Title: Soft Modes Associated with QCD Phase Transitions From Meson Condensation to Colour Superconductitivity

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Abstract:

Soft Modes Associated with QCD Phase Transitions from Meson Condensation to Colour Superconductivity

T. Kunihiro (YITP, Kyoto)

International workshop
"Exotic States of Hot and Dense Matter and their Dual Description"
Perimeter Institute for Theoretical Physics
May 22 - 25, 2007

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and

Correct Relativistic Hydrodynamical Equations for Viscous Fluids

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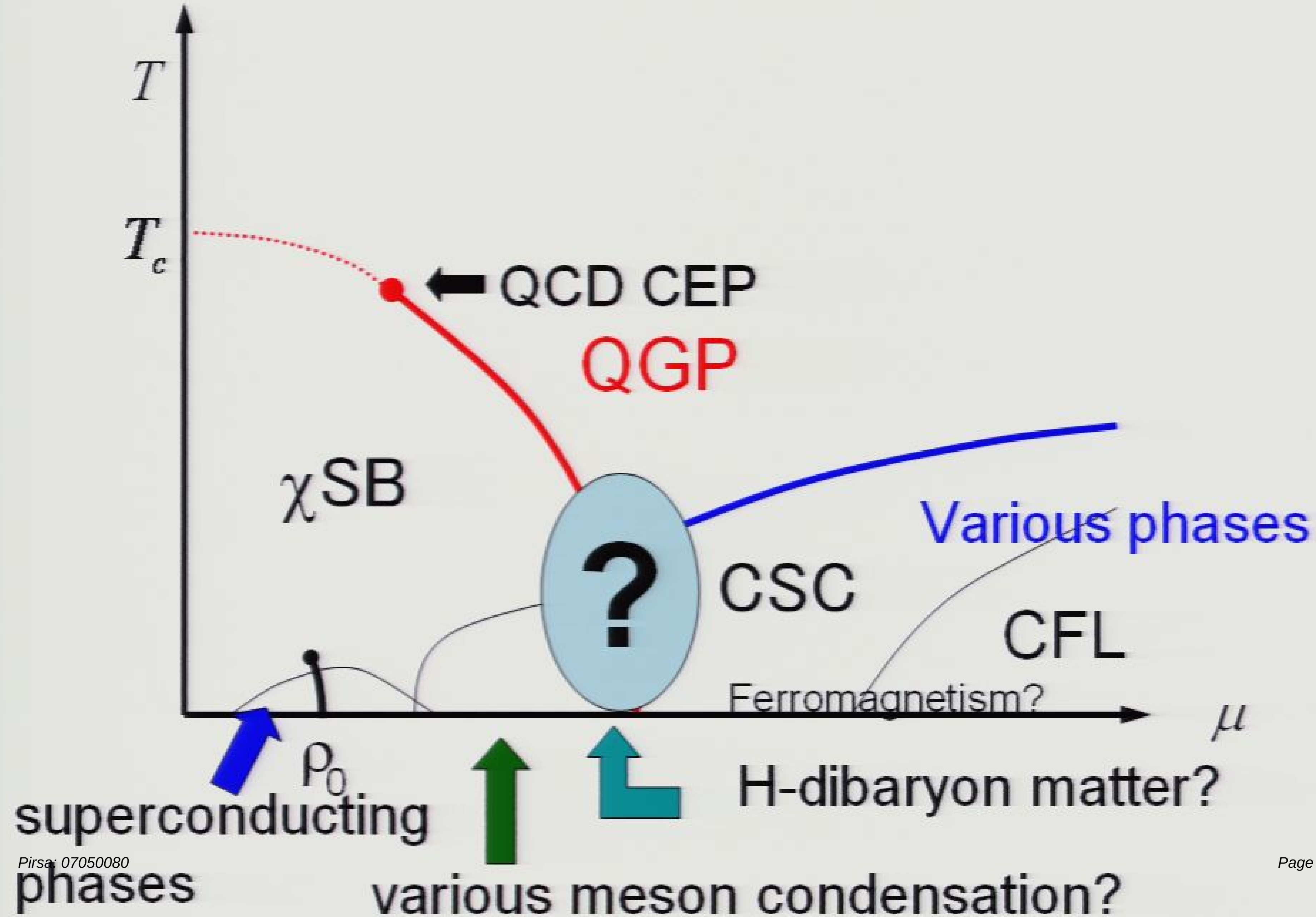
A conjectured QCD phase diagram



QCD CEP

QGP

χSB

CSC

CFL

Various phases

Ferromagnetism?

H-dibaryon matter?

superconducting phases

various meson condensation?

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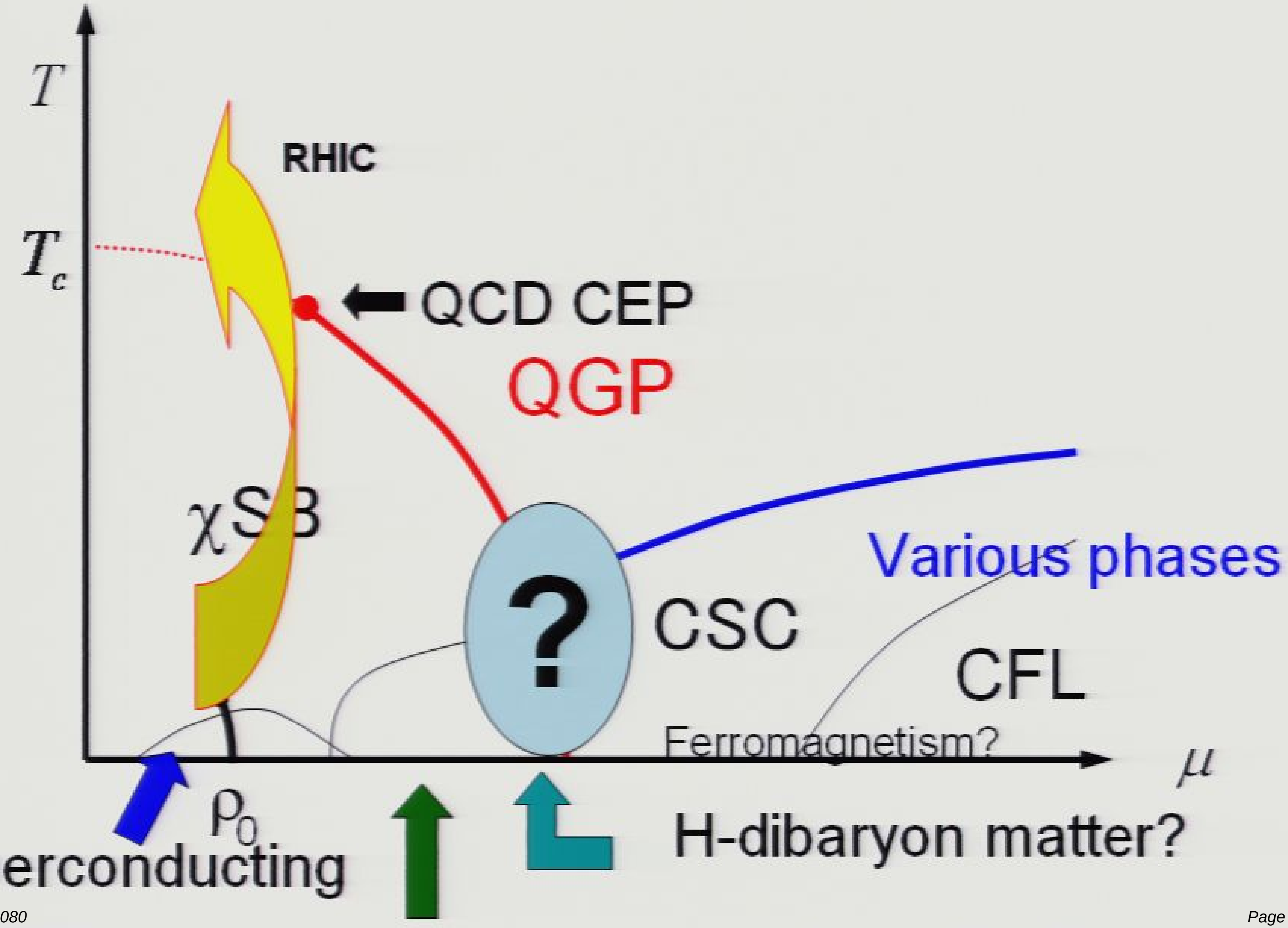

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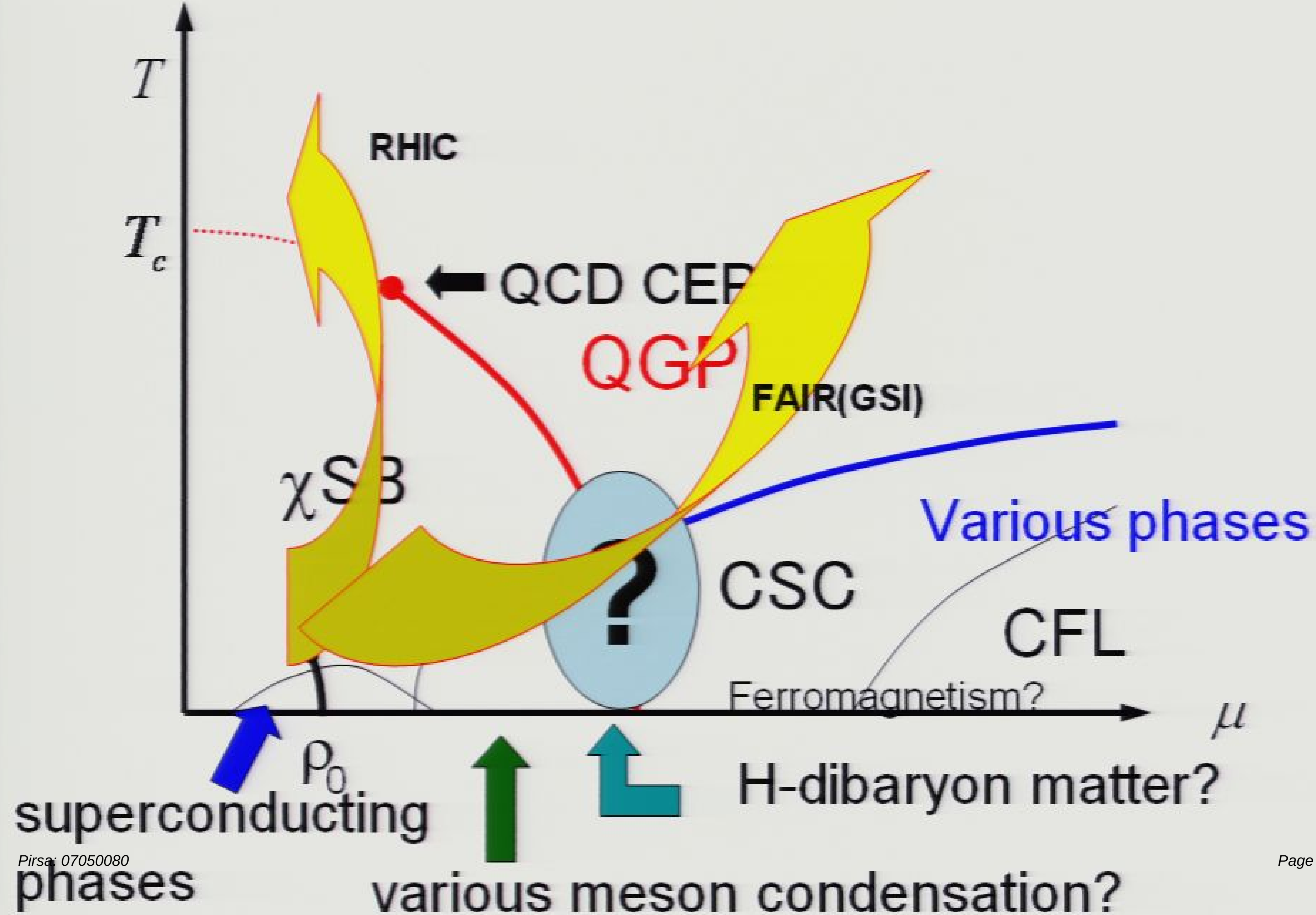

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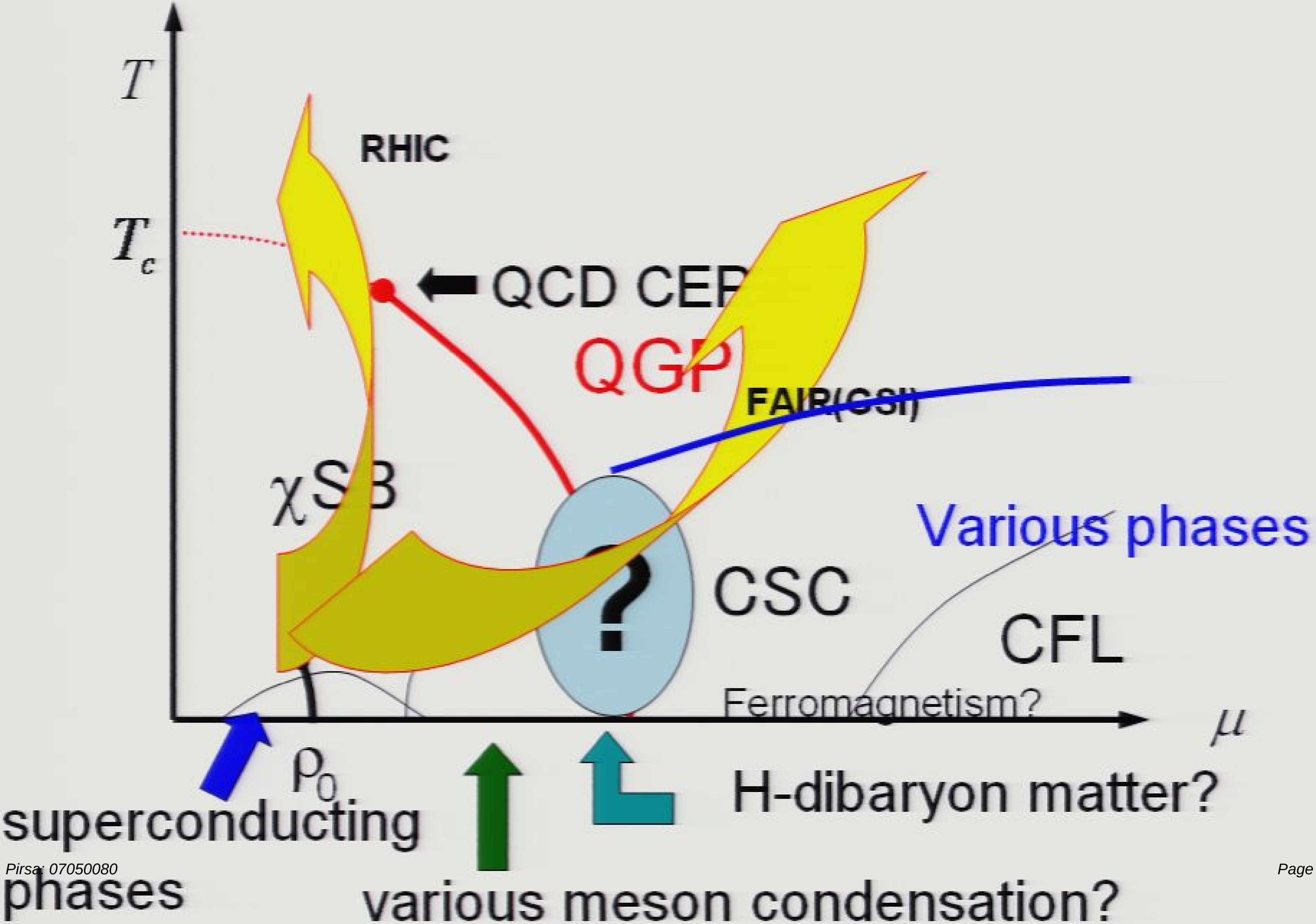

RHIC

QCD CEP

QGP

FAIR(GSI)

χSB

Various phases

CSC

CFL

Ferromagnetism?

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superconducting phases

various meson condensation?

Contents

PART I

1 Pion and Rho meson condensation and their precursory phenomena (history)

2 Color-superconductivity, chiral and/or vector condensations

3 Fluctuation effects of the order parameters and the quark quasi-particle picture

PART II

4 R-G derivation of stable dissipative relativistic fluid dynamical equations

5 Summary

The talk will be, admittedly, heuristic or may be model dependent except for part II; nevertheless we hope that it would help us to get possible ideas on what might happen in hot and/or dense matter.

Tensor force and Pion and Rho Meson Condensation in p-wae

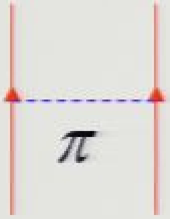
- The year 2007: centennial year of the birth of Hideki Yukawa
- Yukawa introduced the pion in an attempt to describe the nuclear forces in QFT.
- The OPEP (One-Pion-Exchange Potential) contains the Tensor force, a dipole-dipole interaction which is higher order and plays an only minor role in condensed matter physics.
- The tensor force makes nuclear dynamics unique in Nature; it makes the saturation property of nuclear matter, clustering phenomena in nuclei.

OPEP(One-Pion-Exchange Potential)

$$V_{\mathrm{OPEP}}(r) = f^2 m_\pi \frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{3}\Big[(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 Y(m_\pi r) + S_{12} Z(m_\pi r)\Big],$$

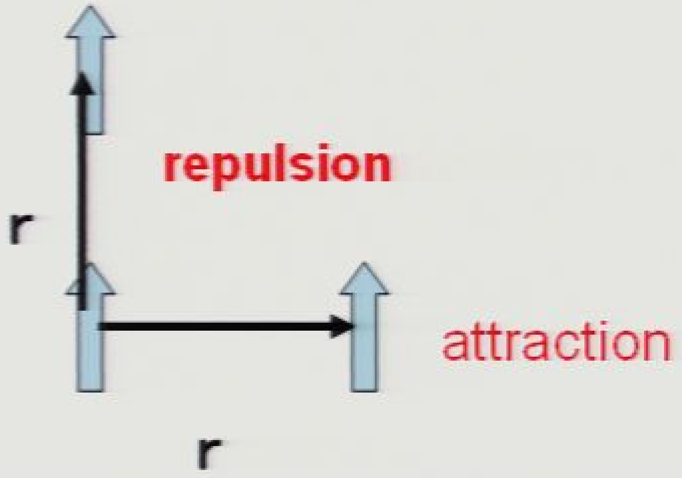
Central Tensor

$$Y(x) = \exp(-x)/x, \; Z(x) = (1 + 3/x + 3/x^2)Y(x),$$

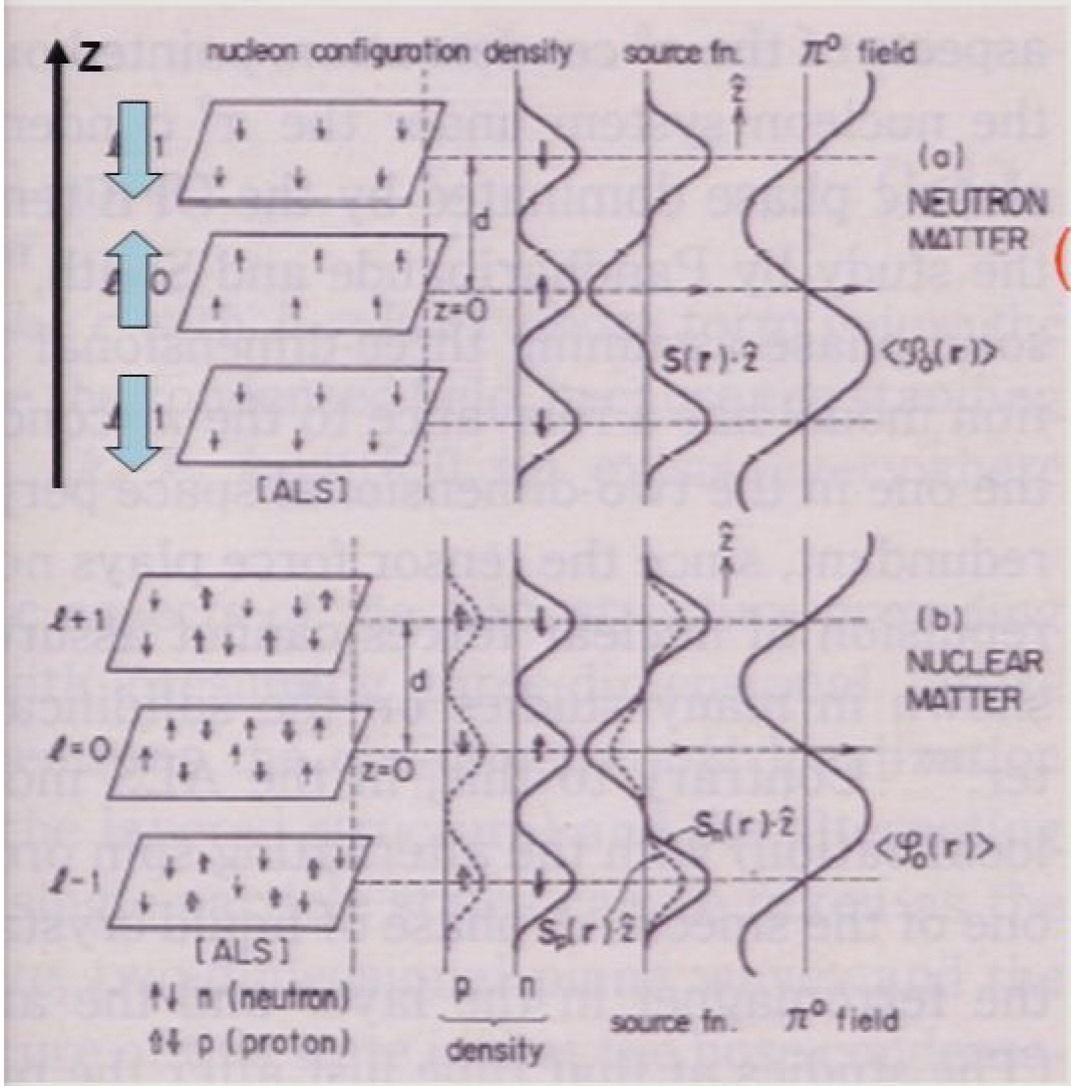


$$S_{12} = 3(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{r}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{r}}) - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$

Tensor operator $(\hat{\boldsymbol{r}} = \boldsymbol{r}/r).$



p-wave Neutral Pion-condensed Baryonic Matter; pion-induced tensor-force dominating phase



