Title: Confinement/Deconfinement Transition in AdS / CFT

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Abstract:

Confinement/Deconfinement Transition in AdS/CFT

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The University of Michigan

Perimeter Institute: Exotic States Workshop

Outline

- $\mathcal{N} = 4$ Thermodynamics from non-extremal D3 branes
- Hawking-Page Phase transition as confinement/deconfinement transition
- The hard wall model and its transition
- Review of the Klebanov-Strassler background
- A black hole in the Klebanov-Tseytlin background
- Drag force in KT at finite temperature
- Hawking-Page transition for the KS background
- Conclusions

$\mathcal{N} = 4$ Thermodynamics from Nonextremal D3 branes

Klebanov, Gubser, Peet, Tseytlin

$$ds^2 = h^{-1/2}(r)[-f(r)dt^2 + dx^i dx_i] + h^{1/2}[f(r)^{-1}dr^2 + r^2 d_5^2]$$

$$h(r) = \frac{R^4}{r^4}, \qquad f(r) = 1 - \frac{r_0^4}{r^4}$$

Temperature: $T = 1/\beta = r_0/\pi R^2$

$$S_{BH} = \frac{A_h}{4G} = \frac{\pi^2}{2} N^2 V_3 T^3$$

Free $U(N)$ $\mathcal{N} = 4$

$$S_0 = \frac{2\pi^2}{3} N^2 V_3 T^3.$$

The Famous 3/4: $S = N^2 f(g_{YM}^2 N) V T^3$

Hawking-Page transition

Hawking,Page; Witten

$$
\begin{aligned}
I &= -\frac{1}{2\kappa^2}\int_{\mathcal{M}} d^5x\sqrt{g}\left(\mathcal{R}-\Lambda\right)\\
\text{on}-\text{shell} &= -\frac{1}{2\kappa^2}\int_{\mathcal{M}} d^5x\sqrt{g}\,\Lambda
\end{aligned}
$$

- Fixing the same temperature

$$
\beta'\sqrt{\frac{r^2}{R^2}+1}=\beta_{b.h.}\sqrt{\frac{r^2}{R^2}+1-\frac{w_4M}{r^2}}
$$

$$
\begin{aligned}
I &= \frac{1}{2\pi G}\lim_{r_{max}\to\infty}\left(V_{b.h.}(r_{max})-V_{AdS}(r_{max})\right)\\
&= \frac{\Omega_3(R^2r_+^3-r_+^5)}{4G(4r_+^2+2R^2}
\end{aligned}
$$

- $\mathcal{N}=4$ SYM on $S^3\times S^1$
- Infinite N

The Hard wall model: Hard IR cutoff

- Motivated by a criterion for confinement in gauge/gravity
- Explained power law behavior in DIS from strings (Polchinski-Strassler)
- Hadronic spectrum (Brodsky-Teramond)
- AdS/QCD

Herzog; Ballon, Boschi-Filho,Braga, LPZ; Kajantie, Tahkokallio, Yee

$$I^{BH}_{total} - I^{AdS}_{total} = \begin{cases} \frac{R^3}{\kappa^2}\,\beta\left(\frac{1}{2z_h^4}\right) & (z_0' < z_h) \\ -\frac{R^3}{\kappa^2}\,\beta\left(\frac{1}{2z_h^4} - \frac{1}{z_0^4}\right) & (z_0' > z_h)\,. \end{cases} \qquad (1)$$

Jump in the number of degrees of freedom

$$G_5 z_0^3 R^2 \sim g^2 \alpha'^4 \,, \qquad (2)$$

$$\sigma = \frac{1}{2\pi\alpha'} \frac{R^2}{z_0^2} \,. \qquad (3)$$

Teper: $\sqrt{\sigma} \sim gN$

From these energies we calculate the entropies as $S = \beta\langle E\rangle + logZ \simeq \beta\langle E\rangle - I_{total}$ and find

$$S_{AdS} = 0 \sim N^0 \qquad (T < T_C) \qquad (4)$$

$$S_{BH} = \frac{R^3 \pi^3}{4G_5} T^3 = \frac{\pi^2 N^2}{2} T^3 \qquad (T > T_C) \,. \qquad (5)$$

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Problems

- Consistent reductions
- The role of 3-form fluxes in the 5-d action, since Λ comes from the 5-form in 10-d
- Is this toy computation true for the honest solutions?

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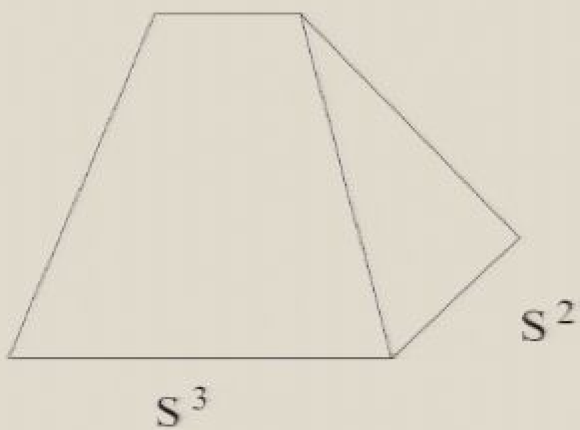
Review of the KS background

- Dual to $\mathcal{N}=1$ supersymmetric $SU(N+M)\times SU(N)$ gauge theory.
- Incoporates logarithmic running of the couplings
- The theory confines in the IR
- Chiral symmetry breaking
- There is an explicit IIB supergravity background: G_{MN}, H_3, F_3, F_5



