

Title: Bottom-up Approach to AdS / QCD

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Abstract:

# AdS/QCD

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## Motivation and plan

- Large  $N_c$ :
  - planar diagrams dominate
  - resonances are infinitely narrow
- Effective theory in terms of resonances is weakly coupled?
- What is this effective theory?
- String theory explicit examples suggest it is a 5d effective theory.
- QCD/phenomenology hint:  $\rho, \rho', \rho'', \dots$  – resonant modes in a cavity (KK)?
- Bottom-up approach (AdS from QCD vs QCD from AdS).

A holographic model: [Erlich, Katz, Son and MS; Da Rold, Pomarol](#)

- ABC of AdS/CFT (holography)
  - A simple model – generic features:
    - chiral symmetry breaking
    - quark-hadron duality, sum rules
    - VMD, etc.
  - Excited meson spectrum:  $m^2 \sim n + S$
- This talk:

# Holographic correspondence: formulation

- Begin with  $S_4[G, q] = \int d^4x \mathcal{L}[G, q]$ .

Generating functional for correlators of an operator  $\mathcal{O}$  (examples of  $\mathcal{O}$ :  $G_{\mu\nu}^a G^{\mu\nu a}$ ,  $\bar{q}q$ ,  $\bar{q}\gamma^\mu t^a q$ , ...):

$$Z_4[\phi_0(x)] = \int \mathcal{D}[G, q] \exp \left\{ iS_4 + i \int_{x^4} \phi_0 \mathcal{O} \right\}.$$

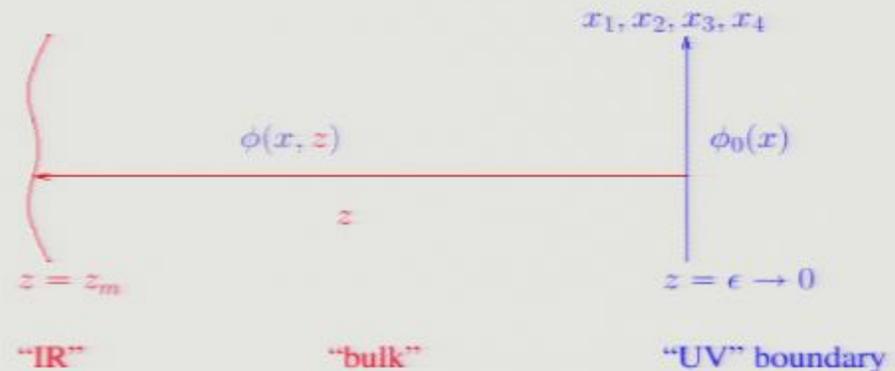
- 5d bulk metric:

$$ds^2 = z^{-2} (-dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu).$$

$$\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1).$$

(Note:  $x^m \rightarrow \lambda x^m$ ).

AdS "slice"



- 

$$Z_5[\phi_0(x)] = \int_{\phi(x, \epsilon) = \phi_0(x)} \mathcal{D}[\phi] e^{iS_5[\phi]}$$

$$Z_4 = Z_5$$

(Generating functnl) [4d sources  $\phi_0(x)$ ] = (Effective action) [fields  $\phi_0(x)$ ].

## Example: conserved current

- Let  $\mathcal{O}$  be a current:  $J^{\mu a} = \bar{q}\gamma^\mu t^a q$ .  
 $\phi_0$ : source for  $J^{\mu a}$  is a vector potential  $V_0^{\mu a}$ . I.e.,

$$Z_4[V] = \int \mathcal{D}[G, q] \exp \left\{ iS_4 + i \int_{x^4} V_0 \cdot J \right\}.$$

- We shall look at

$$\int d^4x e^{iqx} \langle J^{\mu a}(x) J^{\nu b}(y) \rangle = \delta^{ab} (q^\mu q^\nu - q^2 \eta^{\mu\nu}) \Pi(-q^2).$$

In QCD, scale invariance in the UV means  $\Pi(Q^2) \sim \ln(Q^2)$ .

