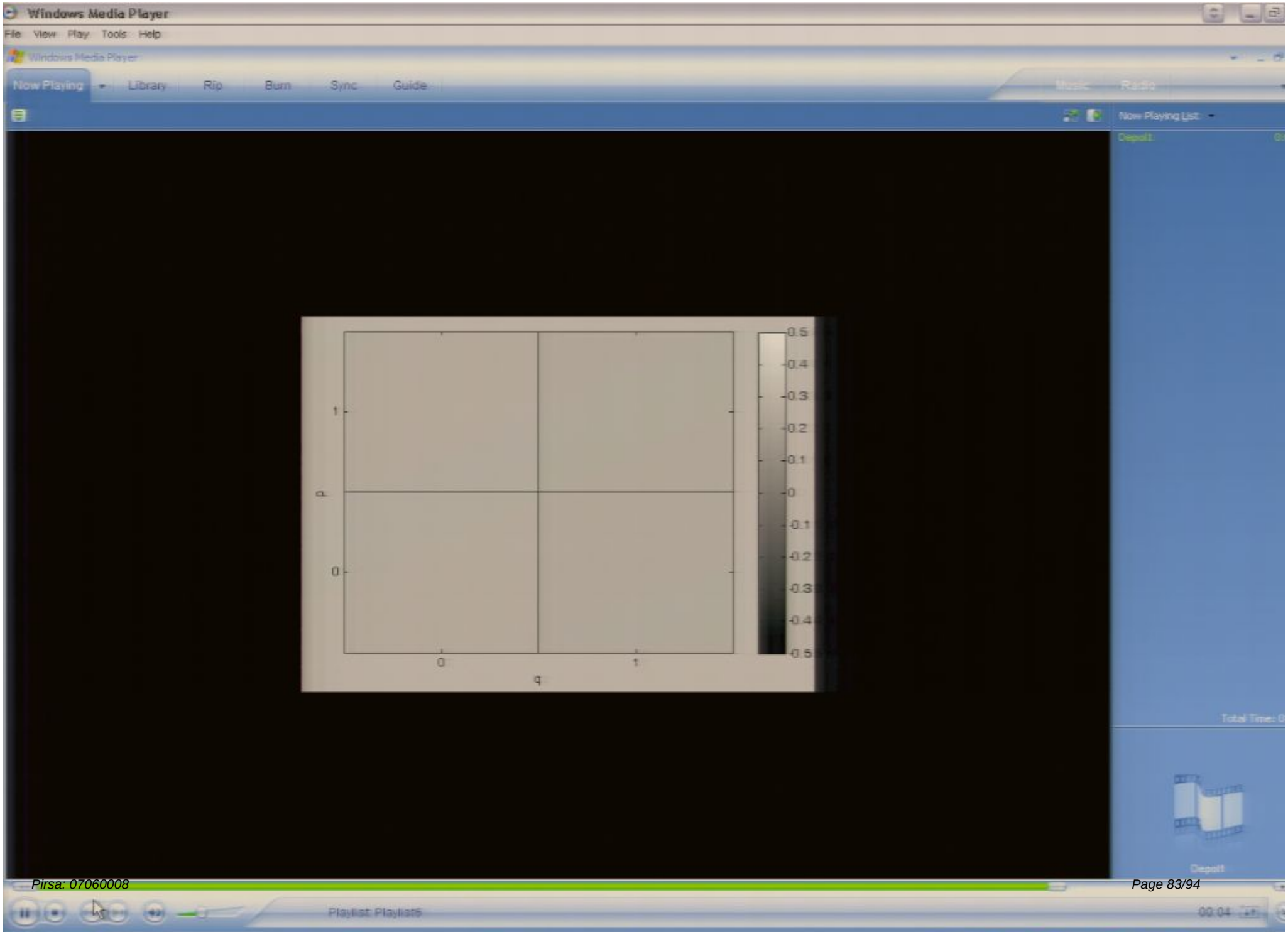
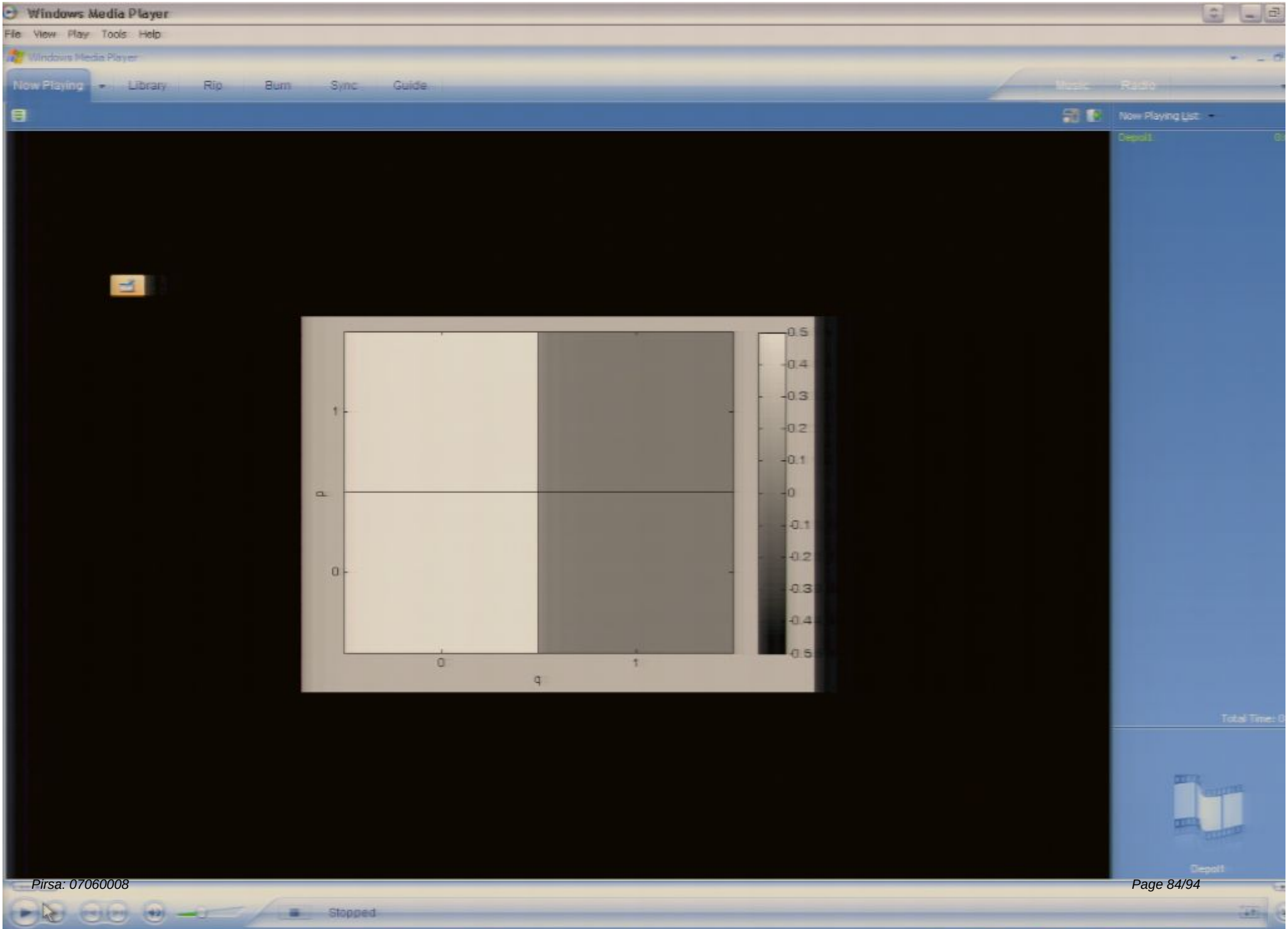
