

Title: Viscous Hydrodynamics and AdS/CFT

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Abstract:

# **Expanding systems in gauge theory/gravity duality**

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23 May 2007

Work with Jorma Louko (Nottingham), T. Tahkokallio (Helsinki)

Janik, Peschanski; Nakamura, Sin, Kim; Kovchegov, Taliotis

Nastase; Shuryak, Sin, Zahed



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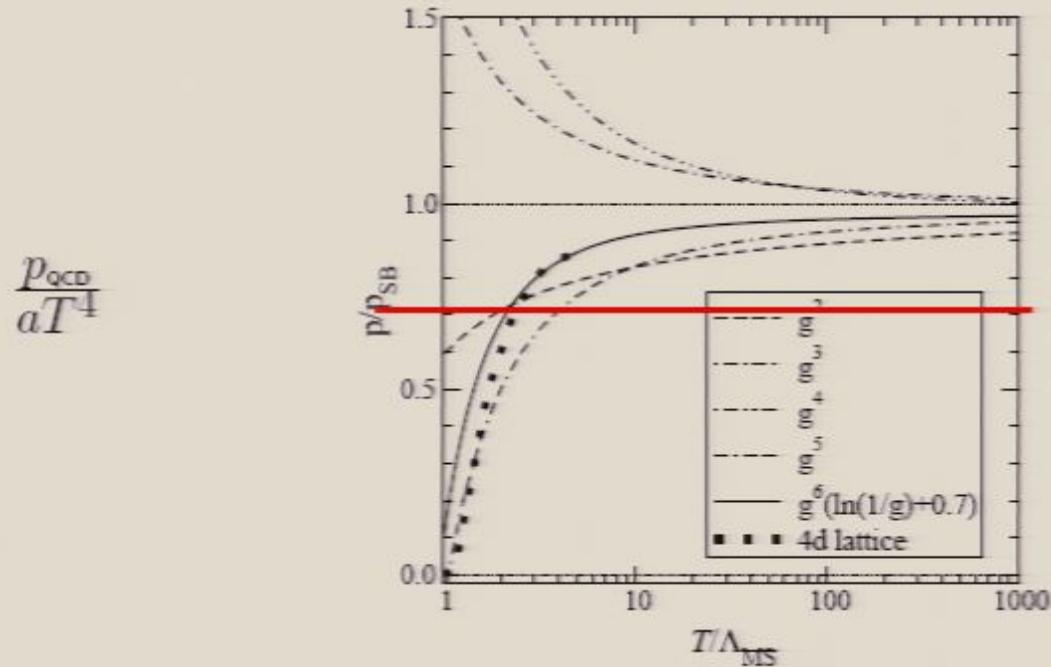
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## The celebrated $3/4$ and $\hbar/4\pi$



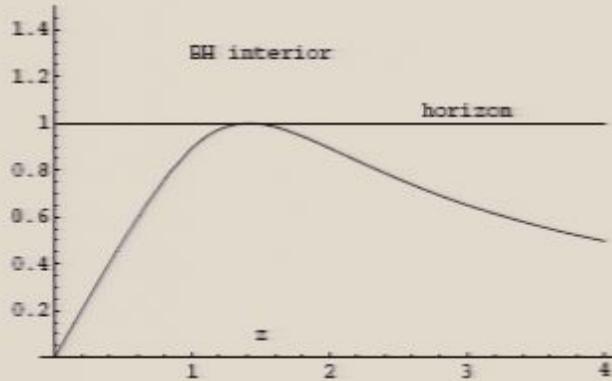
Points: Lattice Monte Carlo, Curves: Perturbation theory

$$\frac{\eta}{s} = \frac{\hbar}{4\pi} \left[ 1 + \frac{135\zeta(3)}{16\sqrt{2}\lambda^{3/2}} + \dots \right] \quad \frac{\eta}{s} \approx \frac{p\tau_c}{s} \approx \frac{T^4\tau_c}{T^3} = T\tau_c \gtrsim \hbar$$

These are static results, derived in AdS<sub>5</sub>/CFT duality from

$$ds^2 = \frac{\mathcal{L}^2}{\tilde{z}^2} \left[ -\left(1 - \frac{\tilde{z}^4}{z_0^4}\right) dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + \frac{1}{1 - \tilde{z}^4/z_0^4} d\tilde{z}^2 \right] \quad T_{\text{Haw}} = \frac{1}{\pi z_0}$$

