Title: Non - Equilibrium Dynamics of Quark-Gluon Plasma

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Abstract:

Heavy Quarks in Strongly Coupled Plasmas: Energy Loss from AdS/CFT

C. Herzog
A. Karch
P. Kovtun
L. Yaffe
C. Kozcaz
+ P. Chesler

see also: Casalderrey-Solana & Teaney
Fries, Gubser, Michalogiorgakis
Liu, Rajagopal, Wiedemann
⋮

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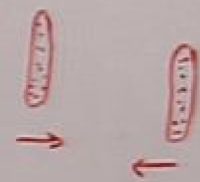
RHIC:



Au + Au
200 GeV/nucleon

time

"splat"
high density partons / "color glass condensate"
weakly coupled non-equilibrium plasma
apparent thermalization
1 fm/c
strongly coupled plasma
expansion, cooling
hadronization
10 fm/c
freezeout

Some probes of QGP dynamics:

elliptic flow – anisotropy of produced hadrons wrt. reaction plane

reflects pressure gradients in non-central events

$\nabla_x p > \nabla_y p$

$\Rightarrow v_x > v_y$

$\Rightarrow$ 

jet quenching: suppression of observed back-to-back high $p_\perp$ jets

charm production: heavy mass $\Rightarrow$ slower thermalization

$\Rightarrow$ less elliptic flow?

jet quenching of $c\bar{c}$

$\Rightarrow$ suppressed production of D mesons (relative to pp, pA)

$\mathcal{N}=4$ supersymmetric Yang-Mills theory:

supersymmetry, AdS/CFT duality =
tools unavailable for QCD

	QCD	$\mathcal{N}=4$ SYM
$T=0$:	confinement, stable particles, scattering, ...	conformal, no particles, no S-matrix

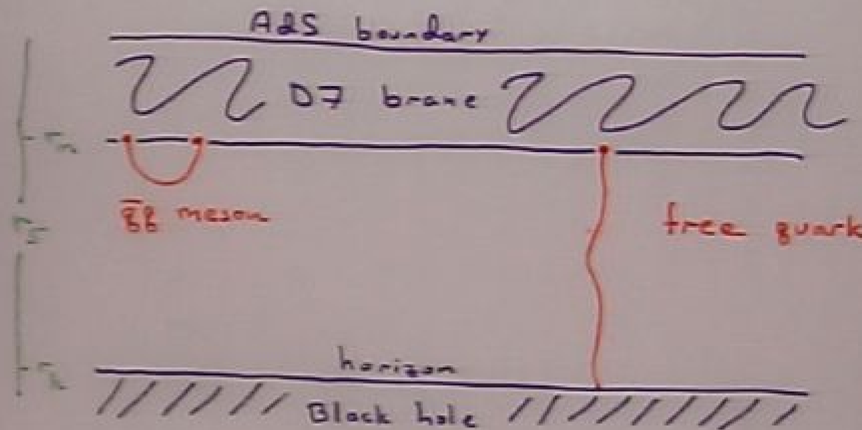
no relation!

	QCD	$\mathcal{N}=4$ SYM
$T>0$:	non-Abelian plasma w. gluons + fundamental rep. matter,	non-Abelian plasma w. gluons + adjoint rep. matter,
	no confinement,	no confinement,
	Debye screening,	Debye screening,
	finite spatial corr. length	finite spatial corr. length

very similar!

Adding flavor (fund. rep. matter) to $\mathcal{N}=4$ SYM

$\Rightarrow$ D7 brane in $AdS_5 \times S^5$ on which open strings can end — Karch, Katz



