Title: Hot Physics at Large N: A Lattice Overview

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Abstract:

Hot Physics at Large N : a lattice overview

Michael Teper (Oxford) – Perimeter Workshop, May '07

- prelude
- $T = T_c$
- $T < T_c$
- $T > T_c$
- conclusions

Lattice – Preamble

Euclidean $R^4 \longrightarrow$ hypercubic lattice on T^4

$x_\mu \bullet - \bullet\, x_\mu + \hat{\mu}\delta x \,:\, A_\mu(x) \in$ SU(N) Lie Algebra

$\longrightarrow$

$x_\mu \bullet - - - \bullet\, x'_m u \,:\, P\left\{ e^{\int_x^{x'} A_\mu dx} \right\} \in$ SU(N) group

$\xrightarrow{x_\mu = a n_\mu}$

$a n_\mu \bullet - - - \bullet\, a n_\mu + a\hat{\mu} \quad : \quad U_\mu(n) \in$ SU(N) group

i.e. SU(N) matrices U_l on each link l

gauge transform. on: $U_\mu(n) \to g(n)U_\mu(n)g^\dagger(n+\hat{\mu})$

$\longrightarrow$ gauge invariant action?

Tr $\prod_{l\in\partial c} U_l$ gauge invariant for any closed curve c

$\longrightarrow$ so

$Z = \int \prod_l dU_l e^{-\beta S}$ where $S = \sum_p \{1 - \frac{1}{N}\text{ReTr}u_p\}$
where u_p is product links around the plaquette p is a suitable, although not unique, SU(N) lattice gauge theory

$\longrightarrow$ symmetries ensure that:

$$\int \prod_l dU_l e^{-\beta S} \quad \xrightarrow{a\to 0} \quad \int DA e^{-\frac{1}{g^2}\int d^4x \text{Tr} F_{\mu\nu}F_{\mu\nu}}$$

with

$\beta = \frac{2N}{g^2(a)} \xrightarrow{a\to 0} \infty$

and we vary the parameter β in order to vary the lattice spacing a

$\longrightarrow$ Monte Carlo:

$$\int \prod_l dU_l \Phi(U) e^{-\beta S} = \frac{1}{n}\sum_{I=1}^{n} \Phi(U^I) + O\left(\frac{1}{\sqrt{n}}\right)$$

- calculating masses from Euclidean correlators:

$\Phi(t)$ a gauge invariant operator

$$\langle \Phi^\dagger(t = an_t)\Phi(0)\rangle = \sum_i |c_i|^2 e^{-aE_i n_t} \overset{t\to\infty}{\simeq} |c|^2 e^{-m a n_t}$$

where am is lightest mass with quantum numbers of Φ in lattice units

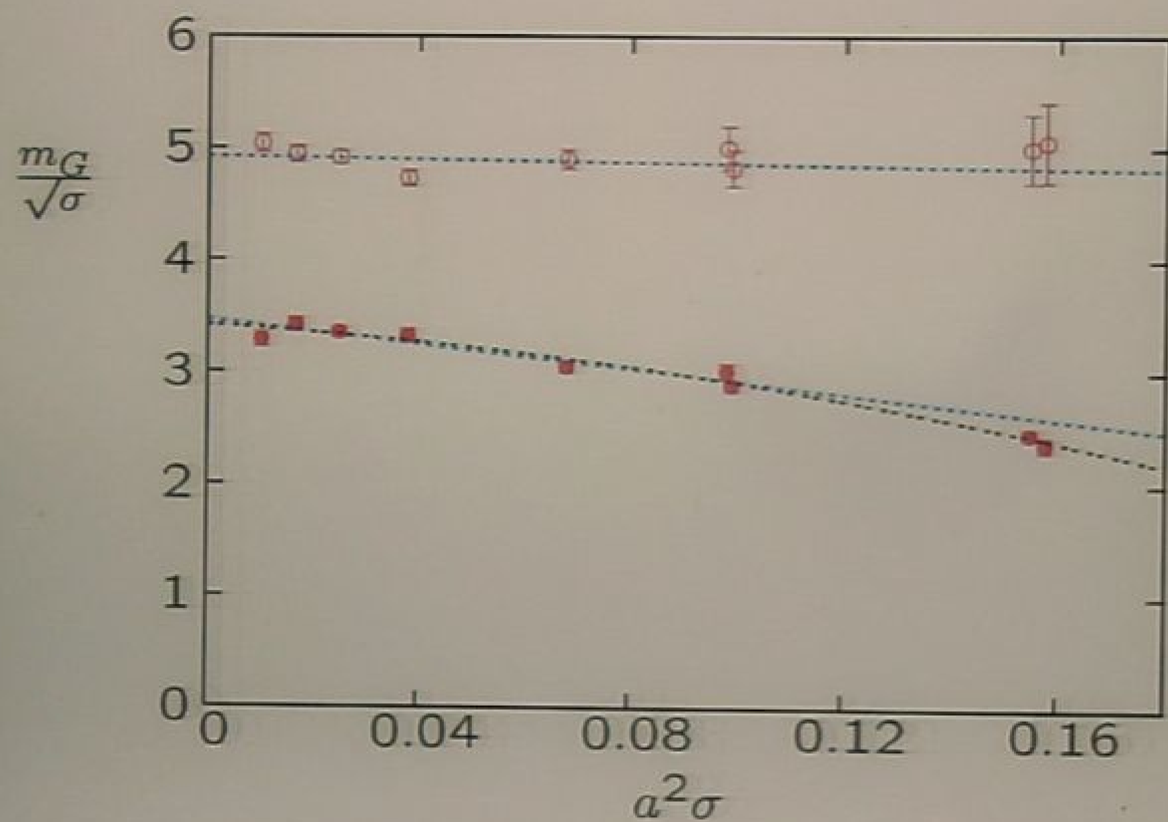
- continuum limit :

$$\frac{am(a)}{a\sqrt{\sigma(a)}} = \frac{m(a)}{\sqrt{\sigma(a)}} = \frac{m(0)}{\sqrt{\sigma(0)}} + c_0 a^2 \sigma + O(a^4)$$

- large N limit :

$$\frac{m(N)}{\sqrt{\sigma(N)}} = \frac{m(\infty)}{\sqrt{\sigma(\infty)}} + \frac{c}{N^2} + O\left(\frac{1}{N^4}\right)$$

Continuum limit mass spectrum: SU(3)



$O(a^2)$ continuum extrapolations:

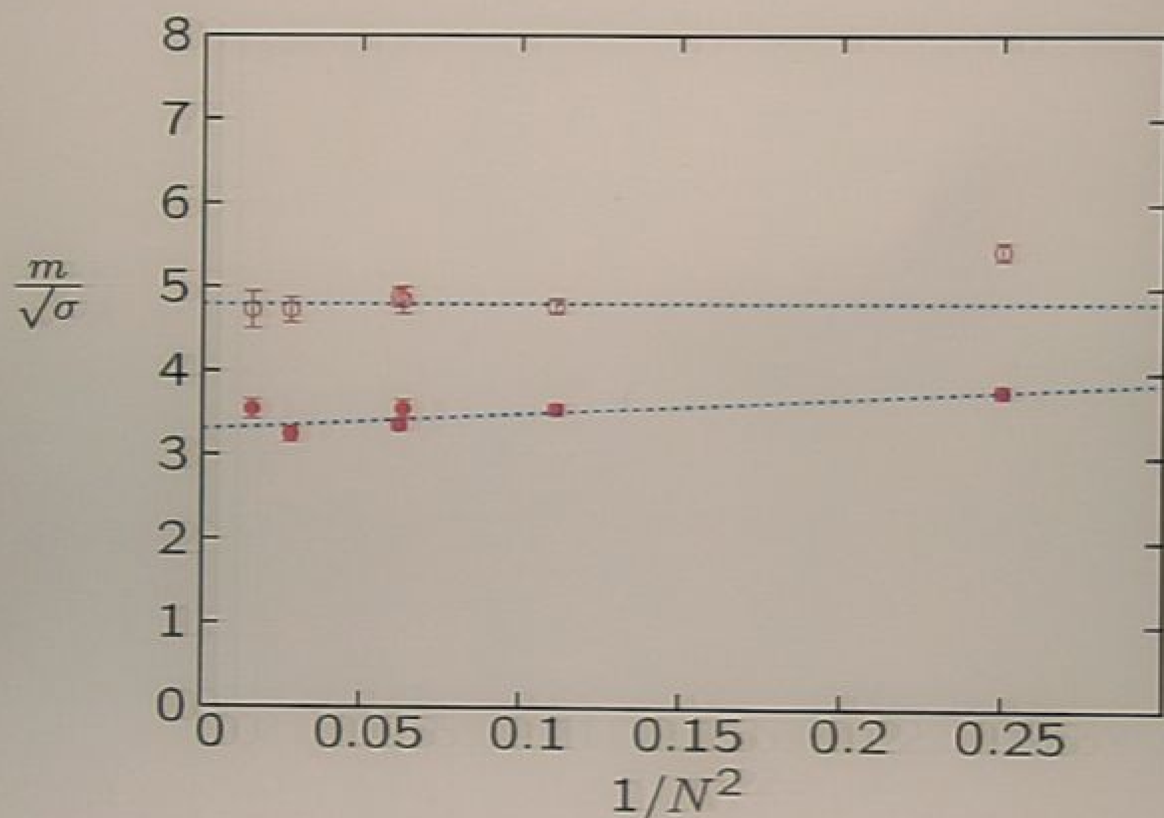
$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.47(4) - 5.52(75)a^2\sigma$$

$$\frac{m_{2^{++}}}{\sqrt{\sigma}} = 4.93(5) - 0.61(1.36)a^2\sigma$$

$O(a^4)$ continuum extrapolation very similar

Mass spectrum: large-N limit

B.Lucini, M.Teper, U.Wenger: hep-lat/0404008



$O(1/N^2)$ extrapolations to $N = \infty$:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}}|_N = 3.31 + \frac{1.90}{N^2}$$

