Title: Isospin Diffusion in Thermal AdS/CFT with flavour

Date: May 24, 2007 04:20 PM

URL: http://pirsa.org/07050071

Abstract:

Motivation + Aim

Combine

AdS/CFT with added flavour

with

AdS/CFT applied to hydrodynamics

Here: Transport processes in the presence of flavour branes

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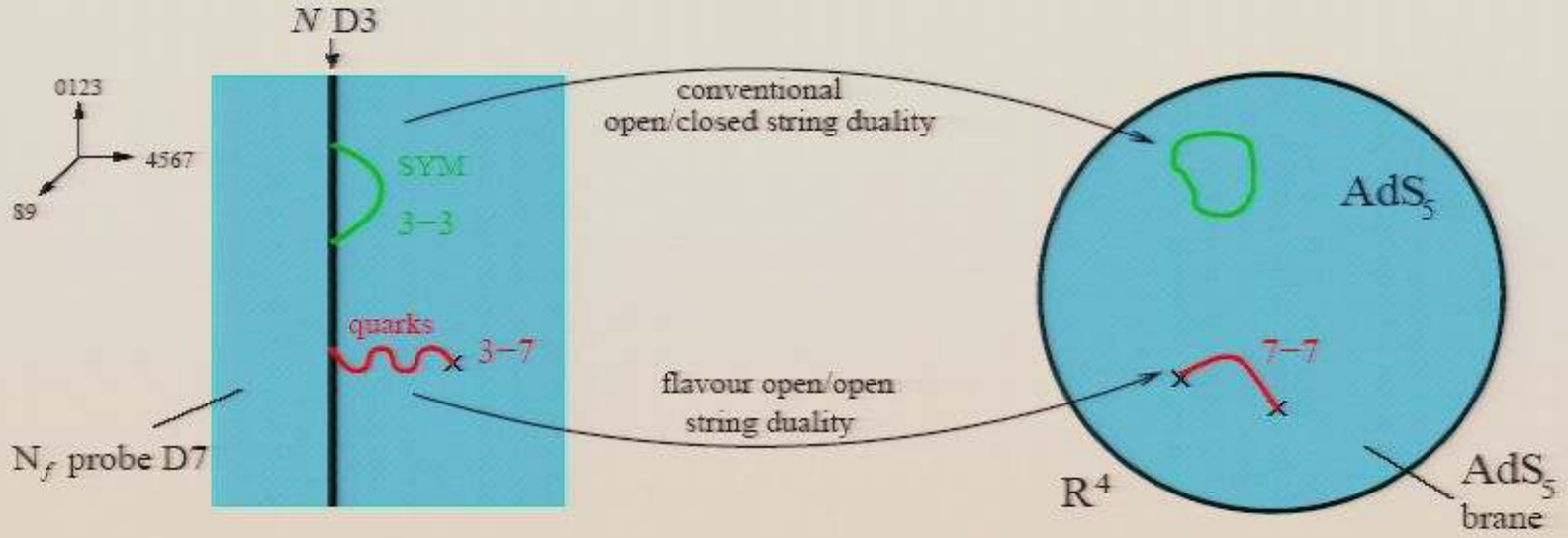
AdS/CFT with flavour



$N \to \infty$ (standard Maldacena limit), N_f small (probe approximation)

duality acts twice:

$\mathcal{N} = 4$ SU(N) Super Yang-Mills theory		IIB supergravity on $AdS_5 \times S^5$
coupled to	$\longleftrightarrow$	+
$\mathcal{N} = 2$ fundamental hypermultiplet		Probe brane DBI on $AdS_5 \times S^3$

Karch, Katz 2002

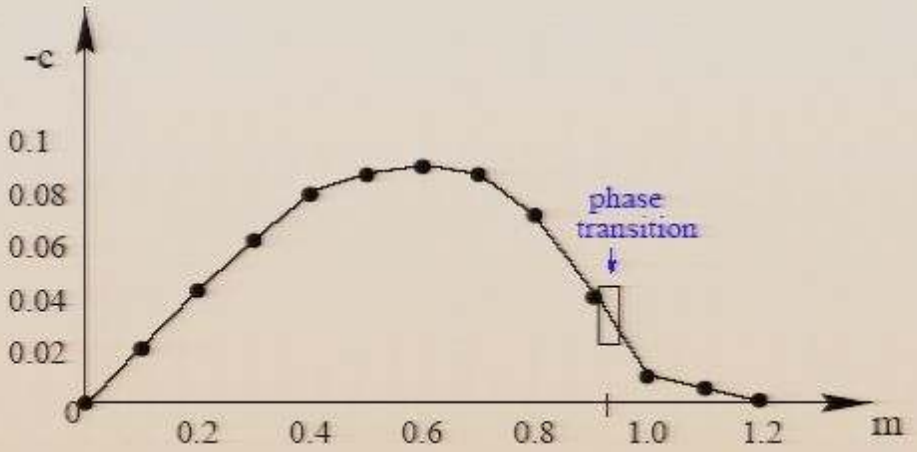
D7 brane embedding in black hole background