Transform  $\tilde{z}^2 = z^2/(1 + z^4/4z_0^4)$ :



Polchinski, cosmicvariance.com /2006/12/07/:  
Physicists have found that some of the properties of this plasma are better modeled (via duality) as a tiny black hole in a space with extra dimensions than as the expected clump of elementary particles in the usual four dimensions of spacetime.

$$\begin{aligned} ds^2 &= \frac{\mathcal{L}^2}{z^2} \left[ -\frac{(1 - z^4/(4z_0^4))^2}{1 + z^4/(4z_0^4)} dt^2 + \left(1 + \frac{z^4}{4z_0^4}\right) (dx_1^2 + dx_2^2 + dx_3^2) + dz^2 \right] \\ &\equiv \frac{\mathcal{L}^2}{z^2} \left\{ \left[ g_{\mu\nu}^{(0)}(x, 0) + \underbrace{g_{\mu\nu}^{(4)}(x) z^4}_{\sim T_{\mu\nu}} + \dots \right] dx^\mu dx^\nu + dz^2 \right\} \end{aligned}$$



$$T_{\mu\nu} = \frac{\mathcal{L}^3}{4\pi G_5} g_{\mu\nu}^{(4)} = \begin{pmatrix} 3aT^4 & 0 & 0 & 0 \\ 0 & aT^4 & 0 & 0 \\ 0 & 0 & aT^4 & 0 \\ 0 & 0 & 0 & aT^4 \end{pmatrix}$$

$$a = \frac{\pi^2 N_c^2}{6} \left[ \frac{3}{4} + \frac{45\zeta(3)}{64\sqrt{2}} \frac{1}{\lambda^{3/2}} + \dots \right]$$

For small  $\lambda \equiv g^2 N_c$ , counting  $2 + 6 + 7/8 \times (4 + 4) = 15 \times$  color massless dofs:

$$a = N_c^2 \frac{\pi^2}{6} \left[ 1 - \frac{3}{2\pi^2} \lambda + \frac{3 + \sqrt{2}}{\pi^3} \lambda^{3/2} + a\lambda^2 \log \lambda + b\lambda^2 + c\lambda^{5/2} + d\lambda^3 \log \lambda + \dots \right]$$

(Nieto computed  $a, b, c, Laine d$  2 years ago, unpublished)

$$\begin{aligned} f(\lambda) = & 1 - \frac{3}{2\pi^2} \lambda + \frac{3 + \sqrt{2}}{\pi^3} \lambda^{3/2} \\ & - \frac{1}{(2\pi)^4} \lambda^2 \left[ 45 + 18\sqrt{2} - 24\gamma + 58 \log 2 + 48 \log \pi - 24 \frac{\zeta'(-1)}{\zeta(-1)} - 24 \log \lambda \right] \\ & - \frac{1}{(2\pi)^5} \lambda^{5/2} \left[ 123 + \frac{11}{8}\sqrt{2} + \left( 3 + \frac{\sqrt{2}}{2} \right) \pi^2 + \left( \frac{53}{2}\sqrt{2} - 264 \right) \log 2 \right. \\ & \left. - \left( 21 + \frac{75}{4}\sqrt{2} \right) \log \left( 1 + \sqrt{2} \right) \right]. \end{aligned}$$

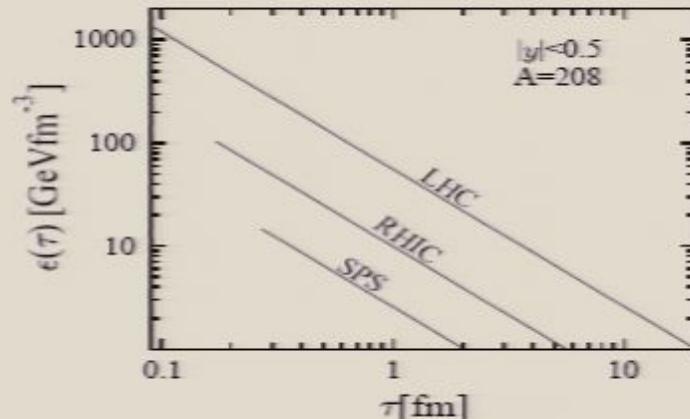
## Expanding matter?

