Title: A Puzzle from Dragging Strings

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Abstract:

A Puzzle from Dragging Strings

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Motivation

Applied string theory

- To understand heavy quark physics at RHIC
  string theory $\longrightarrow$ diffusion constant $\longrightarrow$ RHIC observables
- To provide a theoretical laboratory for studying strongly coupled non-abelian plasmas

Herzog, Karch, Kovtun, Kozcaz, Yaffe, JHEP 0607:013, 2006, hep-th/0605158;
Herzog, JHEP 0609:032,2006, hep-th/0605191;
Herzog and Vuorinen, to appear

Outline

1. Review of the quark energy loss calculations for $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM)
2. Taking the next step: Universal features of energy loss in more general backgrounds

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Setting up the $\mathcal{N} = 4$ SYM calculation

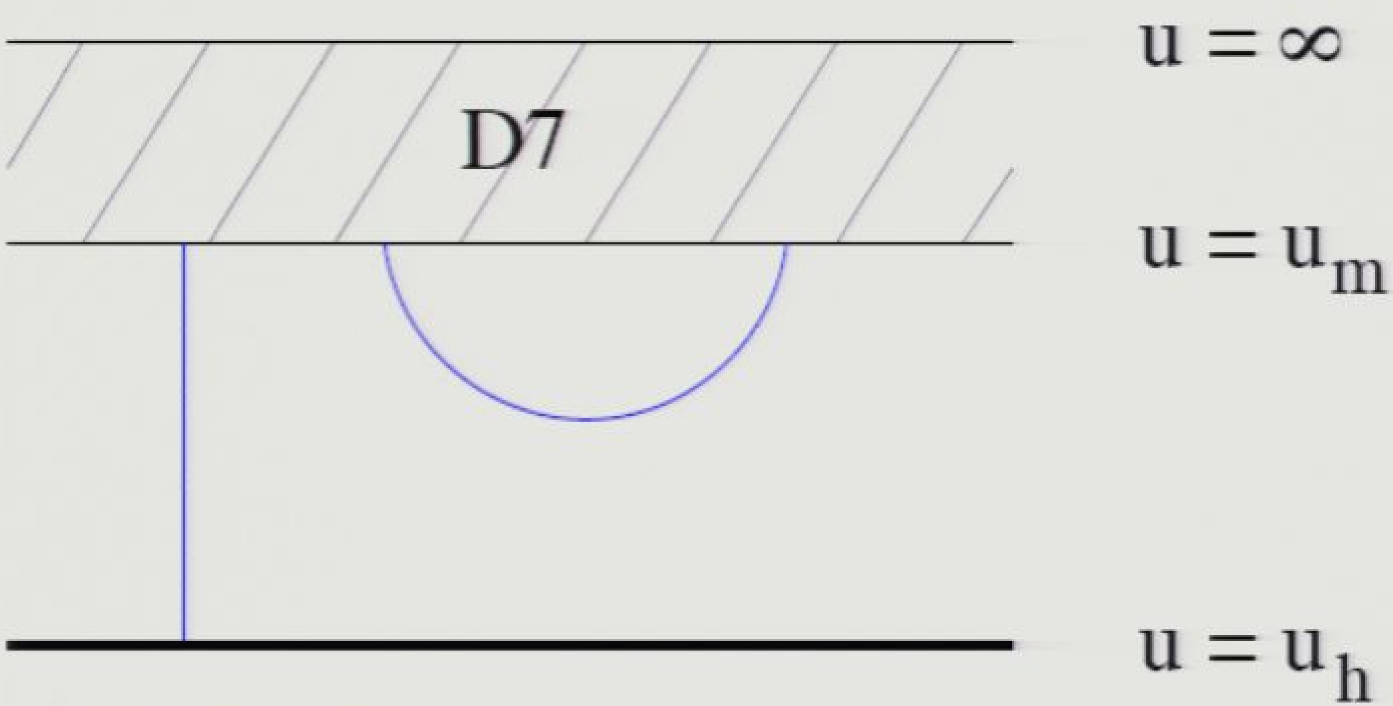
The AdS/CFT correspondence is a strong/weak coupling duality which converts the strongly interacting quark energy loss calculation into classical gravity.