The Ansatz

Buchel; Buchel, Herzog, Klebanov, PZ, Tseytlin; GHKT; PZ, Terrero-Escalante

Metric

$$ds^2 = e^{2z}(-e^{-6x}dX_0^2 + e^{2x}dX_i dX^i) + e^{-2z}ds_6^2\ ,$$

$$\begin{aligned} ds_6^2 &= e^{10y}du^2 + e^{2y}(dM_5)^2\ , \\ (dM_5)^2 &= e^{-8w}e_\psi^2 + e^{2w}(e_{\theta_1}^2 + e_{\phi_1}^2 + e_{\theta_2}^2 + e_{\phi_2}^2) \equiv e^{2w}ds_5^2. \end{aligned}$$

$$\Phi = \Phi(u).$$

p-form fields

$$\begin{aligned} F_3 &= Pe_\psi \wedge (e_{\theta_1} \wedge e_{\phi_1} - e_{\theta_2} \wedge e_{\phi_2})\ , \\ B_2 &= f(u)(e_{\theta_1} \wedge e_{\phi_1} - e_{\theta_2} \wedge e_{\phi_2})\ , \\ F_5 &= \mathcal{F} + *\mathcal{F}\ , \qquad \mathcal{F} = K(u)e_\psi \wedge e_{\theta_1} \wedge e_{\phi_1} \wedge e_{\theta_2} \wedge e_{\phi_2}\ . \end{aligned}$$

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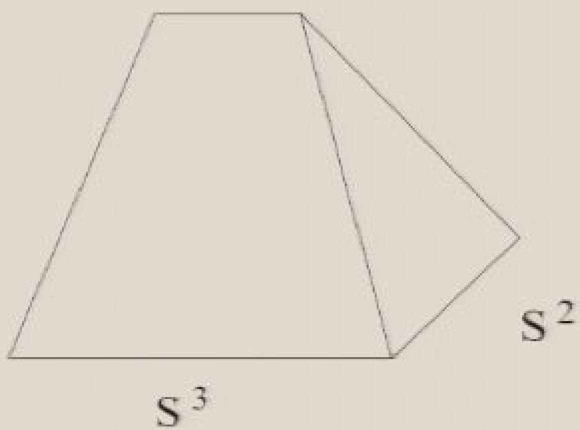

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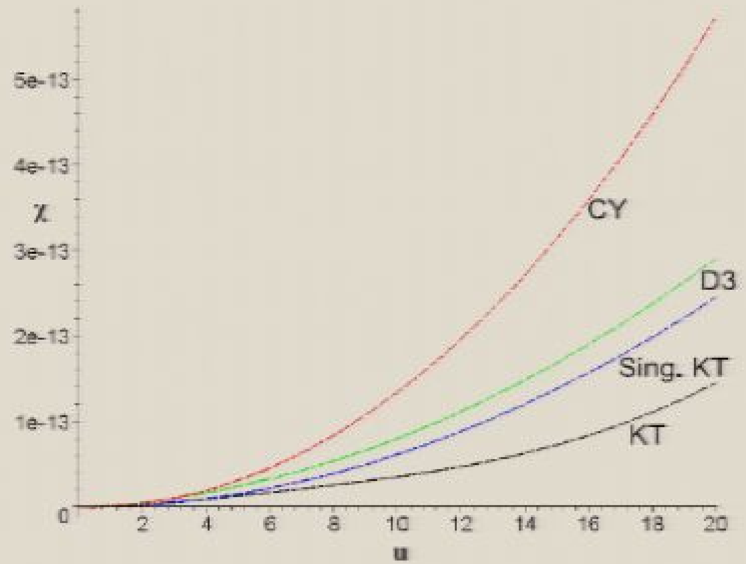
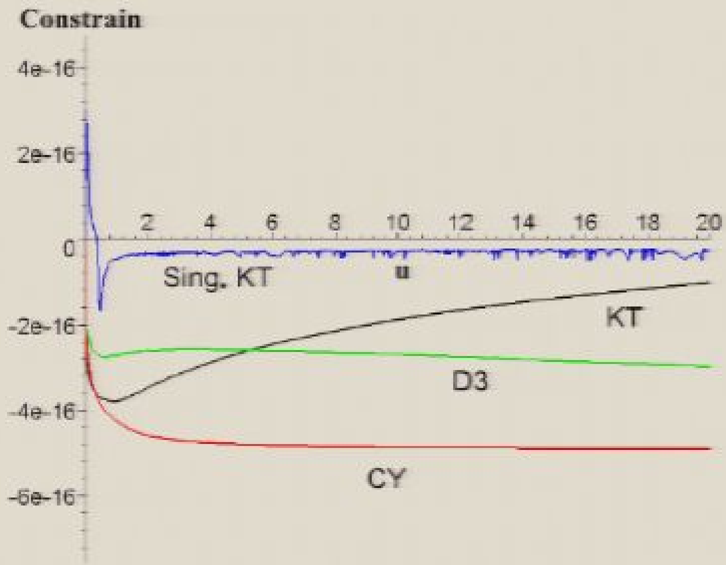
Calibrating the method

Metric functions (z, x, y, w) and matter functions (f, K, Φ)

