Title: Gluon Vortex and Induced Magnetic Fields in Colour Superconductors

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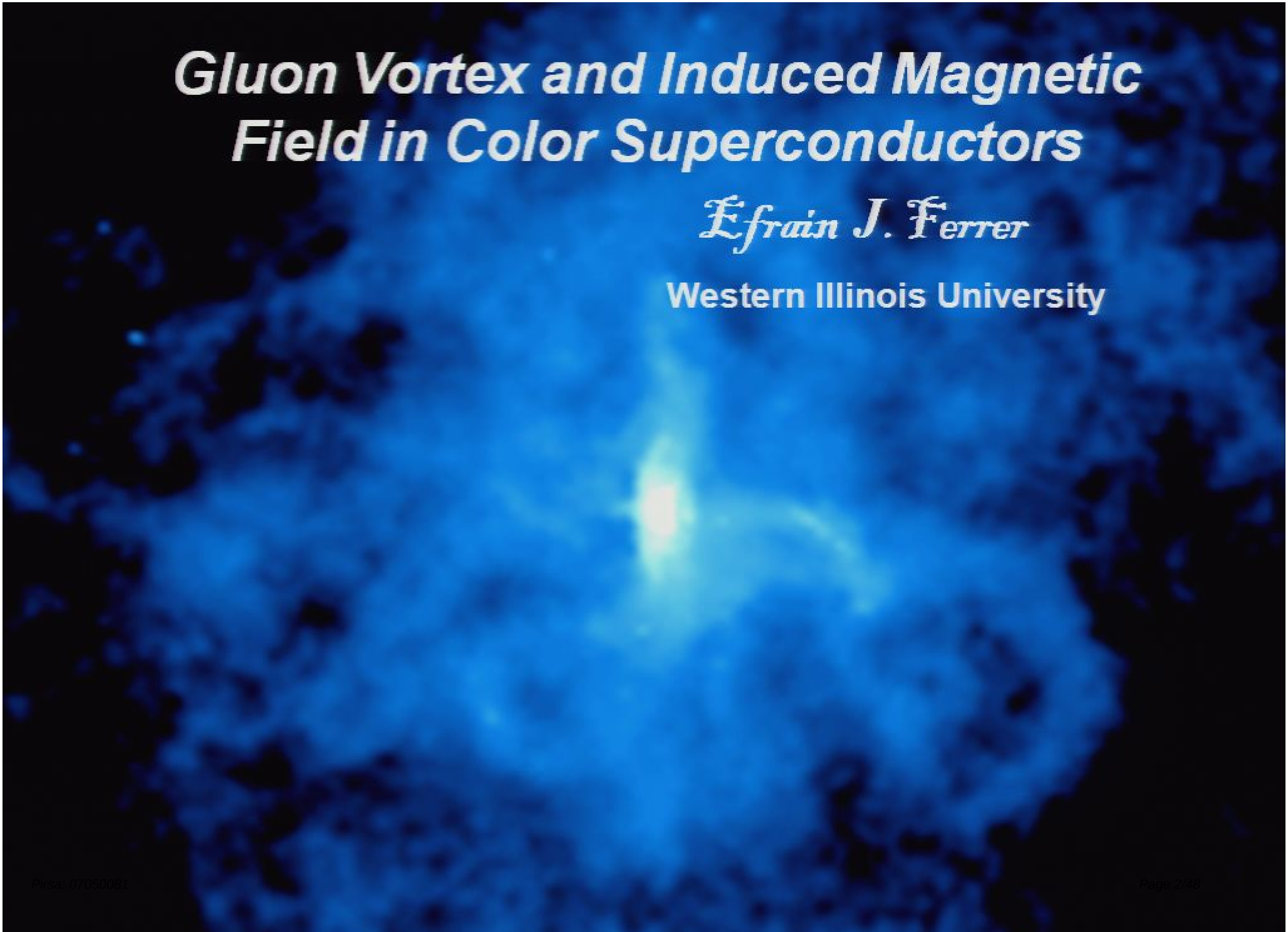
Abstract:

Gluon Vortex and Induced Magnetic Field in Color Superconductors

Efrain J. Ferrer

Western Illinois University



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OUTLINE

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- *Gluons in Magnetized CS*
- *Gluon Vortices and Magnetic Antiscreening*
- *Chromomagnetic Instabilities & $G-\tilde{B}$ Condensates*
- *Conclusions and Future Directions*

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EJF & de la Incera, PRL 97 (2006) 122301

EJF & de la Incera, hep-ph/0705.2403

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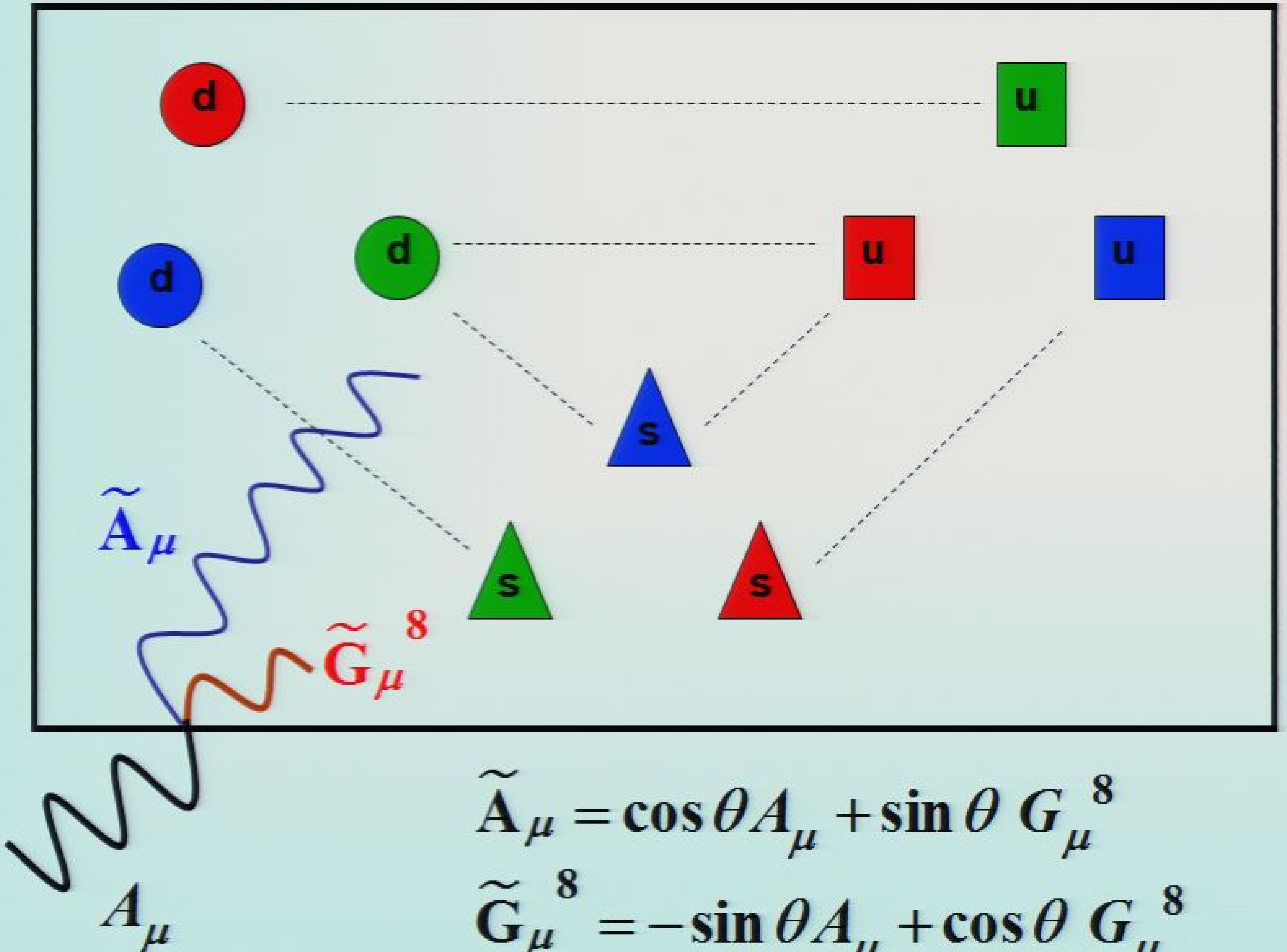
CFL Pairing & Magnetic Field Penetration