1+1+2d Bjorken similarity flow:

$$\epsilon(T) = 3p(T) = 3aT^4, \eta = p'(T)/(4\pi) = aT^3/\pi, a = \pi^2 N_c^2/8, \zeta = 0$$

$$v(t, x) = \frac{x}{t} \equiv \tanh \Theta(\tau, \eta), \quad \Theta(\tau, \eta) = \eta, \quad u^\mu = \frac{x^\mu}{\tau},$$

$$T(t) = \left( T_i + \frac{1}{6\pi\tau_i} \right) \left( \frac{\tau_i}{\tau} \right)^{1/3} - \frac{1}{6\pi\tau}.$$



$$1/\tau^{1/3}, 6\pi \text{ simple, } T_i, \tau_i \text{ (very) hard, } T_i \tau_i \sim \hbar$$



Shuryak-Sin-Zahed

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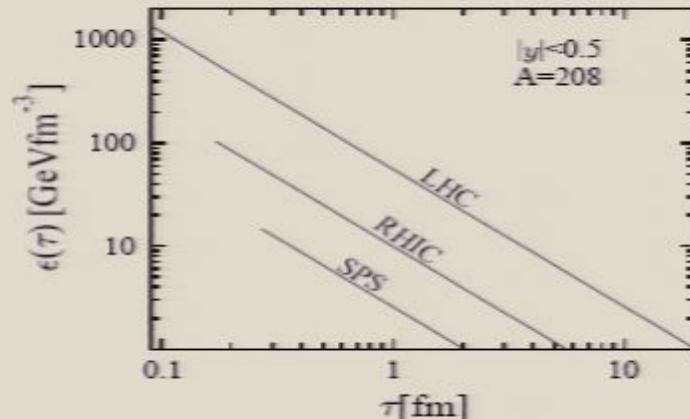
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Shuryak-Sin-Zahed

Search for time-dependent solutions of  $\text{AdS}_{d+1}$ :

$$R_{MN} - \frac{1}{2}Rg_{MN} - \frac{d(d-1)}{2\mathcal{L}^2}g_{MN} = 0$$

for  $d = 4$ ,  $\text{AdS}_5 \times S_5$ , boundary theory:  $\mathcal{N} = 4$  SYM

$$\mathcal{L}^4 = 4\pi g_s N_c \alpha'^2 = g_{\text{YM}}^2 N_c \alpha'^2, \quad \frac{\mathcal{L}^3}{G_5} = \frac{2N_c^2}{\pi}$$

$$ds^2 = \frac{\mathcal{L}^2}{z^2} [-a(\tau, z)d\tau^2 + \tau^2 b(\tau, z)d\eta^2 + c(\tau, z)(dx_2^2 + dx_3^2) + dz^2]$$

$$ds^2 = \frac{\mathcal{L}^2}{z^2} [-a(t, z)dt^2 + b(t, z)d\mathbf{x}^2 + dz^2]$$

for  $d = 2$ ,  $\text{AdS}_3 \times S_3 \times T_4$ , boundary theory: a 2d CFT

$$\mathcal{L}^4 = g_s^2 \frac{16\pi^4 \alpha'^2}{V_4} Q_1 Q_5 \alpha'^2, \quad \frac{\mathcal{L}}{G_3} = 4Q_1 Q_5.$$

$$ds^2 = \frac{\mathcal{L}^2}{z^2} [-a(\tau, z)d\tau^2 + b(\tau, z)\tau^2 d\eta^2 + dz^2].$$