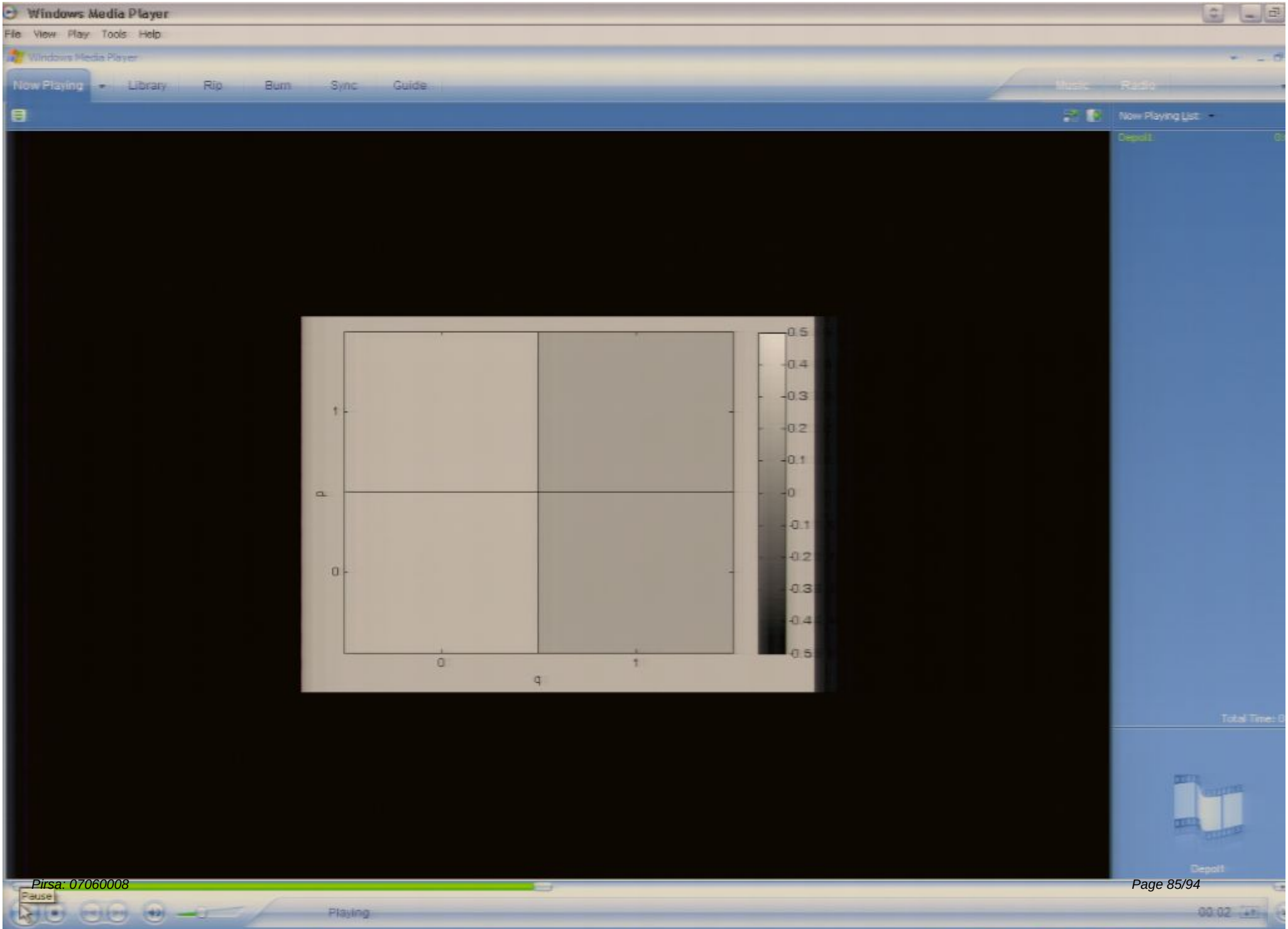
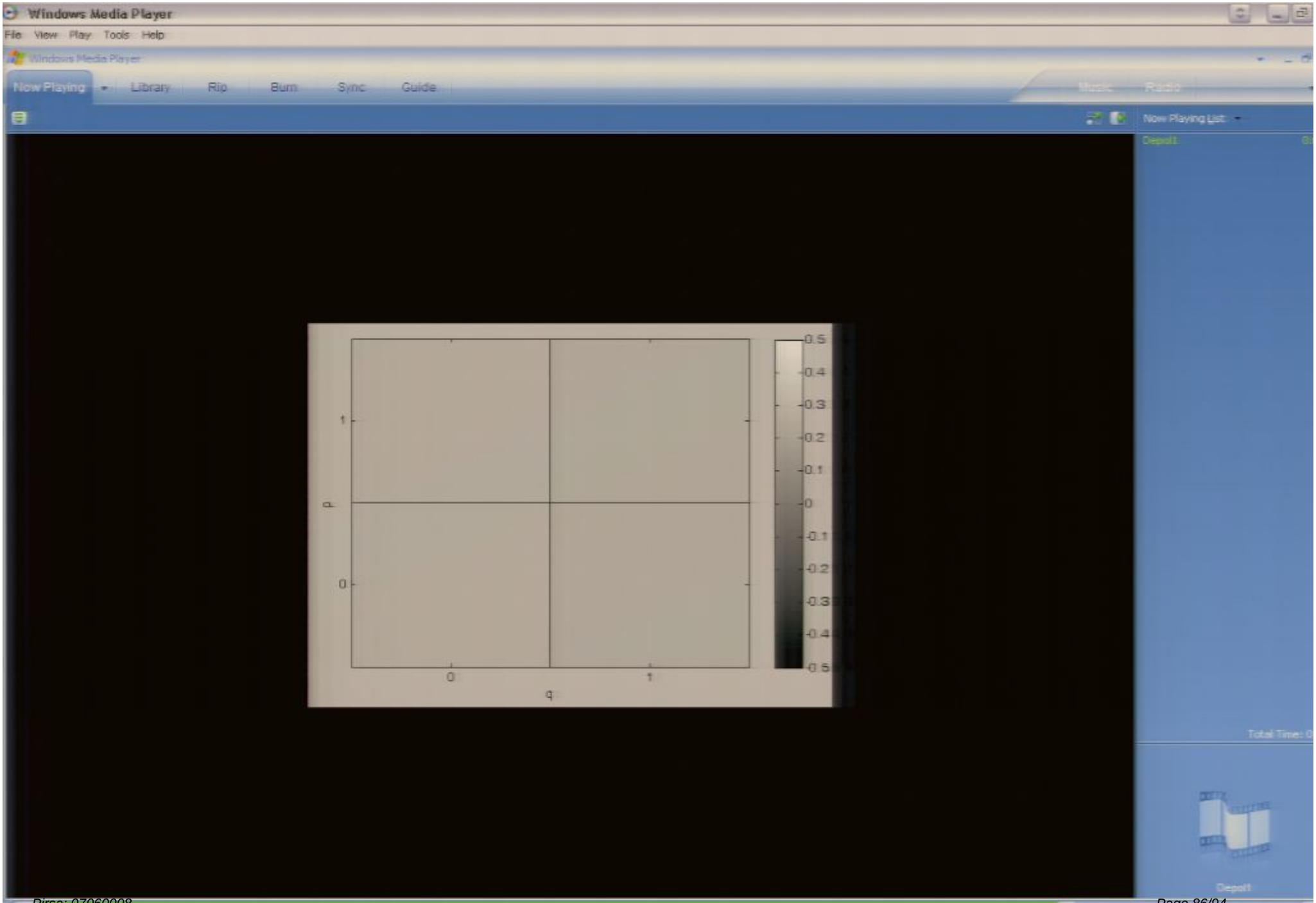
Guide
Now Playing List
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
1
0
p
0
1
q
Playing


