

Title: Effective Potentials, Thermodynamics and Twisted Mass Quarks

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Abstract:

Effective potentials, thermodynamics, and twisted mass

Michael Creutz

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Two powerful non-perturbative tools

- lattice gauge theory
- effective chiral Lagrangians

Combining them can give qualitative insight

- clarifies lattice distortions of chiral symmetry
 - Aoki phase
- stimulates development of new lattice actions
 - domain wall, overlap
- exposes the evils of rooted determinants

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Thermal QCD

- lattice gives quantitative information on deconfinement
- chiral symmetry restoration coincides with deconfinement
- a playground for understanding lattice chiral symmetry



This talk is on lattice artifacts

- interplay of twisted mass Wilson fermions with deconfinement
- intricate phase structure expected
- follow up to hep-lat/9608024

Why care about artifacts?

- understanding lattice limitations
 - failure of rooting
- understanding chiral symmetry
 - how does the anomaly work on the lattice
 - how are masses of confined constituents defined
- gain insight into why chiral gauge theories are so hard
- an amusing non-trivial phase structure

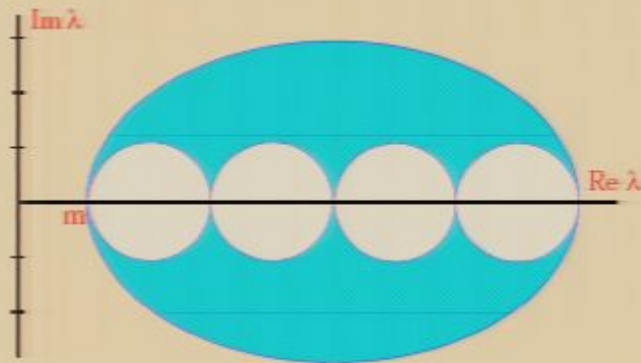


Brief review of Wilson fermions

- lattice makes things periodic in momenta
 - $p_\mu \rightarrow \frac{1}{a} \sin(p_\mu a)$
 - doublers at $p_\mu \sim \pi/a$

Wilson gives doublers a momentum dependent mass

- free case $D_W(p) = 1/a + \frac{2K}{a} \sum_\mu [i\gamma_\mu \sin(p_\mu a) - \cos(p_\mu a)]$
 - fermion mass $m \sim (1/2K - 4)/a$ vanishes at $K = K_c = 1/8$
 - eigenvalues on "nested circles"



Gauge fields on links as usual

- gauge action a sum over plaquettes
 - $S_g = \frac{\beta}{3} \sum_p \text{Re Tr } U_p$
- dynamics fill in the eigenvalue pattern
- real eigenvalues related to topology

Parameters β and K related to bare coupling and quark mass

- $\beta \sim 6/g_0^2$
- $(1/K - 1/K_c) \sim m_q a$
 - $K = 0$ corresponds to infinite quark mass
 - continuum limit: $K \rightarrow K_c + O(m_q a)$

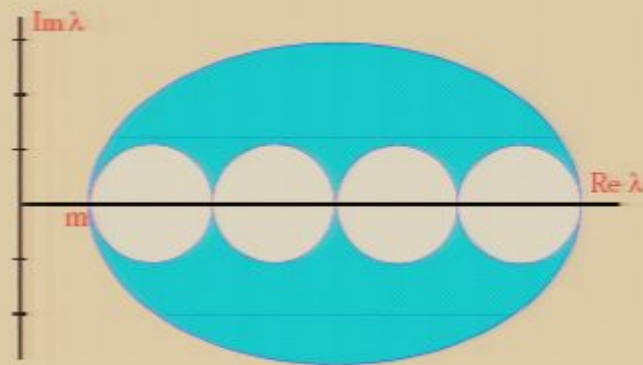


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Concentrate on two flavor QCD

- 3 flavors somewhat more complicated
- restrict discussion to degenerate quarks
- only consider $\Theta_{CP} = 0$

Physical parameters

- Λ_{qcd}
 - sets the scale, use it for units
 - coupling g absorbed via asymptotic freedom
- quark mass $\frac{m}{\Lambda_{qcd}}$
- temperature $\frac{T}{\Lambda_{qcd}}$
- ultraviolet regulator adds a cutoff scale $a\Lambda_{qcd}$, "lattice spacing"



Lattice parameters with Wilson fermions

- coupling $\beta = 6/g_0^2$
- hopping parameter K
- temporal extent N_t
- lattice spacing a sets scale

