Title: Quantum Critical Transport, Duality and M-Theory

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Abstract:

Quantum Critical Transport, Duality and M-theory

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PI workshop, May 25, 2007

MOTIVATION:

- Understand transport in strongly interacting CFT at $T \neq 0$
- Understand degrees of freedom in M-theory

WHY CARE?

- CFT at finite T = theory of quantum criticality

  Interplay between thermal and quantum critical fluctuations

- M-theory = a theory of quantum gravity

  Some parts have field theory definition by AdS/CFT correspondence

- Work in 2+1 dim
- Focus on charge transport : conductivity $\sigma(\omega)$

Kubo formula relates $\sigma(\omega)$ to real-time current-current correlator evaluated in thermal state

$T=0$:

$$C_{\mu\nu}(p) = P_{\mu\nu}\,\Pi(p^2)$$

$$P_{\mu\nu} = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

Lorentz invar. + current conservation

In 2+1 dim., Π has mass dimension = 1

$$\therefore\ \Pi(p^2) = \text{const}\,\sqrt{\vec{k}^2 - \omega^2}$$

Kubo formula: $\boxed{\sigma(\omega) = -\frac{1}{\omega}\,\text{Im}\,\Pi(\omega, \vec{k}=0)}$

$\Pi(\omega,\vec{k}) \sim \sqrt{\vec{k}^2-\omega^2}$,

$\therefore$ In 2+1 dim, there is non-zero conductivity even at $T=0$

$T=0$: $\sigma(\omega)=$ dimensionless constant

What happens when $T \neq 0$?

$T \neq 0$: Lorentz invariance is no more

$$C_{\mu\nu}(\omega,\vec{k}) = P^T_{\mu\nu}\,\Pi^T(\omega,k^2) + P^L_{\mu\nu}\,\Pi^L(\omega,k^2)$$

$$P^T_{ij} = \delta_{ij} - \frac{k_i k_j}{\vec{k}^2}\,, \quad P^L_{\mu\nu} = P_{\mu\nu} - P^T_{\mu\nu}$$ ← rotation invariance + current conservation

$$\Pi^T(\omega,k=0) = \Pi^L(\omega,k=0)$$ otherwise Π^T and Π^L are completely unrelated

$$\Pi^{T,L}(\omega,k^2) = \sqrt{\vec{k}^2-\omega^2}\;K^{T,L}\left(\frac{\omega}{T},\frac{k}{T}\right)$$

non-trivial functional dependence!

Kubo formula:

$$\boxed{\sigma(\omega) = -\frac{1}{\omega}\,\mathrm{Im}\,\Pi^T\left(\frac{\omega}{T}, \frac{k}{T}=0\right)} = -\frac{1}{\omega}\,\mathrm{Im}\,C_{xx}\left(\frac{\omega}{T}, \frac{k}{T}=0\right)$$