- 5d action for  $V_m^a$ ? Let us take

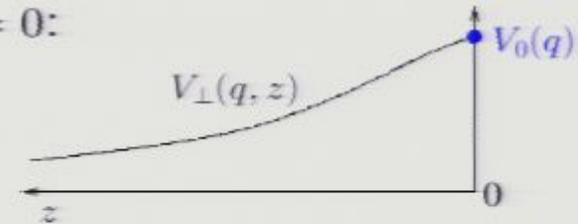
$$S_5 = -\frac{1}{4g_5^2} \int d^5x \sqrt{g} V_{mn}^a V^{amn}.$$

- At tree level, we need to minimize  $S_5$  wrt  $V$  with the b.c.  $V(x, \epsilon) = V_0(x)$ .  
 Then take 2 variational derivatives wrt  $V_0(x)$  and  $V_0(y)$  to find  $\langle J(x) J(y) \rangle$ .

## Example: current (contd)

- $V_5 = 0$  gauge; linearize; Fourier  $x^\mu \rightarrow q^\mu$ ;  $q_\mu V_\perp^\mu = 0$ :

$$\partial_z \left( \frac{1}{z} \partial_z V_\perp \right) + \frac{q^2}{z} V_\perp = 0.$$



- Action to quadratic order in  $V$ , on eqs. of motion (int. by parts):

$$S_5 = -\frac{1}{2g_5^2} \int d^4x \frac{1}{z} V_\mu^a \partial_z V^{a\mu} \Big|_{z=\epsilon}.$$

- Let  $V(q, z)$  be a solution with  $V(q, \epsilon) = 1$ . We need  $V_\perp(q, z) = V_0(q)V(q, z)$ .

$$\Pi(Q^2) = -\frac{1}{g_5^2} \frac{1}{Q^2} \frac{\partial_z V(q, z)}{z} \Big|_{z=\epsilon}. \quad (Q^2 \equiv -q^2)$$

$$V(q, z) = (Qz)K_1(Qz) = 1 + \frac{Q^2 z^2}{2} \ln(Qz) + \mathcal{O}(z^2).$$

Thus

$$\Pi(Q^2) = -\frac{1}{g_5^2} \ln Q + \text{contact terms}$$

## 5d coupling and $N_c$

In QCD



$$\Pi(Q^2) = -\frac{N_c}{24\pi^2} \ln Q^2 + \dots$$

In AdS<sub>5</sub>:

$$\Pi(Q^2) = -\frac{1}{2g_5^2} \ln Q^2 + \dots$$

Thus

$$g_5^2 = \frac{12\pi^2}{N_c}$$

Large  $N_c$   $\Leftrightarrow$  small coupling

## Example: current (endnotes)

- The role of 5d gauge inv:

$$\partial_z \left( \frac{1}{z} \partial_z V_{\parallel} \right) = 0 \quad \Rightarrow \quad \frac{\delta S_{5,\text{eff}}}{\delta (V_0)_{\parallel}} = 0 \quad \text{corresponds to} \quad \partial_{\mu} J^{\mu} = 0.$$

- $\log Q^2$  in UV  $\leftarrow$  scale inv. of the 5d theory near  $z = 0$ .

$$[x^m] = -1; \quad [g_{mn}] = [z^{-2}] = 2 :$$

$$S_5 = -\frac{1}{4g_5^2} \int \underbrace{d^5 x \sqrt{g}}_0 \underbrace{V_{mn}^a V^{amn}}_0$$

$$[g_5] = 0$$

# Dictionary

4d	$\leftrightarrow$	5d
generating func. $W_4$	$\leftrightarrow$	eff. action $S_{5,\text{eff}}$
operator $\mathcal{O}(x)$ ( $\phi_0$ – source)	$\leftrightarrow$	field $\phi(x, z)$ ( $\phi_0$ – boundary value)
scale invariance ( $\log Q$ )	$\leftrightarrow$	scale invariance
$\partial_\mu J^\mu = 0$	$\leftrightarrow$	gauge invariance
large $N_c$	$\leftrightarrow$	small $g_5$
large $Q$	$\leftrightarrow$	small $z$
dimension of $\mathcal{O}$	$\leftrightarrow$	mass of $\phi$

