Title: Spectral Functions in High Temperature QCD

Date: May 24, 2007 11:00 AM

URL: http://pirsa.org/07050066

Abstract:

Transport and spectral functions in high-temperature QCD

Gert Aarts

Physics Department, Swansea University

Dynamics and lattice QCD

PROGRESS

- standard statement:

real time dynamics is difficult on the lattice

nevertheless:

- (very) recent progress in various directions

hep-lat/0703008, 0705.2198 [hep-lat]

Chris Allton, Justin Foley, Simon Hands, Seyong Kim,
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DYNAMICS AND LATTICE QCD

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Dynamics and lattice QCD

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- mesons in a heatbath with nonzero momentum
- charmonium above T_c in *dynamical* ($N_f = 2$) lattice QCD
- transport coefficients

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Dynamics and lattice QCD

PROGRESS

- mesons in a heatbath with nonzero momentum
- charmonium above T_c in *dynamical* ($N_f = 2$) lattice QCD
- transport coefficients:
  - analytical continuation: Maximum Entropy Method
  - standard MEM approach inherently unstable as $\omega \to 0$
  - how to fix this
  - stable and robust first results
  - conductivity: $\sigma/T = 0.4 \pm 0.1$ at $T \sim 1.5T_c$
  - note: only (potentially) applicable in strongly coupled theories, weakly coupled theories is hopeless

(G.A. & J.M.Martinez Resco 2002)

DISCLAIMER

Disclaimer

- $\mathcal{N} = 0$
- nonconformal
- asymptotically free
- $N_c = 3$
- $N_f = 0.2$

OUTLINE

- spectral functions from lattice QCD
- cold start: overlap and domain wall fermions at $T = 0$
- high temperature: spectral functions at non-zero momentum
- a new approach $\Leftarrow$
- first results for the electrical conductivity
- charmonium in $N_f = 2$ QCD on hot, anisotropic lattices
- summary

TRANSPORT COEFFICIENTS

IN QCD

- energy momentum: shear viscosity η, bulk viscosity ξ
- electric charge: electrical conductivity σ — $\partial_t \rho(t, \mathbf{x}) \simeq -\sigma \rho(t, \mathbf{x})$
- global (flavour) charges: diffusion coefficients D — $\partial_t n(t, \mathbf{x}) \simeq D \nabla^2 n(t, \mathbf{x})$

transport coefficients σ, η, ζ, D characterize the dynamics of long wavelength, low frequency fluctuations in the QGP

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transport coefficients σ, η, ζ, D characterize the dynamics of long wavelength, low frequency fluctuations in the QGP

transport coefficients $\sim$ mean free path

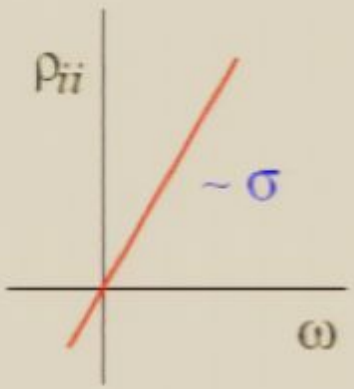
strongly interacting quark-gluon plasma: transport coefficients expected to be small

KUBO RELATIONS

LINEAR RESPONSE

electrical conductivity: $$\sigma = \frac{1}{6}\frac{\partial}{\partial\omega}\rho^{ii}(\omega,\mathbf{0})\Big|_{\omega=0}$$

shear viscosity: $$\eta = \frac{1}{20}\frac{\partial}{\partial\omega}\rho_{\pi\pi}(\omega,\mathbf{0})\Big|_{\omega=0}$$



spectral densities:

$$\rho^{\mu\nu}(\omega,\mathbf{p}) = \int d^4x\, e^{ipx}\langle[j^\mu(x), j^\nu(0)]\rangle_{\rm eq}$$

$$\rho_{\pi\pi}(\omega,\mathbf{p}) = \int d^4x\, e^{ipx}\langle[\pi_{ij}(x), \pi_{ij}(0)]\rangle_{\rm eq}$$

with $j^\mu = \bar\psi\gamma^\mu\psi, \quad \pi_{ij} = T_{ij} - \frac{1}{3}\delta_{ij}T^k_k$

transport coefficients $\sim$ slope of current-current spectral functions at $\omega = 0$

SPECTRAL FUNCTIONS

AND LATTICE CORRELATORS

- use dispersion relation:

$$G_E(i\omega_n) = \int_0^\infty \frac{d\omega}{2\pi} \frac{\rho(\omega)}{\omega - i\omega_n}$$

or

$$G_E(\tau) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega)\rho(\omega)$$

with the kernel

$$K(\tau,\omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

- revived: spectral functions from lattice QCD using the maximum entropy method (MEM)

Asakawa & Hatsuda (2000)

LATTICE QCD

AND SPECTRAL FUNCTIONS

program:

- compute lattice correlators $G_E(\tau)$ numerically
- finite amount of information at $0 \leq \tau/a < N_\tau$
- "invert" integral equation

$$G_E(\tau) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega)\rho(\omega) \qquad K(\tau,\omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

- using Maximum Entropy Method

Zero temperature spectral functions

G.A. and J. Foley [UKQCD], JHEP (2007)

warm-up problem:

meson correlators at zero temperature

- confinement: bound states
- ground state: $G(\tau) \sim \exp(-M\tau) \Leftrightarrow \rho(\omega) \sim \delta(\omega - M)$
- UKQCD and RBC: dynamical QCD with $2+1$ flavours of domain wall fermions, generated on QCDOC

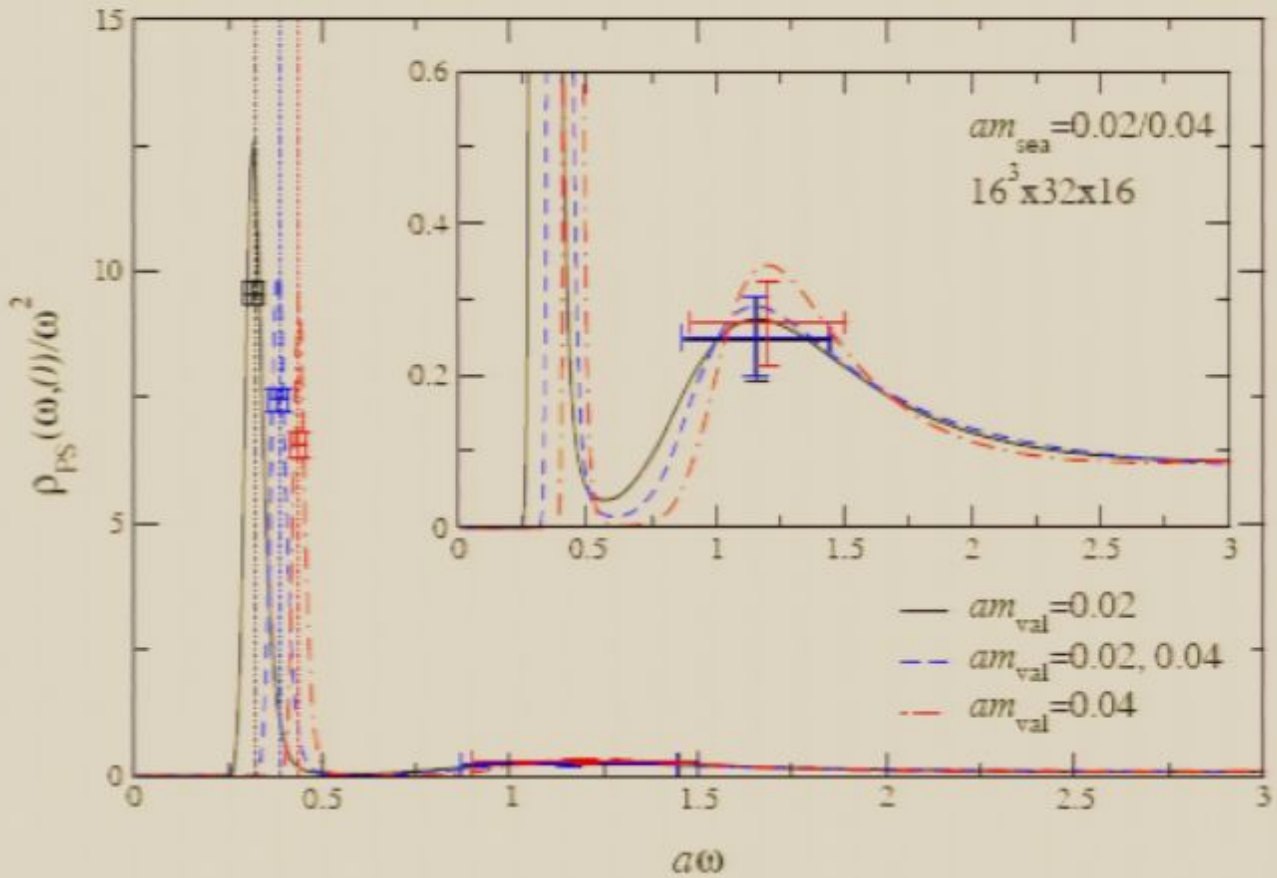
$16^3 \times 32$ with $N_s = 16$ in fifth dimension

$\beta = 2.13$ (Iwasaki action) $a^{-1} \sim 1.6$ GeV

DOMAIN WALL QCD WITH $2+1$ FLAVOURS

G.A. AND J. FOLEY [UKQCD], JHEP (2007)

pseudoscalar channel

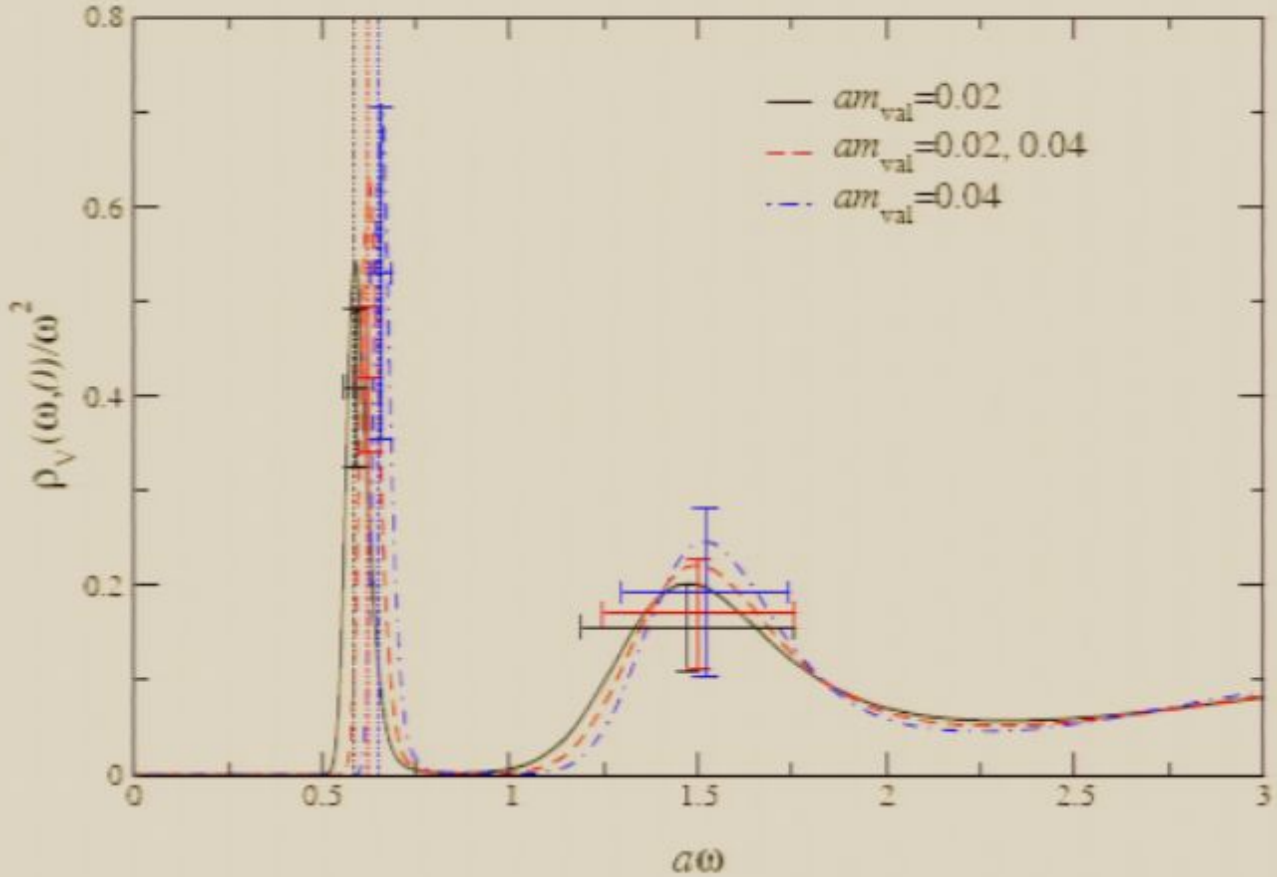


dotted lines: conventional cosh fits

DOMAIN WALL QCD WITH $2+1$ FLAVOURS

G.A. AND J. FOLEY [UKQCD], JHEP (2007)

vector channel



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OVERLAP AND DOMAIN WALL FERMIONS