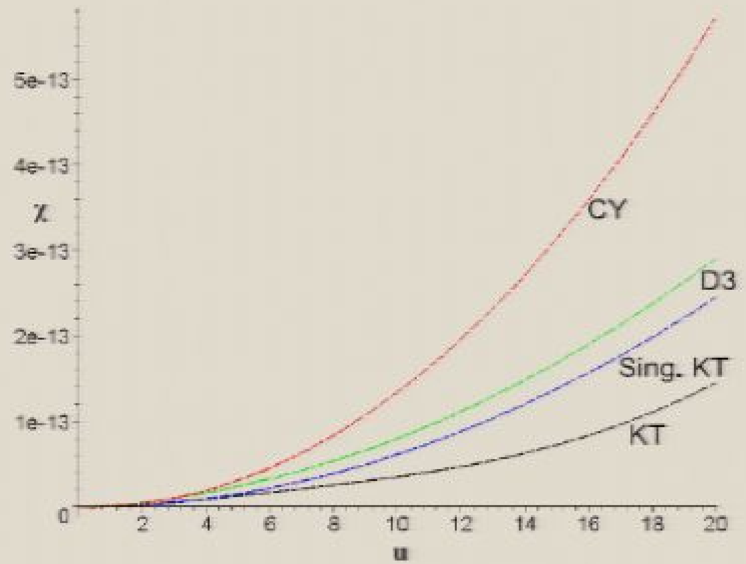
- Klebanov-Witten
- The blown up conifold
- Non-extremal D3 branes: Singular and Regular
- Klebanov-Tseytlin
- A singular generalization of KT
- Perturbative solution with varying flux around nonextremal D3



$$\chi = \sqrt{\sum \left[v_i^{an}(u) - v_i^{num}(u)\right]^2}, \quad i = 1 \cdots 10. \qquad (6)$$



$$5y'^2 - 2z'^2 - 5w'^2 - \frac{1}{8}\Phi'^2 - \frac{1}{4}e^{-\Phi+4z-4y-4w}f'^2$$
$$- e^{8y}(6e^{-2w} - e^{-12w})$$
$$+\frac{1}{4}e^{\Phi+4z+4y+4w}P^2 + \frac{1}{8}e^{8z}(Q+2Pf)^2 - 3a^2 = 0 \ .$$



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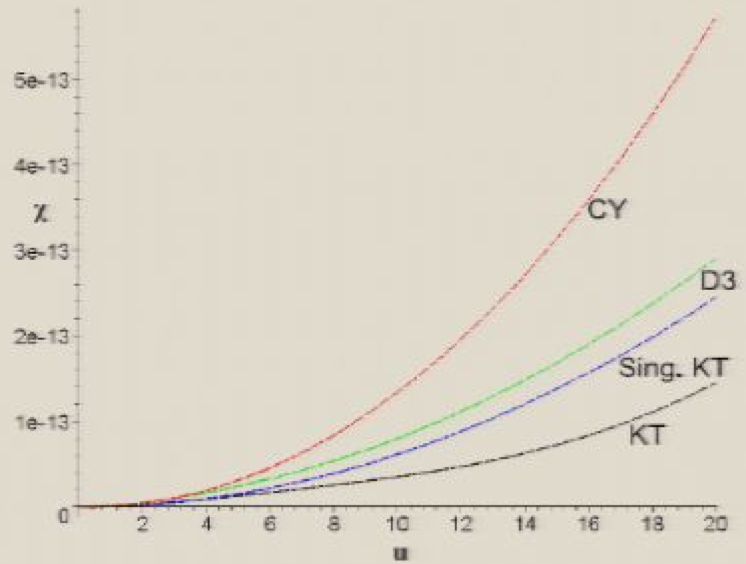

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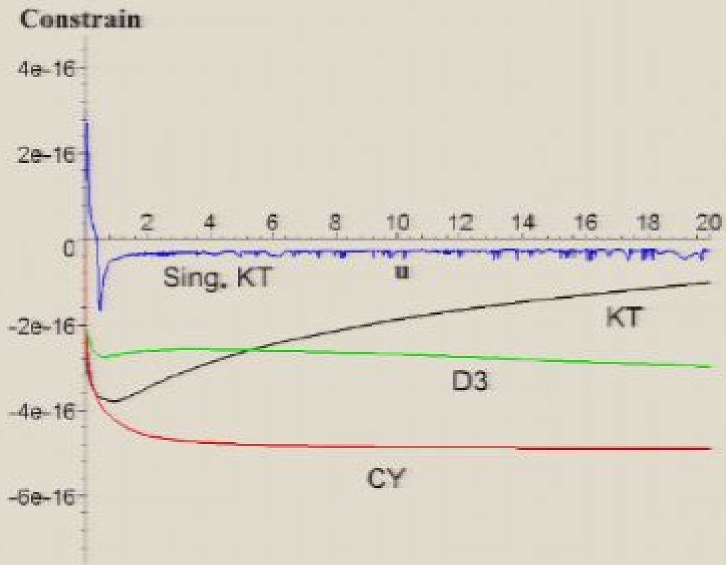

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Comments on perturbation theory in gravitational backgrounds

- Perturbation theory restricts the range of the radial coordinate

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu}$$

- Schwarzschild $\delta g_{00} = 2M/r$ and we are limited to $r \gg M$ for perturbation theory to make sense ($rT \gg 1$).
- For the solution of GHKT we find

$$P^2/K_* \ll 1, \qquad \text{and} \quad u \gg u_c = \frac{1}{8a} e^{-2K_*/P^2}.$$