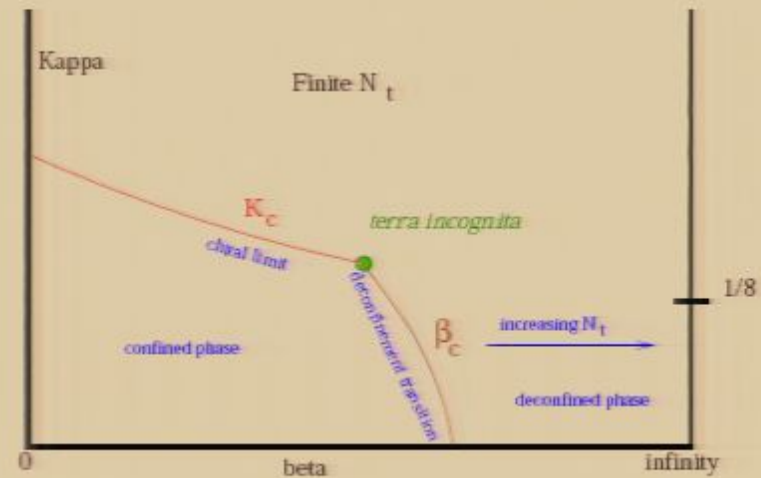
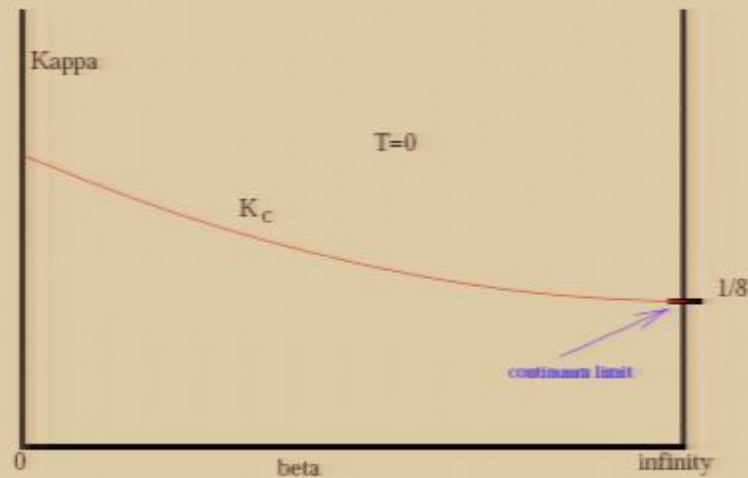
Non-linear mapping $\left\{ \frac{m}{\Lambda_{qcd}}, \frac{T}{\Lambda_{qcd}}, a\Lambda_{qcd} \right\} \longleftrightarrow \{\beta, K, N_t\}$



- well understood

Continuum limit $a\Lambda_{qcd} \rightarrow 0$

- $\beta = 6/g_0^2 \rightarrow \infty$ at a rate tied to Λ_{qcd}
- $K \rightarrow K_c \rightarrow 1/8$ tied to quark mass m
- $N_t = 1/aT \rightarrow \infty$

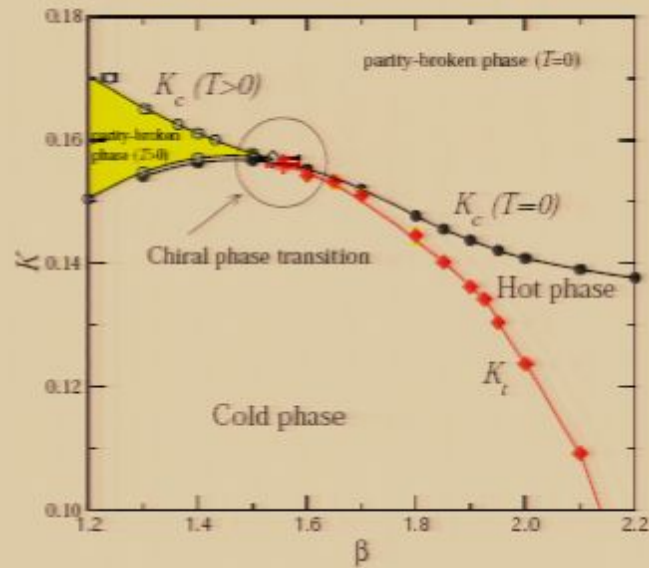


Lattice artifacts

- introduce further structure
- the subject of this talk

CP-PACS (hep-lat/0610038)

