Title: Lattice Finite - Temperature QCD

Date: May 24, 2007 09:00 AM

URL: http://pirsa.org/07050064

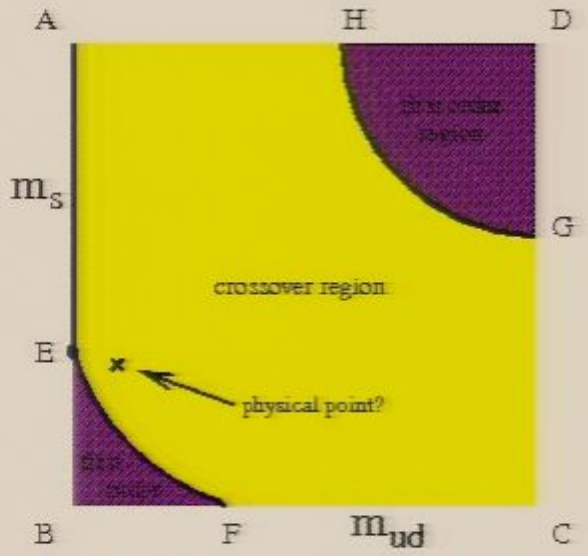
Abstract:

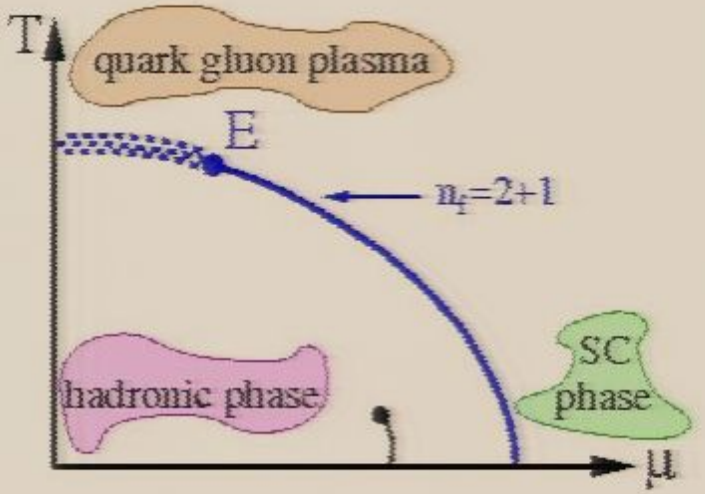
Lattice QCD thermodynamics at μ=0 and $\mu\neq$0

Zoltán Fodor
(University of Wuppertal & University of Budapest)

1. Standard picture of the phase diagram
2. Lattice formulation, fermion doubling, rooting
3. Monte-Carlo techniques in lattice QCD
4. Lattice results at L_t=4 (μ>0)
5. Lattice results at L_t=4,6 (equation of state)
6. Lattice results at L_t=4,6,8,10 (nature and T_c)
7. Conclusions

Standard picture of the phase diagram and its uncertainties





- Chiral phase transition (PT)

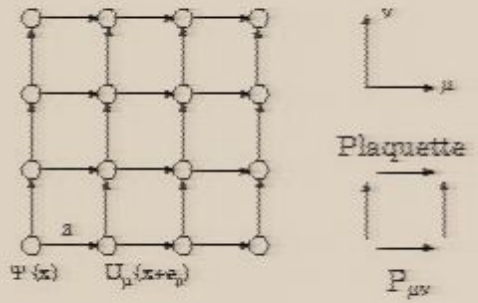
$n_f = 2$ with $m_q = 0$ at $\mu = 0 \Rightarrow 2^{nd}$ order phase transition

$n_f = 3$ with $m_q = 0$ at $\mu = 0 \Rightarrow 1^{st}$ order phase transition

$n_f = 2 + 1$ with physical m_q at $\mu = 0 \Rightarrow$ cross-over
$n_f = 2 + 1$ with physical m_q at $T = 0 \Rightarrow 1^{st}$ order PT
$\Longrightarrow$ results in the standard picture of the QCD phase diagram

$n_f = 2 + 1$ with physical $m_q \Rightarrow$ critical point (E) at μ, T$\neq$0

Lattice action of QCD and Monte-Carlo algorithms



anti-commuting $\psi(x)$ quark fields live on the sites
gluon fields, $A^a_\mu(x)$ are used as links and plaquettes with g_s:

$$U(x,y) = \exp\left(ig_s \int_x^y dx'^\mu \, A^a_\mu(x')\lambda_a/2\right)$$

$$P_{\mu\nu}(n) = U_\mu(n)U_\nu(n+e_\mu)U^\dagger_\mu(n+e_\nu)U^\dagger_\nu(n)$$

$S = S_g + S_f$ consists of the pure gluonic and the fermionic parts

$$S_g = \frac{6}{g_s^2}\sum_{n,\mu,\nu}\left[1 - \frac{1}{2}\mathrm{Tr}(P_{\mu\nu}(n) + P^\dagger_{\mu\nu}(n))\right]$$

fermionic action contains derivatives, differencing scheme:

$$\bar\psi(x)\gamma^\mu\partial_\mu\psi(x) \to \bar\psi_n\gamma^\mu(\psi_{n+e_\mu} - \psi_{n-e_\mu})$$
$$\bar\psi(x)\gamma^\mu D_\mu\psi(x) \to \bar\psi_n\gamma^\mu U_\mu(n)\psi_{n+e_\mu} + \ldots$$

Partition function

$$Z = \int dU d\Psi d\bar{\Psi} e^{-S_E}$$

S_E is the Euclidean action

Parameters:

gauge coupling g

quark masses m_i $(i = 1..N_f)$

(Chemical potentials μ_i)

Volume (V) and temperature (T)

Finite $T \leftrightarrow$ finite temporal lattice extension

$$T = \frac{1}{N_t a}$$

Continuum limit: $a \to 0$

Renormalization: keep the physical spectrum constant

at finite T:

continuum limit $\Longleftrightarrow N_t \to \infty$

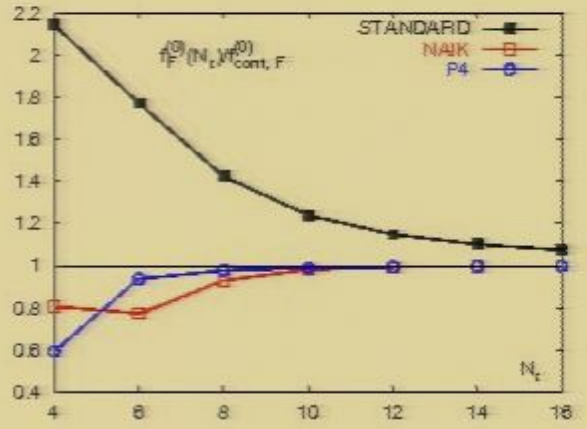
CPU grows like N_t^{13}, thus N_t=10 instead of 6 needs 50-100$\times$ more CPU

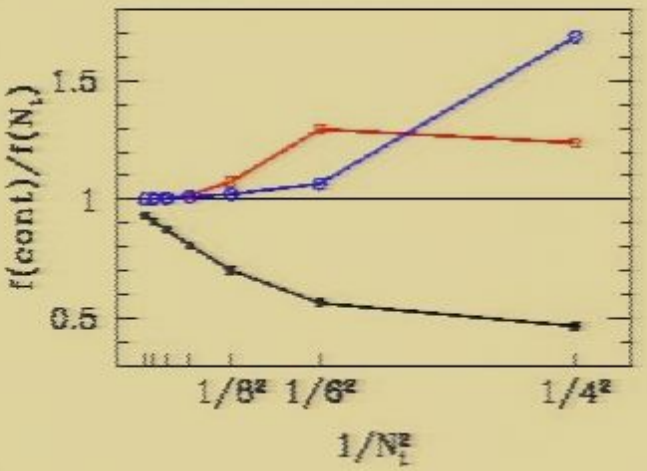
Improved actions

S_E is not unique; many possibilities

from flic to clover and tadpole, hyper-improved and even overimproved improvements

Continuum limit is always important!





[Heller, Karsch, Sturm '99]

Continuum extrapolation from N_t and $N_t + 2$ standard action may be better than using only N_t with improved action

Fermion doubling problem

- naive discretization of the massless Dirac action:

$$S^{-1}(p) = (i/a)\sum_\mu \gamma_\mu \sin(p_\mu a)$$

has 16 zeros within the Brillouin cell (in d dimensions 2^d)
these zeros are at $p_\mu a$=0 and at $\pi \Rightarrow$ altogether 16 modes (doubling)
inclusion of the gauge field does not solve the problem

- solutions to the fermion doubling problem

a. spin diagonalization: staggered fermions (Kogut-Susskind)
cheap, 2a translation invariance, 4 flavors/tastes $\Rightarrow$ rooting $\Rightarrow$ non-local

b. irrelevant operator (doublers mass $\to \infty$): Wilson-fermions
more expensive, chiral symmetry is explicitly broken

c. chiral fermions: $\gamma_5 D + D\gamma_5 = aD\gamma_5 D$ (Ginsparg-Wilson relation)
very nice theoretical properties, very expensive