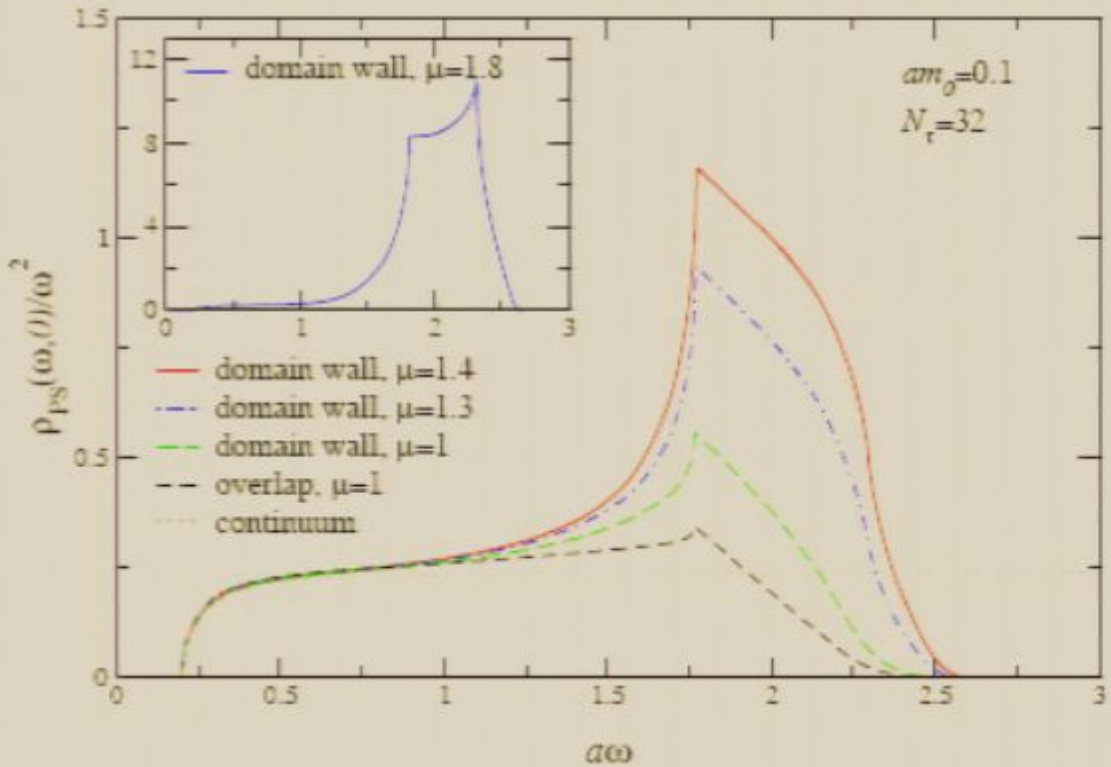
LATTICE ARTEFACTS

lattice artefacts at larger frequencies:

- difference between lattice and continuum dispersion relations
- study in free field limit
- Wilson, staggered, overlap, domain wall, hypercube overlap, ...

OVERLAP AND DOMAIN WALL FERMIONS

FREE FERMION STUDY



- high frequencies: edge of the Brillouin zone
- cusps when $1 < a\omega < 2$
- deviation from continuum spectral functions

finite temperature QCD

MESON SPECTRAL FUNCTIONS

INTEREST

meson spectral functions: euclidean correlators:

$$\rho_H(t,\mathbf{x}) = \langle [J_H(t,\mathbf{x}), J_H^\dagger(0,\mathbf{0})] \rangle \qquad G_H(\tau,\mathbf{x}) = \langle J_H(\tau,\mathbf{x}) J_H^\dagger(0,\mathbf{0}) \rangle$$

$$J_H(\tau,\mathbf{x}) = \bar{q}(\tau,\mathbf{x})\Gamma_H q(\tau,\mathbf{x}) \quad \text{with} \quad \Gamma_H = \{\mathbb{1}, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5\}$$

expectation:

- below T_c: mesons in the heatbath
  simple sharply peaked quasiparticle spectral functions
  at $\omega = \sqrt{p^2 + m^2}$ (+ subdominant continuum)
- (sufficiently far) above T_c: deconfined quarks
  only continuum contribution

QUENCHED QCD WITH STAGGERED FERMIONS

WITH CHRIS ALLTON, JUSTIN FOLEY, SIMON HANDS & SEYONG KIM

quenched QCD below and above T_c

- below T_c:
  $48^3 \times 24, \quad \beta = 6.5, \quad a \sim 0.05$ fm, $\quad T/T_c \sim 0.62$
- above T_c:
  $64^3 \times 24, \quad \beta = 7.192, \quad a \sim 0.02$ fm, $\quad T/T_c \sim 1.5$
  $64^3 \times 16, \quad \beta = 7.192, \quad a \sim 0.02$ fm, $\quad T/T_c \sim 2.25$

100 propagators analyzed

staggered quarks: $ma = 0.01, 0.05, 0.125$
$m/T = 0.24, 1.2, 3$

Meson spectral functions

MOTIVATION

- mesons moving in a heatbath
- hydrodynamic structure
- ...

meson spectral functions at non-zero momentum

Meson spectral functions

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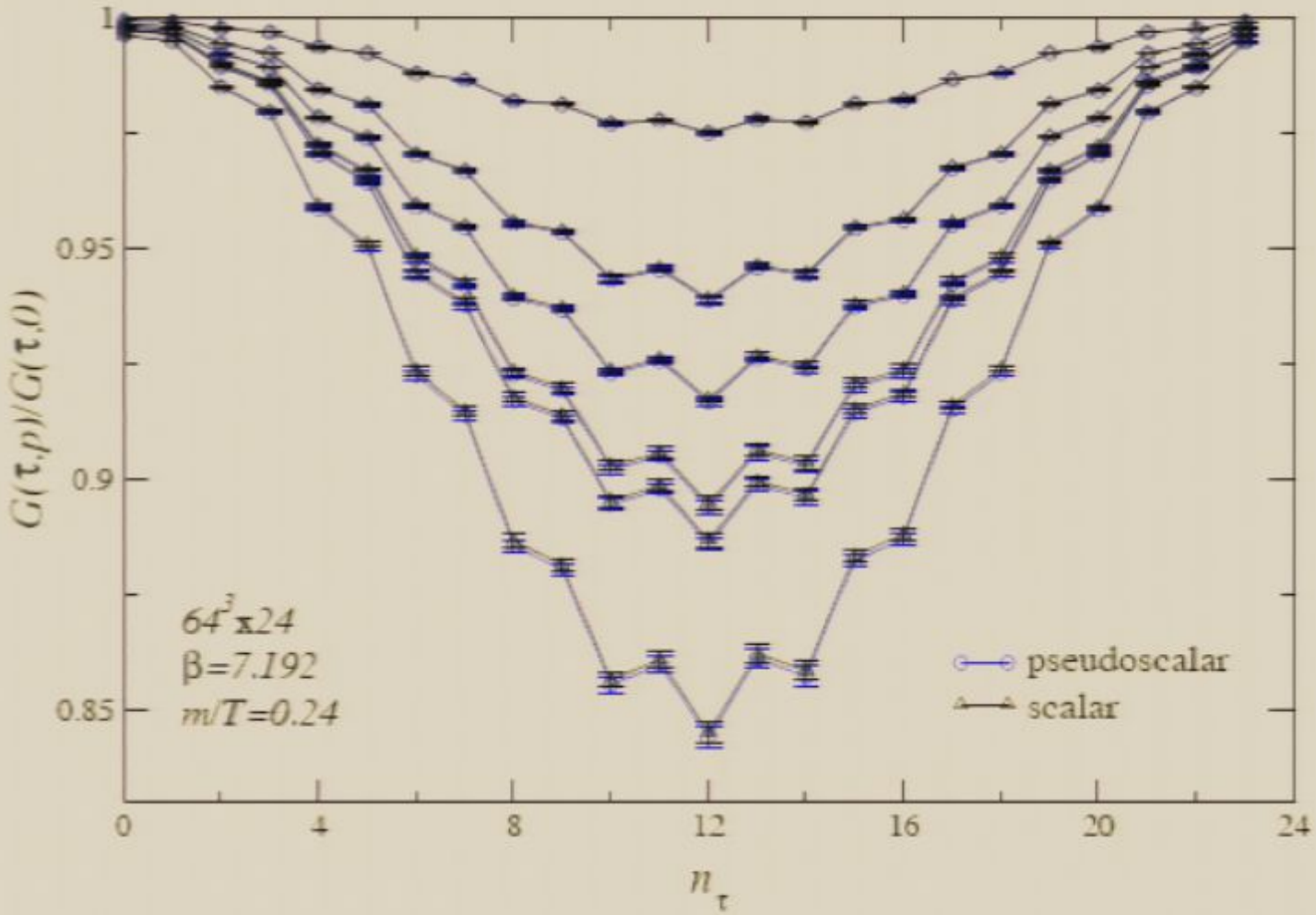
meson spectral functions at non-zero momentum

- momenta on the lattice: $\vec{p} = \frac{2\pi}{L}\vec{n}$
- use twisted boundary conditions
- quark field: $\psi(x_i + L) = e^{i\theta_i}\psi(x_i)$
- meson: two twist angles $\vec{\theta}_1, \vec{\theta}_2$
- meson momentum: $\vec{p} = \frac{2\pi}{L}\vec{n} - \frac{\vec{\theta}_1 - \vec{\theta}_2}{L}$
- ~ 20 momenta in the range $0 < p/T < 5$

HIGH TEMPERATURE QUENCHED QCD

SMALL (BARE) QUARK MASS, $m/T = 0.24$

hot, $T/T_c \sim 1.5$: ratio of correlators



chiral symmetry restoration at non-zero momentum

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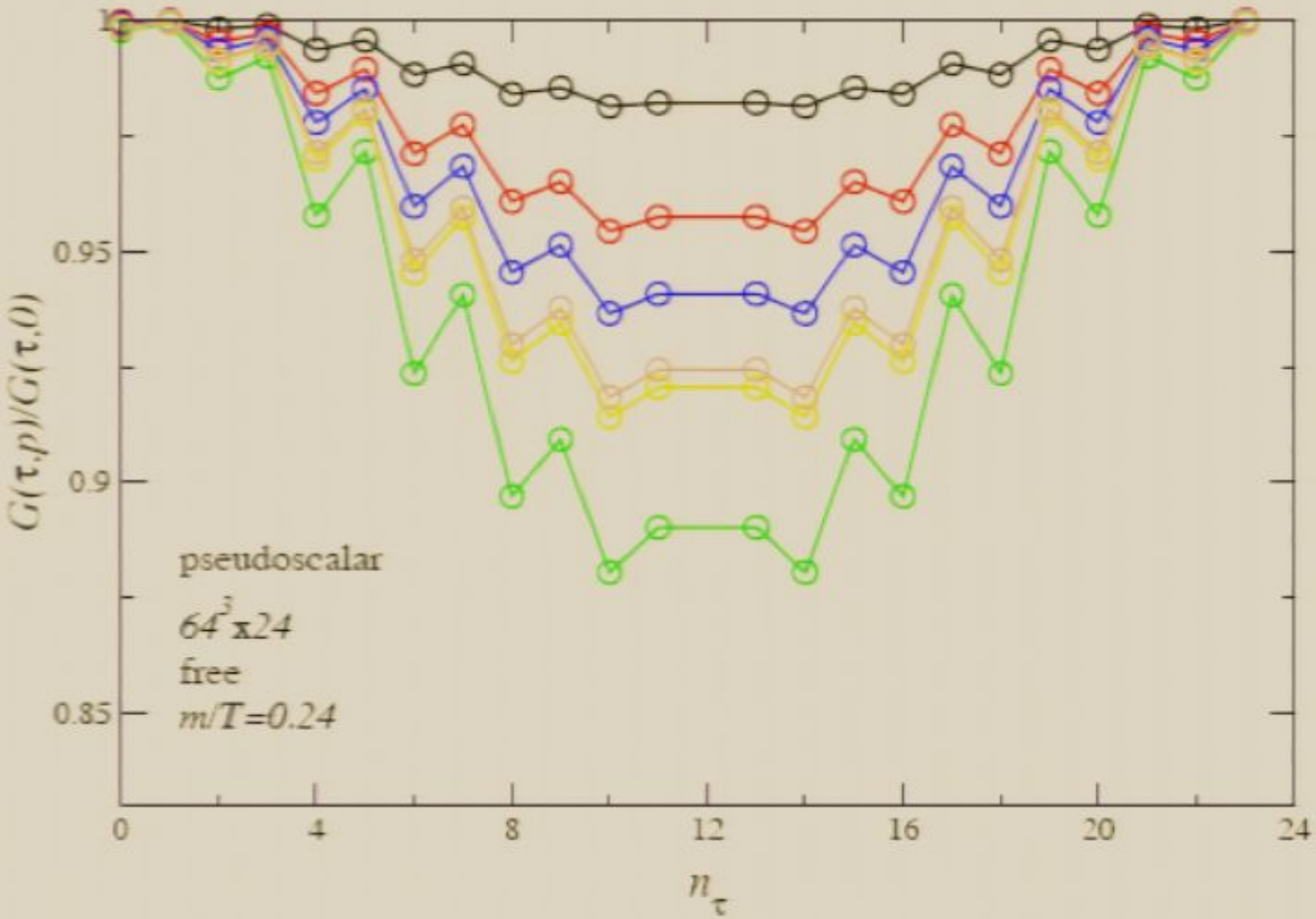