Babington, J.E., Evans, Guralnik, Kirsch 0306018

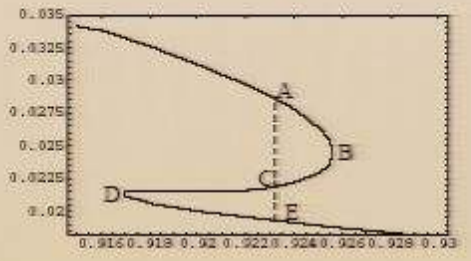
Embeddings

Condensate $c \equiv \langle \bar{\psi}\psi \rangle$ vs. quark mass m

m in units of T



Phase transition at $m_c \approx 0.92$
(1st order)



Kirsch 2004

Isospin chemical potential

Consider a probe of two coincident D7 branes

$SU(2)$ gauge theory on brane probe

Isospin chemical potential:

$$A_0 = \begin{pmatrix} \mu & 0 \\ 0 & -\mu \end{pmatrix}$$

consistent with
equations of motion from Dirac-Born-Infeld action for brane probe

Consider fluctuations about this background:

$$A^a_\nu \to \mu \delta_{\nu 0} \delta^{a3} + A^a_\nu$$

Fluctuations dual to $SU(2)$ flavour current in dual field theory

Fluctuations and vector-vector correlations

1. Establish action and equations of motion of $SU(2)$ gauge field fluctuations about isospin background

   from D7 brane Dirac-Born-Infeld action for embedding with $m_{quark} = 0$

2. Evaluate Green functions for flavour-flavour correlator in momentum space

   Gravity dual calculation of correlator - analytical approach of Policastro, Son, Starinets hep-th/0205052

3. Interpret result by comparing to hydrodynamics

   Frequency-dependent diffusion

Minkowski signature AdS black hole metric

$$ds^2 = \frac{b^2 R^2}{u}\left(-f(u)\,dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + \frac{R^2}{4u^2 f(u)}du^2 + R^2 d\Omega_5^2,$$

$$0 \le u \le 1$$

with

$$f(u) = 1 - u^2, \qquad R^4 = 4\pi g_s N_c \alpha'^2, \qquad b = \pi T.$$

Action

Dirac-Born-Infeld action

$$S_{\mathrm{D7}} = -T_7 \frac{(2\pi\alpha')^2}{4} 2\pi^2 R^3 \int_{u_b=0}^{u_h=1} \mathrm{d}u\, \mathrm{d}^4x\, \sqrt{-g}\, g^{\mu\mu'}\, g^{\nu\nu'}\, F^a_{\mu\nu}\, F^a_{\mu'\nu'}$$

With isospin chemical potential to order $\mathcal{O}(\mu)$:

$$S_{\mathrm{D7}} = -T_7 \frac{(2\pi\alpha')^2}{4} 2\pi^2 R^3 \times \int_{u_b=0}^{u_h=1} \mathrm{d}u\, \mathrm{d}^4x\, \sqrt{-g} g^{\mu\mu'}\, g^{\nu\nu'} \left(4\partial_{[\mu} A^a_{\nu]}\, \partial_{[\mu'} A^a_{\nu']} - 8\delta_{0\nu'} f^{abc} \partial_{[0} A^a_{\mu]}\, A^b_{\mu'}\, \mu^c \right)$$