Checking that this is a black hole

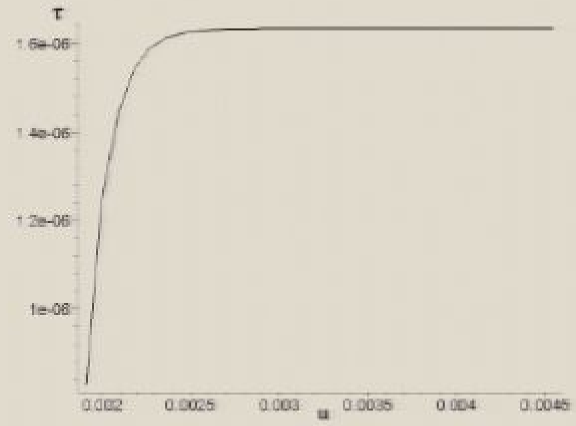
- Asymptotic times: infinite

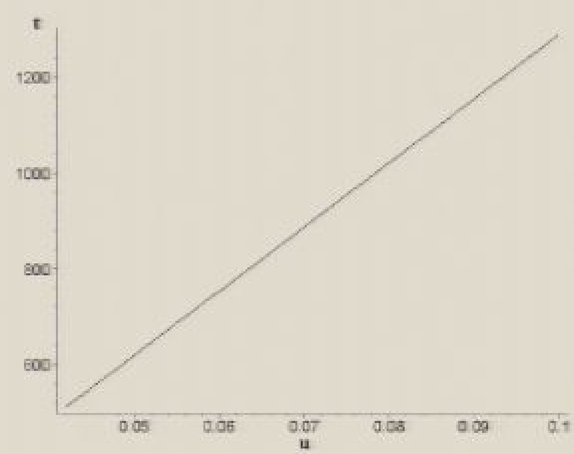
$$t = \int_{u_l}^{u} \frac{e^{5y}}{\sqrt{E^2 e^{6x} - e^{2z}}} e^{-2z+6x} du. \tag{7}$$

- Proper time: finite

$$\tau = \int_{u_l}^{u} \frac{e^{5y}}{\sqrt{E^2 e^{6x} - e^{2z}}} du, \tag{8}$$

- Completeness of the metric
- Analytic cross checks
- Analytic asymptotic behavior ($u \to \infty$ and $u \to 0$)


τ
1.6e-06
1.4e-06
1.2e-06
1e-06
0.002
0.0025
0.003
0.0035
0.004
0.0045
u


τ
1200
1000
800
600
0.05
0.06
0.07
0.08
0.09
0.1
u

Area of the horizon

In the u-coordinates the area of surface defined by a horizon at $u = \text{constant}$ is

$$A = V_5 \exp\left(-2z + 3x + 5y\right). \tag{9}$$

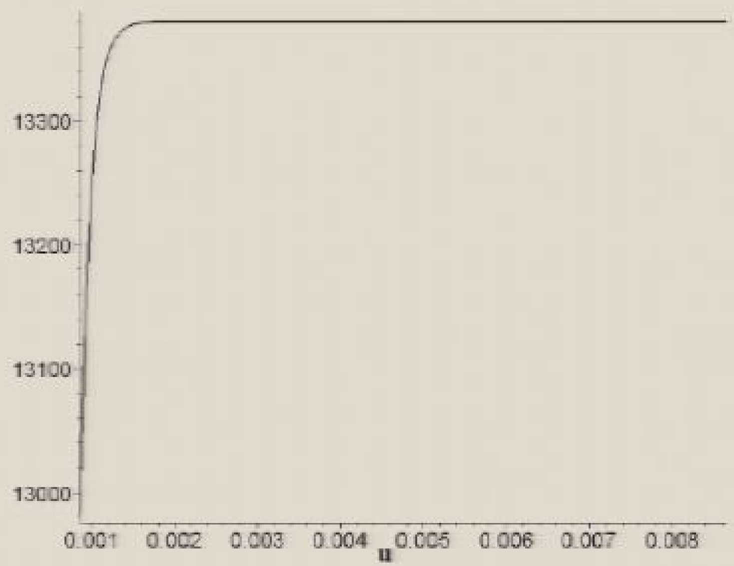
Given that the equation of motion for x has the general solution $x = au$ we are forced into the following situation. If the horizon is at $u \to \infty$, then in order for the area A to be finite we need the following asymptotics for z and y:

$$z \to \alpha\, au + z_*, \qquad y \to \beta\, au + y_*, \tag{10}$$

with the condition that

$$-2\alpha + 5\beta = -3 \tag{11}$$

Area of the horizon



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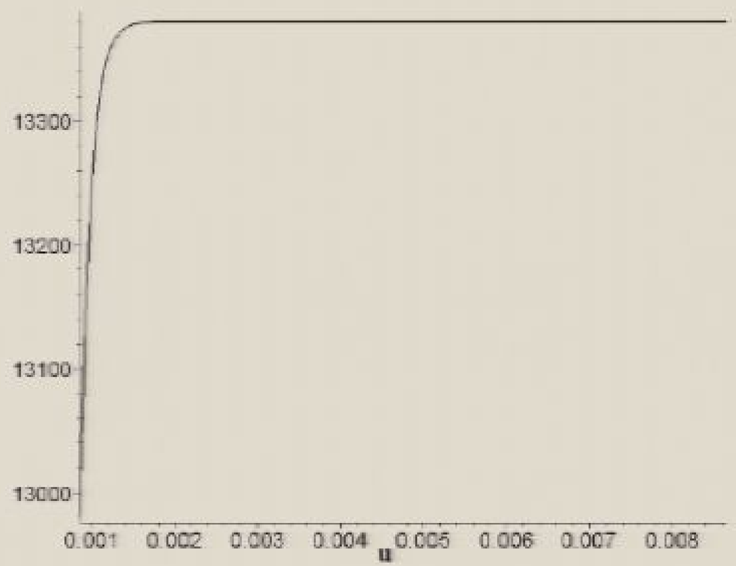

