Title: Highly Excited States, Linear Regge Trajectories and Chiral Symmetry in QCD and AdS/QCD

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Abstract:

Exotic States of Hot and Dense Matter and their Dual Description

Perimeter Institute, May 22–26, 2007

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Misha Shifman and Arkady Vainshtein

William Fine Theoretical Physics Institute
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Work in progress

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Outline

- Introduction and preliminaries
- Linear realization of chiral symmetry: reminder
- The rate of restoration and OPE
- AdS/QCD approach
- Conclusions

Introduction

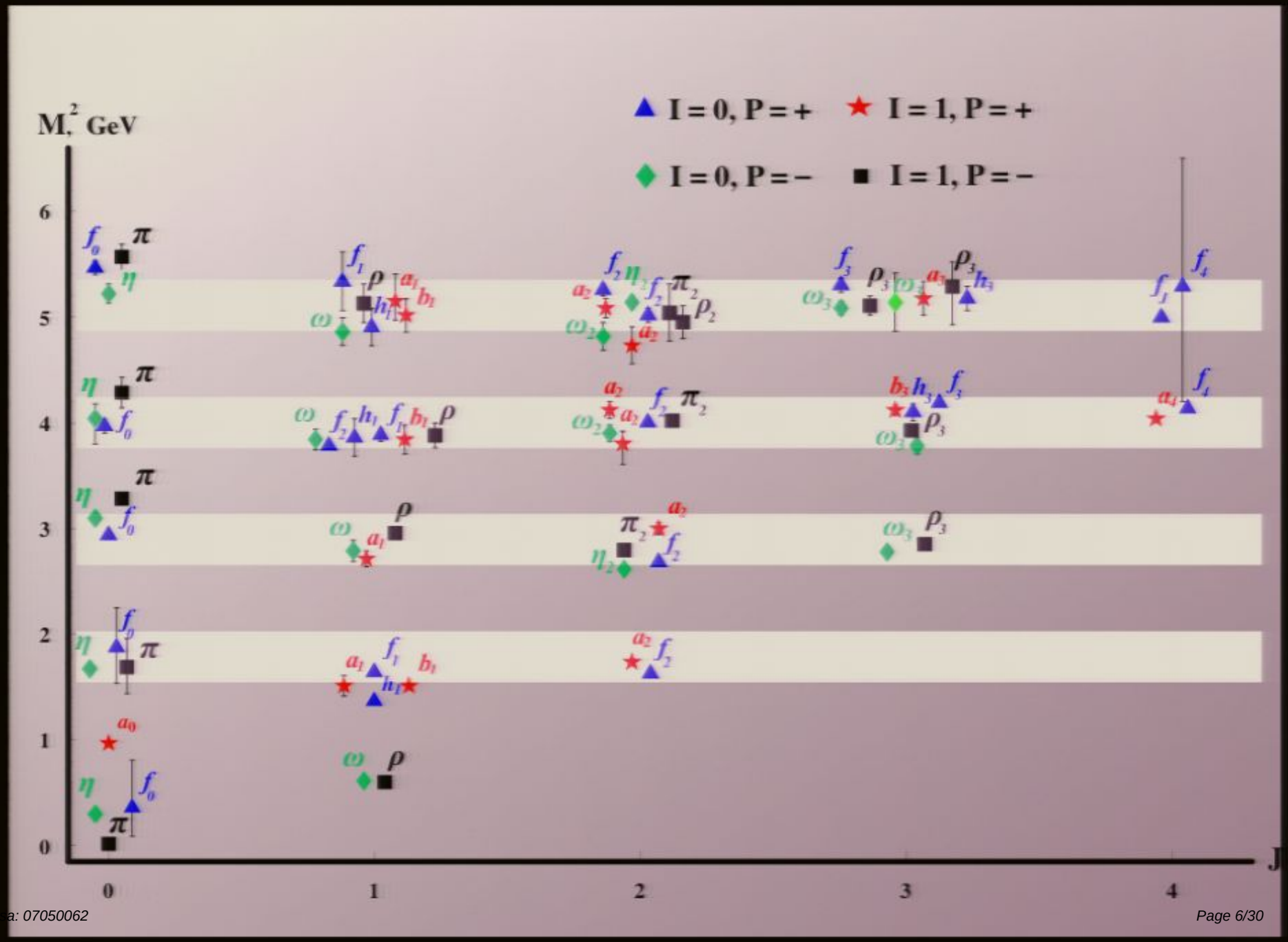
Timeline:

Late 60's and early '70s,
Regge theory, dual resonance models, string theory

'70 to '90s
QCD, most of attention to low-lying states,
methods: soft pions, QCD sun rules, lattice simulations

'80s and early '90s,
Strings as "theory of everything," not about hadrons

Recent,
String/gauge dualities, application to QCD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Preliminaries

$\mathrm{U}(1)_L \times \mathrm{U}(1)_R$

$$V|\pm\rangle = |\pm\rangle\,, \qquad A|\pm\rangle = |\mp\rangle$$

$|\pm\rangle$ are the opposite parity states degenerate in masses.

Generically, for the axial current a^μ

$$\langle +|a^\mu|-\rangle = g(q^2)(p_+ + p_-)_\nu \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)$$

$$= g(q^2)\left[(p_+ + p_-)_\mu - q_\mu \frac{M_+^2 - M_-^2}{q^2}\right]$$

Linear realization (Wigner-Weyl)

$$M_+^2 = M_-^2 \qquad\qquad g_A = g(0) = 1$$

Nonlinear realization (Nambu-Goldstone)

$$g_{\pi+-} = f_\pi^{-1} g_A (M_+^2 - M_-^2)$$

generalized Goldberger-Treiman relation

No constraints on g_A , the axial charge

$A = \int \mathrm{d}^3x\, a^0$ vanishes

Chiral symmetry restoration (χSR)

Imagine $\Delta M_\pm = M_+ - M_-$ is small for high excitations, $\Delta M_\pm \ll M_{\rm had}$. Then,

$$\langle -|a^0|+\rangle = g(q^2)\, 2M_- \frac{\vec{q}^{\,2}}{\vec{q}^{\,2} - (\Delta M_\pm)^2}$$

Although it vanishes at $\vec{q} = 0$ there is the window

$$\Delta M_\pm \ll |\vec{q}| \ll M_{\rm had}$$

where

$$\langle -|a^0|+\rangle = 2M_-\, g(0) = 2M_-\, g_A$$

Restoration of linear realization in the above window for high excitations, $g_A = 1$. Hadron has a core of the size $1/M_{\rm had}$ surrounded by pionic halo of the $1/m_\pi$ size.

"Botched" chiral symmetry restoration

For highly excited states

$$M_{\rm had} \sim 1/R \sim M_{n+1} - M_n$$

For linear Regge trajectories $M^2(J) = (J - J_0)/\alpha'$ and $\Delta M_\pm \sim M_{\rm had}$.

Indeed, there is no degeneracy in parity for the leading trajectories, say, ρ and a_1 . Splitting

