Title: Early Time Dynamics in Heavy Ion Collisions and AdS/CFT Correspondence

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Abstract:

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Yuri Kovchegov
The Ohio State University

based on work done with Anastasios Taliotis, arXiv:0705.1234 [hep-ph]

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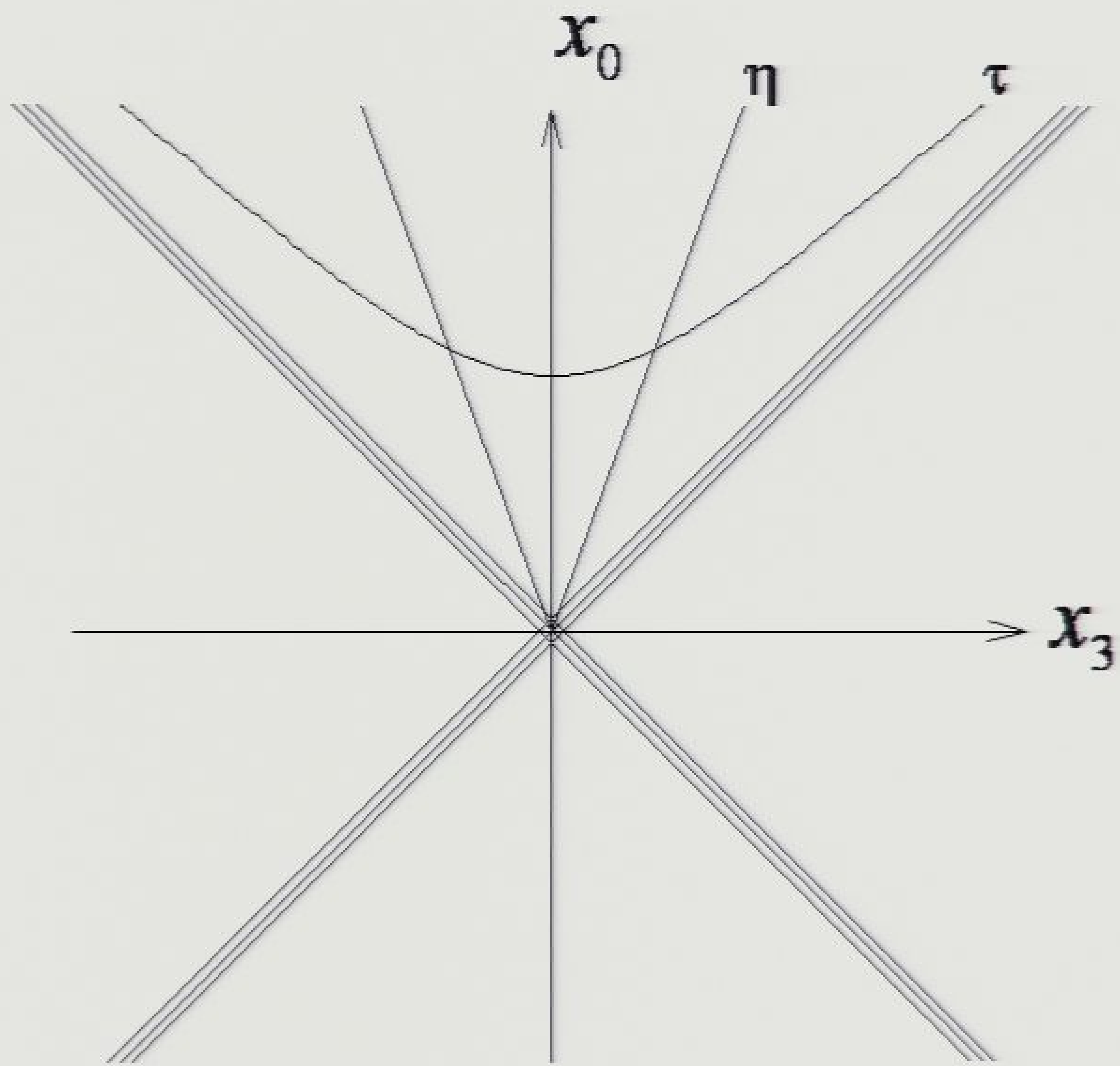
  - Analyzed early-time dynamics and showed that energy density goes to a constant at early times.
  - Have therefore shown that isotropization (and hopefully thermalization) takes place in strong coupling dynamics.
  - Derived a simple formula for isotropization time and used it for heavy ion collisions at RHIC to obtain 0.3 fm/c, in agreement with hydrodynamic simulations.

Notations

We'll be using the following notations:



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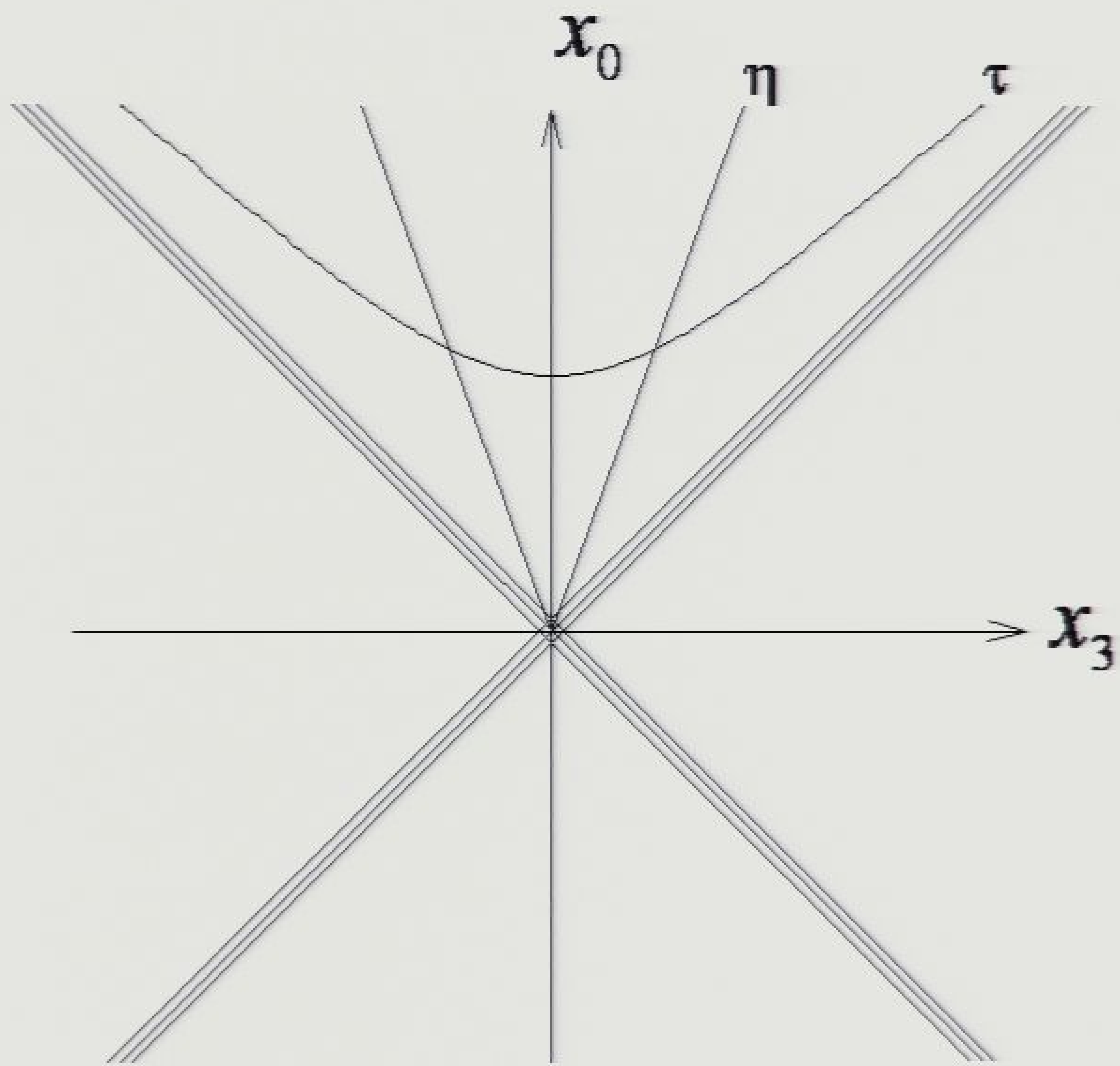