for comparison: free staggered quarks

HIGH TEMPERATURE QUENCHED QCD

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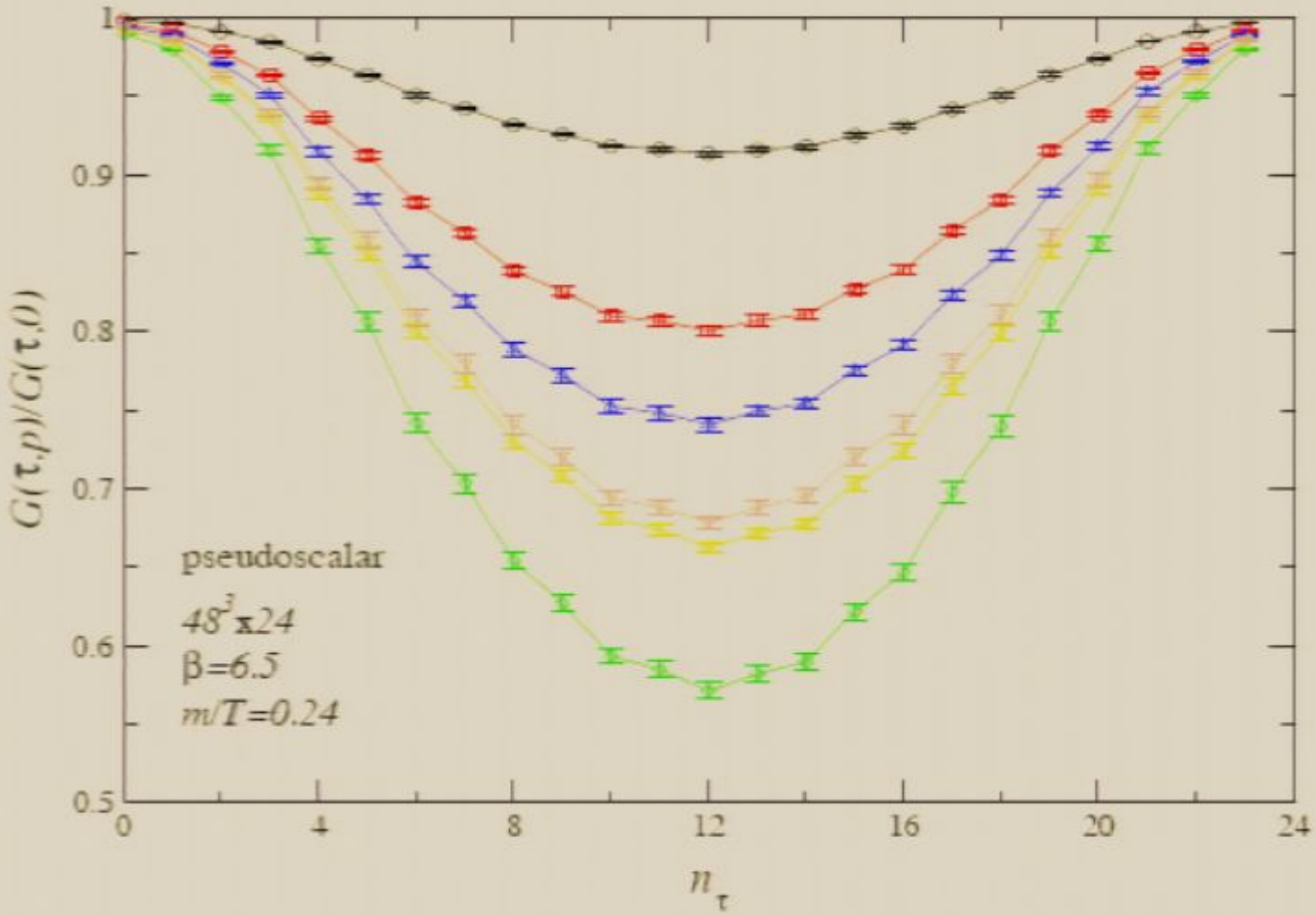
cold, $T/T_c \sim 0.6$



ratio in the confined phase: larger relative effect

MAXIMUM ENTROPY METHOD

RECONSTRUCT $\rho(\omega)$ FROM $G(\tau)$

solve ill-posed inversion problem:

$$G(\tau, \mathbf{p}) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega)\rho(\omega, \mathbf{p})$$

$$K(\tau, \omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

- $G(\tau)$ known at $\mathcal{O}(10)$ data points
- $\rho(\omega)$ needed at $\mathcal{O}(10^3)$ values, $0 < a\omega < a\omega_{\max} \sim 5$

provide (minimal amount of) prior information:

- positivity
- asymptotic behaviour

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Maximum entropy method

RECONSTRUCT $\rho(\omega)$ FROM $G(\tau)$

reconstruct *most probable* spectral function: $P[\rho|GH]$

probability to find ρ, given G and prior information H

identity for conditional probabilities: $P[\rho|GH] = \frac{P[G|\rho H]P[\rho|H]}{P[G|H]}$

- $P[G|\rho H] \sim e^{-L}$ likelihood (χ^2 fit)
- $P[\rho|H] \sim e^{\alpha S}$ prior probability, entropy

entropy term: $S = \int d\omega \left(\rho(\omega) - m(\omega) - \rho(\omega) \log\left[\rho(\omega)/m(\omega)\right]\right)$

- $m(\omega) = m_0\omega^2$: default model, prior information

extremize $P[\rho|GH] \sim \exp(-L + \alpha S)$

STAGGERED FERMIONS

STAGGERING EFFECT

spectral relation reads

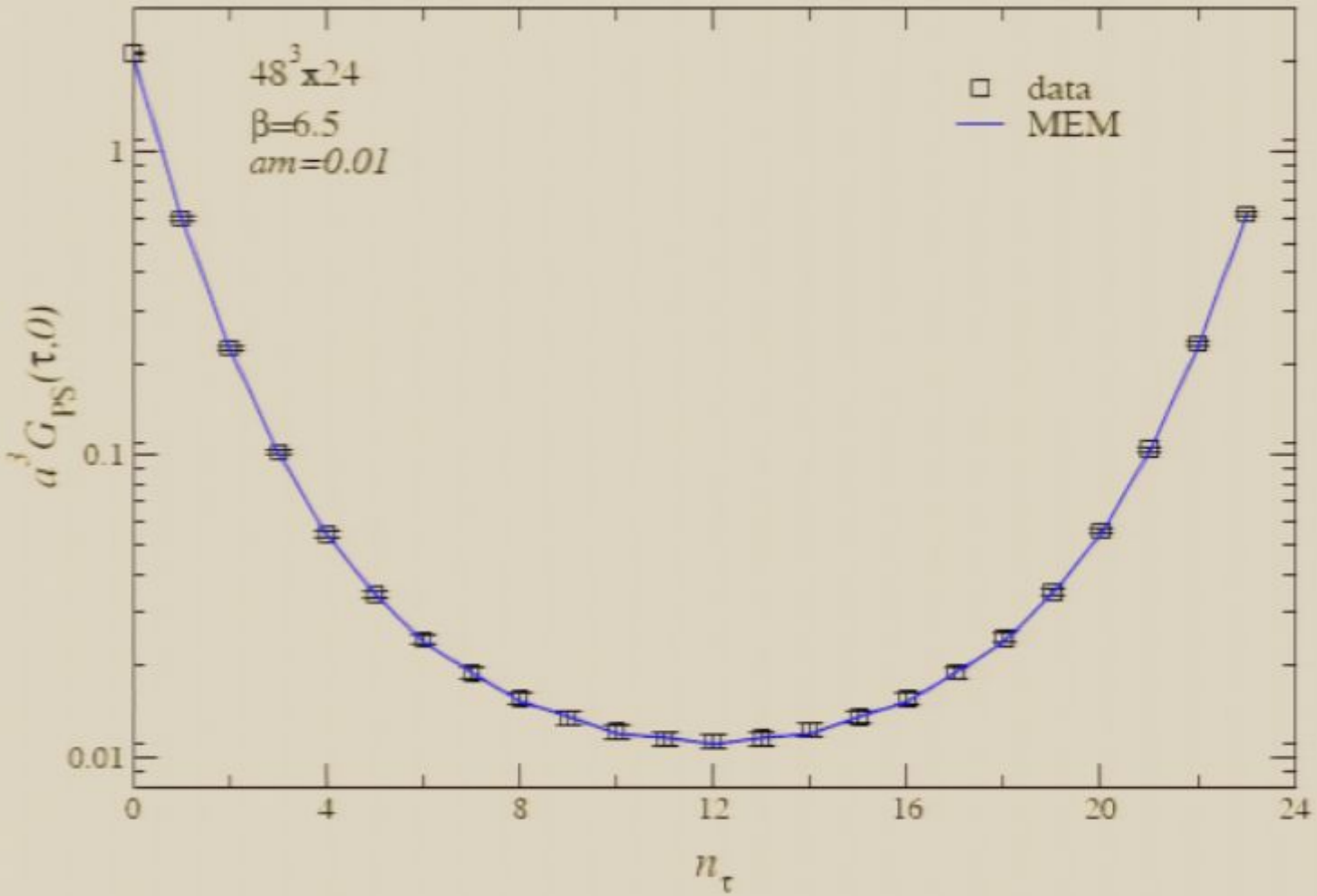
$$G(\tau, \mathbf{p}) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega) \left[\rho(\omega, \mathbf{p}) - (-1)^{\tau/a_\tau} \tilde{\rho}(\omega, \mathbf{p}) \right]$$

- $\rho(\omega, \mathbf{p})$ wanted
- staggered partner $\tilde{\rho}(\omega, \mathbf{p})$: related via $\tilde{\Gamma} = \gamma_4 \gamma_5 \Gamma$ "parity doubler"
- in MEM investigation: independent analysis on even/odd timeslices