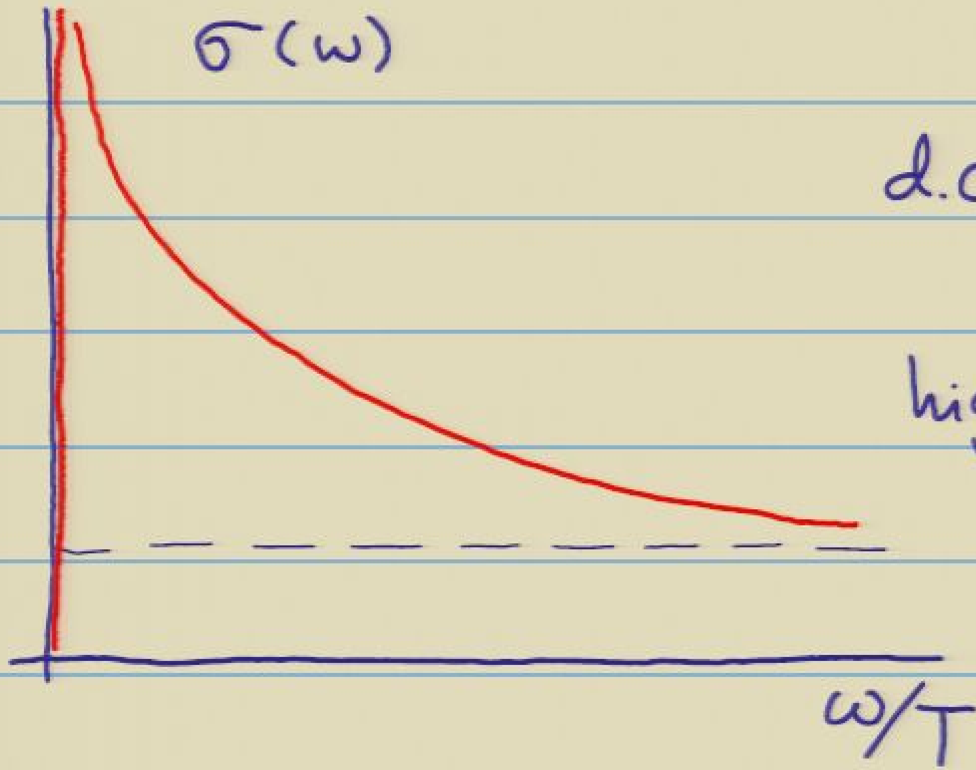
$T \neq 0$: $\sigma(\omega)$ = non-trivial function of $\frac{\omega}{T}$

unlike

$T = 0$, where $\sigma(\omega)$ = just a number

How does $\sigma(\omega)$ look like?