A.B. Migdal, Sawyer-Scalapino ('72)

Pion condensed phase =Alternating-Layer Spin (ALS) structure of the nucleon System

(R.Tamagaki et al (1976~))

cf. PTP suppl.112(1993)

$$(\nabla^2 - m_\pi^{\ 2})\langle\varphi_0(\boldsymbol{r})\rangle = \tilde{f}\,\nabla\cdot\boldsymbol{S}(\boldsymbol{r})$$

$$\boldsymbol{S} = \langle\Phi_{\mathrm{N}}|\psi^\dagger(\xi,t)\tau_3\boldsymbol{\sigma}\psi(\xi,t)|\Phi_{\mathrm{N}}\rangle$$

Pi : longitudinal spin-isospin density wave

⇕

transverse spin-isospin density wave

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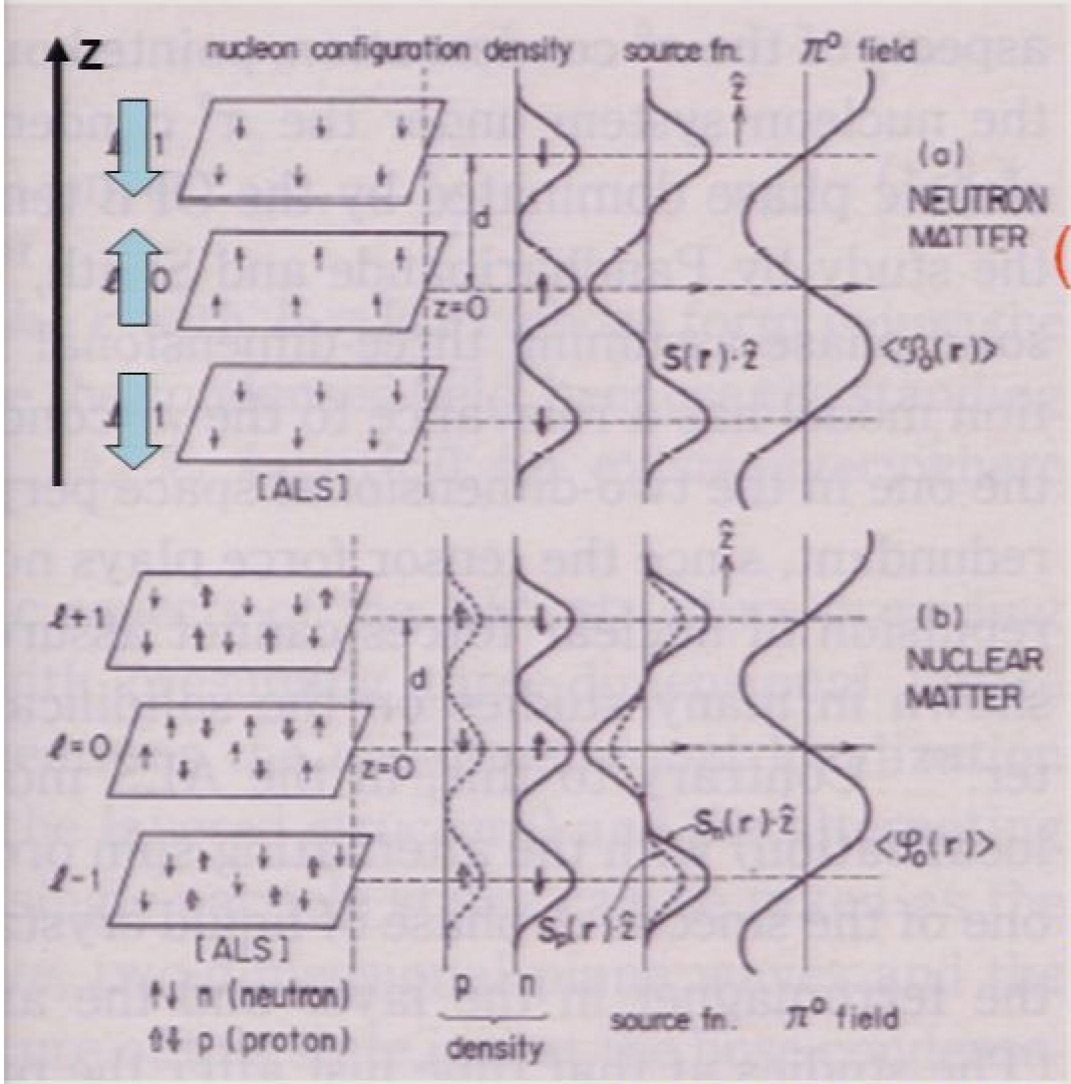

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Rho meson condensation：transverse spin-isospin density wave

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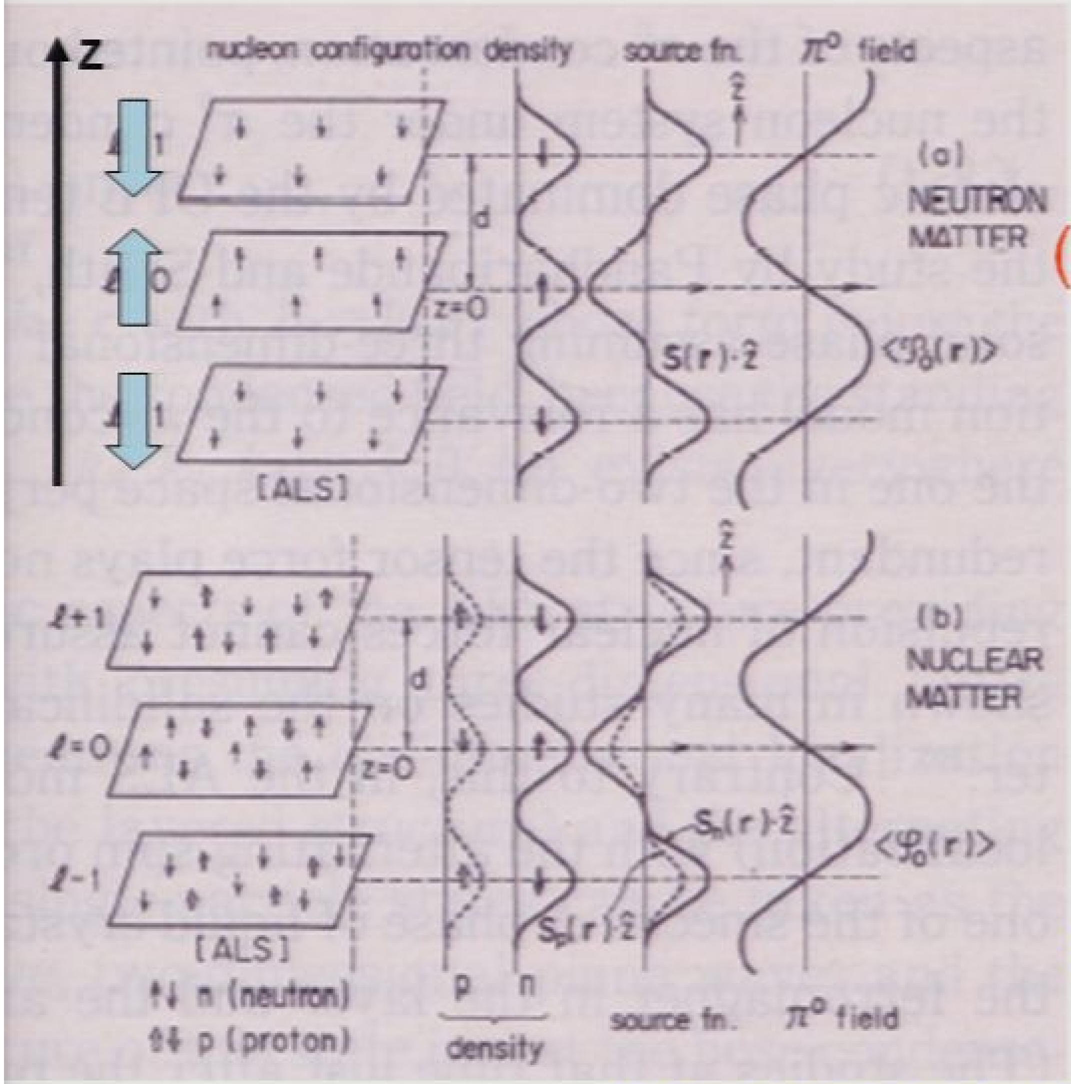

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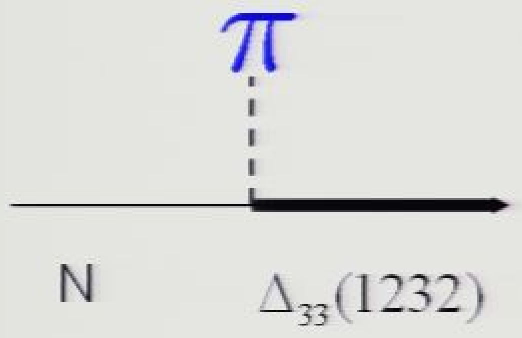
Rho meson condensation: transverse spin-isospin density wave

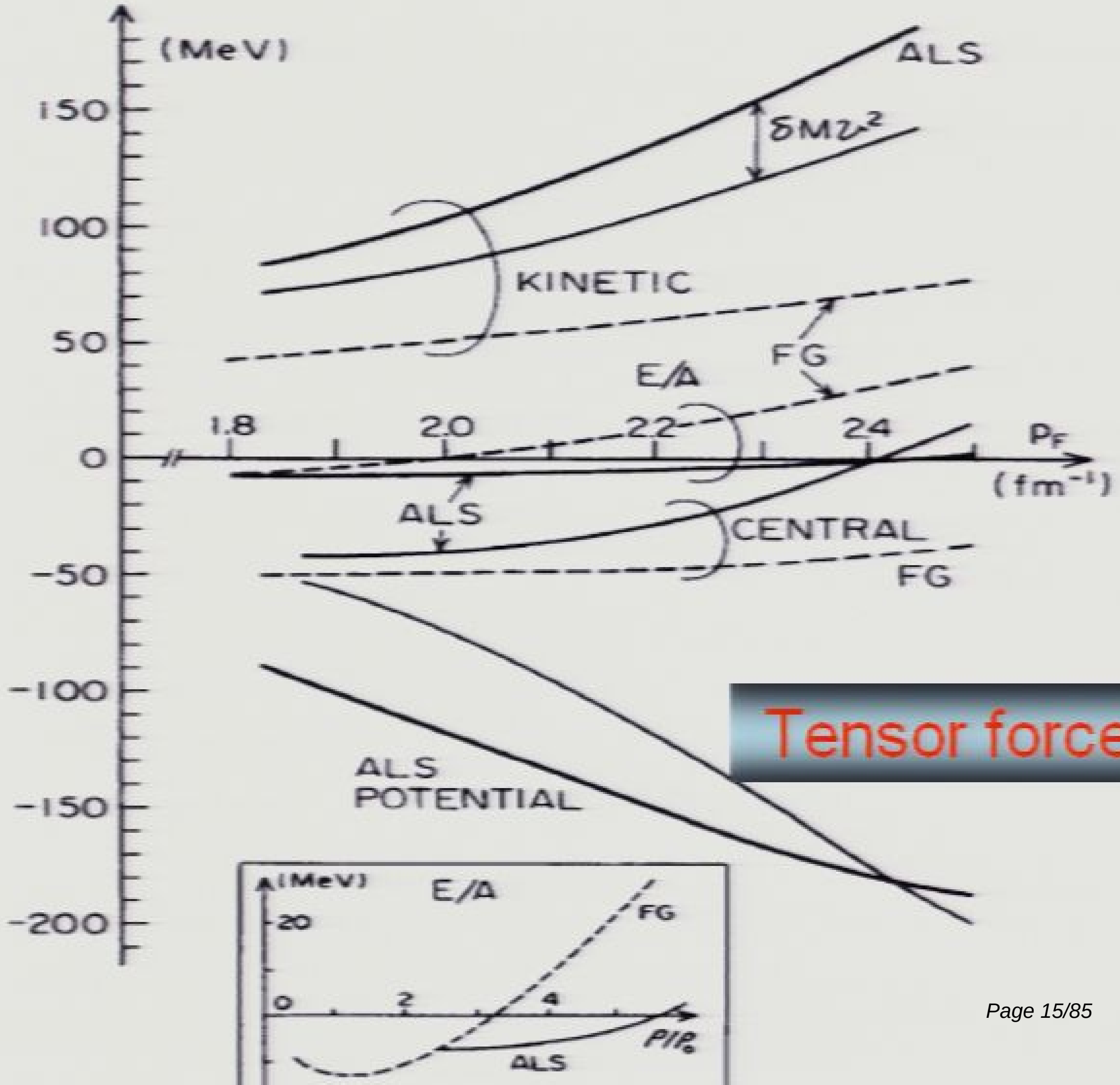
T.K., PTP 60 (1978), 1229; a first work on a vector con. cf. CFL

Effective Force (G0-force*) with the $\Delta_{33}(1232)$ resonance

T.K., T.Takatsuka, R.Tamagaki and T.Tatsumi,
PTP Suppl. 112('93), 123



EOS for pion-condensed N=Z Matter



Tensor force

*D.W.L.Sprung and P.K. Banerjee,NPA168('71);
D.W.L. Sprung,NPA182('72),97.

Role of the vector mesons

- ρ and ω mesons

Tensor coupling > vector (gauge) coupling

dominant ↑

The E.M. formfactor of the nucleon based on the Vector Meson Dominace

$$\bar{\psi}\sigma_{\mu\nu}\tau^{k}\psi(\partial^{\mu}\rho_{k}{}^{\nu}-\partial^{\nu}\rho_{k}{}^{\mu})$$

$$\Rightarrow \quad (\sigma_1\times\mathbf{q})\cdot(\sigma_2\times\mathbf{q})=-S_{12}(\mathbf{q})+\frac{2}{3}\sigma_1\cdot\sigma_2 q^2$$

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cf. Chiral symmetry and its dynamical breaking may be responsible for the VMD according to Hidden Local Symmetry (Bando,Kugo, Yamawaki)

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minus of OPEP

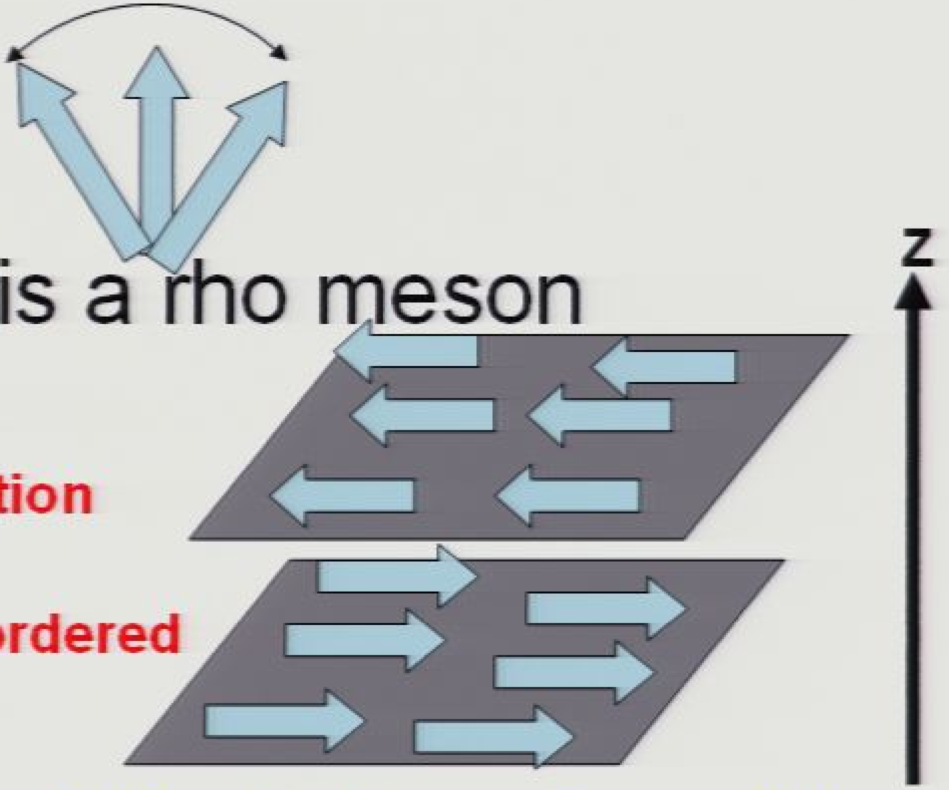
the same sign
as OPEP

Pion to Rho meson condensation

T.K., PTP 60 (1978), 1229;

- The nuclear spin direction oscillates at higher densities,
- And then

bend down, which is a rho meson condensed state.



Rho meson condensation

⇕

Transverse spin-isospin ordered baryonic matter

*** 3-d crystalline structure can be possible by incorporating other isospin components..**

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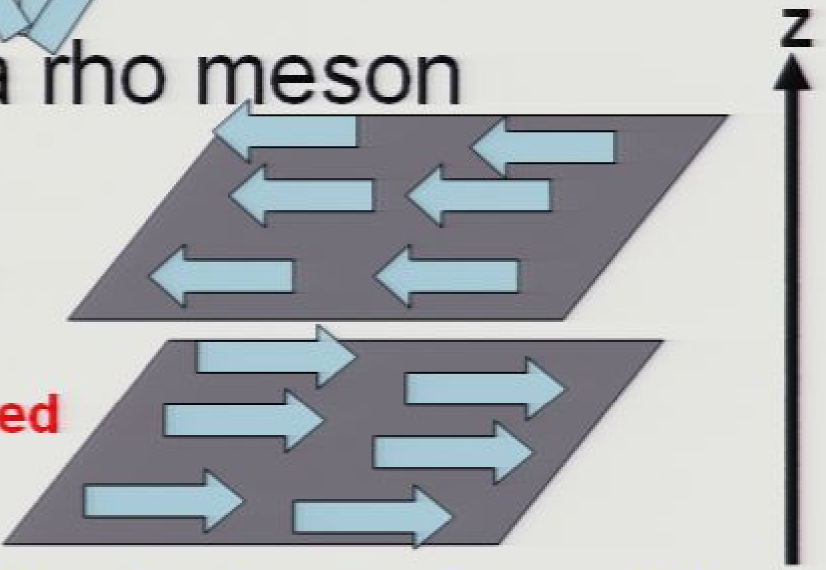


Rho meson condensation

⇕

Transverse spin-isospin ordered baryonic matter

Cf. Gluonic phase in CSC. V.Miransky

*** 3-d crystalline structure can be possible by incorporating other isospin components..**

Softening of the spin-isospin excitation as a precursory phenomenon of the pion condensation:

(T.K. Prog.Theor.Phys. **65** (1981), 1098)

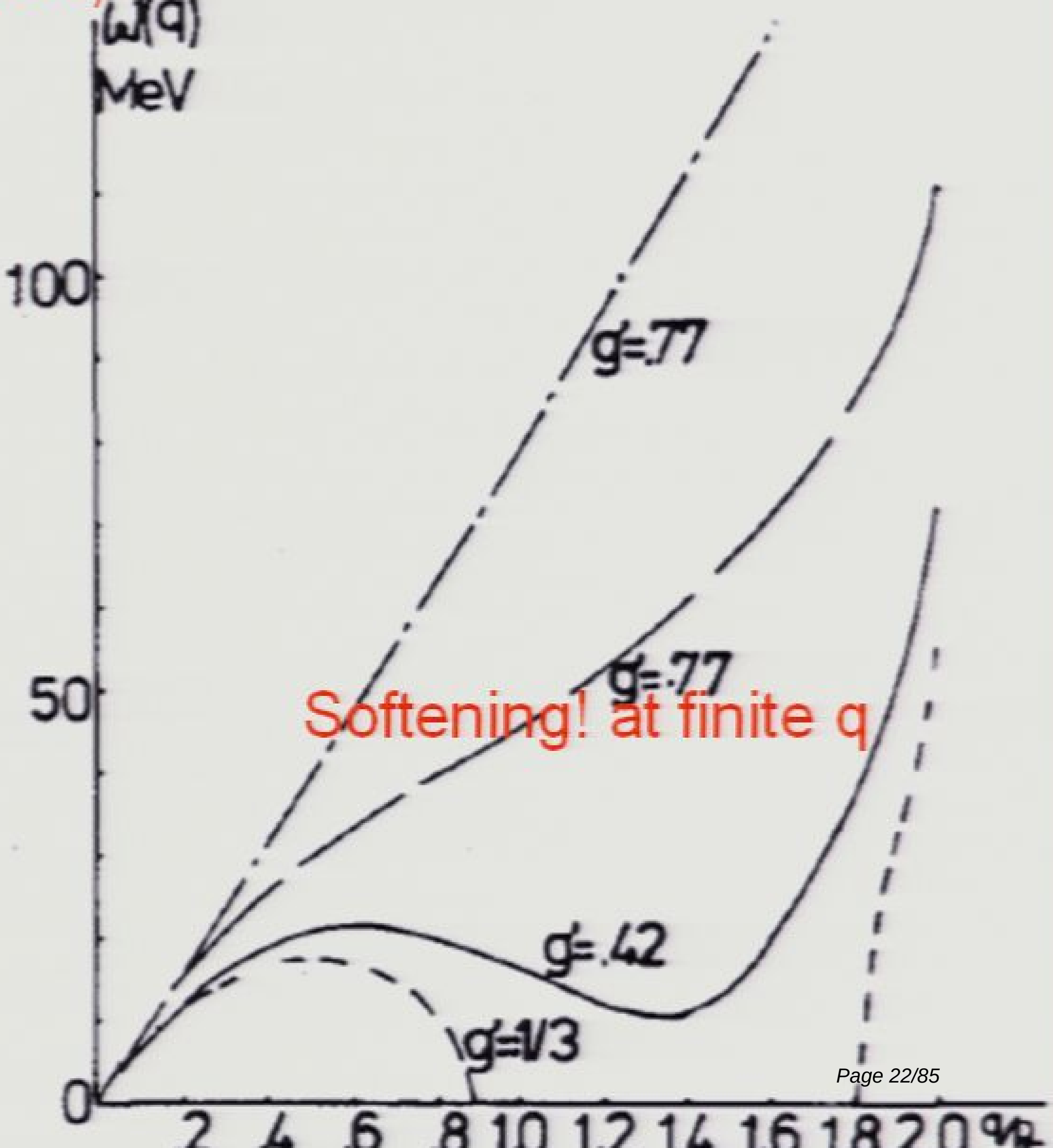
Wave-number dependence of the spin-isospin excitation energy

⇩

L-dependent GT giant Resonances?