$\mu^c = \mu\delta^{3c}$

$m_{quark} = 0$, probe does not extend into 8,9 directions

Calculation method

Son, Starinets 0205051

Action

$$S_{\rm cl} = \int {\rm d}u {\rm d}^4x\, B(u)\, (\partial_u A)^2 + \dots$$

Fourier transform boundary coordinates

Bulk-to-boundary propagator

$$A_\mu(u,\vec{k}) \;=\; f_{\mu i}(u,\vec{k})\, A_i{}^b(\vec{k})$$

Green function

$$G(\omega,\boldsymbol{q}) \;=\; -2\, B(u)\, f(u,-\vec{k})\, \partial_u\, f(u,\vec{k})\Big|_{u\to 0}$$

(for individual components)

R charge current

Policastro, Son, Starinets 0205052

Action:

$$S_{\mathrm{Super-Maxwell}} = -\frac{N^2}{16\pi^2}\int \mathrm{d}u\mathrm{d}^4x\sqrt{-g(u)}F_{\mu\nu}F^{\mu\nu}$$

Fix the gauge $A_u \equiv 0$, choose frame $\vec{k} = (\omega, 0, 0, q)$
Application of calculation method gives

$$G_{03}(\omega, q) = -\frac{{N_c}^2 T}{16\pi}\frac{\omega q}{i\omega - D\boldsymbol{q}^2} + \dots$$

Diffusion coefficient

$$D = \frac{1}{2\pi T}$$

Hydrodynamics

$$G(\omega, \boldsymbol{q}) \propto \frac{1}{i\omega - D\boldsymbol{q}^2}$$

Green function for diffusion equation

$$\partial_0 J_0(t, \boldsymbol{x}) = D \nabla^2 J_0(t, \boldsymbol{x})$$

J_0 current time component

Linear response theory: $\boldsymbol{J} = -D\boldsymbol{\nabla} J_0$

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Policastro, Son, Starinets 0205052

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R charge chemical potential

Son, Starinets 0601127

gauged $U(1)$ supergravity

$U(1)$ chemical potential built into gravity background

$$G_{11}(\omega, q) = \frac{i\kappa {N_c}^2 T^2 \omega}{16(i\omega - Dq^2)} - \frac{i{N_c}^2 T\omega}{16\pi} + \dots$$

Isospin current correlator

Apply calculation method to $SU(2)$ DBI action with isospin chemical potential

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Equations of motion for A_1 and A_2:

$$0 = A_\alpha^{a\prime\prime} + \frac{f'}{f} A_\alpha^{a\prime} + \frac{\mathfrak{w}^2 - f\mathfrak{q}^2}{uf^2} A_\alpha^a + 2i \frac{\mathfrak{w}}{uf^2} f^{abc} \frac{\mu^b}{2\pi T} A_\alpha^c$$

$$\mathfrak{w} = \frac{\omega}{2\pi T}, \qquad \mathfrak{q} = \frac{q}{2\pi T}, \qquad \mathfrak{m} = \frac{\mu}{2\pi T}.$$

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Solving equations of motion

3×5 component equations of motion

coupled through Lorentz and through flavour indices

Decoupling ansatz:

$$X_i = A_i^1 + iA_i^2,$$
$$\widetilde{X}_i = A_i^1 - iA_i^2$$

$$A_i = A_i^a \frac{\sigma^a}{2} = \frac{1}{2}\begin{pmatrix} A_i^3 & A_i^1 - iA_i^2 \\ A_i^1 + iA_i^2 & -A_i^3 \end{pmatrix}$$

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$$0 = X_\alpha'' + \frac{f'}{f} X_\alpha' + \frac{\mathfrak{w}^2 - f\mathfrak{q}^2 \mp 2\mathfrak{m}\mathfrak{w}}{uf^2} X_\alpha, \qquad \alpha = 1, 2$$

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Solving equations of motion

Ansatz

$$X_i = (1-u)^{\beta} F(u), \qquad \widetilde{X}_i = (1-u)^{\widetilde{\beta}} \widetilde{F}(u)$$

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Approximation:

$$\beta \approx \mp\sqrt{\frac{\mathfrak{w}\mathfrak{m}}{2}}, \qquad \widetilde{\beta} \approx \pm i\sqrt{\frac{\mathfrak{w}\mathfrak{m}}{2}}$$

$\widetilde{\beta}$: infalling wave boundary condition
β: modes decay towards horizon

$$F(u) = F_0 + \mathfrak{w}^{1/2} F_{1/2} + \mathfrak{w} F_1 + \mathfrak{q}^2 G_1 + \dots ,$$

insert this into eom and solve order by order in $\mathfrak{q}$, $\mathfrak{w}$

Green Functions

$$G_{0\tilde{0}} = \frac{N_c T}{8\pi} \frac{2\pi T\, \mathfrak{q}^2}{-\sqrt{2\mathfrak{m}\mathfrak{w}} - \mathfrak{q}^2 - \mathfrak{w}\mathfrak{m}\ln 2},$$