- Statement of the AdS/CFT correspondence

$$\mathcal{N} = 4\ SU(N)\ \text{SYM} \sim \begin{array}{c}\text{type IIB string theory in} \\ AdS_5 \times S^5\end{array}$$

- Adding a black hole to AdS_5 is dual to raising the temperature, $u_h = \pi T$.
- Adding a D7-brane that wraps AdS_5 down to some minimal radius u_m is dual to adding a massive $\mathcal{N} = 2$ hypermultiplet.

The geometric dual picture



Classical strings model single quarks and mesons. The mass of the quark is to first approximation $M \sim u_m - u_h$.

What does our AdS/CFT model say?

Two gedanken experiments

$$\frac{dp}{dt} = -\mu p + f$$

- Hit the quark and watch it slow down

$$p(t) = p(0)e^{-\mu t}$$

- Drag the quark at constant velocity and figure out how much force is needed

$$M\mu = f/v$$

Some technical details

- Our line element for the black hole is

$$ds^2 = L^2\left(\frac{du^2}{h} - hdt^2 + u^2 d\vec{x}^2\right) \text{ where } h = u^2\left(1 - \left(\frac{u_h}{u}\right)^4\right)$$

- The classical string is governed by the action

$$S = -T_0 \int d\sigma d\tau \sqrt{-\det g_{ab}}$$

where g_{ab} is the induced metric on the worldsheet and T_0 is the string tension

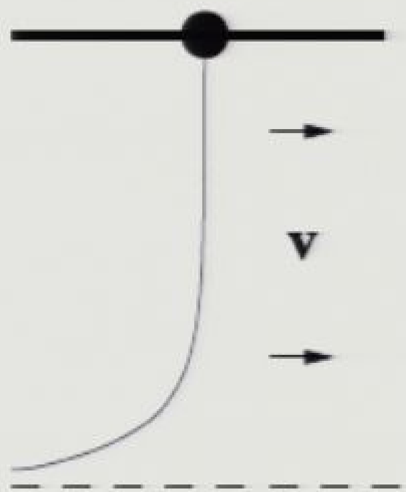
Dragging the string

- There exists an analytic solution corresponding to a single quark moving at constant velocity in response to an electric field.
- This solution has a momentum current

$$\frac{dp}{dt} = -\frac{\pi}{2}\sqrt{\lambda}T^2\frac{v}{\sqrt{1-v^2}}$$

- Assuming a relativistic dispersion relation, one finds

$$\mu = \frac{\pi}{2}\frac{\sqrt{\lambda}T^2}{M_{\text{kin}}}$$



From Strings to Experiment

- This friction coefficient can be converted into a difffusion constant using an Einstein relation (see also Casalderrey-Solana and Teaney).