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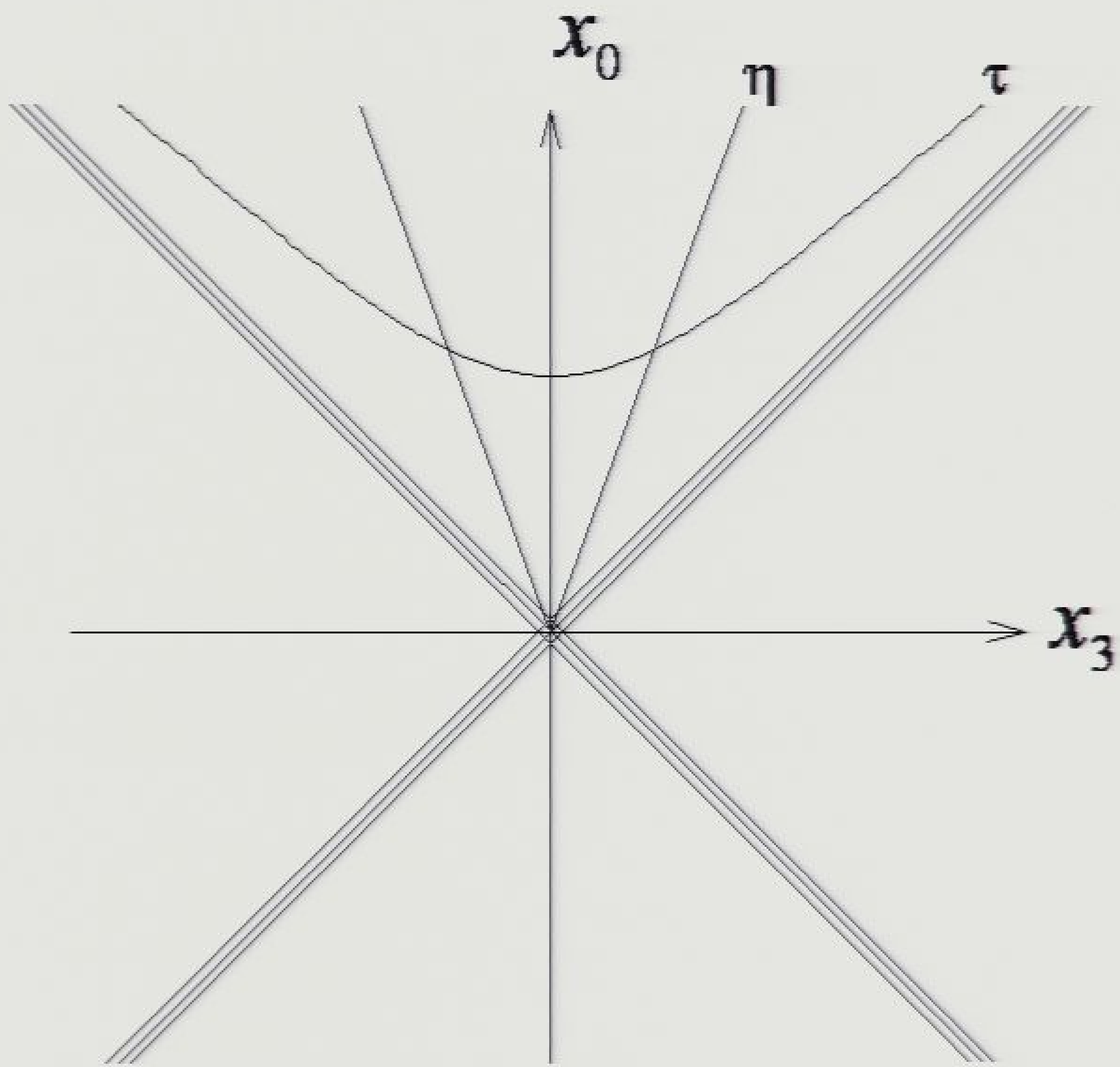
and rapidity

$$\eta = \frac{1}{2} \ln \frac{x_0 + x_3}{x_0 - x_3}$$



Most General Boost Invariant Energy-Momentum Tensor

The most general boost-invariant energy-momentum tensor for a high energy collision of two very large nuclei is (at $x_3 = 0$)

$$T^{\mu\nu} = \begin{array}{c} \begin{pmatrix} \varepsilon(\tau) & 0 & 0 & 0 \\ 0 & p(\tau) & 0 & 0 \\ 0 & 0 & p(\tau) & 0 \\ 0 & 0 & 0 & p_3(\tau) \end{pmatrix} \end{array} \begin{array}{c} \boldsymbol{x_0} \\ \boldsymbol{x_1} \\ \boldsymbol{x_2} \\ \boldsymbol{x_3} \end{array}$$

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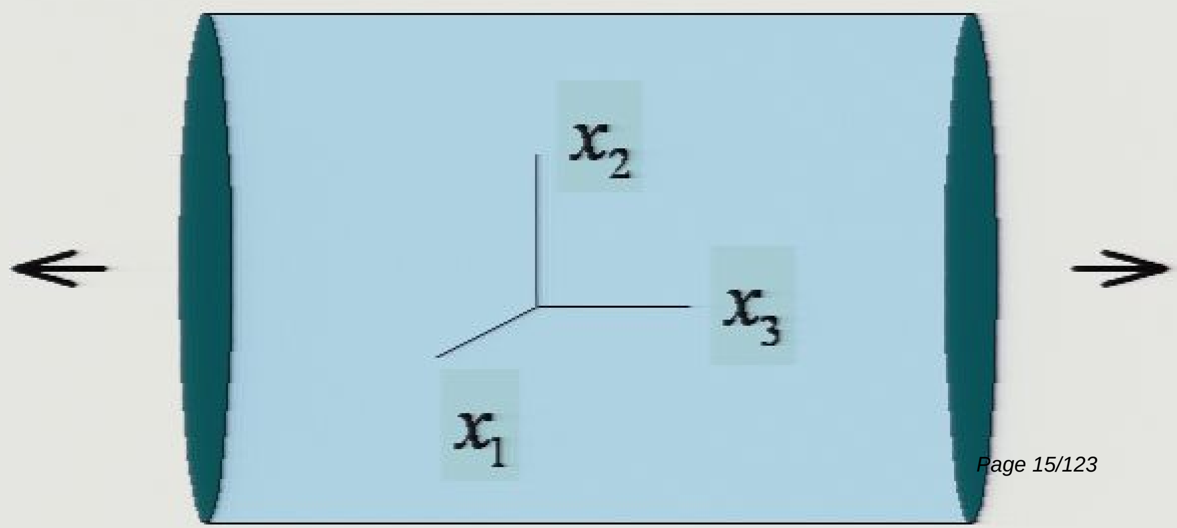