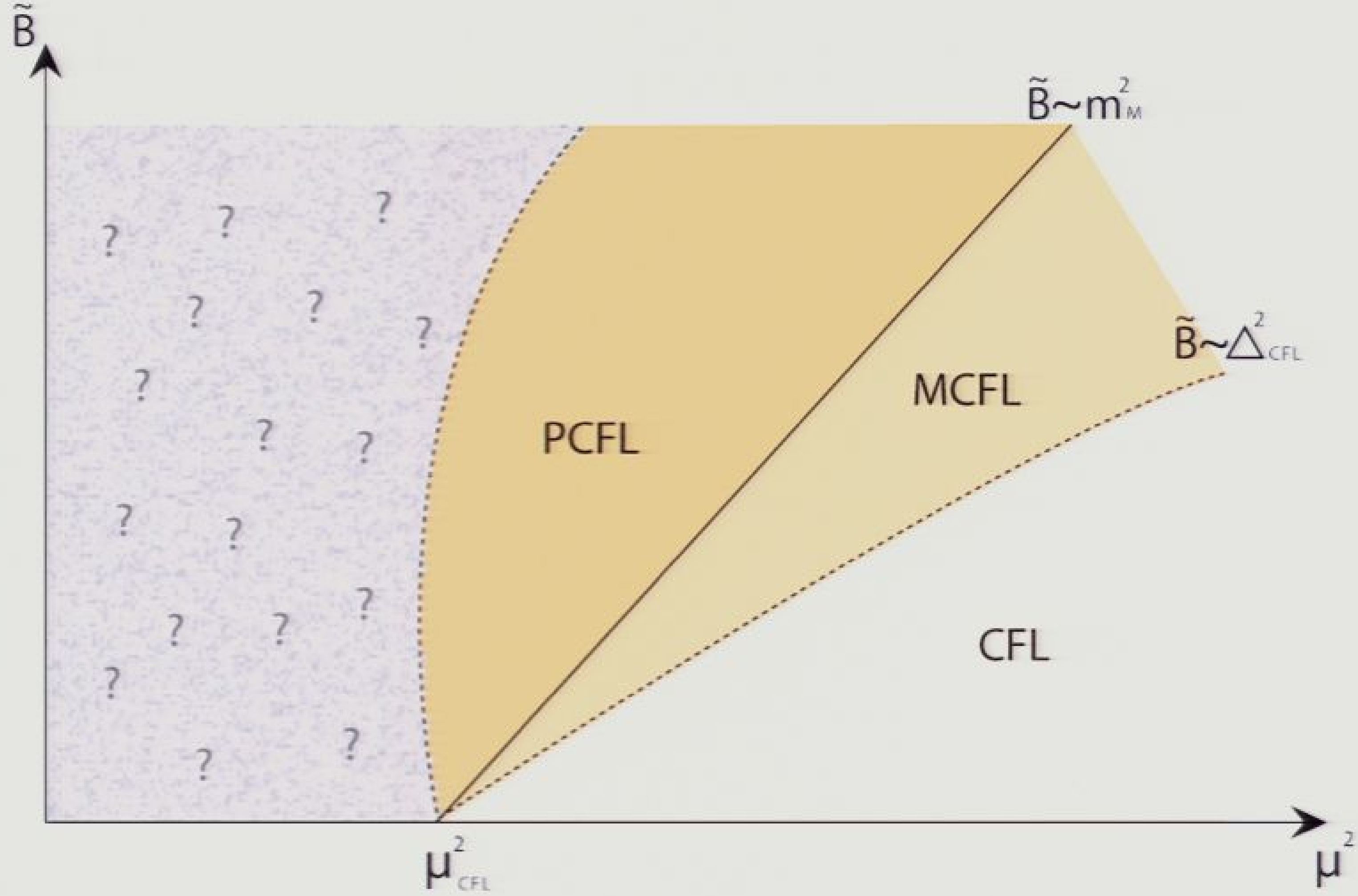


$$\tilde{A}_\mu = \cos\theta A_\mu + \sin\theta\, G_\mu{}^8$$

$$\tilde{G}_\mu{}^8 = -\sin\theta A_\mu + \cos\theta\, G_\mu{}^8$$

Sketch of $\tilde{B}$ vs μ phases of a Color Superconductor with Three-Quark Flavors

EJF & Incera, nucl-ph/0703034



3-Flavor QCD

$B = 0$

$$SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_{e.m}$$

$\downarrow$ $\rightarrow$ **9 Goldstones**

$$SU(3)_{C+L+R} \times \tilde{U}(1)_{em}$$

$B \neq 0$

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_B \times U^{(-)}(1)_A \times U(1)_{e.m.}$$

$\downarrow$ $\rightarrow$ **5 Goldstones**

$$SU(2)_{C+L+R} \times \tilde{U}(1)_{em}$$

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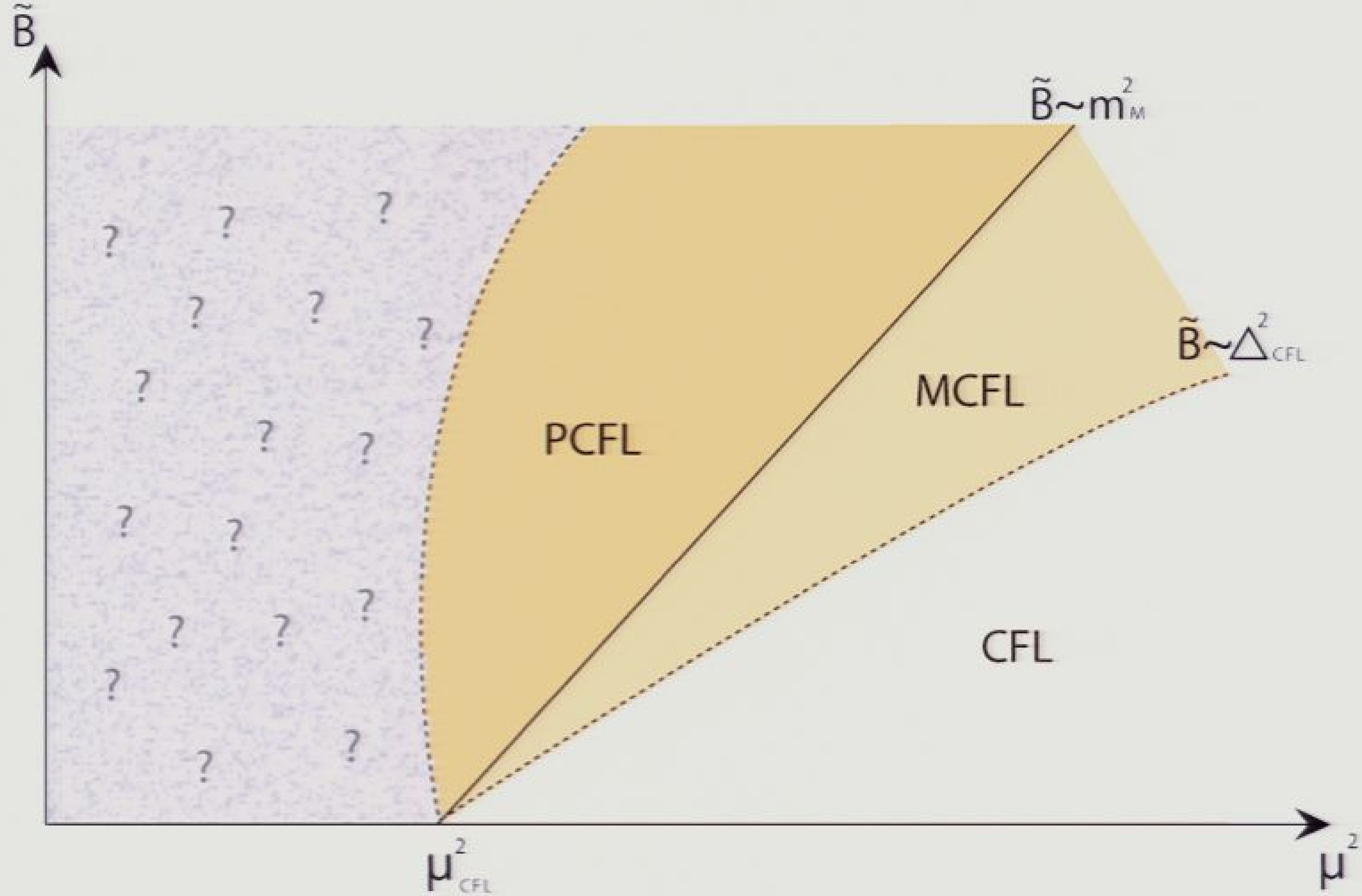