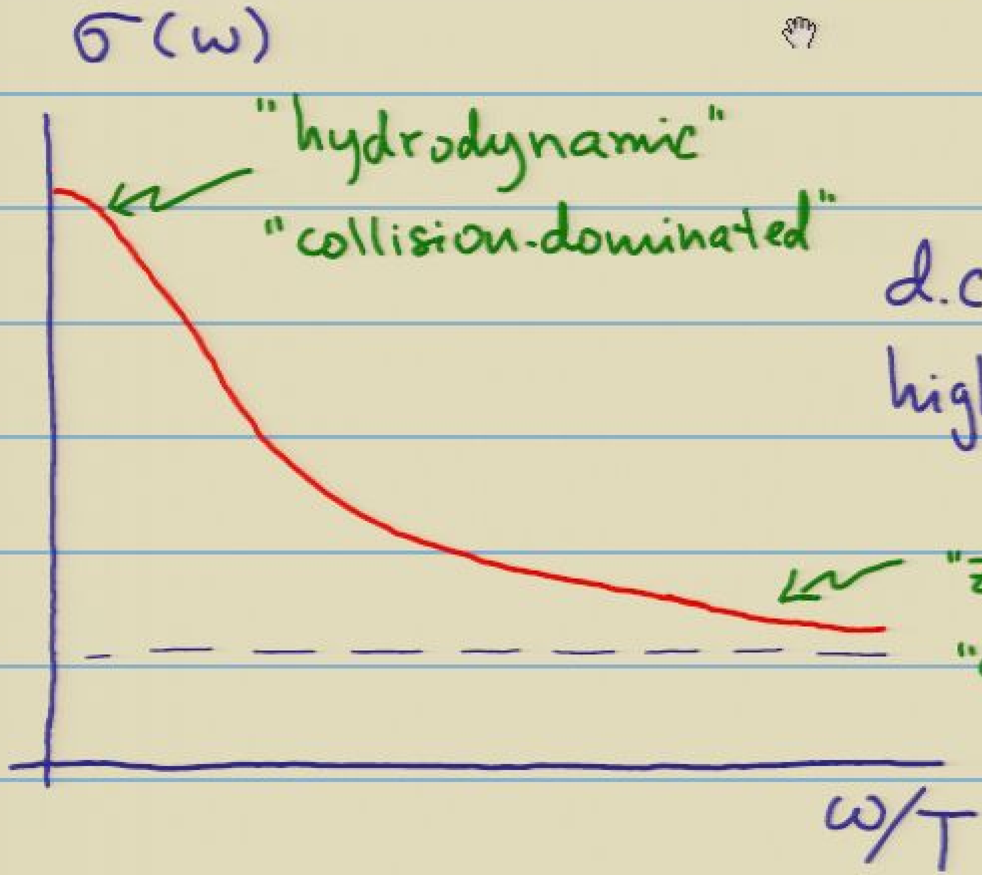
FREE THEORY



d.c. conductivity infinite

high-freq. conductivity constant

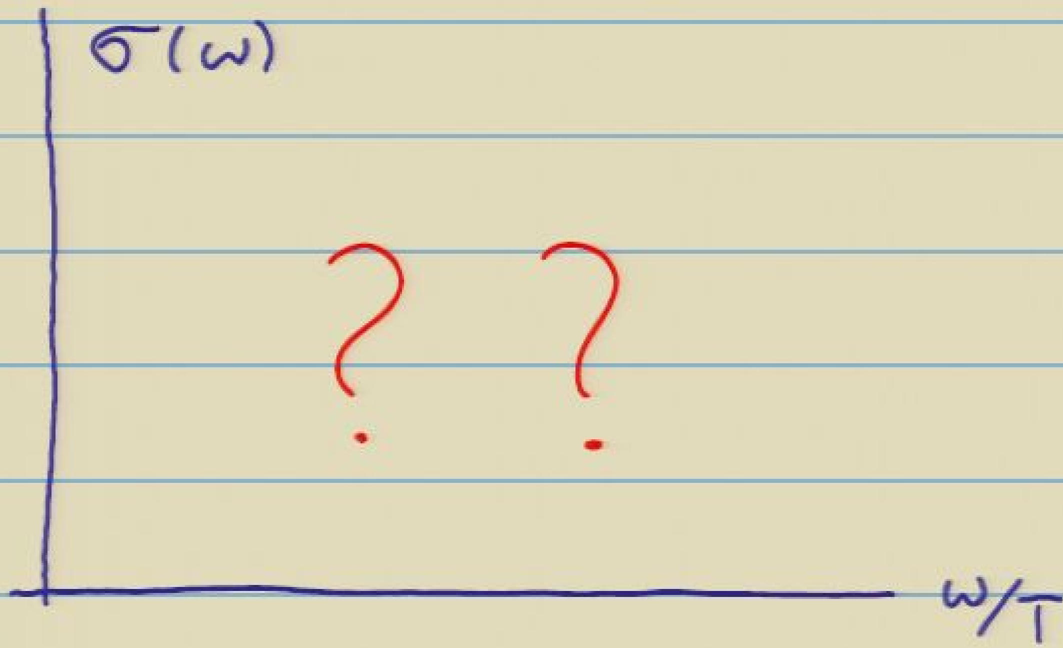
WEAKLY INTERACTING THEORY



d.c. conductivity large $\sim \ell_{mfp}$

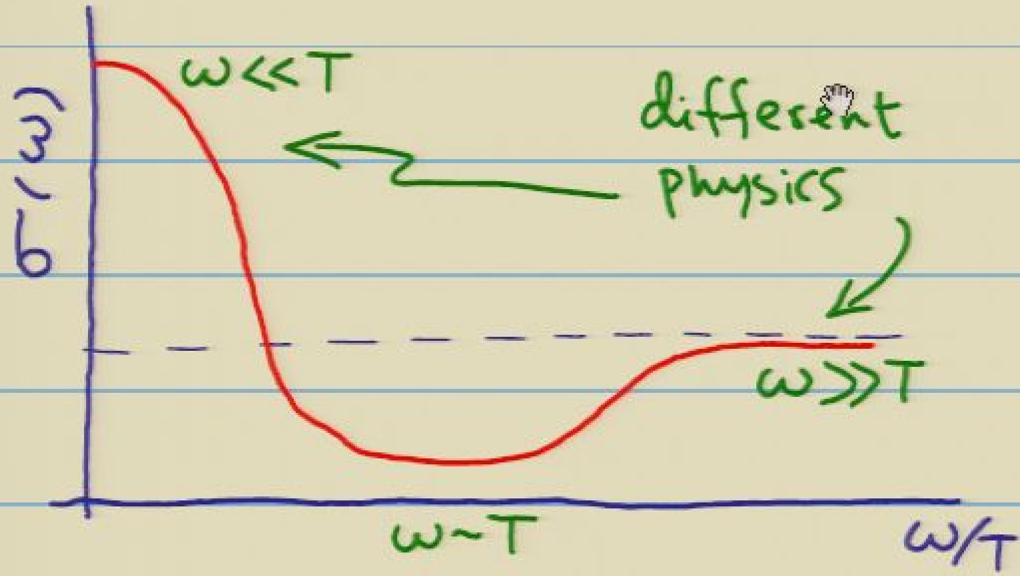
high-freq. conductivity constant

STRONGLY INTERACTING THEORY



In this talk: discuss 2+1 dim CFTs where transport can be studied analytically at strong coupling

Note that d.c. limit $\omega \to 0$ <u>does not</u> commute with $T \to 0$



d.c. conductivity at finite T: $\sigma\left(\frac{\omega}{T} \to 0\right)$

d.c. conductivity at zero T: $\sigma\left(\frac{\omega}{T} \to \infty\right)$

From S. Sachdev's QPT book, p. 188:

"The distinct physical interpretations of $\sigma\left(\frac{\omega}{T} \to \infty\right)$ and $\sigma\left(\frac{\omega}{T} \to 0\right)$ make it clear that, in general, there is no reason for them to have equal values (we cannot of course rule out the existence of exotic models or symmetries that may cause these two to be equal."

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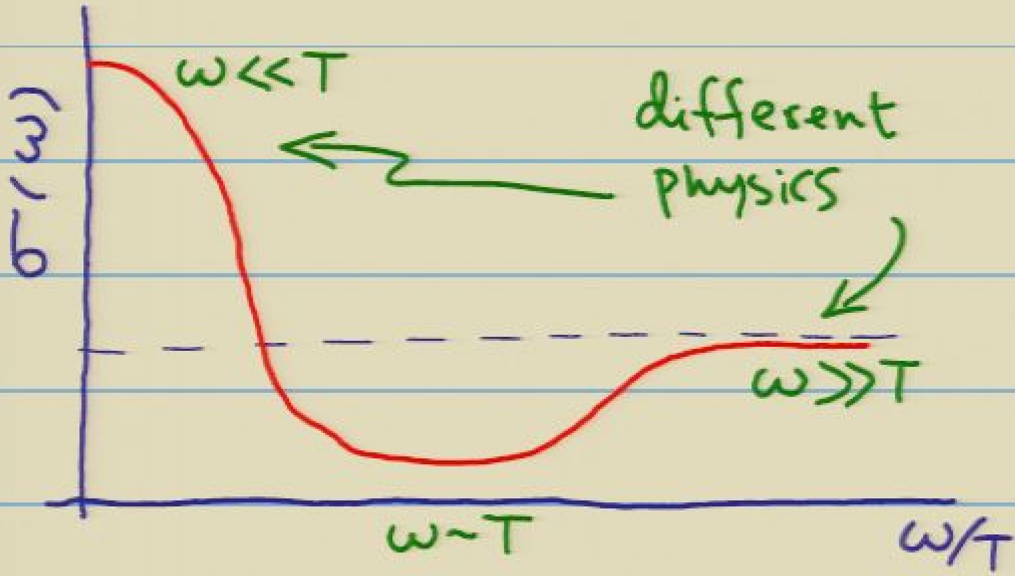

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A model where transport can be studied analytically at strong coupling:

$\boxed{\mathcal{N}=8 \text{ superconformal field theory in } 2+1 \text{ dim}}$

IR fixed point of maximally supersymmetric U(N) Yang-Mills theory in 2+1 dim.