$$d = 2$$

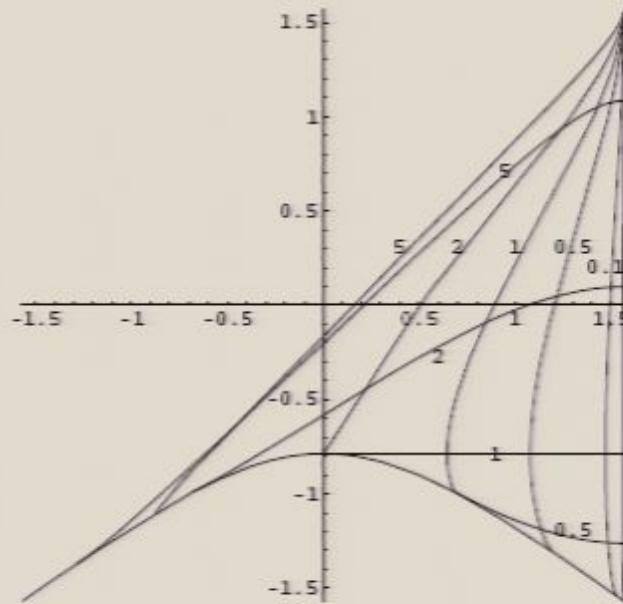
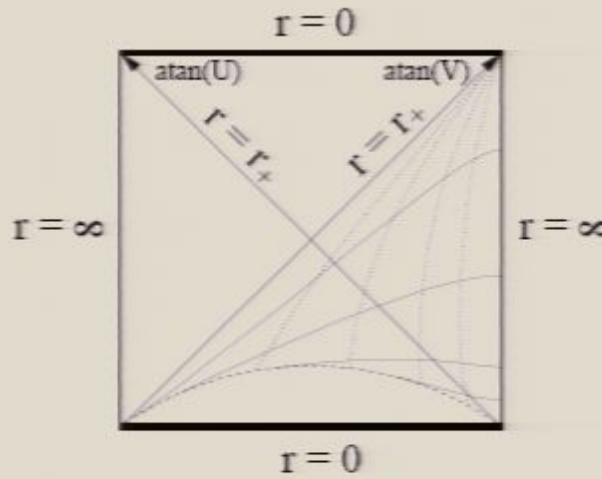
General solution with boundary  $\sim$  Minkowski scales with  $z/\tau$ :

$$ds^2 = \frac{\mathcal{L}^2}{z^2} \left[ -\left(1 - \frac{z^2}{v^2 \tau^2}\right)^2 d\tau^2 + \left(1 + \frac{z^2}{v^2 \tau^2}\right)^2 \tau^2 d\eta^2 + dz^2 \right]$$

Suggests a horizon at  $z = v\tau$  moving with velocity  $v$ . However, structure of  $\text{AdS}_3$  is completely known (BTZ)! Transform  $\tau, z \rightarrow V, U \rightarrow t, r$

$$\begin{aligned} V &= \left( \frac{2\tau - (\sqrt{M} + 1)z}{2\tau + (\sqrt{M} - 1)z} \right) \left( \frac{\tau}{\mathcal{L}} \right)^{\sqrt{M}}, \quad r = \mathcal{L} \sqrt{M} \left( \frac{1 - UV}{1 + UV} \right), \quad M \equiv 1 + \frac{4}{v^2} = M_{\text{BH}} \cdot 8G_3 \\ U &= - \left( \frac{2\tau - (\sqrt{M} - 1)z}{2\tau + (\sqrt{M} + 1)z} \right) \left( \frac{\tau}{\mathcal{L}} \right)^{-\sqrt{M}}, \quad t = \frac{\mathcal{L}}{2\sqrt{M}} \ln \left| \frac{V}{U} \right|. \end{aligned}$$

$$\begin{aligned} \Rightarrow ds^2 &= \mathcal{L}^2 \left[ -\frac{4}{(1 - UV)^2} dV dU + M \left( \frac{1 - UV}{1 + UV} \right)^2 d\eta^2 \right] \\ ds^2 &= - \left( \frac{r^2}{\mathcal{L}^2} - M \right) dt^2 + \frac{dr^2}{r^2 / \mathcal{L}^2 - M} + r^2 d\eta^2 \end{aligned}$$



Boundary is  $r = \infty, z = 0!$

The region  $0 < z < v\tau =$

part of interior of white hole + exterior of black hole.

$$r_m = \frac{2\mathcal{L}}{v} = \mathcal{L}\sqrt{M-1} < r < r_+ = \mathcal{L}\sqrt{M} < r < \infty$$

Matter comes out of a white hole!

We will use  $r_m \int d\eta$  to give the "area" of BH!