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To lowest order, $G_{\tilde{0}0} + G_{0\tilde{0}}$ has pole at

$$\sqrt{\omega} = -i \frac{q^2}{2\pi T \sqrt{2\mu}}$$

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To lowest order, $G_{\tilde{0}0} + G_{0\tilde{0}}$ has pole at

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Green Functions

$$G_{0\tilde{0}} = -\frac{N_c T}{8\pi\sqrt{2\mu}} \frac{q^2\sqrt{\omega}}{\omega + q^2 D(\omega)}, \quad G_{\tilde{0}0} = \frac{N_c T}{8\pi\sqrt{2\mu}} \frac{q^2\sqrt{\omega}}{i\omega - q^2 D(\omega)}$$

with frequency-dependent diffusion coefficient $D(\omega)$

$$D(\omega) = \sqrt{\frac{\omega}{2\mu}} \frac{1}{2\pi T}$$

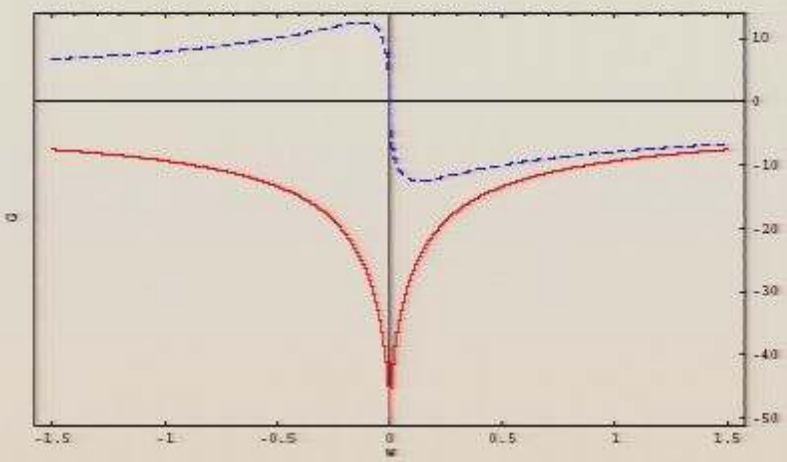
Diffusion equation replaced by

$$\partial_0 J_0(t, \boldsymbol{x}) + \nabla^2 \int_{-\infty}^{\infty} ds\, J_0(s, \boldsymbol{x}) D(t - s) = 0$$

memory function

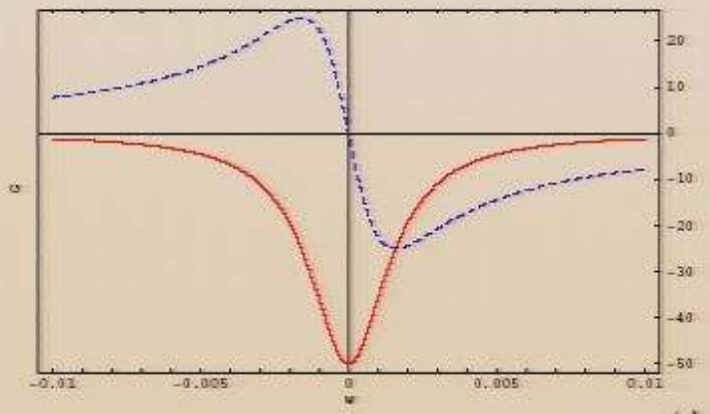
Real and Imaginary part of Green function

$\left(G_{0\tilde{0}} + G_{\tilde{0}0}\right)/T^2$



$\frac{\omega}{T}\, 10^{-4}$

$G_{A_0^3 A_0^3}/T^2$



$\frac{\omega}{T}$

$q/T = 0.1, N_c = 100, \mu/T = 0.2$

Further Green function components

$$G_{3\tilde{3}} = -\frac{N_c T}{8\pi\sqrt{2\mu}} \frac{\omega^{3/2}(\omega-\mu)}{\tilde{Q}(\omega,q)}, \qquad G_{\tilde{3}3} = \frac{N_c T}{8\pi\sqrt{2\mu}} \frac{\omega^{3/2}(\omega+\mu)}{Q(\omega,q)},$$