In reality, coupling to a continuum

L; orbital ang. mom.



Softening! at finite q

Color-superconductivity, chiral and/or vector condensations

Ref's. M. Kitazawa ,T. Koide,Y. Nemoto and T.K.

Prog. of Theor. Phys., **108**, 929(2002)

★ Importance of Vector-type Interaction

- Vector interaction naturally appears in the effective theories.
  - Instanton-anti-instanton molecule model Shaefer,Shuryak ('98)

$$L = G\left\{ \frac{2}{N_C^2}\left[(\bar{\psi}\tau^a\psi)^2 + (\bar{\psi}\tau^a i\gamma_5\psi)^2\right] - \frac{1}{2N_C^2}\left[\underline{(\bar{\psi}\tau^a\gamma^\mu\psi)^2} + (\bar{\psi}\tau^a\gamma^\mu\gamma_5\psi)^2\right]\right\} + L_8$$

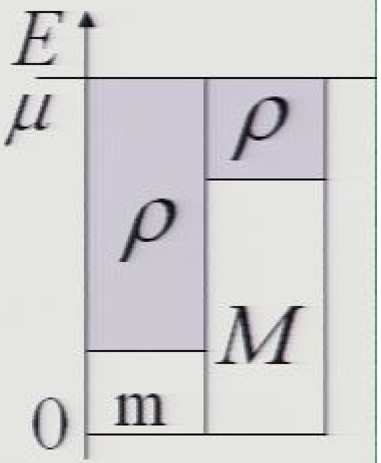
$$\Rightarrow \boxed{G_V / G_S = 1/4}$$

  - Renormalization-group analysis N.Evans et al. ('99)

$$L_{LL}^0 = G_{ll}\underline{\left\{(\bar{\psi}_L\gamma^0\psi_L)^2 - (\bar{\psi}_L\gamma^\mu\psi_L)^2\right\}}$$

$$\boxed{-G_V\left(\bar{\psi}\gamma^\mu\psi\right)^2} \Leftrightarrow \text{density-density correlation}$$

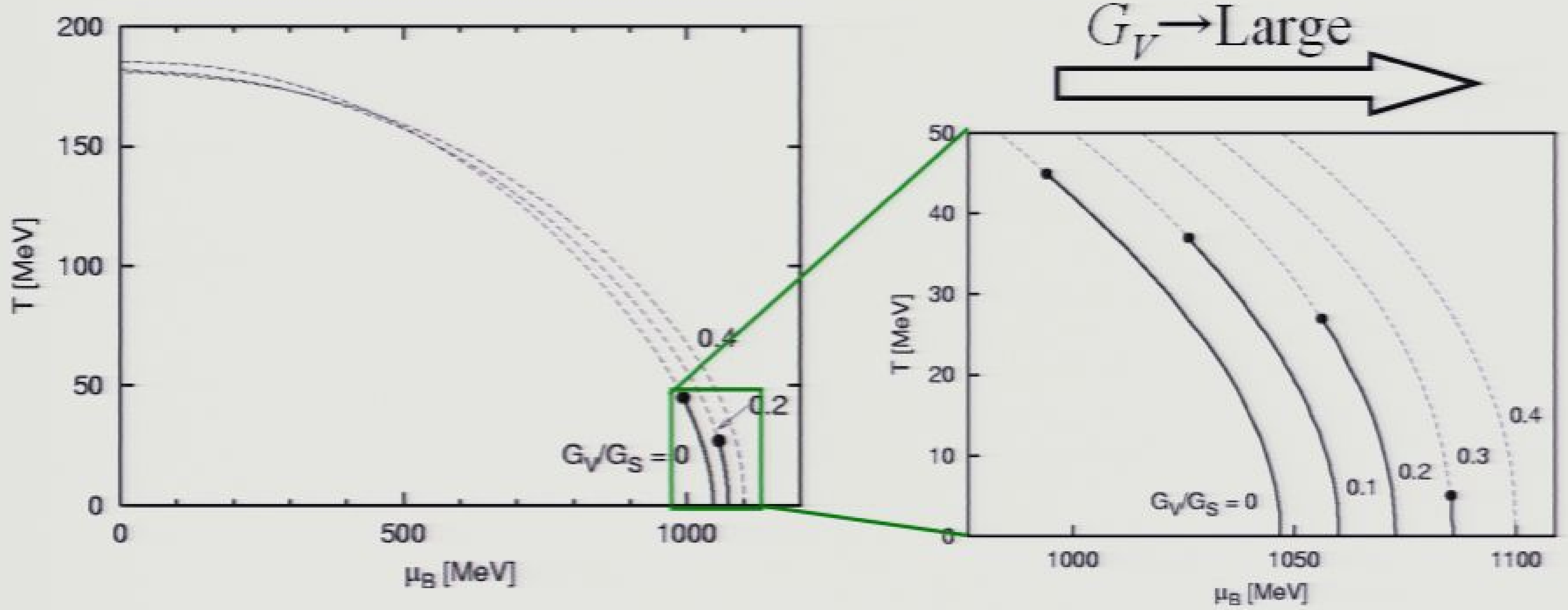
- $-G_V\left(\bar{\psi}\gamma^0\psi\right)^2 \to -G_V\left\langle\bar{\psi}\gamma^0\psi\right\rangle^2 = -G_V\rho^2 \quad \rho = \left\langle\bar{\psi}\gamma^0\psi\right\rangle$



- The importance of the vector interaction is known :
  - Asakawa,Yazaki ('89) Klimt,Luts,&Weise ('90) Buballa,Oertel('96)

Cf. The relevance to baryon(quark) –number susceptibility; T.K., PLB 71 (1991),295

★ Effects of G_V on Chiral Restoration



As G_V is increased,

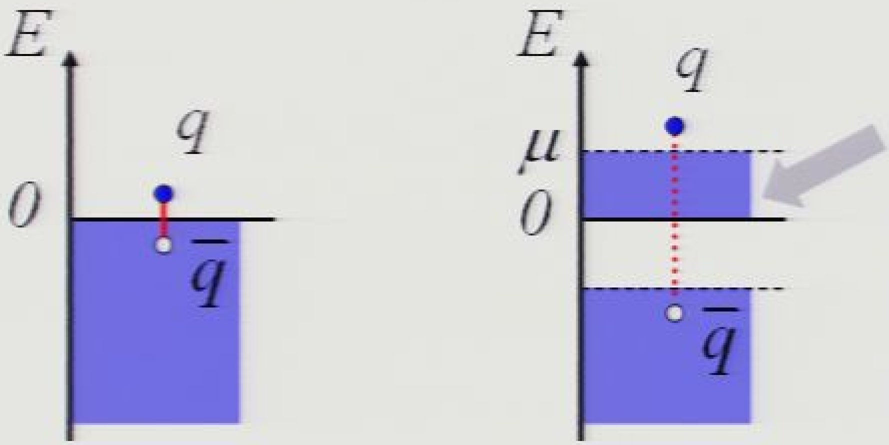
- Chiral restoration is shifted to higher densities.
- The phase transition is weakened.

Asakawa, Yazaki '89 /Klimt, Luts, & Weise '90 /T.K. '91 / Buballa, Oertel '96

What would happen when the CSC joins the game?

★Chiral Restoration at Finite μ

Chiral condensate
($\bar{q}$-q condensate)



Baryon density suppresses the formation of $\bar{q}$-q pairing.

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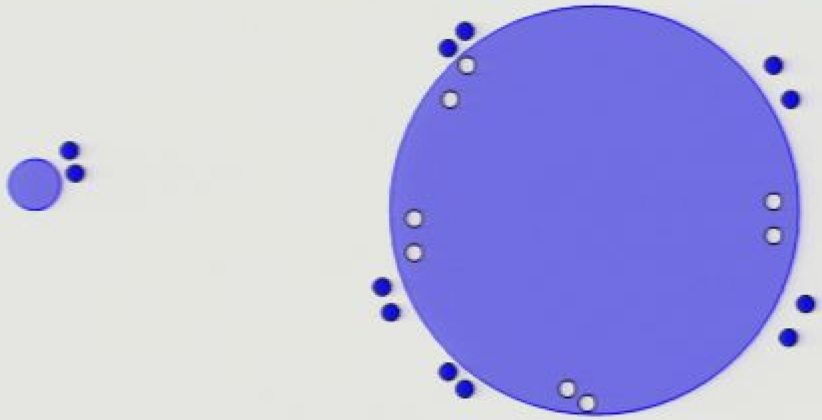

$$g^* = gN_F$$

Small Fermi sphere

Large Fermi sphere leads to strong Cooper instability

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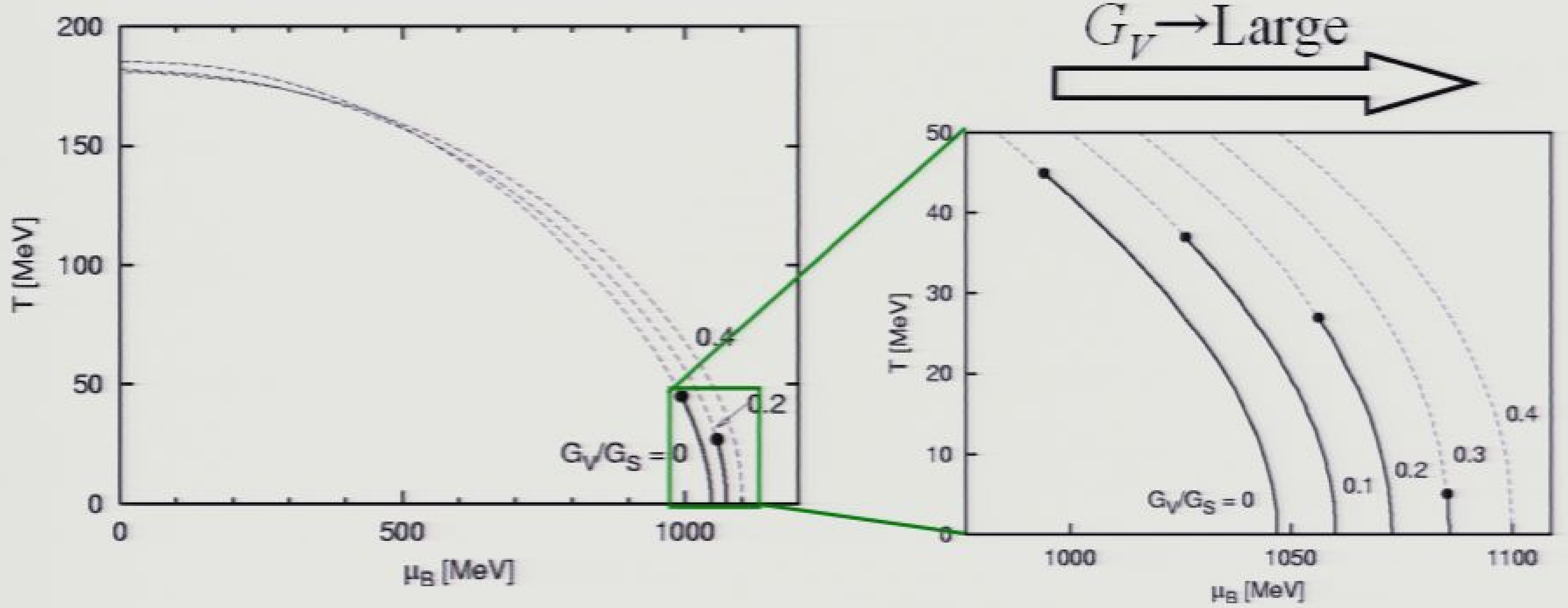

—— First Order
······ Cross Over

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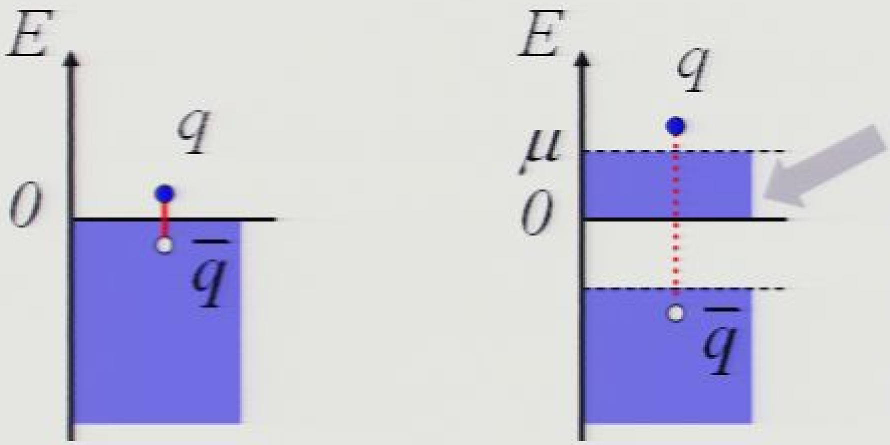

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μ:Small **μ:Large**

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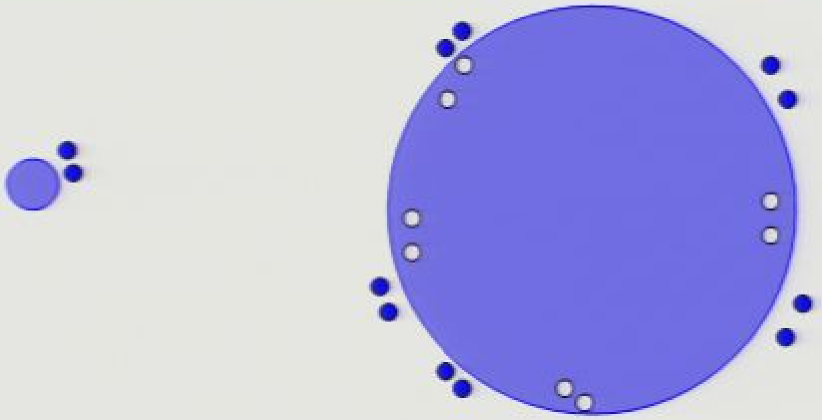

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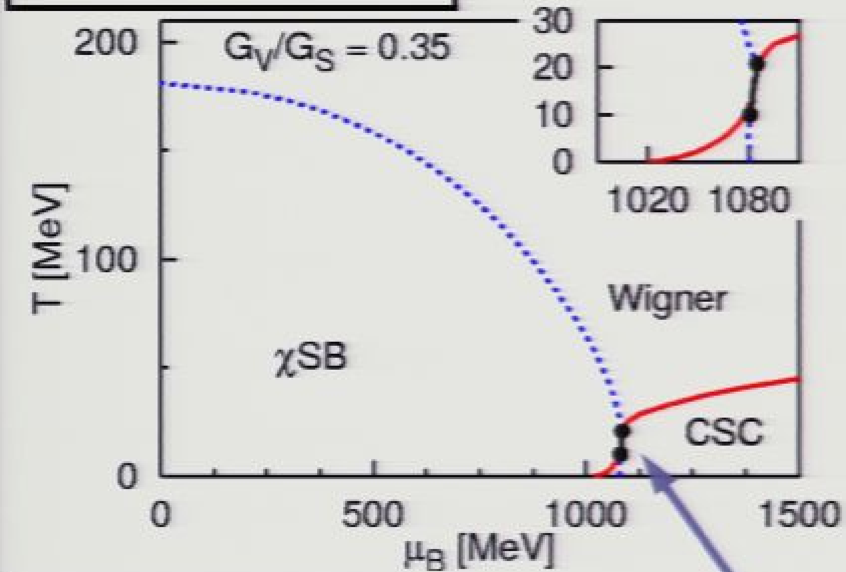
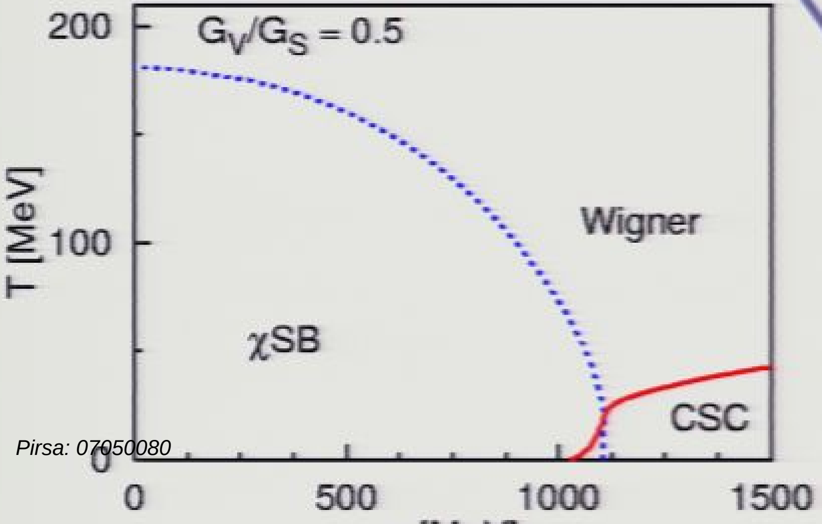
Incorporating CSC

$G_V / G_S = 0.35$



$G_V / G_S = 0.5$



(2) The **first order transition** between χSB and CSC phases **is weakened** and eventually **disappears**.

(3) The region of **the coexisting phase becomes broader**.

Appearance of the coexisting phase becomes robust.

(4) Another end point appears from lower temperature, and hence **there can exist <u>two end points</u>** in some range of G_V!

$0.33 \sim G_V \sim 0.38$

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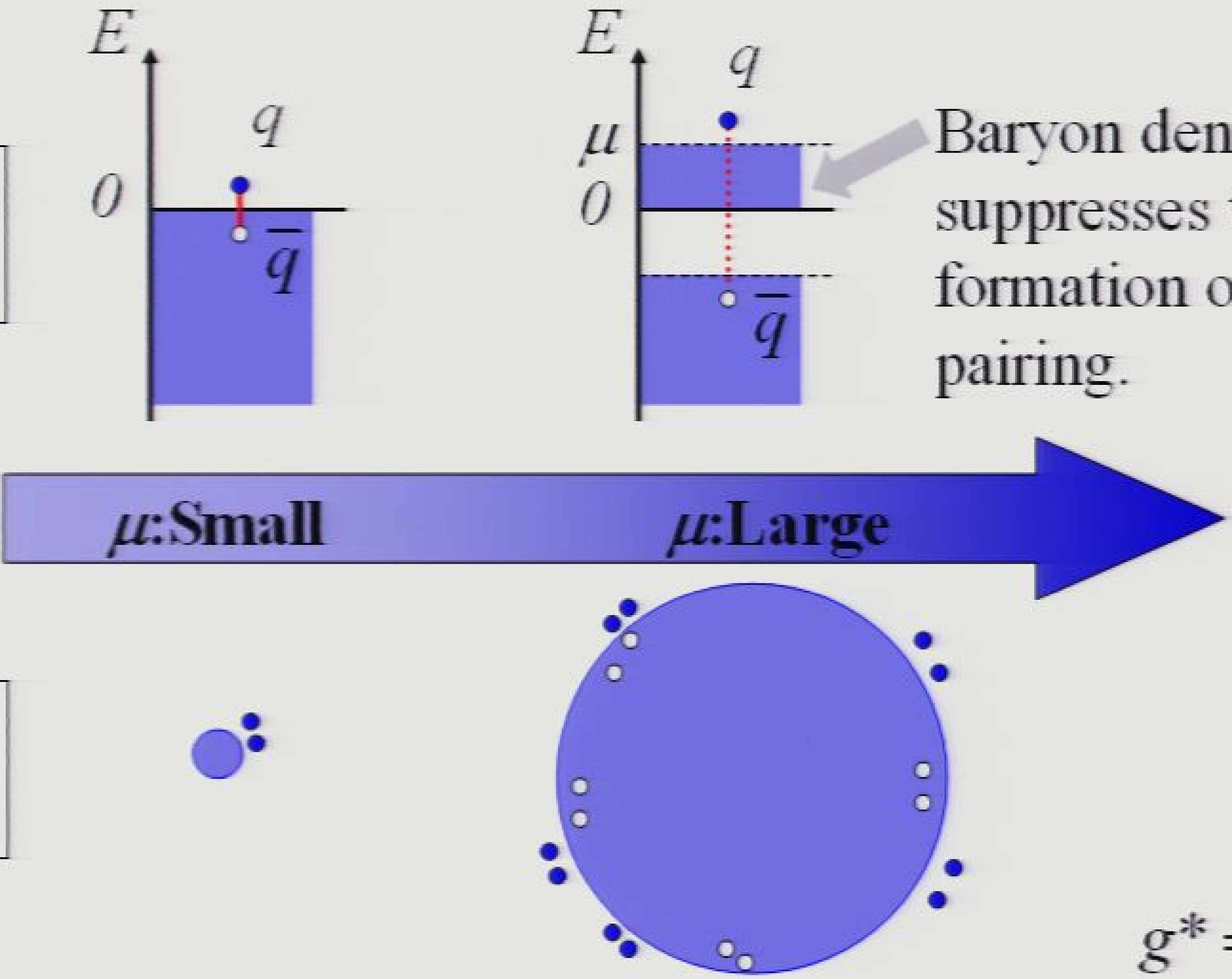

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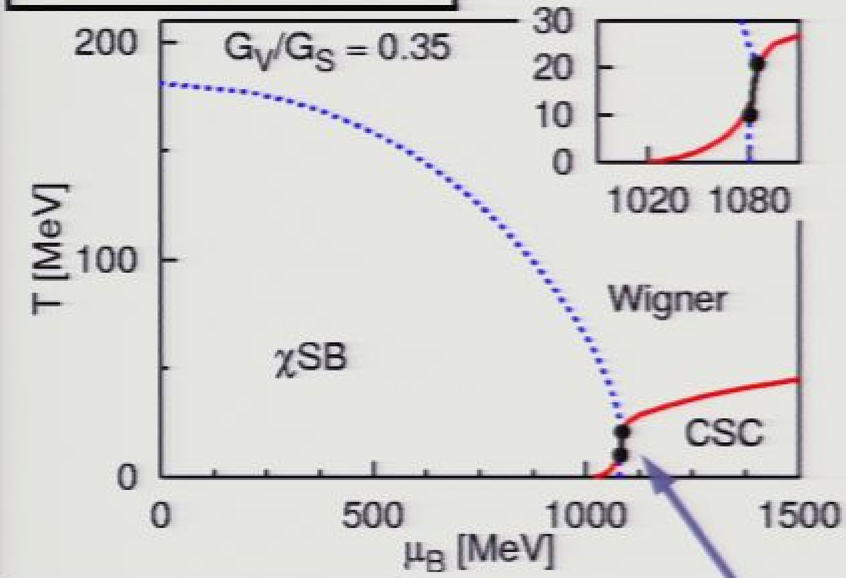