## Dimension of 4d operator and 5d mass

$$\mathcal{L}_5 = \frac{1}{2} \sqrt{g} (g^{mn} \partial_m \phi \partial_n \phi - m_5^2 \phi^2)$$

$z \rightarrow 0$  (at fixed  $q$ , i.e.,  $qz \ll 1$ ) extremum satisfies

$$\partial_z (z^{-3} \partial_z \phi) - z^{-5} m_5^2 \phi = 0$$

$$\phi \sim z^{\Delta_\phi} \quad \text{with} \quad (\Delta_\phi - 4) \Delta_\phi - m_5^2 = 0.$$

$$\underline{\phi z^{-\Delta_\phi} \rightarrow \text{const} = \phi_0} \quad - \quad \text{the source for } \mathcal{O}$$

$$[\phi] = 0 \quad \Rightarrow \quad [\phi_0] = +\Delta_\phi \quad ([x] = -1)$$

$$\text{Thus } [\mathcal{O}] = 4 - \Delta_\phi \equiv \Delta_{\mathcal{O}} \quad \text{and} \quad m_5^2 = \Delta_{\mathcal{O}}(\Delta_{\mathcal{O}} - 4)$$

For example,

$$T_\mu^\mu \quad : \quad m_5^2 = 0; \quad \bar{\psi} \psi \quad : \quad m_5^2 = -3;$$

$$J^\mu \quad : \quad m_5^2 = (\Delta - 3)(\Delta - 1) = 0 \quad - \quad \text{protected.}$$

## Spontaneous symmetry breaking



$$S_5 = \frac{1}{2} \int d^5x \sqrt{g} g^{mn} \partial_m \phi \partial_n \phi + \dots$$

with b.c. at  $z = 0$ :  $\phi z^{-\Delta_\phi} = \phi_0$ . The extremum is a linear combination:

$$\phi_{\text{sol}} = \phi_0 z^{\Delta_\phi} + A z^{\Delta_\mathcal{O}} \quad (\Delta_\phi + \Delta_\mathcal{O} = 4).$$



Vary the source:  $\phi_0 \rightarrow \phi_0 + \delta\phi_0$ :

Freedman et al, Klebanov-Witten

$$\delta S_5 = \int d^4x z^{-3} \delta\phi \partial_z \phi \Big|_{z=0} + \dots = (\Delta_\mathcal{O} - \Delta_\phi) \int d^4x \delta\phi_0 A$$



Compare to  $W_4$ :

$$\delta W_4 = \int d^4x \delta\phi_0 \langle \mathcal{O} \rangle$$

Therefore

$$A = \frac{1}{2\Delta_\mathcal{O} - 4} \langle \mathcal{O} \rangle$$

$A \rightarrow 0$  as  $\phi_0 \rightarrow 0$ : spontaneous symmetry breaking

## The model

4D: $\mathcal{O}(x)$	5D: $\phi(x, z)$	$p$	$\Delta_{\mathcal{O}}$	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A_{L\mu}^a$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A_{R\mu}^a$	1	3	0
$\bar{q}_R^\alpha q_L^\beta$	$(2/z) X^{\alpha\beta}$	0	3	-3

$$S = \int_0^{z_m} d^5x \sqrt{g} \text{Tr} \left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2} (F_L^2 + F_R^2) \right\}$$

Symmetries:  $X \rightarrow LX R^\dagger$ ,  $F_L \rightarrow LF_L L^\dagger$ ,  $F_R \rightarrow RF_R R^\dagger$ .

Boundary conditions at  $z = z_m$ :  $F_{z\mu} = 0$ .

### Chiral symmetry breaking:

$$X_0(z) = \frac{1}{2} M z + \frac{1}{2} \Sigma z^3.$$

Matching to QCD:  $\Sigma^{\alpha\beta} = \langle \bar{q}^\alpha q^\beta \rangle$ . We take  $M = m_q \mathbf{1}$  and  $\Sigma = \sigma \mathbf{1}$ .