r_5 ~ energy scale in dual field theory

$r_5 \nearrow$ boundary : high energy

$\searrow$ horizon : low energy

r_m = minimal radius of D7 brane : mass of fundamental rep. hypermultiplet

$r_m \gg r_h \Rightarrow m \gg T$ heavy quark

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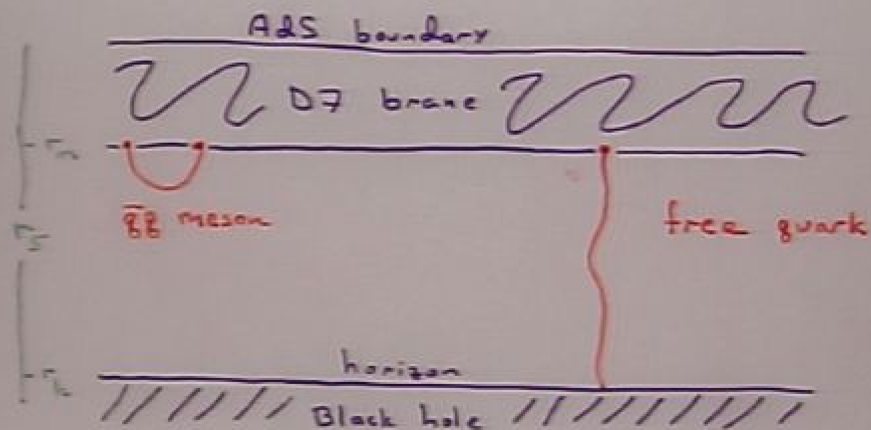
on which open strings can end



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on which open strings can end

r_s ~ ene... le in dual field theory

r_s ... energy

... energy

$r_{...}$ ... fundamental

... hypermultiplet

... heavy quark

Solve classical dynamics of open strings ending on D7 brane, moving in AdS-BH background

$\Rightarrow$ learn about heavy quark dynamics in thermal $\mathcal{N}=4$ SYM

AdS-BH :

$$ds^2 = L^2\left(-h(u)\,dt^2 + \frac{du^2}{h(u)} + u^2\,d\vec{x}^2\right)$$

$$u = r_5/L^2, \quad h(u) = u^2\left(1 - \left(\frac{u_h}{u}\right)^4\right)$$

$$u_h = \pi T$$

String action :

$$S = -T_0 \int d\sigma\, d\tau\, \sqrt{-g}$$

g_{ab} = induced metric

$T_0 = \frac{\sqrt{\lambda}}{2\pi} L^{-2}$

motion in x-u plane, static gauge $\sigma = u$, $\tau = t$

$$\Rightarrow \quad \frac{\partial}{\partial u}\left(h(u)\,u^2 \frac{x'}{\sqrt{-g}}\right) - \frac{u^2}{h(u)} \frac{\partial}{\partial t}\left(\frac{\dot{x}}{\sqrt{-g}}\right) = 0$$

$$-g/L^4 = 1 - \frac{u^2}{h(u)}(\dot{x})^2 + h(u)\,u^2\,(x')^2$$

Solutions:

A. Static, straight

$$E = T_0 L^2 (u_m - u_h)$$
$$= m - \Delta m(T)$$

$m = T_0 L^2 u_m$ = $T=0$ quark mass

$\Delta m(T) = T_0 L^2 u_h = \frac{1}{2}\sqrt{\lambda}\, T$ = thermal mass correction

B. Moving, straight

Local speed of light $c^2(u) = g_{tt} / g_{xx}$
$$= 1 - (u_h/u)^4$$
$$\xrightarrow[u \to u_h]{} 0$$

$\therefore$ $v > c(u)$ for $u_h < u < u_c = u_h / (1-v^2)^{1/4}$

$\Rightarrow$ $-g$ changes sign at u_c

$\Rightarrow$ unphysical solution

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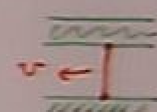

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C. Moving, curved, constant velocity

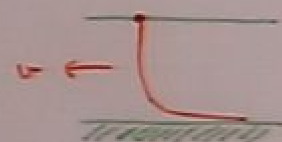
Try $x(u,t) = x(u) + vt$

$\Rightarrow$ time indep EOM $\quad \frac{\partial}{\partial u}\left(h(u)\, u^2 \frac{x'}{\sqrt{-g}} \right) = 0$

$$\text{w.} \quad -g/L^4 = 1 - \frac{u^2 v^2}{h} + h u^2 (x')^2$$

$$\Rightarrow \quad x' = \pm \frac{v}{h(u)} \left(\frac{u_h}{u} \right)^2$$

$$\Rightarrow x(u,t) = x_0 + vt \pm \frac{v}{2u_h} \left[\frac{\pi}{2} - \tan^{-1}\frac{u}{u_h} - \coth^{-1}\frac{u}{u_h} \right]$$