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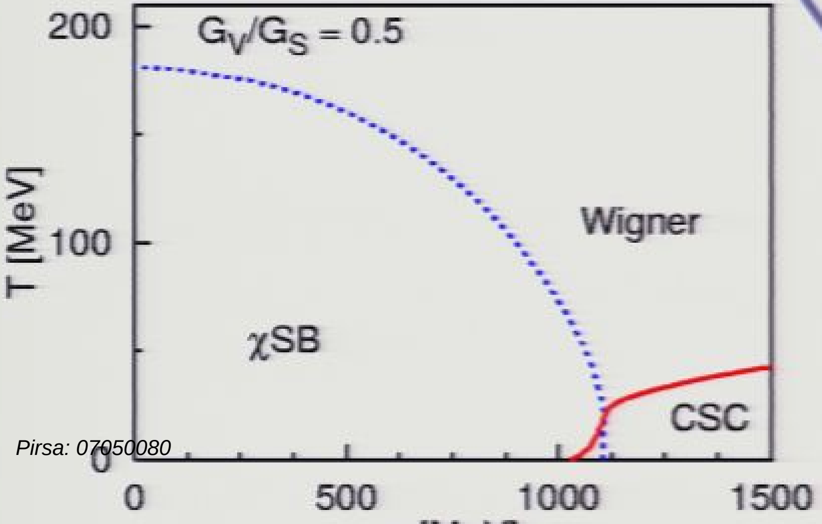

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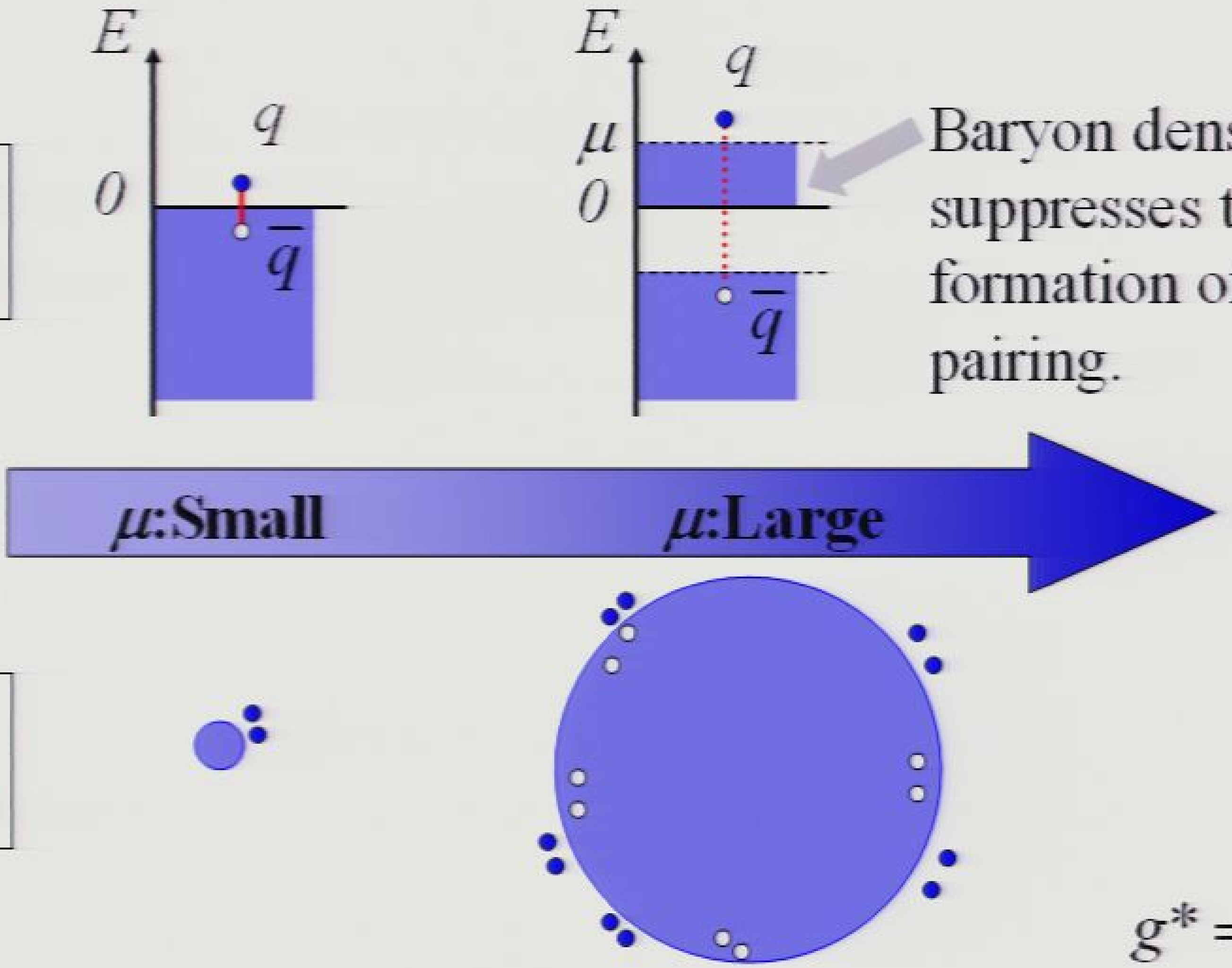

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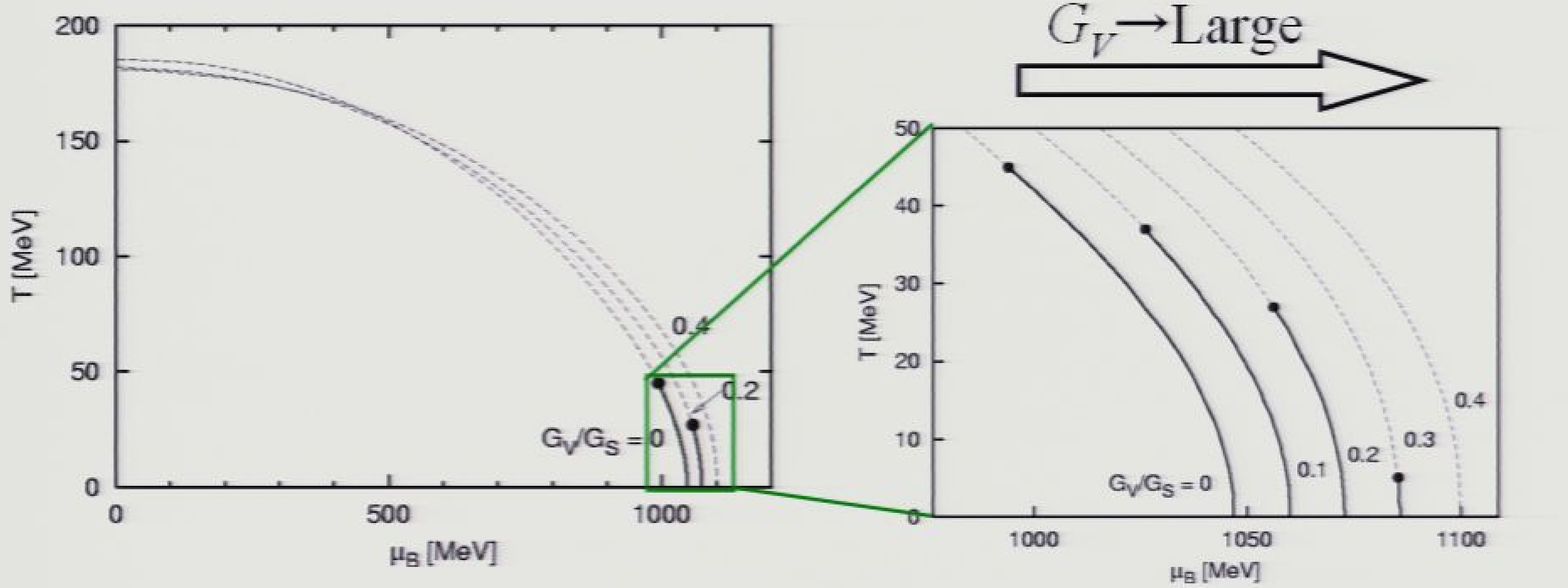

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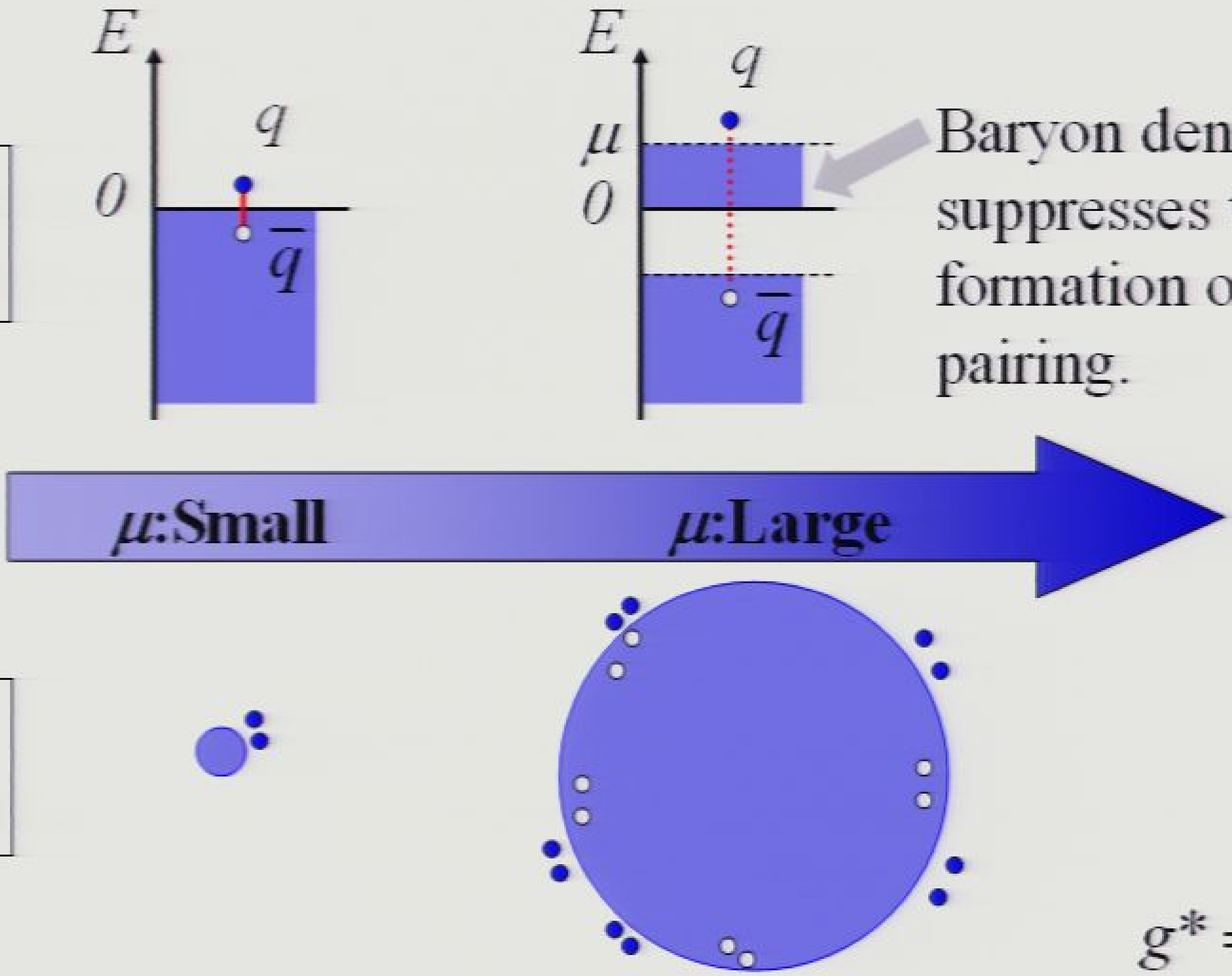

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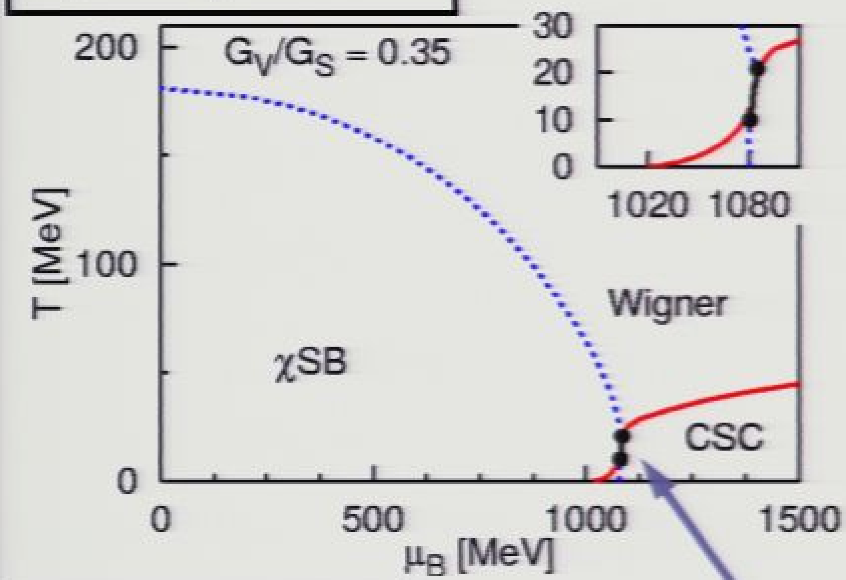

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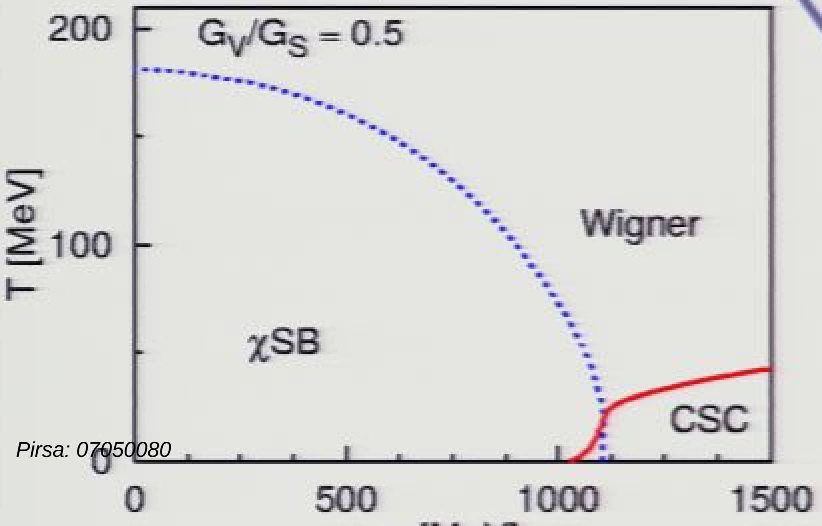

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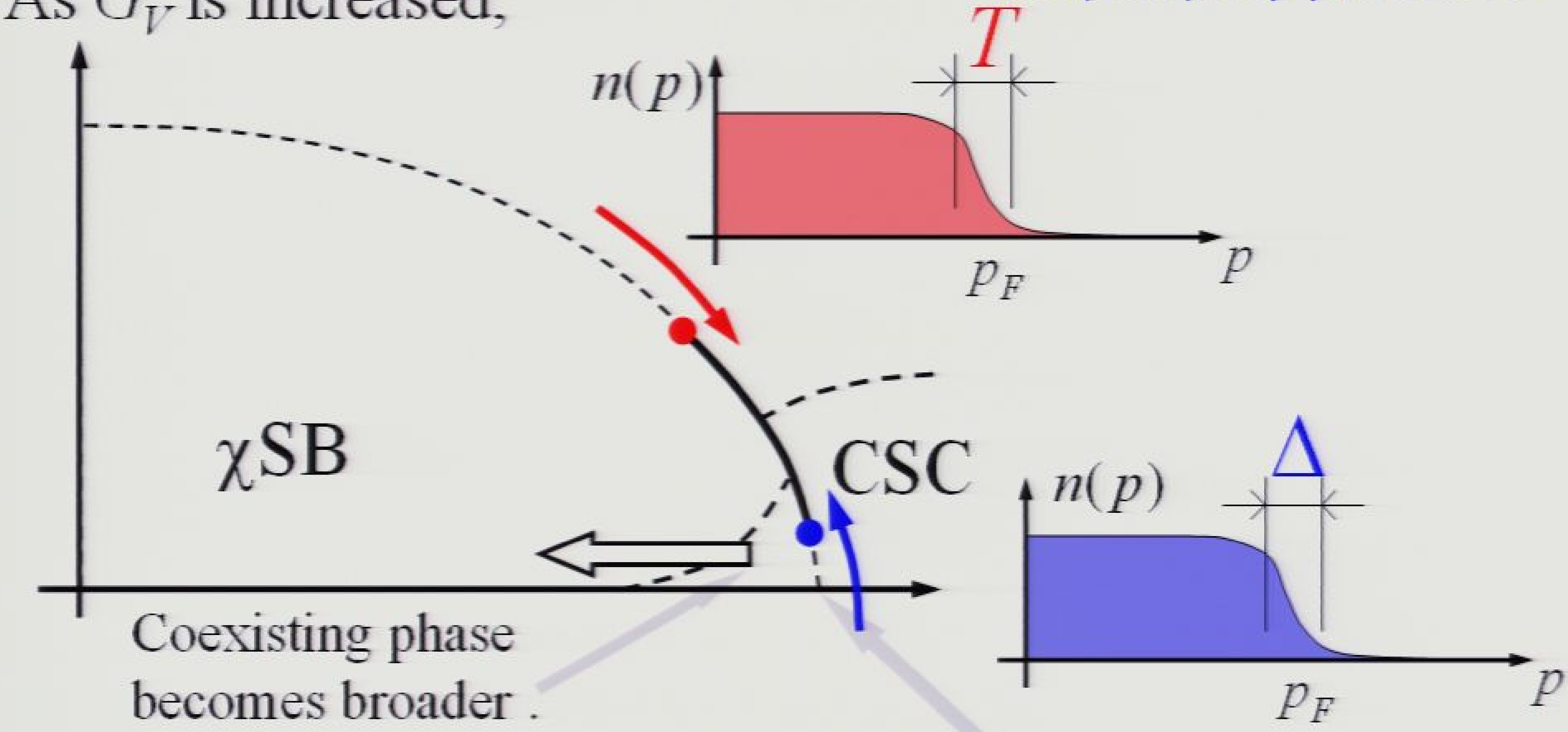
★ End Point at Lower Temperature

M. Kitazawa ,T. Koide,Y. Nemoto and T.K. Prog. of Theor. Phys.**110**, 185 (2003); hep-ph/0307278

As G_V is increased,



Coexisting phase becomes broader .

⇒ Δ becomes larger at the phase boundary between CSC and χSB. ⇒ The Fermi surface becomes obscure.

⇓

This effect plays a role similar to the temperature, and new end point appears from lower T.

★Phase Diagram in 2-color Lattice Simulation

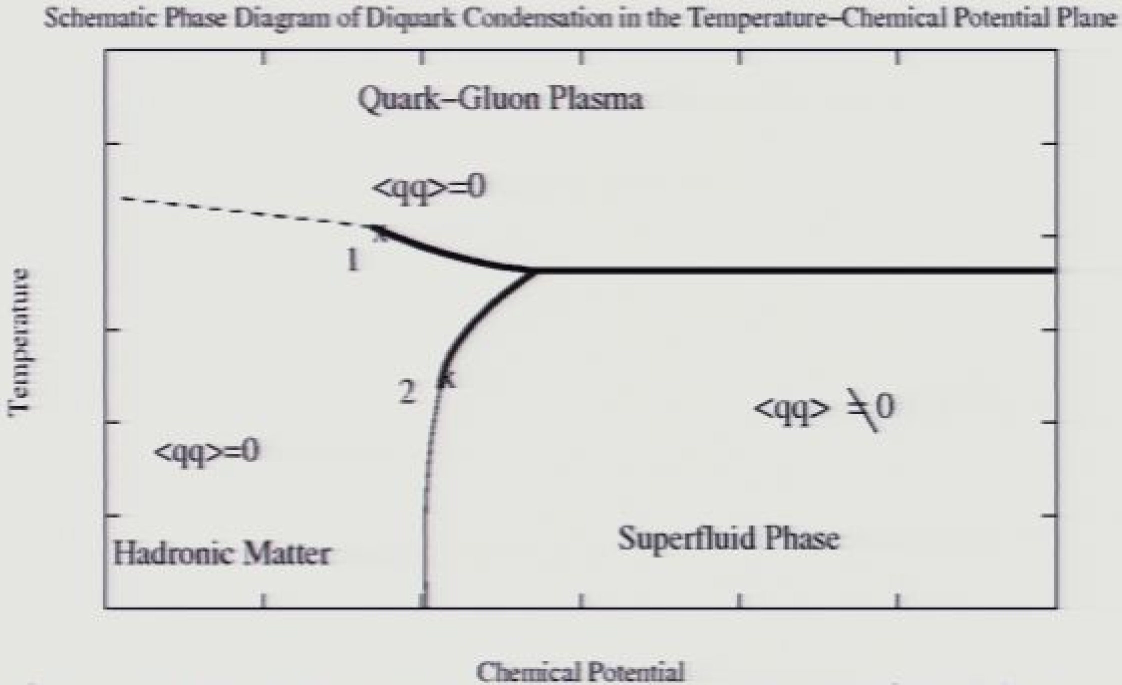


FIG. 1. Schematic Phase Diagram in the T-μ Plane. The thin(thick) line consists of second(first) order transitions. The dashed line denotes a crossover. Point 1 labels a critical point and Point 2 labels a tricritical point. The existence and position of point 1 is a matter of conjecture in the two color model.