### Four free parameters: $m_q$ , $\sigma$ , $z_m$ and $g_5$ .

Compared to three in QCD:  $m_q$ ,  $\Lambda_{\text{QCD}}$  and  $N_c$ .

## Hadrons and QCD sum rule

- Now consider  $q^2 > 0$ :

$$\partial_z \left( \frac{1}{z} \partial_z V_{\perp} \right) + \frac{q^2}{z} V_{\perp} = 0.$$

Normalizable modes:  $\psi_{\rho}(\epsilon) = 0$ ,  $\partial_z \psi_{\rho}(z_m) = 0$ ,  $\int (dz/z) \psi_{\rho}(z)^2 = 1$ .  
Exist only for discrete values of  $q^2 = m_{\rho}^2 > 0$ ,  
where the  $V(q, \epsilon) = 1$  solution is thus not unique.

- Expanding  $V(q, z) = \sum_{\rho} a_{\rho}(q) \psi_{\rho}(z)$ ,

$$\Pi_V(-q^2) = -\frac{1}{g_5^2} \frac{1}{Q^2} \frac{\partial_z V(q, z)}{z} \Big|_{z=\epsilon} = -\frac{1}{g_5^2} \sum_{\rho} \frac{[\psi'_{\rho}(\epsilon)/\epsilon]^2}{(q^2 - m_{\rho}^2) m_{\rho}^2}.$$

$$\text{Decay constants: } F_{\rho} = \frac{1}{g_5} \frac{\psi'_{\rho}(\epsilon)}{\epsilon}.$$

- Automatically satisfy QCD sum rule:

$$\Pi_V(Q^2) = -\frac{1}{2g_5^2} \ln Q^2 + \dots$$

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# Chiral symmetry breaking and pion mass

$$m_\pi^2 f_\pi^2 = 2m_q \sigma + \mathcal{O}(m_q^2)$$

- with  $A = (A_L - A_R)/2$ ,  $A_\mu = A_{\mu\perp} + \partial_\mu \varphi$ ,  $v(z) = m_q z + \sigma z^3$

$$\delta A_\perp : \quad \partial_z (z^{-1} \partial_z A_\perp) + z^{-1} q^2 A_\perp - z^{-3} g_5^2 v^2 A_\perp = 0;$$

$$\Pi_A(-q^2) \xrightarrow{q \rightarrow 0} \frac{f_\pi^2}{-q^2}. \quad \text{As for } V: A(q, \epsilon) = 1. \quad f_\pi^2 = -\frac{1}{g_5^2} \frac{\partial_z A(0, z)}{z} \Big|_{z=\epsilon}.$$

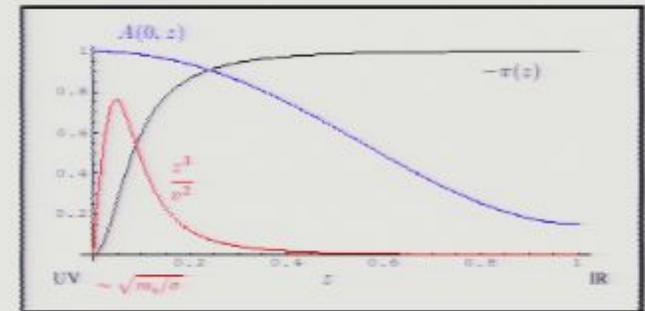
- with  $X = X_0 \exp(i2\pi^a t^a)$

$$\delta A_\parallel : \quad \partial_z (z^{-1} \partial_z \varphi) + z^{-3} g_5^2 v^2 (\pi - \varphi) = 0;$$