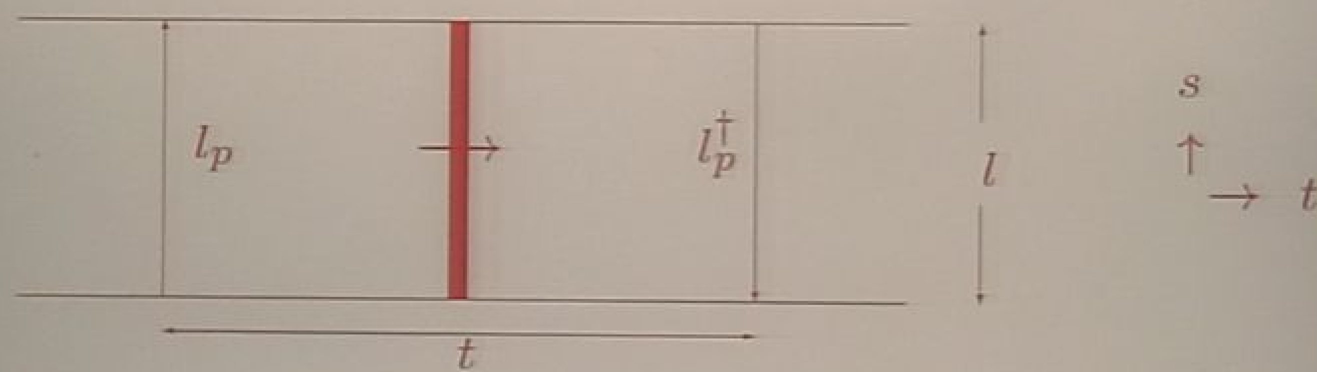
$$\frac{m_{2^{++}}}{\sqrt{\sigma}}|_N = 4.80 + \frac{0.11}{N^2}$$

Linear confinement in SU($N \to \infty$)?

Calculate the mass of a confining flux tube winding around a spatial torus of length l, using correlators of Polyakov loops:

$$\langle l_p^\dagger(t) l_p(0) \rangle \overset{t\to\infty}{\propto} \exp\{-m_p(l)t\}$$

in pictures



where we expect, for linear confinement,

$$m_p(l) = \sigma l - \frac{\pi(D-2)}{6l^2} + O\left(\frac{1}{l^4}\right)$$

- no sources, no Coulomb terms

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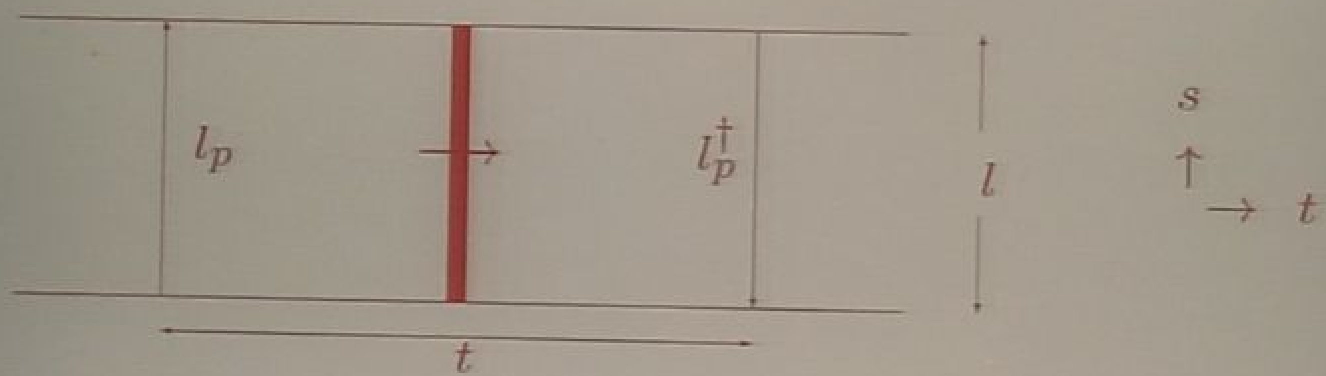

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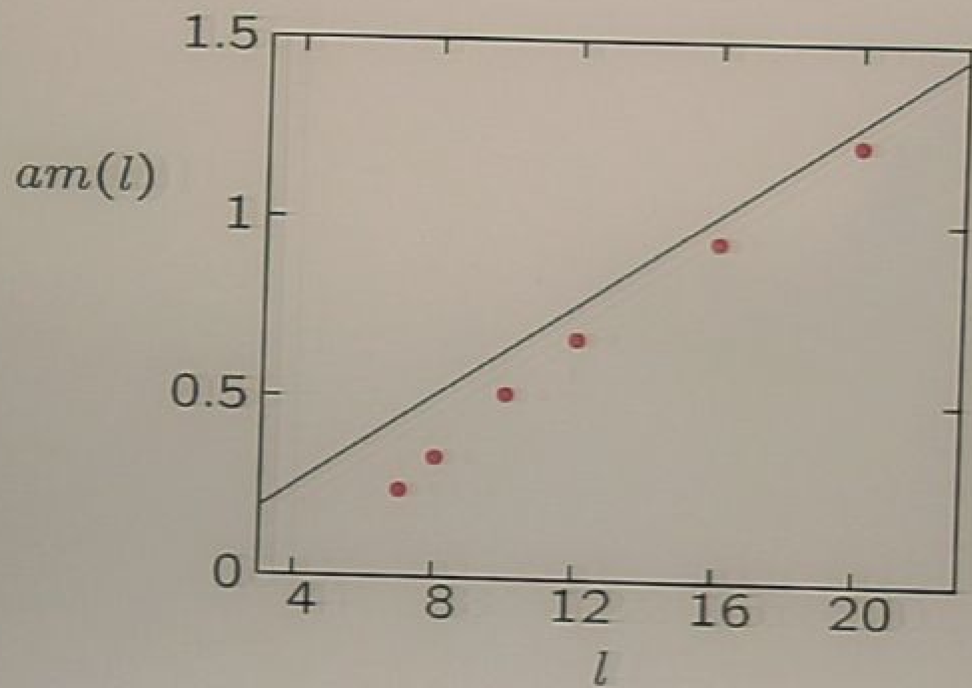
SU(6)

H. Meyer, M. Teper: hep-lat/0411039



indeed we find

$$am(l) \simeq \sigma l$$

over a range of 'string' lengths up to

$$l \simeq 5.0 \times \frac{1}{\sqrt{\sigma}}$$

surely large enough to be asymptotic ...

So :

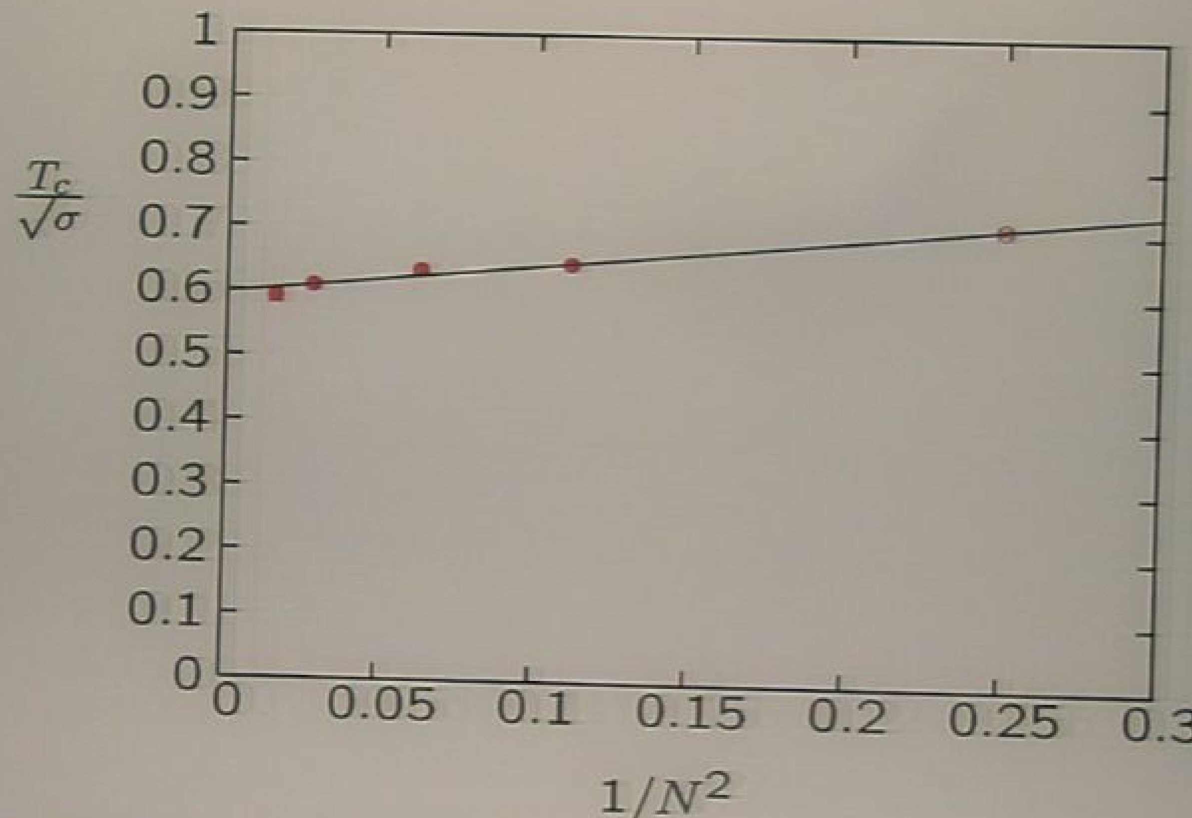
- accurate lattice calculations possible
- accurate continuum extrapolations possible
- SU(3) $\sim$ SU(∞) for many quantities
- linear confinement persists at large N

Motivated by the apparent phenomenological relevance of large-N, let us turn to the detailed properties of SU(N) gauge theories at finite T ...