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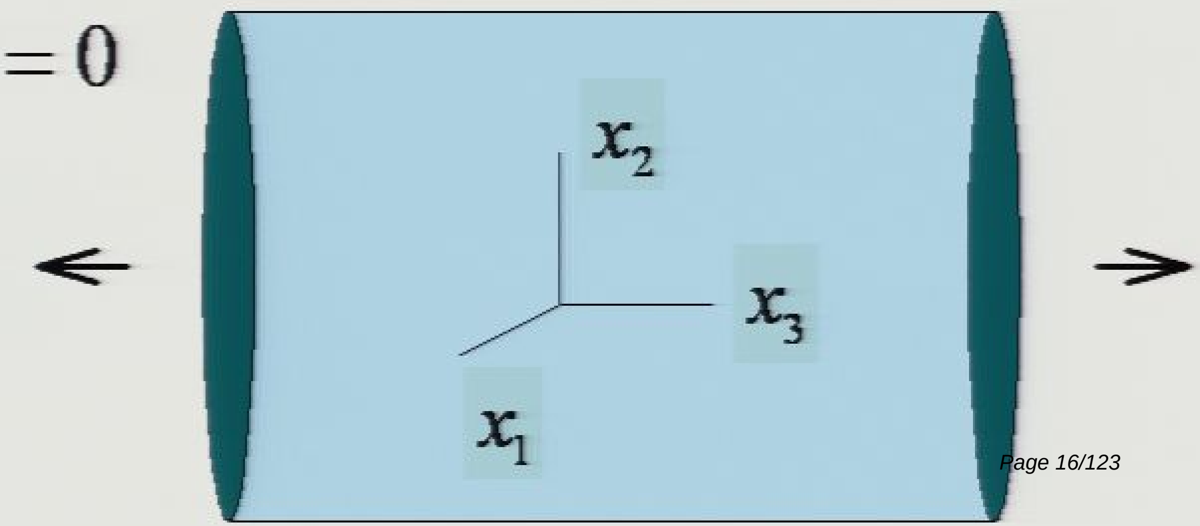
which, due to $\partial_\mu T^{\mu\nu} = 0$

gives $\boxed{\dfrac{d\varepsilon}{d\tau} = -\dfrac{\varepsilon + p_3}{\tau}}$



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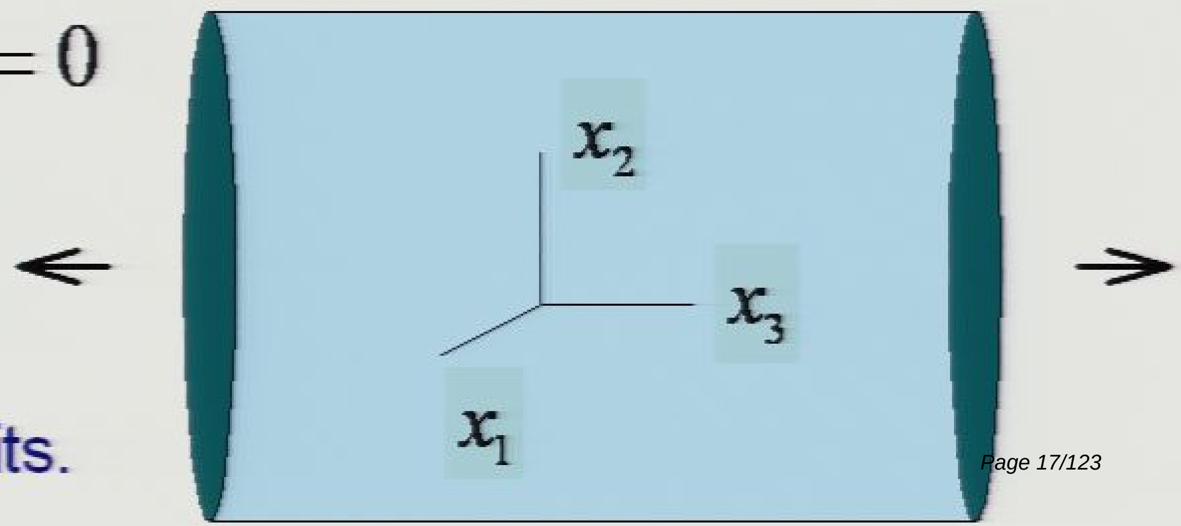

There are 3 extreme limits.

Limit I: “Free Streaming”

Free streaming is characterized by the following “2d” energy-momentum tensor:

$$
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\begin{pmatrix}
\varepsilon(\tau) & 0 & 0 & 0 \\
0 & p(\tau) & 0 & 0 \\
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0 & 0 & 0 & 0
\end{pmatrix}
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such that

$$\frac{d\varepsilon}{d\tau} = -\frac{\varepsilon}{\tau}$$

and

$$\varepsilon \sim \frac{1}{\tau}$$

- The total energy $E \sim \varepsilon\,\tau$ is conserved, as expected for non-interacting particles.

Limit II: Bjorken Hydrodynamics

In the case of ideal hydrodynamics, the energy-momentum tensor is symmetric in all three spatial directions (**isotropization**):

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$$\boxed{\varepsilon \sim \frac{1}{\tau^{4/3}}}$$

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- The total energy E~ $\varepsilon\,\tau$ is **not** conserved, while the total entropy S is conserved.

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If $p_3 > 0$ then, as $\boxed{\dfrac{d\varepsilon}{d\tau} = -\dfrac{\varepsilon + p_3}{\tau}}$, one gets $\varepsilon \sim \dfrac{1}{\tau^{1+\Delta}}$.

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Deviations from the $\varepsilon \sim \frac{1}{\tau}$ scaling of energy density,

like $\varepsilon \sim \frac{1}{\tau^{1+\Delta}}$, $\Delta > 0$ are due to longitudinal pressure

p_3, which does work $p_3 \, dV$ in the longitudinal direction

modifying the energy density scaling with tau.