energy flow: into horizon $\qquad$ out of horizon — discard

near horizon: $\quad x' \sim 1/(u-u_h)$

$x(u)$ diverges logarithmically

energy + momentum flux finite

energy / momentum loss rate:

$$\frac{d}{dt}\begin{pmatrix} E \\ P \end{pmatrix} = -\begin{pmatrix} \pi^1_t \\ \pi^1_x \end{pmatrix}\Bigg|_{u=u_h} = -\frac{T_0 L^2 u_h^2}{\sqrt{1-v^2}} \begin{pmatrix} v^2 \\ v \end{pmatrix} = -\frac{\sqrt{\lambda}}{2}\frac{\pi T^2}{\sqrt{1-v^2}} \begin{pmatrix} v^2 \\ v \end{pmatrix}$$

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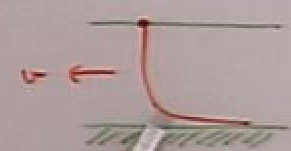

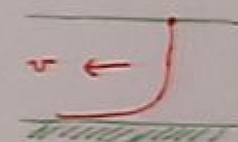

energy flow: [illegible] to horizon $\qquad$ out of horizon — discard

near hor[illegible]: $\quad x' \sim 1/(u-u_h)$

$x(u)$ diverges logarithmically

energy + momentum flux finite

e[illegible] / momentum loss rate:

$$[\text{illegible}] \left(\begin{matrix} \pi^1_t \\ \pi^1_x \end{matrix} \right)\Bigg|_{u=u_h} = -\frac{T_0 L^2 u_h^2}{\sqrt{1-v^2}} \left(\begin{matrix} v^2 \\ v \end{matrix} \right) = -\frac{\sqrt{\lambda}}{2} \frac{\pi T^2}{\sqrt{1-v^2}} \left(\begin{matrix} v^2 \\ v \end{matrix} \right)$$

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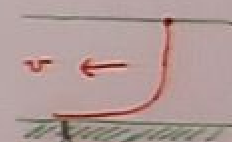

energy flow: into horizon $\qquad\qquad$ out of horizon — discard

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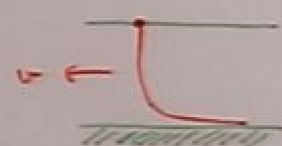

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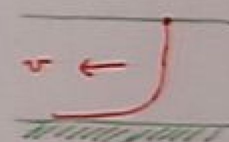

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$$\therefore \quad \frac{dE}{dx} = \frac{dp}{dt} = -\mu p = F_{drag}$$

w. friction coefficient

$$\mu = \frac{\pi}{2} \frac{\sqrt{\lambda}\, T^2}{m_{th}} = \pi T \frac{\Delta m(T)}{m_{th}}$$

N.B. $x'(u) = 0$ only at boundary $(u = \infty)$

$\Rightarrow$ Neumann b.c. at D7 brane not satisfied

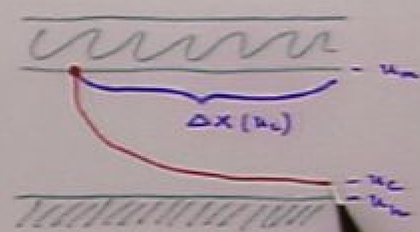
$\Rightarrow$ external force is acting on quark

$$v = \text{const.} \Rightarrow \vec{F}_{ext} = -\vec{F}_{drag}$$

Total string energy = UV + IR divergent

cut off by quark mass m

?



cutoff string energy

$$E \sim -\# \ln(u_c - u_h)$$

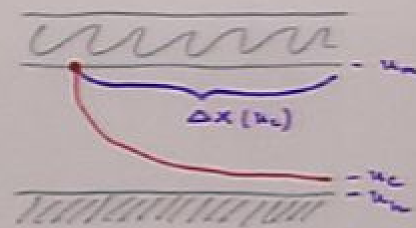
$$\sim \frac{dE}{dx}\,\Delta x(u_c)$$ line divergence in Δx

Total string energy $\supset$

(infinite) work done to ke

quark moving at constant

from $t = -\infty$.



cutoff string energy

$$E \sim -\# \ln(u_c - u_h)$$

$$\sim \frac{dE}{dx}\,\Delta x(u_c) \qquad \text{linear divergence in } \Delta x$$

Total string energy $\supset$

(infinite) work done to keep quark moving at constant velocity from $t = -\infty$.