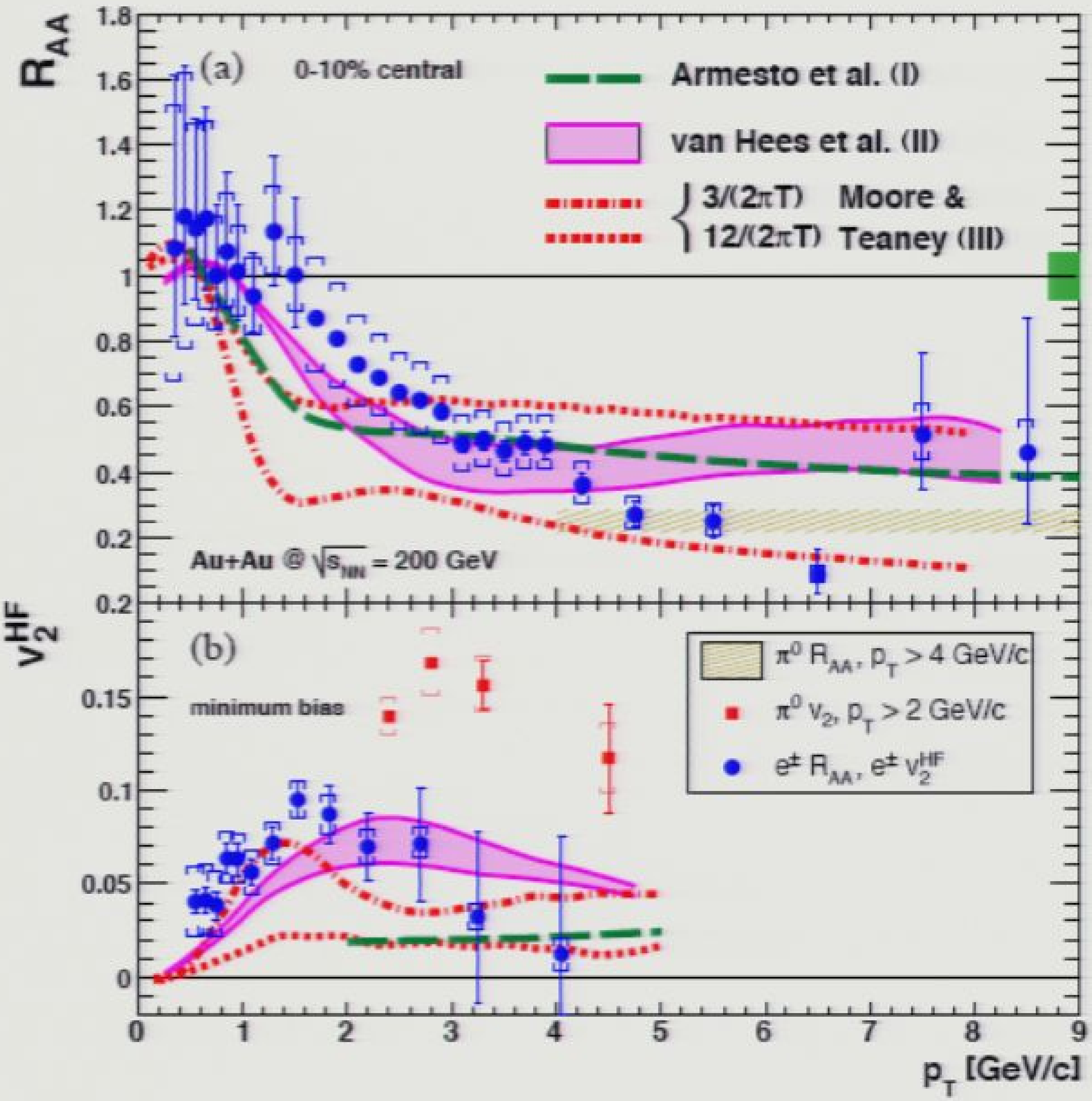
$$D = \frac{T}{\mu M_{\mathrm{kin}}} = \frac{2}{\pi\sqrt{\lambda}T}$$

- Using $\alpha_s \sim 0.5$, we can compute D

$$D_{\mathrm{SYM},\lambda\sim 20} \sim \frac{1}{7T}$$

- We can feed this number into a black box hydrodynamic model of the RHIC collision and see what we get for R_{AA} and v_2.

Data from PHENIX (nucl-ex/0611018)



The Next Step – more general dualities

Consider a space with metric of the form

$$ds^2 = -\alpha(u)dt^2 + \beta(u)du^2 + \gamma(u)\delta_{ij}dx^i dx^j \ .$$

- The metric is asymptotically AdS for $u \gg 1$

$$\alpha \to L^2u^2 \ ; \quad \beta \to \frac{L^2}{u^2} \ ; \quad \gamma \to L^2u^2$$

- The metric has a horizon at $u = u_h$

$$\alpha = (u - u_h)\alpha'(u_h) + \ldots \ ; \quad \frac{1}{\beta} = (u - u_h)\left(\frac{1}{\beta(u_h)}\right)' + \ldots$$

Motivation for such a metric

Ignoring the S^5 directions, metrics of this form dual to

- $\mathcal{N} = 4$ SYM at finite T
- $\mathcal{N} = 4$ SYM at finite T and R-charge chemical potential
- relevant deformations of $\mathcal{N} = 4$ SYM
- speculative AdS/CFT correspondences in $d \neq 4$.