- consequences of taste symmetry and its violation

$$Z = \int \prod_{n,\mu} [dU_\mu(x)] e^{-S_g} \det(M[U])^{n_f/4}$$

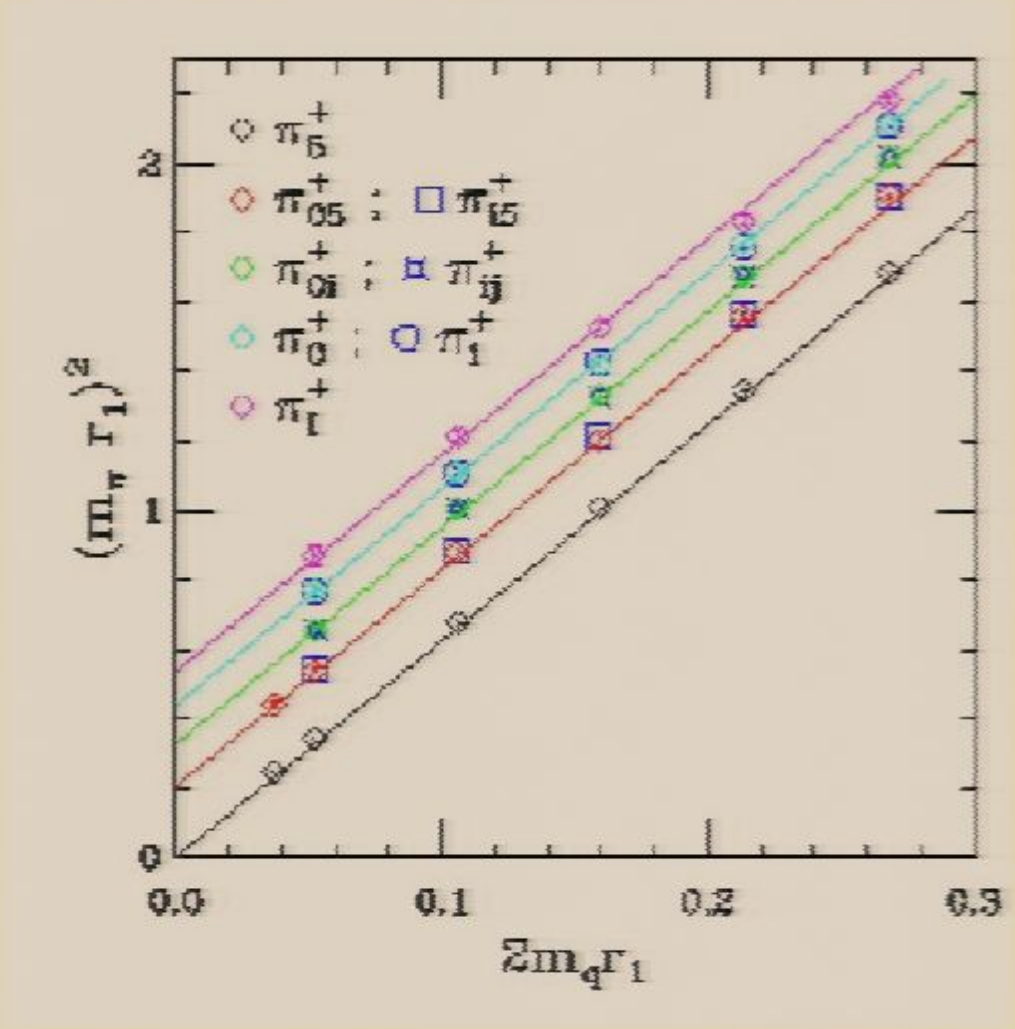
real calculations: one needs n_f=2+1 (two light quarks & strange)
"rooting procedure" leads to non-locality for any lattice spacing
might spoil universality, or it is restored in the continuum limit

lively discussion: staggered fermions are good, bad or just ugly

all large-scale thermodynamics projects are using staggered fermions
MILC: large US collaboration uses for T=0 also staggered fermions

taste violation (we have only 1 pion instead of 3) is severe

MILC result (action is tailored to suppress taste violation)



certainly includes many (perhaps all) of the dynamical effects

→ but one needs other, theoretically cleaner actions

Lattice gauge theory simulations

• Monte-Carlo: n-dimensional integral with importance sampling
e.g. for pure gauge the integration weight is $P = \exp(-S_g)/Z$
probability measure on the phase space of link variables $U_{n\mu}$
Markov process: successive change of $U_{n\mu}$ variables (sweeps)

• Metropolis algorithm for pure gauge (similar to the Ising model)
for $U_{n\mu}$ with a value U_{old} we generate a trial one U_{trial}
accept/reject for a new link value according to the probability

$$P_{acc/rej} = min\,[1, \exp(-\Delta S_g)]$$

a single sweep repeats it for every link of the lattice

• more efficient gauge algorithms for: heatbath, overrelaxation
heatbath:
single variable according to the correct probability distribution
overrelaxation:
sort of reflection of a variable (microcanonical, S is conserved)

Dynamical fermion simulation algorithms

generate a distribution for gauge (!) configurations
in a Monte-Carlo process with weights: $\exp(-S)$

dynamical fermions give bilinear Grassmann terms
note, that this fermion matrix is sparse (few next neighbours)

$$Z = \int \prod_{n\mu} dU_{n\mu} \prod_n d\bar{\psi}_n \psi_n \exp[-S_g(U) - \sum_{rs} \bar{\psi}_r D_{rs} \psi_s]$$

$$= \int \prod_{n\mu} dU_{n\mu} \det D(U) \exp[-S_g(U)]$$

$$= \int \prod_{n\mu} dU_{n\mu} \exp[-S_g(U) - \mathrm{Tr} \log D(U)]$$

result depends exclusively on the gauge field
complete bosonisation of the theory (determinant is nonlocal)
though the effective interaction it is highly non-local

Microcanonical or molecular dynamics evolution

define a fictious Hamiltonian (from the bosonised action)
with fictious momenta $P_{n\mu}$ conjugate to $U_{n\mu}$

$$H_{mc} = \frac{1}{2}\sum_{n\mu} \mathrm{tr} P_{n\mu}^2 + [S_g(U) - \mathrm{Tr}\log\det D(U)]$$

$$= \frac{1}{2}\sum_{n\mu} \mathrm{tr} P_{n\mu}^2 + S_{eff}(U)$$

gluon fields evolve according to the Hamilton equation

$$\frac{dU_{n\mu}(\tau)}{d\tau} = \frac{dH_{mc}}{dP_{n\mu}} = P_{n\mu}(\tau)$$

$$\frac{dP_{n\mu}(\tau)}{d\tau} = -\frac{dH_{mc}}{dU_{n\mu}} = -\frac{dS_{eff}[U(\tau)]}{dU_{n\mu}}$$

above evolution is an energy conserving process: non-ergodicity
renew $P_{n\mu}$ configuration from a random Gaussian ensemble
almost all present simulations with this hybrid Monte-Carlo

- inverse of the fermion matrix is needed

$$\frac{dS_g(U)}{dU_{n\mu}} - \mathrm{Tr}\left[\frac{dD(U)}{dU_{n\mu}} D(U)^{-1}\right]$$

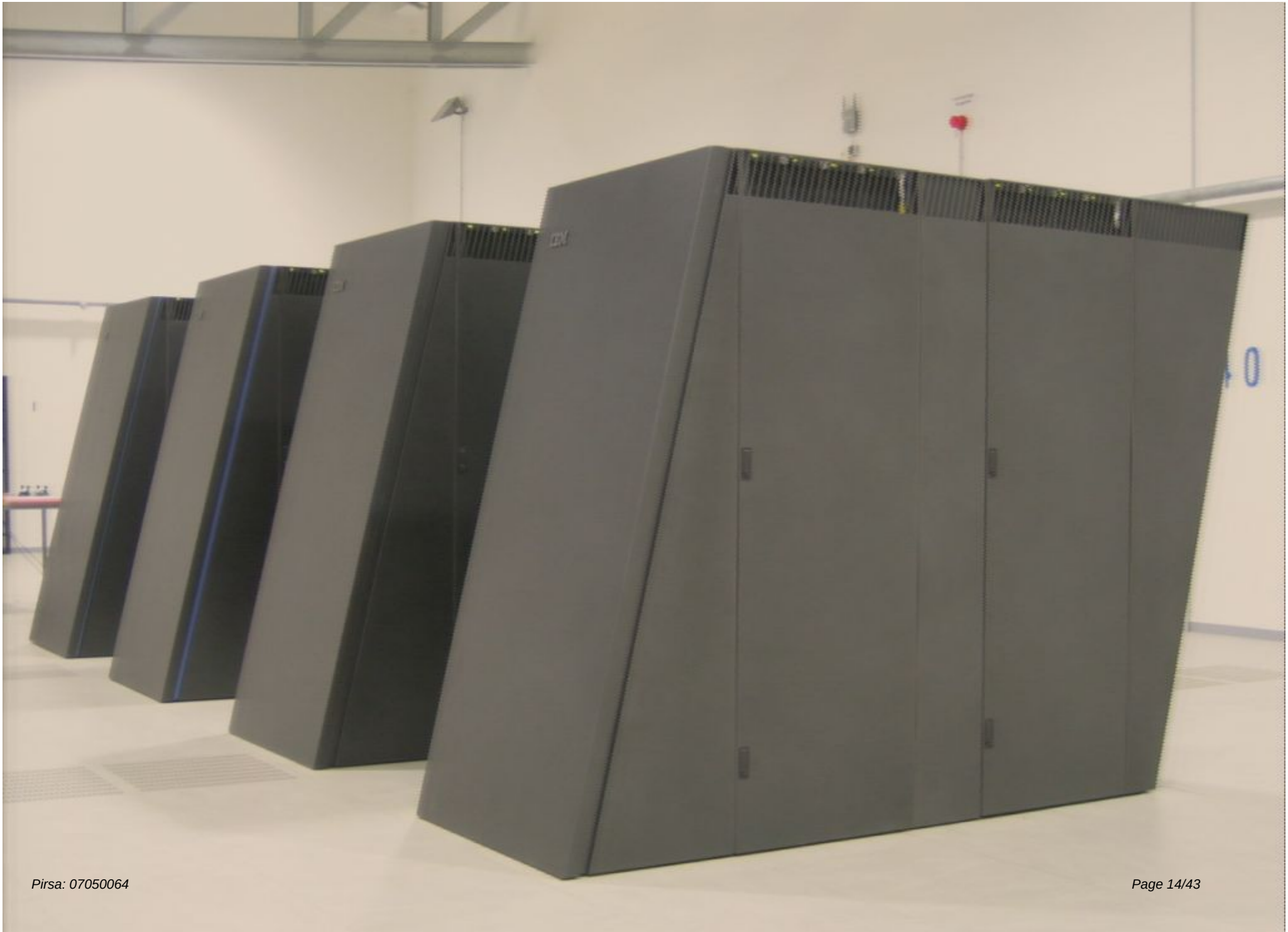
needs the inverse of $D(U)$ of size $V \cdot V = L^4 \cdot L^4$
with Gauss elimination it means $V^3 = L^{12}$ arithmetic operations