- $N_t = 4$

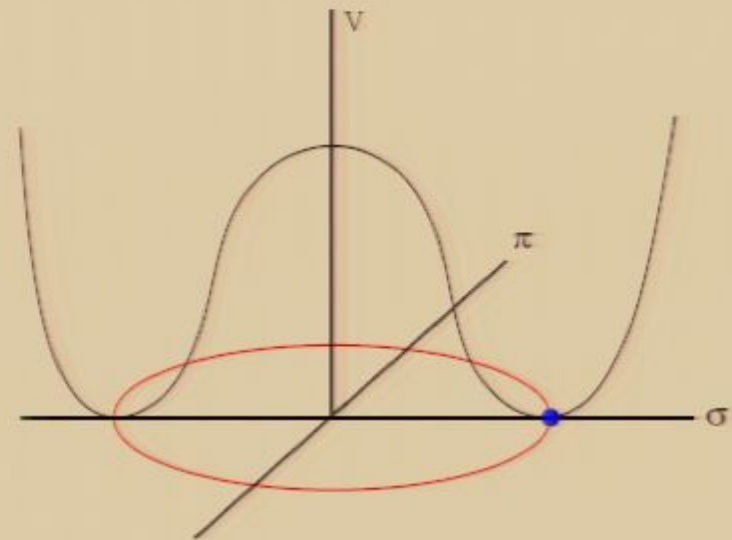


Some recent preliminary APE results (hep-lat/0610112)

- $N_t = 8$ including twisted mass
- more in Regensburg?

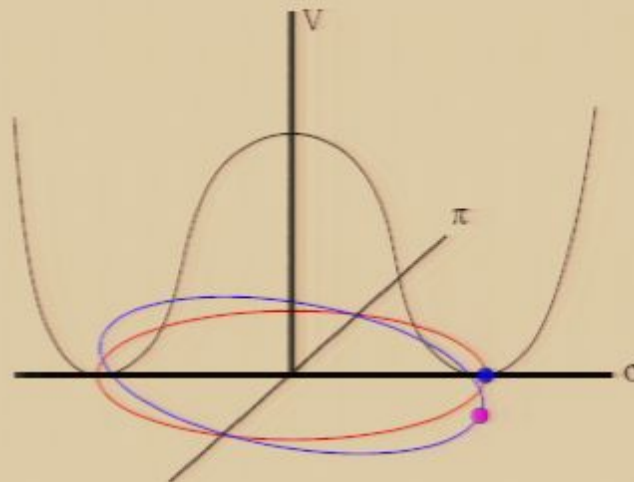
Effective chiral Lagrangians

- $\vec{\pi} \sim i\bar{\psi}\gamma_5\vec{\tau}\psi$
- $\sigma \sim \bar{\psi}\psi$
- $V_0 = \lambda(\sigma^2 + \vec{\pi}^2 - v^2)^2$
- chiral symmetry
 - $O(4)$ rotations of $\Sigma = (\sigma, \vec{\pi})$
 - V_0 invariant
- vacuum spontaneously breaks the chiral symmetry
 - $\langle\sigma\rangle \neq 0$ when $v^2 > 0$
 - pions are Goldstone bosons



Mass term $V_1 = -m\sigma$

- breaks chiral symmetry
- tilts potential to select unique vacuum
- $\langle \sigma \rangle > 0$ $\langle \vec{\pi} \rangle = 0$
- $m_\pi^2 \sim m$



Chiral rotation

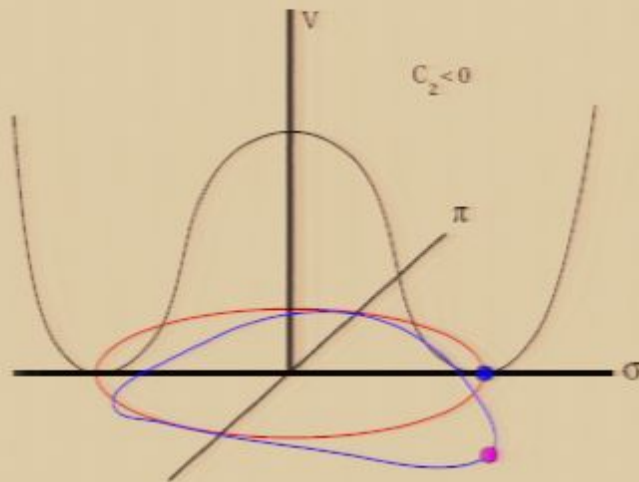
- equivalent physics with $m\sigma \rightarrow m \cos(\theta)\sigma + m \sin(\theta)\pi_3$
- lattice artifacts will break this \rightarrow "twisted mass"