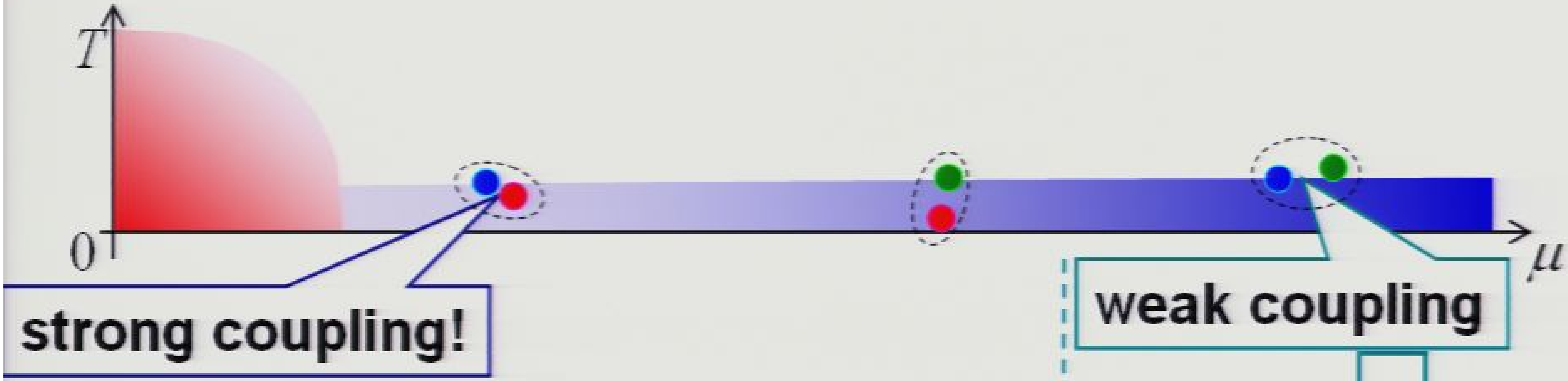
J.B.Kogut et al.Nucl. Phys. B642 (2002), 181; hep-lat/0205019

See also a very recent work in the G-L appraoch with the det-int;
T. Hatsuda, M. Tachibana, N. Yamamoto and G. Baym, PRL97 (2006)

Precursory Phenomena of Color Superconductivity in Heated Quark Matter

Ref. M. Kitazawa, T. Koide, T. K. and Y. Nemoto
Phys. Rev. D70, 956003(2004);
Prog. Theor. Phys. 114, 205(2005),
M. Kitazawa, T.K. and Y. Nemoto,
Phys. Lett.B 631(2005),157
M. Kitazawa and T. K., in preparation

The nature of diquark pairs in various coupling



strong coupling!

weak coupling

$\Delta \sim$ **50-100MeV**

$\Delta / E_F \sim$ **0.1-0.3**

in electric SC

$\Delta / E_F \sim 0.0001$

Mean field approx. works well.

c.f. Talk by Fodor; T_c~140 MeV

Very Short coherence length ξ.

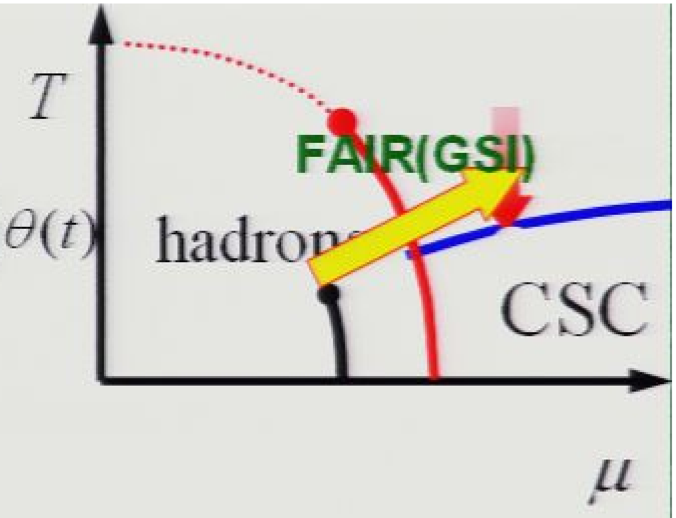
There exist large fluctuations of pair field even at T>0.

- Large pair fluctuations can
  - invalidate MFA.
  - cause precursory phenomena of CSC.

relevant to newly born neutron stars
or intermediate states in heavy-ion collisions (GSI, J-PARC)

Pair Fluctuations in CSC

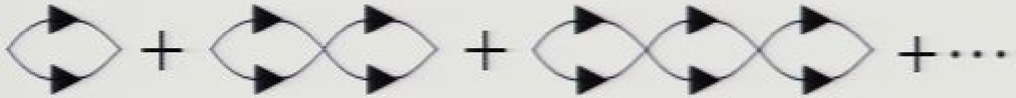
$$D^R(\mathbf{x},t) = -2G_C\left\langle\left[\bar{\psi}(x)i\gamma_5\tau_2\lambda_2\psi^C(x), \bar{\psi}(0)i\gamma_5\tau_2\lambda_2\psi^C(0)\right]\right\rangle\theta(t)$$

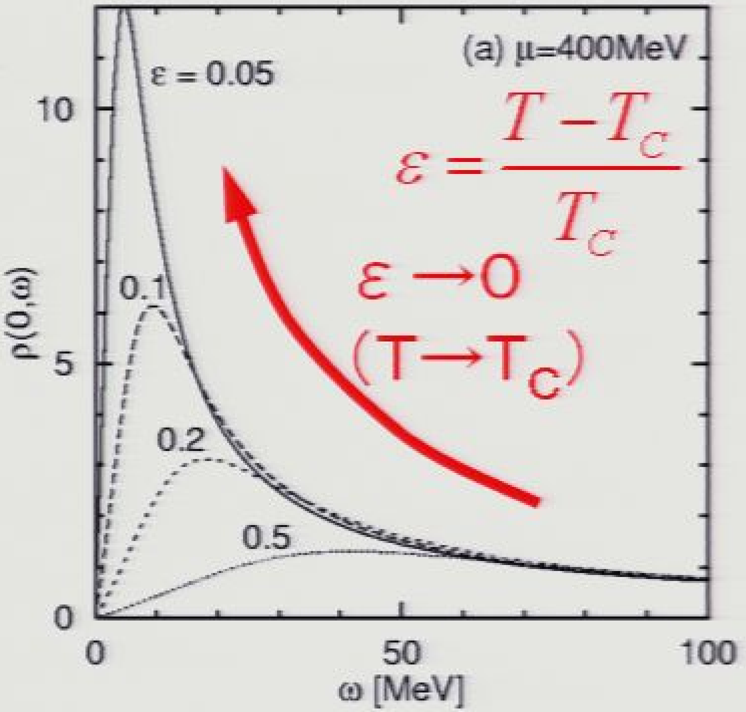


Spectral Function of the diquark excitations

$$\boxed{\rho(\mathbf{k},\omega) = -\frac{1}{\pi}\mathrm{Im}\, D^R(\mathbf{k},\omega)}$$

$$D^R(\mathbf{k},\omega) =$$





- Sharp peak from ε ~ **0.2**

electric SC: ε ~ **0.005**
even in 2d-SC

Existence of large pair fluctuations

M.Kitazawa, T. Koide, T. K., Y. Nemoto, PRD **65**, 091504 (2002)

It may affect various observables even well above T

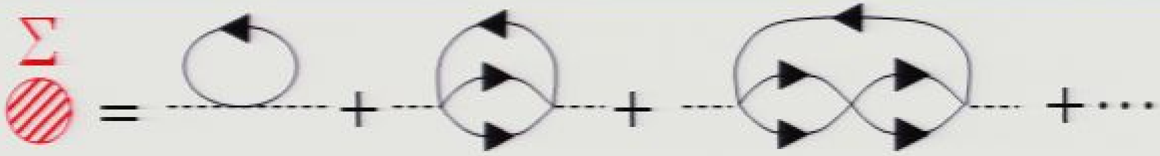
How does the soft mode affect the quark spectra?

---- formation of pseudo gap ----

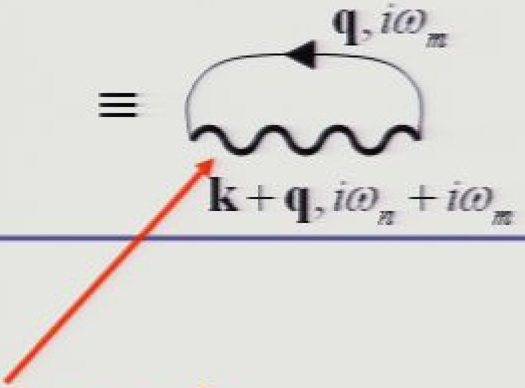
T-matrix Approximation for Quark Propagator

$$G(\mathbf{k},\omega_n)=\frac{1}{G^0(\mathbf{k},i\omega_n)-\Sigma(\mathbf{k},i\omega_n)}\qquad G^0(\mathbf{k},i\omega_n)=\left[(i\omega_n+\mu)\gamma^0-\mathbf{k}\cdot\bar{\gamma}\right]^{-1}=\rightarrow$$

$$\Sigma(\mathbf{k},\omega_n)=$$



$$\equiv$$



$$=T\sum_m\int\frac{d^3\mathbf{q}}{(2\pi)^3}\Xi(\mathbf{k}+\mathbf{q},\omega_n+\omega_m)G^0(\mathbf{q},\omega_m)$$

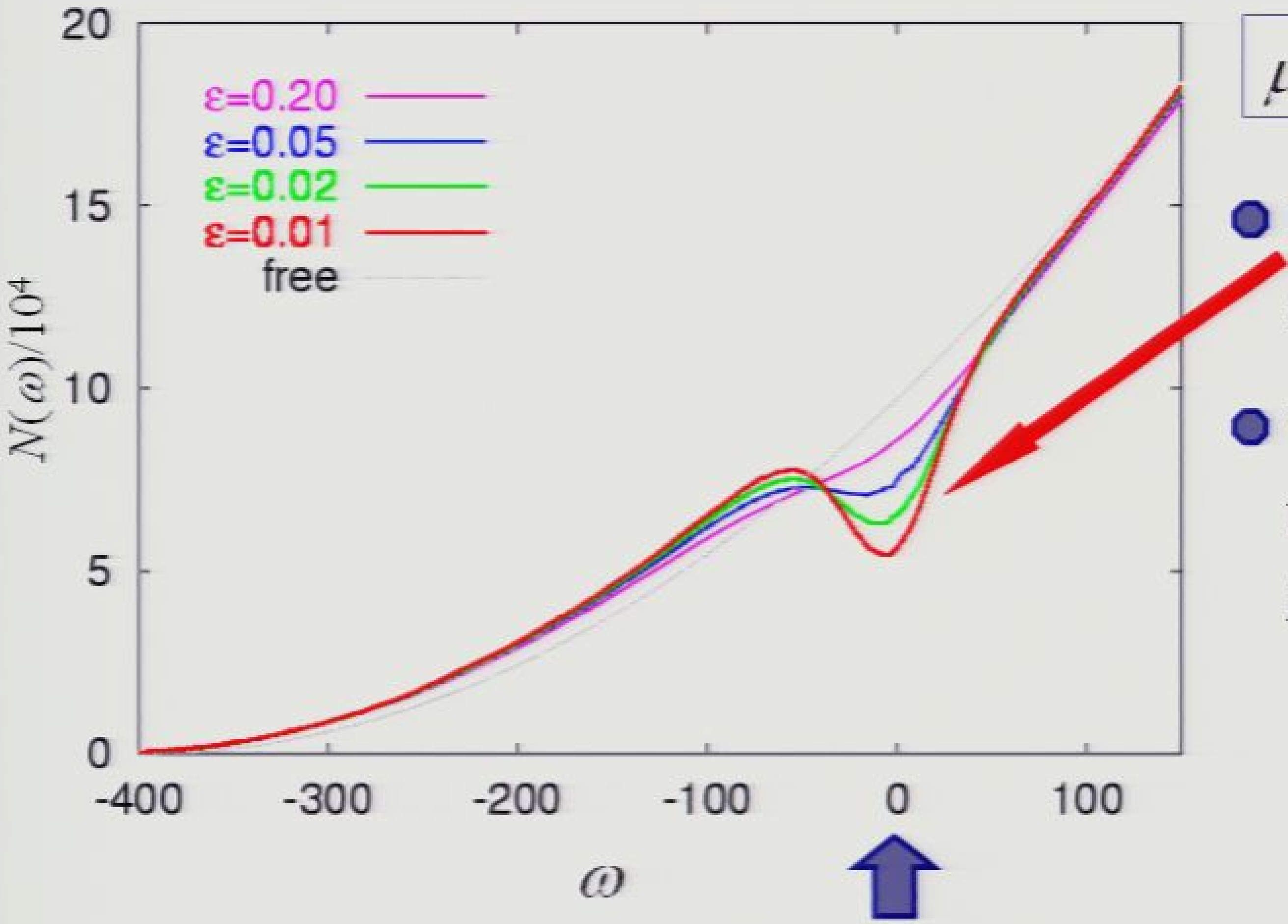
Soft mode

- **Density of States** $N(\omega)$:

$$N=\int d^3x\left\langle\bar{\psi}\gamma^0\psi\right\rangle$$

$$N(\omega)=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\rho^0(\mathbf{k},\omega)\Leftarrow\rho^0(\mathbf{k},\omega)=\frac{1}{4}\mathrm{Tr}\left[\gamma^0\,\mathrm{Im}\,G^R(\mathbf{k},\omega)\right]$$

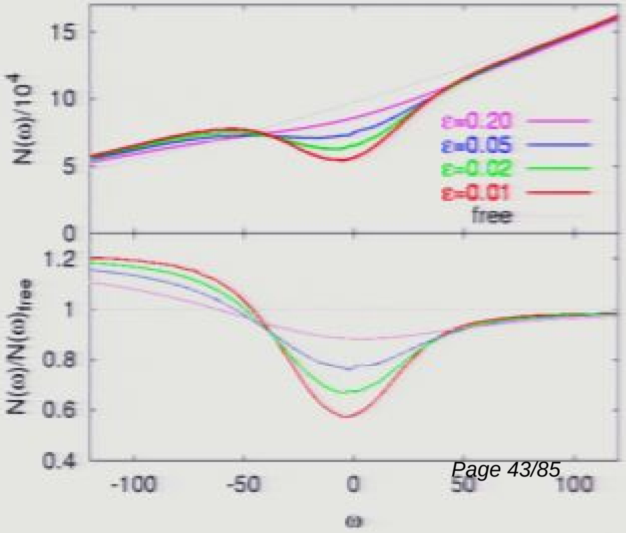
Density of states of quarks in heated quark matter



Fermi energy

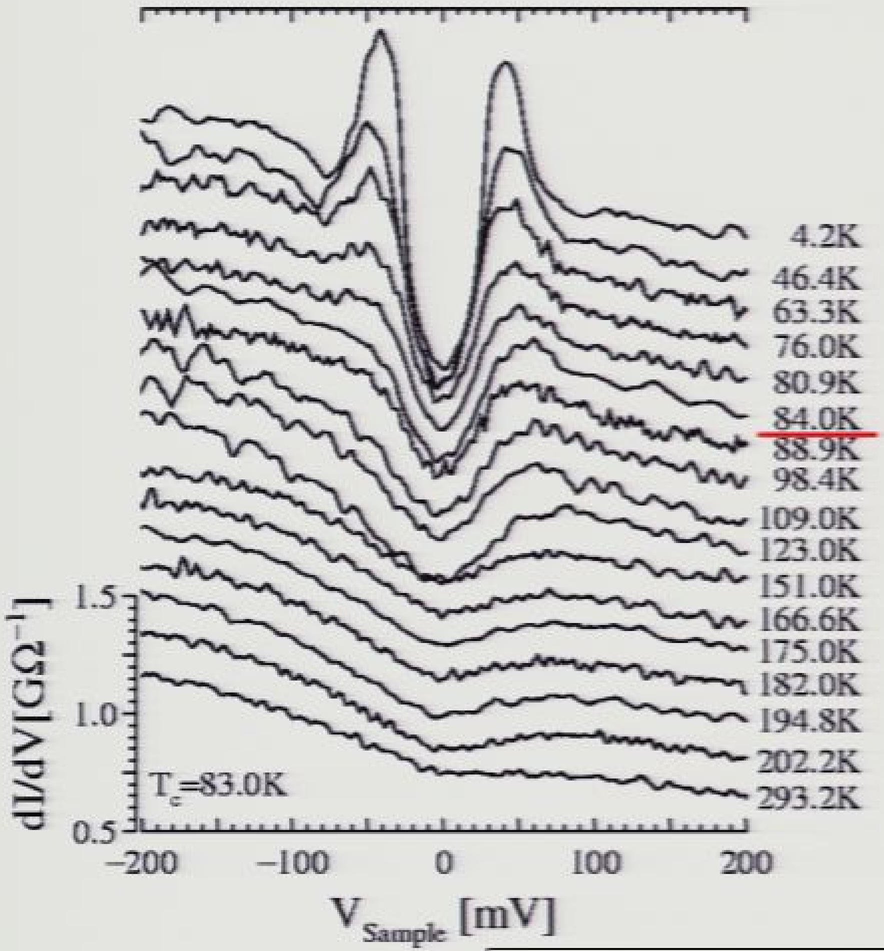
$\mu = 400$ MeV

- **Pseudogap structure appears in $N(\omega)$.**
- The pseudogap survives up to $\varepsilon \sim 0.05$ (5% above T_C).



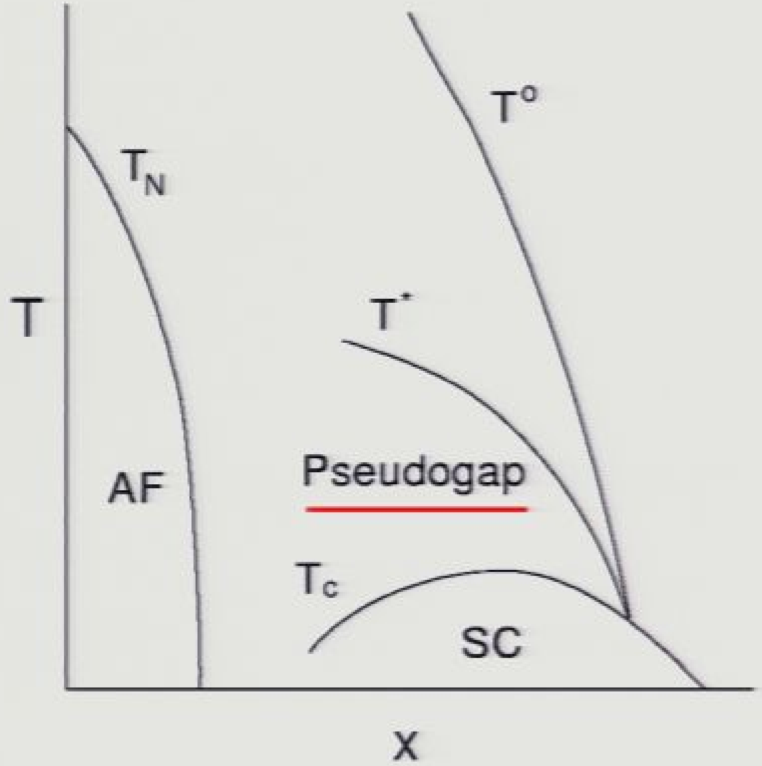
Pseudogap in High T_c Superconductors

:Anomalous depression of the density of state near the Fermi surface in the normal phase.



Renner et al.(‘96)

Conceptual phase diagram of HTSC cuprates



Similarity of QCD matter and Cuprates?

The origin of the pseudogap in HTSC is **still controversial**.

- More accessible observables to see the fluctuations?

➡ dilepton-pair production.
in H-I collisions.

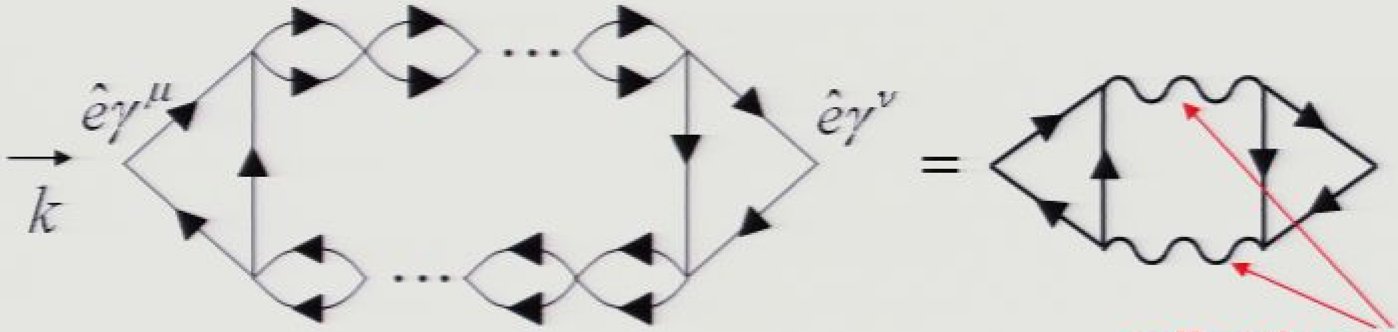
How?

Anomalous Self-energy of Photon; Aslamasov-Larkin term

Sov. Phys. SS **10**,875('68)

Photon Self-Energy Π

$$\Pi^{\mu\nu}_{AL} = \xrightarrow[k]{}$$



$$\hat{e} = \begin{pmatrix} 2/3 & \\ & -1/3 \end{pmatrix}_f$$

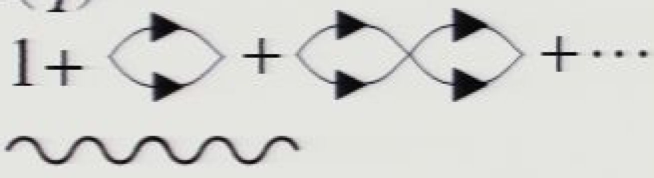
collective pairing oscillation

$$= 3\int d^4q \Gamma^{\mu}(q,q+k)\Xi(q+k)\Gamma^{\nu}(q+k,q)\Xi(q)$$

factor **3** due to color degrees of freedom

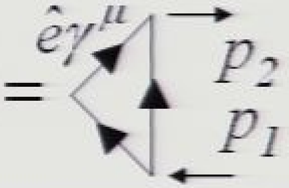
- Pair field (T-matrix):

$\Xi(q) =$



- Vertex:

$\Gamma^{\mu}(p_1,p_2) =$



cf) Maki-Thompson term

- AL term represents the effects of the **virtual Cooper pairs** on the **photon self-energy**.