The general 5d dragging string

- Assuming the string is moving a constant speed $\partial_t x(u,t) = v$, the equation of motion reduces to

$$(\partial_u x)^2 = \frac{\beta C^2(-\alpha + \gamma v^2)}{\gamma\alpha(-\gamma\alpha + C^2)}$$

- In order for the string to stretch from the horizon to the asymptotically AdS boundary, we must have

$$-\alpha(u_c) + \gamma(u_c)v^2 = 0 = -\gamma(u_c)\alpha(u_c) + C^2$$

for some critical radius $u = u_c$.

- We can conclude that the force needed to drag the string is

$$\pi_x^1 = \frac{C}{2\pi\alpha'} = \frac{1}{2\pi\alpha'} v\gamma(r_c) \ .$$

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An Example

- An electrically charged black hole in *AdS* has a metric of the general form given above.
- This metric is dual to a field theory at finite R-charge chemical potential.
- This 5d calculation gives a velocity dependent μ.

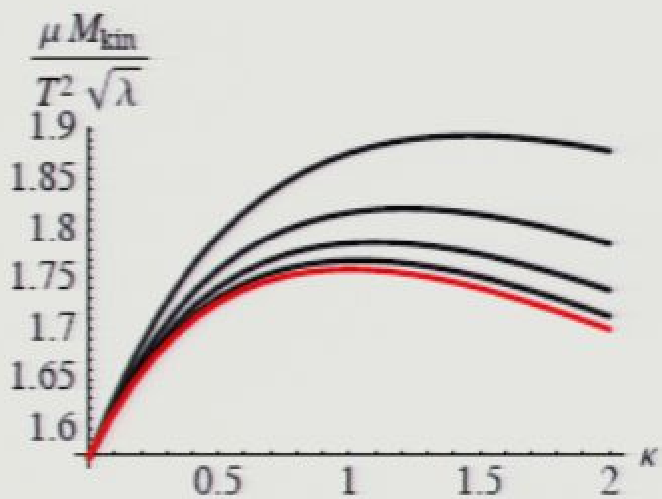

Figure: μ as a function of the "chemical potential" (single charge case) for different values of v. The bottom most curve is the small v limit. As v increases, the damping increases. Also shown from bottom to top are the curves for $v = 0.3, 0.5, 0.7$, and 0.9.

A Puzzle

In hep-th/0605235, Caceres and Guijosa claimed to calculate this same μ for $\mathcal{N} = 4$ SYM at finite R-charge chemical potential, but their result is velocity independent!

One important difference is that CG do the full 10d calculation while I truncated to an effective 5d action.

Two important questions

1. Do my solutions lift to 10d?
2. If so, what is the relation between my solutions and the CG solutions?

Spinning D3-brane background

$$ds_{10}^2 = \sqrt{\Delta} ds_5^2 + \frac{R^2}{\sqrt{\Delta}} \sum_{i=1}^{3} X_i^{-1} \left(d\mu_i^2 + \mu_i^2 (d\phi_i + A^i/R)^2 \right) \ ,$$

where

$$ds_5^2 = -(H_1 H_2 H_3)^{-2/3} h \, dt^2 + (H_1 H_2 H_3)^{1/3} (h^{-1} dr^2 + \frac{r^2}{R^2} d\vec{x}^2) \ ;$$

$$X_i = H_i^{-1} (H_1 H_2 H_3)^{1/3} \ ; \quad A^i = \frac{(1 - H_i^{-1})\sqrt{\nu}}{\ell_i} dt \ ;$$

$$h = -\frac{\nu}{r^2} + \frac{r^2}{R^2} H_1 H_2 H_3 \ ; \quad \Delta = \sum_{i=1}^{3} X_i \mu_i^2 \ ; \quad H_i = 1 + \frac{\ell_i^2}{r^2} \ ;$$

$$\mu_1 = \sin\theta_1 \ ; \quad \mu_2 = \cos\theta_1 \sin\theta_2 \ ; \quad \mu_3 = \cos\theta_1 \cos\theta_2 \ .$$

There is also a RR five-form flux F_5 which does not affect the string dynamics, so we will ignore it.