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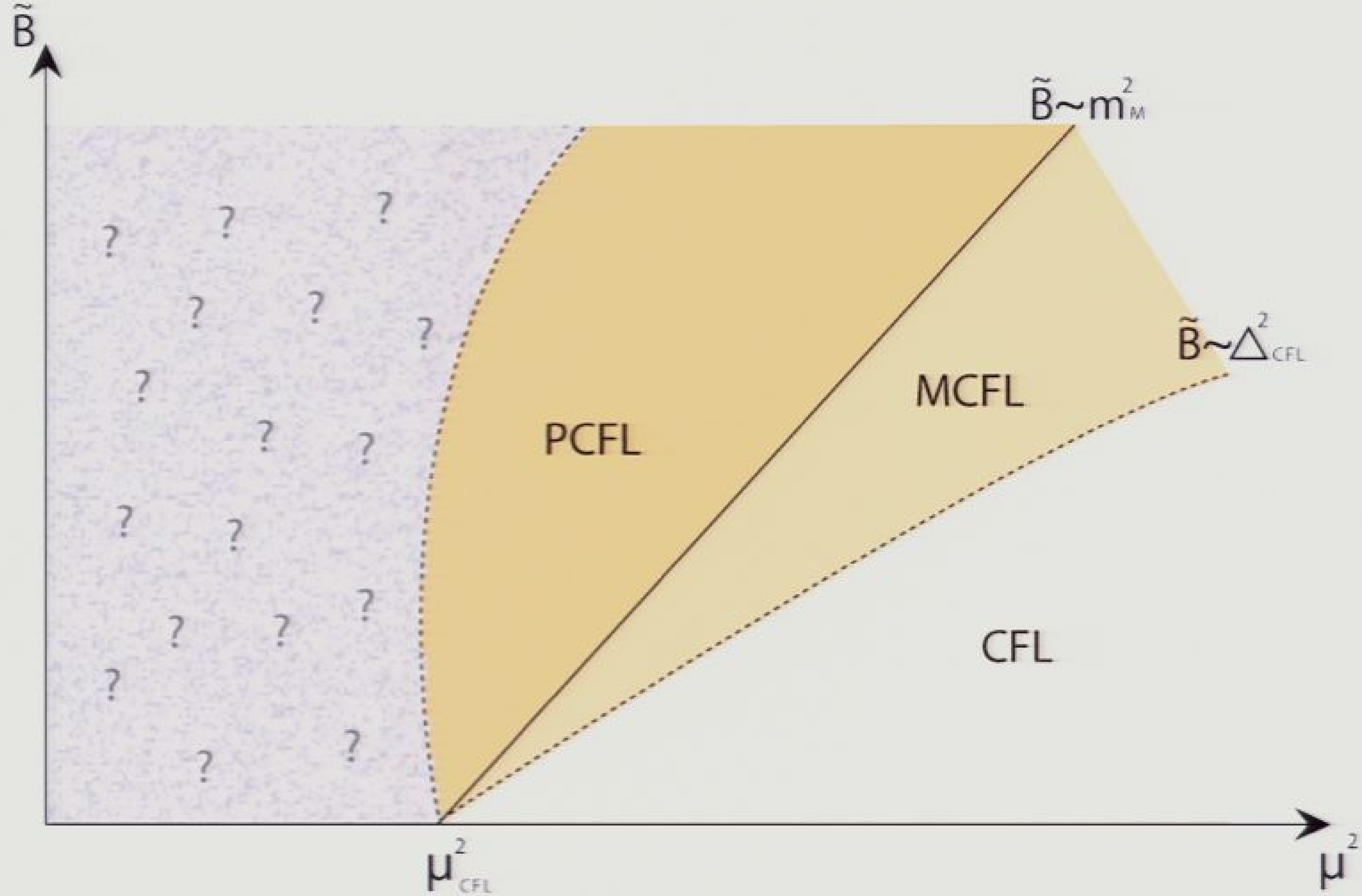

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Magnetic Effects on the Gluons

EJF & de la Incera, Phys. Rev. Lett. 97 (2006) 122301

Because of the modified electromagnetism, gluons are charged in the color superconductor

G^1_μ	G^2_μ	G^3_μ	G^+_μ	G^-_μ	I^+_μ	I^-_μ	$\tilde{G}^8_\mu$
0	0	0	1	-1	1	-1	0

Effective action for the charged gluons within CFL at asymptotic densities

$$\Gamma^c_{eff} = \int d^4x \{ -\frac{1}{4}(\widetilde{f}_{\mu\nu})^2 - \frac{1}{2}|\widetilde{\Pi}_\mu G^-_\nu - \widetilde{\Pi}_\nu G^-_\mu|^2$$
$$-[(m_D^2 \delta_{\mu 0}\delta_{\nu 0} + m_M^2 \delta_{\mu i}\delta_{\nu i}) + i\widetilde{e}\widetilde{f}_{\mu\nu}]G^+_\mu G^-_\nu$$
$$+\frac{g^2}{2}[(G^+_\mu)^2(G^-_\nu)^2 - (G^+_\mu G^-_\mu)^2] + \frac{1}{\lambda}G^+_\mu \widetilde{\Pi}_\mu \widetilde{\Pi}_\nu G^-_\nu \},$$

Where $\widetilde{\Pi}_\mu = \partial_\mu - i\widetilde{e}\widetilde{A}_\mu$

Magnetic Instability for Charged Spin-1 Fields

Assuming that there is an external magnetic field $\tilde{H}$ in the z-direction, one mode becomes unstable when $\tilde{H} > m^2{}_M$

$$\begin{pmatrix} m_M^2 & i\tilde{e}\widetilde{H} \\ -i\tilde{e}\widetilde{H} & m_M^2 \end{pmatrix} \to \begin{pmatrix} m_M^2 + \tilde{e}\widetilde{H} & 0 \\ 0 & m_M^2 - \tilde{e}\widetilde{H} \end{pmatrix}$$

with corresponding eigenvector: $(G_1{}^+, G_2{}^+) \to G(1, i)$

"Zero-mode problem" for non-Abelian gauge fields whose solution is the formation of a vortex condensate of charged spin-1 fields.

Nielsen & Olesen NPB 144 (1978)

Skalozub, Sov.JNP23 (1978);ibid 43 (1986)

Ambjorn & Olesen, NPB315 (1989)

Gibbs Free-Energy:

$$\mathcal{G}_c = \mathcal{F}_{n0} - 2G^\dagger \widetilde{\Pi}^2 G - 2(2\tilde{e}\widetilde{B} - m_M^2)|G|^2 + 2g^2|G|^4 + \frac{1}{2}\widetilde{B}^2 - \widetilde{H}\widetilde{B}$$

Minimum Equations:

$$-\widetilde{\Pi}^2 G - (2\tilde{e}\widetilde{B} - m_M^2)G + 2g^2|G|^2 G = 0,$$

In the approximation: $\tilde{e}\widetilde{H} \approx m_M^2 \gg |G|^2, \tilde{e}(\widetilde{B} - \widetilde{H}_c)$

$$\widetilde{\Pi}^2 G + \tilde{e}\widetilde{H}_c G \approx 0$$

$$2\tilde{e}|G|^2 - \widetilde{B} + \widetilde{H} \approx 0$$

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Minimum Equations:

$$\widetilde{\Pi}^2 G + \widetilde{e}\widetilde{H}_c G \approx 0$$

⇒ $|\widetilde{\Pi}_\mu K_\nu - \widetilde{\Pi}_\nu K_\mu|^2$**=0**

$$2\widetilde{e}|G|^2 - \widetilde{B} + \widetilde{H} \approx 0$$

Minimum Equations:

$$\widetilde{\Pi}^2 G + \widetilde{e}\widetilde{H}_c G \approx 0$$

$\Rightarrow$ $|\widetilde{\Pi}_\mu K_\nu - \widetilde{\Pi}_\nu K_\mu|^2 = 0$

$$2\widetilde{e}|G|^2 - \widetilde{B} + \widetilde{H} \approx 0$$

Magnetic Antiscreening

Minimum Equations:

$$\widetilde{\Pi}^2 G + \widetilde{e}\widetilde{H}_c G \approx 0$$

$\Rightarrow$ $|\widetilde{\Pi}_\mu K_\nu - \widetilde{\Pi}_\nu K_\mu|^2$**=0**

$$2\widetilde{e}|G|^2 - \widetilde{B} + \widetilde{H} \approx 0$$

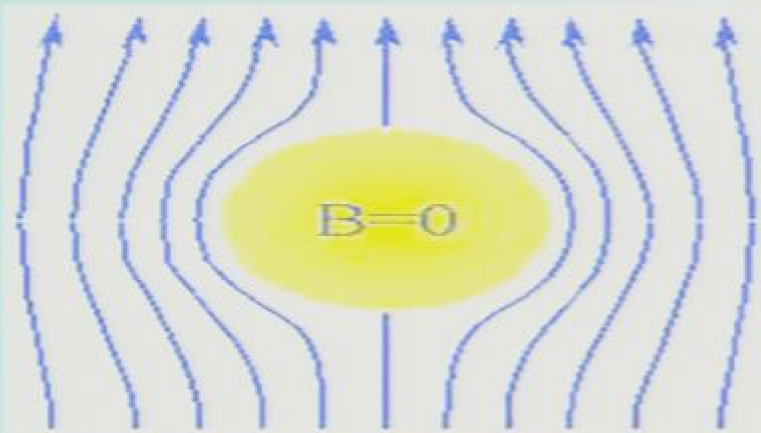
Magnetic Antiscreening

Abrikosov's Equation

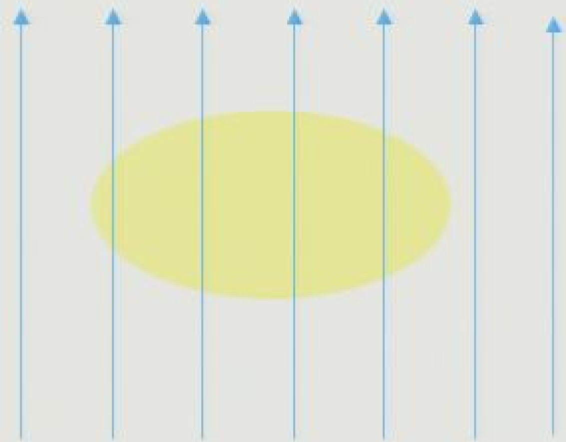
$$[\partial_j^2 - \frac{4\pi i}{\widetilde{\Phi}_0}\widetilde{H}_C x \partial_y - 4\pi^2 \frac{\widetilde{H}_C^2}{\widetilde{\Phi}_0^2} x^2 + \frac{1}{\xi^2}]G = 0, \quad j = x, y$$

$\widetilde{\Phi}_0 \equiv 2\pi/\widetilde{e},$ $\quad$ $\xi^2 \equiv 1/(2\widetilde{e}\widetilde{H}_C - m_M^2) = 1/m_M^2$