Asymptotics

$$
\begin{aligned}
y_\infty &= 1.90027 - 999.942947\,u + 0.0000002\,u^2 \\
&+ 0.00000002\,u^3 + 0.000000002\,u^4\,, \\
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&+ 0.0000001\,u^3 + 0.00000001\,u^4\,.
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$$
z(u) \to -\frac{1}{4}\ln\left[4(u - u_{sing})\right] + \cdots \qquad (12)
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f_1 = \pm P \mathrm{e}^{(\Phi_0 + 4y_0 + 4w_0)}\,.
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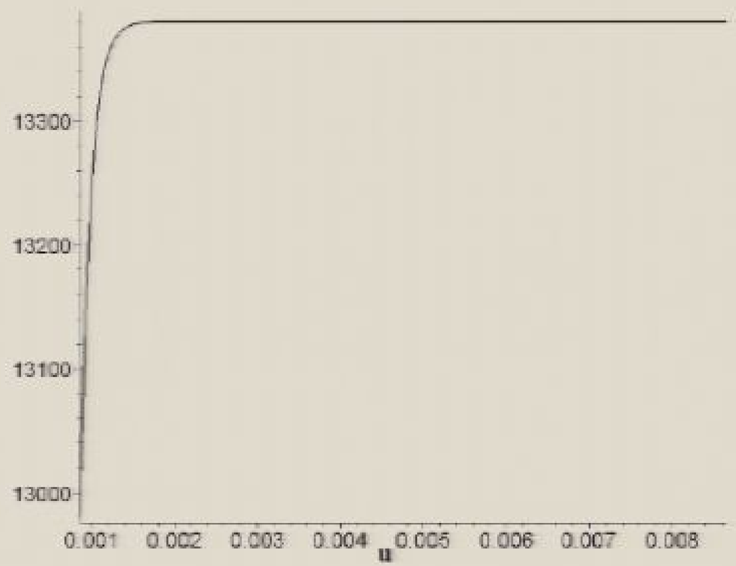
Area of the horizon



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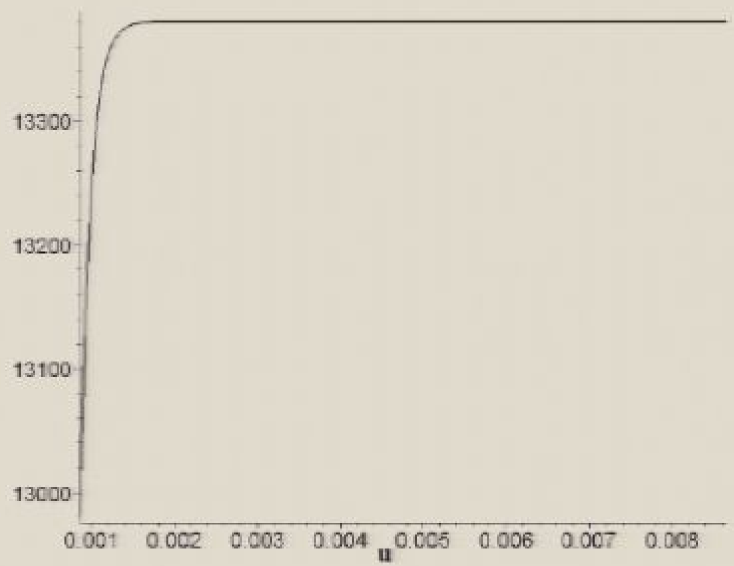

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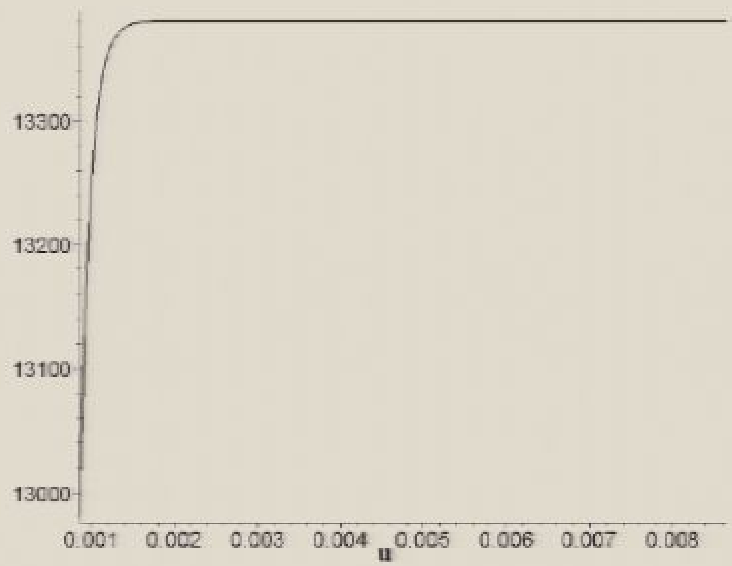

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Drag force in this background: Preliminary Results