## Energy-momentum tensor in boundary CFT

8

$$ds^2 = \frac{\mathcal{L}^2}{z^2} [g_{\mu\nu} dx^\mu dx^\nu + dz^2]$$

$$g_{\mu\nu} = g_{\mu\nu}^{(0)}(\tau) + g_{\mu\nu}^{(2)}(\tau)z^2 + \dots \quad g_{\mu\nu}^{(0)} = \text{diag}(-1, \tau^2)$$

$$T_{\mu\nu} = \frac{\mathcal{L}}{8\pi G_3} [g_{\mu\nu}^{(2)} - g_{\mu\nu}^{(0)} \text{Tr}(g_{\mu\nu}^{(2)})].$$

$$T_\nu^\mu = \begin{pmatrix} -\epsilon(\tau) & 0 \\ 0 & p(\tau) \end{pmatrix}, \quad \epsilon(\tau) = p(\tau) = \frac{\mathcal{L}}{4\pi G_3} \frac{1}{v^2 \tau^2} = \pi Q_1 Q_5 \left( \frac{1}{\pi v \tau} \right)^2$$

Unproblematic; but what is entropy density and  $T$ :  $S = s(T)V = p'(T)V$ ?

$$\text{Try: } S = \frac{A}{4G_3} = \frac{r_m \int d\eta}{4G_3} = \frac{\mathcal{L}}{4G_3} \frac{2}{v\tau} \times \int \tau d\eta, \quad V = \int \tau d\eta$$

$$\Rightarrow s(T) = \frac{\mathcal{L}}{2G_3 v \tau} \quad T(\tau) = \frac{1}{\pi v \tau} = \frac{\sqrt{M-1}}{2\pi \tau}$$

The  $\tau$  dependent coordinate singularity at  $r_m = 2\mathcal{L}/v$  was used to get the expected  $T$ !

Effectively: scale static  $T_H$ ,  $s$  by  $\mathcal{L}/\tau$ ,  $T_{BTZ} = \frac{\sqrt{M}}{2\pi \mathcal{L}} \rightarrow \frac{\sqrt{M}}{2\pi \tau}$

## Thermodynamics:

9

$$\epsilon(T) = p(T) = \pi Q_1 Q_5 T^2(\tau), \quad s(T) = 2\pi Q_1 Q_5 T(\tau)$$
$$T = \frac{1}{\pi v \tau}, \quad \pi T \tau = \frac{1}{v} \gtrsim \hbar \rightarrow v \lesssim 1, M \gtrsim 4.$$

Compare with ideal BE-FD 1+1d gas with  $N_b = N_f = 4Q_1 Q_5$ :

$$\epsilon(T) = p(T) = (N_b + \frac{1}{2} N_f) \frac{\pi}{6} T^2 = \pi Q_1 Q_5 T^2, \quad s(T) = 2\pi Q_1 Q_5 T$$

Just the same, no 3/4

Thermalisation: smallest  $\tau_i$  for which  $T = T_i \frac{\tau_i}{\tau}$

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## Rotating metric

The time dependent form of the standard rotating BTZ metric is

$$\begin{aligned} ds^2 = & \frac{\mathcal{L}^2}{z^2} \left( -\left\{ 1 - \frac{(M-1)z^2}{2\tau^2} + \frac{1}{16} \left[ (M-1)^2 - (J/\mathcal{L})^2 \right] \frac{z^4}{\tau^4} \right\} d\tau^2 - \frac{Jz^2}{\mathcal{L}\tau} d\tau d\eta \right. \\ & \left. + \left\{ 1 + \frac{(M-1)z^2}{2\tau^2} + \frac{1}{16} \left[ (M-1)^2 - (J/\mathcal{L})^2 \right] \frac{z^4}{\tau^4} \right\} \tau^2 d\eta^2 + dz^2 \right). \end{aligned}$$

$$T_{\mu\nu} = \frac{\mathcal{L}}{16\pi G_3 \tau^2} \begin{pmatrix} M-1 & -(J/\mathcal{L})\tau \\ -(J/\mathcal{L})\tau & (M-1)\tau^2 \end{pmatrix} = (\epsilon + p)u_\mu u_\nu + pg_{\mu\nu}^{(0)}.$$

$$\epsilon = p = \frac{\mathcal{L}}{16\pi G_3 \tau^2} \sqrt{(M-1)^2 - (J/\mathcal{L})^2}, \quad u^\mu = (\cosh(\eta + \phi), \sinh(\eta + \phi))$$