Lattice artifacts

Break chiral symmetry

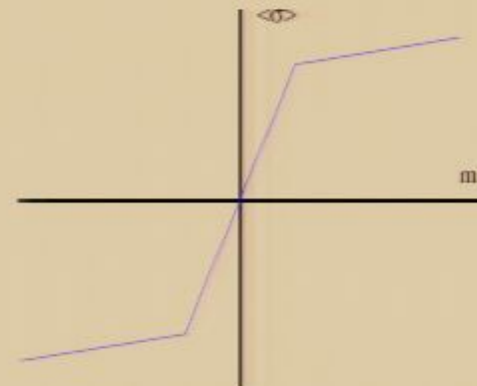
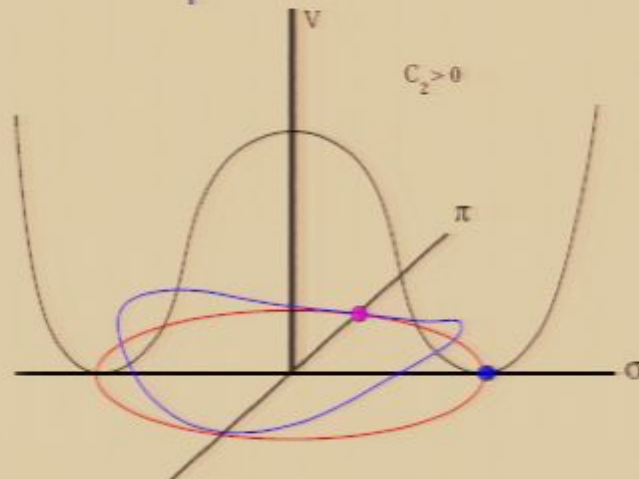
- lowest correction additively renormalizes mass
- $K_c \rightarrow K_c(\beta)$
- $m \rightarrow (1/K - 1/K_c(\beta))$
- model next correction with $c_2\sigma^2$
 - sign of c_2 can depend on gauge action
 - $c_2 < 0$ chiral transition goes first order
 - no exact Goldstone bosons

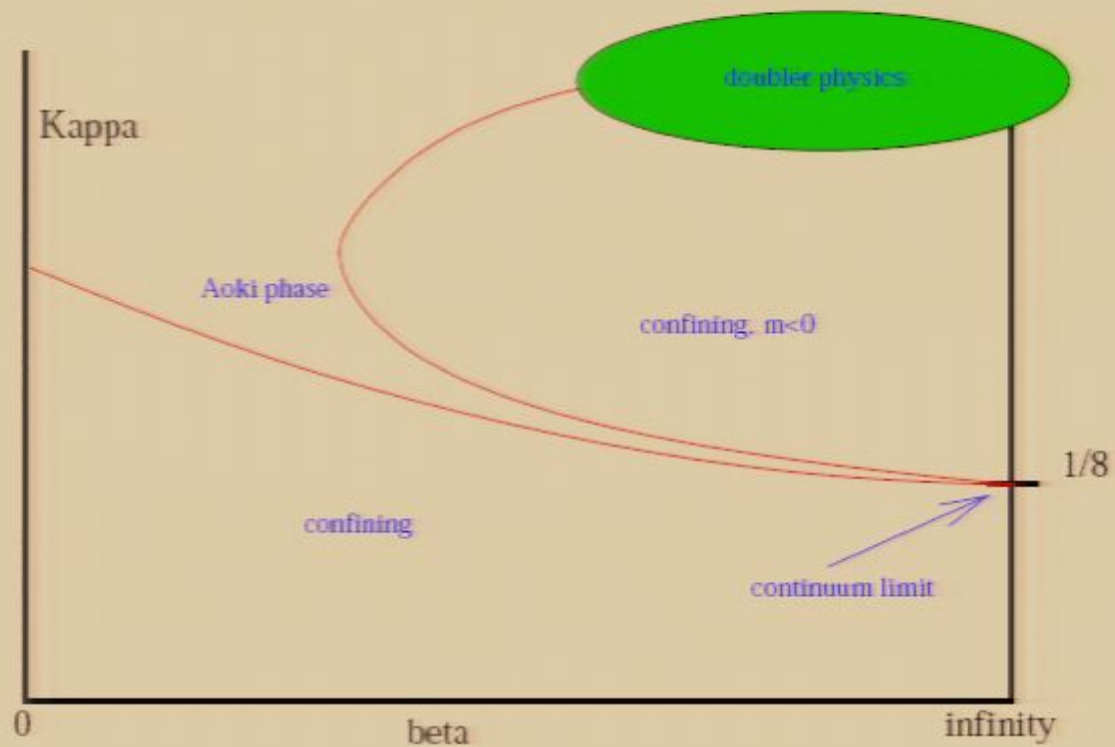
(MC; Sharpe and Singleton)



If $c_2 > 0$ chiral transition splits into two second order transitions

- separated by phase with $\langle \vec{\pi} \rangle \neq 0$
 - breaks parity and flavor spontaneously
 - two Goldstone bosons from flavor breaking
 - third massless pion only at critical point
- the "Aoki phase"