Toward a resolution

- For the single charge backgrounds, the relevant part of the 10d metric is

$$ds^2 = f[-\alpha dt^2 + \beta du^2 + \gamma dx^2] + \frac{1}{f}\epsilon(d\psi + \phi dt)^2$$

- Take $\dot{x} = v$ *and* $\dot{\psi} = \omega$. Having a string that stretches from the boundary to the horizon now requires.

$$\begin{aligned} 2\pi\alpha'\pi^1_x &= vf(u_c)\gamma(u_c) \\ 2\pi\alpha'\pi^1_\psi &= \frac{\epsilon(u_c)}{f(u_c)}(\phi(u_c) + \omega) \end{aligned}$$

where

$$f^2(-\alpha + v^2\gamma) + \epsilon(\phi + \omega)^2\Big|_{r=r_c} = 0$$

- The Caceres and Guijosa strings have $\omega = 0$ and hence nonzero π^1_ψ!

Strings with no torque

- Let's impose the condition $\pi^1_\psi = 0$.
- Then the force required to drag the string becomes

$$\pi^1_x = \frac{1}{2\pi\alpha'} v f(u_c)\gamma(u_c)$$

where

$$\alpha(u_c) - v^2\gamma(u_c) = 0$$

is the same condition that u_c had to satisfy for the 5d solutions!

- π^1_x is equal to the force in the 5d case if $f(u_c) = 1$.
- Conclusion: My 5d solutions uplift to strings with no torque in the case $f(u_c) = 1$; caveat.

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Digression on string frame vs. Einstein frame

- For these R-charged backgrounds, there is a canonical choice of effective 5d metric, and for this choice $f(u_c)$ is not generically equal to one.
- However, in several cases, it is possible to remedy this problem by multiplying the metric by an overall function of u.
- The effective 5d metric has three dilaton like scalar fields; this multiplication is like converting between string and Einstein frame in ten dimensions.
- One case where $f(u_c) = 1$ is where all the R-charges are equal and the scalar fields have vanishing expectation value.

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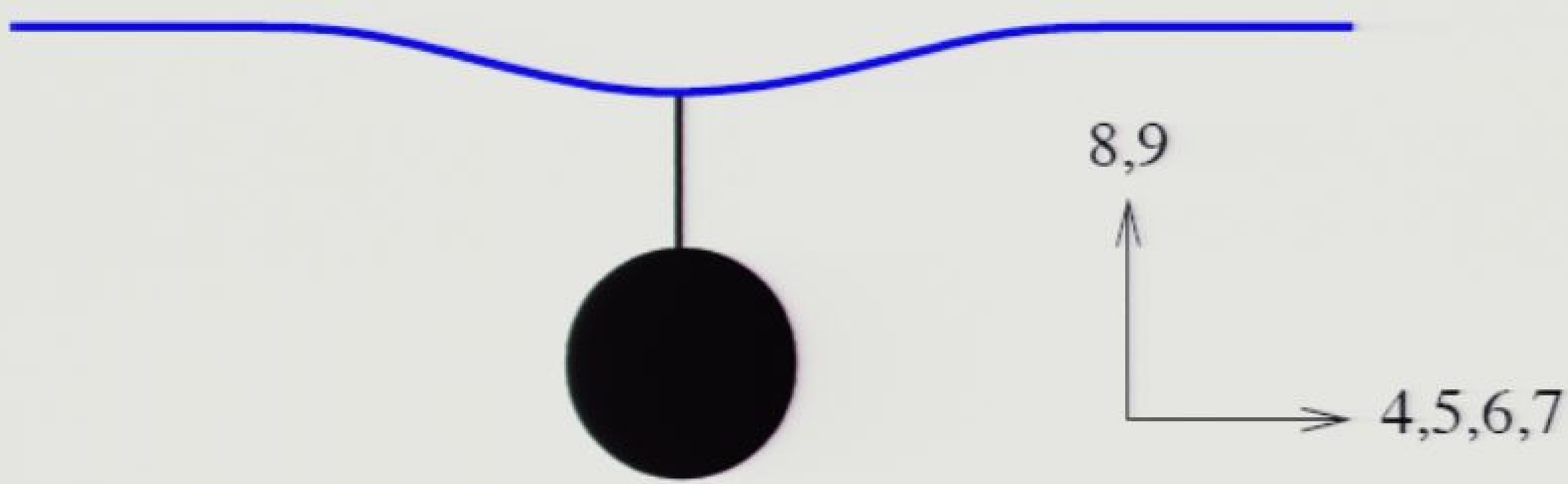
Which strings are better?