Seiberg, hep-th/9705117

Why study this particular CFT?

- Toy model to compute $\sigma(\omega)$ at strong coupling using AdS/CFT
- This theory describes N coincident M2 branes in M-theory

Itzhaki, Maldacena, Sonnenschein, Yankielowicz, hep-th/9802042

Comments:

- The results will apply to a large class of CFTs, not just to the $\mathcal{N}=8$ SCFT.

- AdS/CFT correspondence will be used as a tool to study field theory.

Philosophy: applied string/M theory, analogous to applying AdS/CFT to heavy-ion physics

Setup

$\mathcal{N}=8$ SCFT in 2+1d, large N $\longleftrightarrow$ supergravity on $AdS_4 \times S^7$

thermal state in CFT $\longleftrightarrow$ black hole in AdS_4

SO(8) global symmetry $\longleftrightarrow$ isometry of S^7

conserved currents J^a_μ $\longleftrightarrow$ gauge fields A^a_μ in AdS_4

$$SO(8) \Rightarrow \quad \langle J^a_\mu J^b_\nu \rangle = C_{\mu\nu} \delta^{ab}$$

suffices to look at <u>abelian</u> gauge fields in AdS_4

Action: $S = -\frac{1}{4g^2} \int d^4x \sqrt{-g}\, F^{\mu\nu} F_{\mu\nu} \qquad \frac{1}{g^2} \sim N^{3/2}$

Recap:

- We need conductivity $\sigma(\omega)$
- $\sigma(\omega)$ is evaluated from $C_{\mu\nu}(\omega)$ (Kubo formula)
- $C_{\mu\nu}(\omega)$ is evaluated from AdS/CFT by the standard prescription: $C_{\mu\nu}(\omega,k) \sim \dfrac{\delta^2 S_{Maxw.}}{\delta A^0_\mu \, \delta A^0_\nu}$

* * *

At $\vec{k}=0$ can solve Maxwell eq-s analytically:

$$\Pi^T(\omega,k=0) = \Pi^L(\omega,k=0) = \frac{-i\omega}{g^2}$$

Kubo formula: $\boxed{\sigma(\omega) = \frac{1}{g^2}}$

$\sigma(\omega)$ is an ω-independent constant, for all ω/T.

This is very unusual

Physical reason why $\sigma(\omega) = \text{const}$?

- CFT currents live in 2+1 dim
- dual AdS_4 gauge fields live in 3+1 dim
- abelian gauge fields in 3+1 dim have electric-magnetic duality

EM duality in the 3+1 dim bulk implies

$$\boxed{\Pi^T(\omega,k)\,\Pi^L(\omega,k) = \frac{-1}{g^2}(\omega^2 - k^2)}$$

Herzog, PK, Sachdev, Son, hep-th/0701036

on the 2+1 dim boundary. Take $k \to 0$: $\sigma(\omega) = 1/g^2$ for all ω.

EM duality of the dual 3+1 dim theory implies ω-independent conductivity in 2+1 dim CFT at $T \neq 0$.

Comments on $\boxed{\Pi^T(\omega,k)\,\Pi^L(\omega,k) = \text{const}(\omega^2 - \vec{k}^2)}$, $T \neq 0$

- Very non-trivial relation at $T\neq 0$: a priory no reason to expect any connection between Π^T and Π^L

- Besides AdS/CFT, no other examples are known where $\Pi^T\Pi^L \sim p^2$

- Assuming AdS/CFT: there is a large class of exotic 2+1 dim CFTs where $\Pi^T\Pi^L \sim p^2$. These are field theories whose AdS dual contains $S = \frac{-1}{4g^2}\int d^4x \sqrt{-g}\, F^{\mu\nu}F_{\mu\nu}$ details of metric not important