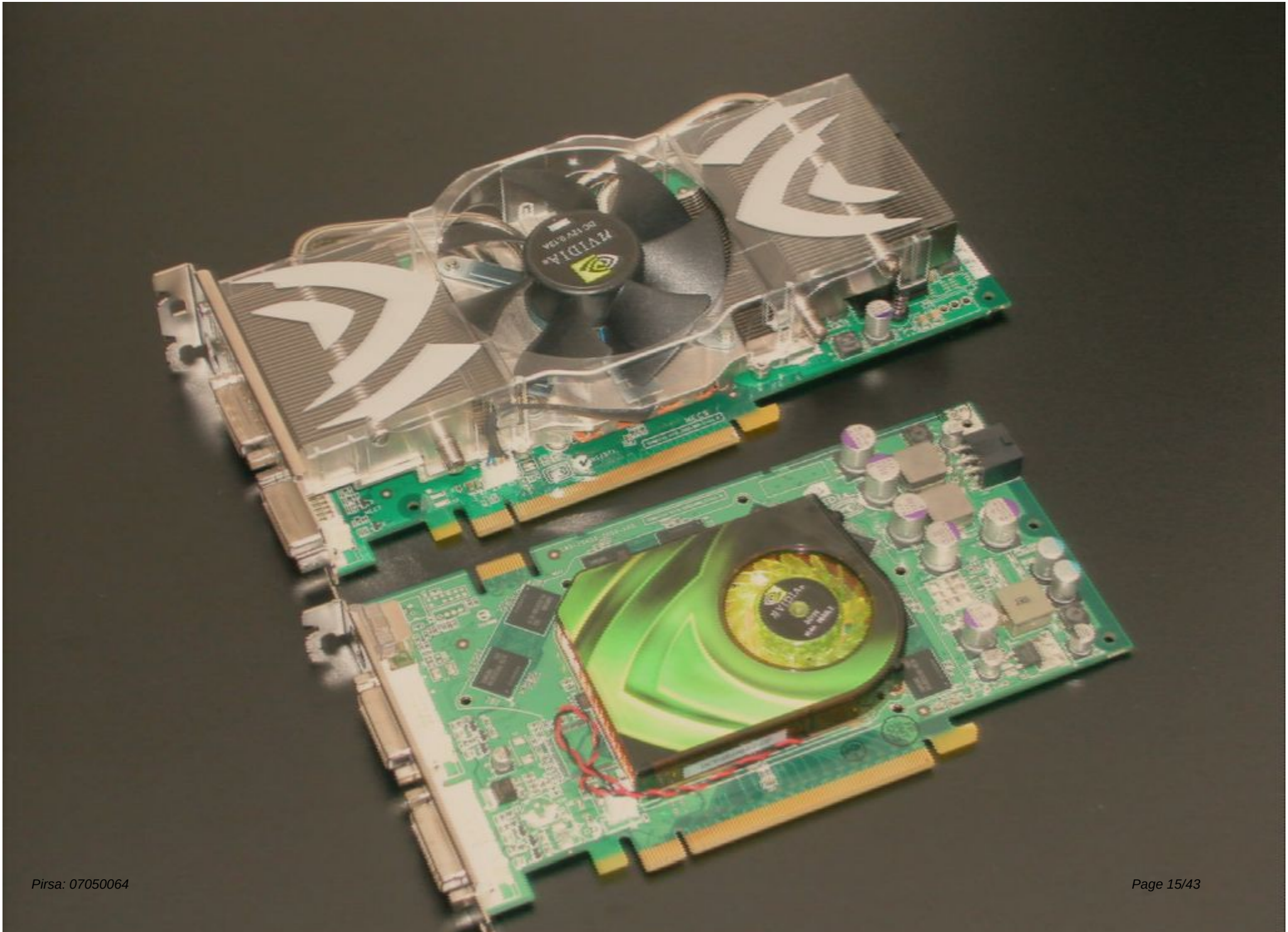
important trick: estimation by ξ_r complex Gaussian noise

$$\mathrm{Tr}\left[\frac{dD(U)}{dU_{n\mu}} D(U)^{-1}\right] \approx \sum_{rs} \xi_r^* \left[\frac{dD(U)}{dU_{n\mu}} D(U)^{-1}\right]_{rs} \xi_s$$

for $V \to \infty$ it fluctuates with relative order of $1/\sqrt{V}$
computing $D(U)^{-1}$ is replaced by computing $D(U)^{-1}\xi$

$\Rightarrow$ solving the linear equation $D(U)x = \xi$ is easy
effective recursive algorithms of $\mathcal{O}$(V) operations
most popular for lattice QCD: conjugate gradient (CG)
100-1000 steps are enough (much faster than inversion)





differencing scheme for quarks:

$$\bar{\psi}(x)\gamma^{\mu}\partial_{\mu}\psi(x) \rightarrow \bar{\psi}_n\gamma^{\mu}(\psi_{n+e_\mu} - \psi_{n-e_\mu})$$
$$\bar{\psi}(x)\gamma^{\mu}D_{\mu}\psi(x) \rightarrow \bar{\psi}_n\gamma^{\mu}U_{\mu}(n)\psi_{n+e_\mu} + \ldots$$

the chemical potential acts as the
fourth component of an imaginary, constant vector potential

fermionic part as a bilinear expression: $S_f = \bar{\psi}_n M_{nm}\psi_m$

Euclidean partition function gives Boltzman weights

$$Z = \int \prod_{n,\mu}[dU_\mu(x)][d\bar{\psi}_n][d\psi_n]e^{-S_g - S_f} = \int \prod_{n,\mu}[dU_\mu(n)]e^{-S_g}\det(M[U])$$

Metropolis step for importance sampling:

$$P(U \rightarrow U') = \min\left[1, \exp(-\Delta S_g)\det(M[U'])/\det(M[U])\right]$$

for μ=0 the determinant is real, for $\mu \neq 0$ it is complex
⇒ no probability interpretation, no Monte-Carlo method

Overlap improving multi-parameter reweighting

Z. Fodor and S.D. Katz, Phys. Lett. B534 (2002) 87

$$Z(m,\mu,\beta) = \int \mathscr{D}U \exp[-S_g(\beta,U)] \det M(m,\mu,U) =$$

$$\int \mathscr{D}U \exp[-S_g(\beta_0,U)] \det M(m_0,\mu=0,U)$$

$$\left\{ \exp[-S_g(\beta,U)+S_g(\beta_0,U)] \frac{\det M(m,\mu,U)}{\det M(m_0,\mu=0,U)} \right\}$$

first line = measure, field configurations of the Monte-Carlo
curly bracket = can be measured on each configuration, weight

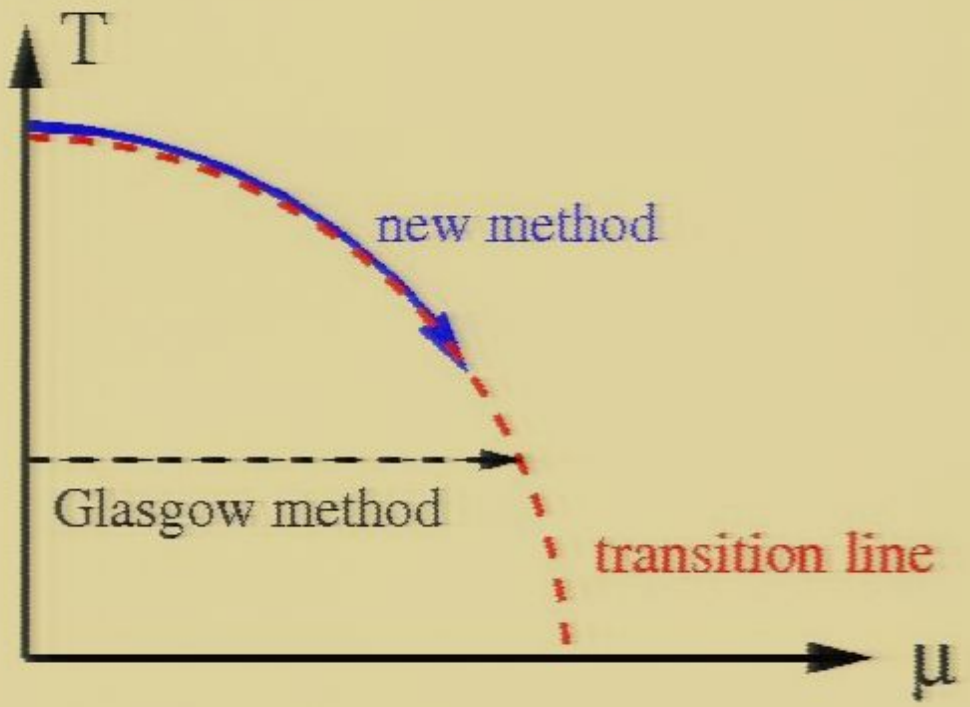
simultaneously changing several parameters: better overlap
e.g. transition configurations are mapped to transition ones

expectation value of an observable O:

$$\langle O \rangle_{\beta,\mu,m} = \frac{\sum w(\beta,\mu,m) O(\mu,m)}{\sum w(\beta,\mu,m)}$$

observables to get the transition points at $\mu \neq 0$ (susceptibilities)

Comparison with the Glasgow method



one parameter reweighting

single parameter (μ)