Mahato, PZ, Terrero-Escalante

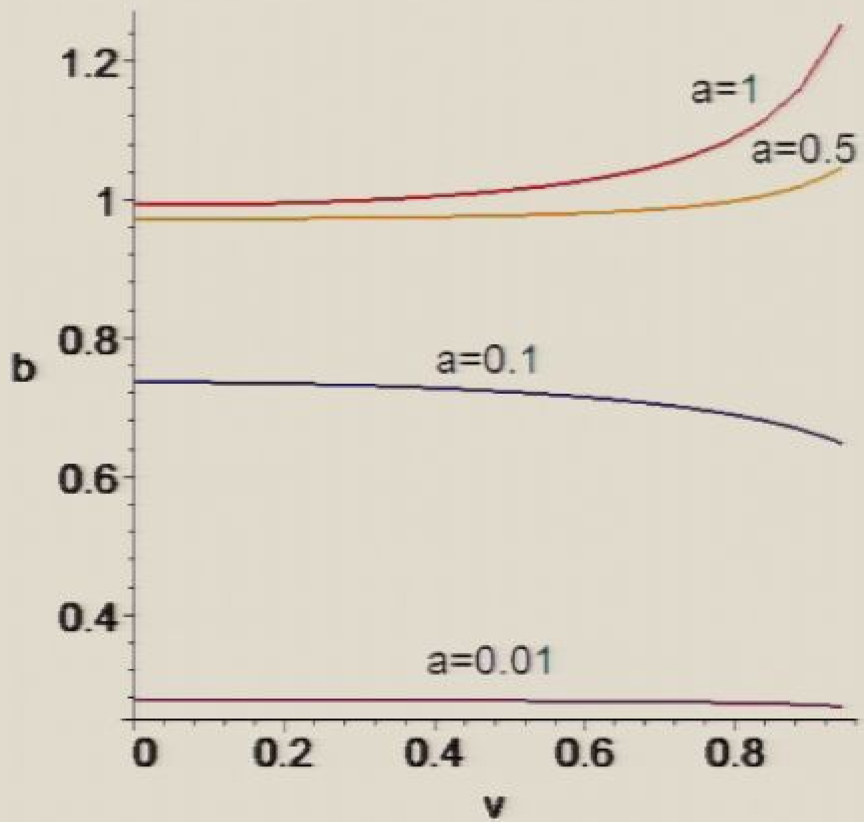


Figure: Behavior of μ for a representative set of temperatures

a	P	$u_{90\%}$	$3-2\alpha+5\beta$
1	4.26906025	2.19361453	-9×10^{-15}
0.5	2.87343889	4.387229	-7×10^{-15}
0.1	1.094389139	21.9361453	-2×10^{-15}
0.01	0.316430028	219.361453	10^{-15}

ϕ_*	u_{sing}	$f_1+P\mathrm{e}^{\phi_0+4y_0+4w_0}$
-3×10^{-7}	3×10^{-10}	0.0001257
-2×10^{-7}	3×10^{-11}	0.0000065
-1.2×10^{-7}	4×10^{-10}	0.00000002
-1.4×10^{-8}	2×10^{-9}	0.000000002

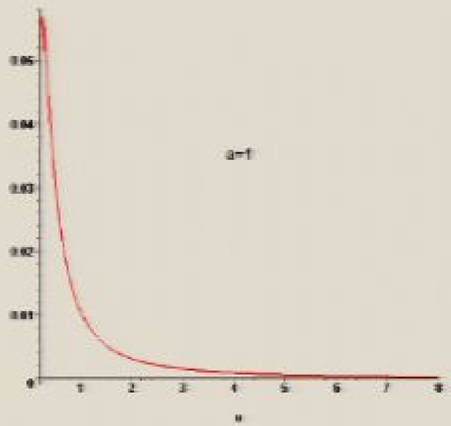
- velocity dependence on the relativistic region
- Can compute analytically

$$\mu(v << 1) \approx \sqrt{2a}\mathrm{e}^{\frac{\Phi_0}{2}} \tag{13}$$

and

$$\mu(v \approx 1) \rightarrow \mathrm{e}^{\frac{\Phi(u)}{2}}\mathrm{e}^{2\,z_*}\mathrm{e}^{2\,(\alpha+1)au} \tag{14}$$

- Can argue about the T^2 dependence but mainly in the nonrelativistic region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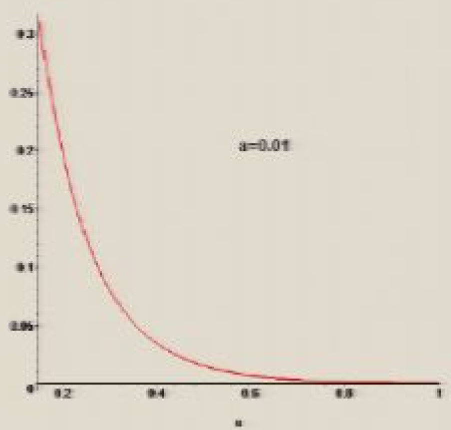
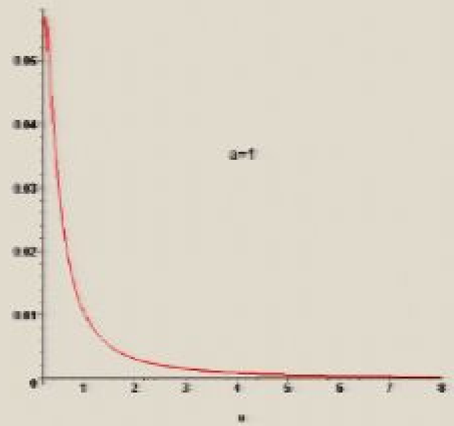

a=0.01

a	P	$u_{90\%}$	$3-2\alpha+5\beta$
1	4.26906025	2.19361453	-9×10^{-15}
0.5	2.87343889	4.387229	-7×10^{-15}
0.1	1.094389139	21.9361453	-2×10^{-15}
0.01	0.316430028	219.361453	10^{-15}

ϕ_*	u_{sing}	$f_1+P\mathrm{e}^{\phi_0+4y_0+4w_0}$
-3×10^{-7}	3×10^{-10}	0.0001257
-2×10^{-7}	3×10^{-11}	0.0000065
-1.2×10^{-7}	4×10^{-10}	0.00000002
-1.4×10^{-8}	2×10^{-9}	0.000000002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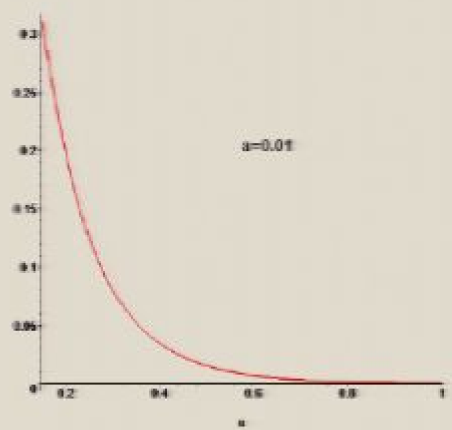
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Drag force in this background: Preliminary Results