Deconfining temperature in D=3+1

B.Lucini, M.Teper, U.Wenger: hep-lat/0307017,0502003

$$L_s^3 L_t \Rightarrow T = \frac{1}{a(\beta)L_t} \quad \text{if} \quad L_s \gg L_t$$



2nd order ∘ ; 1st order •

⇒

$$\frac{T_c}{\sqrt{\sigma}} = 0.597(4) + \frac{0.45(3)}{N^2}$$

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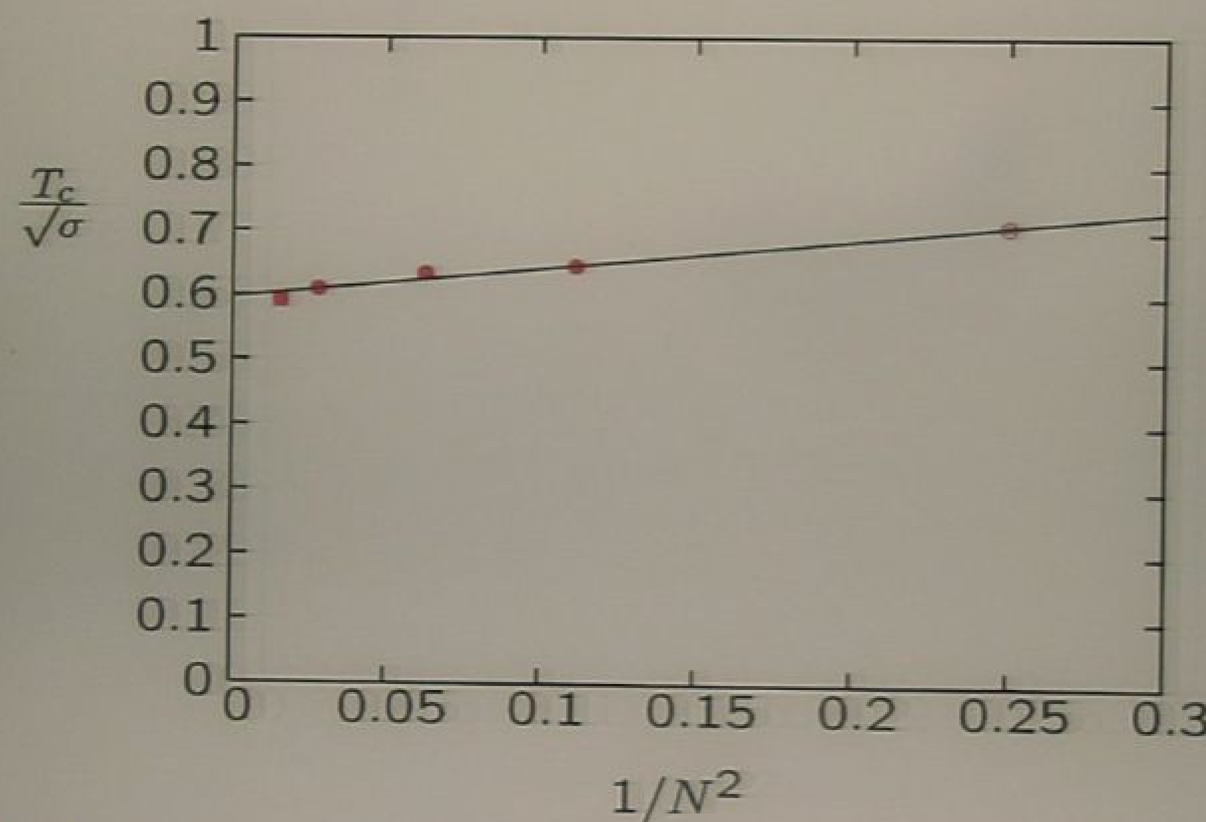

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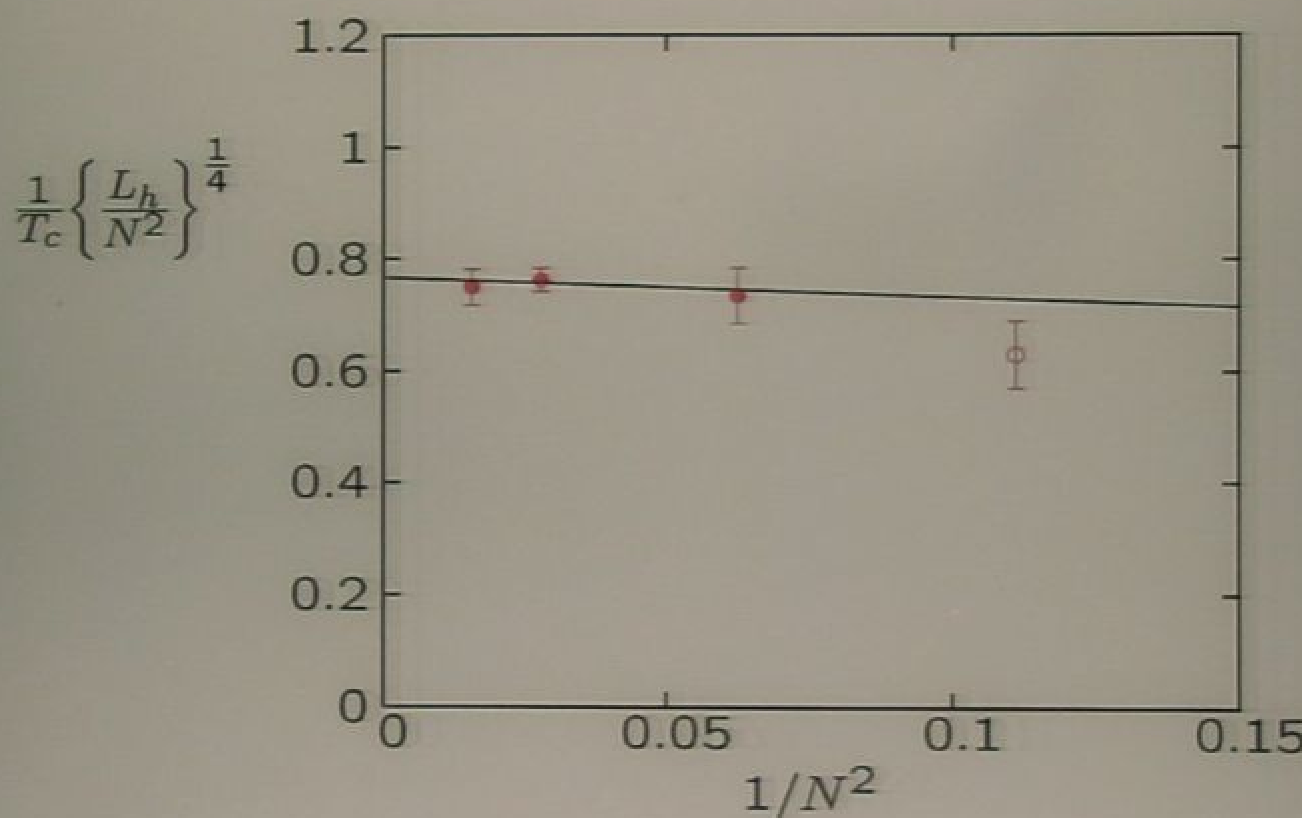
Confinement-deconfinement latent heat

B.Lucini, M.Teper, U.Wenger: hep-lat/0307017,0502003



$\Rightarrow$

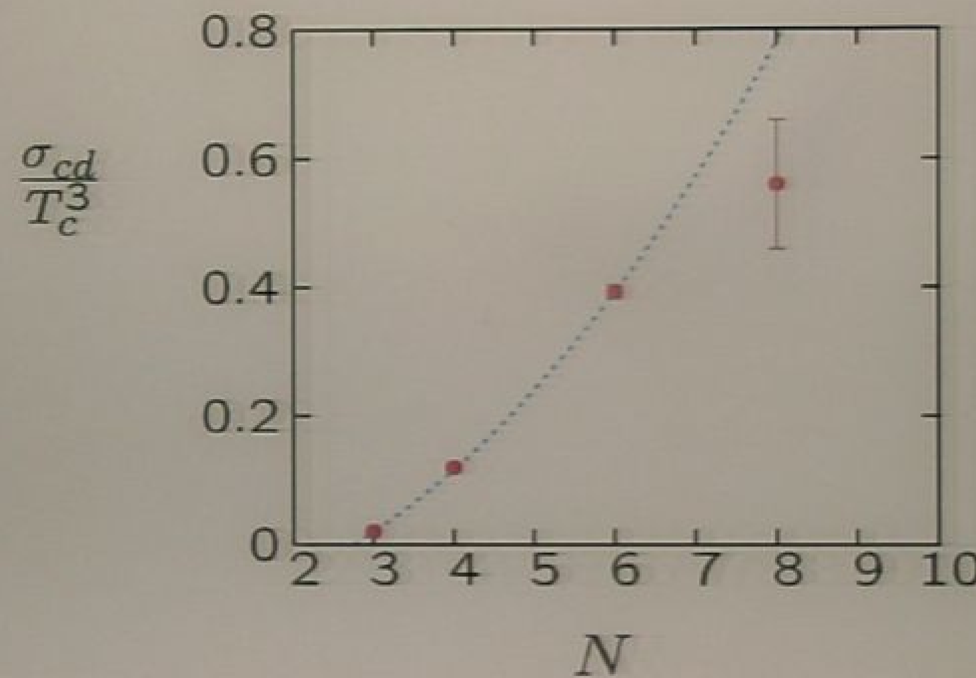
large-N deconfinement is 'normal' first order