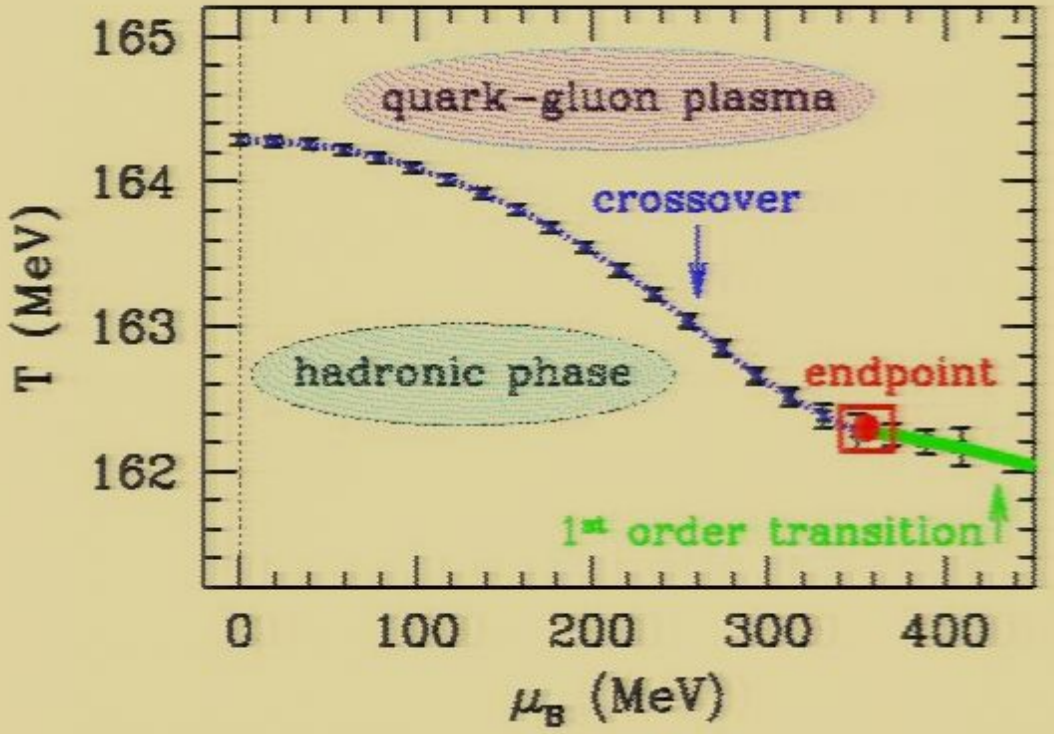
purely hadronic configurations

New method

two parameters (μ and β)

transition configurations

- T as a function of the baryonic chemical potential μ_B



- lattice result for physical quark masses at $L_t = 4$

endpoint: $T_E = 162 \pm 2$ MeV, $\mu_E = 360 \pm 40$ MeV

at μ_B=0 transition temperature: $T_c = 164 \pm 2$ MeV.

$T/T_c = 1 - C\mu_B^2/T_c^2$ wit C=0.0032(1)

as expected: E moves closer to μ=0, T_c and C decreases

$\mu \neq 0$ multi-parameter reweighting with Taylor expansion

C.R. Allton et al., Phys. Rev. D66 074507,'02, D68 014507,'03

$$Z(m,\mu,\beta) = \int \mathscr{D}U \exp[-S_g(\beta,U)] \det M(m,\mu,U) =$$

$$\int \mathscr{D}U \exp[-S_g(\beta_0,U)] \det M(m_0,\mu=0,U)$$

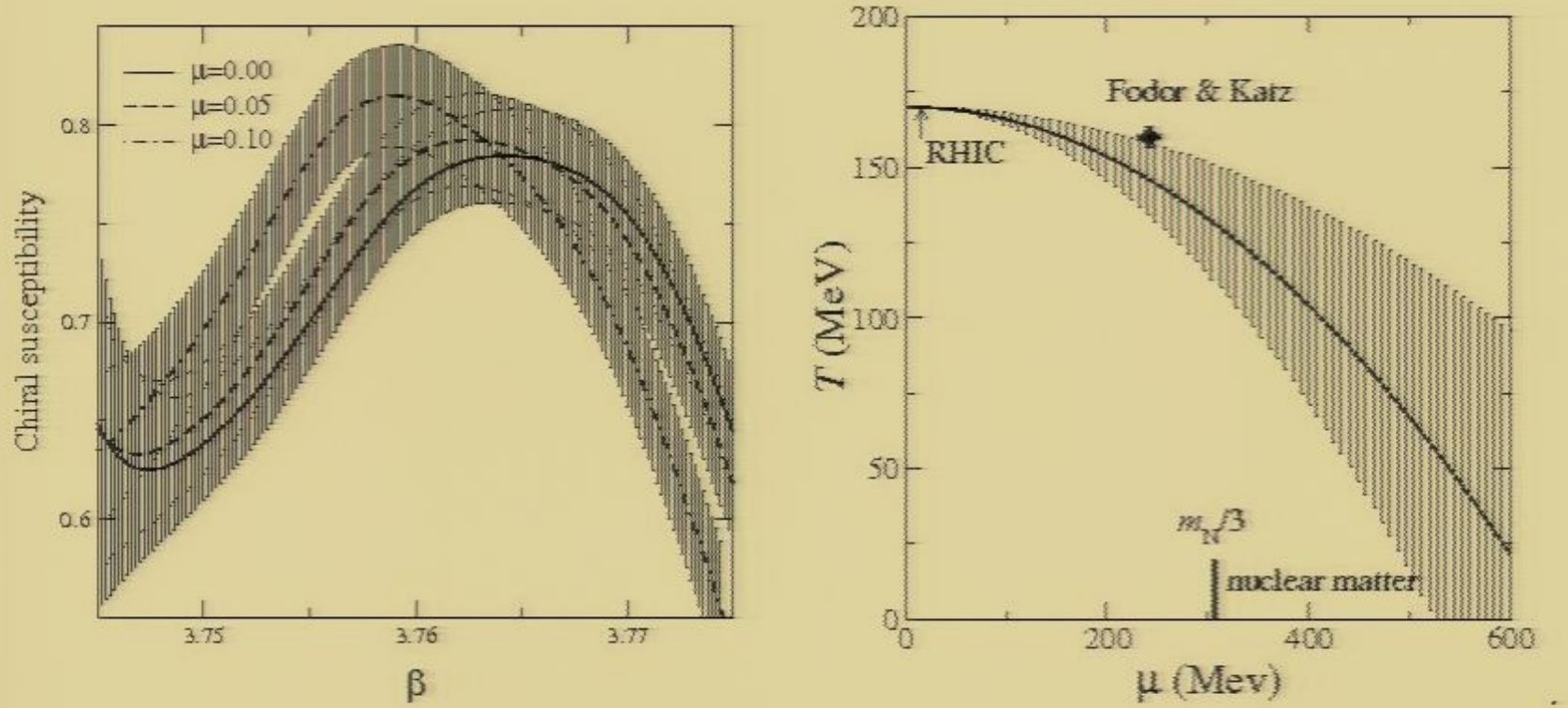
$$\left\{ \exp[-S_g(\beta,U) + S_g(\beta_0,U)] \frac{\det M(m,\mu,U)}{\det M(m_0,\mu=0,U)} \right\}$$

instead of evaulating determinants expand them in μ or exp(μ):

$$\ln\left(\frac{\det M(\mu)}{\det M(0)}\right) = \sum_{n=1}^{\infty} \frac{\mu^n}{n!} \frac{\partial^n \ln\det M(0)}{\partial \mu^n} \equiv \sum_{n=1}^{\infty} R_n \mu^n$$

faster than the complete evaluation of the determinants
only valid for somewhat smaller μ values than the full technique

• trace out the transition points $\beta_c(\mu)$ in 2 flavour QCD
by looking for the susceptibility peaks of Polyakov or $\langle\bar{\psi}\psi\rangle$
convert it into physical units (T and μ_B in MeV)



⇒ curvature is consistent with other results

presence of higher order terems in the Taylor expansion

⇒ uncertainties at small T and large μ

QCD phase diagram from imaginary chemical potential

P.deForcrand, O.Philipsen, Nucl. Phys. B642 290,'02; B673 170, '03

M.D'Elia, M.P.Lombardo, Phys. Rev. D67 014505,'03

fermion determinant: real for imaginary chemical potential (μ_I)
$\Rightarrow$ no sign problem, no need for reweighting

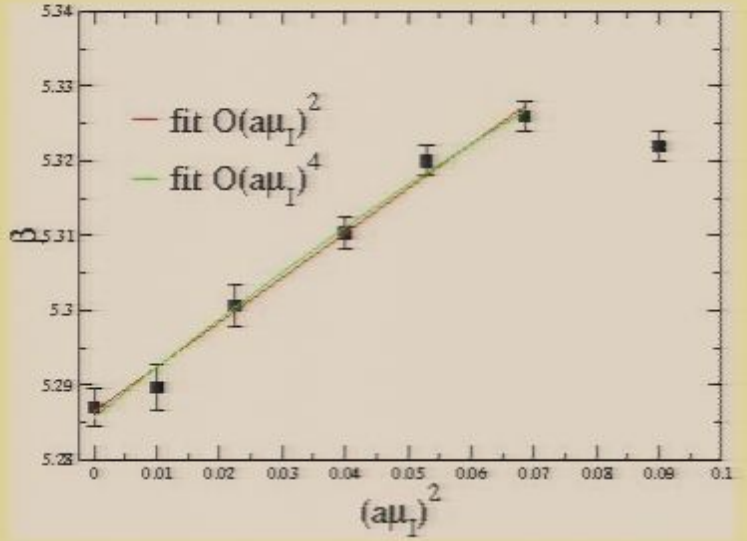
directly obtain the (β_c, μ_I) transition line
analytically continue it to get the physical (β_c, μ) line

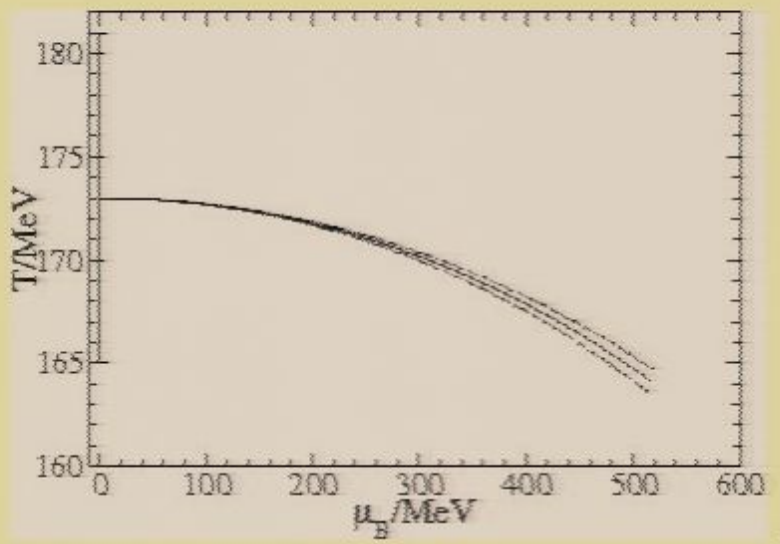
transition line (β_c, μ_I) is given by the susceptibility-peak

$$\chi = VN_t\langle(\mathcal{O} - \langle\mathcal{O}\rangle)^2\rangle, \qquad \partial\chi/\partial\beta = 0 \qquad \partial^2\chi/\partial\beta^2 < 0$$

on finite V the analytic $\chi(\mu_I, \beta)$ can be measured
using the implicitely given $\beta_c(\mu_I)$ one gets

$$\partial\beta_c/\partial\mu = -i\partial\beta_c/\partial\mu_I$$





- curvature is consistent with other results

$$T_c(\mu)/T_c(0) = 1 - 0.500(67)(\mu/\pi T_c)^2$$

- mass dependence in n_f=3 QCD for the critical endpoint:

$$m_c(\mu)/m_c(0) = 1 + 0.84(36)(\mu/\pi T_c)^2$$

- the equation of state can be determined, too

Density of states (DOS) method

Constrained simulations:

Force some observable to have a given value

this way configurations with all values of the observable present

overlap problem not so serious

For any observable:

$$< O >= \int dx \, \langle O f(U) \rangle_x \rho(x) \Big/ \int dx \, \langle f(U) \rangle_x \rho(x)$$

ρ, the density of states is the constrained partition function for some observable ϕ

$$\rho(x) \equiv Z_\phi(x) = \int \mathcal{D}U \, g(U) \, \delta(\phi - x).$$

Possible choices for ϕ:

ϕ =PI (Bhanot et.al, '87; Karliner et.al,'88; Azooiti et.al,'90; Luo, '01; Takaishi, '04)

$\phi = \Theta$ (complex phase) (Gocksch, '88)

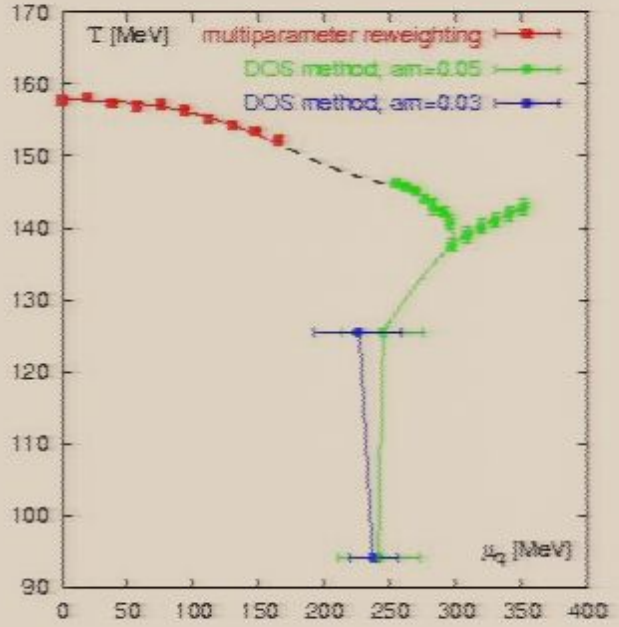
$\Phi = n_q$ (Ambjorn et. al., '02)

Our choice: $\phi = P$ $\quad g = |\det M| \exp\{-S_G\}, \quad f = \exp\{i\theta\}$

Results for QCD at large μ

Z. Fodor, S.D. Katz, C. Schmidt, JHEP 0703:121,2007 [hep-lat/0701022]

$N_f = 4$ staggered QCD on 6^4, $8 \cdot 6^3$ lattices



existence of a triple point around $\mu_q \approx$ 300 MeV and T $\lesssim$ 135 MeV

Note, L_t=6 lattices: smallest T is 73 MeV (if m_ρ fixes the scale)

Mass dependence checked:

small T transition point does not depend on pion mass

Equation of state from lattice simulations

energy density (ϵ) and pressure (p) from partition function:

$$\epsilon(T) = \frac{T^2}{V}\frac{\partial(\log Z)}{\partial T} \qquad p(T) = T\frac{\partial(\log Z)}{\partial V}.$$

T, V are varied by a, take derivative with respect of a

$$\frac{\epsilon - 3p}{T^4} = -\frac{L_t^3}{L_s^3} a \frac{d(\log Z)}{da}$$

the pressure ($p \propto \log[Z]$) along the LCP by the integral method:

$$\frac{p}{T^4} = L_t^4 \int d(\beta, m \cdot a) \left(\frac{\partial(\log Z)}{\partial \beta}, \frac{\partial(\log Z)}{\partial(m \cdot a)} \right)$$

Renormalization of the pressure

We want $p(T=0)=0$ and $\epsilon(T=0)=0 \rightarrow$
Simulations at both

$T>0$ $(N_t \ll N_s)$ and $T=0$ $(N_t \gtrsim N_s)$

are necessary and then subtraction:

$$\frac{p}{T^4} = \frac{p_T}{T^4} - \frac{p_0}{T^4}; \qquad \frac{\epsilon}{T^4} = \frac{\epsilon_T}{T^4} - \frac{\epsilon_0}{T^4}$$

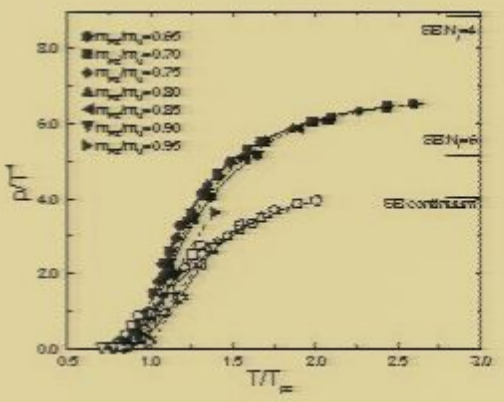
numerical precision needed for the subtraction increases with N_t^4
$\rightarrow$ CPU costs grow faster ($\mathcal{O}(1/a^{13})$) than for $T=0$ simulations

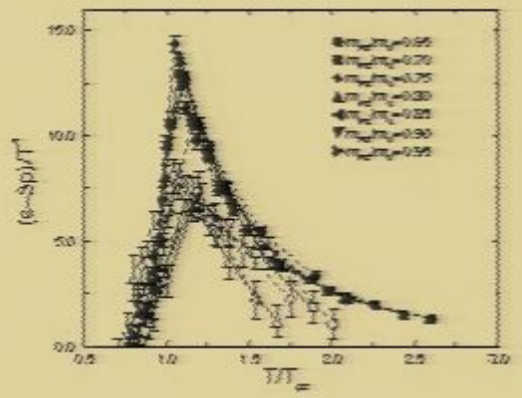
Today

- $N_t = 4$ is easy
- $N_t = 6$ is difficult
- $N_t = 8$ is a challenge

Previous lattice results

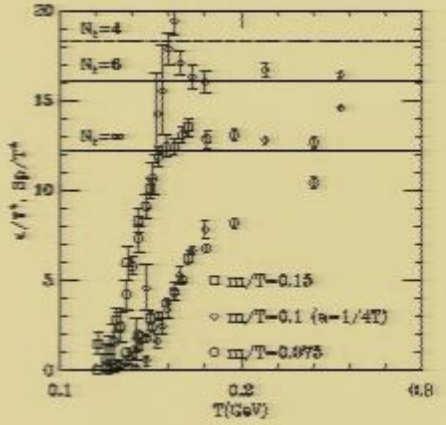
Wilson fermions: slower

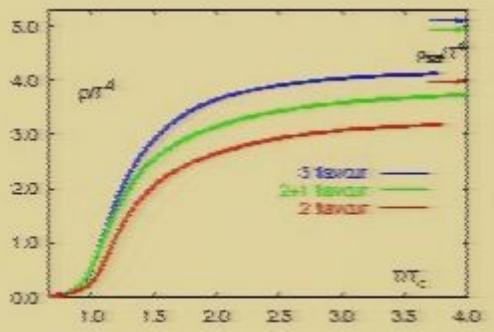




[Ali-Khan et al, '01]

Staggered fermions: faster





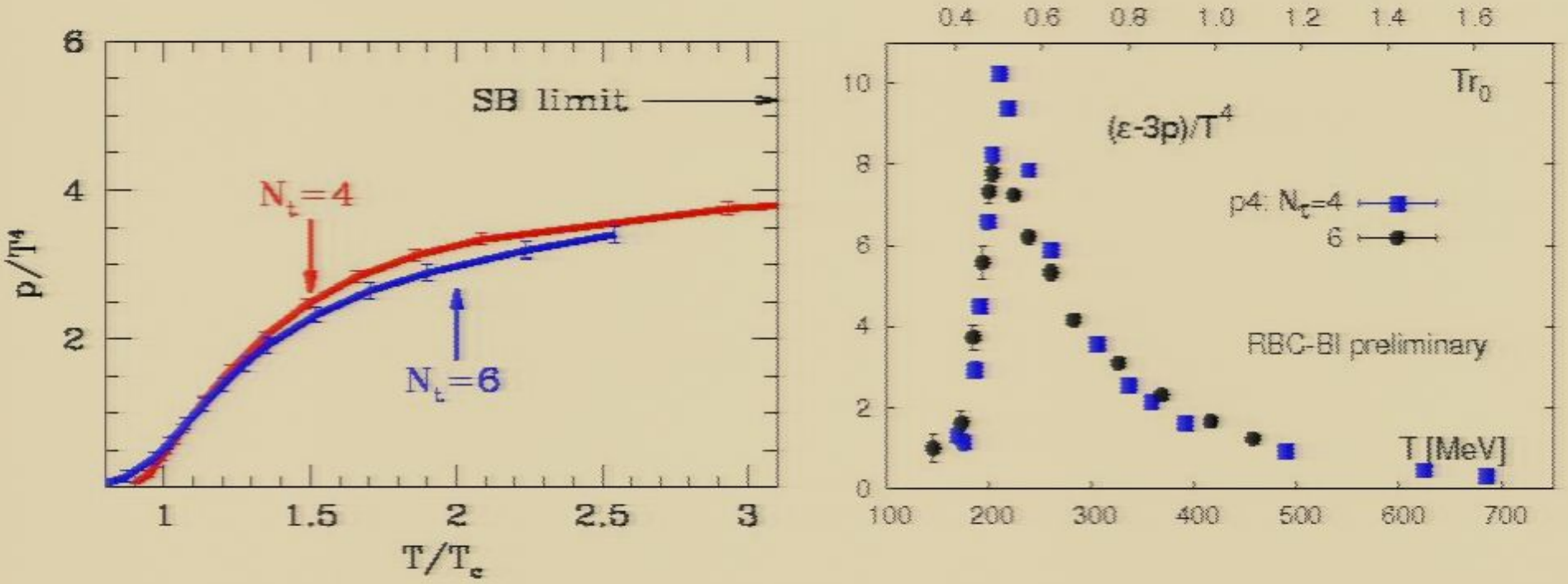
[Bernard et al, '96]