first: results below T_c

SPECTRAL FUNCTIONS FROM MEM

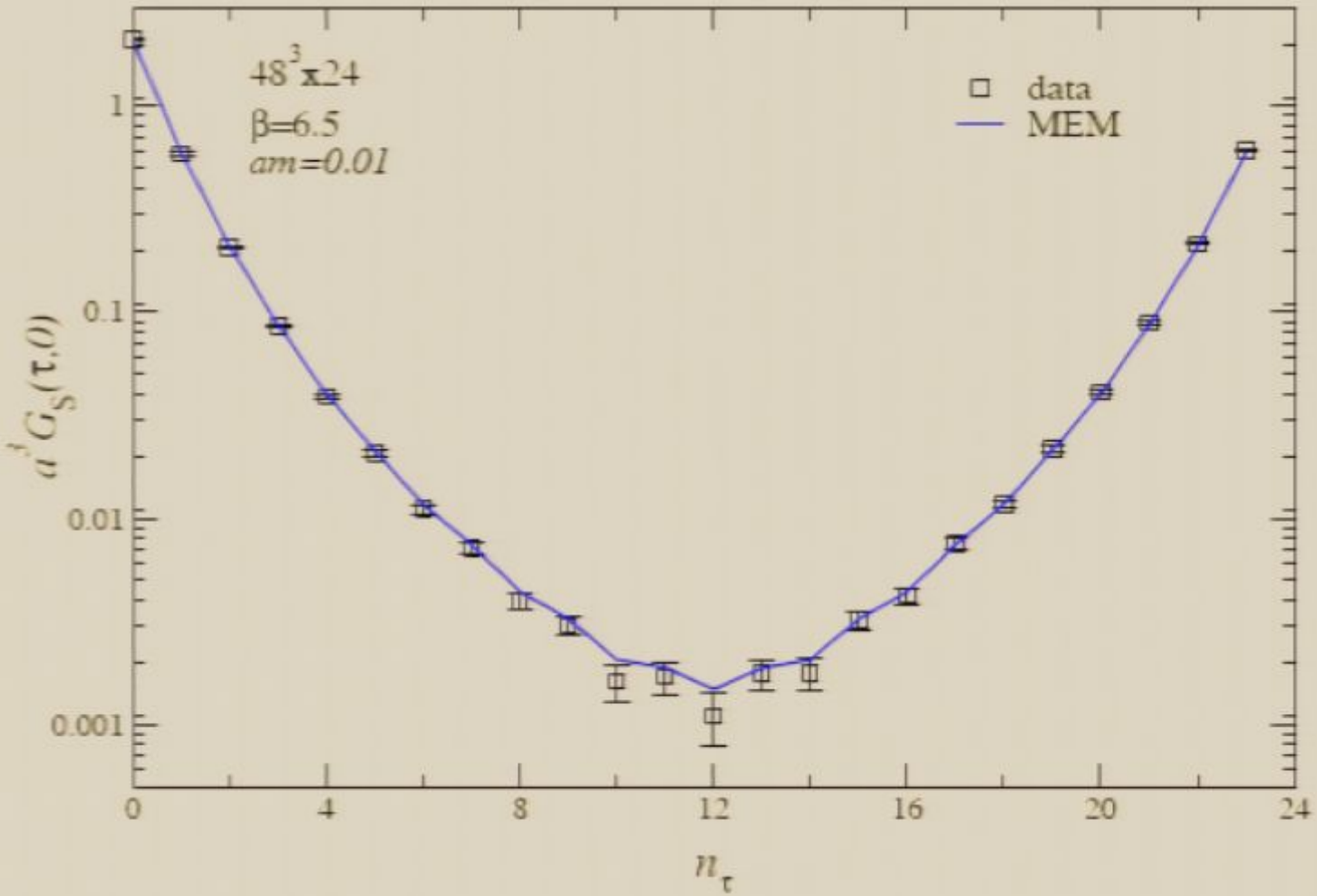
ROLE OF STATISTICAL ERROR



reconstructed pseudoscalar correlator

SPECTRAL FUNCTIONS FROM MEM

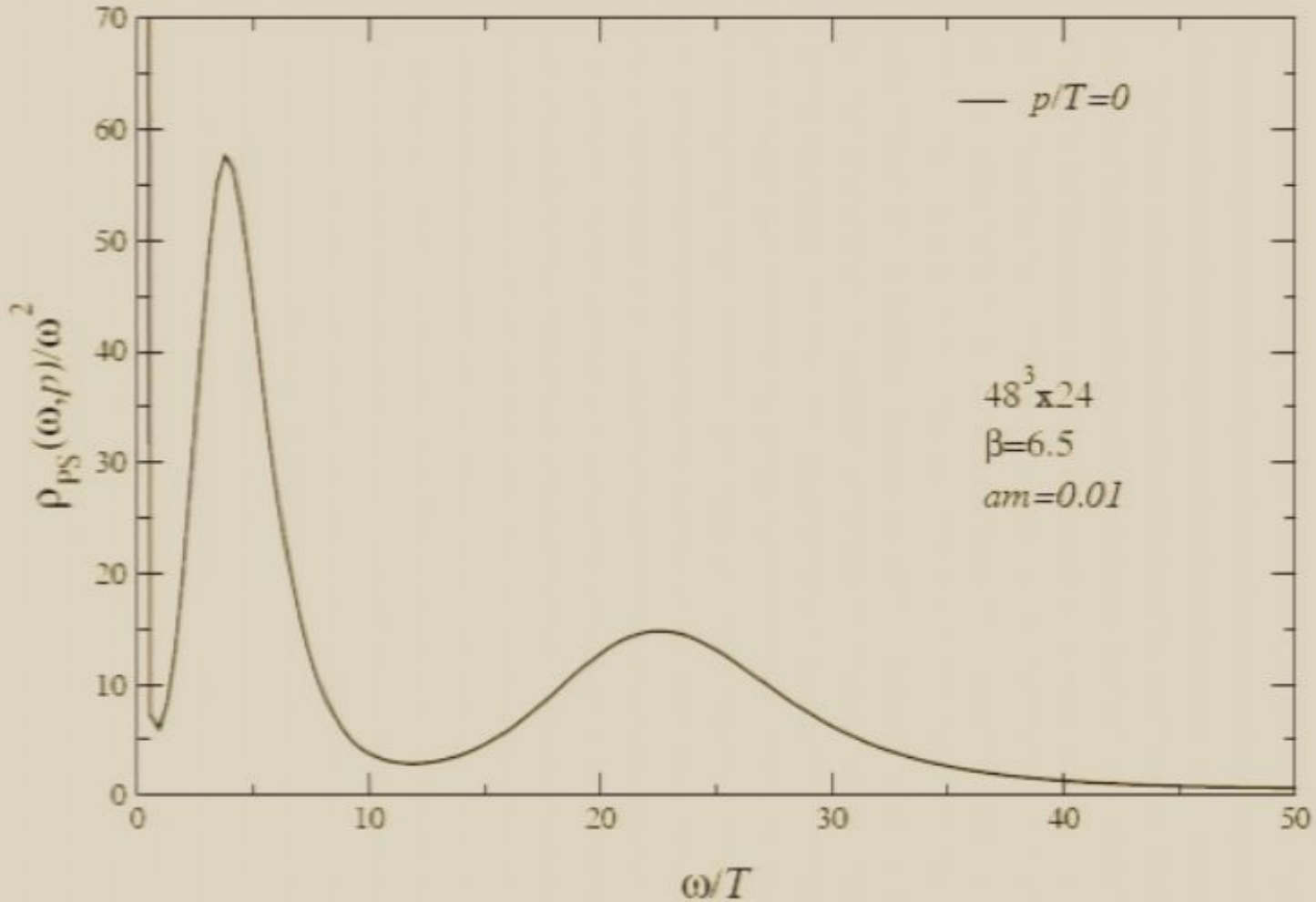
ROLE OF STATISTICAL ERROR



reconstructed scalar correlator: larger statistical uncertainty

MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

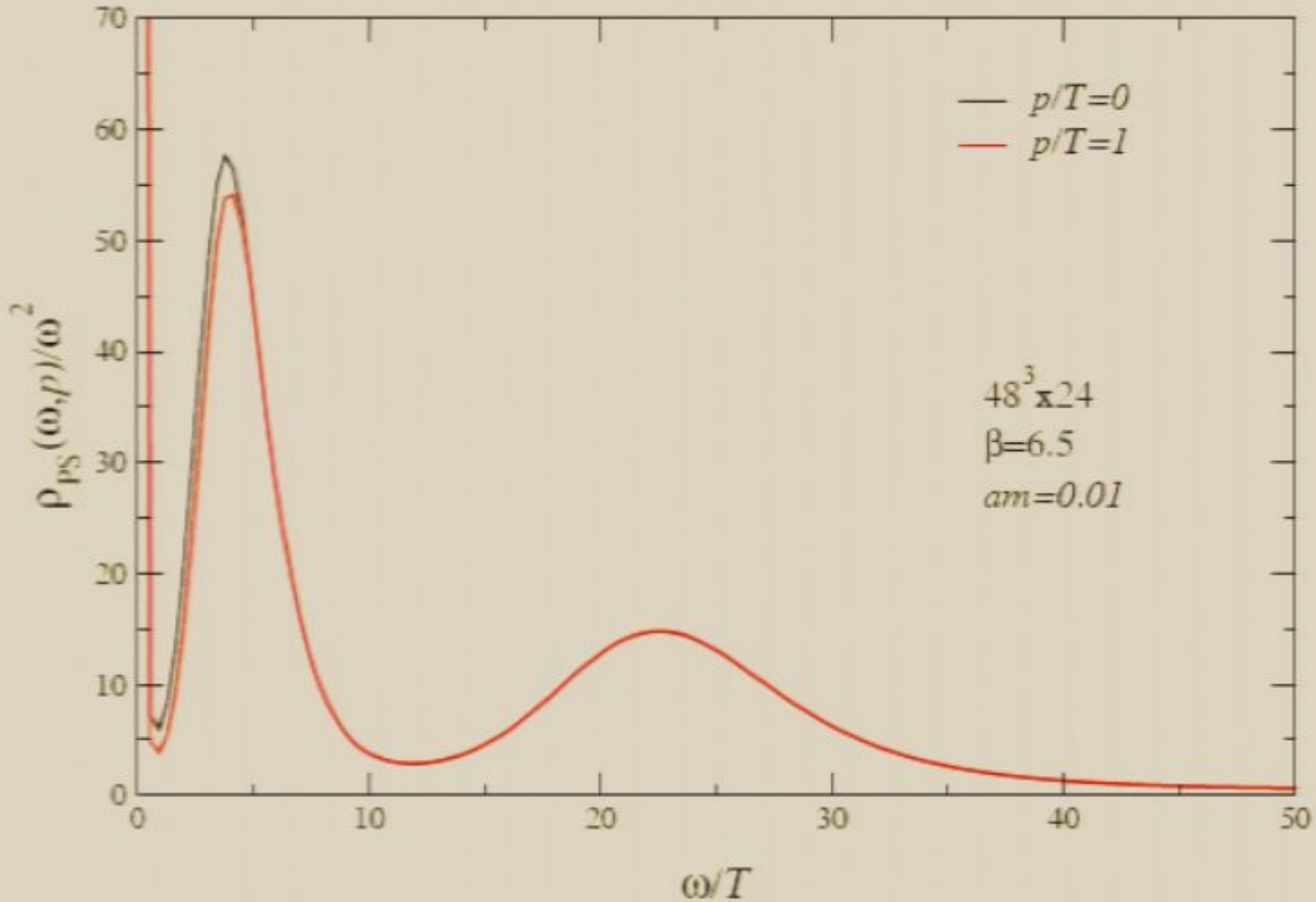
PSEUDOSCALAR, BELOW T_c



zero momentum bound state

MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

PSEUDOSCALAR, BELOW T_c



increase momentum: moving quasiparticle

MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

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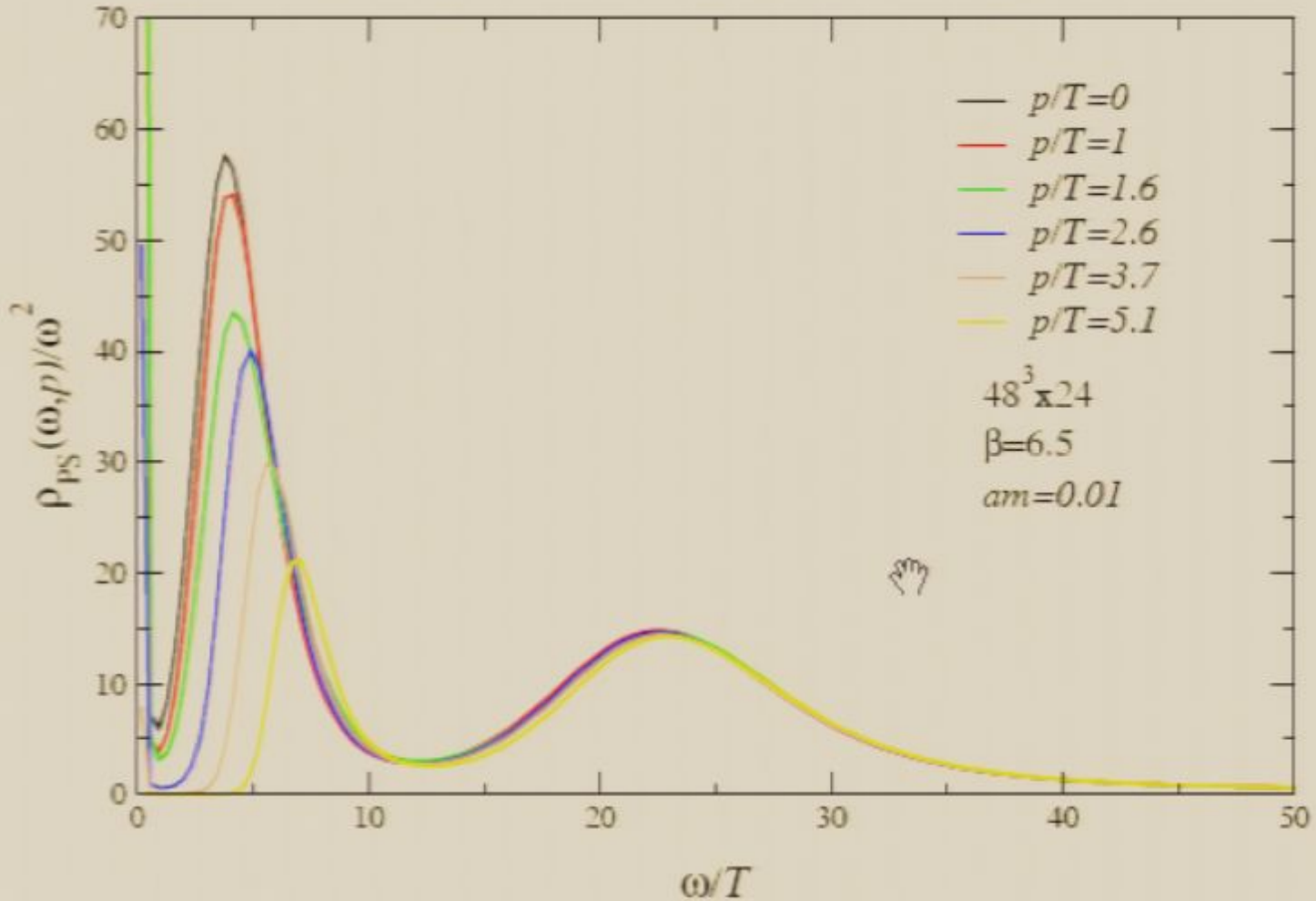