$$M_+^2 - M_-^2 = \Delta J_0/\alpha' \sim \Lambda^2$$

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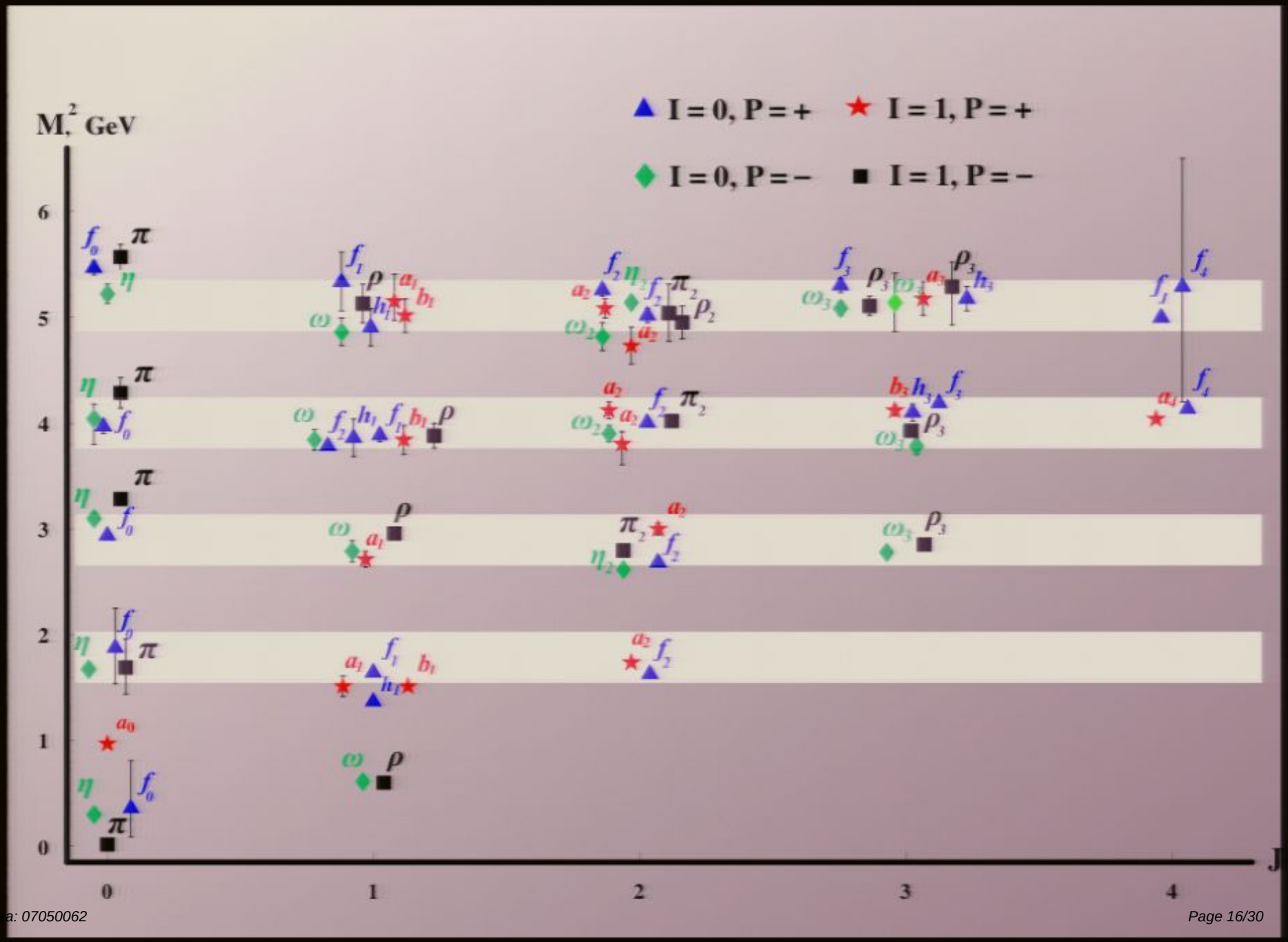
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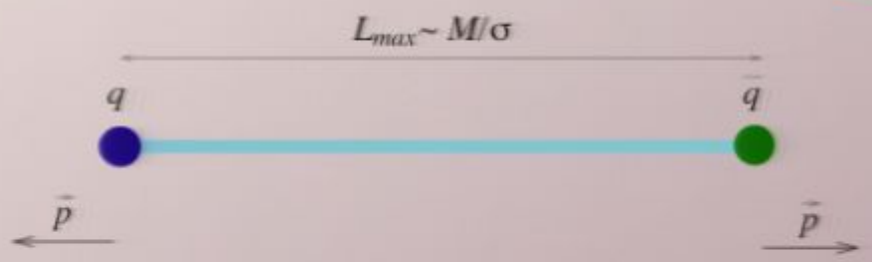
Quasiclassical string