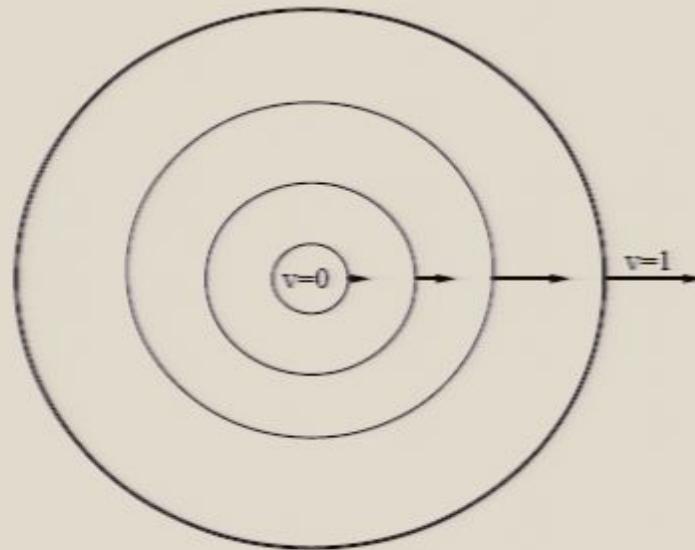
Metric with  $J \neq 0$  describes a flow boosted with the velocity

$$v_{\text{boost}} = \tanh \phi = \frac{J/\mathcal{L}}{\sqrt{(M-1)^2 - (J/\mathcal{L})^2}}$$

## Similarity expansion in 1+3d

$$T_{\mu\nu} = (\epsilon + p) \frac{x_\mu x^\nu}{\tau^2} + p g_{\mu\nu}$$

Fixed time  $t$ :



$$\mathbf{v} = \frac{\mathbf{x}}{t} \theta(t - |\mathbf{x}|), \quad u^\mu = (\gamma, \gamma \mathbf{v}) = \frac{x^\mu}{\tau}, \quad \tau = \sqrt{t^2 - \mathbf{x}^2}$$

$$\epsilon'(\tau) + \frac{3}{\tau}(\epsilon + p) = 0 \quad p = \epsilon/3 \quad \epsilon(\tau) = \frac{\epsilon_0}{\tau^4} = \frac{\epsilon_0}{(t^2 - \mathbf{x}^2)^2}$$

Gravity dual of spherical similarity expansion would be a time dependent solution of <sup>12</sup>

$$R_{MN} + 4g_{MN} = 0$$

with the symmetries (coordinates =  $t, r, \theta, \phi, z$ )

$$ds^2 = \frac{\mathcal{L}^2}{z^2} [-a(t, r, z)dt^2 + b(t, r, z)dt dr + c(t, r, z)dr^2 + g(t, r, z)d\Omega_2^2 + dz^2],$$

which expanded near boundary

$$g_{\mu\nu}(x, z) = \eta_{\mu\nu} + \dots + g_{\mu\nu}^{(4)}(x)z^4 + \dots$$

leads to  $T_{\mu\nu} = (\epsilon + p)x_\mu x_\nu / \tau^2 + pg_{\mu\nu}$ ,  $\epsilon = 3p = 3aT^4(t)$ :

$$T_{\mu\nu} = \frac{\mathcal{L}^3}{4\pi G_5} g_{\mu\nu}^{(4)} = \begin{pmatrix} 4p\frac{t^2}{\tau^2} - p & 4p\frac{tr}{\tau^2} & 0 & 0 \\ 4p\frac{tr}{\tau^2} & 4p\frac{r^2}{\tau^2} + p & 0 & 0 \\ 0 & 0 & r^2p & 0 \\ 0 & 0 & 0 & r^2p\sin^2\theta \end{pmatrix}$$

Can "only" do 1+1 dim or  $r = 0$  in 1+3d, 2 functions, 2 variables:

$$ds^2 = \frac{\mathcal{L}^2}{z^2} [-\epsilon(t, z)dt^2 + b(t, z)d\mathbf{x}^2 + dz^2]$$