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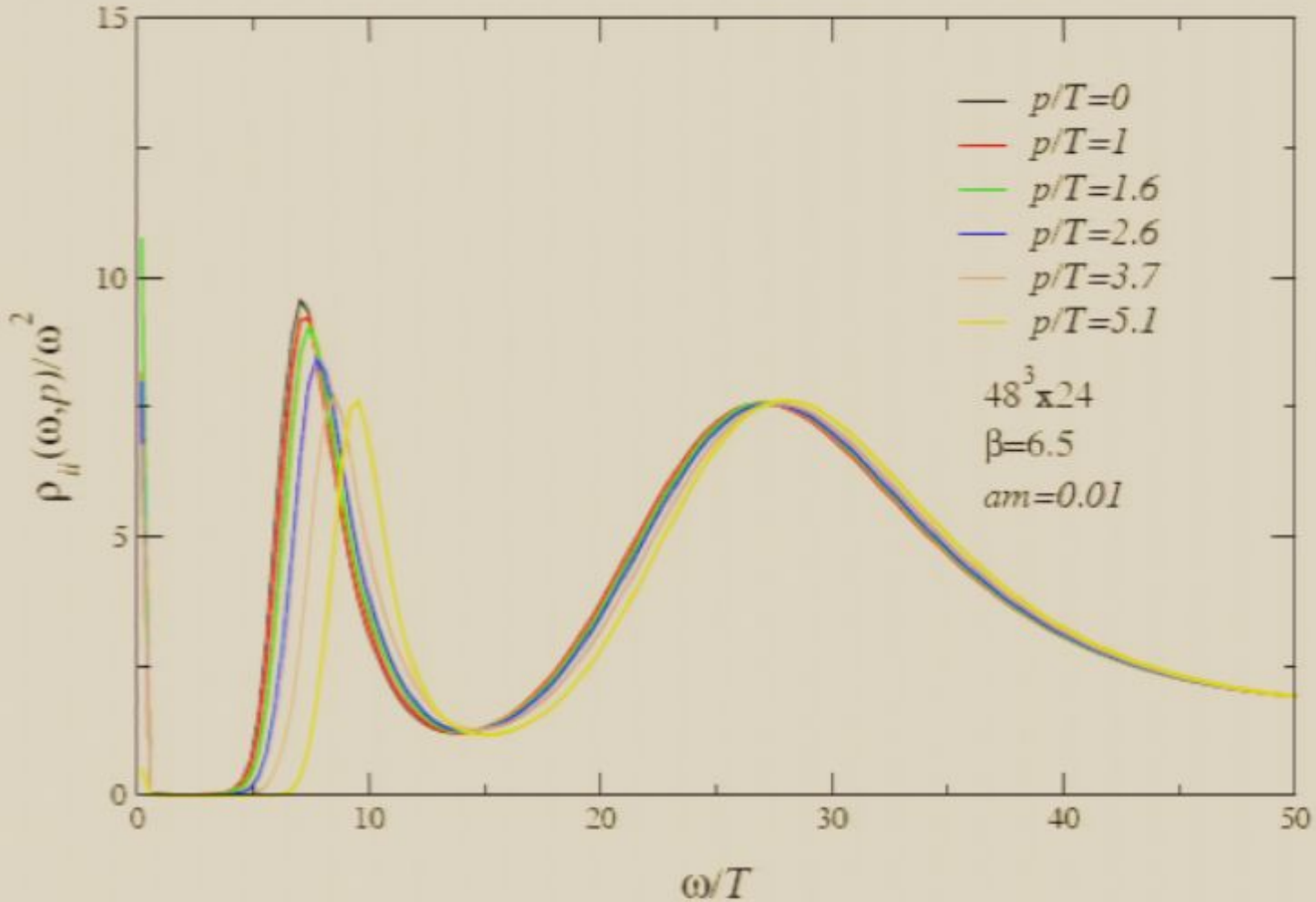


heavier quasiparticle in the vector channel

Momentum dependent spectral functions

BELOW T_c

- moving mesons below T_c

Momentum dependent spectral functions

BELOW T_c

- moving mesons below T_c

all fine, but we found serious problems in applying MEM above T_c

recent progress (hep-lat/0703008): modification of the algorithm

motivated by the quest for transport coefficients

- reduction step

Maximum Entropy Method

Bryan's algorithm: reduction step

view kernel $K(\omega_n, \tau_i)$ as $N_\omega \times N$ matrix
singular value decomposition $K = UWV^T$

- U: $N_\omega \times N$ matrix, $U^T U = \mathbb{1}_{N\times N}$
- $W = \text{diag}(\xi_1, \ldots, \xi_N)$
- V: $N \times N$ matrix, $VV^T = V^T V = \mathbb{1}_{N\times N}$

reflection symmetry: $K(\omega, 1/T - \tau) = K(\omega, \tau)$, use $N = N_\tau/2$

subspace spanned by vectors: $u_i(\omega_n) = U_{ni}$, $i = 1, \ldots, N$

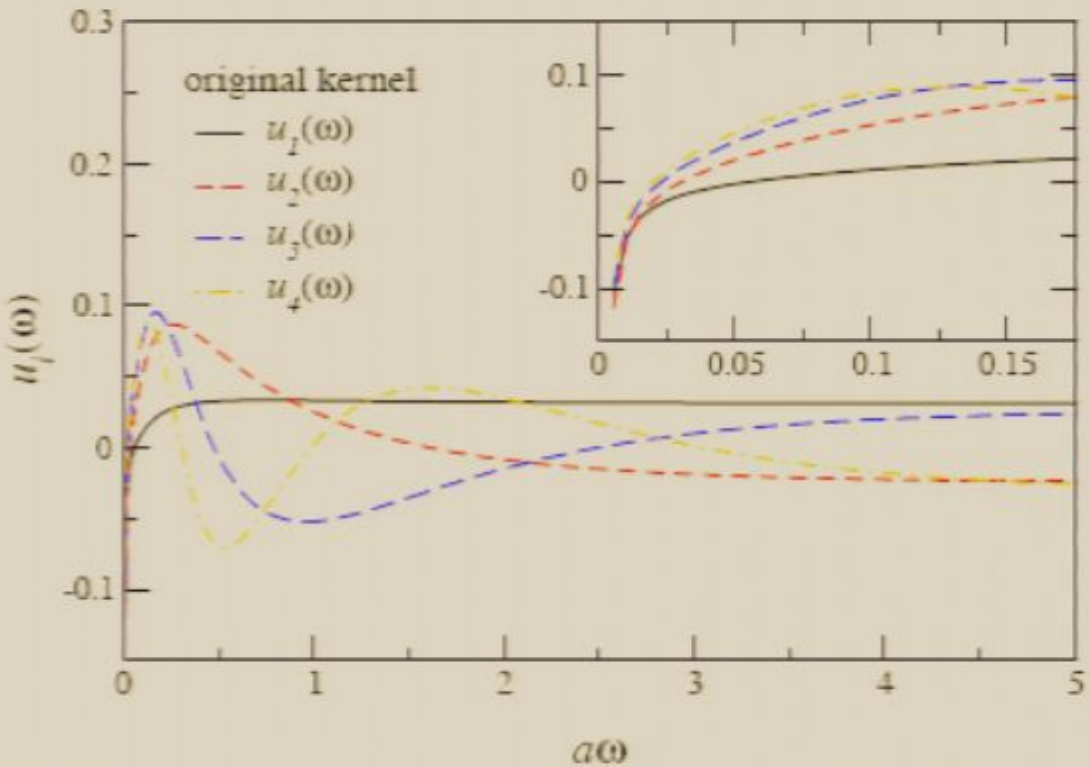
normalized: $\langle u_i | u_j \rangle \equiv \sum_{n=1}^{N_\omega} u_i(\omega_n) u_j(\omega_n) = \delta_{ij}$.

write: $\rho(\omega) = m(\omega) \exp f(\omega)$ $\qquad f(\omega) = \sum_{i=1}^{N} c_i u_i(\omega)$

Maximum Entropy Method

Bryan's algorithm: reduction step

basis functions $u_i(\omega)$, $a\omega_{\max} = 5$, $N_\omega = 1000$, $N_\tau = 24$



$$\lim_{\omega \to 0} K(\omega, \tau) = \frac{2T}{\omega} + \frac{\omega}{T}\left[\frac{1}{6} - \tau T(1 - \tau T)\right] + \mathcal{O}\left(\frac{\omega^3}{T^3}\right)$$

Maximum Entropy Method

HEP-LAT/0703008

- basis functions are divergent when $\omega \to 0$
- MEM does not always converge
- when converges: small ω region numerically unstable

easy modification:

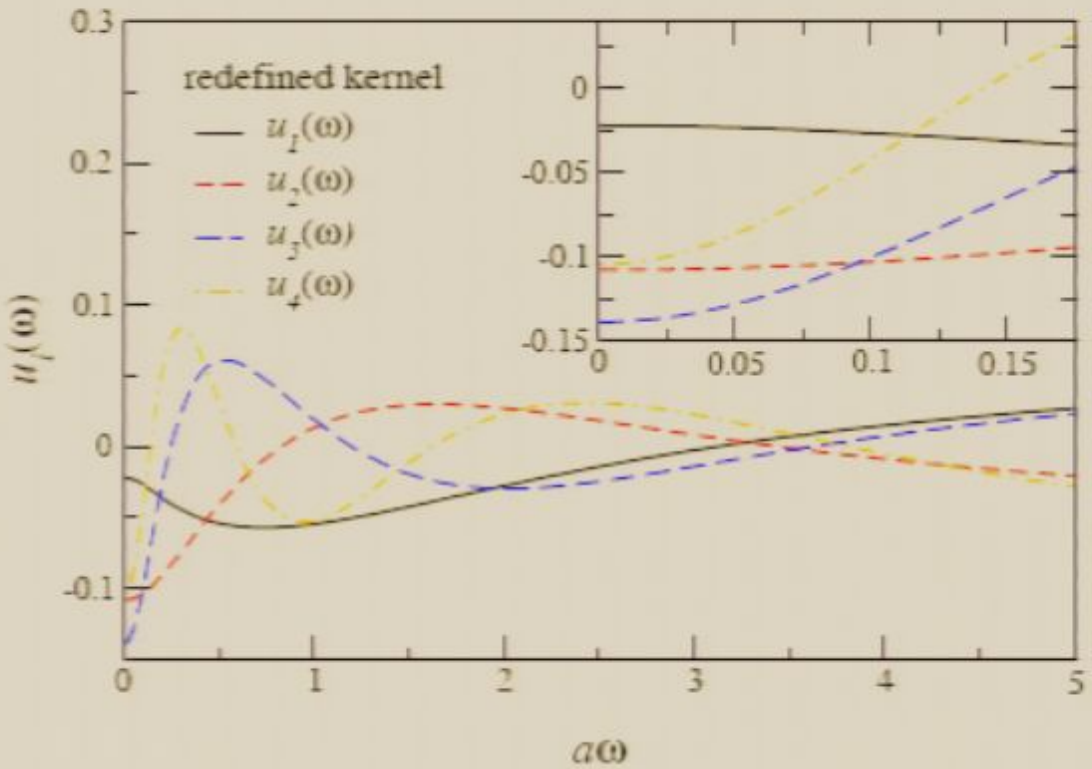
$$\overline{K}(\omega,\tau) = \frac{\omega}{2T} K(\omega,\tau) \qquad \overline{\rho}(\omega) = \frac{2T}{\omega}\rho(\omega)$$

- note that $K(\omega,\tau)\rho(\omega) = \overline{K}(\omega,\tau)\overline{\rho}(\omega)$.
- finite kernel when $\omega \to 0$: $\overline{K}(0,\tau) = 1$
- reconstruct $\overline{\rho} \sim \rho/\omega$
- reduction to different subspace

Maximum Entropy Method

MODIFIED REDUCTION STEP

new basis functions $\overline{u}_i(\omega)$, $a\omega_{\max} = 5$, $N_\omega = 1000$, $N_\tau = 24$



finite when $\omega \to 0$ write: $\overline{\rho}(\omega) = \overline{m}(\omega) \exp \sum_{i=1}^{N} \overline{c}_i \overline{u}_i(\omega)$

Maximum Entropy Method

MODIFIED REDUCTION STEP

$$\overline{\rho}(\omega) = \overline{m}(\omega) \exp \sum_{i=1}^{N} \overline{c}_i \overline{u}_i(\omega)$$

- non-zero intercept: excluded by traditional default model $\overline{m}(\omega) \sim \omega$
- new default model: $\overline{m}(\omega) = \overline{m}_0(b + a\omega)$
- $b > 0$: parameter to assess default model dependence