[Karsch, Laermann, Peikert, 2000]

Ongoing project: Bielefeld-Brookhaven-Columbia-Riken

Equation of state and scaling

Y.Aoki, Z.Fodor, S.D.Katz, K.K.Szabo, JHEP, 0601, 089, 2006. F.Karsch, hep-ph/0701210



$\rightarrow N_t = 8$ is needed for final continuum-extrapolated result

recent L_t=4,6 results also from the MILC collaboration: hep-lat/0611031

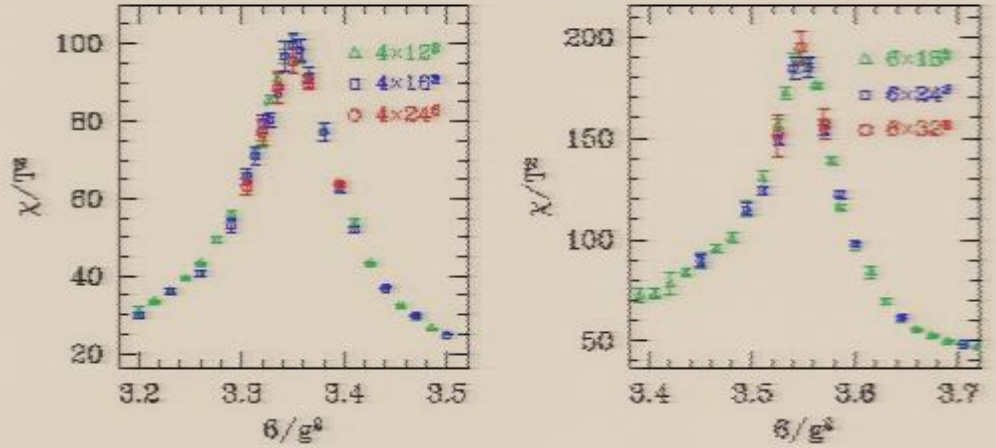
- The nature of the QCD the transition

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

Symanzik improved gauge, stout improved fermionic action
masses: fixed to their physical value m_π=140 MeV, m_K=500 MeV

Finite size scaling of the chiral susceptibility: $\chi=(T/V)\partial^2 \log Z/\partial m^2$

first order transition $\Longrightarrow$ peak width $\propto$ 1/V, peak height $\propto$ V
cross over $\Longrightarrow$ peak width $\approx$ constant, peak height $\approx$ constant



for aspect ratios 3–6 (an order of magnitude larger volumes)
volume independent scaling $\Longrightarrow$ cross-over
do we get the same result (cross-over) in the continuum limit?

- How to get rid of the discretization errors?

a. susceptibility for fixed physical volumes in the continuum
b. finite size analysis of the continuum extrapolated values

renormalize the susceptibility the same way as the free energy

$$f(T) \propto \log Z(T\neq 0)/V_4 - \log Z(T=0)/\bar{V}_4$$

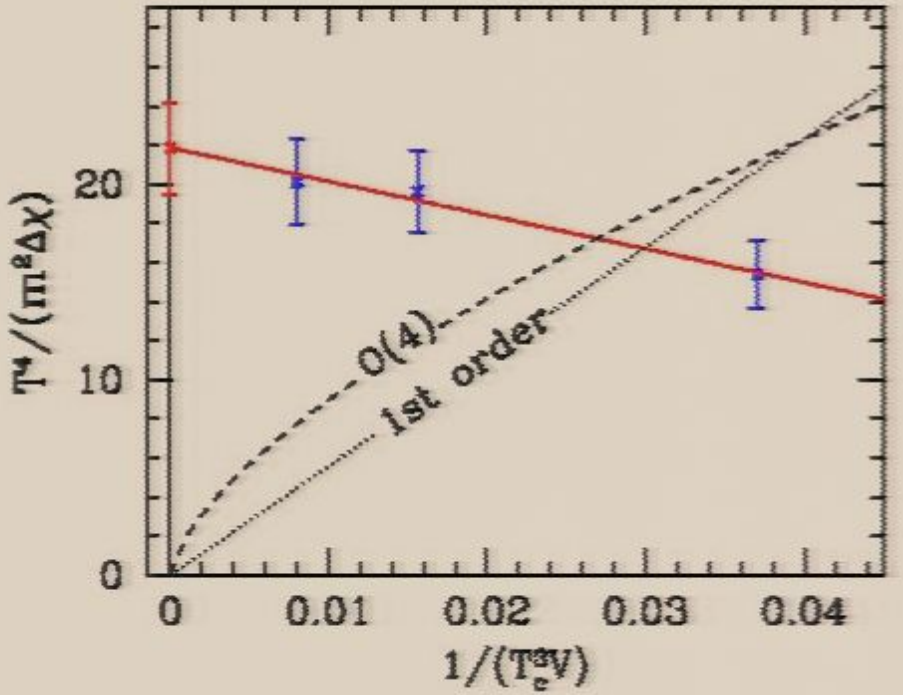
$p(T)$ has a continuum limit and we can use $m_r = Z_m \cdot m$

$$\chi_r(T) = \partial^2/(\partial m_r^2)\,[\log Z(T\neq 0)/V_4 - \log Z(T=0)/\bar{V}_4]$$

construct a quantity in continuum: Z_m drops out from $m^2\partial^2/\partial m^2$

$$\Longrightarrow m_r^2 \cdot \chi_r(T) = m^2 \cdot [\chi(T\neq 0) - \chi(T=0)]$$

- finite size study of continuum extrapolated $m^2\Delta\chi$ (N_t=4: off)



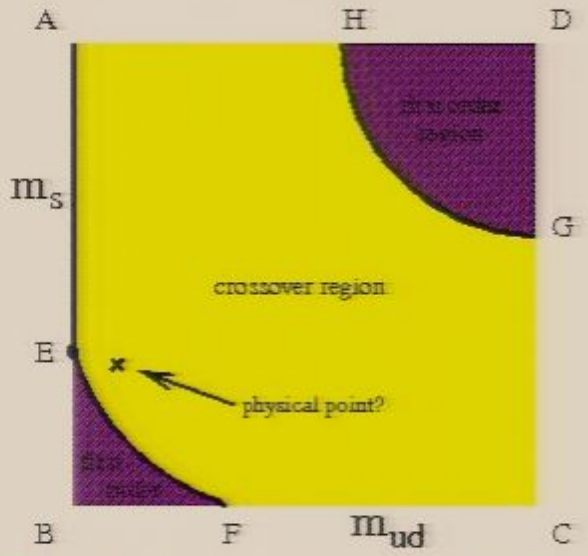
the result is consistent with an approximately constant behavior for a factor of 5 difference within the volume range

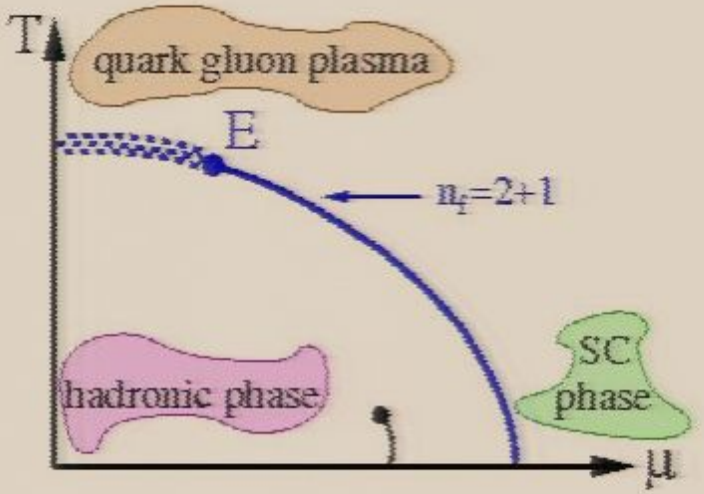
chance probability for 1/V is 10^{-19} for O(4) is $7 \cdot 10^{-13}$

continuum result with physical quark masses in staggered QCD:

the QCD transition at μ=0 is a cross-over

Standard picture of the phase diagram and its uncertainties





- Chiral phase transition (PT)

$n_f = 2$ with $m_q = 0$ at $\mu = 0 \Rightarrow 2^{nd}$ order phase transition

$n_f = 3$ with $m_q = 0$ at $\mu = 0 \Rightarrow 1^{st}$ order phase transition

$n_f = 2 + 1$ with physical m_q at $\mu = 0 \Rightarrow$ cross-over
$n_f = 2 + 1$ with physical m_q at $T = 0 \Rightarrow 1^{st}$ order PT
$\Longrightarrow$ results in the standard picture of the QCD phase diagram

$n_f = 2 + 1$ with physical $m_q \Rightarrow$ critical point (E) at μ, T$\neq$0

The transition temperature

Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B. 643 (2006) 46 [hep-lat/0609068]

$T = 0$:

set the physical scale and locate the physical point

Three quantities are needed (m_π and m_K for the quark masses)

Several possibilities for the third quantity

- string tension (not existing in full QCD)
- static quark potential at intermediate distances ($r_0^2 \cdot dV/dr$=1.65)
- directly measurable quantities (e.g. f_K)

Further quantities are predictions (e.g. r_0, f_π, m_{K^*})

$T > 0$:

cross-over $\rightarrow$ different definitions give different T_c

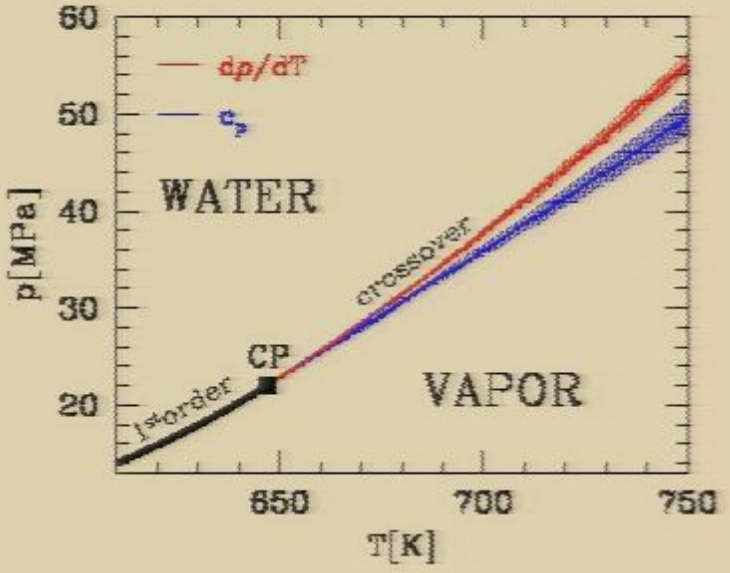
Possible choices:

- Chiral susceptibility
- Quark number susceptibility
- Polyakov-loop

T>0 Simulations

No well defined T_c

Example of water-steam transition

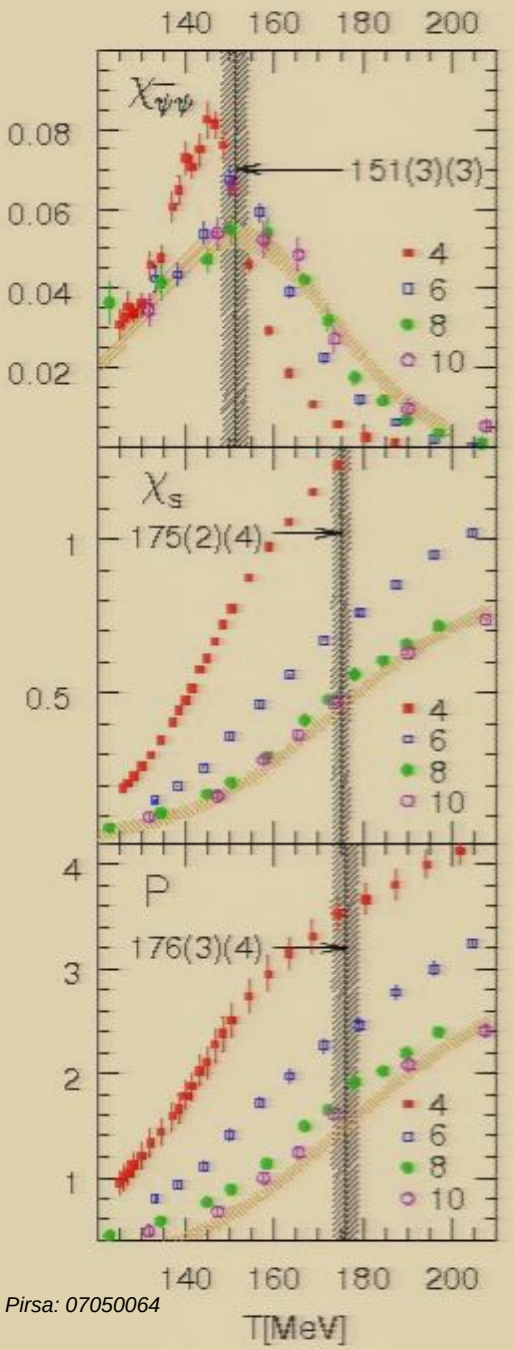


above the critical point c_p and $d\rho/dT$ give different T_cs.

Our choices in QCD

$\frac{m^2\Delta\chi}{T^4}$ $\rightarrow$ chiral transition

Quark number susceptibility
Polyakov loop
$\rightarrow$ de-confinement transition



Chiral susceptibility

T_c=151(3)(3) MeV

ΔT_c=28(5)(1) MeV

Quark number susceptibility

T_c=175(2)(4) MeV

ΔT_c=42(4)(1) MeV

Polyakov loop

T_c=176(2)(4) MeV

ΔT_c=38(5)(1) MeV

Recent result of the Bielefeld-Brookhaven-Columbia-Riken collaboration

M. Cheng et.al, Phys. Rev. D74 (2006) 054507 [hep-lat/0608013]

Transition temperature from $\chi_{\bar{\psi}\psi}$ and Polyakov loop, from both quantities

T_c=192(7)(4) MeV, $\Longrightarrow$ Contradicts to our result

Main differences from our work

only $N_t = 4$ & 6 (cutoff: $a \approx$0.3 fm & 0.2 fm or $a^{-1} \approx$700 MeV & 1 GeV)

scale is set by r_0 instead of f_K

no renormalization, χ/T^2 is used

$\rightarrow$ this can only explain ≈ 10 MeV difference

What is the reason for this discrepancy?

Their last concluding remark: it is desirable to

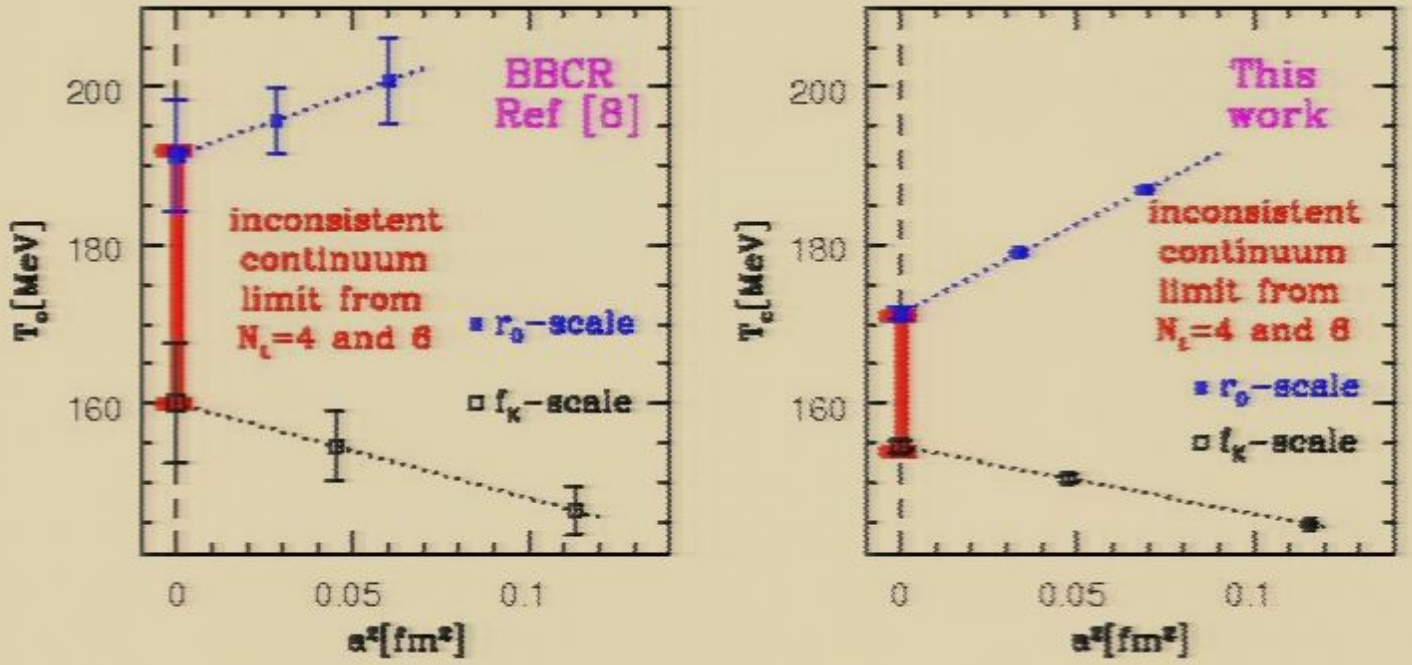
"obtain a reliable independent scale setting for the transition temperature from an observable not related to properties of the static potential".

What if they used f_K to set the scale?

We repeated some of their $T = 0$ simulations to determine f_K

Alternatively

We can use r_0 and only $N_t = 4,6$



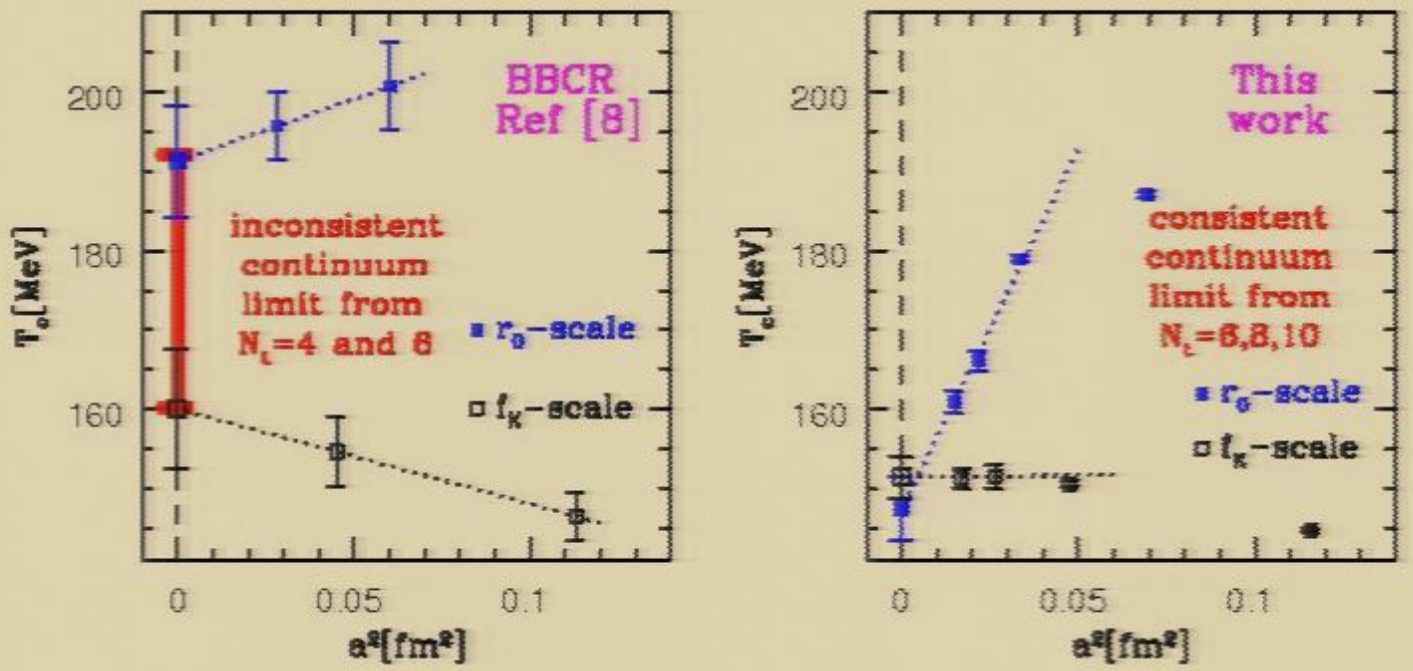
Continuum limits from $N_t = 4,6$ are inconsistent!