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$$Q(\omega,q) = i\omega - q^2 D(\omega), \qquad \tilde{Q}(\omega,q) = \omega + q^2 D(\omega)$$

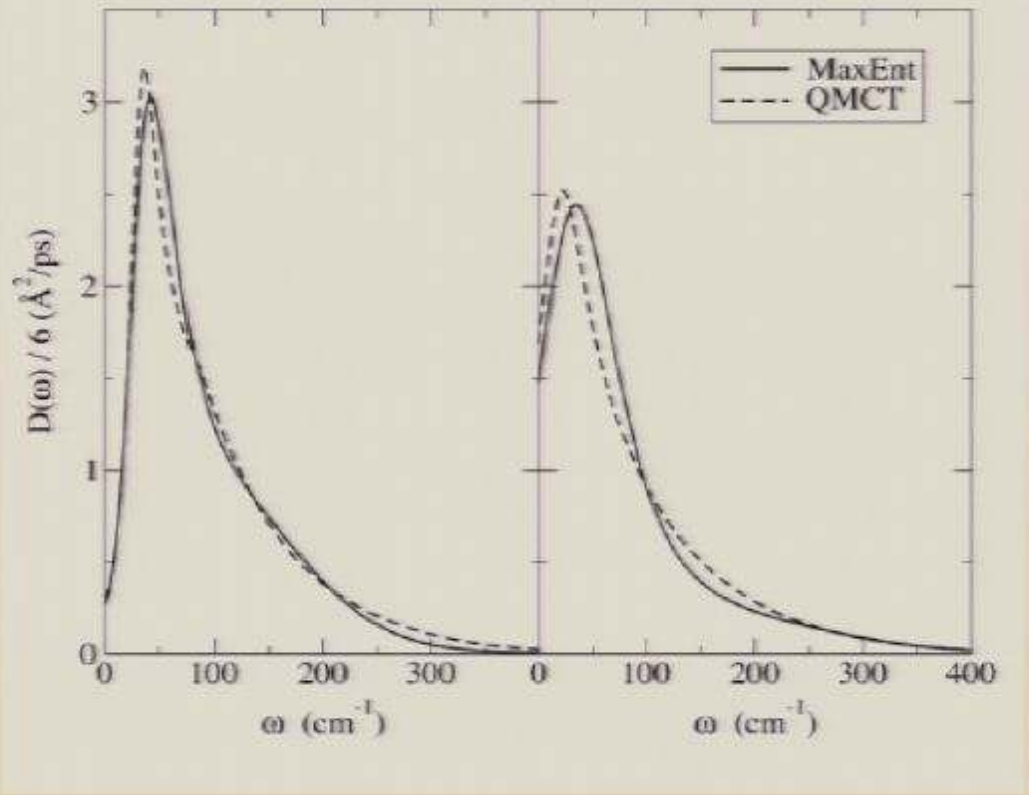
$$G_{1\tilde{1}} = G_{2\tilde{2}} = \frac{\sqrt{2} N_c T}{8\pi}\sqrt{\mu\omega}, \quad G_{\tilde{1}1} = G_{\tilde{2}2} = -\frac{i\sqrt{2} N_c T}{8\pi}\sqrt{\mu\omega}$$

Components pointing along isospin in flavour space:

$$G_{A_0^3 A_0^3} = \frac{N_c T}{4\pi} \frac{q^2}{i\omega - D_0 q^2}$$

Comparison to quantum liquid theory

Frequency-dependent diffusion



E. Rabani et al: Diffusion in para-Hydrogen, PRL 87 (2001) 265702.

Interpretation

$SU(2)$ isospin: u- and d-quarks in Yang-Mills plasma

Chemical potential: Energy necessary to flip isospin

Diffusion frequency dependent:

Relaxation of isospin fluctuations in plasma non-exponential

Interpretation

cf. Nick Evans' talk: Bose-Einstein condensation

Apreda, J.E., Evans, Guralnik 0504151

In presence of instanton on D7 brane within supergravity:

Isospin chemical potential leads to Bose-Einstein instability (even at $T = 0$)