The mass M_n

$$M_n = 2p + \sigma r$$

Quantization

$$\int_0^{\ell_*} p(r)\, dr = \pi n \qquad p(r) = (M_n - \sigma r)/2 \qquad \ell_* = \frac{M_n}{\sigma}$$

gives

$$M_n^2 = 4\pi\sigma n \sim \Lambda^2 n$$

When $L \neq 0$

$$n \to n_r + L$$

Chiral symmetry: linear realization

$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$

$$[q_L]^{if}_{\alpha}\,, \qquad [q_R]^{i\bar{f}}_{\dot{\alpha}} \qquad\qquad q_{L,R} \;=\; (1 \mp \gamma_5)q/2$$

Symmetry transformations

$$q^f_L \to L^f_g\, q^g_L\,, \qquad q^f_R \to R^{\bar{f}}_{\bar{g}}\, q^{\bar{g}}_R \qquad\qquad L, R \in \mathrm{SU}(N_f)$$

Classically QCD has also $\mathrm{U}(1)_L \times \mathrm{U}(1)_R \equiv \mathrm{U}(1)_B \times \mathrm{U}(1)_A$ invariance

$$q_L \to \mathrm{e}^{i\eta_L}\, q_L\,, \qquad q_R \to \mathrm{e}^{i\eta_R}\, q_R$$

Although $\mathrm{U}(1)_A$ is broken at quantum level large N_c argumentation implies $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$ asymptotic symmetry.

Having in mind bilinear in quarks interpolating fields two types of representations of $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$

(i) Nonsinglet of $\mathrm{U}(1)_A$

$$M^f_{\bar{f}} \sim \bar{q}_{R\,\bar{f}}\, q^f_L$$

$\{N_f, N_f\}$ representation contains $2N_f^2$ real fields.
Reflection of space coordinates, P, transforms $q^{if}_{L\,\alpha}$ to $q^{i\bar{f}}_{R\,\dot{\alpha}}$ and vice versa, i.e.

$$PM = M^\dagger$$

The Hermitian and anti-Hermitian parts represent N_f^2 fields of the opposite parity.

(ii) Singlets of $\mathrm{U}(1)_A$

$$\left[V^L\right]_g^f = [\bar{q}_L]_g\,[q_L]^f \qquad \left[V^R\right]_g^f = [\bar{q}_R]_g\,[q_R]^f$$

N_f^2 real fields in each. They are related by reflection of space coordinates, P, so the sum and difference are the fields of opposite parity.

Strictly speaking the trace part can split from the adjoint but the OZI rule at large N_c implies that this splitting is small.

Thus, the chiral multiplets lead to the $2N_f^2$ degeneracy. Each parity contains one adjoint and one singlet of $\mathrm{SU}(N_f)_V$

Twist of interpolating fields is larger for spin zero:

$$M^f_{\bar f} = \bar q_{\bar f} \frac{1-\gamma_5}{2} q^f \qquad \left[V^{L,R}\right]^f_g = \bar q_g \gamma^\mu \frac{1 \mp \gamma_5}{2} q^f \partial_\mu (\mathrm{Tr}\, G^2)$$

Twist is 3 and 6 are shown.

For spin 1 the leading twist is 2

$$\left[V^{L,R}_\mu\right]^f_g = \bar q_g \gamma_\mu \frac{1 \mp \gamma_5}{2} q^f \qquad \left[M_{\mu\nu}\right]^f_{\bar f} = \bar q_{\bar f} \sigma_{\mu\nu} \frac{1-\gamma_5}{2} q^f$$

For higher spins are similar with extra derivatives.