$N = 3$ 'weakly' first order

Confinement-deconfinement wall tension

B.Lucini, M.Teper, U.Wenger: hep-lat/0502003

aT=0.2 :



fit :

$$\frac{\sigma_{cd}}{T_c^3} = 0.0138N^2 - 0.104$$

$\Rightarrow$

interface tension small

…ement wall tension

B.Lucini, M.Teper, U.Wenger: hep-lat/0502003

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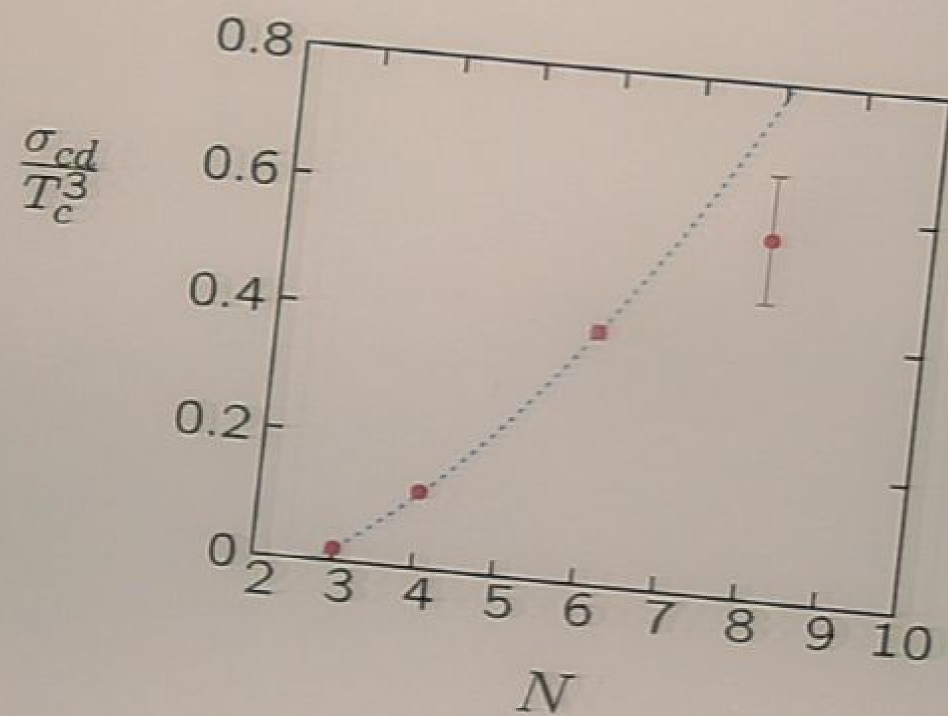

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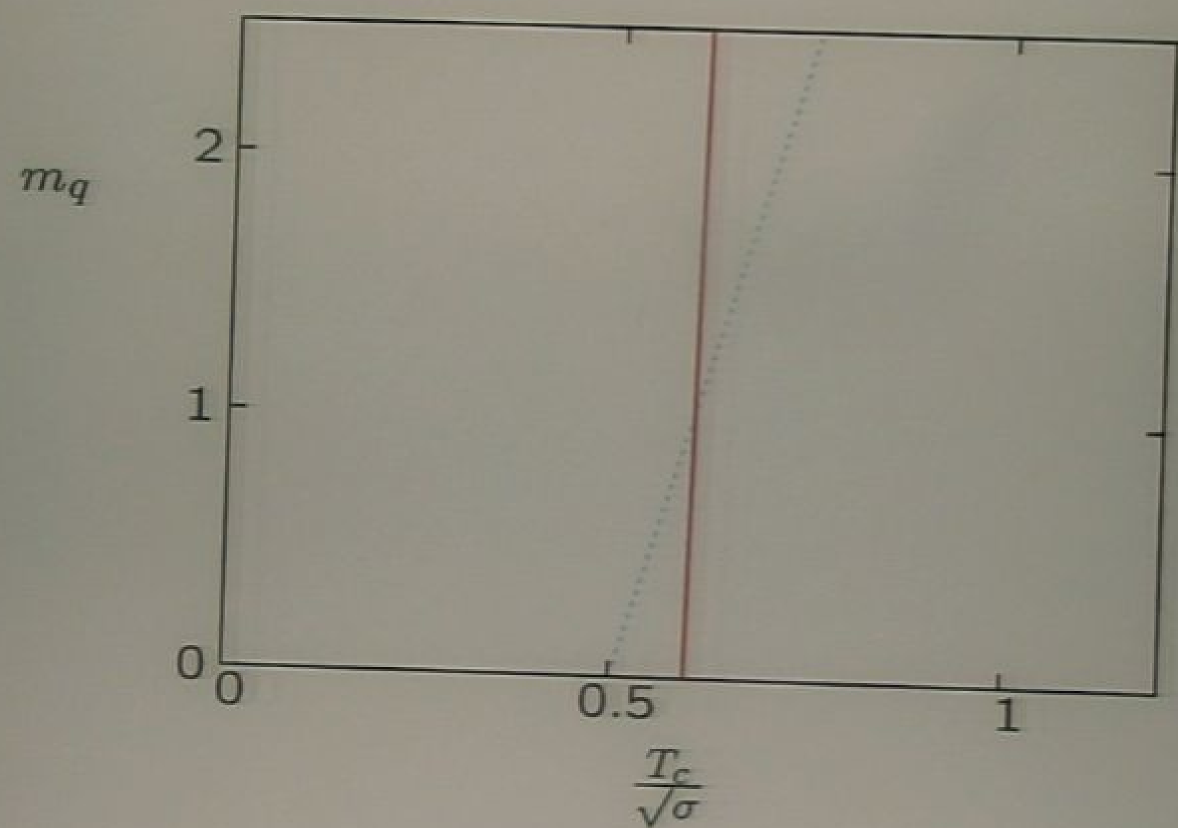
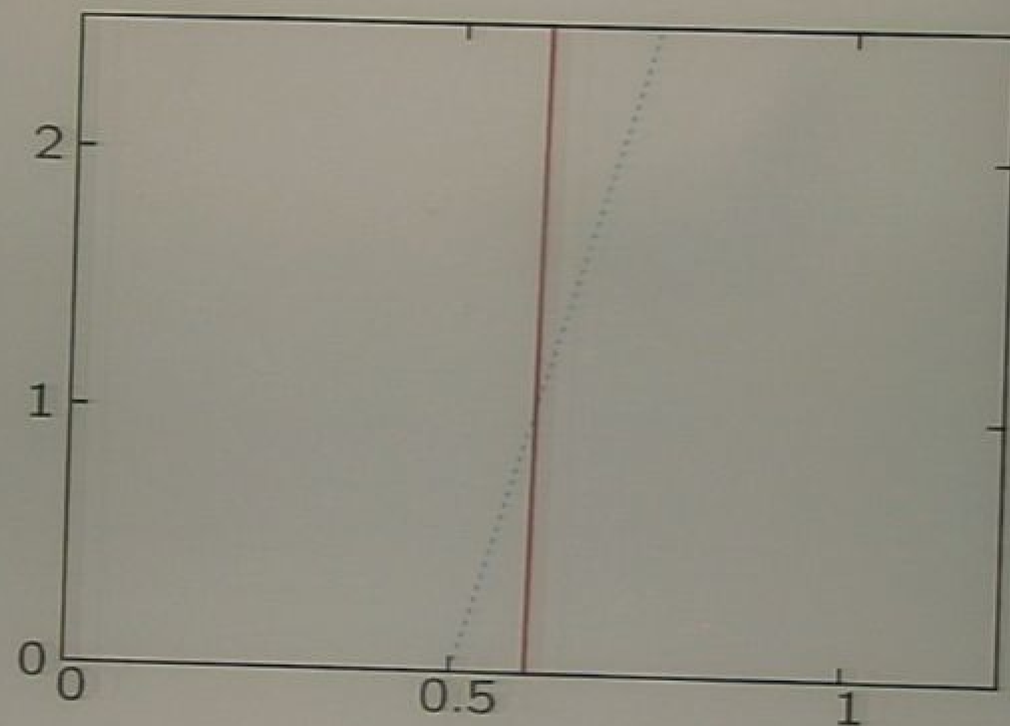
Confinement-deconfinement in QCD_∞?



—— $T = T_c$ ind of m_q at $N = \infty$

Confinement-deconfinement in QCD_∞?