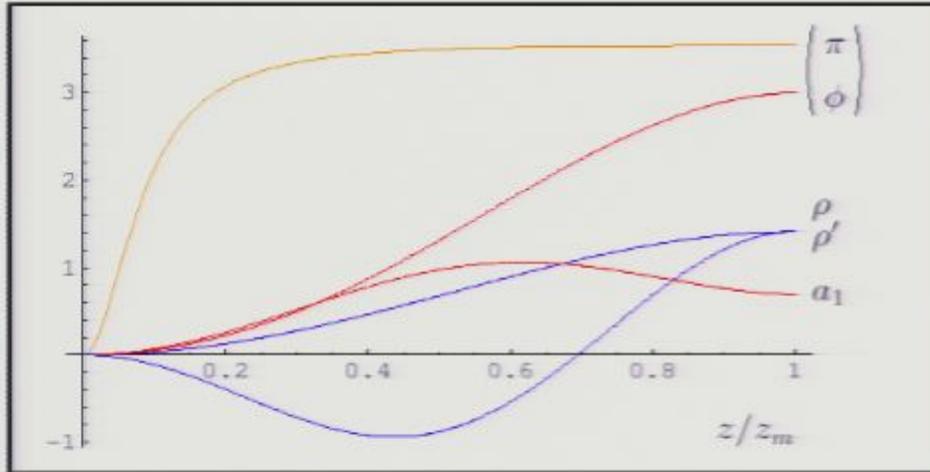
$$\delta A_z : \quad -q^2 \partial_z \varphi + z^{-2} g_5^2 v^2 \partial_z \pi = 0.$$

For  $q \rightarrow 0$ :  $\varphi(z) = A(0, z) - 1$ ,  $\pi(z) = -1$ .  
 $\phi(0) = 0$ , but  $\pi(0) = 0$  requires  $q \neq 0$  and

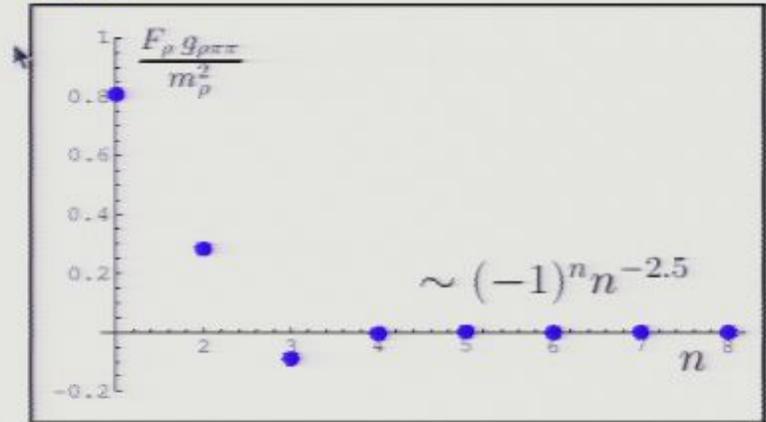
$$\pi(z') = q^2 \int_0^{z'} dz \frac{z^3}{v(z)^2} \cdot \frac{1}{g_5^2 z} \partial_z A(0, z) = -q^2 f_\pi^2 \frac{1}{2m_q \sigma} = -1 \quad \text{for } z \gg \sqrt{m_q/\sigma}.$$



# Meson wavefunctions and VMD

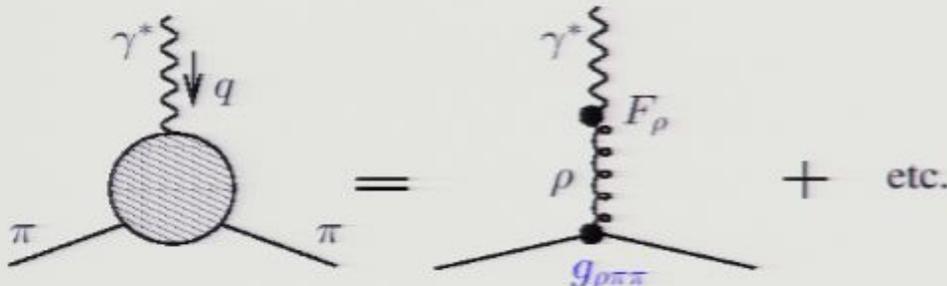


## VMD



Couplings:

$$g_{\rho\pi\pi} = g_5 \int dz \psi_\rho(z) \underbrace{\left( \frac{\phi'^2}{g_5^2 z} + \frac{v^2 (\pi - \phi)^2}{z^3} \right)}_{|\text{pion}|^2 \leftarrow \text{no nodes}}$$