D. Moving finite mass quark, no external force

$\Rightarrow$ non-stationary, decelerating solutions

Late times $\Rightarrow$ small velocity

= linearize about static solution

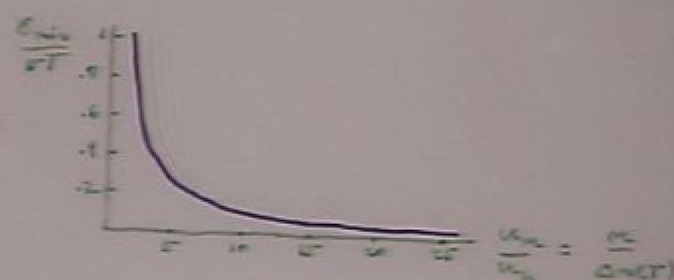
$$\Rightarrow \quad \frac{\partial}{\partial u}\left(h(u)\, u^2 x'\right) = \frac{u^2}{h(u)}\, \ddot{x}$$

near horizon: $x(u_h(1+\epsilon), t) = \underset{\substack{\text{outgoing} \\ \text{(into BH)}}}{F(t + \tfrac{1}{4}\ln\epsilon)} + \underset{\substack{\text{incoming} \\ \text{(from BH)}}}{G(\cancel{t - \tfrac{1}{4}}\ln\epsilon)}$

$e^{-\gamma t}$ time dependence, Neumann b.c. at $u = u_m$

$\Rightarrow$ discrete quasi-normal modes (QNM)

Slowest mode:



γ_{min} = friction coeff. μ (in $v \to 0$ limit)

Numerical QNM results:

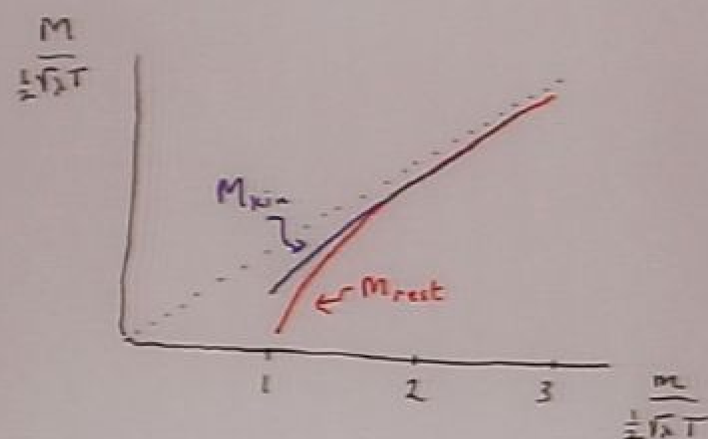
- yield μ for arbitrary $O(1)$ $m/\Delta m(T)$,
- agree w. analytic result for $m/\Delta m(T) \gg 1$.

Thermal dispersion relation

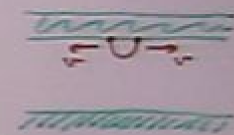
significant corrections if $m/\sqrt{\lambda}T = O(1)$

$$E = M_{rest}(T) + \frac{p^2}{2M_{kin}(T)} + O(p^4)$$

$$= \sqrt{p^2 + M_{kin}^2} + (M_{rest} - M_{kin}) \qquad v = O(1)$$



E. Separating quark + antiquark



- mimics dynamics of $c\bar{c}$ jets
- avoids IR sensitivity of analytic + QNM analysis
- requires numerical solution of non-linear PDE
- must choose initial string configuration
  (not just q & $\bar{q}$ initial position + velocity)