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$\leftrightarrow$ deviations from $\varepsilon \sim \dfrac{1}{\tau}$

Limit III: Color Glass at Early Times

In CGC at very early times $\varepsilon \sim \log^2 \frac{1}{\tau}, \qquad \tau Q_s << 1$ (Lappi, '06)

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AdS/CFT Approach

Start with the metric in Fefferman-Graham coordinates in AdS_5 space

$$ds^2 = \frac{1}{z^2}\left[-A(\tau, z)\, d\tau^2 + \tau^2\, B(\tau, z)\, d\eta^2 + C(\tau, z)\, dx_\perp^2 + dz^2\right]$$

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Expand the 4d metric near the boundary of the AdS space

$$\tilde{g}_{\mu\nu}(x,z) = \tilde{g}^{(0)}_{\mu\nu}(x) + z^2\,\tilde{g}^{(2)}_{\mu\nu}(x) + z^4\,\tilde{g}^{(4)}_{\mu\nu}(x) + \ldots$$

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$$\langle T_{\mu\nu}\rangle = \frac{N_c^2}{2\,\pi^2}\,\tilde{g}^{(4)}_{\mu\nu}(x)$$

Iterative Solution

General solution of Einstein equations is not known and is hard to obtain. One first assumes a specific form for energy density

$$\varepsilon = \varepsilon(\tau)$$

and the solves Einstein equations perturbatively order-by-order in z:

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At the order z^4 it gives the following familiar conditions:

$$\partial_\mu \langle T^{\mu\nu} \rangle = 0 \quad \text{and} \quad \langle T^\mu_\mu \rangle = 0$$

Iterative Solution

We begin by expanding the coefficients of the metric

$$ds^2 = \frac{1}{z^2}\left[-A(\tau,z)\,d\tau^2 + \tau^2\,B(\tau,z)\,d\eta^2 + C(\tau,z)\,dx_\perp^2 + dz^2\right]$$

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$$A(\tau,z) = e^{a(\tau,z)}, \quad B(\tau,z) = e^{b(\tau,z)}, \quad C(\tau,z) = e^{c(\tau,z)}$$

into power series in z:

$$a(\tau,z) = \sum_{n=0}^{\infty} a_n(\tau)\,z^{4+2n}, \quad b(\tau,z) = \sum_{n=0}^{\infty} b_n(\tau)\,z^{4+2n}, \quad c(\tau,z) = \sum_{n=0}^{\infty} c_n(\tau)\,z^{4+2n}.$$

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To illustrate their structure let me display one of them:

$$a_2(\tau) = -\frac{1}{384}\left[a_0\,\tau^{\Delta-4}\,(4\,\Delta^2 - \Delta^4) + a_0^2\,\tau^{2\,\Delta}\,8\,(8 + 8\,\Delta + 3\,\Delta^2)\right]$$

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(only if $\Delta > -4$!)

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Assuming power-law scaling $\varepsilon \sim \tau^{\Delta}$ the above conditions lead to

$$-4 \leq \Delta \leq 0.$$

The above conclusion about which term dominates at what time is safe!

Late Time Solution: Scaling

At late times the perturbative (in z) series becomes

$$a(\tau, z) = \#\, z^4\, \tau^{\Delta} + \#'\, z^8\, \tau^{2\,\Delta} + \ldots$$

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$$a(\tau, z) = \# z^4 \tau^{\Delta} + \#' z^8 \tau^{2\Delta} + \dots$$

Janik and Peschanski ('05) were the first to observe it and looked for the full solution of Einstein equations at late proper time as a function of the scaling variable

$$v = (-a_0)^{1/4} z \tau^{\Delta/4}$$

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Here $a_0 < 0$ is the normalization of the energy density

$$\epsilon(\tau) = -\frac{N_c^2}{2\pi^2} \, a_0 \, \tau^{\Delta}$$

Janik and Peschanski's Late Time Solution

The late time solution reads (in terms of scaling variable v, for v fixed and τ going to infinity):

$$a(v) = \frac{1}{2}\left(1 - \frac{1}{D}\right)\ln(1 + D\,v^4) + \frac{1}{2}\left(1 + \frac{1}{D}\right)\ln(1 - D\,v^4)$$

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with $D = \sqrt{\frac{3\Delta^2 + 8\Delta + 8}{24}}$

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At this point Janik and Peschanski fixed the power Δ by requiring that the curvature invariant has no singularities:

$$\mathcal{R} = R_{\mu\nu\rho\sigma}\,R^{\mu\nu\rho\sigma} < \infty$$

Late Time Solution: Branch Cuts

$$a(v) = \frac{1}{2}\left(1 - \frac{1}{D}\right)\ln(1 + D\,v^4) + \frac{1}{2}\left(1 + \frac{1}{D}\right)\ln(1 - D\,v^4)$$

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This is not your run of the mill singularity: this is a branch cut! This means that the metric becomes complex and multivalued for $1 - D\,v^4 \leq 0$! Since the metric has to be real and single-valued we conclude that the metric (and the curvature invariant) <u>do not exist</u> for $1 - D\,v^4 \leq 0$. That is unless the coefficients in front of the logarithms are integers!

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If the coefficients in front of the logarithms are integers, functions A, B and C would be single-valued and real.

Late Time Solution: Fixing the Power

Requiring the coefficients in front of the logarithms to be integers l,m,n

$$\frac{1}{2}\left(1+\frac{1}{D}\right) = n$$

$$\frac{1}{2}\left(1+\frac{\Delta+1}{D}\right) = m$$

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after simple algebra (!) one obtains that the only allowed power is $\Delta = -\frac{4}{3}$, giving the Bjorken hydrodynamic scaling of the energy density, reproducing the result of Janik and Peschanski

$$\boxed{\epsilon(\tau) = -\frac{N_c^2}{2\pi^2}\, a_0\, \frac{1}{\tau^{4/3}} \propto \frac{1}{\tau^{4/3}}}$$

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Iterative Solution

We begin by expanding the coefficients of the metric

$$ds^2 = \frac{1}{z^2}\left[-A(\tau,z)\,d\tau^2 + \tau^2\,B(\tau,z)\,d\eta^2 + C(\tau,z)\,dx_\perp^2 + dz^2\right]$$

$$A(\tau,z) = e^{a(\tau,z)}, \quad B(\tau,z) = e^{b(\tau,z)}, \quad C(\tau,z) = e^{c(\tau,z)}$$

into power series in z:

$$a(\tau,z) = \sum_{n=0}^{\infty} a_n(\tau)\,z^{4+2n}, \quad b(\tau,z) = \sum_{n=0}^{\infty} b_n(\tau)\,z^{4+2n}, \quad c(\tau,z) = \sum_{n=0}^{\infty} c_n(\tau)\,z^{4+2n}.$$

Iterative Solution: Power-Law Scaling

Assuming power-law scaling $\varepsilon \sim \tau^{\Delta}$

we iteratively obtain coefficients in the expansion

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To illustrate their structure let me display one of them:

$$a_2(\tau) = -\frac{1}{384}\left[a_0\, \tau^{\Delta-4}\,(4\,\Delta^2 - \Delta^4) + a_0^2\, \tau^{2\,\Delta}\, 8\,(8 + 8\,\Delta + 3\,\Delta^2)\right]$$

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(only if $\Delta > -4$!)