These strings have different boundary conditions at large u and are not comparable.

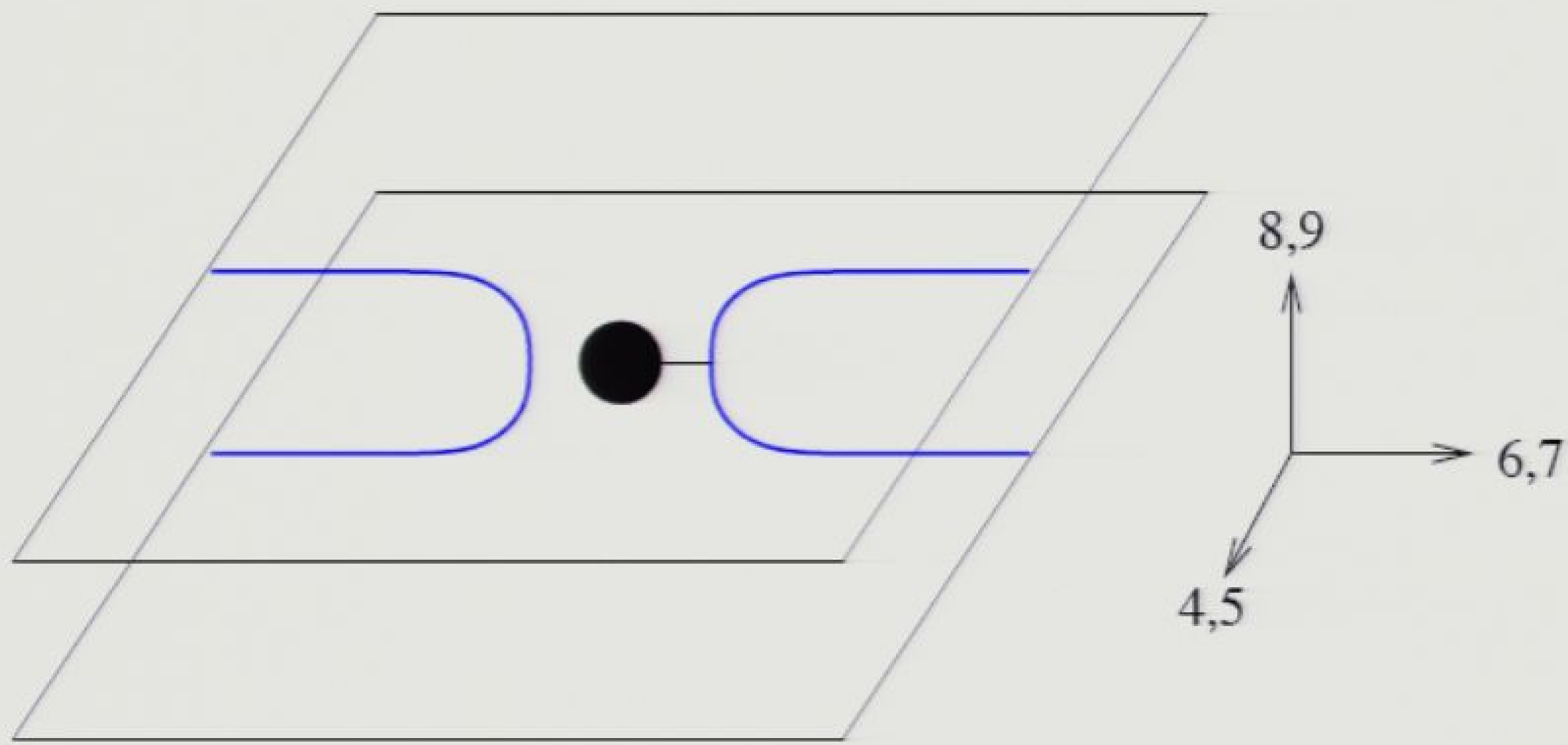
- The Caceres and Guijosa strings are not spinning and require forces applied in both the x and ψ directions.
- My strings are spinning but require no force in the ψ direction.