Higgs VEV $\Leftrightarrow$ Instanton size

$$A_m = \frac{Q^2 \bar{\sigma}_{mn} y_n}{y^2(y^2 + Q^2)}$$

$$V(Q) = -T_7 \frac{32\pi^2 \alpha'^2}{g_s} \mu^2 Q^2$$

y_m coordinates in 4,5,6,7 directions

Conclusion

- Two coincident D7 brane probes in AdS black hole background
- Isospin chemical potential $A_0 = \begin{pmatrix} \mu & 0 \\ 0 & -\mu \end{pmatrix}$
- leads to effects beyond linear response theory when comparing to hydrodynamics

  due to terms with $\partial_t \to \mu$ in equations of motion
- frequency-dependent isospin diffusion
- in simplest possible case ($\omega < \mu < T$): $D \propto \sqrt{\frac{\omega}{\mu}}$

Outlook

- more involved embeddings ($m_{quark} \neq 0$)
- find $D(\omega)$ for all values of ω, q
- generally look for frequency dependence in gravity dual description of plasma
- Within gauge/gravity duality descriptions of hydrodynamics, there are phenomena beyond linear response theory.

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Further Green function components

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$$G_{0\tilde{3}} = -\frac{N_c T}{8\pi\sqrt{2\mu}} \frac{\sqrt{\omega}\, q(\omega-\mu)}{\tilde{Q}(\omega,q)}, \qquad G_{\tilde{0}3} = \frac{N_c T}{8\pi\sqrt{2\mu}} \frac{\sqrt{\omega}\, q(\omega+\mu)}{Q(\omega,q)}$$

$$Q(\omega,q) = i\omega - q^2 D(\omega), \qquad \tilde{Q}(\omega,q) = \omega + q^2 D(\omega)$$

$$G_{1\tilde{1}} = G_{2\tilde{2}} = \frac{\sqrt{2} N_c T}{8\pi}\sqrt{\mu\omega}, \quad G_{\tilde{1}1} = G_{\tilde{2}2} = -\frac{i\sqrt{2} N_c T}{8\pi}\sqrt{\mu\omega}$$

Components pointing along isospin in flavour space:

$$G_{A_0^3 A_0^3} = \frac{N_c T}{4\pi} \frac{q^2}{i\omega - D_0 q^2}$$

Green Functions

$$G_{0\tilde{0}} = -\frac{N_c T}{8\pi\sqrt{2\mu}} \frac{q^2\sqrt{\omega}}{\omega + q^2 D(\omega)}, \quad G_{\tilde{0}0} = \frac{N_c T}{8\pi\sqrt{2\mu}} \frac{q^2\sqrt{\omega}}{i\omega - q^2 D(\omega)}$$

with frequency-dependent diffusion coefficient $D(\omega)$

$$D(\omega) = \sqrt{\frac{\omega}{2\mu}} \frac{1}{2\pi T}$$

Diffusion equation replaced by

$$\partial_0 J_0(t, \boldsymbol{x}) + \nabla^2 \int_{-\infty}^{\infty} ds\, J_0(s, \boldsymbol{x}) D(t - s) = 0$$

memory function

Further Green function components

$$G_{3\tilde{3}} = -\frac{N_c T}{8\pi\sqrt{2\mu}} \frac{\omega^{3/2}(\omega-\mu)}{\tilde{Q}(\omega,q)}, \qquad G_{\tilde{3}3} = \frac{N_c T}{8\pi\sqrt{2\mu}} \frac{\omega^{3/2}(\omega+\mu)}{Q(\omega,q)},$$

$$G_{0\tilde{3}} = -\frac{N_c T}{8\pi\sqrt{2\mu}} \frac{\sqrt{\omega}\, q(\omega-\mu)}{\tilde{Q}(\omega,q)}, \qquad G_{\tilde{0}3} = \frac{N_c T}{8\pi\sqrt{2\mu}} \frac{\sqrt{\omega}\, q(\omega+\mu)}{Q(\omega,q)}$$

$$Q(\omega,q) = i\omega - q^2 D(\omega), \qquad \tilde{Q}(\omega,q) = \omega + q^2 D(\omega)$$

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Components pointing along isospin in flavour space:

$$G_{A_0^3 A_0^3} = \frac{N_c T}{4\pi} \frac{q^2}{i\omega - D_0 q^2}$$

Outlook

- more involved embeddings ($m_{quark} \neq 0$)
- find $D(\omega)$ for all values of ω, q
- generally look for frequency dependence in gravity dual description of plasma
- Within gauge/gravity duality descriptions of hydrodynamics, there are phenomena beyond linear response theory.