Windows Media Player
File View Play Tools Help
Now Playing
Library
Rip
Burn
Sync
Guide
Now Playing List
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
1
0
p
0
1
q
Playing


Windows Media Player
File View Play Tools Help
Now Playing
Library
Rip
Burn
Sync
Guide
Now Playing List
p
q
0
1
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
Playlist: Playlist6


Windows Media Player
File View Play Tools Help
Now Playing
Library
Rip
Burn
Sync
Guide
Now Playing List
1
0
p
0
1
q
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
Stopped


Windows Media Player
File View Play Tools Help
Now Playing
Library
Rip
Burn
Sync
Guide
Now Playing List
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
1
0
p
0
1
q
Pause
Playing


Windows Media Player
File View Play Tools Help
Now Playing
Library
Rip
Burn
Sync
Guide
Now Playing List
0.5
0.4
0.3
0.2
0.1
0
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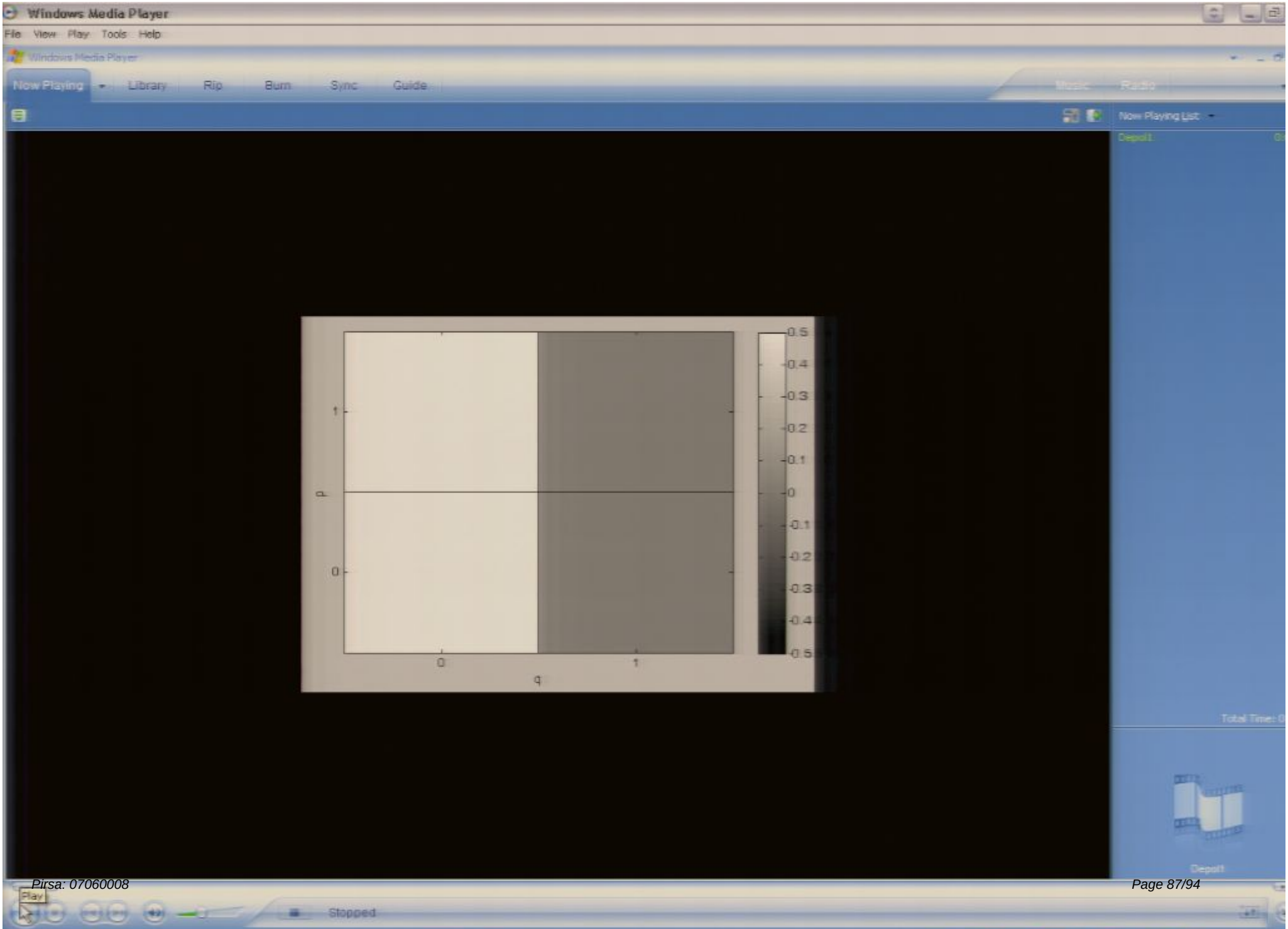
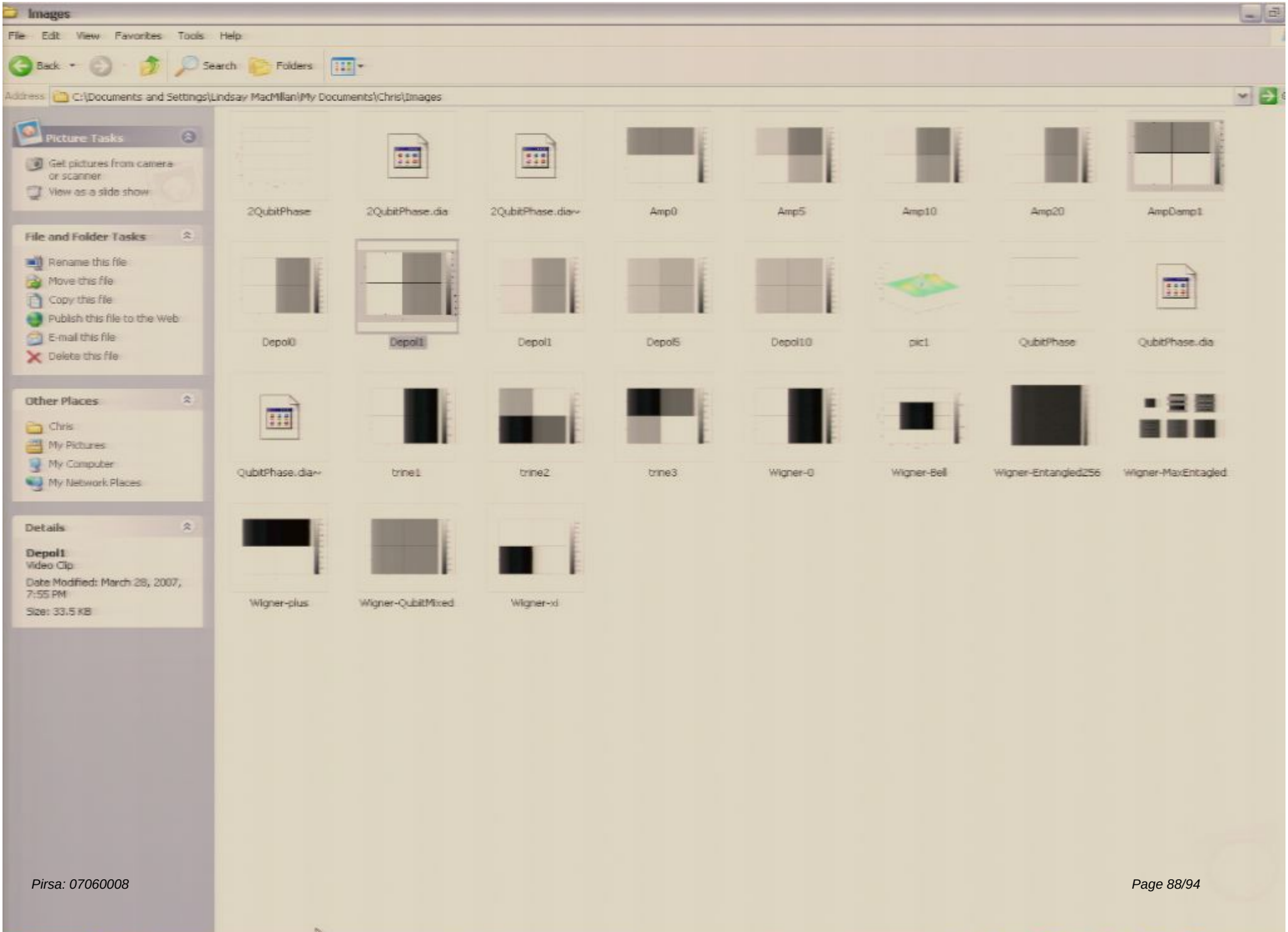
-0.2
-0.3
-0.4
-0.5
1
0
p
0
1
q
Pause
Playlist: Playlist6