D7-branes for CG

- A finite T and R-charge chemical potential analog of the D7-brane in AdS_5 was studied by Albash, Filev, Johnson, Kundu, hep-th/0605175.
- At its locus of closest approach to the horizon, the D7-brane is point-like in the squashed S^5.
- The strings of Caceres and Guijosa can end consistently on such a D7-brane.



A proposal for D7-branes for my strings



Some preliminary details

- At its locus of closest approach to the horizon, the D7-brane is an S^3 in the squashed S^5.
- My strings can spin around this S^3.
- At $T = 0$ and zero chemical potential, the profile for these D7-branes is

$$\xi = \int_C^x \frac{C^3 dx}{\sqrt{x^6 - C^6}} + \xi_0 ,$$

  C is a chiral condensate, ξ_0 the mass term.
- At $T = 0$ and zero chemical potential, this D7-brane is higher in energy than a pair of the old D7's

$$\Delta V = \frac{C^4 \sqrt{\pi}\Gamma(-2/3)}{6\Gamma(-1/6)} > 0$$

  but maybe the story changes for $T > 0$ and $\kappa > 0$ or in other backgrounds, maybe metastable.

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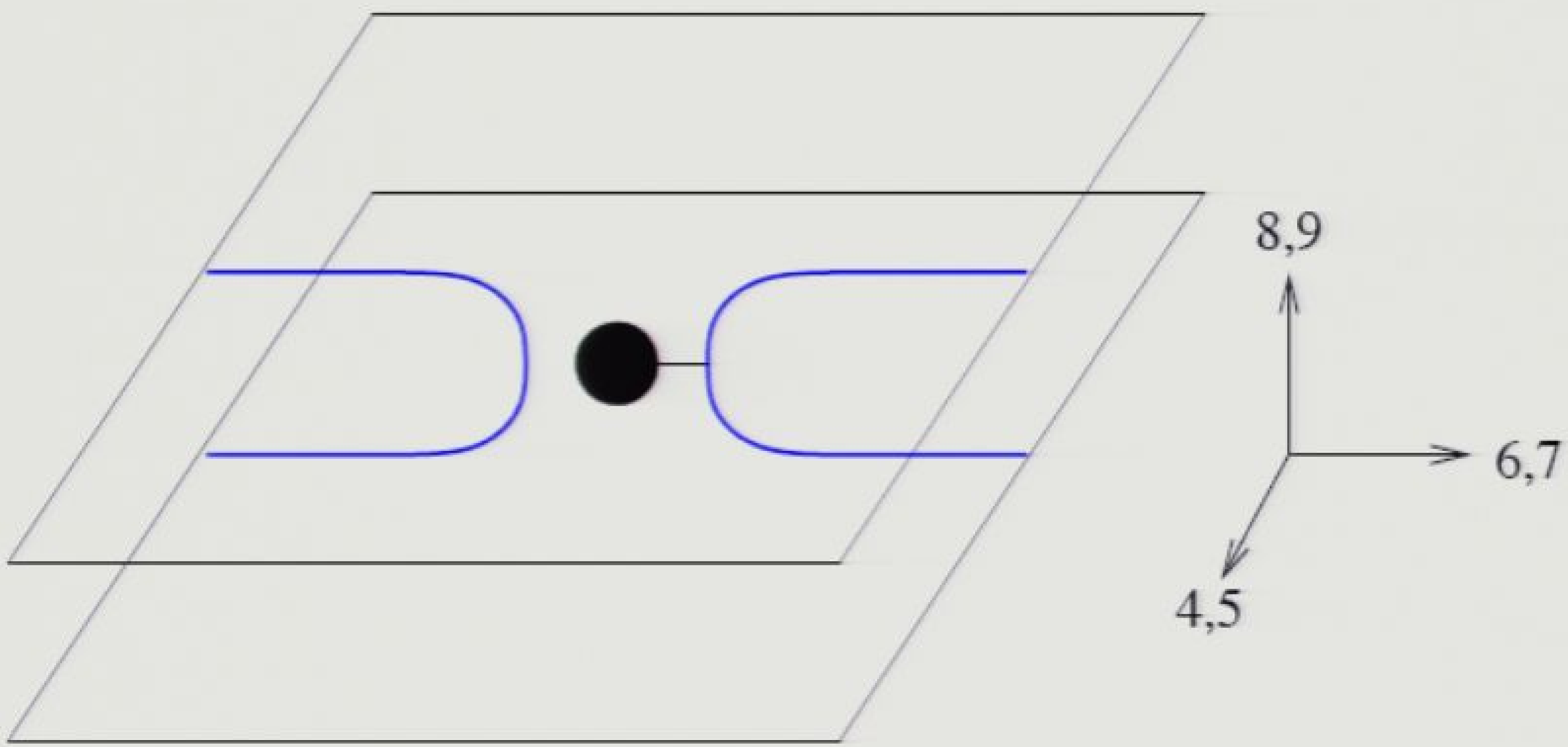