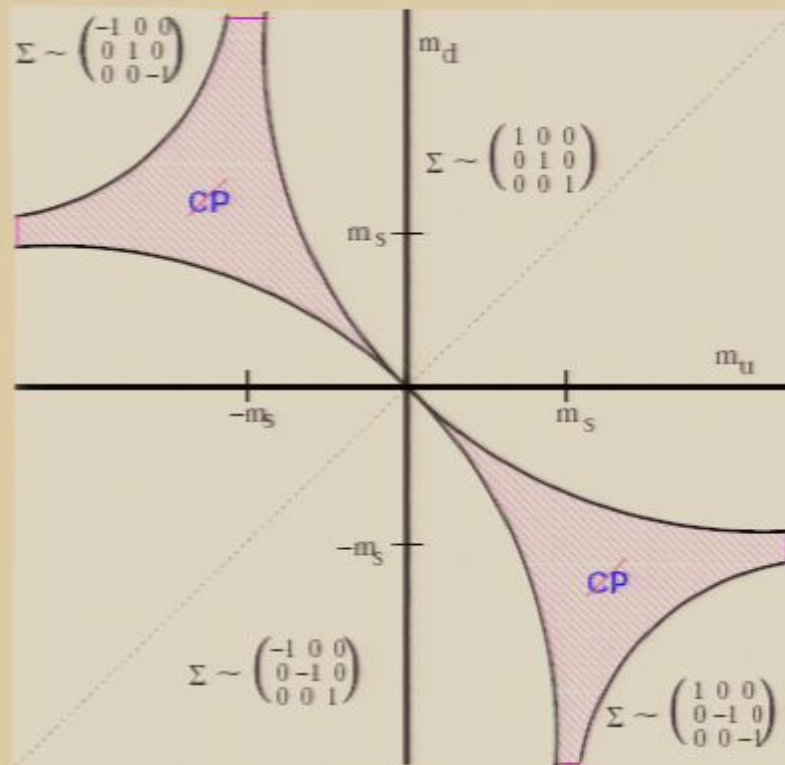


Add c_2 term to the effective model

- $$V(\vec{\pi}, \sigma, L) = \lambda(\sigma^2 + \vec{\pi}^2 - v^2)^2 - (1/K - 1/K_c(\beta))\sigma + c_2\sigma^2$$

Comment

- Aoki phase survives continuum limit if $m_u \neq m_d$
 - at fixed $m_s \neq 0$:



Twisted Mass

The c_2 term breaks the equivalence of different chiral directions

- no longer equivalent physics with $m\sigma \rightarrow m \cos(\theta)\sigma + m \sin(\theta)\pi_3$
- equivalent to giving up and down quarks opposite phases
 - $m\bar{\psi}\psi \rightarrow \frac{1}{2}(m\bar{\psi}_L\psi_R + m^*\bar{\psi}_R\psi_L)$
 - $m_u \rightarrow e^{i\theta}m_u \quad m_d \rightarrow e^{-i\theta}m_d$
- on the lattice add new mass term
 - $\mu i\bar{\psi}\tau_3\gamma_5\psi \sim \mu\pi_3$

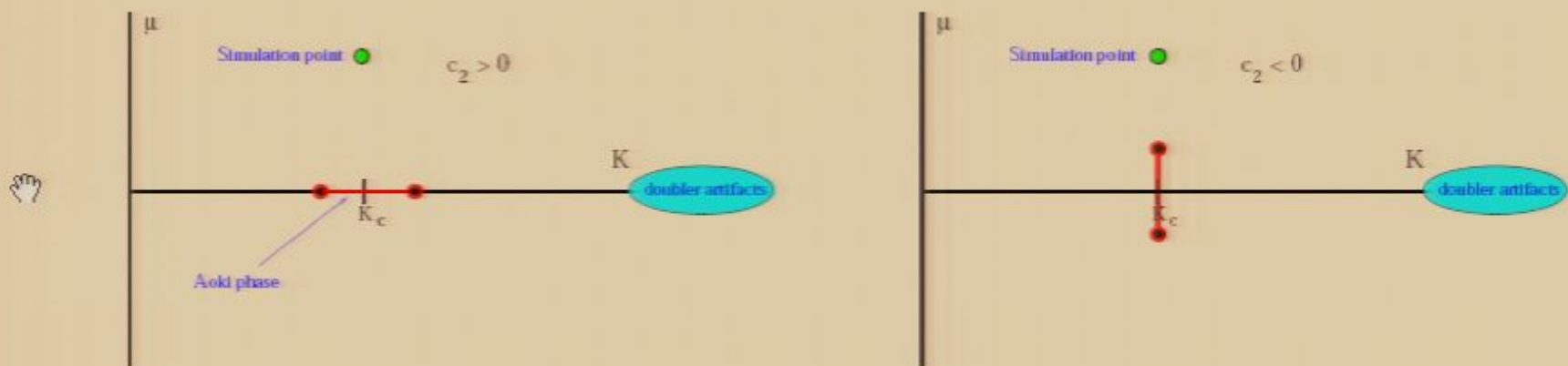
$$V(\vec{\pi}, \sigma) = \lambda(\sigma^2 + \vec{\pi}^2 - v^2)^2 - (1/K - 1/K_c(\beta))\sigma + c_2\sigma^2 - \mu\pi_3$$