Conventional Superconductor

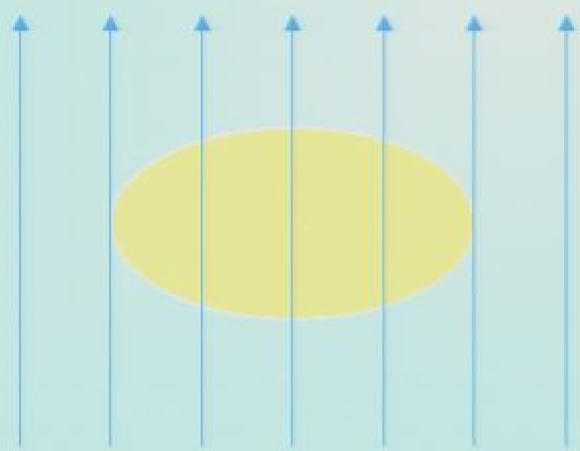


$H < H_c$



$H \geq H_c$

Color Superconductor



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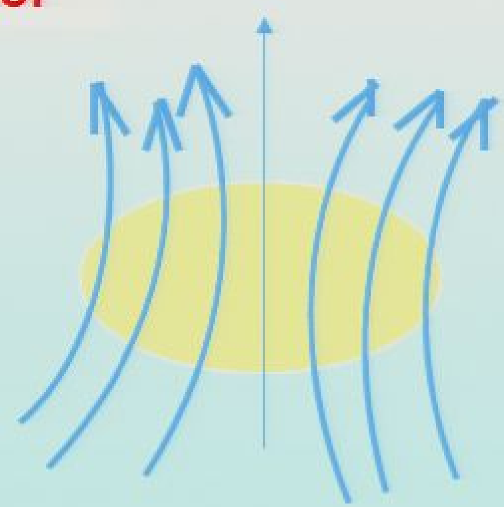
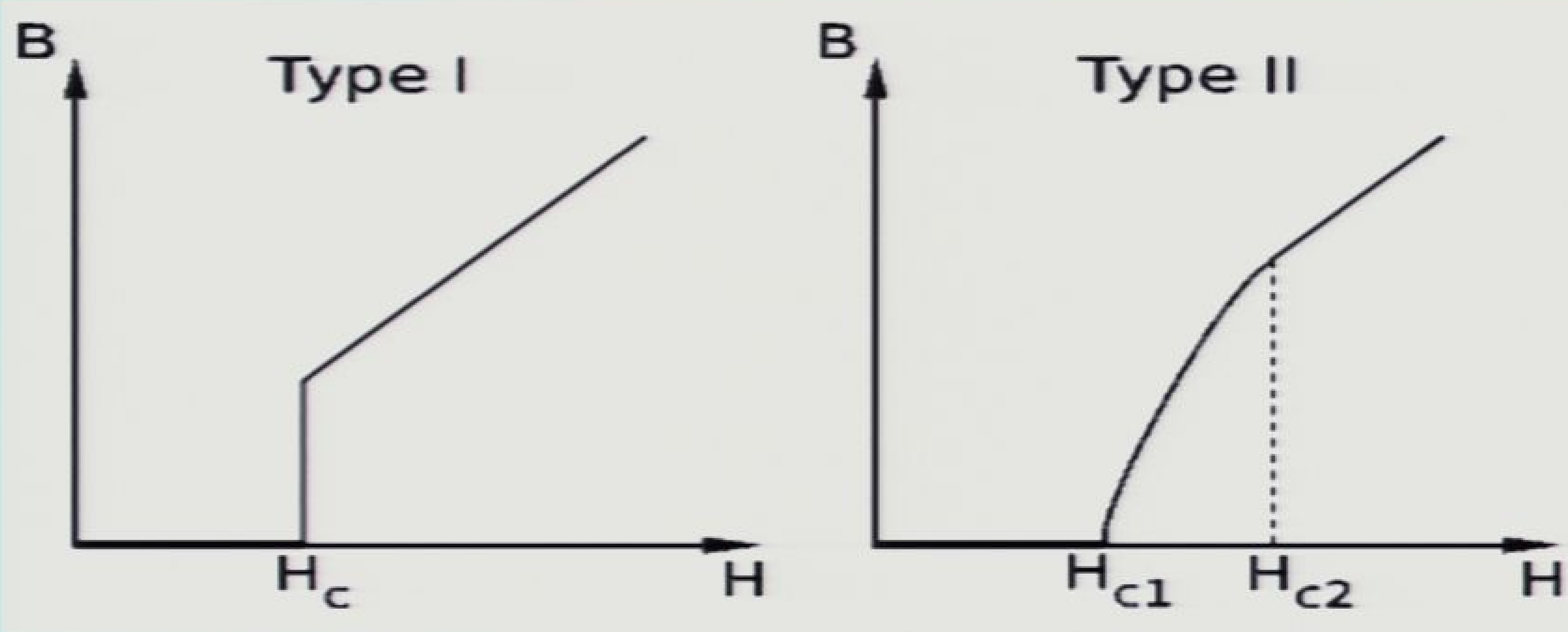


$H \geq H_c$

Variation of internal magnetic field (B) with applied external magnetic field (H) for Type I, Type II, and Color Superconductors



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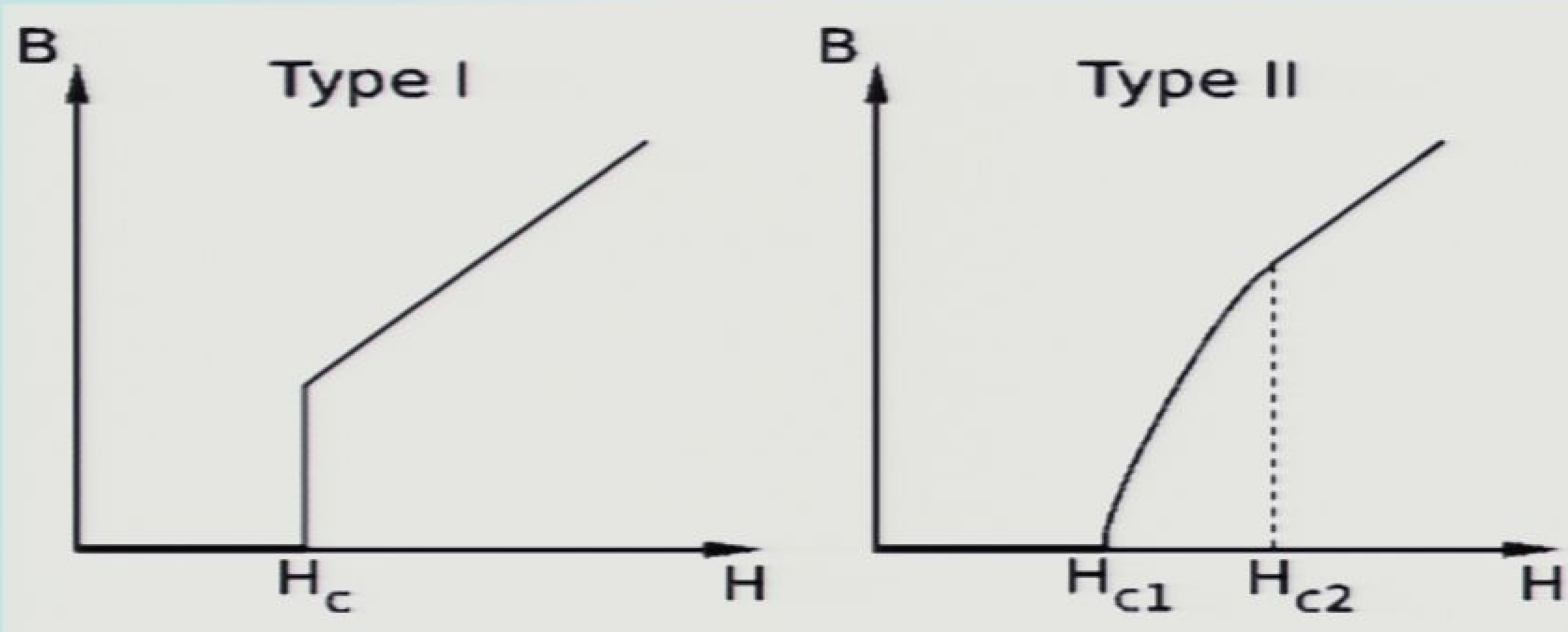