Windows Media Player
File View Play Tools Help
Now Playing
Library
Rip
Burn
Sync
Guide
Now Playing List
1
0
p
0
1
q
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
Stopped

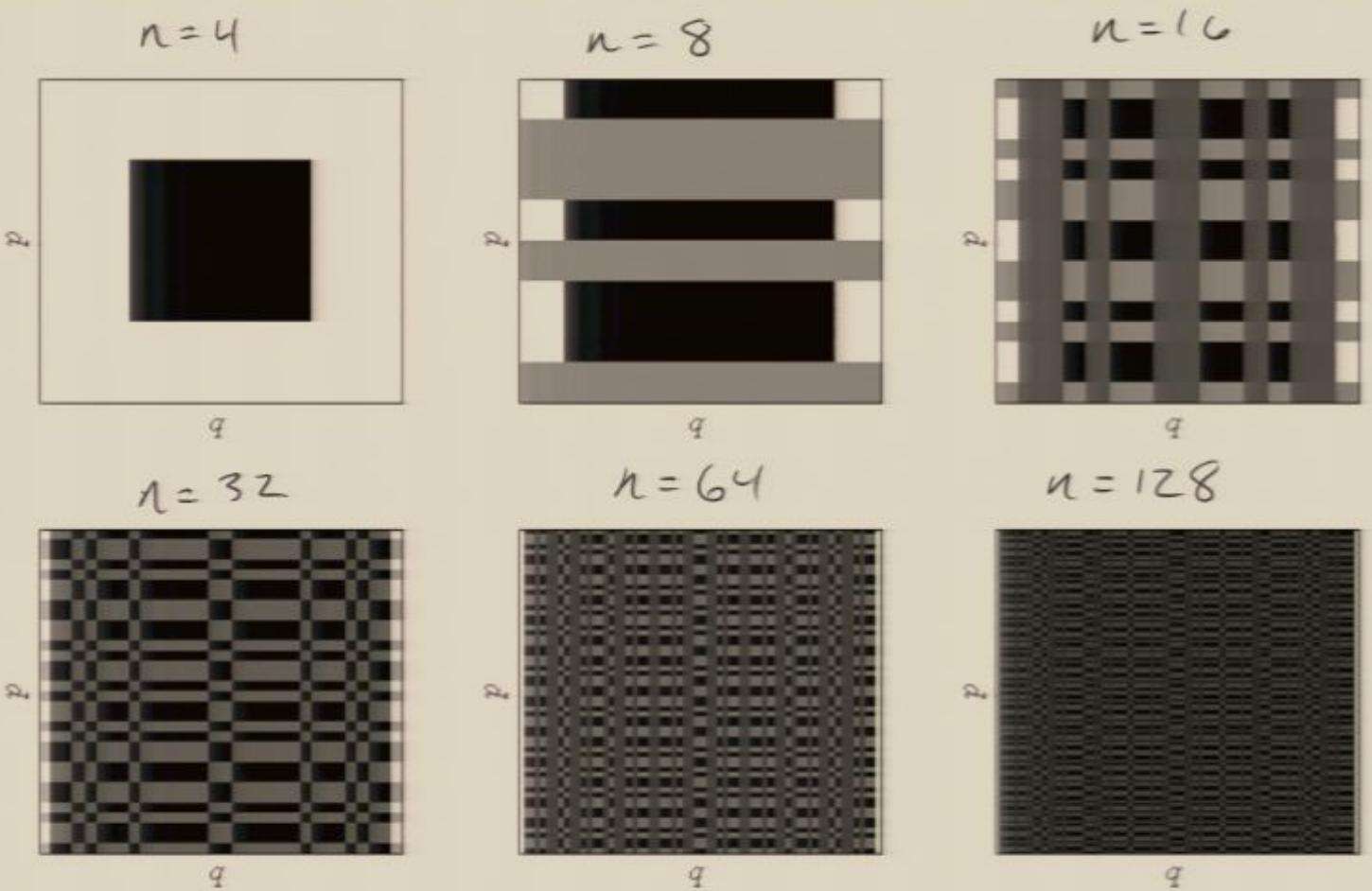

Images
File Edit View Favorites Tools Help
Back Search Folders
Address C:\Documents and Settings\Lindsay MacMillan\My Documents\Chris\Images
Picture Tasks
Get pictures from camera or scanner
View as a slide show
File and Folder Tasks
Rename this file
Move this file
Copy this file
Publish this file to the Web
E-mail this file
Delete this file
Other Places
Chris
My Pictures
My Computer
My Network Places
Details
Depol1
Video Clip
Date Modified: March 28, 2007, 7:55 PM
Size: 33.5 KB
2QubitPhase
2QubitPhase.dia
2QubitPhase.dia~
Amp0
Amp5
Amp10
Amp20
AmpDamp1
Depol0
Depol1
Depol1
Depol5
Depol10
pic1
QubitPhase
QubitPhase.dia
QubitPhase.dia~