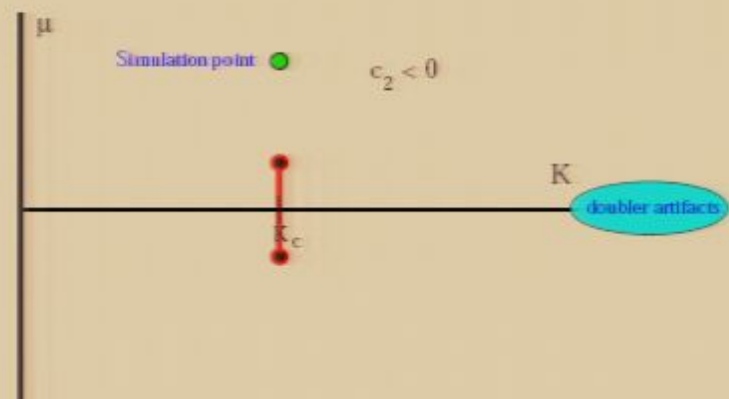
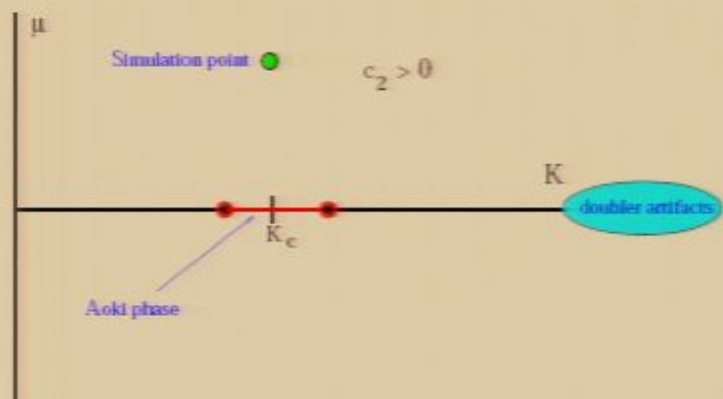


"Magnetic field" conjugate to the Aoki phase order parameter

Motivations

- $O(a)$ lattice artifacts cancel
- fermion determinant positive
- faster than overlap or domain wall
- avoid diseases of staggered quarks
- allows continuation around Aoki phase





Difficulties

- need to know K_c
- odd N_f ?
 - $m \leftrightarrow -m$ symmetry broken by anomalies
- mass needs to be larger than c_2 artifacts
- chiral symmetry still not exact



Deconfinement

Wilson-Polyakov line

- $L(\vec{x}) = \text{Re Tr } \mathcal{T} \exp(i \int_0^{1/T} A_0(\vec{x}) dt)$ continuum
- $L(\vec{x}) = \text{Re Tr } \mathcal{T} \prod_t U_{t+1,t}(\vec{x})$ lattice
- symmetry of pure gauge theory under $L \rightarrow e^{2\pi i/3} L$
- $\langle L \rangle$ vanishes at low temperature and $m = \infty$
- symmetry broken by quarks when $m < \infty$

Quark condensate $\langle \bar{\psi}\psi \rangle \sim \langle \sigma \rangle$

- vanishes at high temperature and $m = 0$
- $V(\vec{\pi}, \sigma)$ evolves to a single minimum.

Both $\langle L \rangle$ and $\langle \bar{\psi}\psi \rangle$ show rapid change over a small region of T

- first order for large m
 - like 3 state Potts model
- probably second order for $m = 0$

- like an $SO(4)$ sigma model

- first order not ruled out

(D'Elia, Di Giacomo, Pica)

- either crossover or phase transition for intermediate m



Effective model for the Wilson line

(Svetitsky, Yaffe, Gocksch, Ogilvie, Pisarski ...)

- complex field $L(x)$

$$V(L) = \alpha_1 |L|^4 + \alpha_2 (T_c - T) |L|^2 - \alpha_3 \text{Re}(L^3) - \alpha_4 \text{Re}L$$

- α_2 drives the transition
- α_3 term reduces symmetry to Z_3
 - drives transition towards first order
- α_4 term models fermion breaking of Z_3
 - can soften transition to a crossover

Changing to lattice parameters $T_c - T \rightarrow \beta_c(K, N_t) - \beta$



$$V(L) = \alpha_1 |L|^4 + \alpha_2 (\beta_c(K, N_t) - \beta) |L|^2 - \alpha_3 \text{Re}(L^3) - \alpha_4 \text{Re}L$$

Merging the models

(Mocsy, Sannino, Tuominen, Ratti, Thaler, Roessner, Weise, ...)