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Early Time Solution: Scaling

Let us apply the same strategy to the early-time solution: using perturbative (in z) solution at early times give the following series

$$\begin{aligned} a(\tau, z) &= \# z^4 \tau^{\Delta} + \#' z^6 \tau^{\Delta-2} + \#'' z^8 \tau^{\Delta-4} + \ldots \\ &= z^4 \tau^{\Delta} \left(\# + \#' \frac{z^2}{\tau^2} + \#'' \frac{z^4}{\tau^4} + \ldots \right) \end{aligned}$$

While no single scaling variable exists, it appears that the series expansion is in

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such that

$$a(\tau, u) = \tau^{\Delta+4} u^4 \left(\# + \#' u^2 + \#'' u^4 + \ldots \right)$$

Early Time Solution: Ansatz

Keeping u fixed and taking $\tau \to 0$, we write the following ansatze for the metric coefficients:

$$A(\tau, u) = e^{a(\tau,u)} = e^{\tau^{\Delta+4}\alpha(u)} = 1 + \tau^{\Delta+4}\alpha(u) + o(\tau^{2\Delta+8})$$

$$B(\tau, u) = 1 + \tau^{\Delta+4}\beta(u) + o(\tau^{2\Delta+8})$$

$$C(\tau, u) = 1 + \tau^{\Delta+4}\gamma(u) + o(\tau^{2\Delta+8})$$

with α, β and γ some unknown functions of u.

Early-Time General Solution

Solving Einstein equations yields

$$
\begin{aligned}
A(\tau,u) &= 1 + a_0\,\tau^{4+\Delta}\,u^4\,F\left(-1-\frac{\Delta}{2},-\frac{\Delta}{2};3;u^2\right),\\
B(\tau,u) &= 1 + a_0\,\tau^{4+\Delta}\,u^4\left[(\Delta+1)\,F\left(-1-\frac{\Delta}{2},-\frac{\Delta}{2};3;u^2\right)\right.\\
&\qquad\left.-\frac{\Delta\,(\Delta+2)}{6}\,u^2\,F\left(1-\frac{\Delta}{2},-\frac{\Delta}{2};4;u^2\right)\right],\\
C(\tau,u) &= 1 + a_0\,\tau^{4+\Delta}\,u^4\,\frac{\Delta+2}{12}\left[-6\,F\left(-1-\frac{\Delta}{2},-\frac{\Delta}{2};3;u^2\right)\right.\\
&\qquad\left.+\Delta\,u^2\,F\left(1-\frac{\Delta}{2},-\frac{\Delta}{2};4;u^2\right)\right].
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where F is the hypergeometric function.

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Hypergeometric functions have a branch cut for u>1.
We have branch cuts again!

Allowed Powers of Proper Time

However, now hypergeometric functions are not in the exponent. The only way to avoid branch cuts is to have hypergeometric series terminate at some finite order, becoming a polynomial.

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Hence, at early times the physically allowed powers are:

$$\boxed{-1 \leq \Delta \leq 0}$$

Early Time Solution: Terminating the Series

Finally, we see that the hypergeometric series in the solution

$$A(\tau,u) = 1 + a_0\,\tau^{4+\Delta}\,u^4\,F\left(-1-\frac{\Delta}{2},-\frac{\Delta}{2};3;u^2\right),$$

$$B(\tau,u) = 1 + a_0\,\tau^{4+\Delta}\,u^4\left[(\Delta+1)\,F\left(-1-\frac{\Delta}{2},-\frac{\Delta}{2};3;u^2\right)\right.$$
$$\left.-\frac{\Delta\,(\Delta+2)}{6}\,u^2\,F\left(1-\frac{\Delta}{2},-\frac{\Delta}{2};4;u^2\right)\right],$$

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$$\left.+\Delta\,u^2\,F\left(1-\frac{\Delta}{2},-\frac{\Delta}{2};4;u^2\right)\right].$$

terminates only for $\boxed{\Delta = 0}$ in the physically allowed

range of $-1 \le \Delta \le 0$.

Early Time Solution

The early-time scaling of the energy density in this strongly-coupled medium is

$$\boxed{\epsilon(\tau) \to \text{constant} \qquad \text{as} \qquad \tau \to 0.}$$

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This leads to the following energy-momentum tensor, reminiscent of CGC at very early times:

$$T^{\mu\nu} = \begin{array}{cc} \begin{pmatrix} \varepsilon(\tau) & 0 & 0 & 0 \\ 0 & \varepsilon(\tau) & 0 & 0 \\ 0 & 0 & \varepsilon(\tau) & 0 \\ 0 & 0 & 0 & -\varepsilon(\tau) \end{pmatrix} & \begin{array}{c} x_0 \\ x_1 \\ x_2 \\ x_3 \end{array} \end{array}$$

Early Time Solution: Log Ansatz

One can also look for the solution with the logarithmic ansatz (sort of like fine-tuning):

$$\epsilon(\tau) = -\frac{N_c^2}{2\pi^2}\left[a_0 \ln^{\delta}\left(\frac{1}{\tau}\right) + a_1 \ln^{\delta-1}\left(\frac{1}{\tau}\right)\right]$$