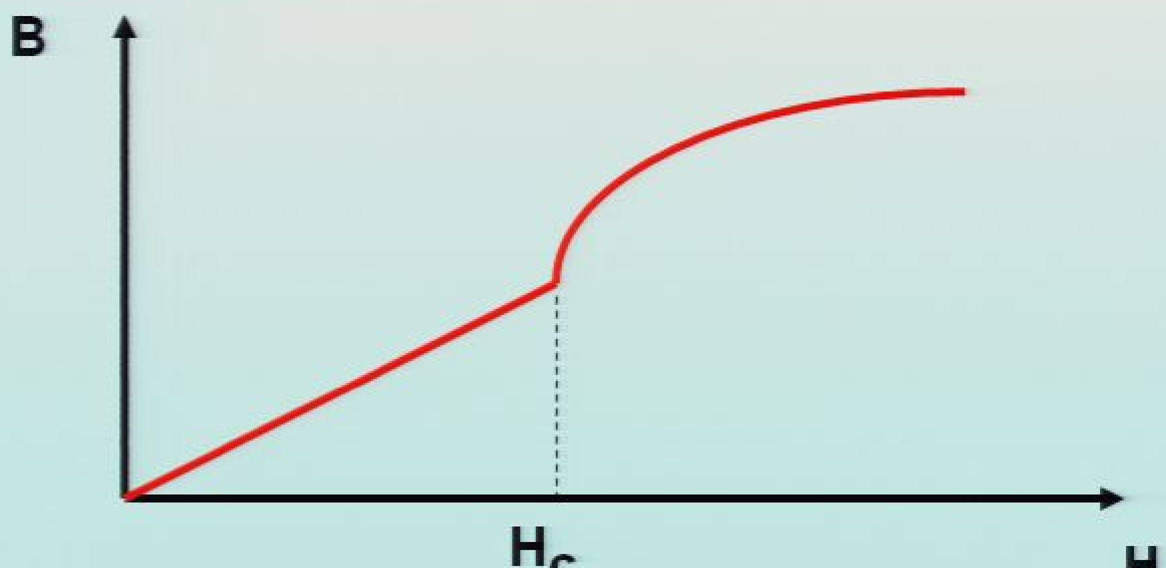

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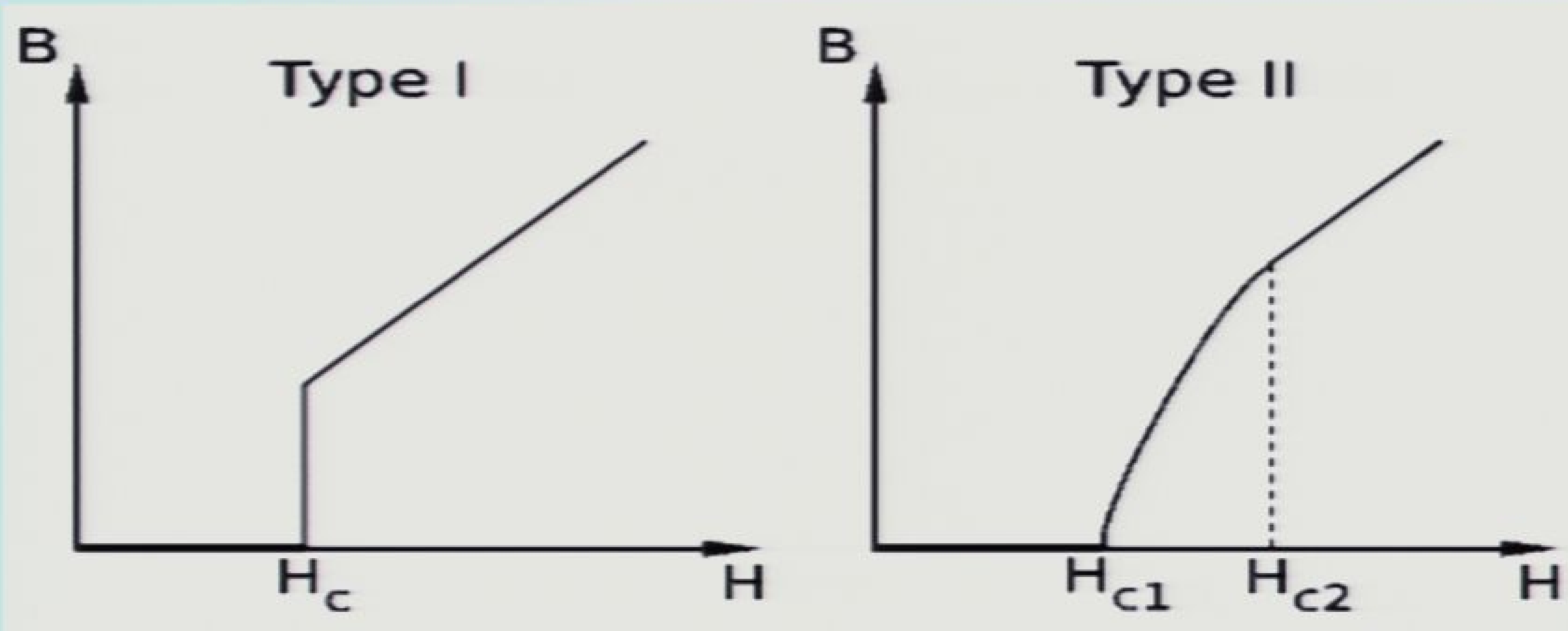
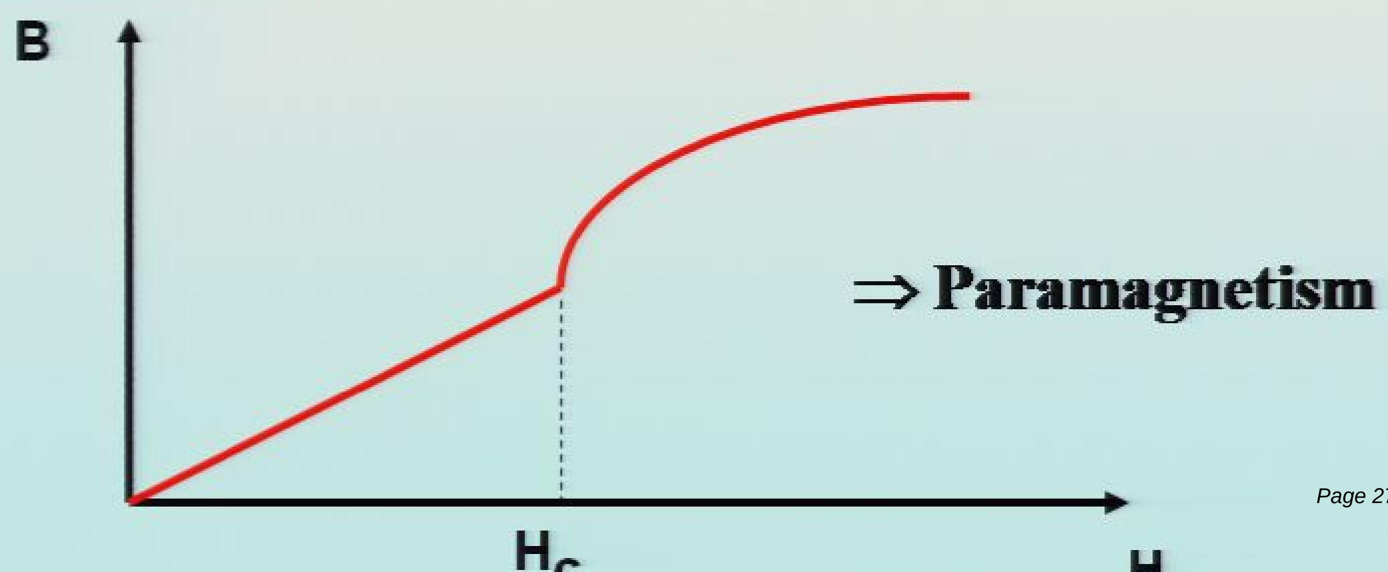




Linearizing the minimum equation for G* around the critical field

$$[\partial_j^2 - \frac{4\pi i}{\widetilde{\Phi}_0}\widetilde{H}_C x\partial_y - 4\pi^2\frac{\widetilde{H}_C^2}{\widetilde{\Phi}_0^2}x^2 + \frac{1}{\xi^2}]G = 0, \quad j = x, y$$

where

$$\tilde{\Phi}_0 = \frac{2\pi}{\tilde{e}}, \quad \xi^2 = 1/(2\tilde{e}\tilde{H} - m^2{}_M) = \frac{1}{m^2{}_M}$$

to find the solution

$$G_k = \exp\left[-iky\right]\exp\left[-\frac{(x-x_k)^2}{2\xi^2}\right]$$

$$x_k = \frac{k\Phi_0}{2\pi\tilde{H}}$$

Vortex Solution

From the experience with conventional type II superconductivity, it is known that the inhomogeneous condensate solutions prefer periodic lattice domains to minimize the energy. Then, putting on periodicity in the *y*-direction:

$$\Delta y = b \quad \Rightarrow \quad k = 2\pi n / b, \quad n = 0, \pm 1, \pm 2, \ldots$$

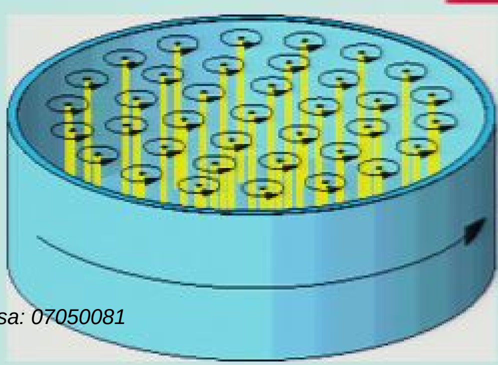
Vortex lattice,
First image 1967

The periodicity is also transferred to the *x*-direction:

$$x_n = k_n \xi^2 = 2\pi n \xi^2 / b$$

Then, the general solution is given by the superposition:

$$\overline{G}(x,y) = [1/\sqrt{2}\tilde{e}\xi] e^{-\frac{1}{2\xi^2}x^2} \vartheta_3(u,q)$$



The vortex lattice induces a magnetic field that forms a fluxoid along the z-direction. The magnetic flux through each periodicity cell in the vortex lattice is quantized

$$\widetilde{B} \Delta x \Delta y = 2\pi / \tilde{e}$$

Chromomagnetic Instabilities & $G-\tilde{B}$ Condensates in 2SC

EJF & de la Incera, hep-ph/0705.2403

$$\{G_\mu^{(1)}, G_\mu^{(2)}, G_\mu^{(3)}, K_\mu, K_\mu^\dagger, \widetilde{G}_\mu^8, \widetilde{A}_\mu\}$$

$$K_\mu \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} G_\mu^{(4)} - iG_\mu^{(5)} \\ G_\mu^{(6)} - iG_\mu^{(7)} \end{pmatrix}$$