Nambu-Goto action, $\sigma = u$, $\tau = t$

$\Rightarrow$ PDEs w. moving boundary $u_m > u > u_{min}(t)$

$\therefore$ Bad choice

Polyakov action

$$S = T_0 \int d\sigma\, d\tau\, \underbrace{\sqrt{-\gamma}\,\gamma^{\alpha\beta}}_{\text{world sheet metric}}\, \underbrace{G_{\mu\nu}}_{\text{spacetime metric}}\, \underbrace{\partial_\alpha X^\mu\, \partial_\beta X^\nu}_{\text{embedding}}$$

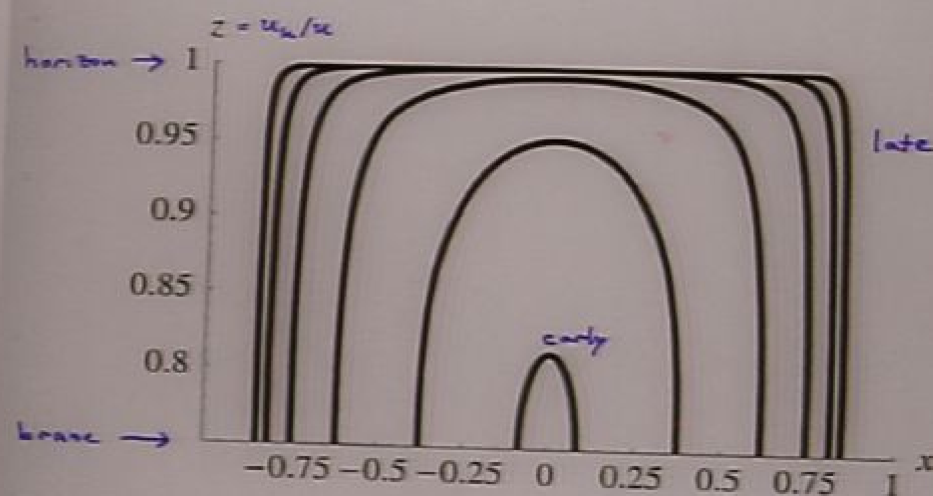
$\Rightarrow$ EOM $\partial_\alpha\left(G_{\mu\nu}\sqrt{-\gamma}\,\gamma^{\alpha\beta}\partial_\beta X^\nu\right) = \frac{1}{2}\sqrt{-\gamma}\,\gamma^{\alpha\beta}\frac{\partial G_{\nu\rho}}{\partial X^\mu}\,\partial_\alpha X^\nu\,\partial_\beta X^\rho$

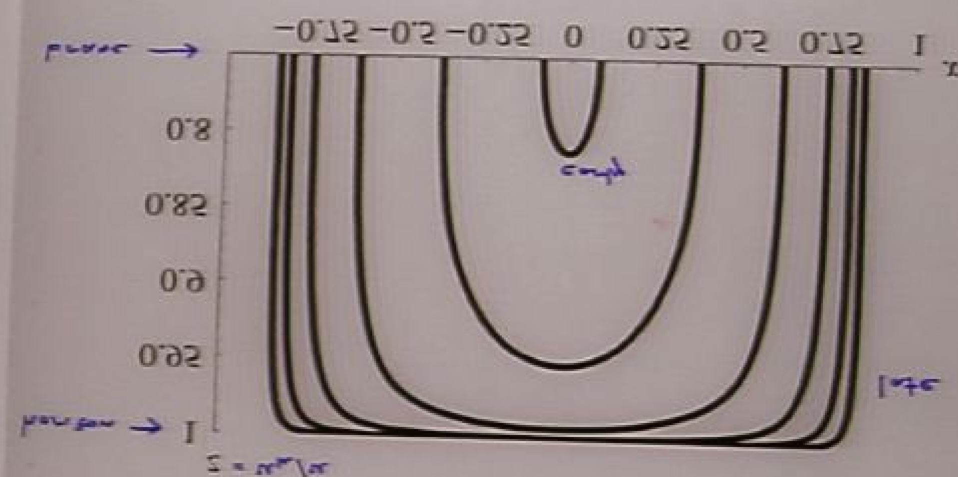
+ constraint $G_{\mu\nu}\,\partial_\alpha X^\mu\,\partial_\beta X^\nu = \frac{1}{2}\gamma_{\alpha\beta}\,\gamma^{\gamma\delta}\,G_{\mu\nu}\,\partial_\gamma X^\mu\,\partial_\delta X^\nu$

world sheet metric $\gamma^{\alpha\beta}$: arbitrary — but affects numerical stability