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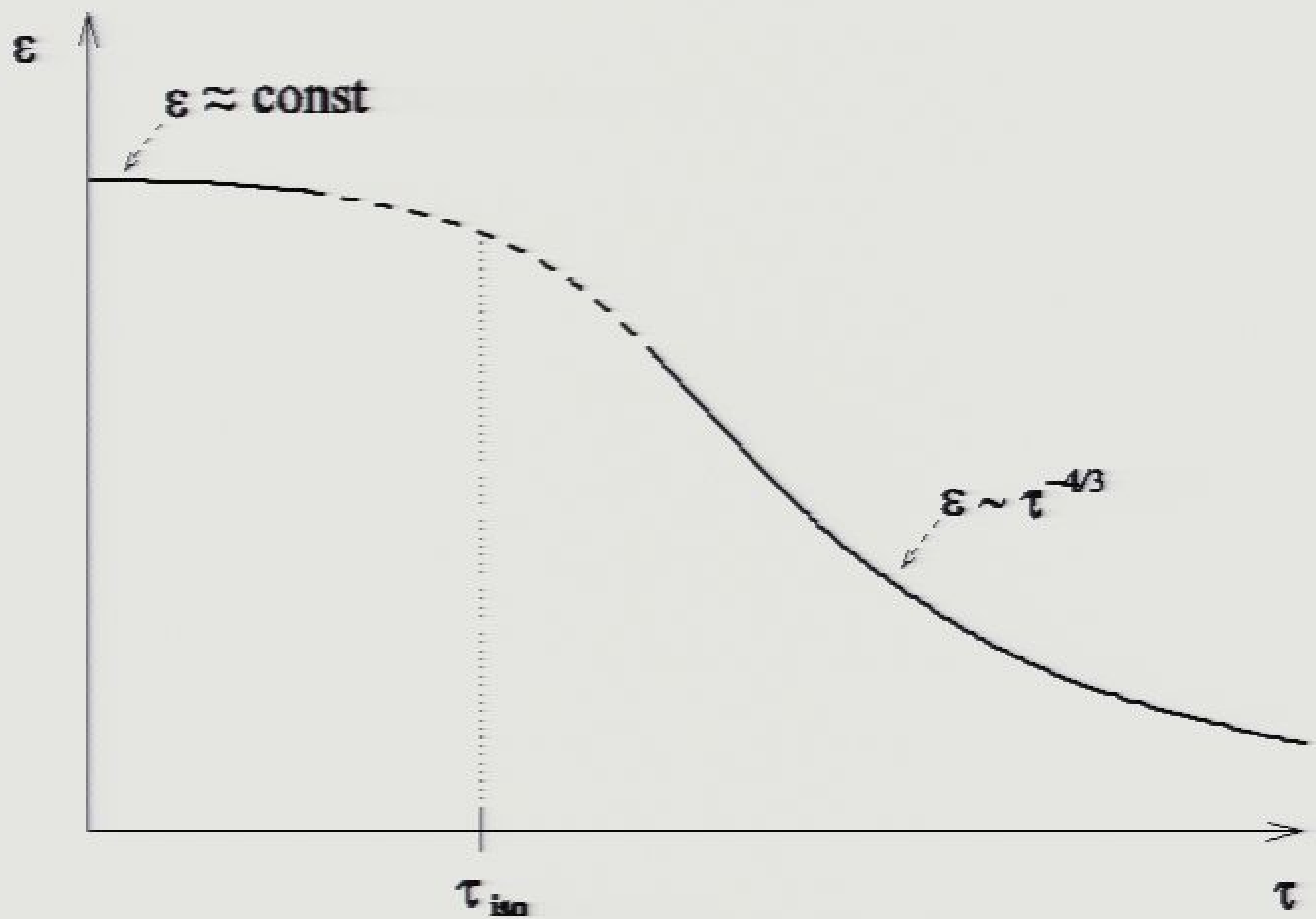
The approach to a constant at early times could be logarithmic! (More work is needed to sort this out.)

Isotropization Transition: the Big Picture

We summary of our knowledge of energy density scaling with proper time for the strongly-coupled medium at hand:



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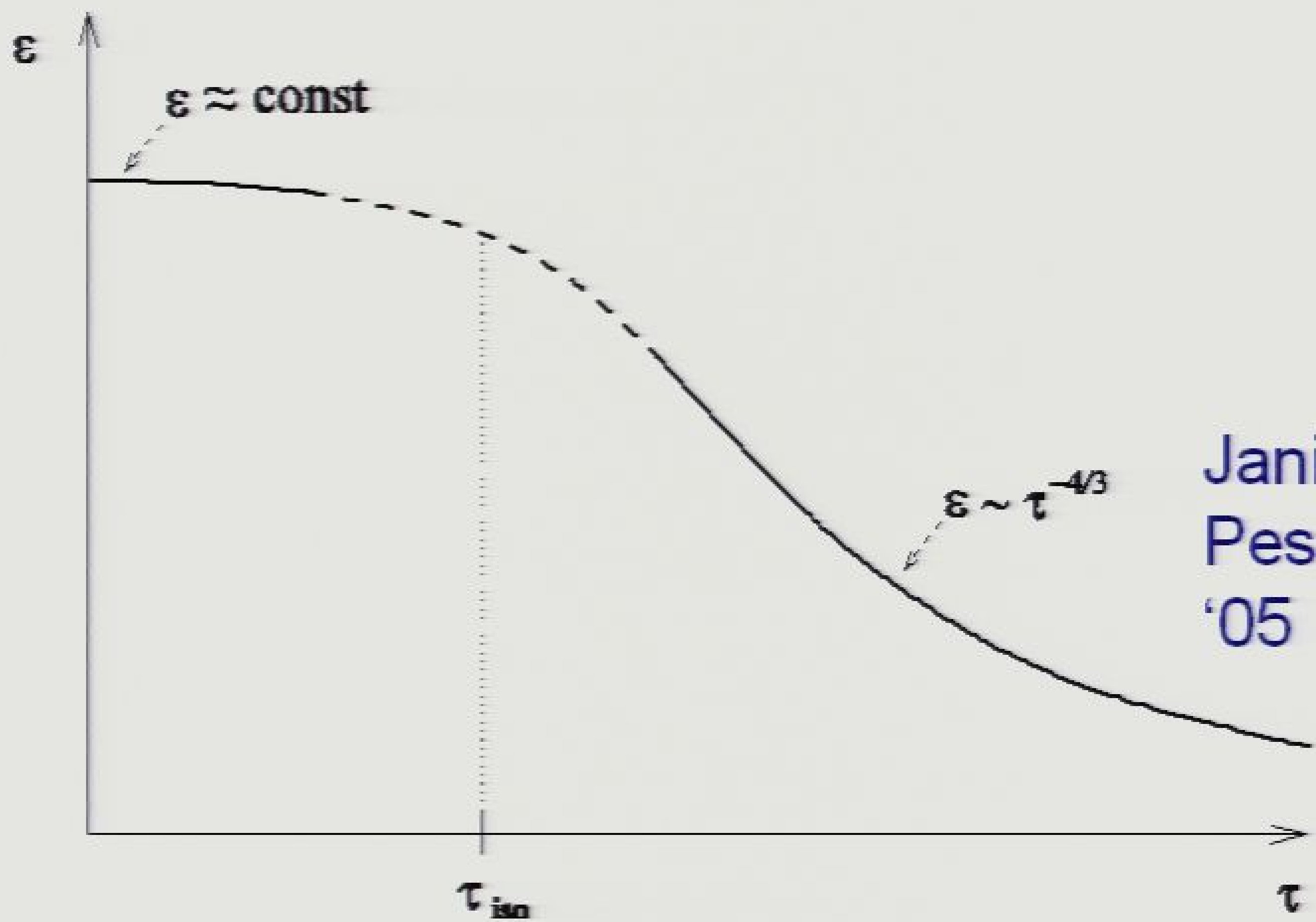