G_μ^1	G_μ^2	G_μ^3	K_μ	$K_\mu^\dagger$	$\widetilde{G}_\mu^8$
0	0	0	1/2	-1/2	0

$$\widetilde{e} = e\cos\theta$$

Neutrality Conditions

$$\frac{\partial \Omega}{\partial \mu_B} = J_0^{(B)}$$

$$\frac{\partial \Omega}{\partial \mu_e} = 0$$

$$\frac{\partial \Omega}{\partial \mu_8} = 0 \qquad \frac{\partial \Omega}{\partial \mu_3} = 0$$

$$\mu_8 = (\sqrt{3} g / 2) \langle G_0^{(8)} \rangle \qquad \mu_3 = (g / 2) \langle G_0^{(3)} \rangle$$

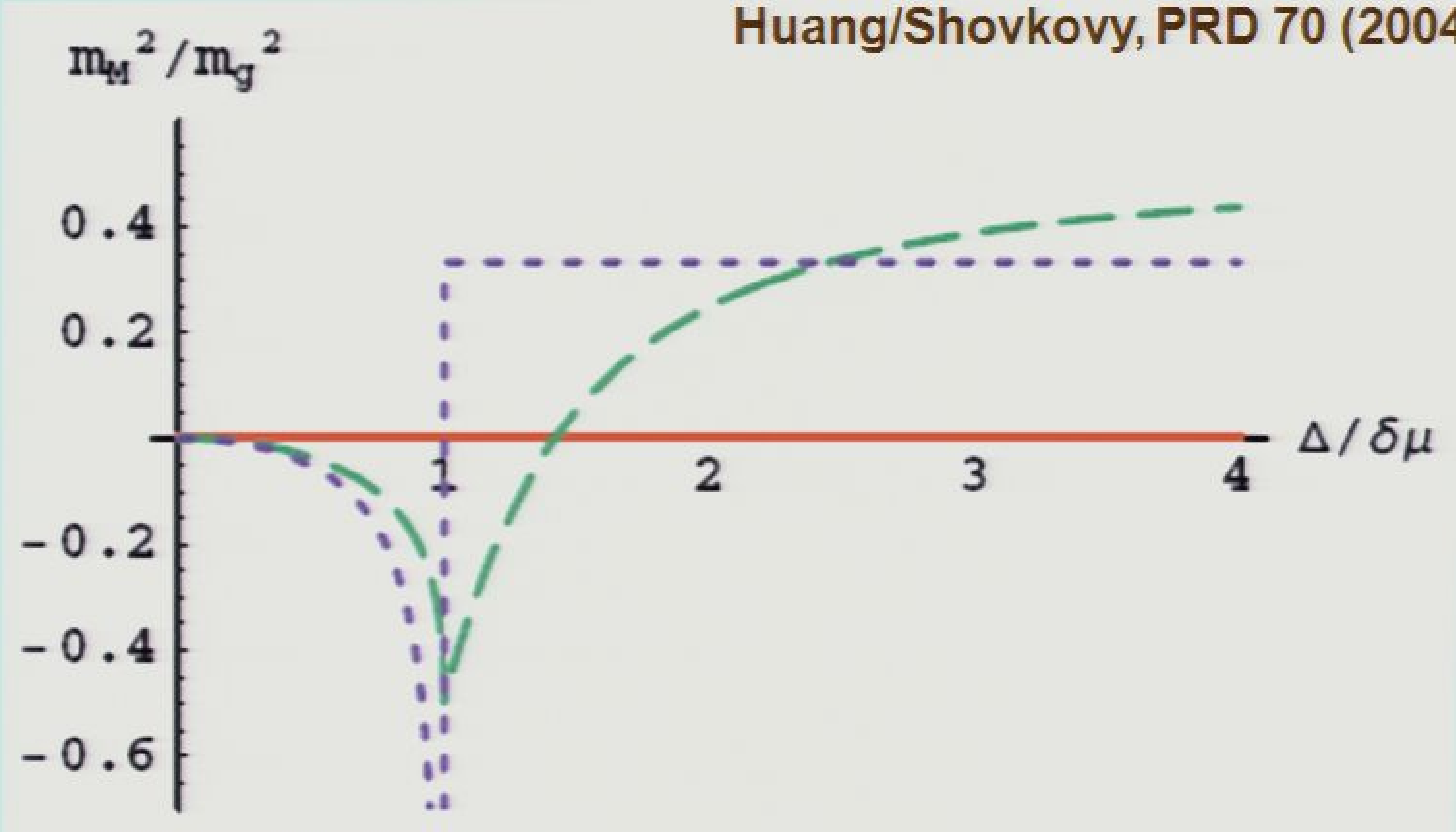
Stable Phase:

$$\mu_3 = 0, \qquad \mu_8 << \mu_e < \mu, \qquad \mu_8 \sim \Delta / \mu$$

Huang/Shovkovy, PLB 564 (2003) 205

Meissner Screening Masses
& Chromomagnetic Instabilities
in Neutral Dense Two-Flavor Quark Matter

Huang/Shovkovy, PRD 70 (2004) 051501



$$m_M^2 = \frac{2\alpha_s\overline{\mu}^2}{3\pi}\left[1-\frac{2\delta\mu^2}{\Delta^2}\right],$$

At $\Delta > \delta\mu > \Delta/\sqrt{2}$ → ***Tachyonic Mode of Charged Gluons***

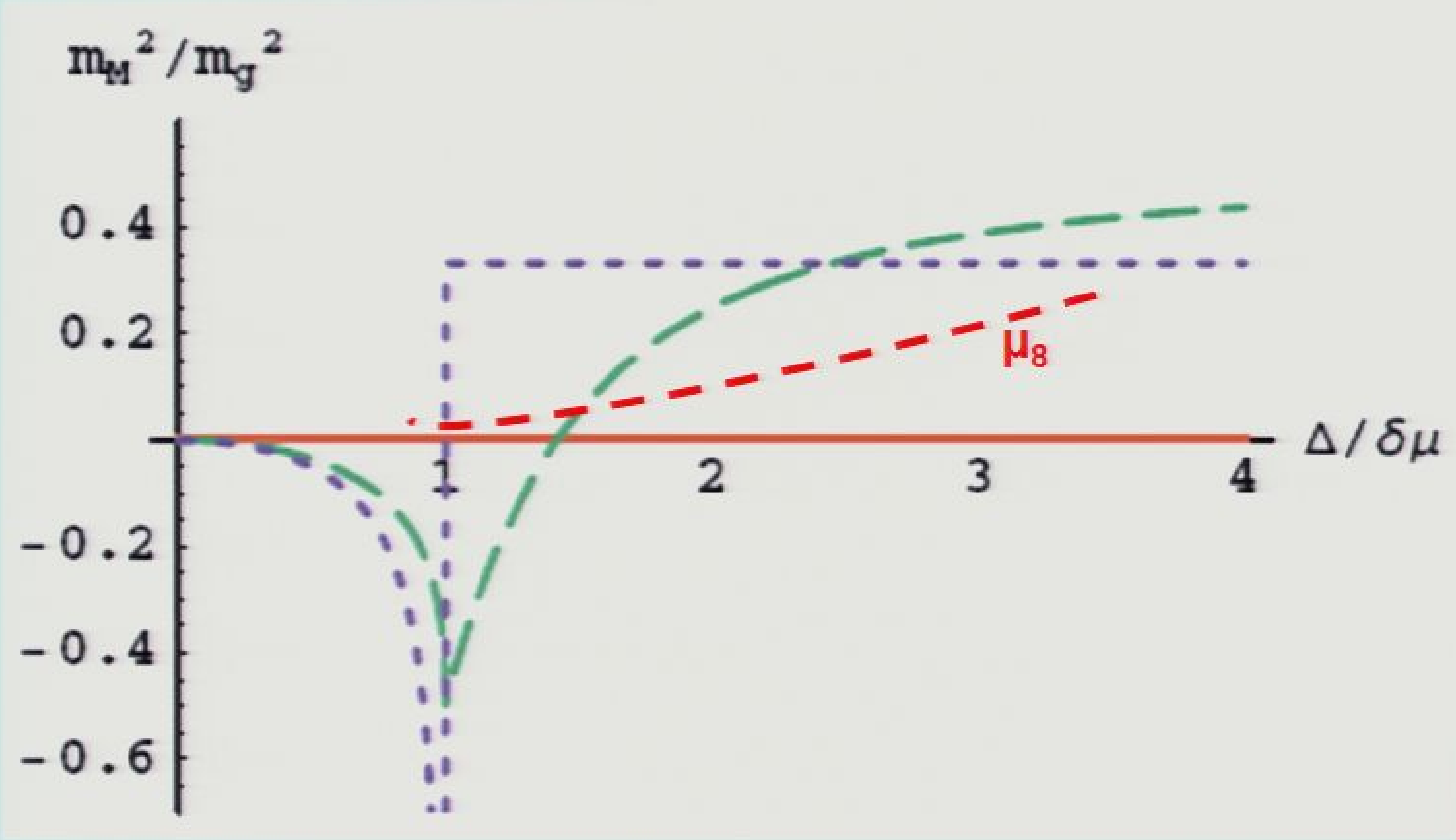
Effective Action

$$\Gamma^g_{eff} = \int d^4x\{-\frac{1}{4}(\widetilde{f}_{\mu\nu})^2 - \frac{1}{2}|\widetilde{\Pi}_\mu K_\nu - \widetilde{\Pi}_\nu K_\mu|^2$$
$$- [m^2_M \delta_{\mu i}\delta_{\nu i} - (\mu_8 - \mu_3)^2\delta_{\mu\nu} + i\widetilde{q}\widetilde{f}_{\mu\nu}]K_\mu K^\dagger_\nu$$
$$+ \frac{g^2}{2}[(K_\mu)^2(K^\dagger_\nu)^2 - (K_\mu K^\dagger_\mu)^2] + \frac{1}{\lambda}K^\dagger_\mu \widetilde{\Pi}_\mu \widetilde{\Pi}_\nu K_\nu\}$$

$$\langle K_\mu \rangle \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} \overline{G}_\mu \\ 0 \end{pmatrix},$$

$$\overline{G}_\mu \equiv G(x,y)(1,-i,0,0), \qquad \langle \widetilde{f}_{12} \rangle = \widetilde{B}$$