Isospin sum rule for  $\pi$  formfactor:

$$G(q^2 = 0) = \sum_\rho \frac{F_\rho g_{\rho\pi\pi}}{m_\rho^2} = 1$$

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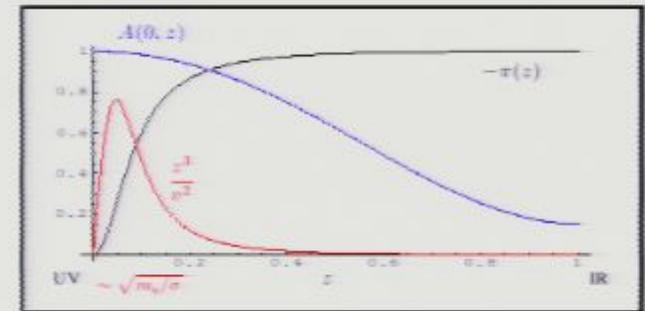
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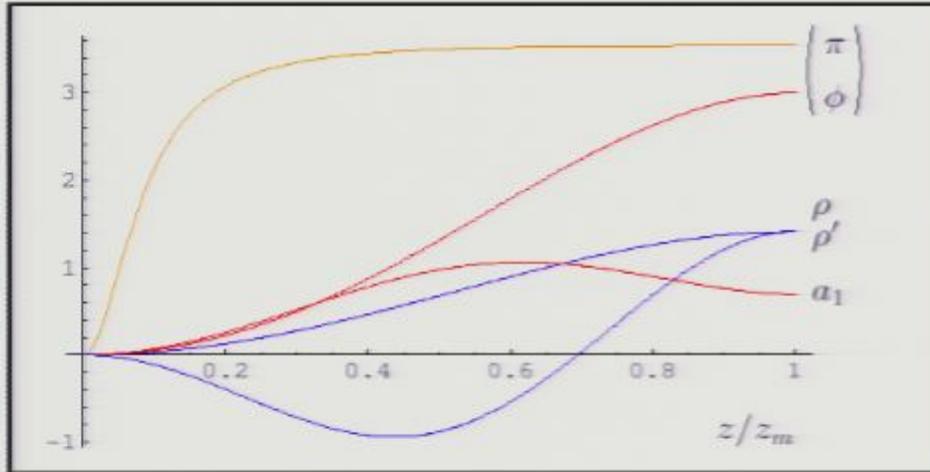
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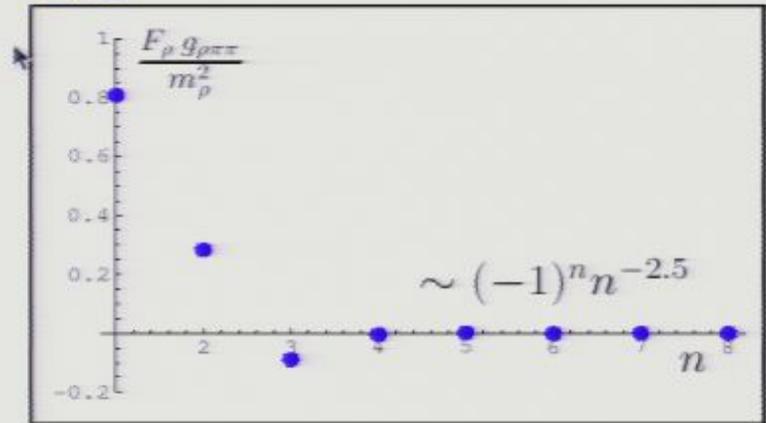
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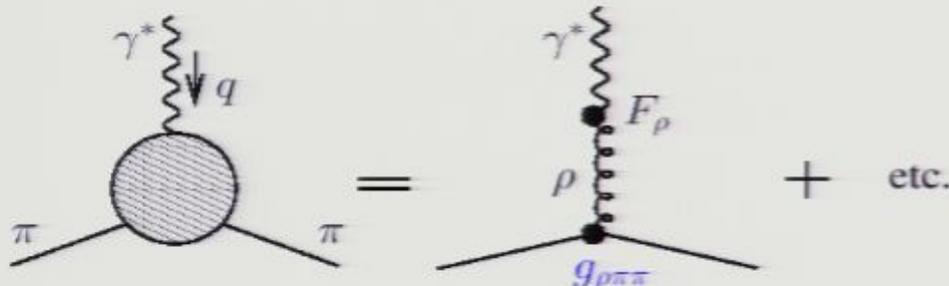