m_q

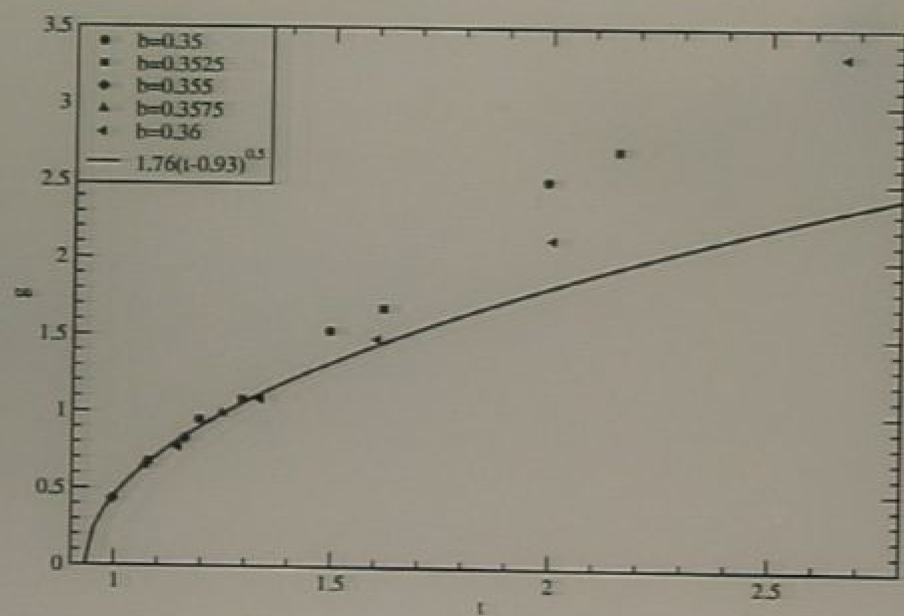


$\frac{T_c}{\sqrt{\sigma}}$

—— $T = T_c$ ind of m_q at $N = \infty$

Chiral symmetry restoration as $T \to T_c$ at large N

R. Narayanan, H. Neuberger: hep-th/0605173

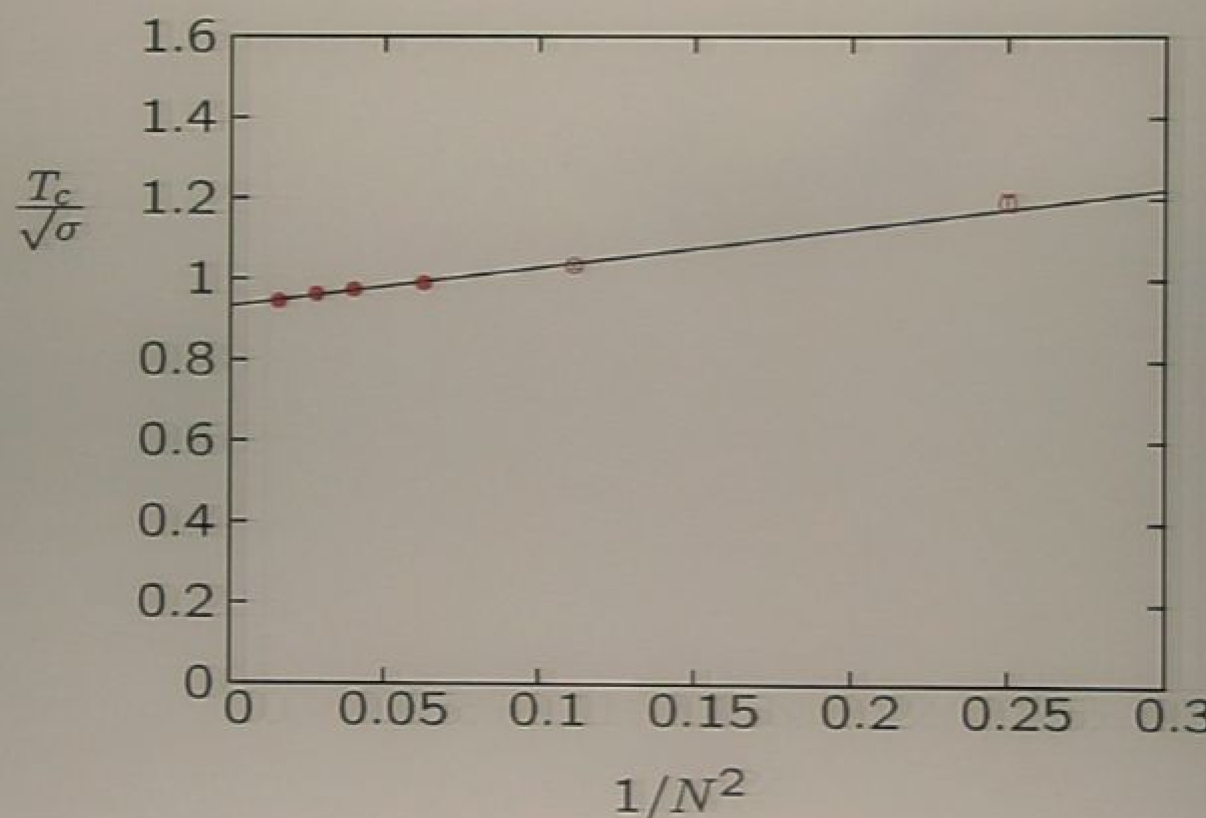


the gap in the eigenvalue spectrum of the Dirac operator, D, at $\lambda \simeq 0$ for $N = 23$ to $N = 53$.

D=3+1 ⟶ D=2+1

$\frac{T_c}{g^2N}$ J. Liddle, M.Teper : hep-lat/0509082; in preparation

$\frac{\sqrt{\sigma}}{g^2N}$ B. Bringoltz, M.Teper : hep-th/0611286



2nd order ∘ ; 1st order •

⇒

fit : $\frac{T_c}{\sqrt{\sigma}} = 0.933(4) + \frac{0.96(8)}{N^2}$ preliminary

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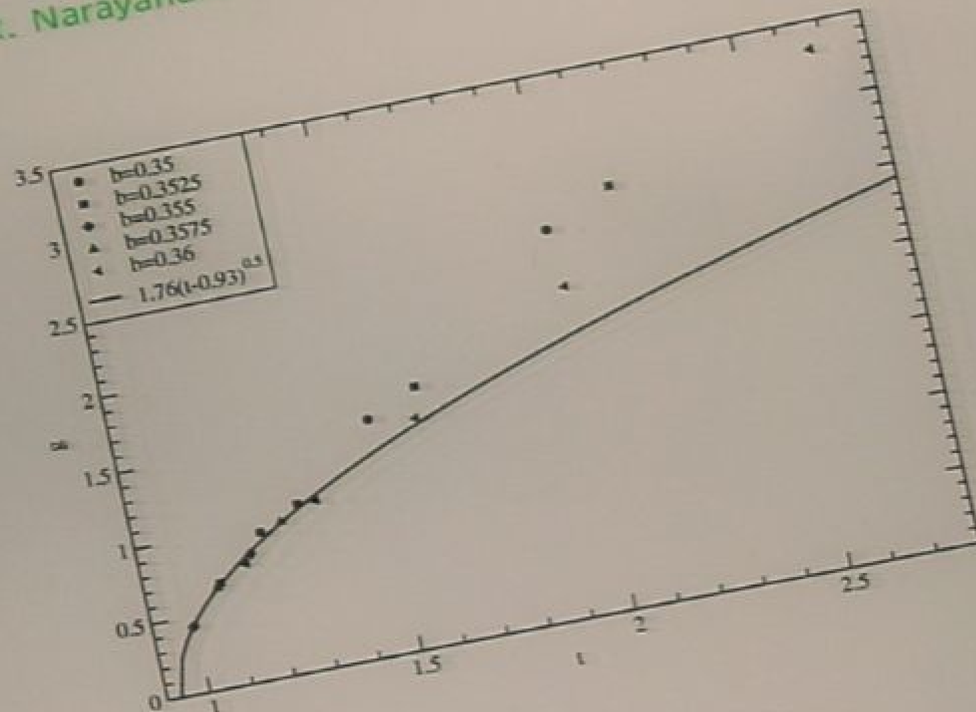

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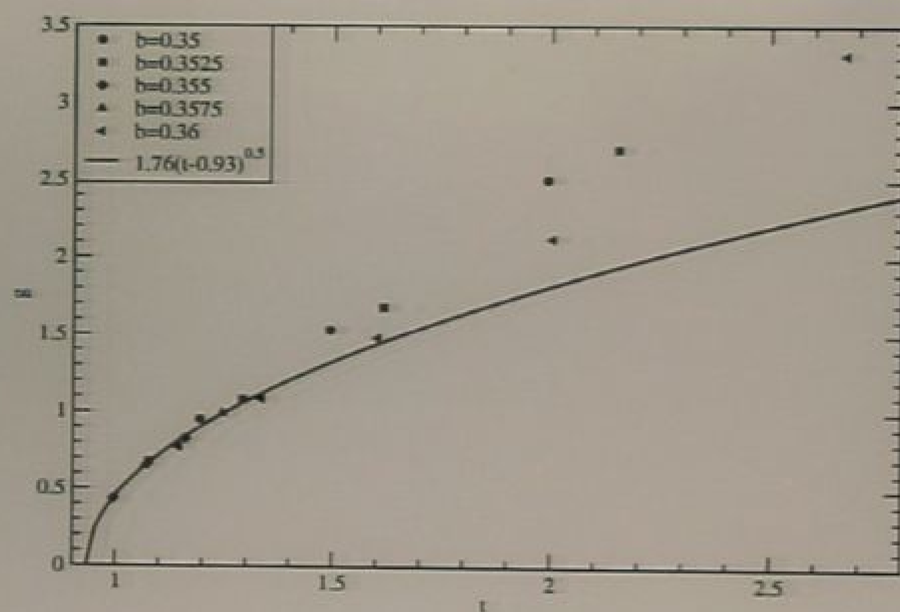