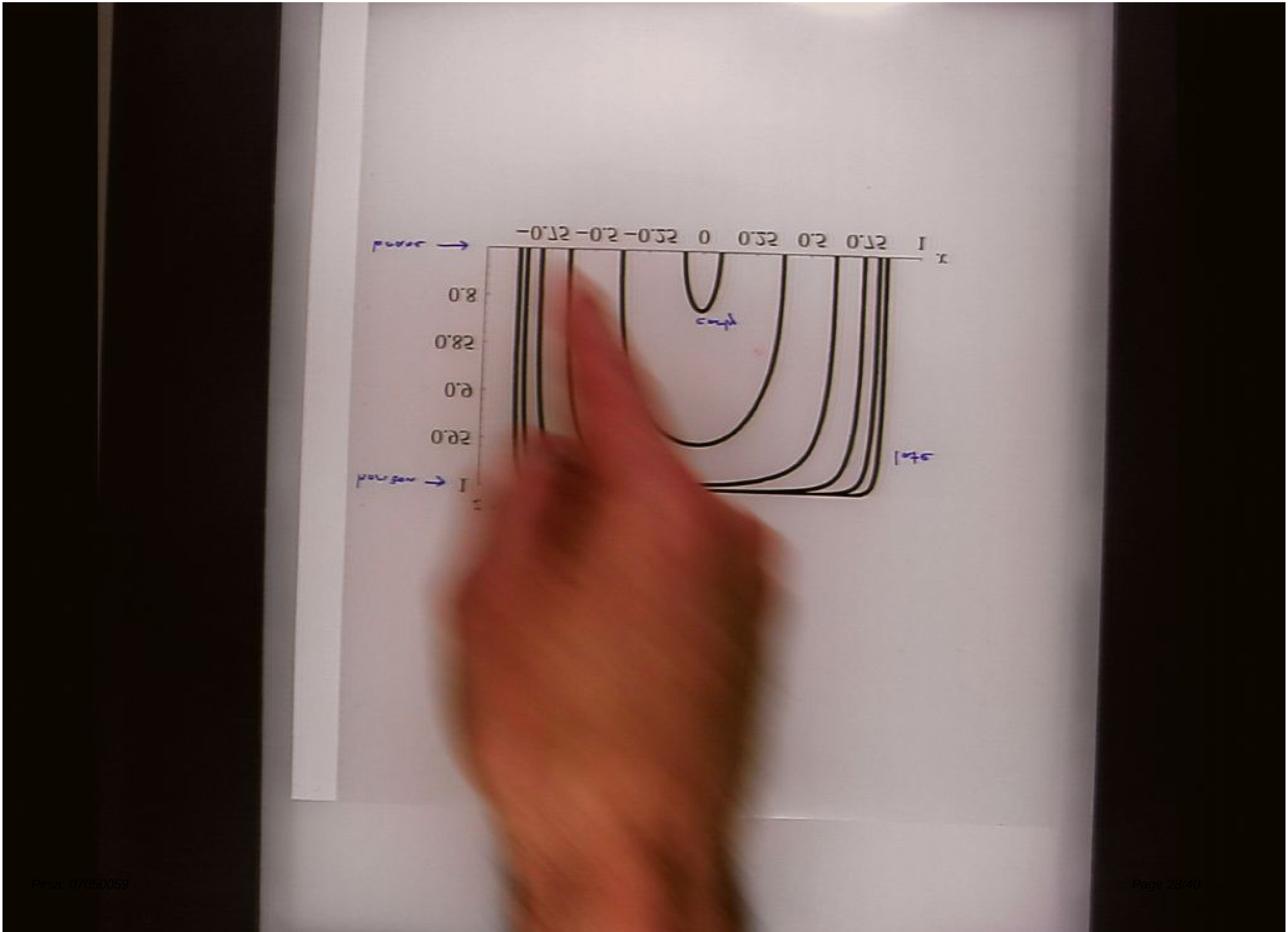
good choice: "stretched conformal"