Janik, Peschanski '05

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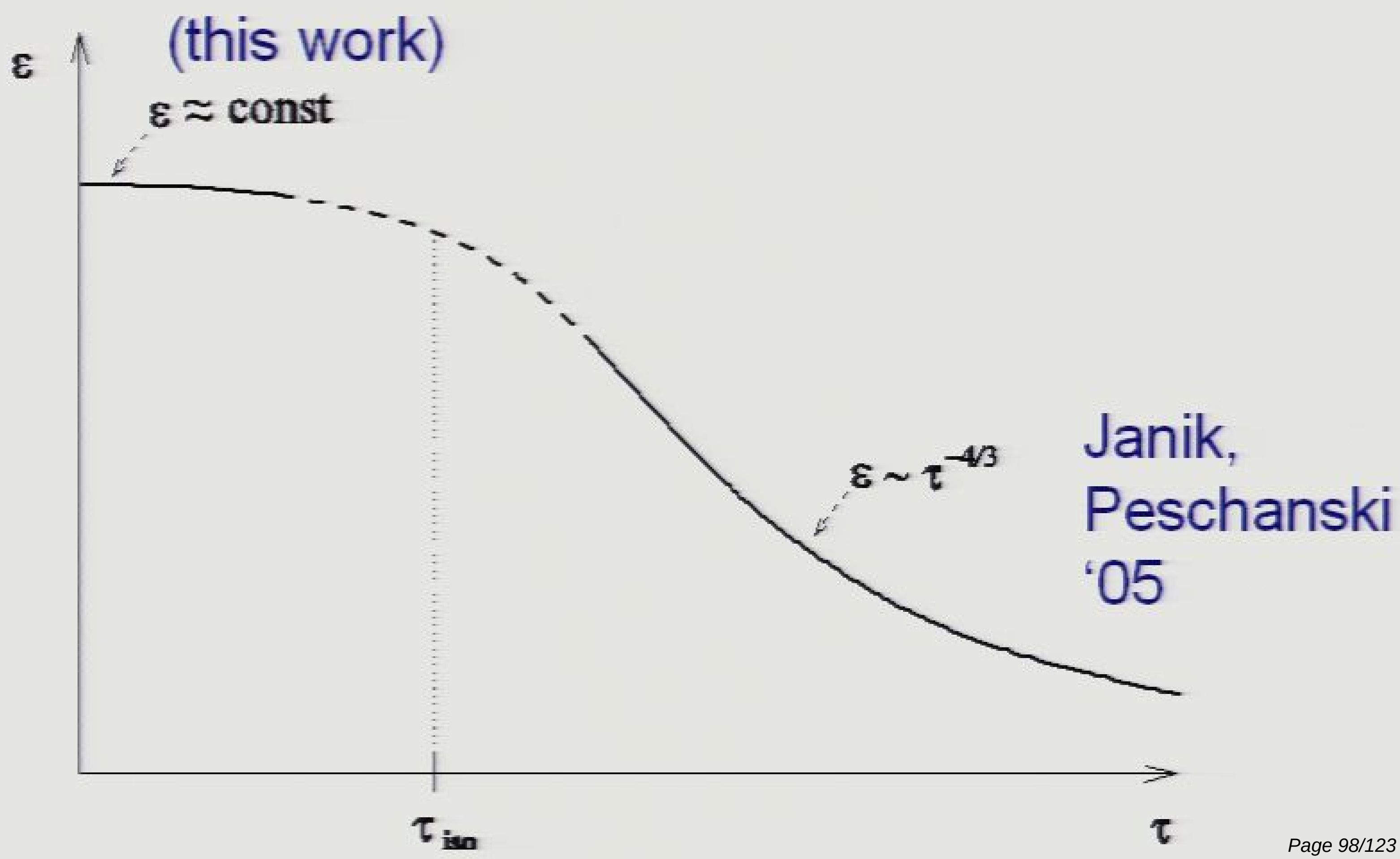

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$$a_2(\tau) = -\frac{1}{384}\left[a_0\,\tau^{\Delta-4}\,(4\,\Delta^2 - \Delta^4) + a_0^2\,\tau^{2\,\Delta}\,8\,(8+8\,\Delta+3\,\Delta^2)\right]$$

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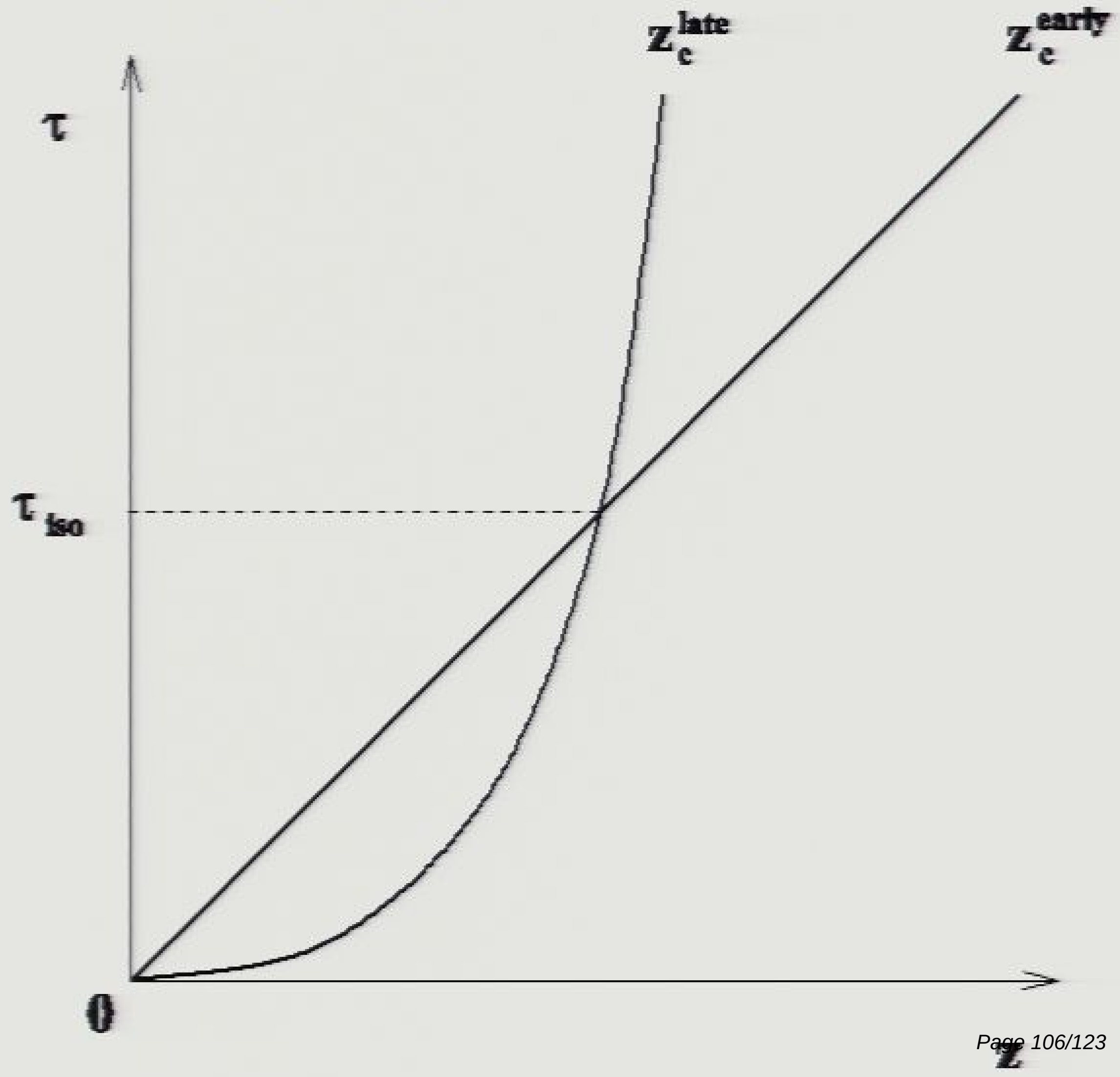
Isotropization Transition: Time Estimate

We plot both branch cuts in the (z, τ) plane:



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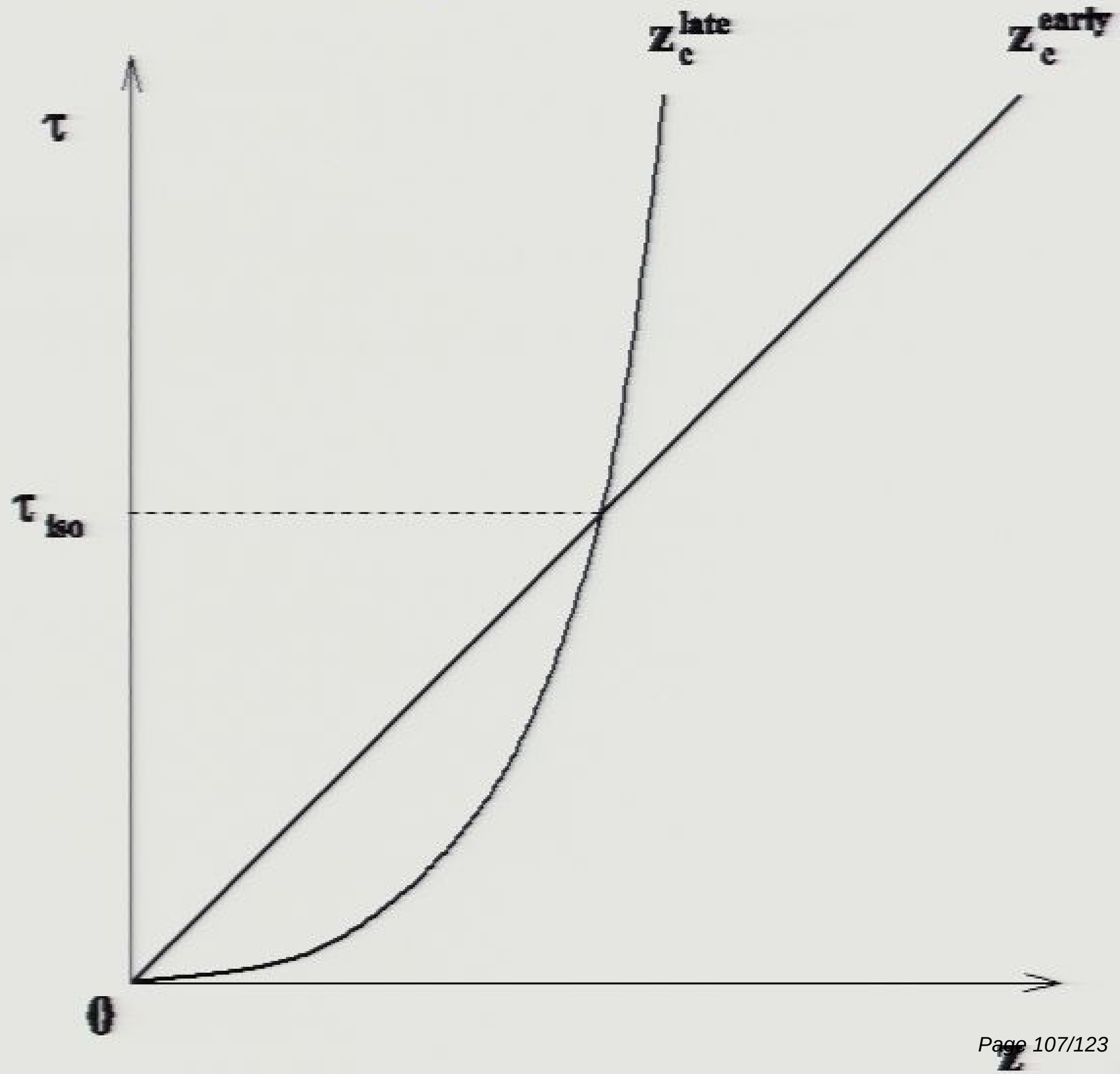

The intercept is at the "isotropization time"

$$\boxed{\tau_{\text{iso}} = \left(\frac{3}{-a_0}\right)^{\frac{3}{8}}}$$

Isotropization Transition: Time Estimate

In terms of more physical quantities we re-write the above estimate as

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in good agreement with hydrodynamics!

Isotropization Transition Estimate: Self-Critique

An AdS/CFT skeptic would argue that our estimate

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is easy to obtain from dimensional reasoning. If one has a conformally invariant theory with $\epsilon(\tau) = \frac{\epsilon_0}{\tau^{\frac{4}{3}}}$, the only scale in the theory is given by ε_0 . Making a scale with dimension of time out of it gives $\tau_{iso} \sim \varepsilon_0^{-3/8}$.

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- Analyzed early-time dynamics and showed that energy density goes to a constant at early times.

- Have therefore shown that isotropization (and hopefully thermalization) takes place in strong coupling dynamics.

- Derived a simple formula for isotropization time and used it for heavy ion collisions at RHIC to obtain 0.3 fm/c, in agreement with hydrodynamic simulations.

Early Time Solution: Terminating the Series

Finally, we see that the hypergeometric series in the solution

$$A(\tau,u) = 1 + a_0\,\tau^{4+\Delta}\,u^4\,F\left(-1-\frac{\Delta}{2},-\frac{\Delta}{2};3;u^2\right),$$

$$B(\tau,u) = 1 + a_0\,\tau^{4+\Delta}\,u^4\left[(\Delta+1)\,F\left(-1-\frac{\Delta}{2},-\frac{\Delta}{2};3;u^2\right)\right.$$
$$\left.-\frac{\Delta\,(\Delta+2)}{6}\,u^2\,F\left(1-\frac{\Delta}{2},-\frac{\Delta}{2};4;u^2\right)\right],$$

$$C(\tau,u) = 1 + a_0\,\tau^{4+\Delta}\,u^4\,\frac{\Delta+2}{12}\left[-6\,F\left(-1-\frac{\Delta}{2},-\frac{\Delta}{2};3;u^2\right)\right.$$
$$\left.+\Delta\,u^2\,F\left(1-\frac{\Delta}{2},-\frac{\Delta}{2};4;u^2\right)\right].$$

terminates only for $\boxed{\Delta = 0}$ in the physically allowed

range of $-1 \le \Delta \le 0$.

Iterative Solution

We begin by expanding the coefficients of the metric

$$ds^2 = \frac{1}{z^2}\left[-A(\tau,z)\,d\tau^2 + \tau^2\,B(\tau,z)\,d\eta^2 + C(\tau,z)\,dx_\perp^2 + dz^2\right]$$

$$A(\tau,z) = e^{a(\tau,z)}, \quad B(\tau,z) = e^{b(\tau,z)}, \quad C(\tau,z) = e^{c(\tau,z)}$$

into power series in z:

$$a(\tau,z) = \sum_{n=0}^{\infty} a_n(\tau)\,z^{4+2n}, \quad b(\tau,z) = \sum_{n=0}^{\infty} b_n(\tau)\,z^{4+2n}, \quad c(\tau,z) = \sum_{n=0}^{\infty} c_n(\tau)\,z^{4+2n}.$$

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