$$\begin{aligned} V(\vec{\pi}, \sigma, L) = & \lambda(\sigma^2 + \vec{\pi}^2 - v^2)^2 - (1/K - 1/K_c(\beta))\sigma + c_2\sigma^2 - \mu\pi_3 \\ & + \alpha_1|L|^4 + \alpha_2(\beta_c(K) - \beta)|L|^2 - \alpha_3\text{Re}(L^3) - \alpha_4\text{Re}L \\ & + \alpha_5|L|^2(\sigma^2 + \vec{\pi}^2) \end{aligned}$$

The α_5 term couples the chiral fields and the loop field

- jump in $|L|$ can turn off the chiral symmetry breaking



Two unknown functions: $K_c(\beta)$ and $\beta_c(K)$

- for simplicity ignore K_c dependence on N_t before deconfinement

Increasing quark mass increases deconfinement temperature

Continuum physics independent of twisting

- an "inverted umbrella" or "conical" structure in m, μ, T space
- symmetric under rotations about T axis
- possibly a crossover

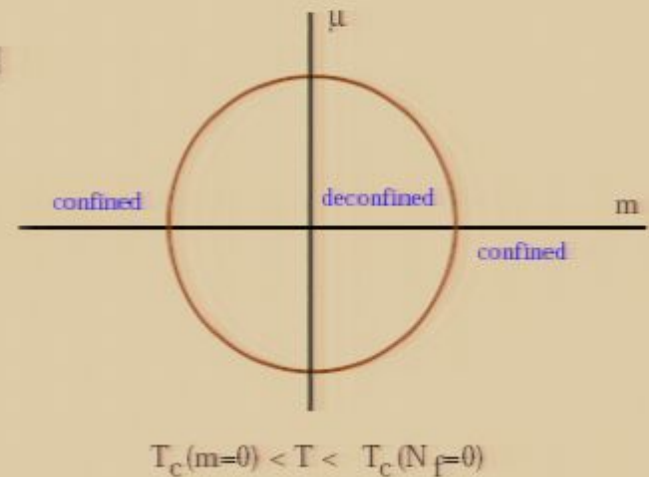


Consider an intermediate temperature where

- massless theory deconfined
- massive one still confined
- horizontal slice through the "cone"

Continuum physics independent of twisting

- $m \leftrightarrow -m$ connected



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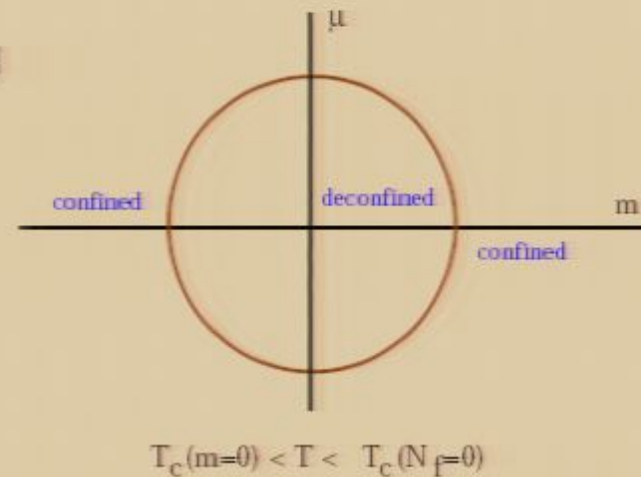