- Compare with momentum transport: there is a large class of field theories with viscosity $\frac{\eta}{s} = \frac{1}{4\pi}$

P.K., Son, Starinets, hep-th/0405231

- The relation $\Pi^T(\omega,k)\,\Pi^L(\omega,k) = \text{const}\,(\omega^2 - k^2)$ should be viewed as another example of universality in AdS/CFT

- Outside of AdS/CFT: in abelian theories near the critical point, particle-vortex duality $\Rightarrow$ $\Pi^T(\omega,k)\,\tilde{\Pi}^L(\omega,k) = \text{const}\,(\omega^2 - \vec{k}^2)$

in the dual theory

CHARGE TRANSPORT IN EXTERNAL MAGNETIC FIELD

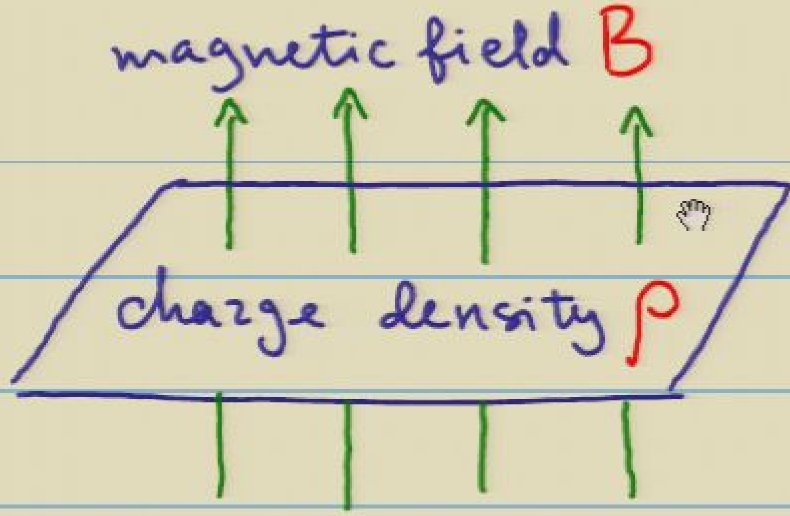
$$\boxed{\sigma_{ij}(\omega) = -\frac{1}{\omega}\,\mathrm{Im}\,C_{ij}(\omega, k=0)}$$

conductivity becomes a tensor, with $\sigma_{xy} \neq 0$: this is Hall effect

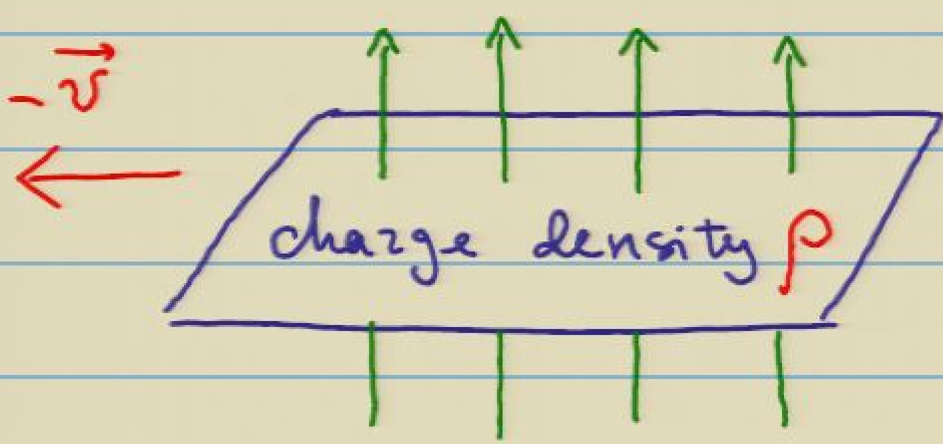
(What we computed earlier was $\sigma = \sigma_{xx} = \sigma_{yy}$)