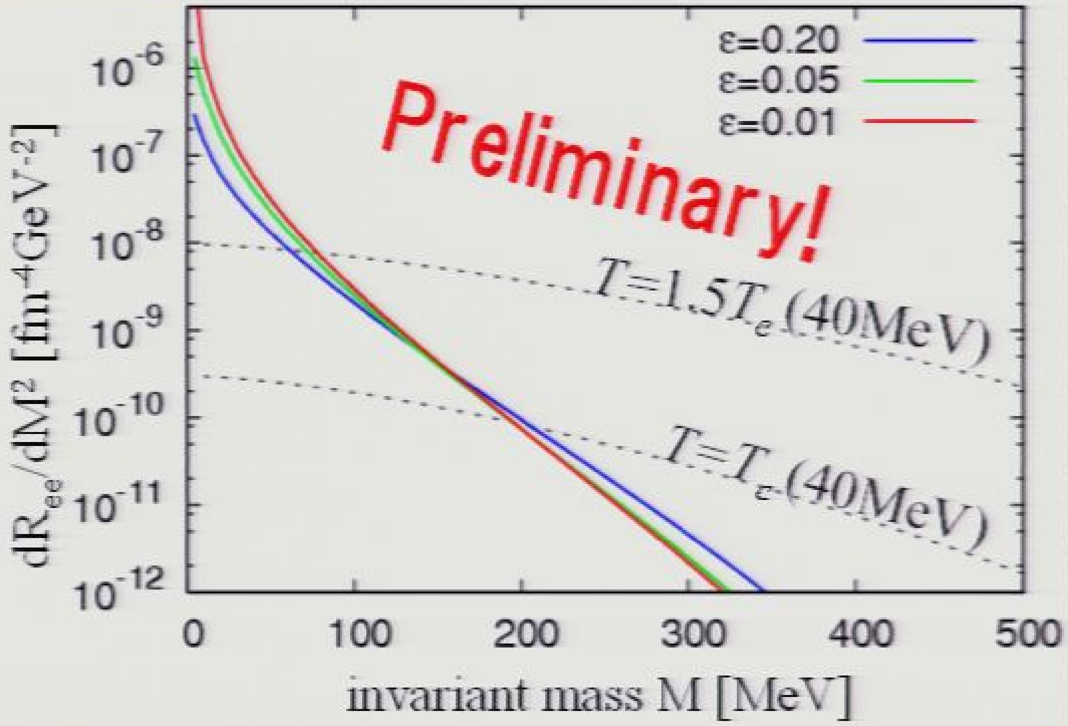
Dilepton-pair Production

$$\frac{dR_{ee}}{d^4q} = -\frac{\alpha}{12\pi^4 Q^2}\,\mathrm{Im}\,\Pi^{R\mu}_{\mu}\,\frac{1}{e^{q^0/T}-1}$$

-per invariant mass

$$\frac{dR_{ee}}{dM^2} = \int\frac{d^3q}{2q^0}\frac{dR_{ee}}{d^4q}$$

— with AL-term
----- from free quarks



M. Kitazawa and T.K., (2005),unpublished

- Prominent enhancement at M<150MeV.
- The peak becomes sharp as $\varepsilon \rightarrow 0$.

C.f. For the lepton –pair production from the CFL phase, Jaikumar, Rapp, Zahed, PRC65,055205('02)

Precursory Hadronic Mode
and
Single Quark Spectrum
above
Chiral Phase Transition

Interest in the particle picture in QGP

RHIC experiments

robust collective flow

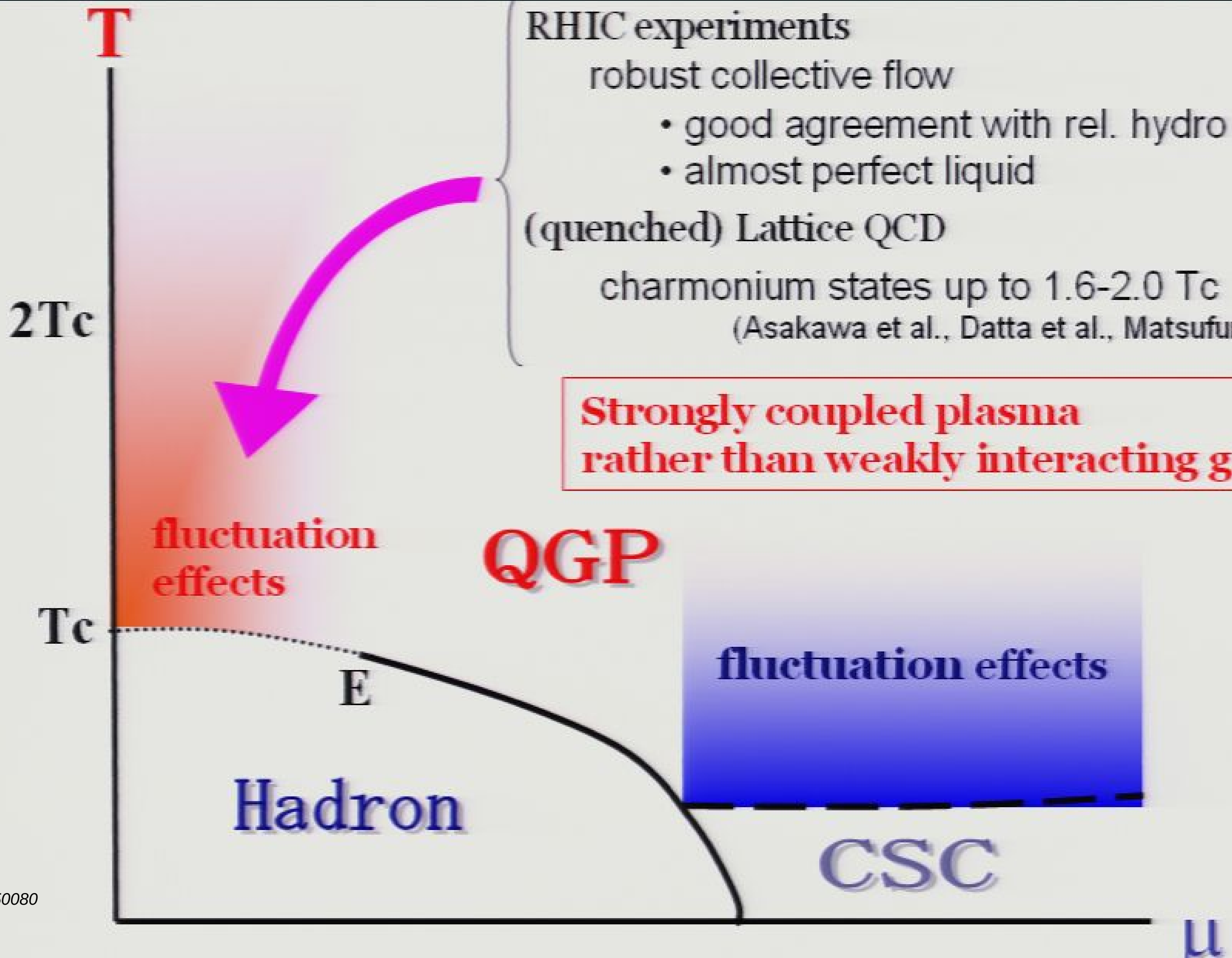
- good agreement with rel. hydro models
- almost perfect liquid

(quenched) Lattice QCD

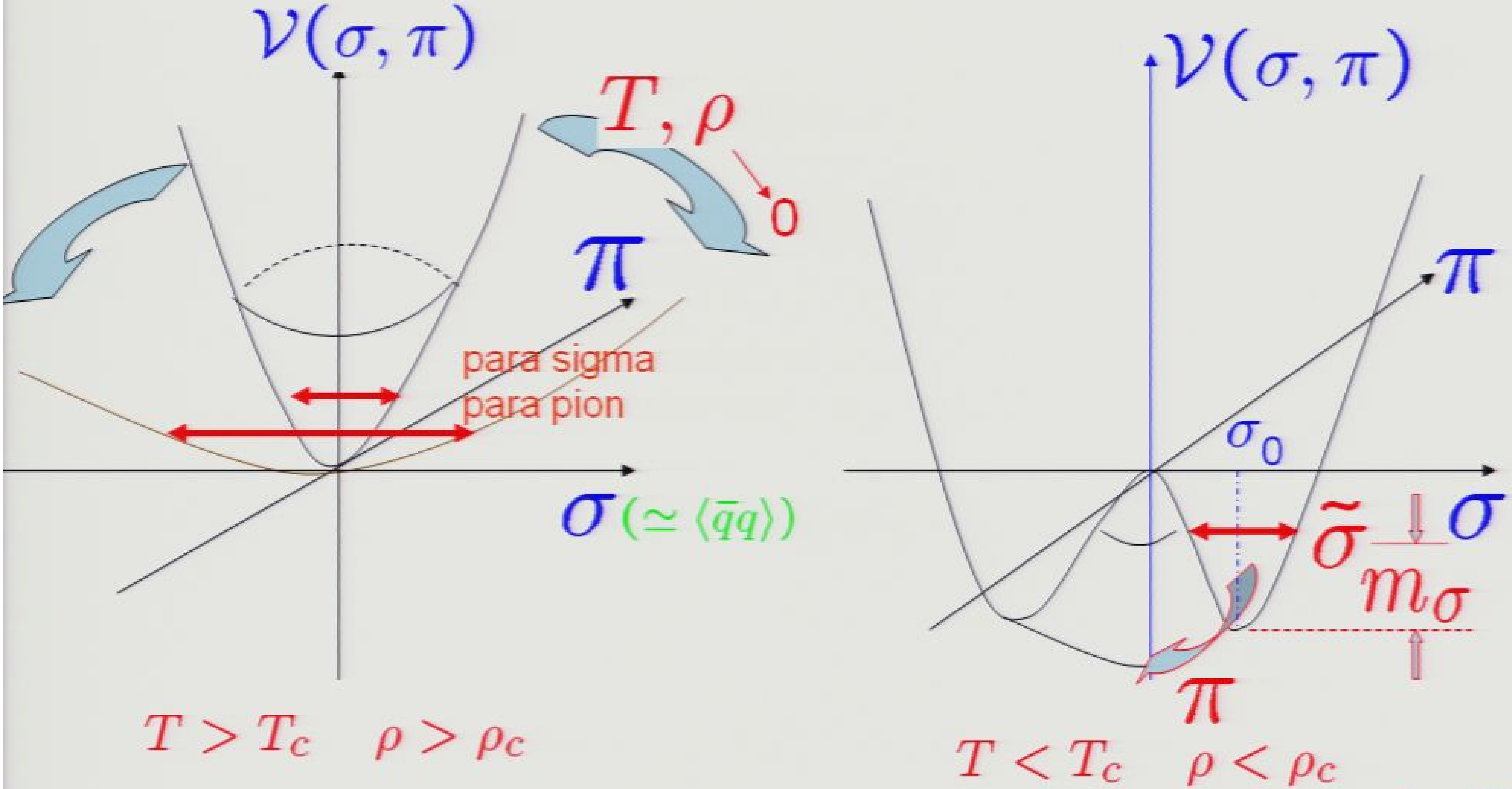
charmonium states up to 1.6-2.0 Tc

(Asakawa et al., Datta et al., Matsufuru et al. 2004)

Strongly coupled plasma rather than weakly interacting gas



Chiral Transition and the sigma mode (meson)



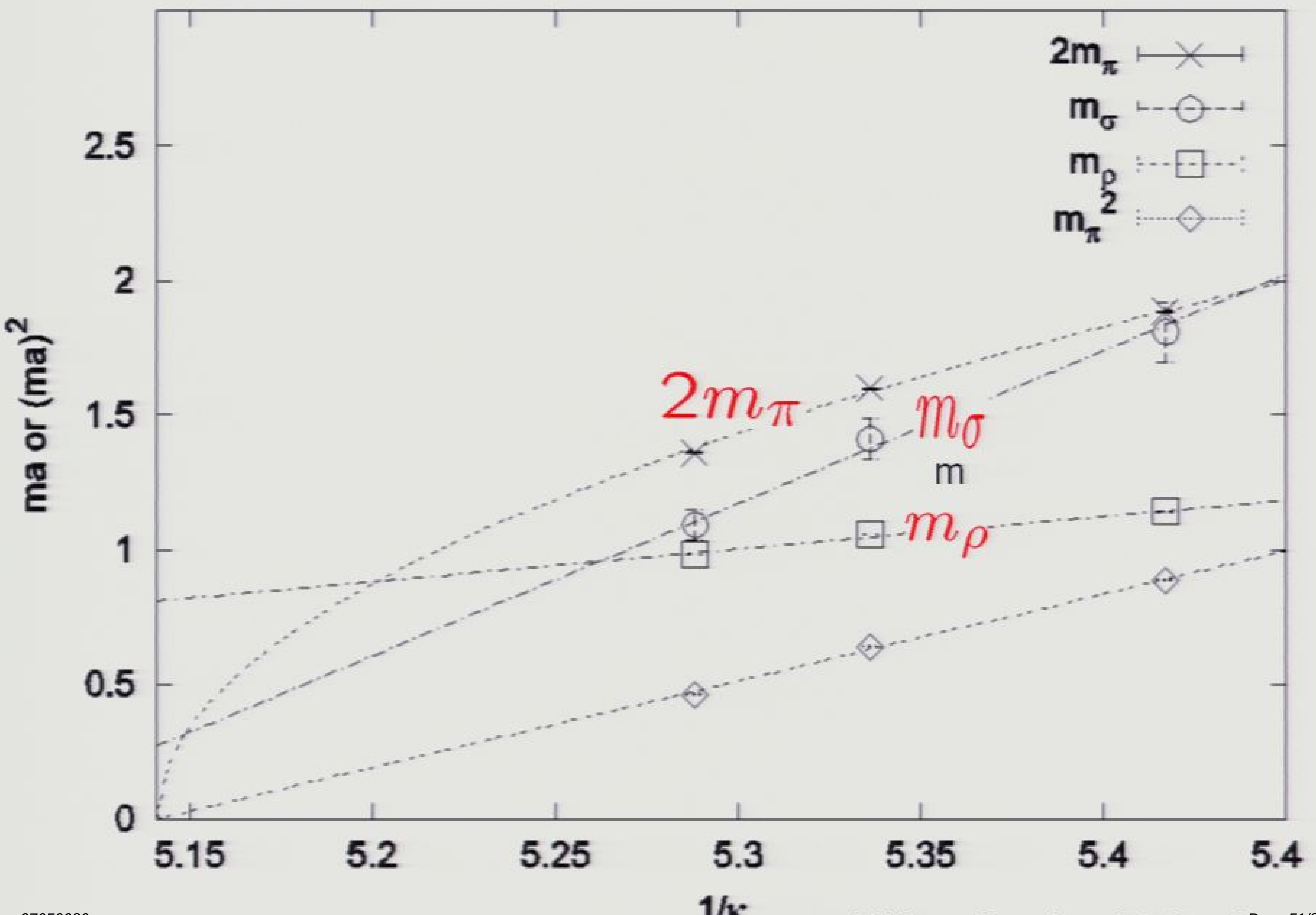
$$\sigma = \sigma_0 + \tilde{\sigma}$$

c.f. Higgs particle in WSH model

ϕ , Higgs field ➡ $\phi = \langle\phi\rangle + \tilde{\phi}$

Higgs particle

Lattice Calculations in full QCD of the sigma mass

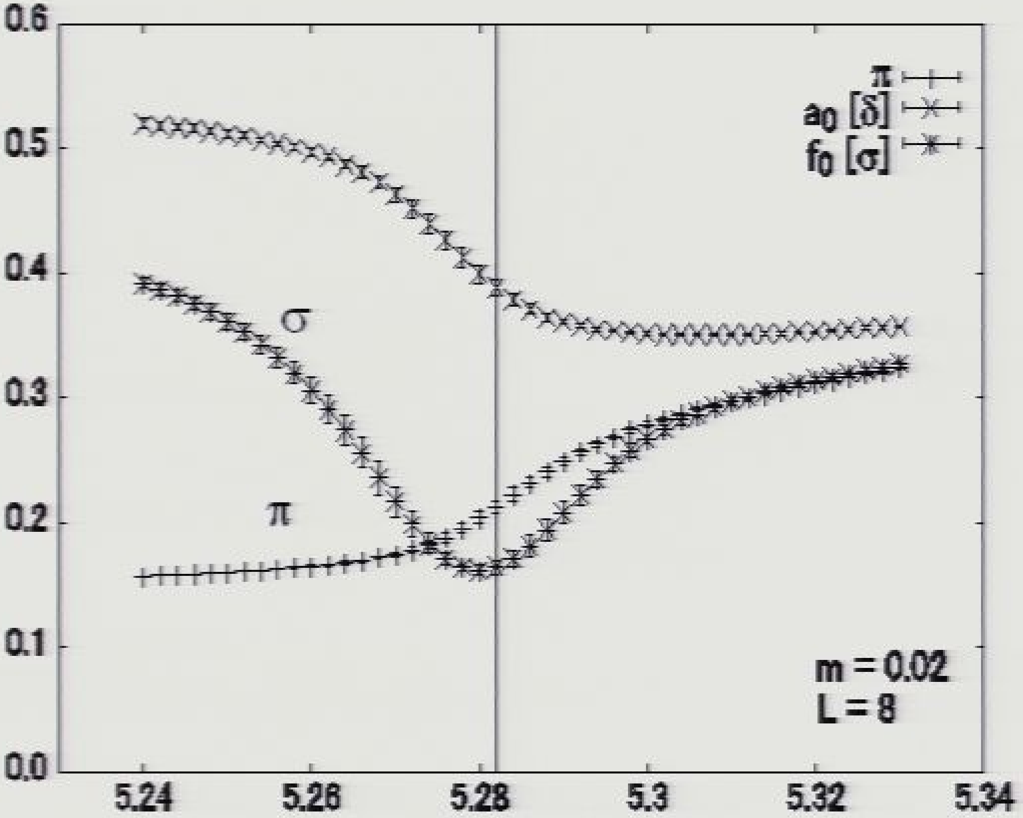


The SCALAR collaboration, PRD70, 034504(2004) : Wilson fermion, Z_2 noize method

Cf. Lattice Calculation of the *generalized masses*

F. Karsch, Lect. Note Phys. **583** (2002), 209. $N_f = 2,\, 8^3 \times 4$; Staggered fermion

$$m_\sigma^2 = \chi_\sigma^{-1}$$
$$\chi_\sigma = \langle (\bar{q}q)^2 \rangle$$



the softening of the σ with increasing T

and

a degeneracy of the σ and π at high T

What is the significance of the σ in hadron physics?

Confinement and chiral transion in Lattice QCD

(F.Karsch, Lect. Notes 583 (2002), 209)

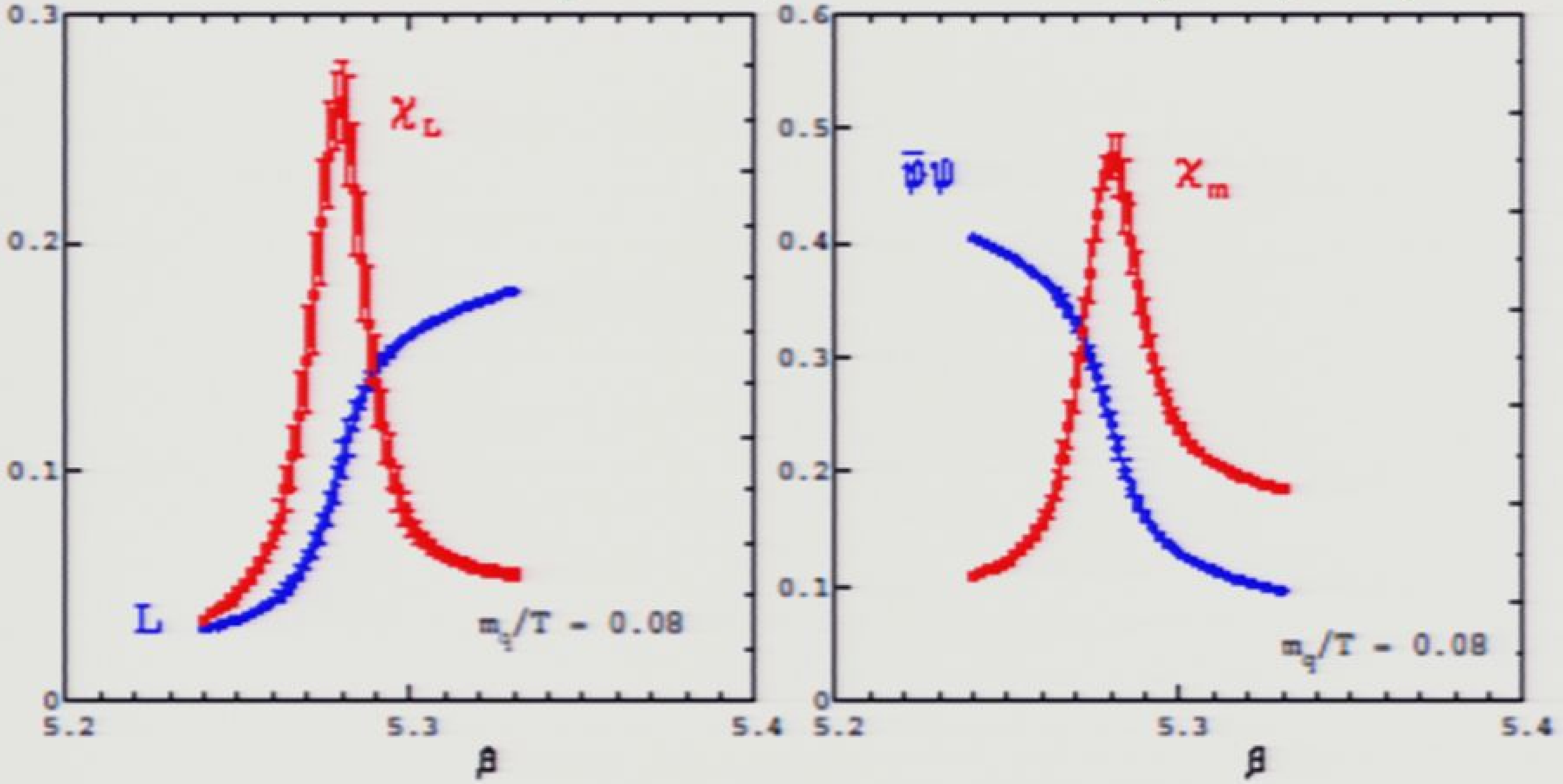


Fig. 2. Deconfinement and chiral symmetry restoration in 2-flavour QCD: Shown is $\langle L\rangle$ (left), which is the order parameter for deconfinement in the pure gauge limit ($m_q \to \infty$), and $\langle\bar\psi\psi\rangle$ (right), which is the order parameter for chiral symmetry breaking in the chiral limit ($m_q \to 0$). Also shown are the corresponding susceptibilities as a function of the coupling $\beta = 6/g^2$.