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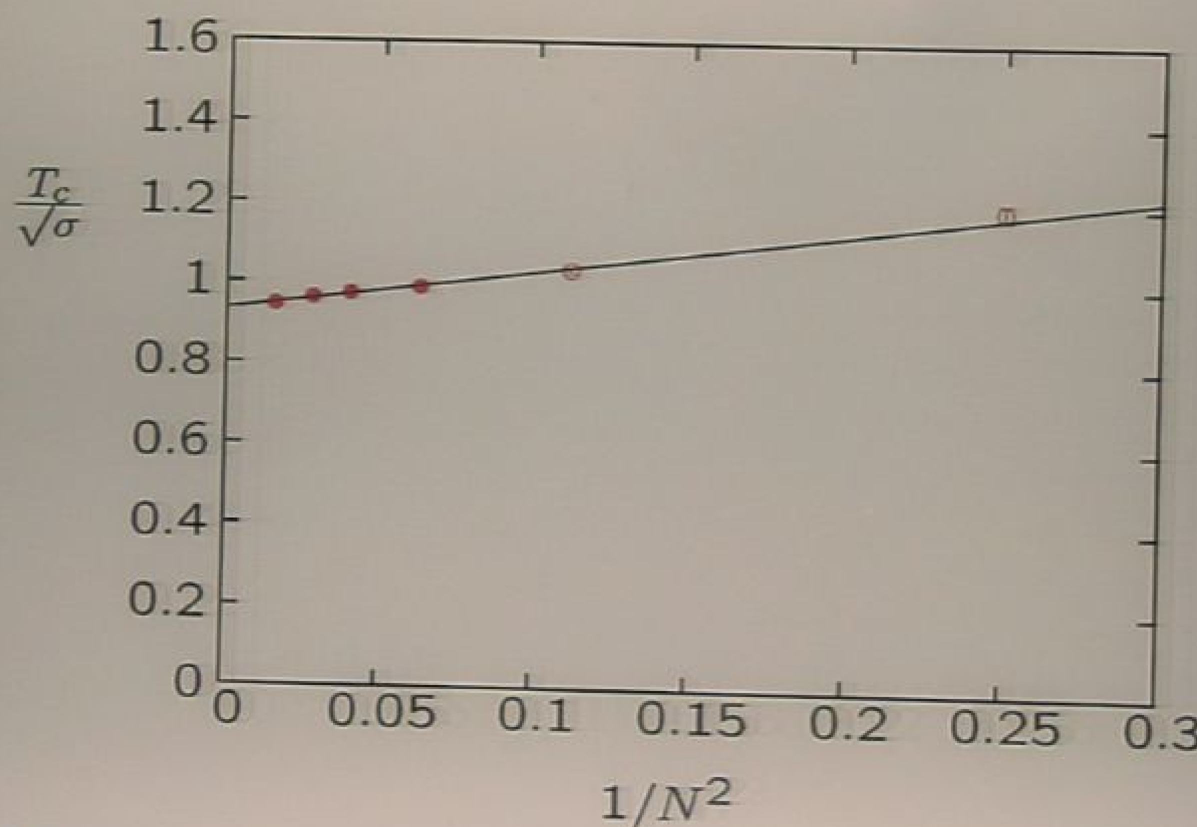


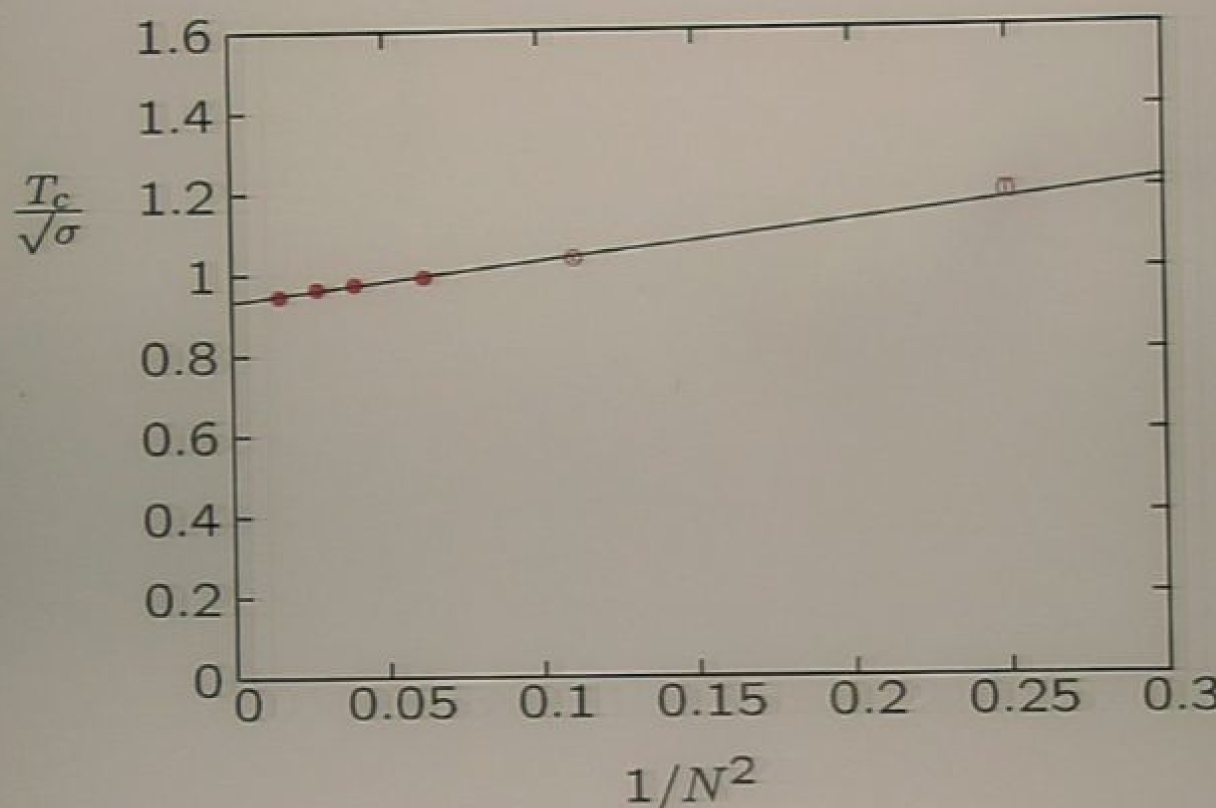
2nd order ○ ; 1st order ●

$\Rightarrow$

fit : $\frac{T_c}{...}$ = 0.903(4) + 0.06(8)...

$\frac{T_c}{g^2N}$ J. Liddle, M. Teper : hep-lat/0509082; in preparation

$\frac{\sqrt{\sigma}}{g^2N}$ B. Bringoltz, M. Teper : hep-th/0611286



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Single or multiple deconfining transitions?

B.Lucini, M.Teper, U.Wenger: hep-lat/0502003

let l_p be the Polyakov loop (fund repn), then

$\langle l_p \rangle = 0$; confined

$\langle l_p \rangle = z \in Z_N$; deconfined

i.e. deconfinement $\leftrightarrow Z_N$ ssb

$\Rightarrow$

is there one transition or several?

e.g.

$$SU(4): \qquad Z_4 \overset{T=T_c}{\longrightarrow} Z_2 \overset{T=T_{c'}}{\longrightarrow} 1$$

corresponding to

$T = T_c$: k=2 strings break – but not k=1

$T = T_{c'}$: k=1 strings break

NO : there is only one transition

N counting of free energies (heuristic)

$$
\begin{aligned}
Z &= e^{-\frac{F}{T}} = \sum_n e^{-\frac{F_n}{T}} && (1)\\
&= \sum_{c=singlet} e^{-\frac{F_c}{T}} + \sum_{g=gluons} e^{-\frac{F_g}{T}} && (2)\\
&= e^{-\frac{F_c}{T}} + e^{-\frac{F_g}{T}} && (3)
\end{aligned}
$$

and at $T = T_c$ we have

$$F_c = F_g$$

but

$F_g \sim N^2$ colour singlet entropy $\sim N^0$

so reason that $T_c \not\to 0$ as $N \to \infty$ is that

$$E_c = hadron\, masses + E_{vac}$$

and

$$E_{vac} \sim -N^2 \sim gluon\, condensate$$

so

$F_g = -E_{vac}$ at $T = T_c$

Strongly Coupled Gluon Plasma - at large N?

B. Bringoltz, M. Teper: hep-lat/0506034

Consider

$$Z(T,V) = \exp\left\{-\frac{F}{T}\right\} = \exp\left\{-\frac{fV}{T}\right\} = \int DU \exp\left(-\beta S_W\right).$$

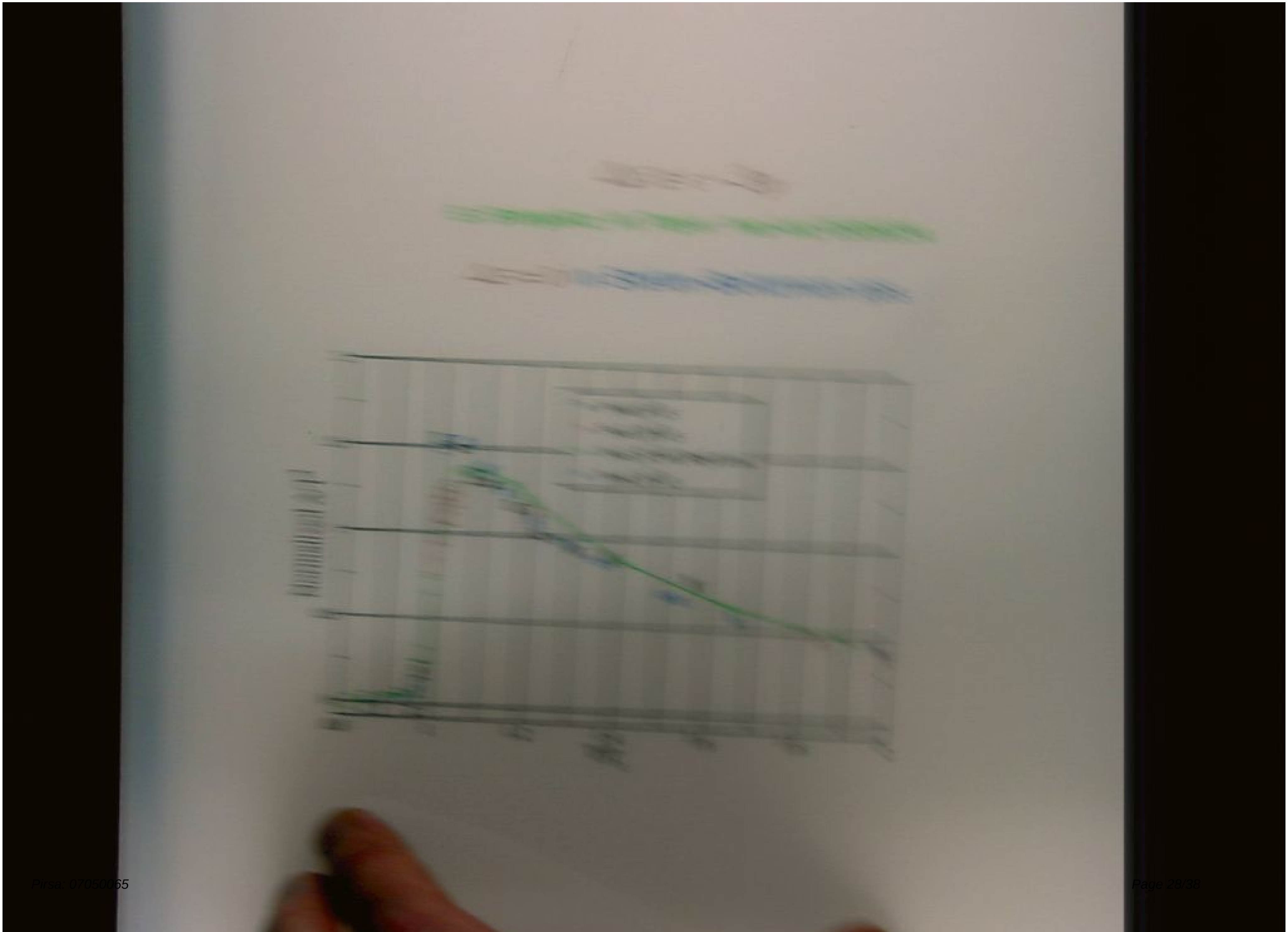
now $p = T\frac{\partial}{\partial V}\log Z(T,V) = \frac{T}{V}\log Z(T,V) = \frac{T}{V}\int_{\beta_0}^{\beta} d\beta' \frac{\partial \log Z}{\partial \beta'}$