**Solution in 1+3 dimensions:**  $r = r(t)$

$$ds^2 = [-a(t, z)dt^2 + b(t, z)d\mathbf{x}^2 + dz^2]/z^2, \quad R_{MN} + 4g_{MN} = 0$$

$$a(t, z) = \frac{\left[\left(1 - \frac{r''}{4r}z^2\right)^2 - \left(\frac{r''}{4r} - \frac{r'^2}{4r^2}\right)^2 z^4 - \frac{1}{4r^4 z_0^4} z^4\right]^2}{\left[\left(1 - \frac{r'^2}{4r^2}z^2\right)^2 + \frac{1}{4r^4 z_0^4} z^4\right]} \stackrel{r(t) \neq 1}{=} \frac{(1 - z^4/(4z_0^4))^2}{1 + z^4/(4z_0^4)}$$

$$b(t, z) = r^2 \left[ \left(1 - \frac{r'^2}{4r^2}z^2\right)^2 + \frac{1}{4r^4 z_0^4} z^4 \right] \stackrel{r(t) \neq 1}{=} 1 + \frac{z^4}{4z_0^4}$$

Again a time dependent solution with a "horizon" at  $a(t, z) = 0$ :

$$z_{H\pm}^2 = \frac{4r^2}{rr'' \pm \sqrt{4/z_0^4 + (r'^2 - rr'')^2}} \stackrel{r(t) \neq 1}{=} 2z_0^2, \quad \pi T_H = \frac{1}{z_0}$$

Is there a coordinate transformation transforming away the  $t$ -dependence?

Boundary metric now is RW:

$$g_{\mu\nu}(x, 0) = (-1, r^2(t), r^2(t), r^2(t))$$

Brane gravity adds a brane and Einstein with  $G_4$  to determine  $r(t)$ .

$$T_{\mu\nu}:$$

$$g_{\mu\nu}(t, z) = g_{\mu\nu}^{(0)}(t) + g_{\mu\nu}^{(2)}(t)z^2 + g_{\mu\nu}^{(4)}(t)z^4 + \dots,$$

$$T_{\mu\nu} = \frac{\mathcal{L}^3}{4\pi G_5} \left[ g_{\mu\nu}^{(4)} - \frac{1}{8} g_{\mu\nu}^{(0)} [(\text{Tr } g_{(2)})^2 - \text{Tr } g_{(2)}^2] - \frac{1}{2} (g_{(2)} g_{(0)}^{-1} g_{(2)})_{\mu\nu} + \frac{1}{4} \text{Tr } g_{(2)} \cdot g_{(2)\mu\nu} \right]$$

$$T_{tt} = \epsilon(t) = \frac{3N_c^2}{8\pi^2} \left( \frac{1}{z_0^4 r^4} + \frac{r'^4}{4r^4} \right) \sim \frac{T_0^4}{r^4(t)} + \frac{1}{t^4} \sim \underbrace{T^4(t)}_{\text{radiation}} + \underbrace{t^{-4}}_{\text{curvature}},$$

$$T_1^1(t) = \frac{1}{3} \epsilon(t) - \underbrace{\frac{N_c^2 r'^2 r''}{8\pi^2 r^3}}_{\text{trace anomaly}/3}$$

$\epsilon = 3p \sim T^4$  are known,  $T(t) = ?$ ,  $s = p'(T) = ?$ ,  $r(t) = ?$ . Obvious that  $r(t) = t/t_0$  works nicely:

$$\epsilon(t) = \frac{3\pi^2 N_c^2}{8} \frac{1}{t^4} \left( \frac{t_0^4}{\pi^4 z_0^4} + \frac{1}{4} \right) \Rightarrow T(t) = \frac{1}{\pi z_0} \frac{t_0}{t} \quad \text{if } t_0 \gg z_0$$

$T \sim 1/\tau^{1/3}$  follows if expansion is in 1d, thermalisation in 3d.



## Conclusions

1. Lattice QCD can determine purely Euclidian quantities, no real time.
2. AdS/CFT can compute real time correlators, but using time independent gravitational backgrounds (linear response)

$$\langle e^{\int d^4x \mathcal{O}(x) \phi_0(x,0)} \rangle_{\text{FT}} = e^{-\int d^4x dz L(\phi(x,z))}$$

$\Rightarrow$  correlators between states at  $t = -\infty$  and  $t = +\infty$ .