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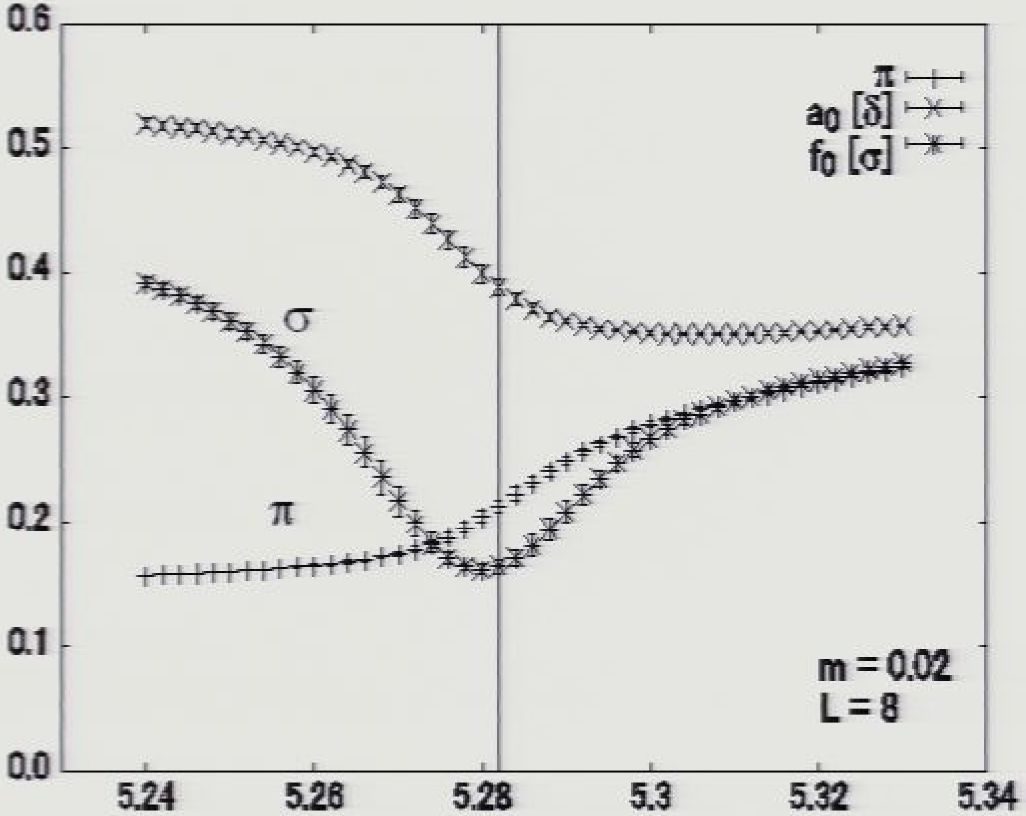

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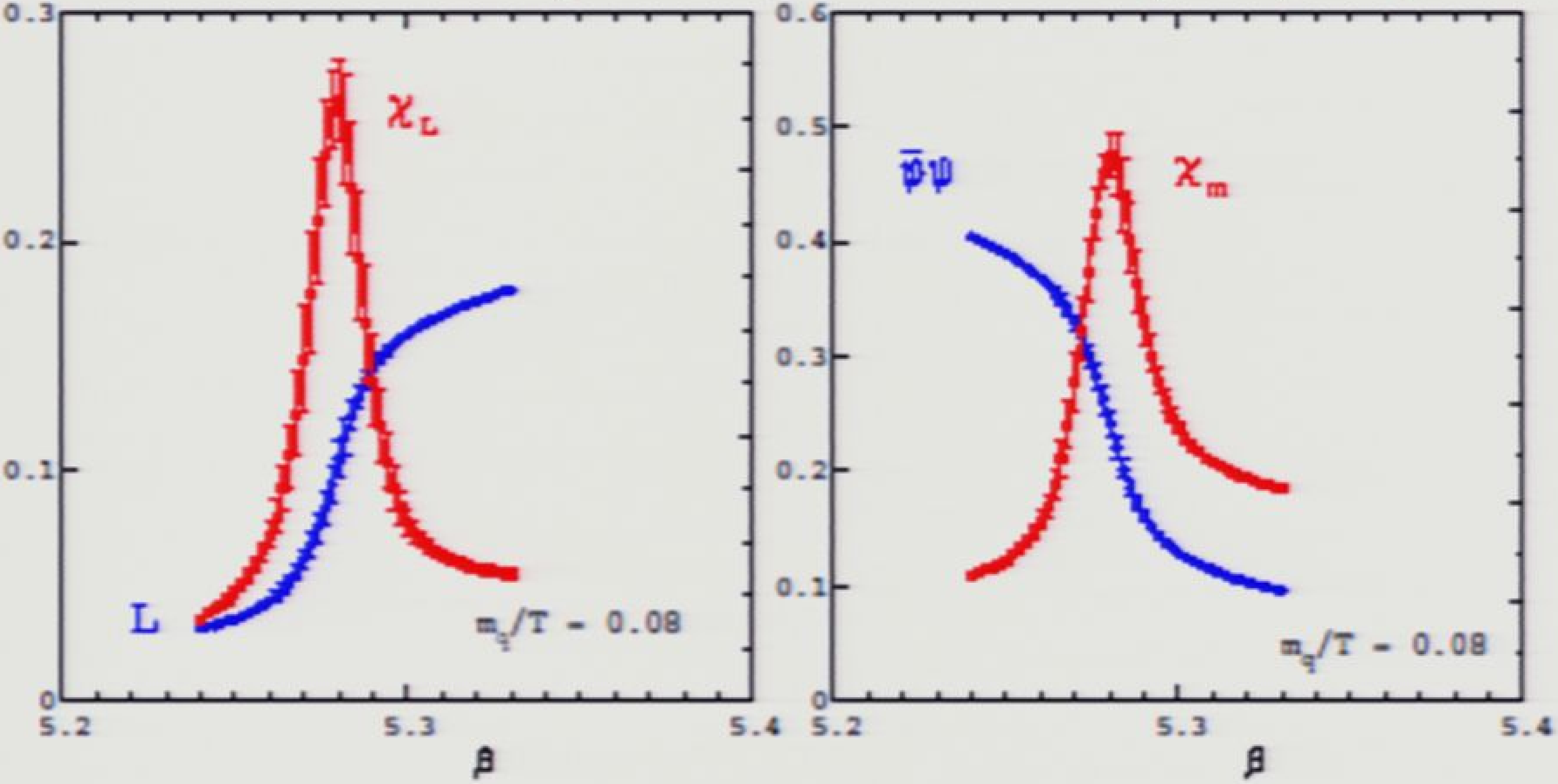

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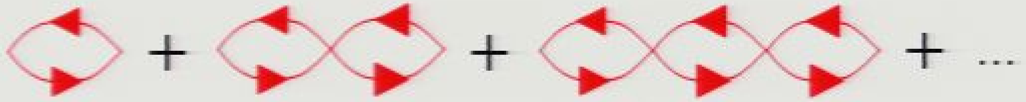
The spectral function of the degenerate hadronic ``para-pion" and the ``para-sigma" at T>Tc for the chiral transition: Tc=164 MeV

T. Hatsuda and T.K. (1985)

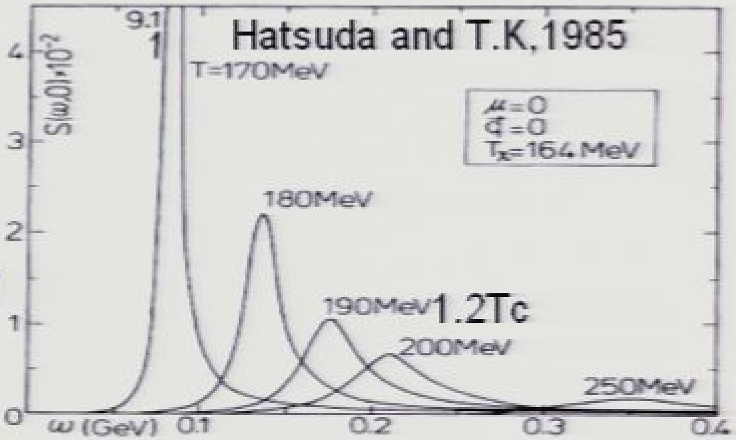
- response function in RPA

$D(\mathbf{k},\omega) =$



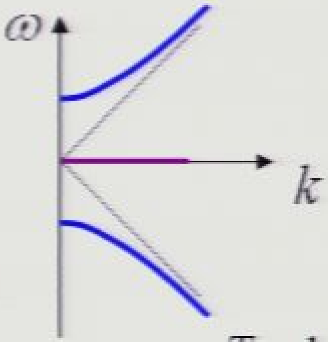
- spectral function

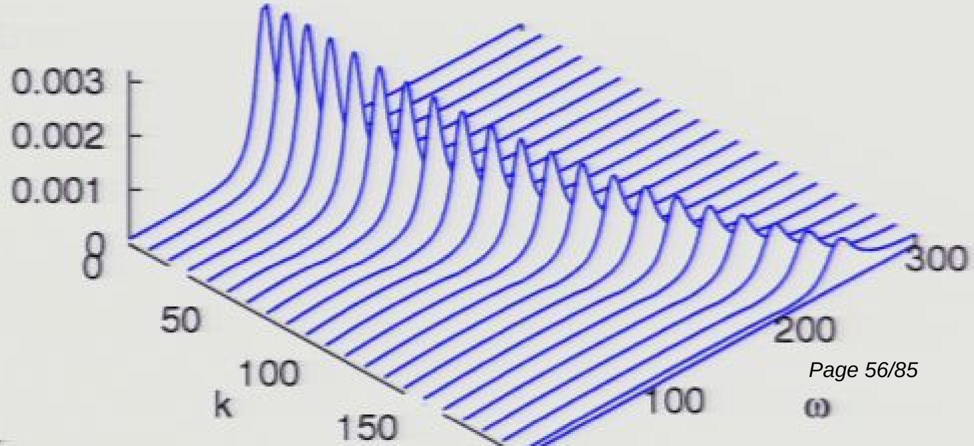
$$A(\mathbf{k}\omega) = -\frac{1}{\pi}\mathrm{Im}\,D(\mathbf{k}\omega)$$



$T \to T_c$, they become elementary modes with small width!

sharp peak in time-like region





M.Kitazawa, Y.Nemoto and T.K. (05)

Quark Spectral Function near T_C

---- effects of the chiral soft modes on quark spectra ----

- **Quark self-energy**

$\Sigma(\mathbf{k}, i\omega_n) =$

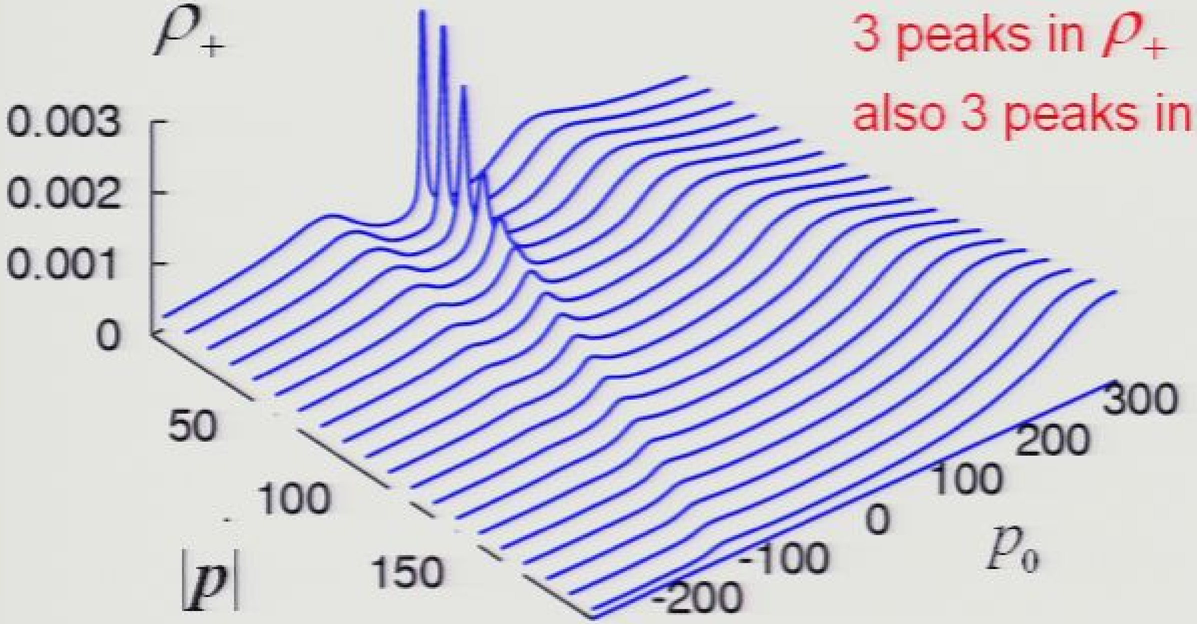
- **Spectral Function**

$$A(\mathbf{p}, p^0) = \underset{\text{quark}}{\underline{\rho_+(\mathbf{p}, p^0)}}\Lambda_+\gamma^0 + \underset{\text{antiquark}}{\underline{\rho_-(\mathbf{p}, p^0)}}\Lambda_-\gamma^0$$

$$\Lambda_\pm(p) = \frac{1}{2}(1 \pm \gamma^0\vec{\gamma}\cdot\hat{p})$$

for free quarks,

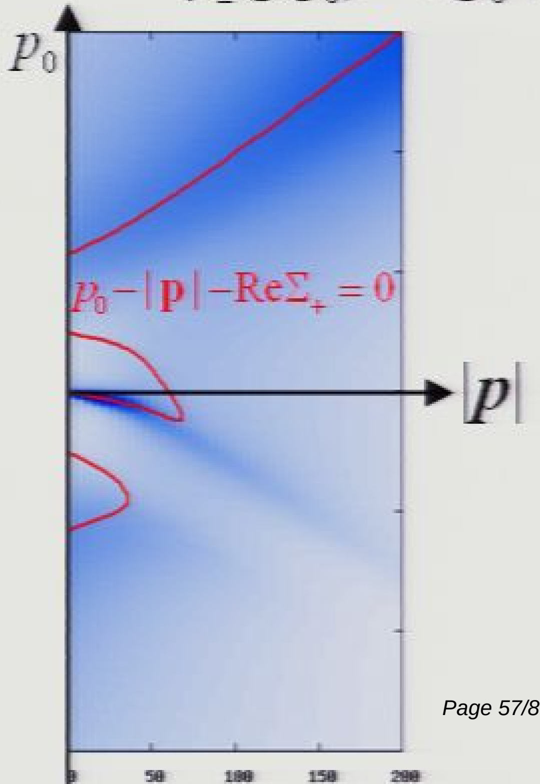
$\rho_\pm(\mathrm{p}, p_0) \propto \delta(p_0 \mp |\mathrm{p}|)$



3 peaks in ρ_+

also 3 peaks in ρ_-

$T = 1.05T_C, \mu = 0$



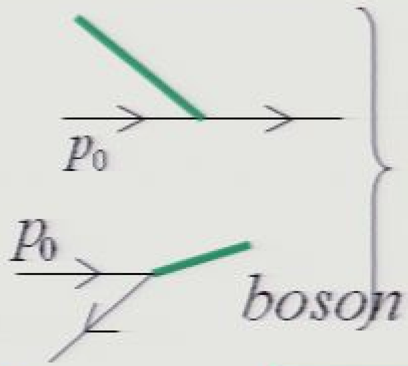
Resonant Scatterings of Quark for CHIRAL Fluctuations

"quark hole": annihilation mode of a thermally excited quark

"antiquark hole": annihilation mode of a thermally excited antiquark

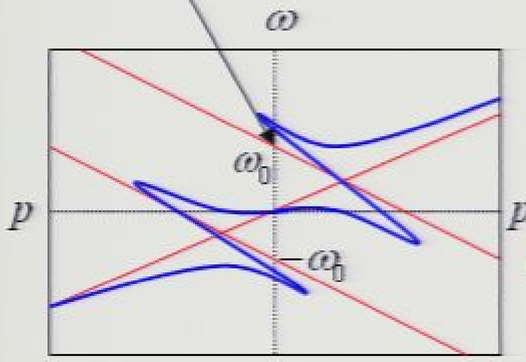
(Weldon, 1989)

bosonic soft modes



lead to quark-"antiquark hole" mixing

the 'mass' of the elementary modes



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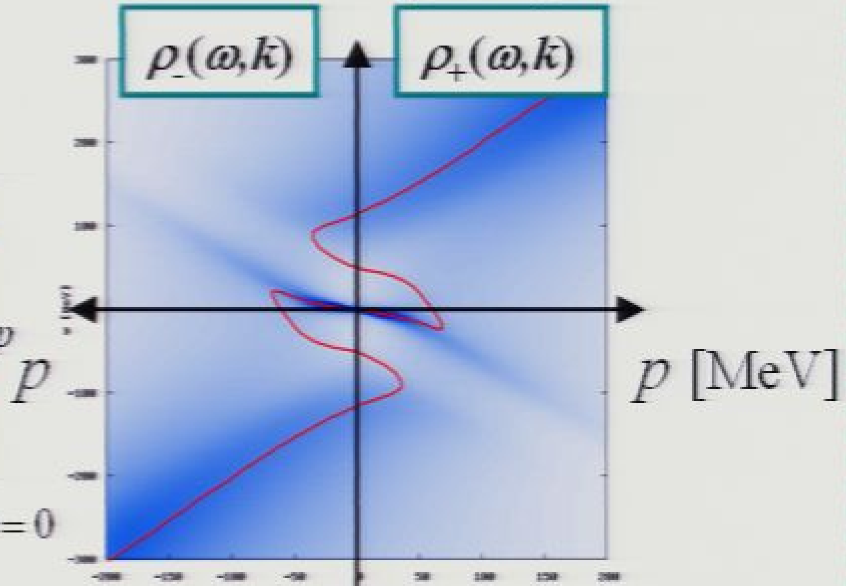
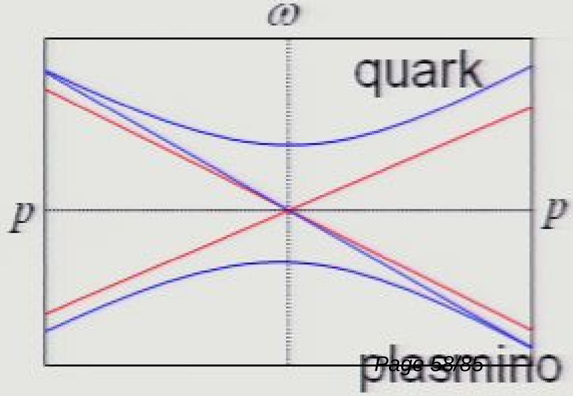


cf: hot QCD

(HTL approximation)

(Klimov, 1981)



Quark at very high T (T>>Tc)

- 1-loop (g<<1) + HTL approx. ($p, \omega, m_q \ll$ T)

$$\Sigma(\omega, p) =$$

thermal mass

$$m_T^2 = \frac{1}{6} g^2 T^2$$

dispersion relations

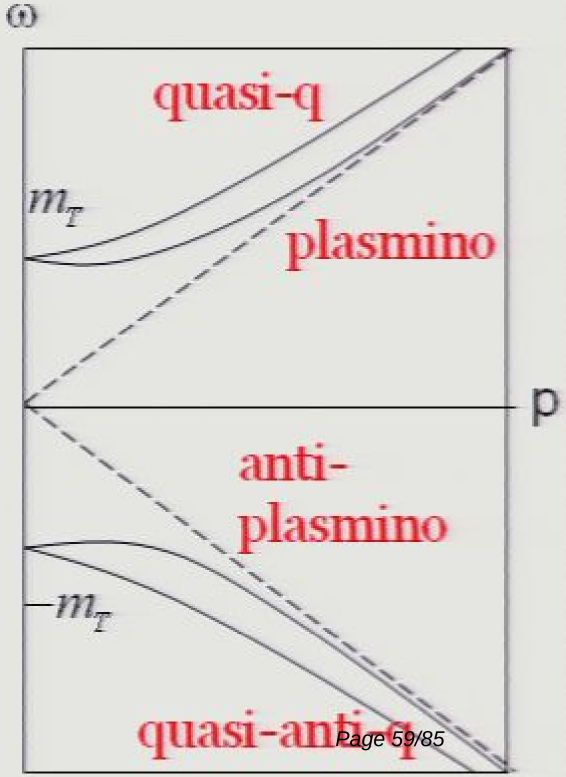
quark spectrum $D_+(\omega, p) = 0$

$$\omega = E_q, -E_{\text{plasmino}}$$

anti-quark spectrum

$$D_-(\omega, p) = 0$$

$$\omega = -E_p, E_{\text{plasmino}}$$



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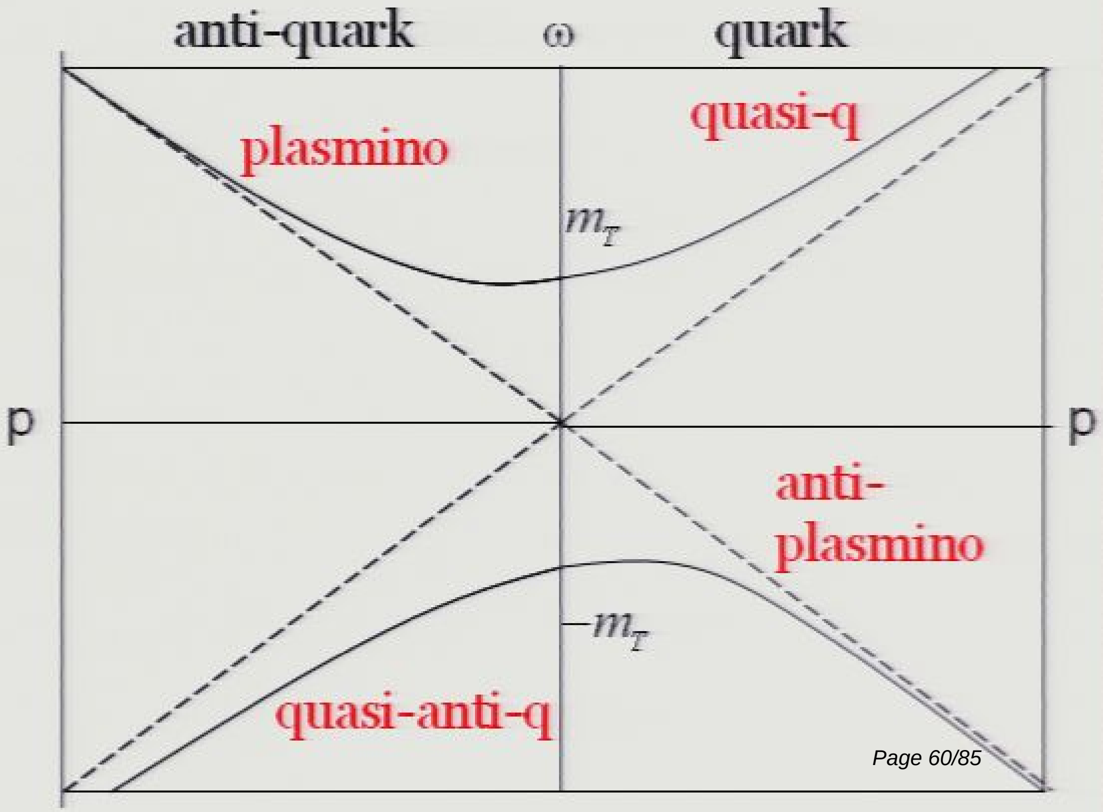