What if they used f_K to set the scale?

We repeated some of their $T = 0$ simulations to determine f_K

Alternatively

We can use r_0 and only $N_t = 4,6,8,10$



Continuum limits from $N_t = 6,8,10$ are consistent!

Conclusion: Continuum limit from $N_t = 4,6$ is not safe

Conclusions

lattice thermodynamics: important (already/soon full) results

nature of the transition: analytic transition (cross-over)

T_c discrepancies between groups: resolve it in the continuum

equation of state: still needs a continuum extrapolation

μ>0 results are quite far from the continuum limit (N_t=4)

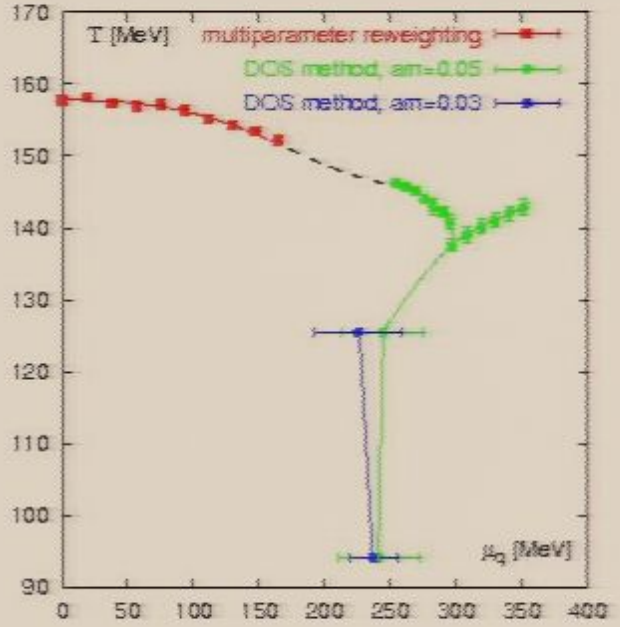
"all" results are within the staggered formalism (non-locality)

$\Longrightarrow$ closer to the continuum + non-staggered fermions

Results for QCD at large μ

Z. Fodor, S.D. Katz, C. Schmidt, JHEP 0703:121,2007 [hep-lat/0701022]

$N_f = 4$ staggered QCD on 6^4, $8 \cdot 6^3$ lattices



existence of a triple point around $\mu_q \approx$ 300 MeV and T $\lesssim$ 135 MeV

Note, L_t=6 lattices: smallest T is 73 MeV (if m_ρ fixes the scale)

Mass dependence checked:

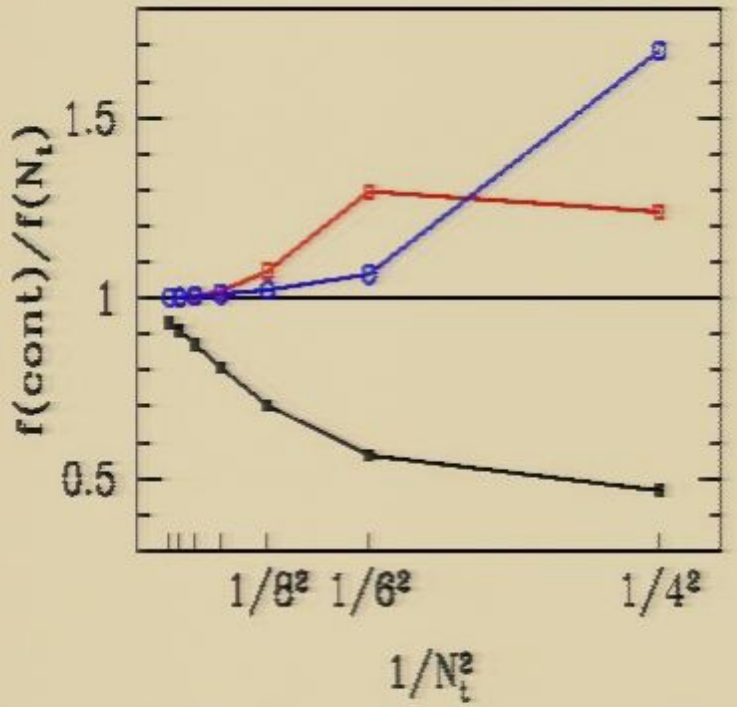
small T transition point does not depend on pion mass

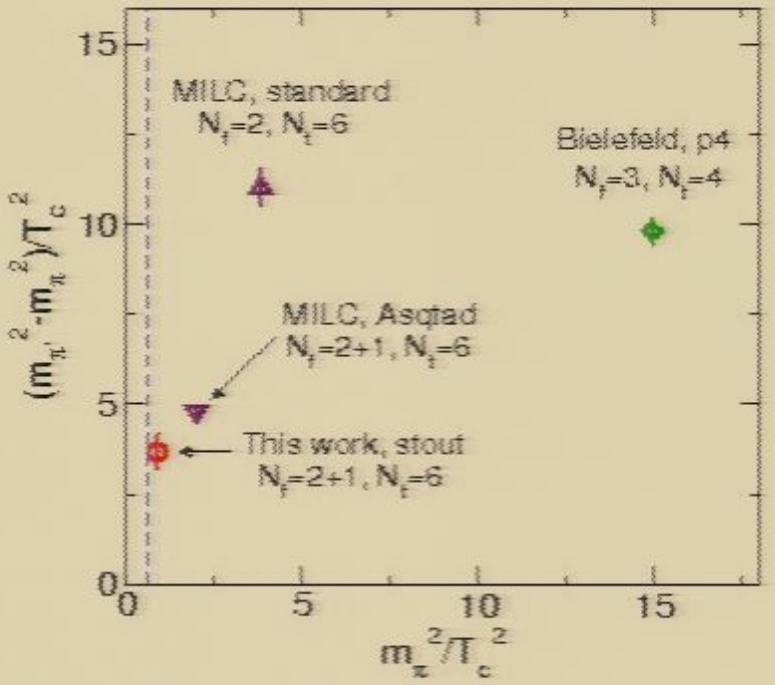
The order of the QCD transition

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675 [hep-lat/0611014]

Lattice action:

Symanzik improved gauge, stout improved fermionic action





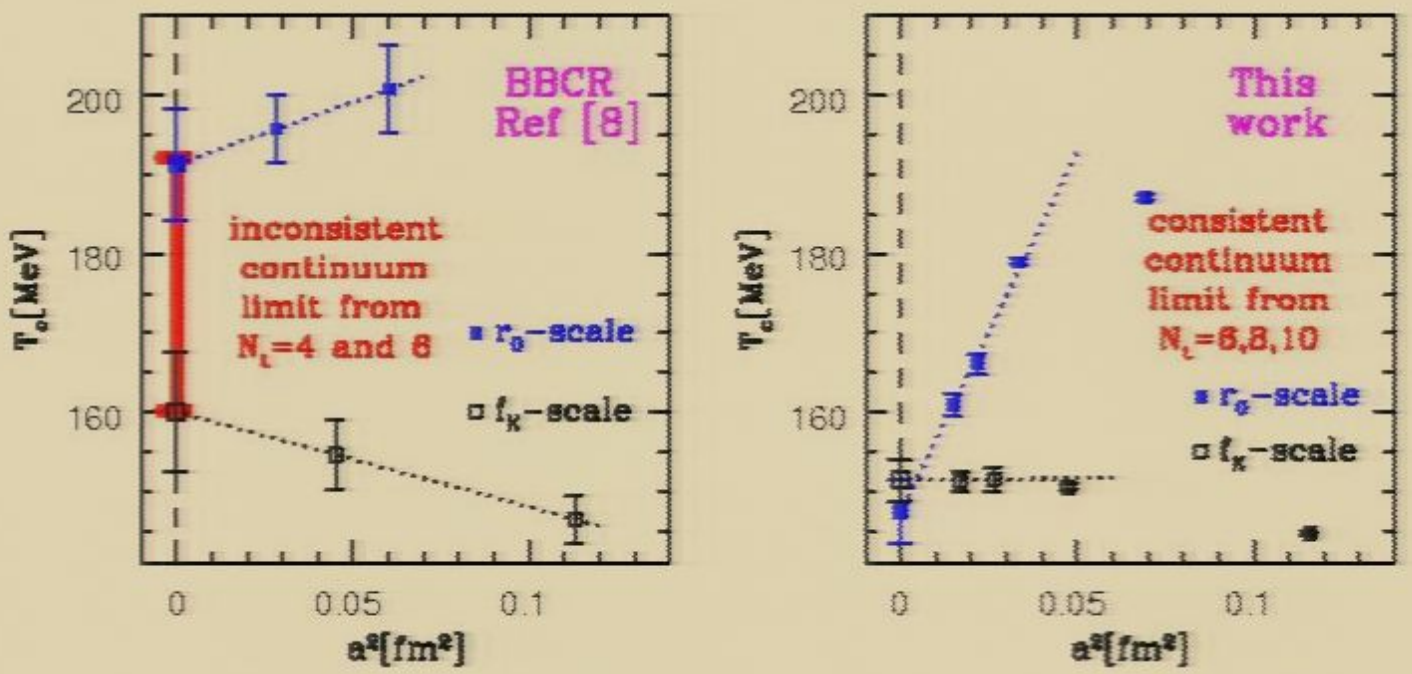
N_t=4,6 don't scale reduce unphysical non-degeneracy of π's

What if they used f_K to set the scale?

We repeated some of their $T = 0$ simulations to determine f_K

Alternatively

We can use r_0 and only $N_t = 4,6,8,10$



Continuum limits from $N_t = 6,8,10$ are consistent!

Conclusion: Continuum limit from $N_t = 4,6$ is not safe