trine1
trine2
trine3
Wigner-0
Wigner-Bell
Wigner-Entangled256
Wigner-MaxEntagled
Wigner-plus
Wigner-QubitMixed
Wigner-xi

Example 1 - Microsoft Office OneNote 2003

Example 1

May 31, 2007
10:51 AM

GHZ state — $\frac{1}{\sqrt{n}}\left(|00\cdots\rangle + |11\cdots\rangle\right)$



Remarks - Microsoft Office OneNote 2003

Remarks

May 31, 2007
5:50 PM

1. Which vector representation is the best/most appropriate discrete analog of the continuous Wigner representation?

2. In one of these representations, can we write

$$\frac{d}{dt}\rho = \frac{1}{i\hbar}\left(H \star \rho - \rho \star H\right) = \text{classical stuff} + \text{quantum stuff} \; ?$$

discrete star product now

what is classical evolution in compact space?

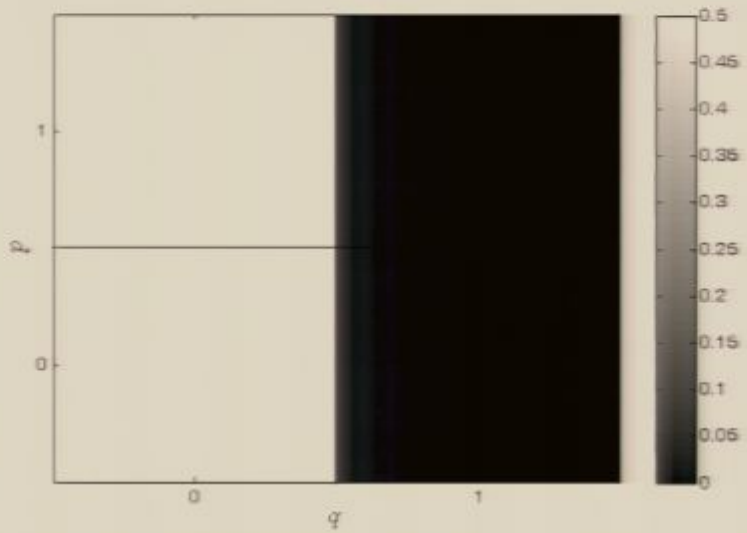
3. Model classical system of spins with finite precision in preparations and measurements.