Weakly First-order Phase Transition



$$m_M^2 = \frac{2\alpha_s \bar{\mu}^2}{3\pi}[1 - \frac{2\delta\mu^2}{\Delta^2}],$$ ***At*** $\Delta > \delta\mu > \Delta/\sqrt{2}$ → ***Tachyonic Mode of Charged Gluons***

Free-Energy:

$$\mathcal{F}_g = \frac{\widetilde{B}^2}{2} - 2\overline{G}^*\widetilde{\Pi}^2\overline{G} + 2g^2|\overline{G}|^4$$
$$-2[2\tilde{q}\widetilde{B} + (\mu_8 - \mu_3)^2 - m_M^2]|\overline{G}|^2$$

Minimum Equations:

$$\frac{\partial \mathcal{F}_g}{\partial \mu_3} = 0 \quad \Rightarrow \quad \mu_3 = \mu_8$$

$$\frac{\partial \mathcal{F}_g}{\partial \overline{G}^*} = 0$$

$$-\widetilde{\Pi}^2\overline{G} - (2\tilde{q}\widetilde{B} + |m_M^2|)\overline{G} + 2g^2|\overline{G}|^2\overline{G} = 0$$

Linear Equations

$$\delta\mu \simeq \delta\mu_c \quad \Rightarrow \quad |\widetilde{\Pi}_\mu K_\nu - \widetilde{\Pi}_\nu K_\mu|^2 \approx 0$$

$$-\widetilde{\Pi}^2\overline{G} - (2\tilde{q}\widetilde{B} + |m_M^2|)\overline{G} + 2g^2|\overline{G}|^2\overline{G} = 0$$

$$\widetilde{\Pi}^2\overline{G} + \tilde{q}\widetilde{B}\overline{G} \simeq 0$$

$$2\tilde{q}|\overline{G}|^2 - \widetilde{B} \simeq 0$$

$$|\overline{G}|^2 \simeq \Lambda_{g/\tilde{q}}|m_M^2|/2\tilde{q}^2 + \mathcal{O}(m_M^4)f(x,y)$$

$$\tilde{q}\widetilde{B} \simeq \Lambda_{g/\tilde{q}}|m_M^2| + \mathcal{O}(m_M^4)g(x,y)$$

Linear Equations

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$$|\widetilde{\Pi}_\mu K_\nu - \widetilde{\Pi}_\nu K_\mu|^2 \approx 0$$

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$\Downarrow$

$$\widetilde{\Pi}^2\overline{G} + \tilde{q}\widetilde{B}\overline{G} \simeq 0$$

$$2\tilde{q}|\overline{G}|^2 - \widetilde{B} \simeq 0$$

$\Rightarrow$

$$\overline{\mathcal{F}}_g \simeq -2(g^2 - \tilde{q}^2)|\overline{G}|^4$$

$$|\overline{G}|^2 \simeq \Lambda_{g/\tilde{q}}|m_M^2|/2\tilde{q}^2 + \mathcal{O}(m_M^4)f(x,y)$$

$$\tilde{q}\widetilde{B} \simeq \Lambda_{g/\tilde{q}}|m_M^2| + \mathcal{O}(m_M^4)g(x,y)$$

Gluon-Condensate Solution

$$\widetilde{\Pi}^2\overline{G} + \tilde{q}\widetilde{B}\overline{G} \simeq 0$$

$$[\frac{1}{r}\partial_r(r\partial_r) + \frac{1}{r^2}\partial_\theta^2 + \frac{1}{\xi^2}(1 - i\partial_\theta) - \frac{r^2}{4\xi^4}]G(r,\theta) = 0$$

$$\widetilde{A}_i = -(\widetilde{B}/2)\epsilon_{ij}x_j \qquad \xi^2 \equiv 1/\Lambda_{g/\tilde{q}}|m_M^2|$$

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$$\widetilde{A}_i = -(\widetilde{B}/2)\epsilon_{ij}x_j \qquad \xi^2 \;\equiv\; 1/\Lambda_{g/\tilde{q}}|m_M^2|$$

$$r \ll \xi \qquad \frac{r^2}{4\xi^4} \ll \frac{1}{\xi^2} \qquad G(r,\theta) \sim R(r)e^{i\chi}$$

$$[r\partial_r(r\partial_r) + \frac{r^2}{\xi^2}]R(r) = 0$$

$$G(r) = (1/\sqrt{2}\tilde{q}\xi)J_0(r/\xi)\exp i\chi$$

Gluon-Condensate Solution

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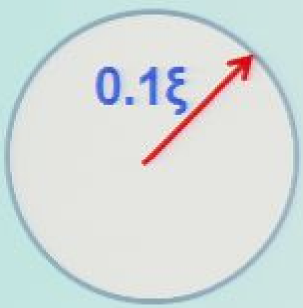
$$G(r) = (1/\sqrt{2}\widetilde{q}\xi)J_0(r/\xi)\exp i\chi$$

$$|\overline{G}|^2 \simeq \frac{1}{2\widetilde{q}^2\xi^2} - \frac{r^2}{4\widetilde{q}^2\xi^4}$$

Condensate Free-Energy

Inhomogeneous Condensate: *EJF & de la Incera, hep-ph/0705.2403*



$$\overline{F}_g \approx -\frac{\pi \left| m_M{}^2 \right|}{200\tilde{q}^2}$$

Homogeneous Condensate: *Gorbar/Hashimoto/Miransky, PLB 632 (2006) 305*

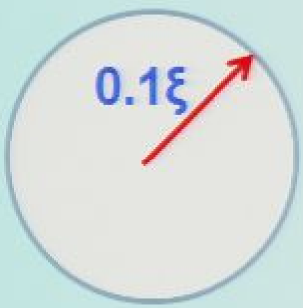
$$\overline{F}_g \approx -\frac{\left(\frac{g^2}{\tilde{q}^2} - 1\right) \pi^2 \left| m_M{}^2 \right|^2}{200\, \alpha_s{}^3 m_g{}^2}$$

$$\alpha_s < 1$$

$$\alpha_s \simeq 1$$

Condensate Free-Energy

Inhomogeneous Condensate: *EJF & de la Incera, hep-ph/0705.2403*



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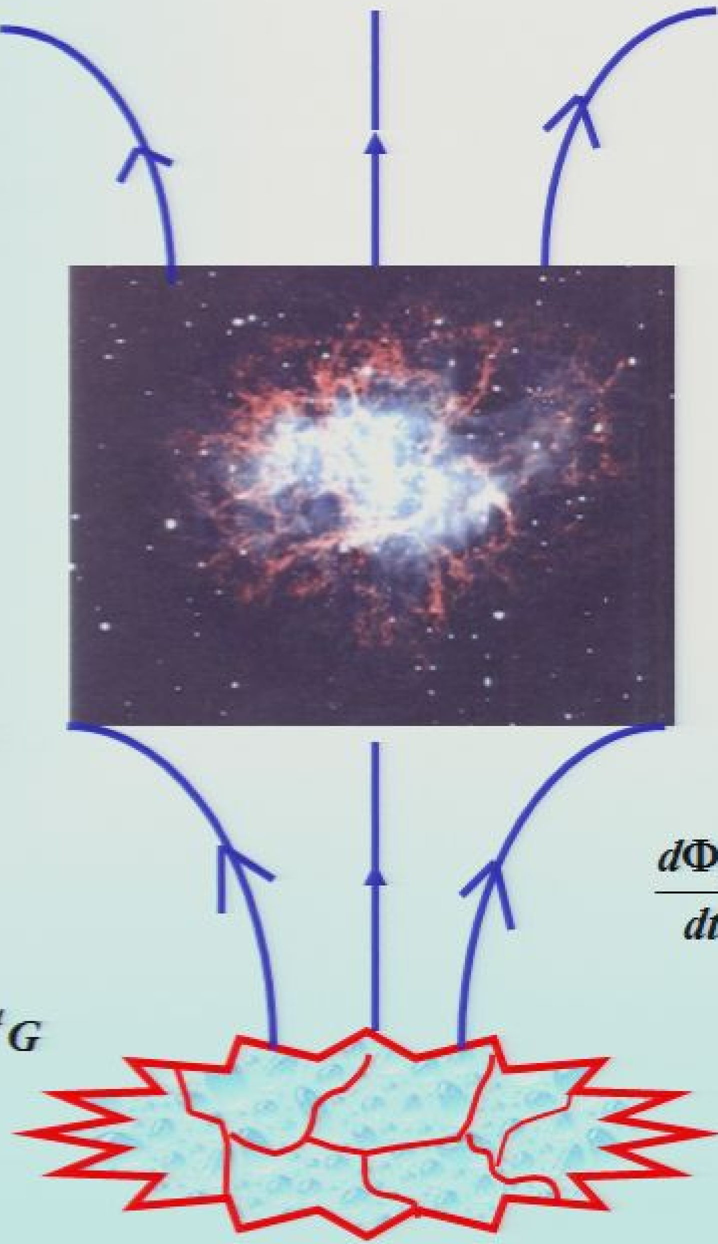
Origin of Stellar Magnetic Fields

$B = 10^8\,G$

~ 12 billion Yrs.