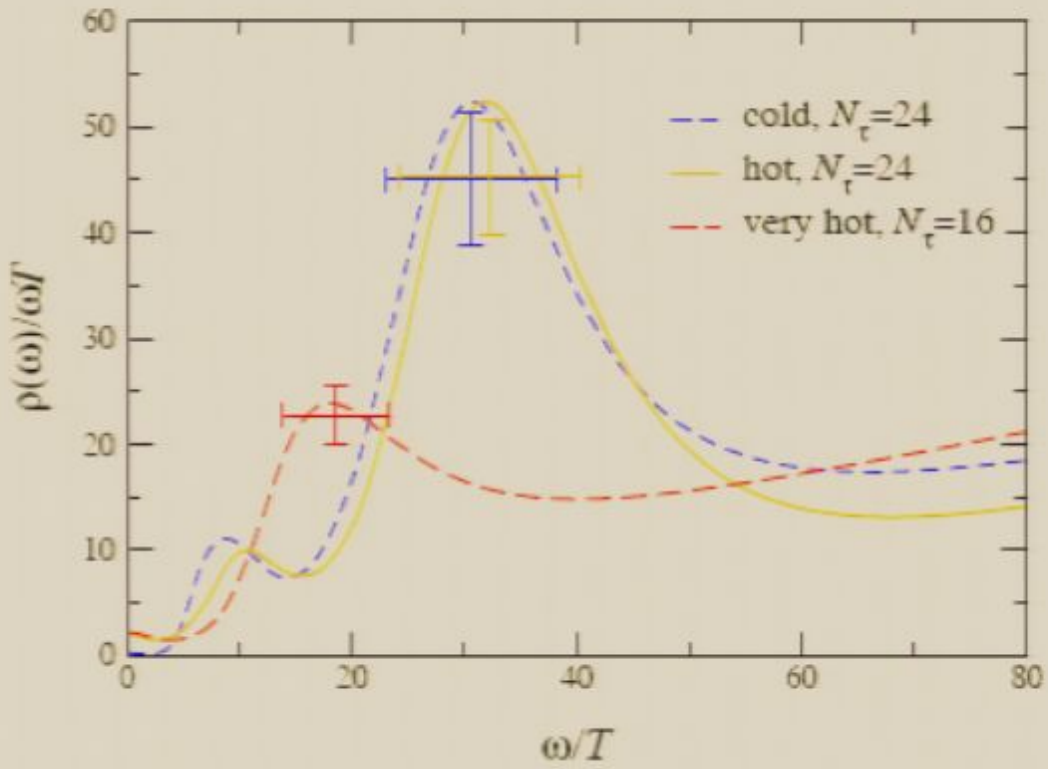
conductivity: $$\frac{\sigma}{T} = \lim_{\omega \to 0} \frac{\rho(\omega)}{6\omega T} = \frac{\overline{\rho}(0)}{12T^2}$$

with $\rho(\omega) = \sum_{i=1}^{3} \rho^{ii}(\omega)$ in vector channel

CONDUCTIVITY

NEW METHOD

reconstruction of $\rho(\omega)/\omega T$

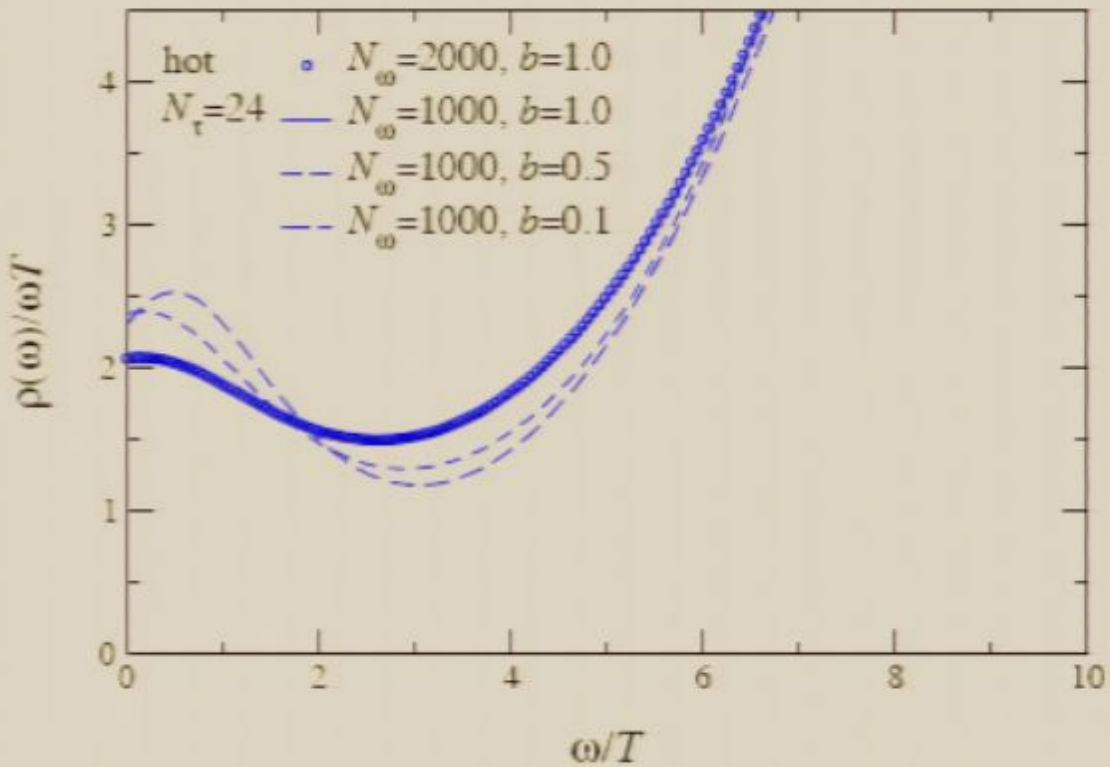


nonzero intercept in deconfined phase: conductivity

CONDUCTIVITY

NEW METHOD

hot: $T/T_c = 1.5$

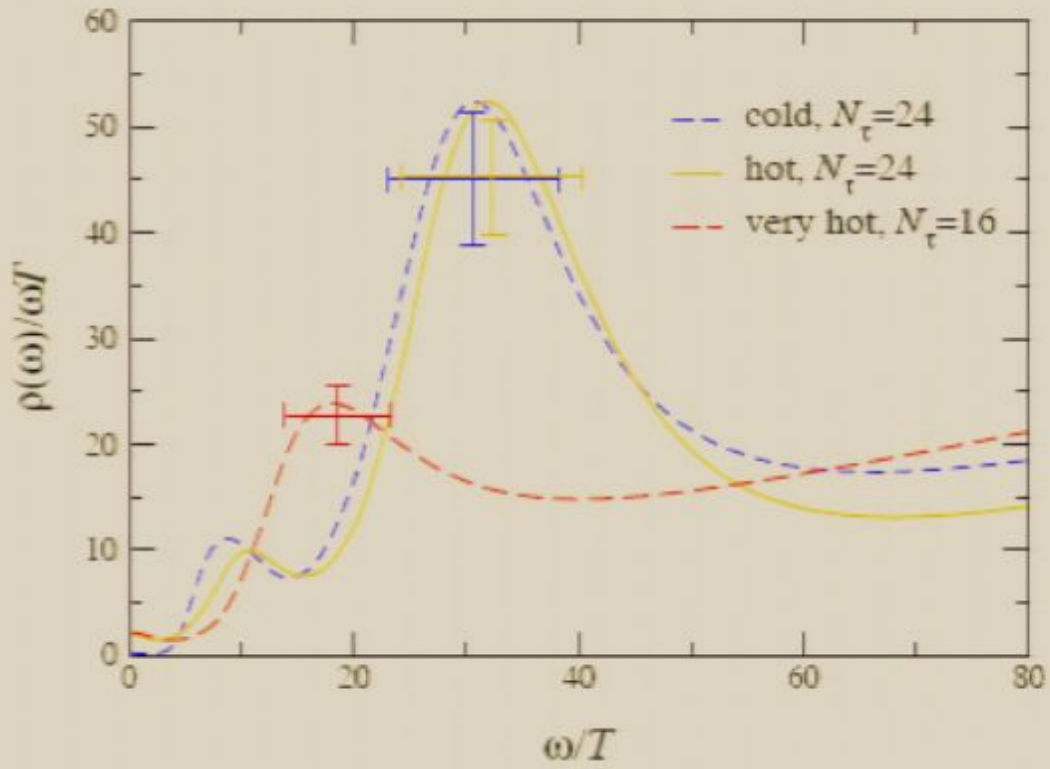


blow-up of small energy region
variations of default model: robust signal

CONDUCTIVITY

NEW METHOD

reconstruction of $\rho(\omega)/\omega T$



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CONDUCTIVITY

NEW METHOD

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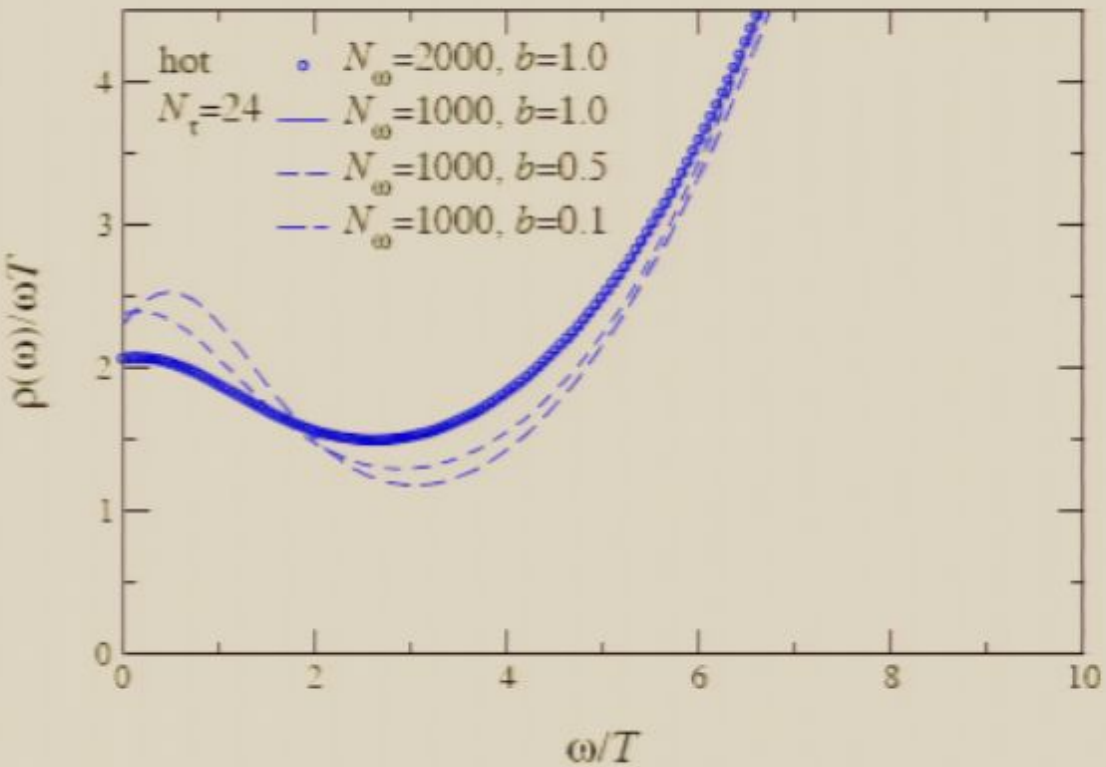

blow-up of small energy region
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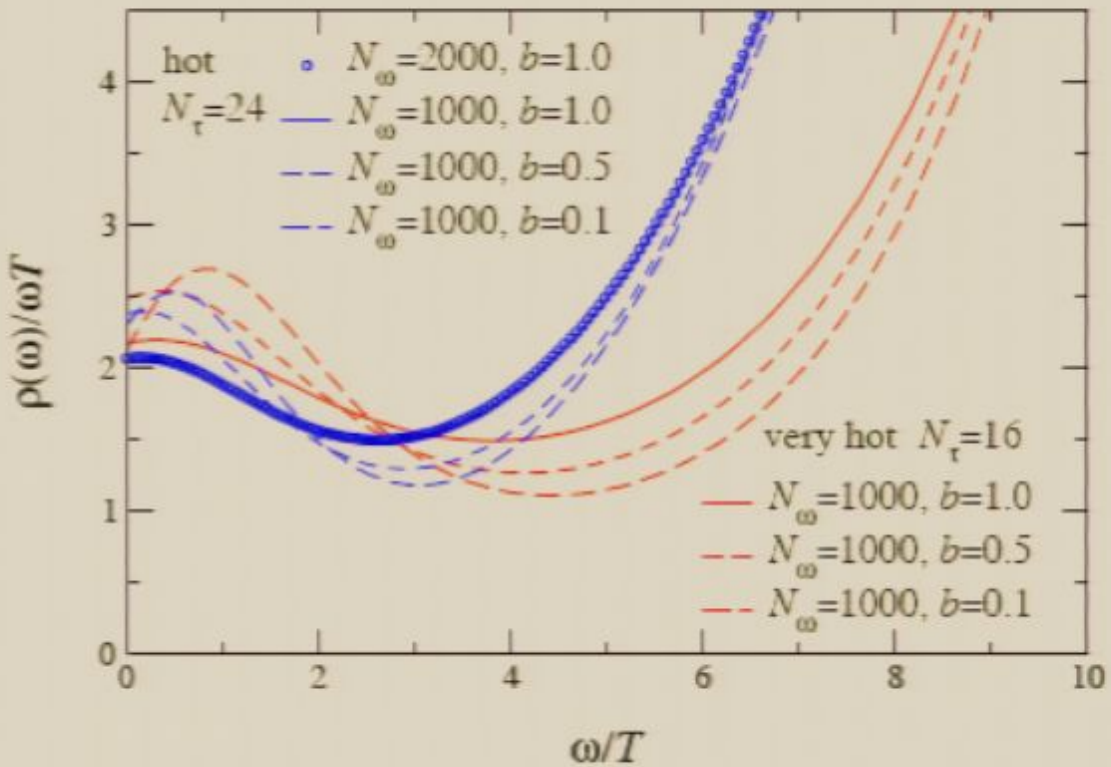
CONDUCTIVITY

NEW METHOD

hot: $T/T_c = 1.5$ and 2.25



blow-up of small energy region
variations of default model: robust signal

CONDUCTIVITY

NEW METHOD

- conductivity $\sigma/T = 0.4 \pm 0.1$
- systematic error due to MEM reconstruction
- in line with the notion of sQGP

recall:

weakly coupled: $\sigma/T \sim 1/\alpha_s^2 \gg 1$

strongly coupled: $\sigma/T \sim 1$

- relevant for charge transport in heavy ion phenomenology

CHARMONIUM

IN THE QGP

- fate of heavy quark bound states: charmonium
- J/ψ suppression: signal for deconfinement?
- quenched lattice QCD studies have shown that J/ψ and η_c may survive to $T \sim 2T_c$

Asakawa & Hatsuda, Datta, Karsch, Petreczky & Wetzorke, Umeda et al, ...

what happens in dynamical QCD?