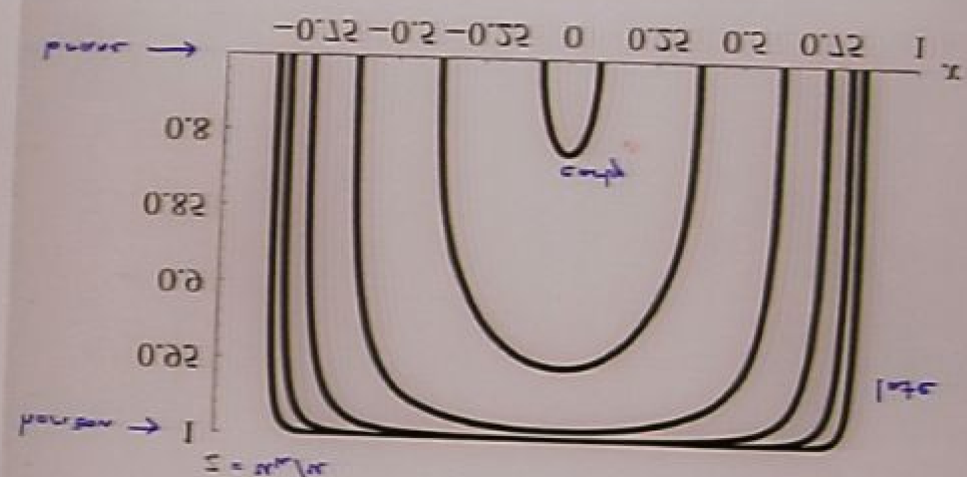
$$[\gamma^{\alpha\beta}] = \begin{bmatrix} -s(\sigma,\tau) & 0 \\ 0 & s(\sigma,\tau)^{-1} \end{bmatrix} e^{\omega(\sigma,\tau)}$$


z
horizon
1
0.95
0.9
0.85
0.8
brane
late
early
−0.75
−0.5
−0.25
0
0.25
0.5
0.75
1
x



x
-0.75 -0.5 -0.25 0 0.25 0.5 0.75 1
0.8
0.85
0.9
0.95
1
brane
horizon
early
late