• In a relativistic theory, σ_{xy} is fixed by boost invariance

D.C. conductivity



$$\vec{j} = 0$$
$$\vec{B} = (0,0,B)$$
$$\vec{E} = 0$$



$$\vec{j} = \rho\vec{v}$$
$$B = (0,0,B)$$
$$\vec{E} = -\vec{v}\times\vec{B}$$
$$\Downarrow$$
$$\vec{E} = -\frac{B}{\rho}\,\vec{j}\times\hat{z}$$

$$\therefore\quad j_i = \sigma_{ij}E_j\ ,\qquad \boxed{\sigma_{xy} = \frac{\rho}{B}\ ,\ \sigma_{xx} = 0}$$

CHARGE TRANSPORT IN EXTERNAL MAGNETIC FIELD

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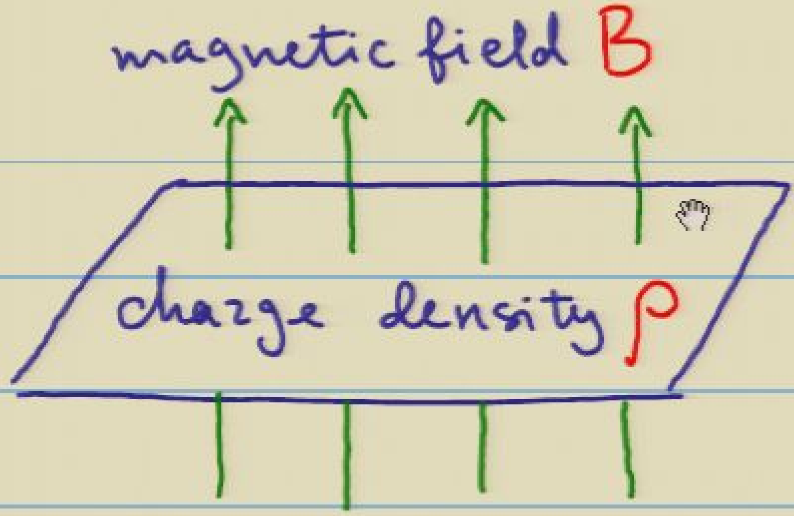

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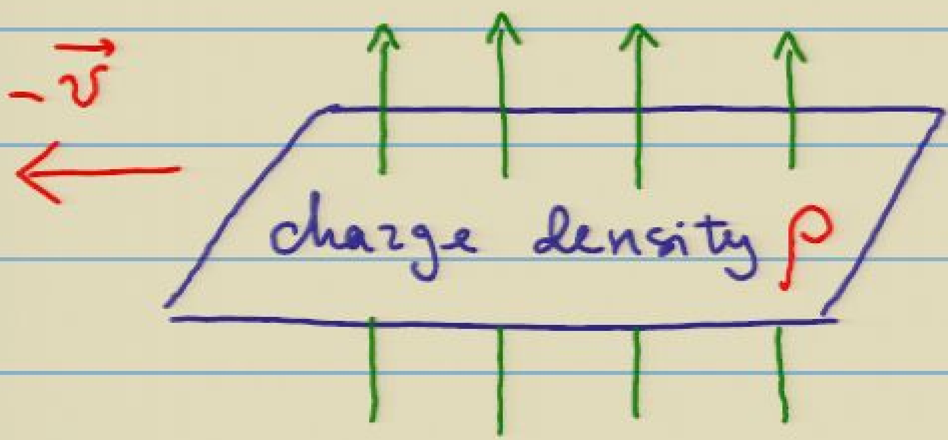