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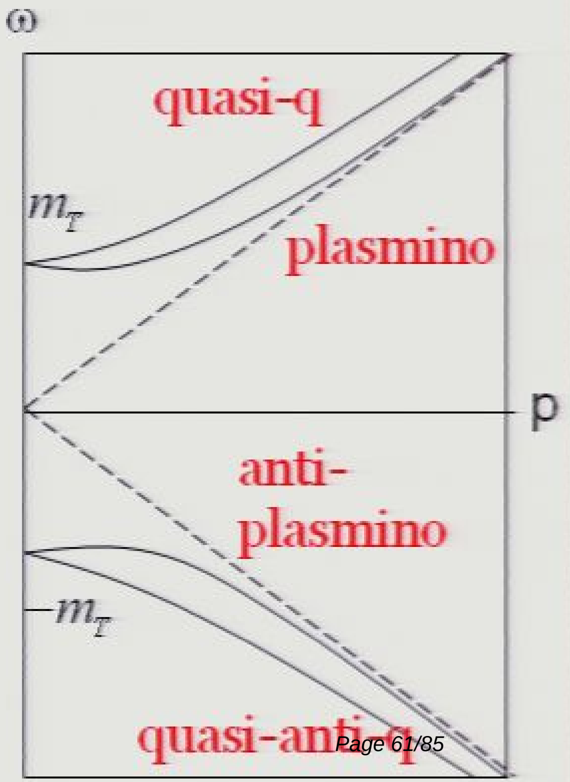

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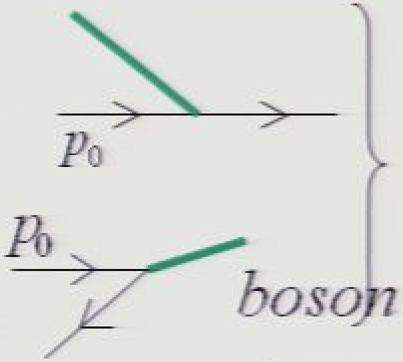

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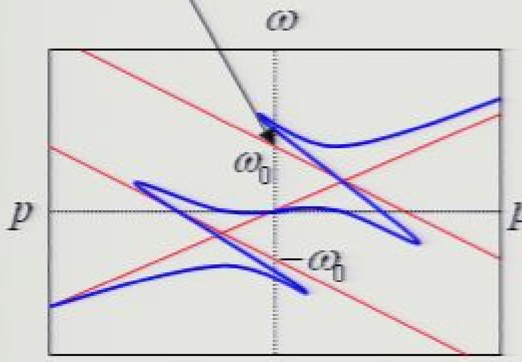

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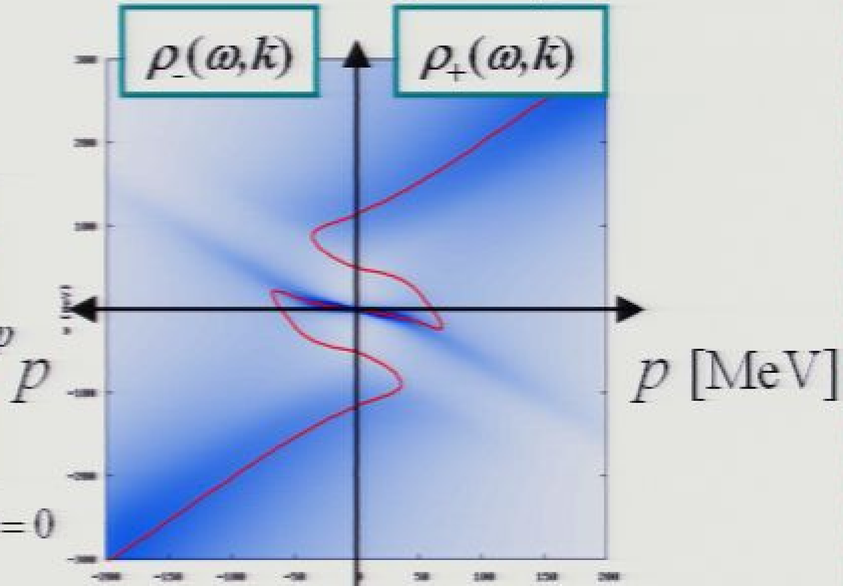

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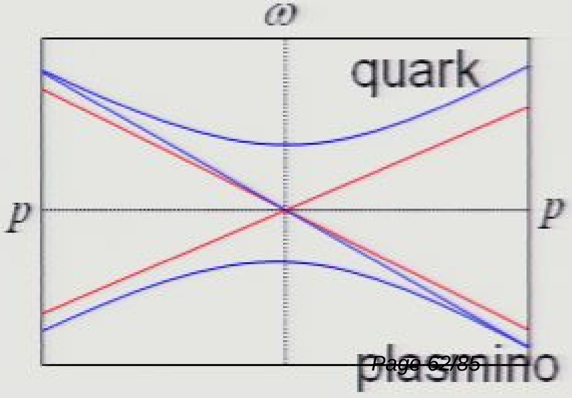

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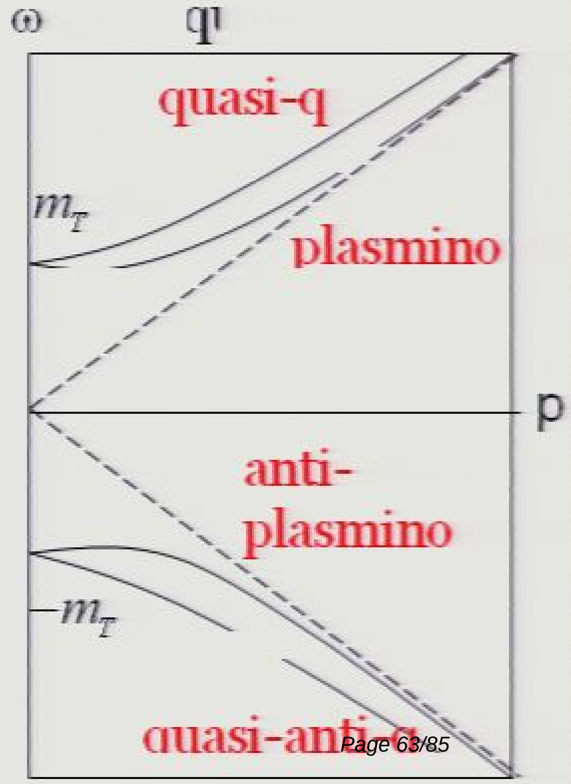

Dilepton production through the chiral fluctuations

Dilepton production rate

$$\frac{d\Gamma}{d^4q} = \frac{-\alpha\,\mathrm{Im}\,\Pi^{R\mu}_{\mu}(q_0,q)}{12\pi^4 q^2(\exp[q_0/T]-1)}$$

photon self-energy

Ex:

$\Pi_{\mu\nu} =$

c.f.: in HTL
Braaten, Pisarski, Yuan (1990)

Y. Nemoto et al, in preparation

Dilepton production through the chiral fluctuations

Dilepton production rate

$$\frac{d\Gamma}{d^4q} = \frac{-\alpha \,\mathrm{Im}\,\Pi^{R\mu}_{\mu}(q_0,q)}{12\pi^4 q^2(\exp[q_0/T]-1)} \leftarrow \text{photon self-energy}$$

Ex:

$\Pi_{\mu\nu} =$

c.f.: in HTL
Braaten, Pisarski, Yuan (1990)

Ex: Aslamazov-Larkin (AL) term

Aslamazov, Larkin (1968)

anomalous electric conductivity
"para-conductivity"

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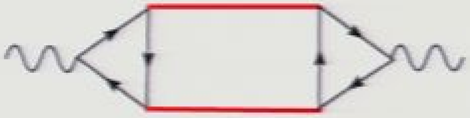

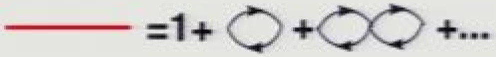

Y. Nemoto et al, in preparation

Dilepton production through the chiral fluctuations

Dilepton production rate

$$\frac{d\Gamma}{d^4q} = \frac{-\alpha\,\mathrm{Im}\,\Pi^{R\mu}_{\mu}(q_0,q)}{12\pi^4 q^2(\exp[q_0/T]-1)} \longleftarrow$$

photon self-energy

Ex:

$\Pi_{\mu\nu} =$

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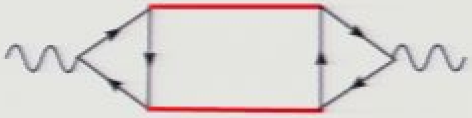
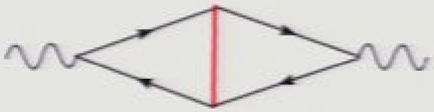


Ex: Maki-Thompson (MT) term

Maki (1968), Thompson (1969)

$\Pi_{\mu\nu} =$



AL > MT at weak coupling
AL >> MT at strong coupling

Strinati et al. (2002)

Y. Nemoto et al, in preparation

Extension to finite m_q

There is a critical mass m_c at which the quark spectral properties change from the collective nature to a free quark nature.

K. Mitsutani, Y.Nemoto, M. Kitazawa, T.K., in preparation

Stable (and correct) Relativistic Hydrodynamics for Viscous Fluids as derived by the renormalization-group method

K. Tsumura , T.K. and K. Ohnishi

Ref. K. Tsumura, K. Ohnishi and T.K.
Phys. Lett. B646 (2007) 134-140
and
K. Tsumura and T.K. in preparation

Introduction

- Relativistic hydrodynamics is widely and successfully used in various fields of physics; H-I coll. , Astro. Physics
- Dissipative hydrodynamics?

Near freeze-out and/or hadron corona,
we need dissipative hydrodynamics or kinetic description, depending I.C.

T. Hirano et al, Phys. Lett. B636 (2006), 299;
See also, C.Nonaka& S.A. Bass, Phys.Rev.C75:014902,2007; talk yesterday

Even if shear viscosity ~ 0, how about bulk viscosity?
The system is expanding? c.f. shear viscosity or shear flow ~0?

Expanding QGP ⇨ Anomalous viscosity due to coupling to chromomagnetic fields; Asakawa, Bass and Muller, P.R.L.96:252301,('06)

- An analysis using dissipative hydrodynamics is needed even to show the dissipative effects are negligible.
- Before going into sophisticated quantum theories, such as AdS/CFT , let us see how the relativistic hydrodynamical equations for a viscous fluid has a firm foundation or not in the classical level.

Ambiguity in hydrodynamical flow

Fluid dynamics = a system of balance equations

- $\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N^\mu = 0.$ **energy−momentum :** $T^{\mu\nu}$ **number :** N^μ
- If dissipative, there arises an ambiguity and instabilities even apart from Acaucality.

W.A. Hiskock and L. Lindblom, PRD31, 725 ('85)

Eckart frame no dissipation in the number flow; Thermal forces contain Du.

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu - (p + \xi X)\,\Delta^{\mu\nu} + \lambda T\, u^\mu \tilde{X}^\nu + \lambda T\, u^\nu \tilde{X}^\mu + 2\,\eta\, X^{\mu\nu},$$

$$N^\mu = n\, u^\mu,$$

⇒ Describing the flow of matter. $\Delta_p^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \equiv \Delta^{\mu\nu},$ $\xi = \varsigma$

Landau frame no dissipation in energy flow

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu - (p + \xi X)\,\Delta^{\mu\nu} + 2\,\eta\, X^{\mu\nu},$$

$$N^\mu = n\, u^\mu - \lambda \underline{\frac{nT}{\epsilon + p}} X^\mu.$$

Thermal forces contain only spatial derivatives.

⇒ describing the energy flow.

with transport coefficients: $\xi \quad \lambda \quad \eta$

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Characterization of the frame when the dissipative flow exists

For the dissipative terms, $\delta T^{\mu\nu}$ and δN^{μ}:

$$\text{Eckart's constraints :} \quad \begin{cases} 1.\ u_\mu\, u_\nu\, \delta T^{\mu\nu} = 0, \\ 2.\ u_\mu\, \delta N^{\mu} = 0, \\ 3.\ \Delta_{\mu\nu}\, \delta N^{\nu} = 0, \end{cases}$$

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imposed to the solution of Boltzmann eq.

H.Grad('49), ('58),
C. Marle ('69),
J. M. Stewart ('71)
c.f. N.G. van Kampen ('87)

What is the correct fluid dynamical equation in the particle (or Eckart) frame?

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The constraints 1 and 5 compatible? No! ⇒ Leads to a different eq.'s!

What is the correct fluid dynamical equation in the particle (or Eckart) frame?

The separation of scales in the relativistic heavy-ion collisions

Liouville → Boltzmann → Fluid dyn.

pQCD finite-T field th. With phase tr. (3-cr.)

Slower dynamics

Possible role of relativistic dissipative fluid dynamics,
Consistently connected kinetic (Boltzmann) equation

K. Tsumura, K. Ohnishi and T.K., PLB646 ('07),134

on the basis of the RG method; Chen-Goldenfeld-Oono('95),T.K.('95)

C.f. Y. Hatta and T.K. ('02)
K.Tsumura and TK ('05)

Boltzmann eq. → Navier−Stokes eq.

Derivation of the relativistic fluid dynamical equation from the rel. Boltzmann eq. --- an RG-reduction of the dynamics

K. Tsumura, T.K. K. Ohnishi; Phys. Lett. B646 (2007) 134-140

c.f. Non-rel. Y.Hatta and T.K., Ann. Phys. 298 ('02), 24; T.K. and K. Tsumura, J.Phys. A:39 (2006), 8089

Ansatz of the origin of the dissipation= the spatial inhomogeneity

$\boldsymbol{a}_p^\mu$ would become a macro velocity

$$\tau \equiv \boldsymbol{a}_p^\mu x_\mu, \quad \sigma^\mu \equiv \left(g^{\mu\nu} - \frac{\boldsymbol{a}_p^\mu \boldsymbol{a}_p^\nu}{\boldsymbol{a}_p^2} \right) x_\nu \equiv \boldsymbol{\Delta}_p^{\mu\nu} x_\nu \qquad x^\mu \Longrightarrow \tau \quad \sigma^\mu$$

$$\frac{\partial}{\partial\tau} = \frac{1}{\boldsymbol{a}_p^2} \boldsymbol{a}_p^\mu \partial_\mu \equiv \boldsymbol{D},$$ time-like derivative $$\boldsymbol{\Delta}_p^{\mu\nu} \frac{\partial}{\partial\sigma^\nu} = \boldsymbol{\Delta}_p^{\mu\nu} \partial_\nu \equiv \boldsymbol{\nabla}^\mu$$ space-like derivative

$$\boxed{\boldsymbol{D}\tau = 1, \quad \boldsymbol{D}\sigma^\mu = 0, \quad \boldsymbol{\nabla}^\mu \tau = 0, \quad \boldsymbol{\nabla}^\mu \sigma^\nu = \boldsymbol{\Delta}_p^{\mu\nu}.} \qquad \boxed{[\boldsymbol{D}, \boldsymbol{\nabla}^\mu] = \frac{1}{\boldsymbol{a}_p^2} \boldsymbol{a}_p^\nu \boldsymbol{\Delta}_p^{\mu\rho} [\partial_\nu, \partial_\rho] = 0,}$$

Rewrite the Boltzmann equation as,

$$\Longrightarrow \quad \frac{\partial}{\partial\tau} f_p(\tau, \sigma) = \frac{1}{p \cdot \boldsymbol{a}_p} C[f]_p(\tau, \sigma) - \frac{1}{p \cdot \boldsymbol{a}_p} p \cdot \boldsymbol{\nabla} f_p(\tau, \sigma)$$

perturbation

1st order $$\frac{\partial}{\partial\tau} \tilde{f}_p^{(1)} = \sum_q A_{pq} \tilde{f}_q^{(1)} + F_p$$

$$A_{pq} \equiv \frac{1}{p \cdot \boldsymbol{a}} \frac{\partial}{\partial f} C[f]_p \Big| \qquad F_p \equiv -\frac{1}{p \cdot \boldsymbol{a}_p} p \cdot \boldsymbol{\nabla} f_p^{\mathrm{eq}}$$

‘Eckart frame’

$$a_p^{\mu} = \frac{m}{p \cdot u}\, u^{\mu}$$

Then,

$$J^{\mu\alpha}_{\rm hydro.} = \begin{cases} (\epsilon + \underline{3\,\xi_{\rm E}\, X)}\, u^{\mu}\, u^{\nu} - (p + \xi_{\rm E}\, X)\, \Delta^{\mu\nu} \\ \qquad\qquad + \lambda_{\rm E}\, T\, u^{\mu}\, \tilde{X}^{\nu} + \lambda_{\rm E}\, T\, u^{\nu}\, \tilde{X}^{\mu} + 2\, \eta_{\rm E}\, X^{\mu\nu} \; \boxed{\alpha = \nu} \\ m\, n\, u^{\mu} \; \boxed{\alpha = 4} \end{cases}$$

$$T^{\mu\nu} = \epsilon\, u^{\mu}\, u^{\nu} - (p + \xi\, X)\, \Delta^{\mu\nu} + \lambda\, T\, u^{\mu}\, \tilde{X}^{\nu} + \lambda\, T\, u^{\nu}\, \tilde{X}^{\mu} + 2\, \eta\, X^{\mu\nu},$$

$$N^{\mu} = n\, u^{\mu},$$

$$\boxed{\begin{aligned} \xi_{\rm E} &\equiv \left\{ \frac{1}{3} \left(\frac{4}{3} - \gamma \right)^{-1} \right\}^{2} \xi_{\frac{\pi}{2}}, \\ \lambda_{\rm E} &\equiv \lambda_{\frac{\pi}{2}}, \\ \eta_{\rm E} &\equiv \eta_{\frac{\pi}{2}} \end{aligned}}$$

C.f. This is different from the Eckart nor Marle-Stewart equation, though, which includes the time-like derivative in the thermal force.

Our theory can be stable in contrast to theirs,

because of no time-like D derivative in the thermal force

$$\partial_\mu J^{\mu\alpha}_{\text{hydro}} = 0,$$

$$J^{\mu\alpha}_{\text{hydro}} \equiv \sum_p \frac{1}{p^0} p^\mu \varphi^\alpha_{0p} \left\{ f^{\text{eq}}_p - \left[A^{-1} \bar{Q} F\right]_p \right\} = J^{\mu\alpha}_0 + \Delta J^{\mu\alpha},$$

$$J^{\mu\alpha}_0 \equiv \sum_p \tfrac{1}{p^0} p^\mu \varphi^\alpha_{0p} f^{\text{eq}}_p$$

$$\underline{\Delta J^{\mu\alpha} \equiv -\sum_p \tfrac{1}{p^0} p^\mu \varphi^\alpha_{0p} \left[A^{-1} \bar{Q} F\right]_p} \Longleftarrow$$

the dissipative terms!

The distribution function;

$$\underline{f(\tau_0) = f^{\text{eq}} - A^{-1} \bar{Q} F - A^{-2} \bar{Q} H - A^{-1} \bar{Q} I}$$

Landau frame:

If we set, $a^\mu_p = u$

$$\Delta J^{\mu\alpha} = \begin{cases} -\zeta\, \Delta^{\mu\nu} X + 2\eta X^{\mu\nu} & \alpha = \nu \\ -T \lambda z \hat{h}^{-1} X^\mu & \alpha = 4. \end{cases}$$

$$\begin{aligned} X &\equiv -\nabla_\mu u^\mu, \\ X_\mu &\equiv \nabla_\mu \ln T - \hat{h}^{-1} \nabla_\mu \ln(nT), \\ X_{\mu\nu} &\equiv \frac{1}{2}\left(\Delta_{\mu\rho} \Delta_{\nu\sigma} + \Delta_{\mu\sigma} \Delta_{\nu\rho} - \frac{2}{3} \Delta_{\mu\nu} \Delta_{\rho\sigma}\right) \nabla^\rho u^\sigma. \end{aligned}$$

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$$\boldsymbol{a}_p^\mu = \frac{m}{p\cdot u}\,u^\mu$$

Then,

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Summary and concluding remarks

PART I

- QCD phase diagram may have richer structure than expected and imagined; owing to various correlations and their mutual interplay such as vector and axial vector correlations as well as the scalar and diquark ones.
- The fluctuations of the order parameter can give rise to the collective (soft) modes, which in turn affect the quark properties especially its quasi-particle picture, in the critical regions of the QCD phase transitions.
- In generic, spectra of fermion coupled to a massive boson may show a multiple-peak structure at T~m_B, which may have relevance to other contexts; for instance, neutrino spectra in the era of leptogenesis?

PART II

- The Eckart equation seems not compatible with the underlying Relativistic Boltsmann equation.
- The RG method gives a consistent fluid dynamical equation for the particle (Eckart) frame, which is new.
- The new equation may be derived also

  in a phenomenological way. (not shown)
- new equation is different in the terms containing the bulk viscosity, which should be important for the description of expanding fluids like the early universe and the QCD matter created by rel. H-I collisions.
- The linear analysis of the new equations show that these equations even for 'Eckart frame' can be stable for a wide class of EOS and the transport coefficients.
- There are many things to do, of course, to establish the relativistic fluid dynamics for a viscous fluid even in the classical level.