but $\frac{\partial \log Z}{\partial \beta} = -\langle S_W\rangle = N_p\langle u_p\rangle$

so $a^4[p(T) - p(0)] = 6\int_{\beta_0}^{\beta} d\beta'(\langle u_p\rangle_T - \langle u_p\rangle_0)$.

i.e. $\frac{p(T)}{T^4} = 6L_t^4\int_{\beta_0}^{\beta} d\beta'(\langle u_p\rangle_T - \langle u_p\rangle_0)$.

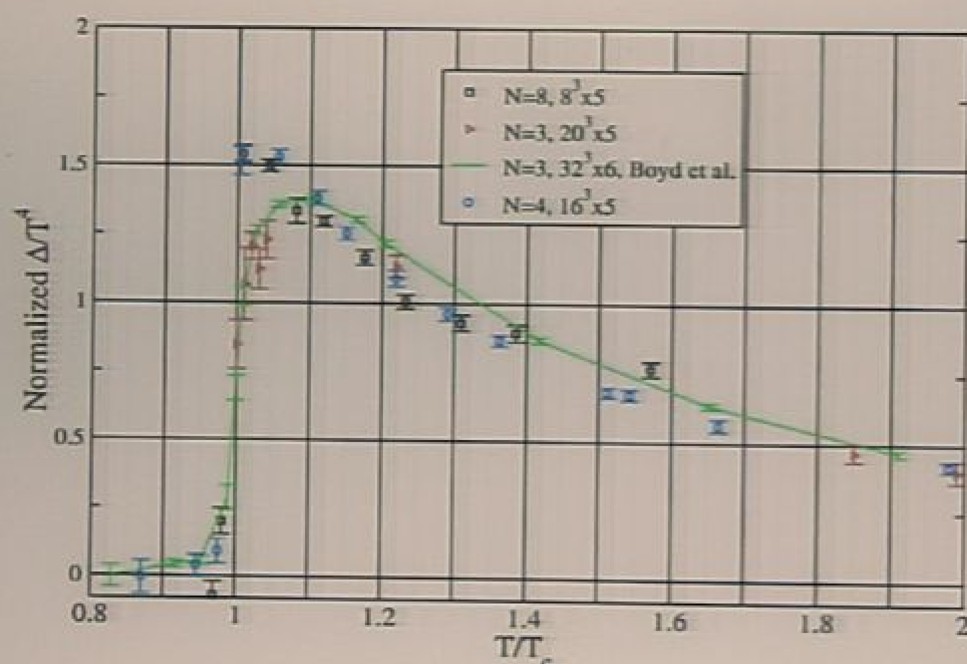
similarly $(\epsilon - 3p)/T^4 = 6L_t^4(\langle u_p(\beta)\rangle_0 - \langle u_p(\beta)\rangle_T) \times \frac{\partial\beta}{\partial \log(a(\beta))}$.



$\Delta \equiv \epsilon - 3p$

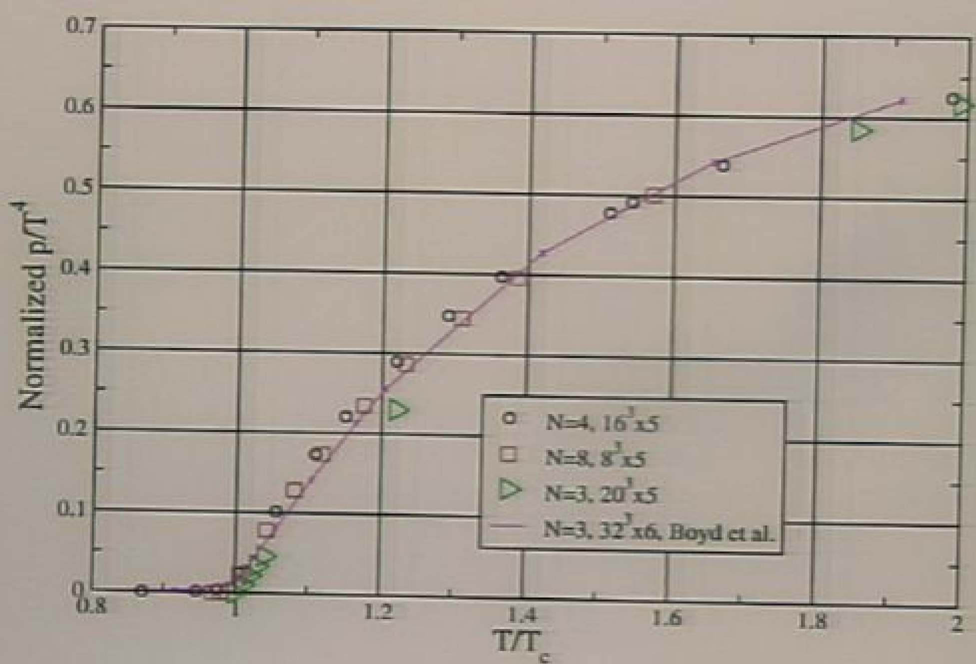
B. Bringoltz, M.Teper: hep-lat/0506034

$\Delta = 0$ in Stefan-Boltzman gas



Strong Gluon Plasma - high-T pressure anomaly

B. Bringoltz, M.Teper: hep-lat/0506034



$\Rightarrow$

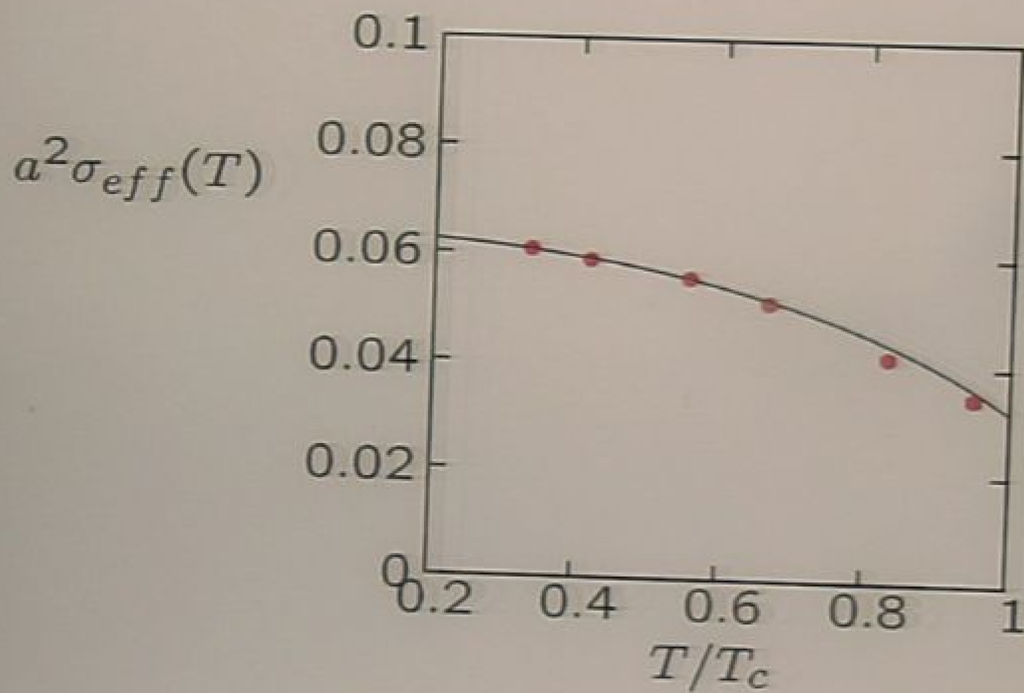
SGP is a large-N phenomenon: dynamics must survive at $N = \infty$

$\Rightarrow$

- not (colour singlet) hadrons above T_c
- not topology (instantons)

D=3+1 SU(6)

H.Meyer, M.Teper: hep-lat/0411039



Nambu-Goto fit shown:

$$a^2\sigma_{eff}(l = 1/aT) = \frac{am_l(l)}{l} = a^2\sigma\left(1 - \frac{2\pi}{3}\frac{1}{\sigma l^2}\right)^{\frac{1}{2}}$$

$\xrightarrow{T\to 0}$

$$\sigma_{eff}(T) = \sigma - \frac{\pi}{3T^2} + O(\frac{1}{T^4})$$

Searching for the Hagedorn transition : SU(12)