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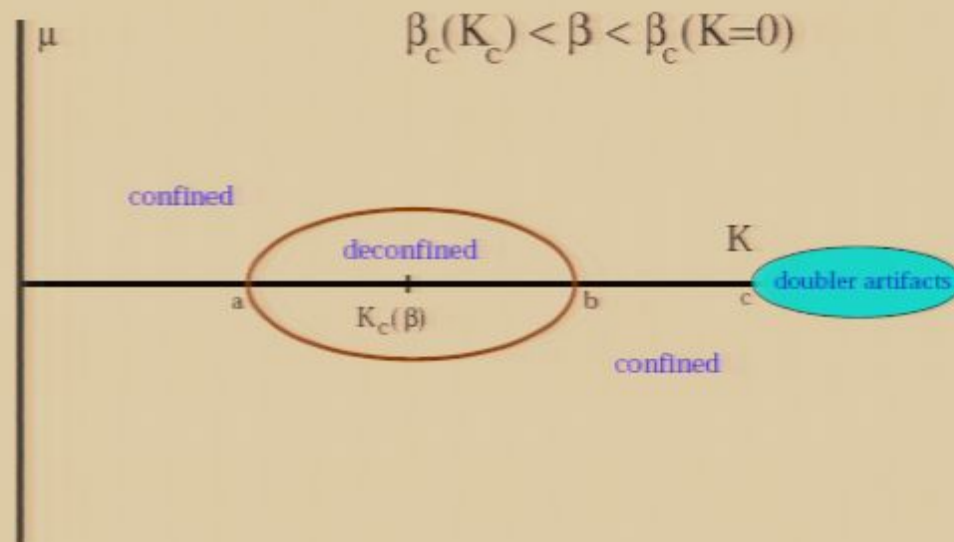
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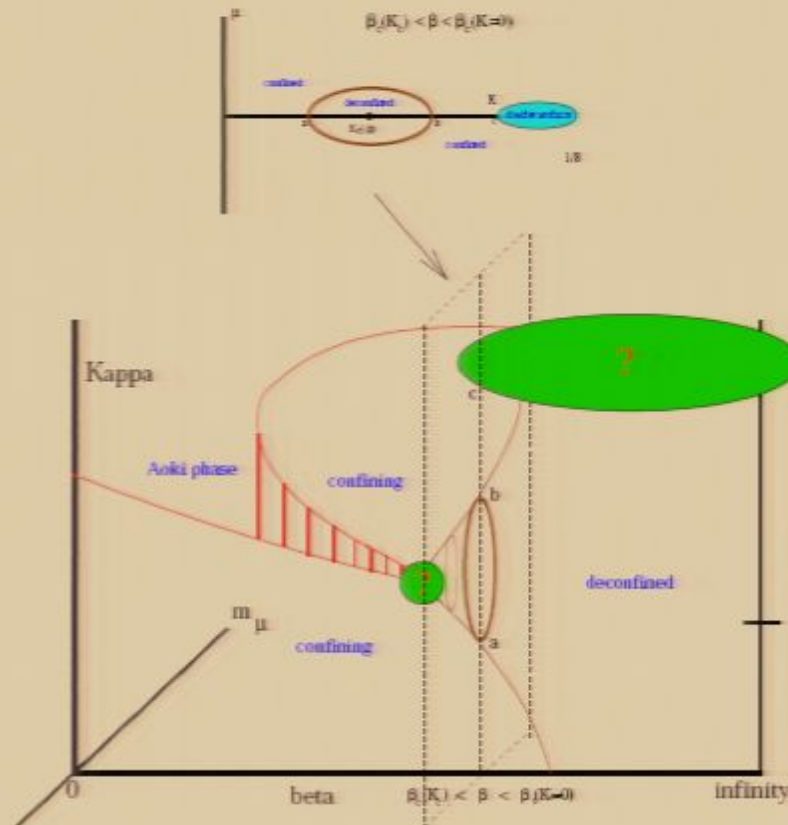
Lattice distorts the equivalent physics circle



- Aoki phase washed out in deconfined region
 - $V(\sigma, \pi)$ has a simple and unique minimum

Deconfinement becomes a "conical" structure in β, K, m_μ space

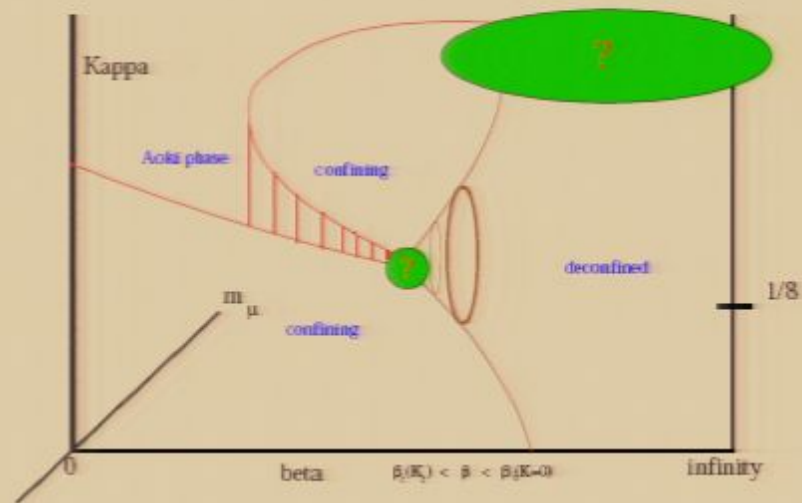
- at fixed β , deconfinement at constant $\sqrt{c\mu^2 + (1/K - 1/K_c)^2}$



Conclusions

Complex phase structure tied to lattice artifacts

- should study $K > K_c$ in more detail
 - second deconfinement line expected
- twisted mass connects phases
 - at fixed β : elliptical structure in μ, K plane



Questions

$N_f = 3$?

- parity broken phase becomes physical: $m < 0 \Rightarrow \Theta = \pi$
- possible twists in λ_3 and λ_8 directions
 - i.e: $m_u \sim e^{2\pi i/3}$, $m_d \sim e^{-2\pi i/3}$, $m_s \sim 1$

$N_f = 1$?

- no chiral symmetry
 - quark condensate loses meaning
- critical kappa and zero quark mass decoupled
- parity broken phase at sufficiently negative mass
- nothing to twist?