Example 1 - Microsoft Office OneNote 2003

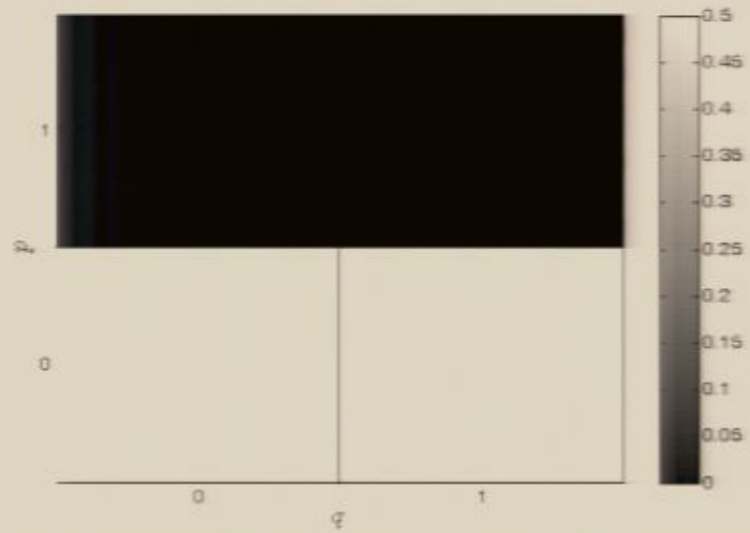
Example 1

May 31, 2007
10:51 AM

$|0\rangle$



$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$



trine measurement

$|1\rangle$

Example 1 - Microsoft Office OneNote 2003

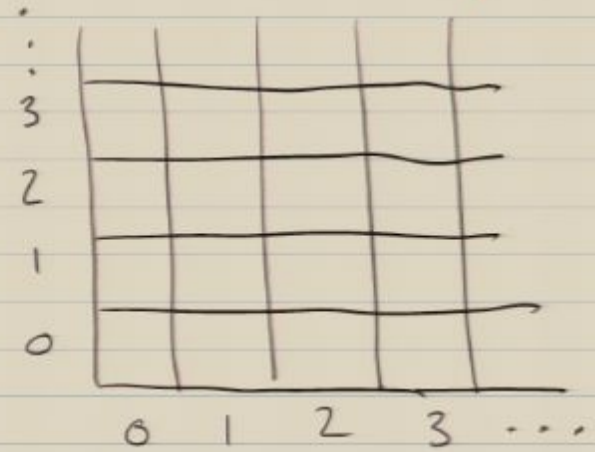
Example 1

May 31, 2007
10:51 AM

Wootters (87)

$\Sigma = \mathbb{Z}_n \times \mathbb{Z}_n$ ← discrete phase space

n prime



Each $\alpha \in \Sigma$

$\alpha = (q, p)$

The inner product on $\mathbb{C}^{\Sigma}$ is

$$\langle A, B \rangle = \sum_{\alpha} A_\alpha B_\alpha$$

an multiplication is

$$(A \star B)_\alpha = \sum_{\beta\gamma} A_\beta B_\gamma \Gamma_{\alpha\beta\gamma}$$

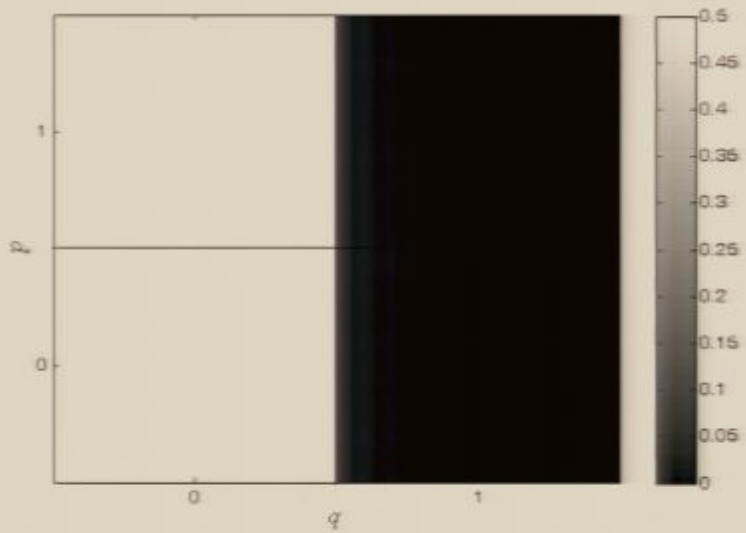
$\alpha = (q_1, p_1)$

Example 1 - Microsoft Office OneNote 2003

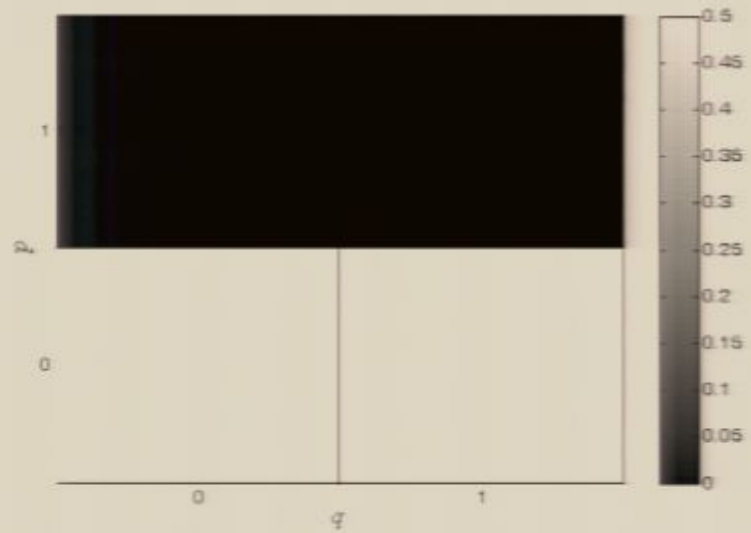
Example 1

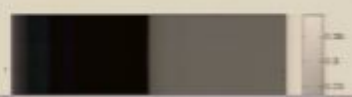
May 31, 2007
10:51 AM

$|0\rangle$



$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$





trine measurement

Remarks - Microsoft Office OneNote 2003

Remarks

May 31, 2007
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