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Things to do

1. Clarify the interpretation of these D7-branes in terms of adding $\mathcal{N} = 2$ hypermultiplets to $\mathcal{N} = 4$ SYM.
2. Stability issues, both for D7-brane embeddings and these spinning strings.
3. ...

The transverse "S^5" directions have important physical effects on dragging strings in more complicated supergravity backgrounds.

Toward a resolution

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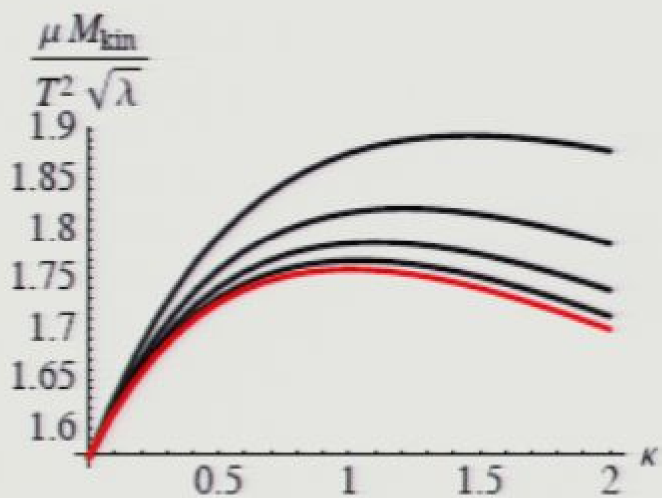

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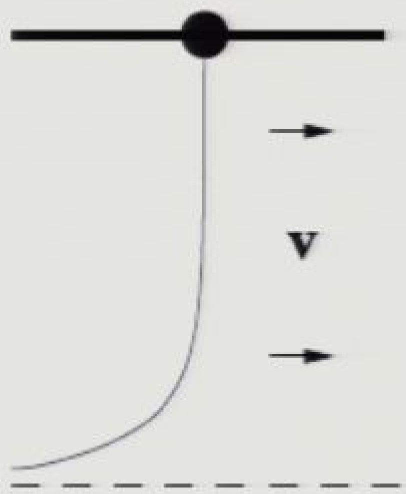

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