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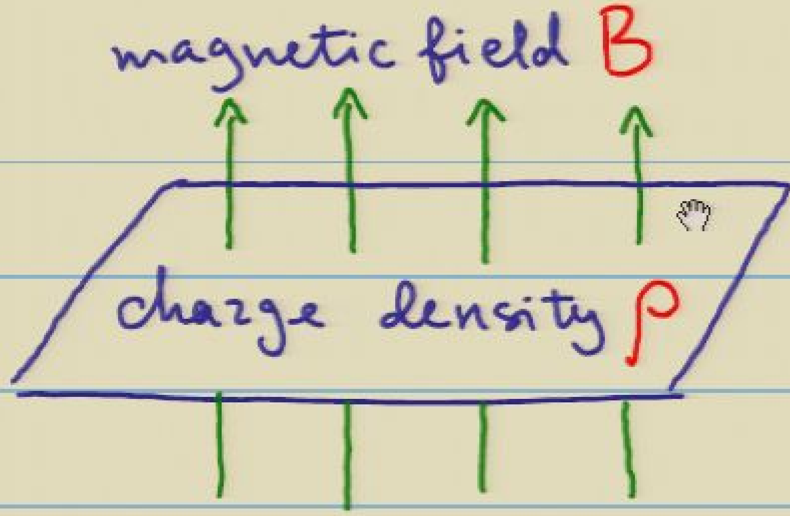
DOES AdS/CFT KNOW ABOUT THE HALL EFFECT?

- Need background charge density in CFT (= chemical potential) } → electrically charged b.h. in AdS_4

- Need background magnetic field in CFT } → magnetically charged b.h. in AdS_4

Need to solve Maxwell equations in 3+1 dim. bulk, with both electric and magnetic charges.

D.C. conductivity



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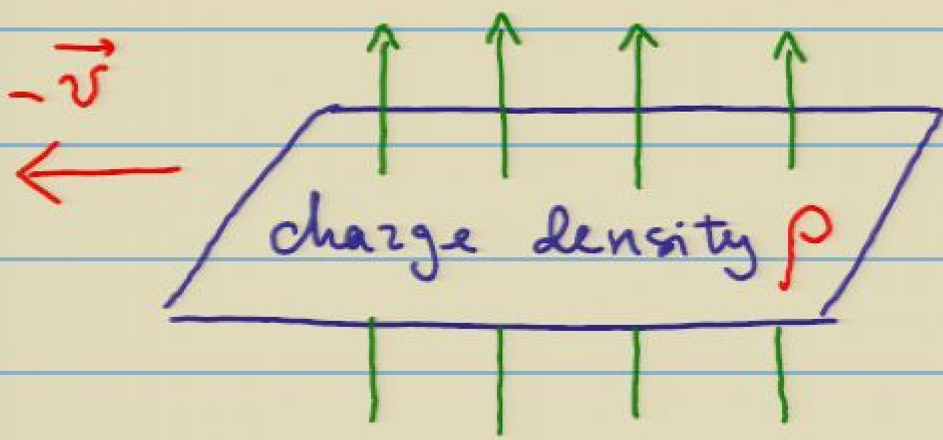

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Linear perturbations of b.h. + Kubo formula $\Rightarrow$ $\sigma_{xy}(\omega=0) = \frac{\rho}{B}$

$\sigma_{xx}(\omega=0) = 0$

P.K., S.Hartnoll, arXiv: 0704.1160

Note that the d.c. limit $\omega \to 0$ does not commute with $B \to 0$.

(At $B=0$, we found $\sigma_{xx}(\omega\to 0) =$ finite, $\sigma_{xy}(\omega \to 0) = 0$)

Thoughts about the future

- Cross-over between $\frac{\omega}{T} \ll \frac{B}{T^2}$ and $\frac{\omega}{T} \gg \frac{B}{T^2}$.

- Action of the EM duality in 3+1 dim bulk on the complex conductivity $\sigma_{xy} + i\sigma_{xx}$

- Electric and magnetic charges are quantized. Quantum Hall?

- Flux compactifications of string theory produce a landscape of AdS_4 vacua $\rightsquigarrow$ a landscape of universality classes. Find critical Ising model?

Conclusion

AdS/CFT was arguably useful in application to strongly interacting QCD matter. Can it be useful in application to condensed matter?

No Signal

VGA-1