CHARMONIUM

WITH TRINLAT: SKULLERUD, OKTAY, PEARDON + ALLTON

what happens in dynamical QCD?

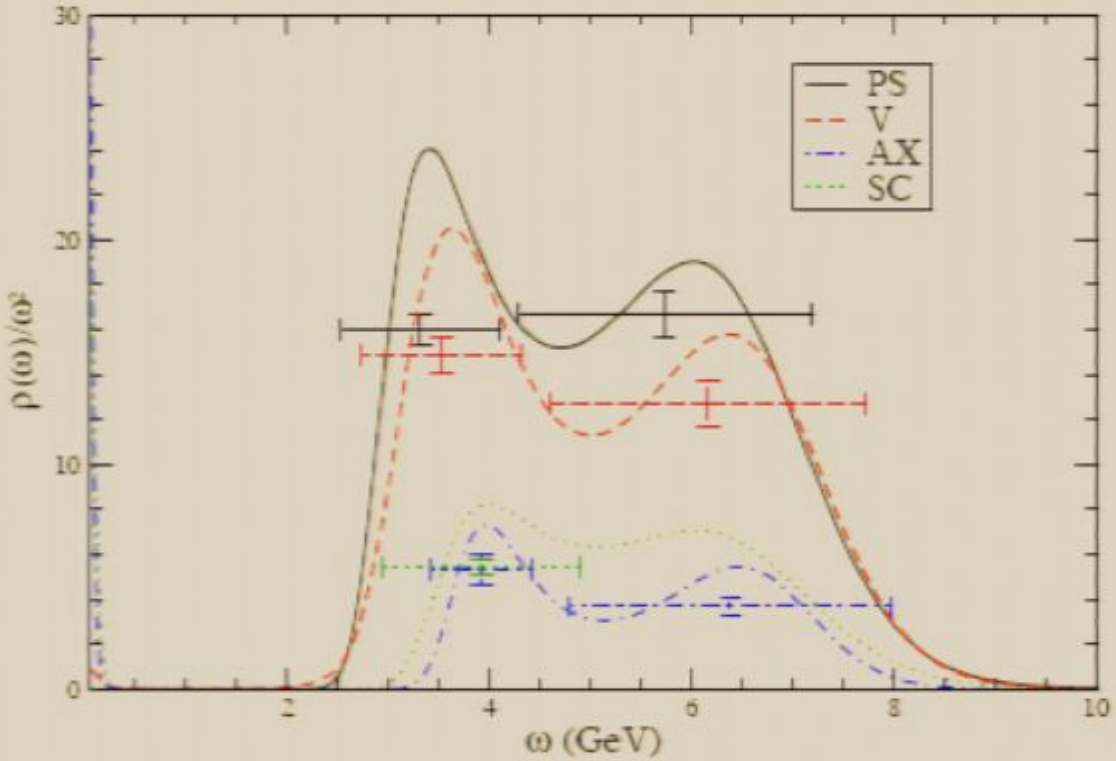
- TrinLat: $N_f = 2$ QCD on highly anisotropic lattices
- $\xi = a_s/a_\tau \approx 6$, $1/a_\tau \approx 7$ GeV, $m_\pi/m_\rho \approx 0.54$
- $8^3 \times N_\tau$ and $12^3 \times N_\tau$
- temperature scan in steps of $\Delta T \approx 7$ GeV:

$$N_\tau = 32, 31, 30, 29, 28, 24, 20, 18, 16$$
$$T/T_c = 1.05, 1.08, 1.12, 1.16, 1.2, 1.4, 1.68, 1.86, 2.1$$

CHARMONIUM

WITH TRINLAT: ARXIV:0705.2198

$T/T_c = 1.05$ ($N_\tau = 32$)



spectral functions in pseudoscalar (PS), vector (V), axial-vector (AV) and scalar (SC) channels

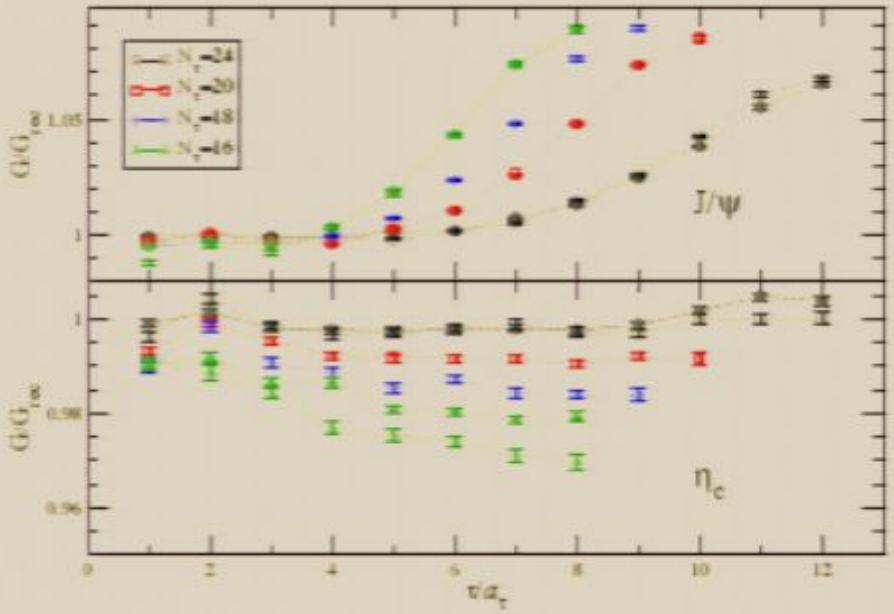
CHARMONIUM

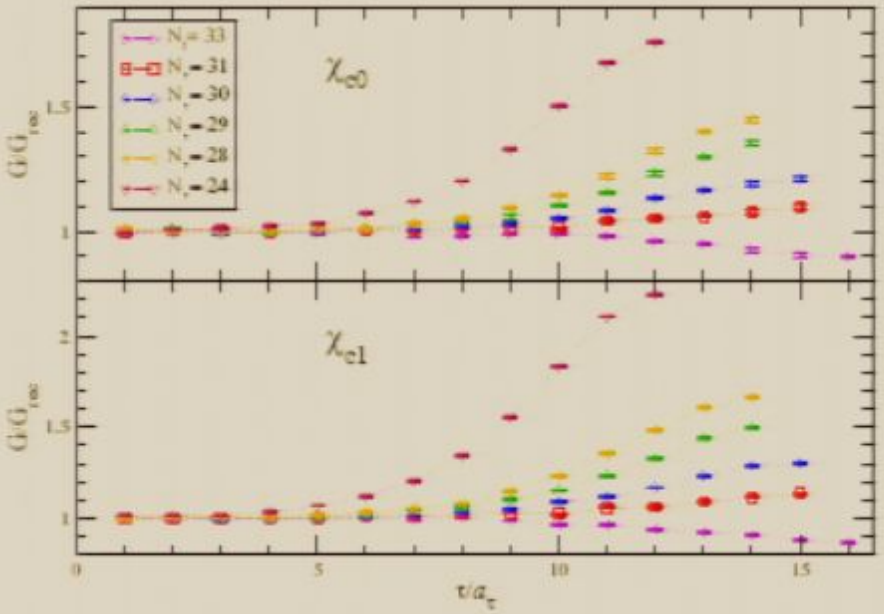
WITH TRINLAT: ARXIV:0705.2198

temperature dependence in correlators:

S-waves:
vector, pseudoscalar

P-waves:
scalar, axial vector





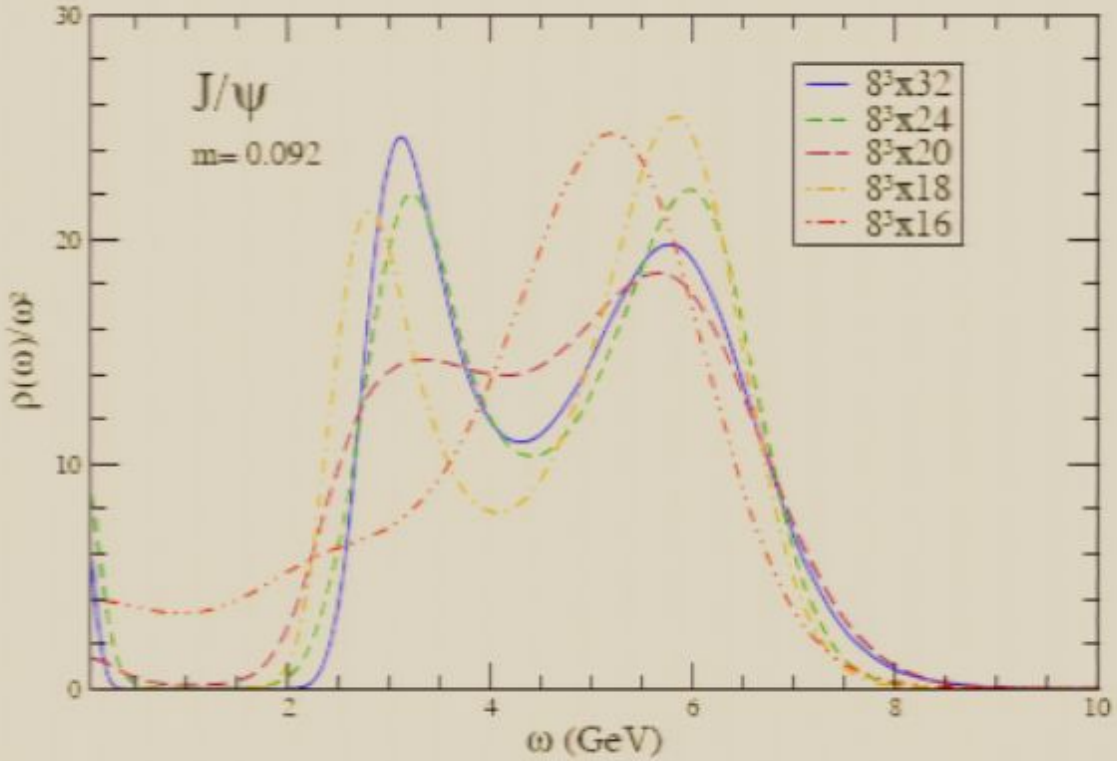
S-waves: a few percent

P-waves:
significant T dependence

CHARMONIUM

WITH TRINLAT: ARXIV:0705.2198

vector:

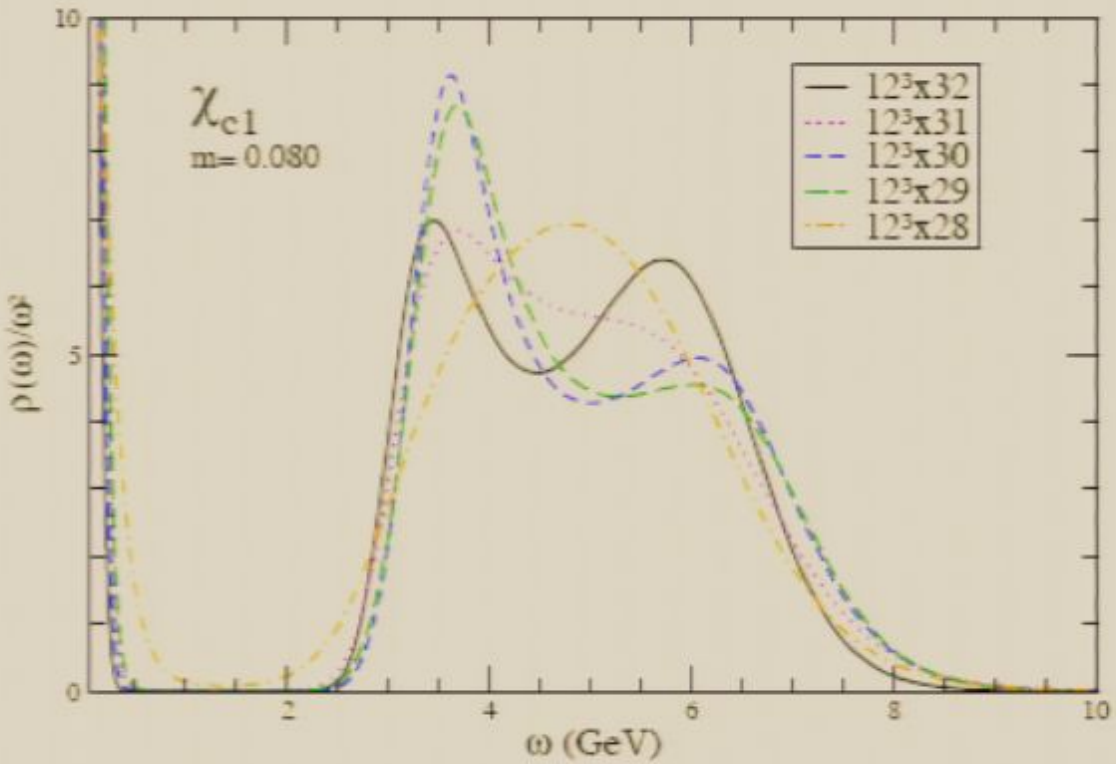


groundstate peak seems to disappear at $N_\tau \sim 20 - 16$ or $T \sim 1.7 - 2T_c$