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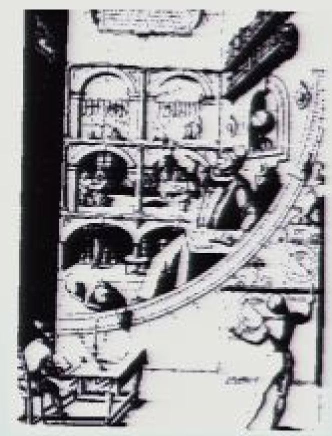

$B = 10^{-4}\,G$

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$$\frac{d\Phi_S}{dt} = -\frac{1}{4\pi\sigma}\int_S \nabla \times (\nabla \times \vec{B}) \cdot d\vec{S} \approx 0$$

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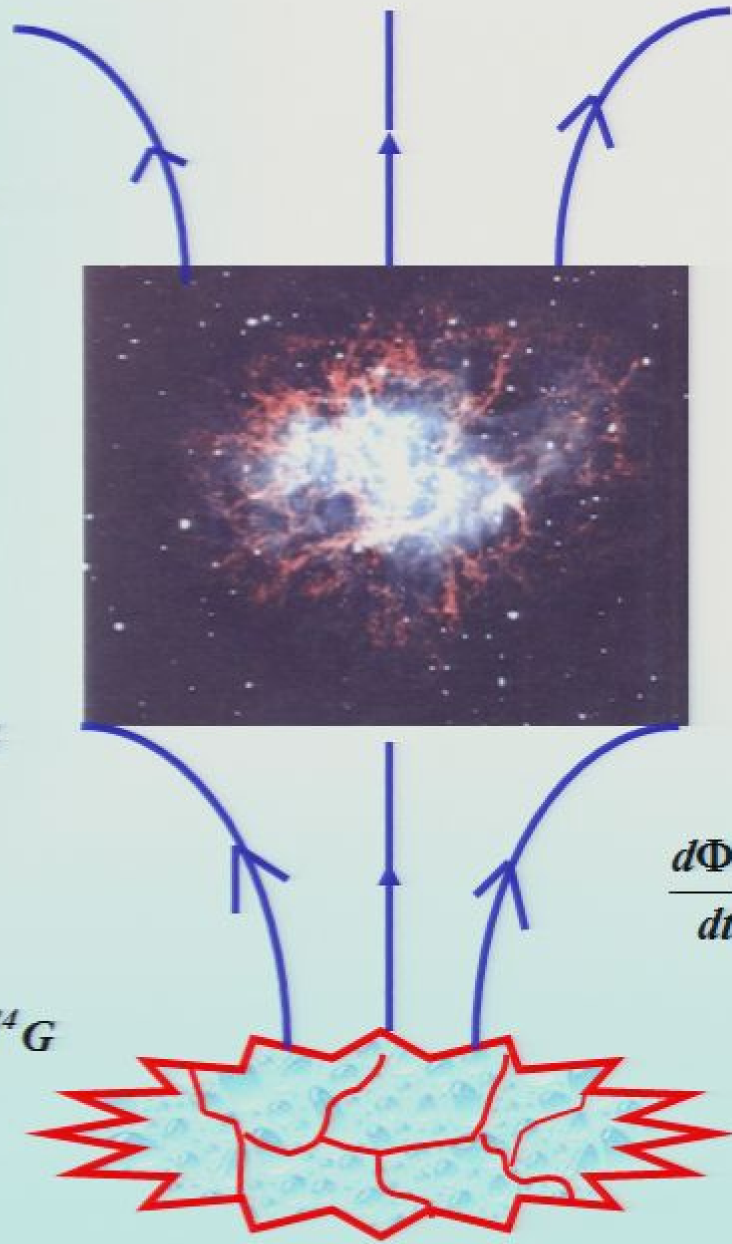

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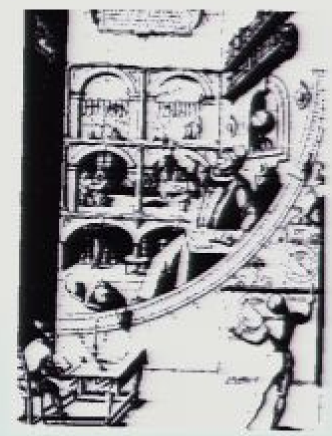

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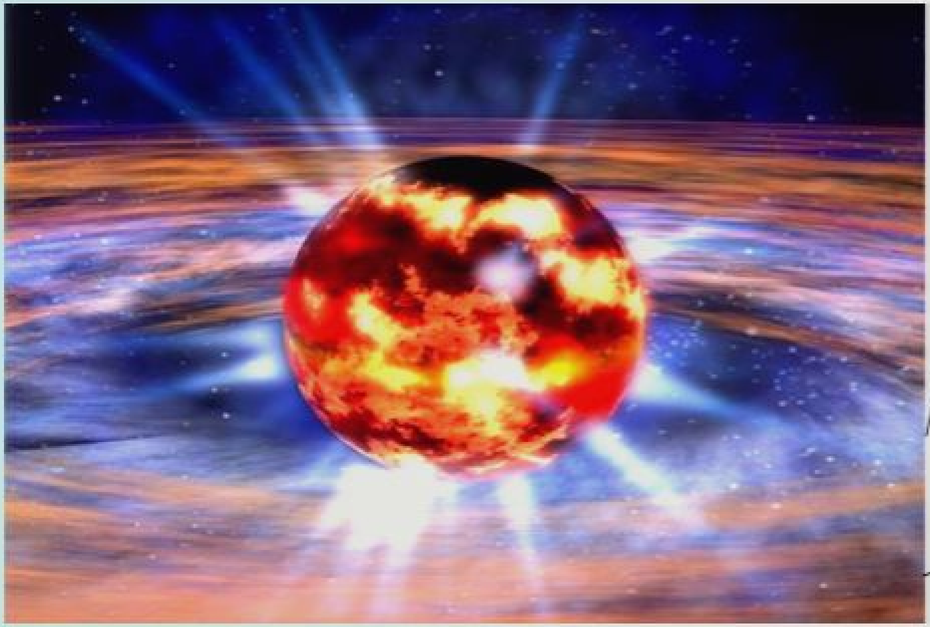
Difficulties of the Standard Magnetar Model



Supernova remnants associated with magnetars should be an order of magnitude more energetic.

↓

Recent calculations indicate that their energies are similar.

When a magnetar spins down, the rotational energy output should go into a magnetized wind of ultra-relativistic electrons and positrons that radiate via synchrotron emission.

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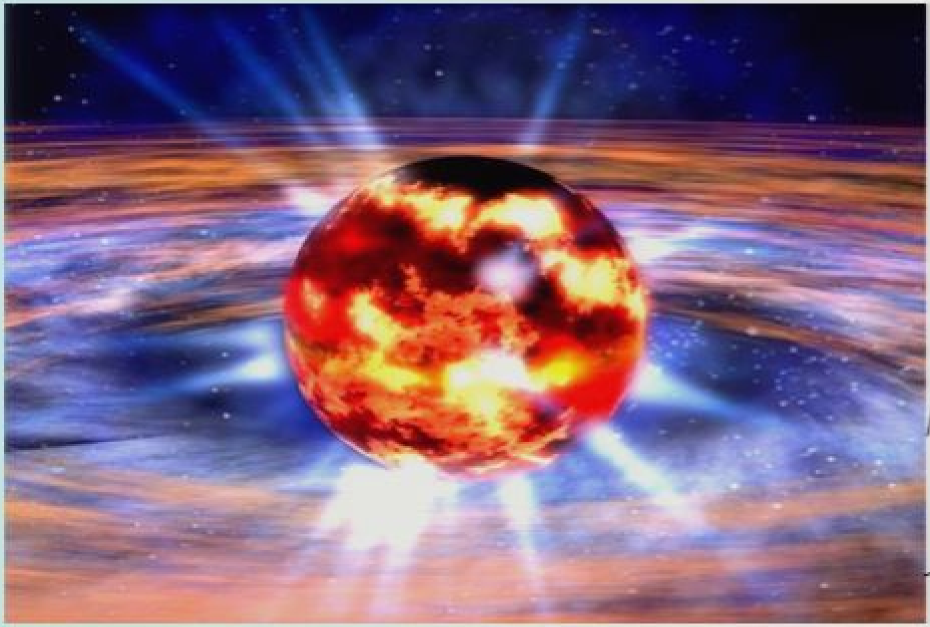

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Possible Alternative:

B induced by the CS core.

Conclusions and Future Directions

- *Color Superconductivity at moderate densities can be a source of stellar magnetic fields.*

  Magnetars $\longleftrightarrow$ *CS Cores*

- *Numerically solving the nonlinear equation, looking for the realization of the vortex state*

- *Exploring even lower densities (finding the corrections to the coefficients).*

- *Three-flavor system: vortex state in gCFL?*