Mahato, PZ, Terrero-Escalante

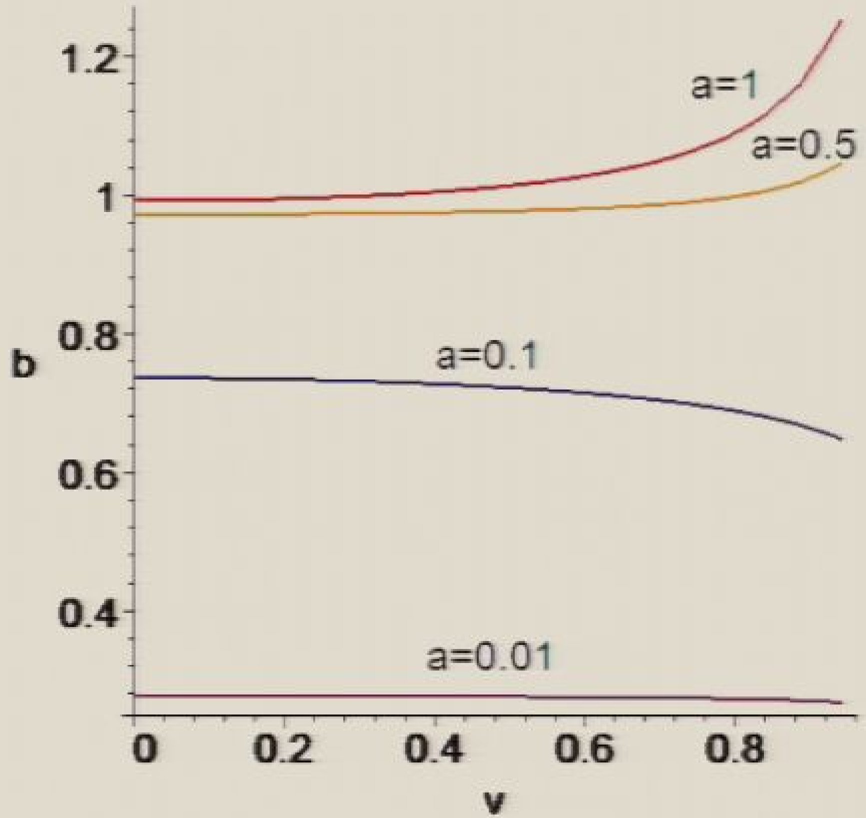


Figure: Behavior of μ for a representative set of temperatures

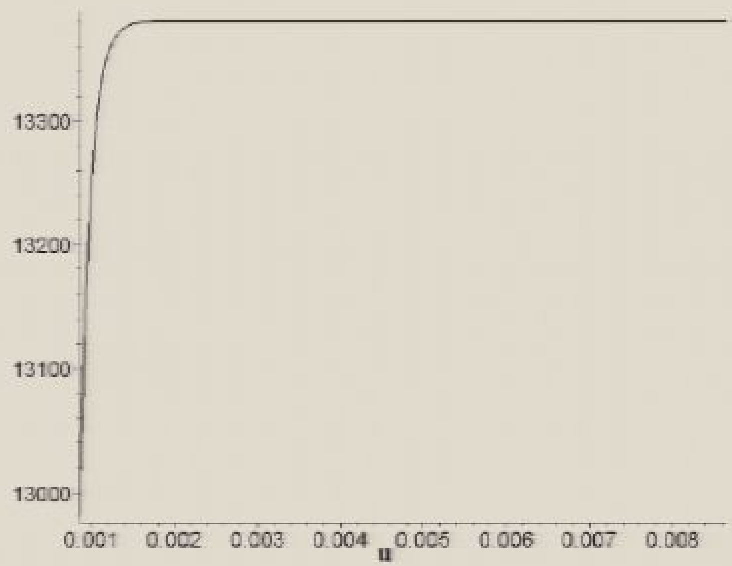
Asymptotics

$$
\begin{aligned}
y_\infty &= 1.90027 - 999.942947\,u + 0.0000002\,u^2 \\
&+ 0.00000002\,u^3 + 0.000000002\,u^4\,, \\
z_\infty &= -0.000113 - 999.857345\,u + 0.0000004\,u^2 \\
&+ 0.0000001\,u^3 + 0.00000001\,u^4\,.
\end{aligned}
$$

$$
z(u) \to -\frac{1}{4}\ln\left[4(u - u_{sing})\right] + \cdots \qquad (12)
$$

$$
f_1 = \pm P \mathrm{e}^{(\Phi_0 + 4y_0 + 4w_0)}\,.
$$

Area of the horizon



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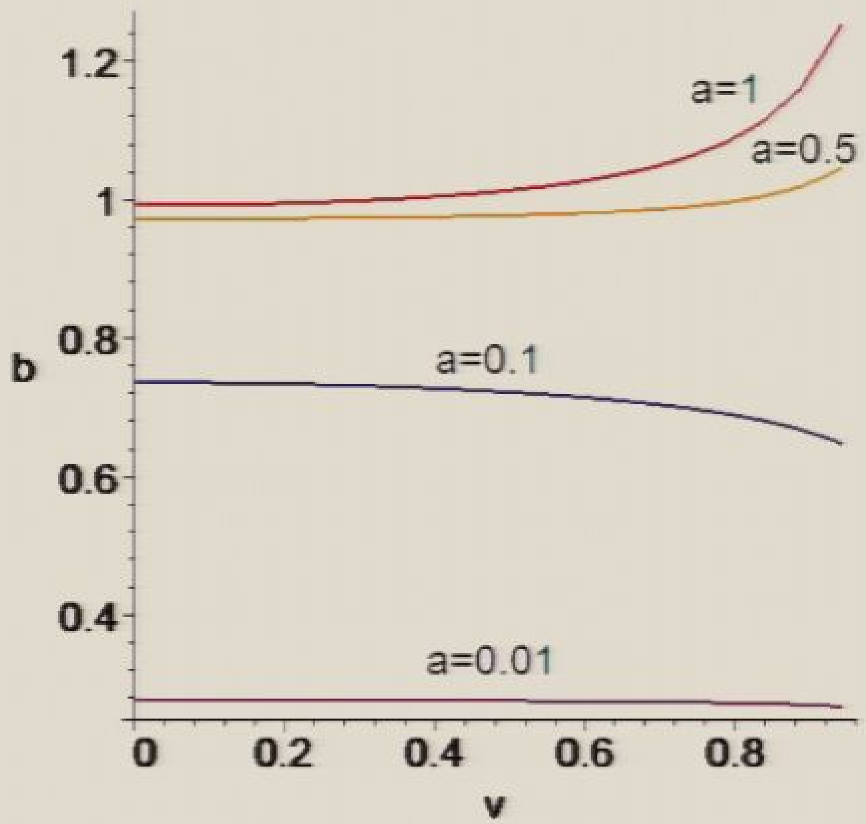
Drag force in this background: Preliminary Results