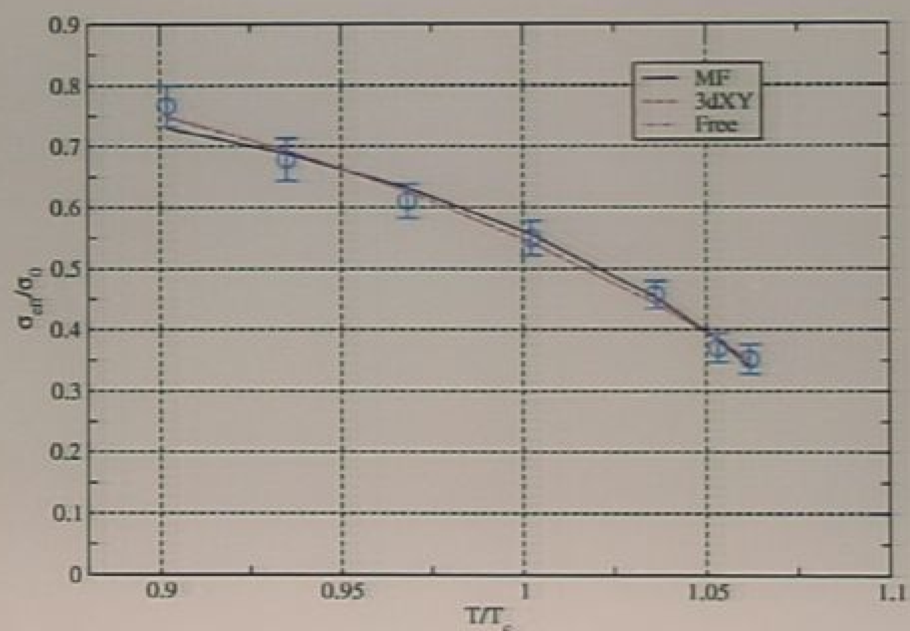
B. Bringoltz, M.Teper: hep-lat/0508021

use strong metastability of the 1st order deconfining transition to stay in the confining phase for

$$T > T_c$$

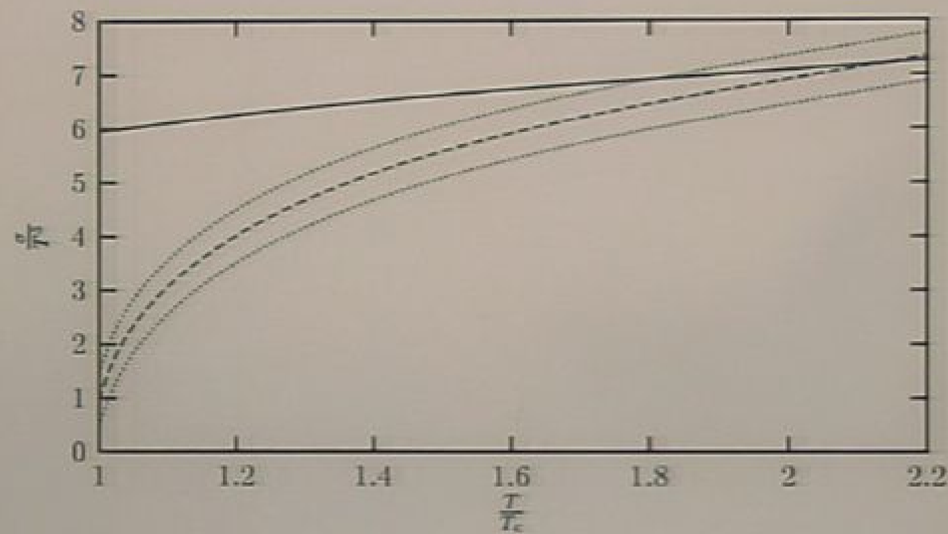
and try to extrapolate to

$$\lim_{T \to T_H} \sigma_{eff}(T) = 0$$



The 't Hooft string tension ...

F. Bursa, M. Teper: hep-lat/0505025



SU(4) 't Hooft string tension in units of T (with 2-loop perturbative result using $g^2(T) \simeq g^2_{MFI}(a)$).

$\Rightarrow$

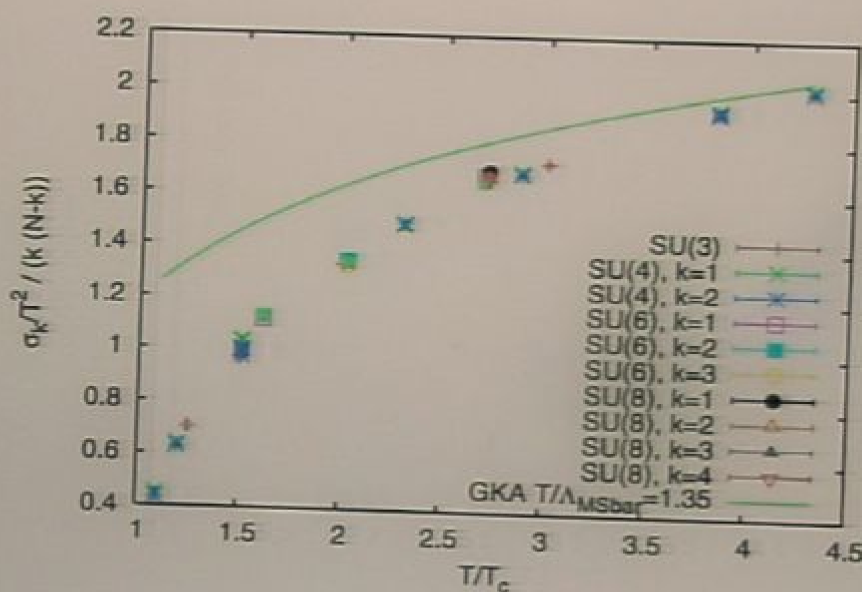
hint of approximate duality

$T \leftrightarrow \frac{1}{T}$

between the confining and 't Hooft string tensions

also ...

de Forcrand, Lucini, Noth: hep-lat/0510081



$\Rightarrow$

tension very small at $T = T_c$

small deconf-deconf wall tensions at $T = T_c$

$\Rightarrow$

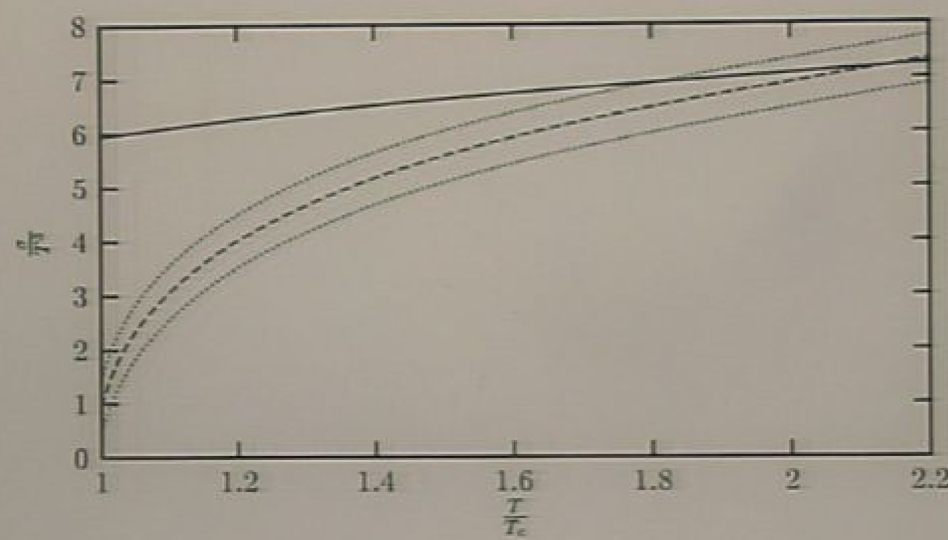
$\sigma_{cd} \overset{T=T_c}{=} 2\sigma_{k=N/2}$

$\Leftrightarrow$

small conf-deconf wall tension at $T = T_c$

The 't Hooft string tension ...

F. Bursa, M. Teper: hep-lat/0505025



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In Summary:

- $SU(\infty)$ is linearly confining in both $D = 3+1$ and $D = 2+1$ and for many quantities $SU(3) \simeq SU(\infty)$.

- in particular, the 'strong coupling gluon plasma' (e.g. pressure anomaly) is a large-N phenomenon

some quantities that have been calculated at finite T and for 'all' N:

- $T_c/\sqrt{\sigma}$ in $D = 3+1, 2+1$;
- L_h/T_c^D latent heat in $D = 3+1, 2+1$;
- σ_{CD} surface tension in $D = 3+1$;
- $\sigma_{DD'}$ 't Hooft tension in $D = 3+1$;
- effective string tension for $T < T_c$
- spatial k-string tensions for $T > T_c$
- electric screening masses for $T > T_c$

some observations:

- the transition is first order for $N \geq 3$ in $D = 3 + 1$, and for $N \geq 4$ in $D = 2 + 1$

- at this point the $O(N^2)$ gluon plasma free energy is entirely balanced by the $O(N^2)$ confining vacuum energy (gluon condensate)

- we have gone beyond T_c, about halfway to 'T_H', in the confined metastable phase

- the 't Hooft tension decreases rapidly as $T \to T_c^+$ suggesting some approximate duality with the (Wilson) string tension, with a dual Hagedorn transition just below T_c

- topological fluctuations vanish as $\exp\{-cN\}$ at all T in the deconfined phase (calorons?)

- a single phase transition at larger N where the whole Z_N group is broken

- a cascade of finite volume transitions at $N \to \infty$ that are essentially deconfining transitions on ever more dimensionally reduced gauge + adjoint scalar theories some observations:

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• adjoint Polyakov loops make an elegant order param-