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## Comparison with experiment

Observable	Measured	Model	Units
$m_\pi$	$139.6 \pm 0.0004$	$139.6^*$	MeV
$m_\rho$	$775.8 \pm 0.5$	$775.8^*$	MeV
$m_{a_1}$	$1230 \pm 40$	1363	MeV
$f_\pi$	$92.4 \pm 0.35$	$92.4^*$	MeV
$F_\rho^{1/2}$	$345 \pm 8$	329	MeV
$F_{a_1}^{1/2}$	$433 \pm 13$	486	MeV
$g_{\rho\pi\pi}$	$6.03 \pm 0.07$	4.48	—
$m_{\rho'}$	$1465 \pm 25$	1450	MeV

\* fitted

- $N_c \Rightarrow g_5 = \sqrt{12\pi^2/N_c} = 2\pi.$
- $m_\rho = 2.405/z_m \Rightarrow z_m = (323 \text{ MeV})^{-1}.$
- $f_\pi$  and  $m_\pi \Rightarrow \sigma = (327 \text{ MeV})^3$  and  $m_q = 2.29 \text{ MeV}.$

## Linear confinement: $m_n^2 \sim n$

- Problem for high  $n$ :  $m_n^2 \sim n^2$ .
- High  $n$  mesons are large  $\Rightarrow$  IR regime. Thus high  $n \Leftrightarrow$  large  $z$ .

- $$I = \int d^5x \sqrt{g} e^{-\Phi} \mathcal{L},$$
 A. Karch et al.

Instead of hard cutoff at  $z_m$  —  $\Phi \rightarrow z^2$  at large  $z$

- Mode equation

$$\partial_z \left( e^{-B} \partial_z v_n \right) + m_n^2 e^{-B} v_n = 0,$$



$B \equiv \Phi(z) - A(z)$ , with  $e^{A(z)} = z^{-1}$  from  $ds^2 = e^{2A}(dx \cdot dx - dz^2)$ .

Substitute  $v_n = e^{B/2} \psi_n$

$$-\psi_n'' + V(z)\psi_n = m_n^2 \psi_n, \quad V(z) = \frac{1}{4}(B')^2 - \frac{1}{2}B''.$$

- With  $\Phi = z^2$ :  $V = z^2 + \frac{3}{4z^2}$  — 2d harmonic oscillator (radial eqn.,  $m = 1$ ).

$$m_n^2 = 4(n+1)$$

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## Higher spins

$$I = \frac{1}{2} \int d^5x \sqrt{g} e^{-\Phi} \left\{ \nabla_N \phi_{M_1 \dots M_S} \nabla^N \phi^{M_1 \dots M_S} + M^2(z) \phi_{M_1 \dots M_S} \phi^{M_1 \dots M_S} + \dots \right\}$$

Gauge inv.:  $\delta \phi_{M_1 \dots M_S} = \nabla_{(M_1} \xi_{M_2 \dots M_S)}$ .