3. How does one handle processes starting at some  $t = 0$ ? "Gauge invariant" formulation = ? Can one derive new results for  $t$  dependent processes from AdS/CFT?

$$T_{\mu\nu}:$$

$$g_{\mu\nu}(t, z) = g_{\mu\nu}^{(0)}(t) + g_{\mu\nu}^{(2)}(t)z^2 + g_{\mu\nu}^{(4)}(t)z^4 + \dots,$$

$$T_{\mu\nu} = \frac{\mathcal{L}^3}{4\pi G_5} \left[ g_{\mu\nu}^{(4)} - \frac{1}{8} g_{\mu\nu}^{(0)} [(\text{Tr } g_{(2)})^2 - \text{Tr } g_{(2)}^2] - \frac{1}{2} (g_{(2)} g_{(0)}^{-1} g_{(2)})_{\mu\nu} + \frac{1}{4} \text{Tr } g_{(2)} \cdot g_{(2)\mu\nu} \right]$$

$$T_{tt} = \epsilon(t) = \frac{3N_c^2}{8\pi^2} \left( \frac{1}{z_0^4 r^4} + \frac{r'^4}{4r^4} \right) \sim \frac{T_0^4}{r^4(t)} + \frac{1}{t^4} \sim \underbrace{T_0^4(t)}_{\text{radiation}} + \underbrace{t^{-4}}_{\text{curvature}},$$

$$T_1^1(t) = \frac{1}{3} \epsilon(t) - \underbrace{\frac{N_c^2 r'^2 r''}{8\pi^2 r^3}}_{\text{trace anomaly}/3}$$

$\epsilon = 3p \sim T^4$  are known,  $T(t) = ?$ ,  $s = p'(T) = ?$ ,  $r(t) = ?$ . Obvious that  $r(t) = t/t_0$  works nicely:

$$\epsilon(t) = \frac{3\pi^2 N_c^2}{8} \frac{1}{t^4} \left( \frac{t_0^4}{\pi^4 z_0^4} + \frac{1}{4} \right) \Rightarrow T(t) = \frac{1}{\pi z_0} \frac{t_0}{t} \quad \text{if } t_0 \gg z_0$$

$T \sim 1/\tau^{1/3}$  follows if expansion is in 1d, thermalisation in 3d.



**Solution in 1+3 dimensions:**  $r = r(t)$

$$ds^2 = [-a(t, z)dt^2 + b(t, z)d\mathbf{x}^2 + dz^2]/z^2, \quad R_{MN} + 4g_{MN} = 0$$

$$a(t, z) = \frac{\left[\left(1 - \frac{r''}{4r}z^2\right)^2 - \left(\frac{r''}{4r} - \frac{r'^2}{4r^2}\right)^2 z^4 - \frac{1}{4r^4 z_0^4} z^4\right]^2}{\left[\left(1 - \frac{r'^2}{4r^2}z^2\right)^2 + \frac{1}{4r^4 z_0^4} z^4\right]} \stackrel{r(t) \neq 1}{=} \frac{(1 - z^4/(4z_0^4))^2}{1 + z^4/(4z_0^4)}$$

$$b(t, z) = r^2 \left[ \left(1 - \frac{r'^2}{4r^2}z^2\right)^2 + \frac{1}{4r^4 z_0^4} z^4 \right] \stackrel{r(t) \neq 1}{=} 1 + \frac{z^4}{4z_0^4}$$

Again a time dependent solution with a "horizon" at  $a(t, z) = 0$ :

$$z_{H\pm}^2 = \frac{4r^2}{rr'' \pm \sqrt{4/z_0^4 + (r'^2 - rr'')^2}} \stackrel{r(t) \neq 1}{=} 2z_0^2, \quad \pi T_H = \frac{1}{z_0}$$

Is there a coordinate transformation transforming away the  $t$ -dependence?

Boundary metric now is RW:

$$g_{\mu\nu}(x, 0) = (-1, r^2(t), r^2(t), r^2(t))$$

Brane gravity adds a brane and Einstein with  $G_4$  to determine  $r(t)$ .

$$T_{\mu\nu}:$$

$$g_{\mu\nu}(t, z) = g_{\mu\nu}^{(0)}(t) + g_{\mu\nu}^{(2)}(t)z^2 + g_{\mu\nu}^{(4)}(t)z^4 + \dots,$$

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