Mahato, PZ, Terrero-Escalante



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-1.4×10^{-8}	2×10^{-9}	0.000000002

- velocity dependence on the relativistic region
- Can compute analytically

$$\mu(v << 1) \approx \sqrt{2a}\mathrm{e}^{\frac{\Phi_0}{2}} \tag{13}$$

and

$$\mu(v \approx 1) \rightarrow \mathrm{e}^{\frac{\Phi(u)}{2}}\mathrm{e}^{2\,z_*}\mathrm{e}^{2\,(\alpha+1)au} \tag{14}$$

- Can argue about the T^2 dependence but mainly in the nonrelativistic region

Hawking-Page transition: Preliminary Results

PZ, Terrero-Escalante

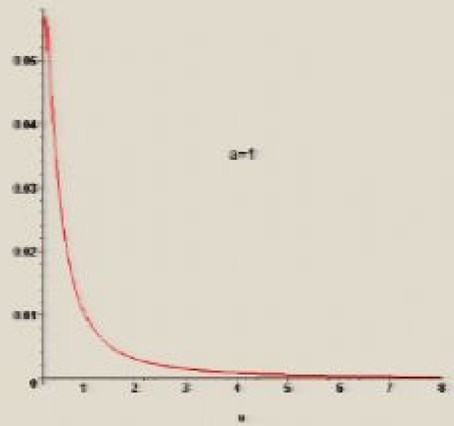
- Both backgrounds have the same physical temperature

$$\beta' \ \exp\left(p - \frac{x}{2} + A\right)|_{\tau_R} = \beta_{b.h.} \exp(2z - 3a\,u)|_{u_R} \quad (15)$$

$$\tau_R = \frac{3}{2} \ln \left|\frac{2^{5/6}}{3^{1/2}} \epsilon^{-2/3} u_R^{-1/4}\right| \quad (16)$$

- Consider only the difference of actions

$$I = \beta_{b.h.} \lim_{\tau_R \to \infty} \left(\frac{\exp(2z - 3a\,u)_{b.h.}}{\exp\left(p - \frac{x}{2} + A\right)_{KS}} \int_0^{\tau_R} d\tau(\mathrm{KS}) - \int_{u_h}^{u_R} du(\mathrm{BlackHole}) \right) \quad (17)$$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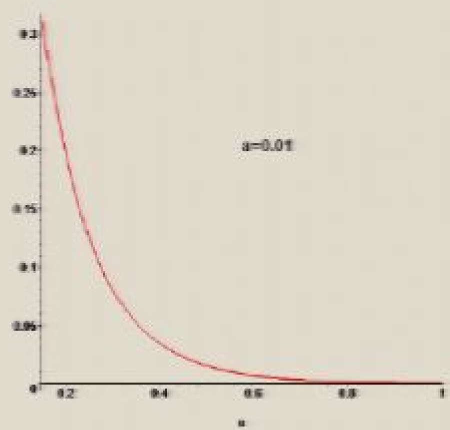
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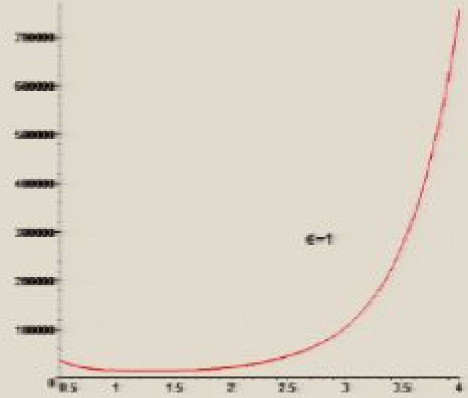
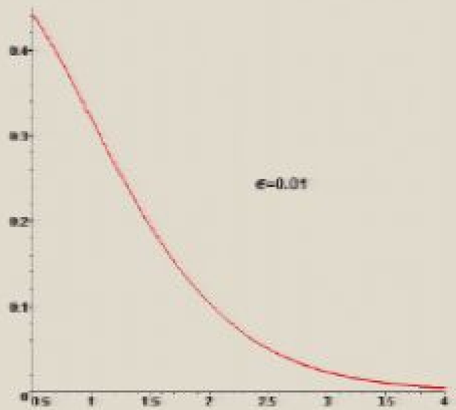

ε=1
0
0.5
1
1.5
2
2.5
3
3.5
4


0.4
0.3
0.2
0.1
0
ε=0.01
1
2.5
4

Expectations

- The relevant parameter: $a\ \epsilon^{-8/3}$.
- The jump in the number of the degrees of freedom should be there.

Conclusions

- The full phase space diagram: connection to weak coupling results and chemical potential
- The actual black hole in the KS background: A chance at chiral symmetry breaking versus confinement/deconfinement
- The introduction of dynamical fundamental flavor at the backreacted level.
- Supergravity is limited: Direct string calculation