For baryon of the maximal spin $N_c/2$ twist $t = N_c$

$$\left[B^L_{\alpha_1 \ldots \alpha_{N_c}}\right]^{f_1 \ldots f_{N_c}} = \epsilon_{i_1 \ldots i_{N_c}} q^{i_1 f_1}_{L\alpha_1} \cdots q^{i_{N_c} f_{N_c}}_{L\alpha_{N_c}}$$

The parity transformation relates it to $\left[B^R_{\dot\alpha_1 \ldots \dot\alpha_{N_c}}\right]^{\bar f_1 \ldots \bar f_{N_c}}$

But to get the chirally invariant mass we need mirror baryons $\tilde B^{\bar f_1 \ldots \bar f_{N_c}}_L$, $\tilde B^{f_1 \ldots f_{N_c}}_R$. In terms of interpolating operators it requests $t = 2N_c$.

The rate of chiral symmetry restoration and OPE

$$\Pi_{S,P} = i \int d^4x e^{iqx} \langle 0|T\{j_{S,P}(x) j_{S,P}(0)\}|0\rangle$$

$$j_S = \bar{q}q, \quad j_P = i\bar{q}\gamma_5 q$$

From OPE (both in 2D and 4D)

$$\Pi_S - \Pi_P \sim \frac{g^2 \langle \bar{q}q \rangle^2}{Q^4} + \ldots$$

In 4D each individual resonance gives

$$\frac{\Lambda^2 M_n^2}{Q^2 + M_n^2} \equiv \Lambda^2 - \frac{\Lambda^2 Q^2}{Q^2 + M_n^2}$$

and would-be chiral pair (implying local duality)

$$\frac{\Lambda^2 Q^2 \, \delta M_n^2}{(Q^2 + M_n^2)^2} \quad \Rightarrow \quad \delta M_n^2 \sim \frac{\Lambda^4}{M_n^2}$$

In 2D by the same token we get $\delta M_n^2 \sim \Lambda^2$

The heuristic argument: quark helicity is of importance near the turning point in the string picture, $p \sim \Lambda$.
In momentum space it is the volume $\sim \Lambda^{D-1}$, thus

$$\delta M_n \sim \Lambda \left(\frac{\Lambda}{M_n}\right)^{D-1}$$

However, there is no problem to maintain OPE with different behavior of the splitting once local duality is abandoned. So above estimates should be viewed as a kind of lower bound for the splitting.

AdS/QCD approach

I'll discuss two different calculations: by Sakai & Sugimoto and by Karch, Katz, Son & Stephanov.

In both the holographic description of hadrons with the fifth coordinate z is used.

In the first the authors placed N_f test D8 -- $\overline{\text{D8}}$ brane pairs in the background of N_c D4 branes compactified on SUSY breaking S_1.

In the second, bottom-up approach, the author fix the metric and dilaton field to get linear dependence of M^2 on n_r and L.

$$S^{\rm SS} = \kappa \int d^4x dz \, {\rm Tr}\, [K^{-1/2} F_{\mu\nu}^2/2 + K F_{\mu z}^2]$$

$$K = 1 + z^2 \quad \kappa = \lambda N_c / 108\pi^3$$

$$S^{\mathrm{KKSS}} = \int d^4x dz\, \mathrm{e}^{\Phi(z)} \sqrt{g} \left[-|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right]$$

$$\Phi = z^2 \qquad\qquad g_5^2 = 12\pi^2/N_c$$

Eigenvalues in QM of the hologhaphic coordinate z give M^2 in 4D. Characteristic z grows with n as $\sqrt{n}$.

Splitting between ρ and a_1 states in KKSS is due to X field containing pion and its scalar partner. The X contribution diminishes with n : chiral symmetry restores.

It does not happen in the SS approach where the pion is built in together with vector and axial fields. No asymptotic linear realization then.

Phenomenologically it looks better: no convergence for ρ and a_1 leading trajectories is visible.

Conclusions

In respect to asymptotic linear realization of the chiral symmetry jury is still out.

However the phenomenological linearity of the Regge trajectories is an evidence that we do not see the symmetry restoration in the visible range of high excitation.