CHARMONIUM

WITH TRINLAT: ARXIV:0705.2198

axial vector:

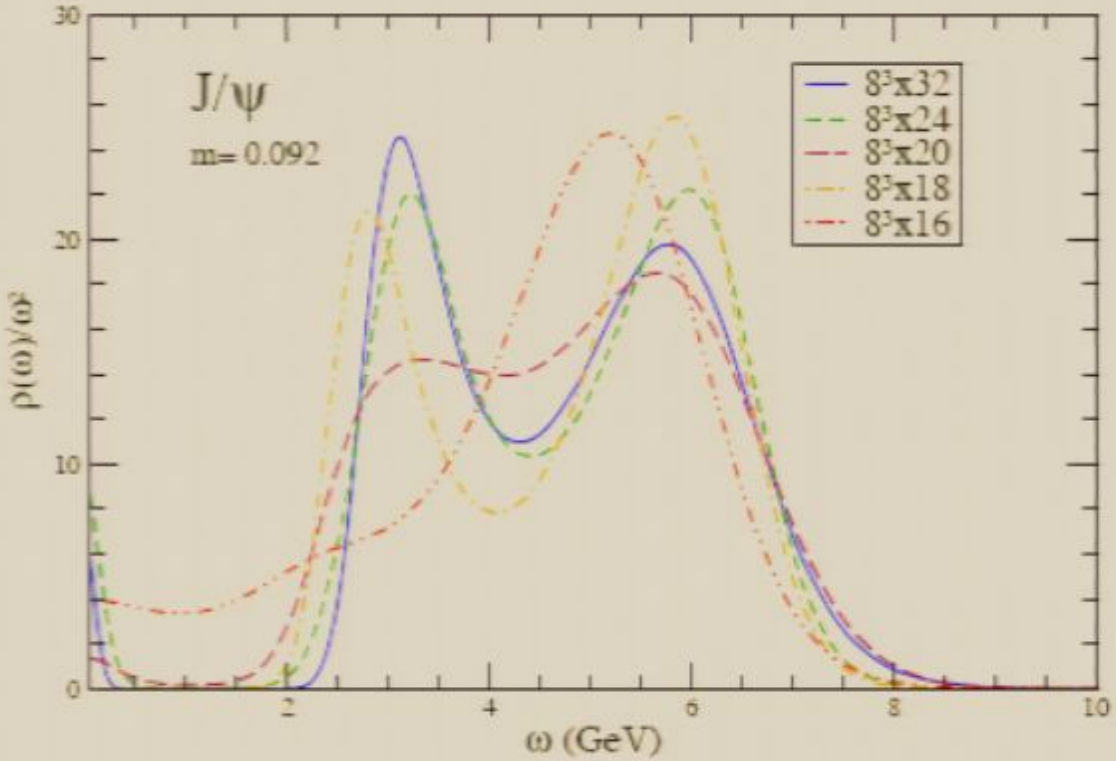


groundstate peak seems to disappear at $N_\tau \sim 28$ or $T \sim 1.2T_c$

CHARMONIUM

WITH TRINLAT: ARXIV:0705.2198

vector:



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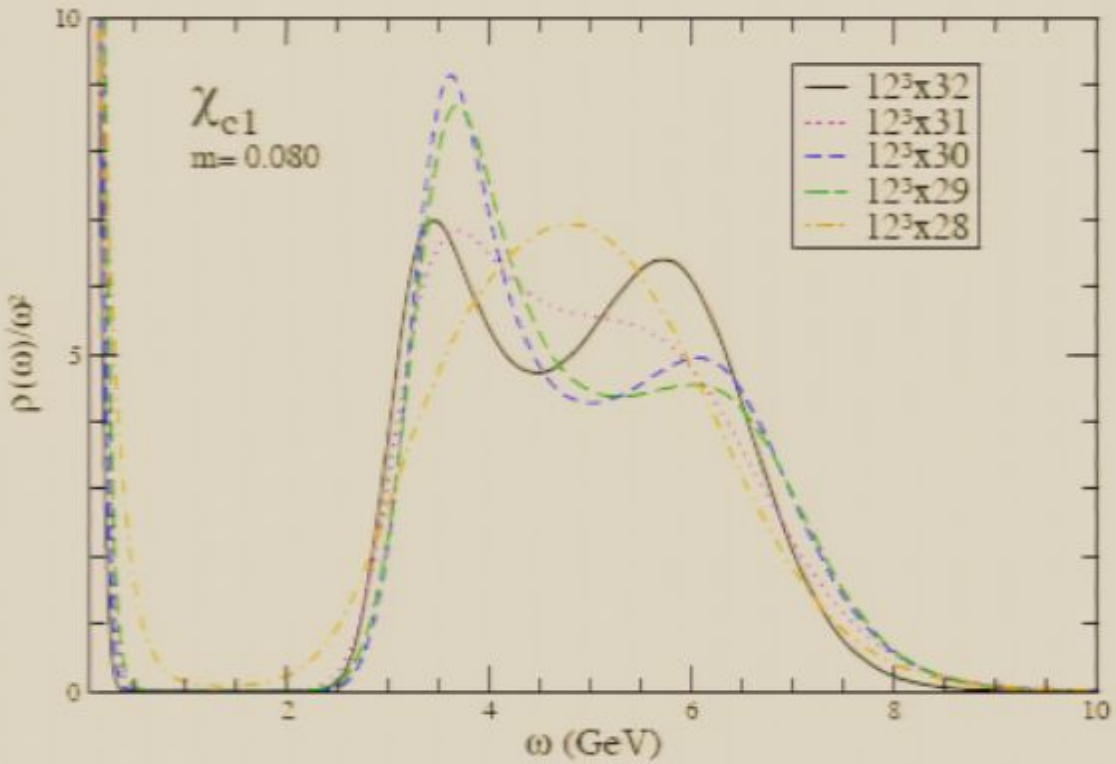
CHARMONIUM

WITH TRINLAT: ARXIV:0705.2198

axial vector:



groundstate peak seems to disappear at $N_\tau \sim 28$ or $T \sim 1.2T_c$

Summary

- lattice QCD and spectral functions
- domain wall QCD at zero temperature

SUMMARY

- lattice QCD and spectral functions
- domain wall QCD at zero temperature
- meson spectral functions at high temperature
- below T_c: moving bound states
- above T_c: modification of algorithm necessary
- first MEM result for conductivity in deconfined phase: $\sigma/T = 0.4 \pm 0.1$
- charmonium in dynamical lattice QCD
- S-waves survive, P-waves don't (confirms quenched studies)

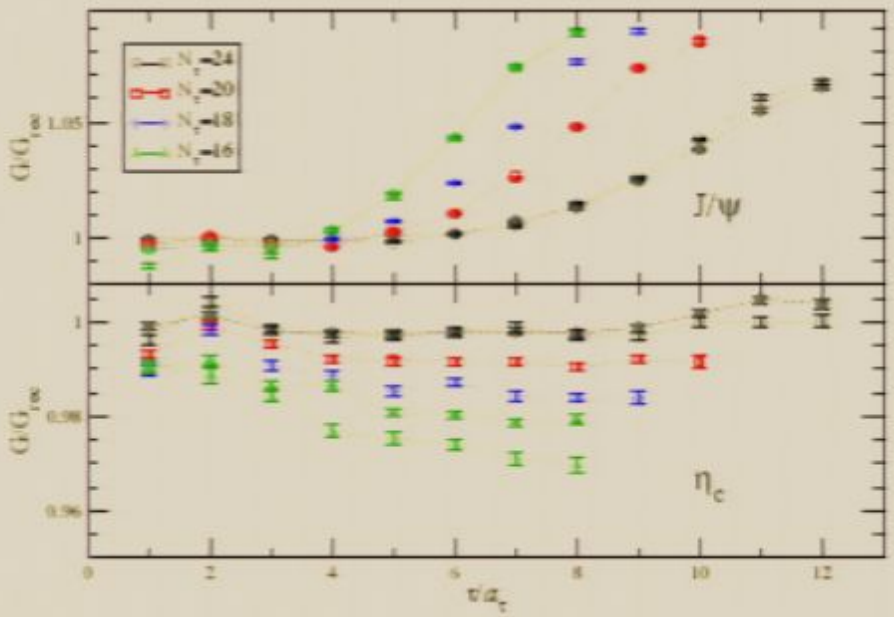
CHARMONIUM

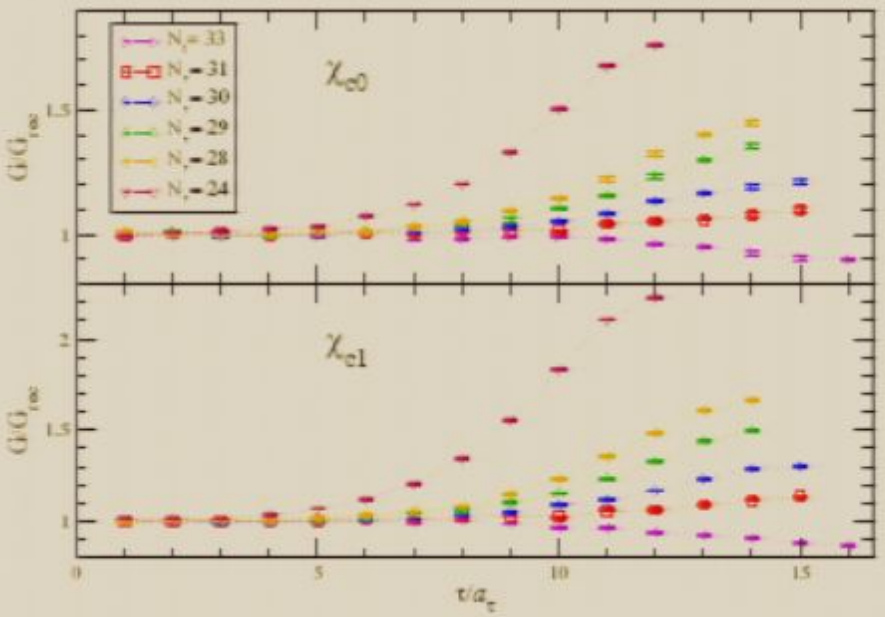
WITH TRINLAT: ARXIV:0705.2198

temperature dependence in correlators:

S-waves:
vector, pseudoscalar

P-waves:
scalar, axial vector





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CHARMONIUM

WITH TRINLAT: ARXIV:0705.2198

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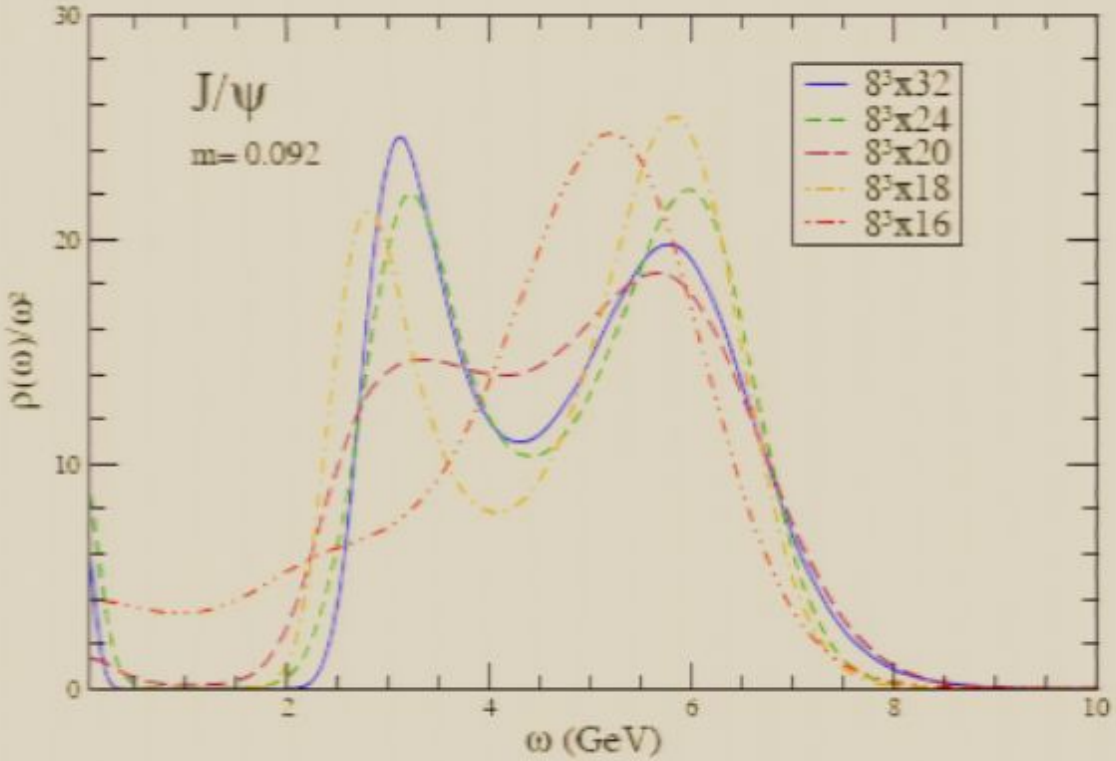

groundstate peak seems to disappear at $N_\tau \sim 20 - 16$ or $T \sim 1.7 - 2T_c$