In the axial gauge  $\phi_{z \dots} = 0$ :  $\delta \phi_{z \dots} = \nabla_z \xi_{\dots} + \nabla_{(\dots} \xi_{\dots)z} = \xi'_{\dots} - 2(S-1)A' \xi_{\dots} = 0$ .

and there is residual gauge inv.:  $\xi(x, z)_{\dots} = e^{2(S-1)A(z)} \tilde{\xi}_{\dots}(x)$

under which a rescaled field  $\tilde{\phi}_{\dots} = e^{-2(S-1)A} \phi_{\dots}$  just shifts:  $\delta \tilde{\phi}_{\dots} = \partial_{(\dots} \tilde{\xi}_{\dots)}$

Thus residual shift requires

$$I = \frac{1}{2} \int d^5x e^{5A} e^{-\Phi} \left\{ e^{4(S-1)A} e^{-2A(1+S)} \partial_N \tilde{\phi}_{\mu_1 \dots \mu_S} \partial^N \tilde{\phi}_{\mu_1 \dots \mu_S} \right\}.$$

which means  $B = \Phi - (2S-1)A$  and  $V = z^2 + \frac{S^2-1/4}{z^2} + 2(S-1)$  and

$$m^2 = 4(n+S).$$

## Linear confinement: $m_n^2 \sim n$

- Problem for high  $n$ :  $m_n^2 \sim n^2$ .
- High  $n$  mesons are large  $\Rightarrow$  IR regime. Thus high  $n \Leftrightarrow$  large  $z$ .

- $I = \int d^5x \sqrt{g} e^{-\Phi} \mathcal{L}$ , A. Karch et al.

Instead of hard cutoff at  $z_m$  —  $\Phi \rightarrow z^2$  at large  $z$

- Mode equation

$$\partial_z \left( e^{-B} \partial_z v_n \right) + m_n^2 e^{-B} v_n = 0,$$



$B \equiv \Phi(z) - A(z)$ , with  $e^{A(z)} = z^{-1}$  from  $ds^2 = e^{2A}(dx \cdot dx - dz^2)$ .

Substitute  $v_n = e^{B/2} \psi_n$

$$-\psi_n'' + V(z)\psi_n = m_n^2 \psi_n, \quad V(z) = \frac{1}{4}(B')^2 - \frac{1}{2}B''.$$

- With  $\Phi = z^2$ :  $V = z^2 + \frac{3}{4z^2}$  — 2d harmonic oscillator (radial eqn.,  $m = 1$ ).

$$m_n^2 = 4(n+1)$$

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## Higher spins (contd.)

$$m^2 = 4(n + S).$$

(Classical) Nambu-Gotto string also predicts:  $\frac{dm^2}{dn} = \frac{dm^2}{dS}$ .

Note:  $B = \Phi - (2S - 1)A \Rightarrow$  no  $z^2$  should be in  $A$ , or else  $m_n \sim nS$

Since  $\tilde{\phi} \dots \rightarrow \text{const}$  at boundary is a solution  $\Rightarrow [\mathcal{O}^{\dots}] = 4 - [\tilde{\phi} \dots]$

Since  $[\tilde{\phi} \dots] = [\phi \dots] - 2(S - 1) = 2 - S$  we find

$$[\mathcal{O}^{\dots}] = 2 + S$$

i.e., twist = 2.

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# Outlook

- Chiral symmetry and high resonances (restoration? Weinberg's "mended")
- Baryons
  - Dirac fields (Teramond-Brodsky, Hong-Inami-Yee)
  - Skyrmions/instantons (Hong-Rho-Yee-Yi, Hata-Sugimoto-Yamata)
  - finite density (Domokos-Harvey)
- chiral anomaly (WZW) (Hill-Zachos, Sakai-Sugimoto)
- strange mesons (Shock-Wu)
- (scalar) glueball spectrum (Polchinski-Strassler, Boschi-Filho-Braga)
- higher order chiral Lagrangian (DaRold-Pomarol, Hirn-Sanz)
- running of  $\alpha_s$  — log corrections to warp factor in UV
- power corrections to OPE — higher order terms in 5d
- Structure functions
- thermodynamics, transport ... (Herzog, Kajantie-Tahkokallio-Yee, Nakano-Teraguchi-Wen)