-0.75 -0.5 -0.25 0 0.25 0.5 0.75 1
x
0.8
0.85
0.9
0.95
1

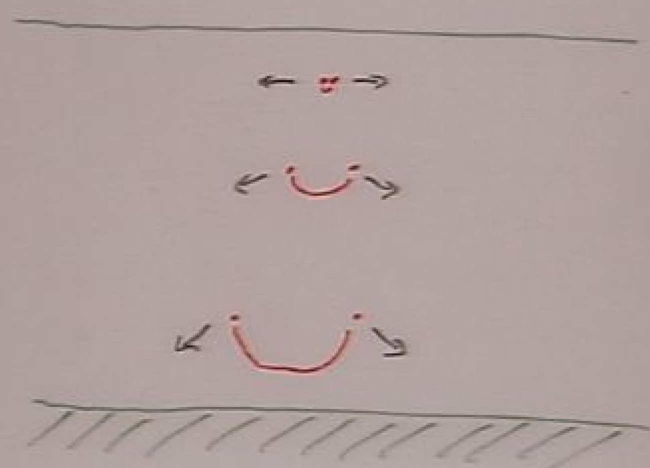


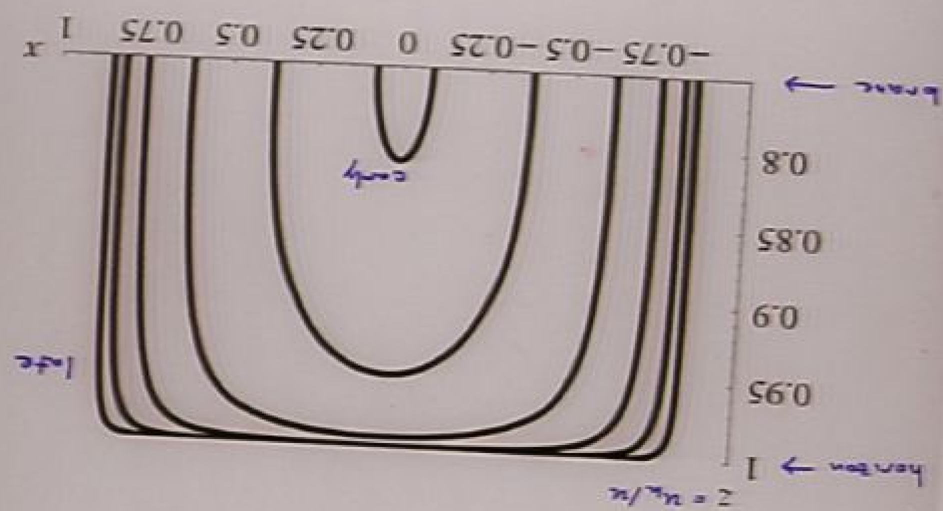
$$M_{kin} \geq \frac{1}{2}\Delta m(T) = \frac{1}{4}\sqrt{\lambda}\, T$$

$$\Rightarrow \quad \mu = \pi T \frac{\Delta m(T)}{M_{kin}} \leq 2\pi T$$

upper limit agrees very well with light $q\bar{q}$ dynamics.

A. Karch
C. Jensen




x
1
0.75
0.5
0.25
0
−0.25
−0.5
−0.75
0.8
0.85
0.9
0.95
1
brane
horizon
early
late
z = u_h/u

$$M_{kin} \geq \frac{1}{2}\,\Delta m(T) = \frac{1}{4}\sqrt{\lambda}\,T$$

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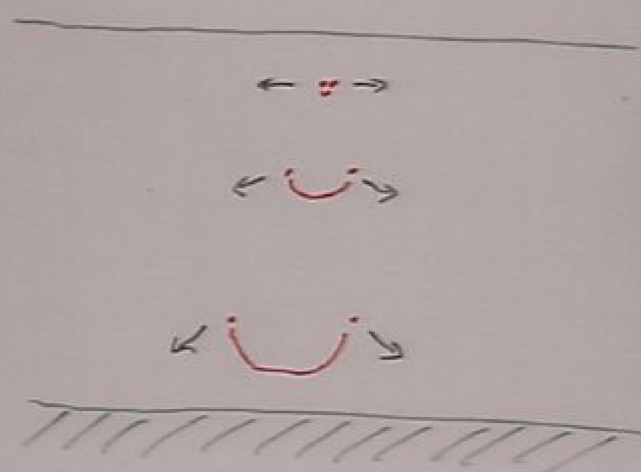


Wakes

w. P. Chesler

Where does energy from quark go?

Examine $\langle T^{\mu\nu}(x)\rangle_{\text{moving quark}}$

$$\langle T^{\mu\nu}\rangle = \frac{\delta}{\delta h_{\mu\nu}} S[h]\Big|_{\text{on-shell}}$$

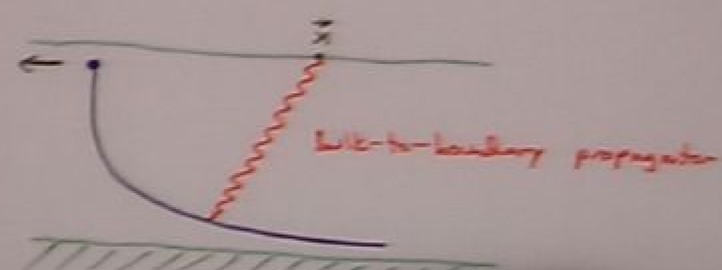
$h_{\mu\nu}$ ← boundary metric

$$S = S_{\text{Einstein}} + S_{\text{DBI}} + S_{\text{GH}} + S_{\text{counterterm}}$$

∴ Need solution of linearised gravitational fluctuations in AdS-BH background with trailing string as source.

$$\mathcal{O}^{AB}_{CD}\, h_{AB}(x) = \rho_{CD}(x)$$ ← 5-d string stress-energy

↑ 2nd order linear diff. op.



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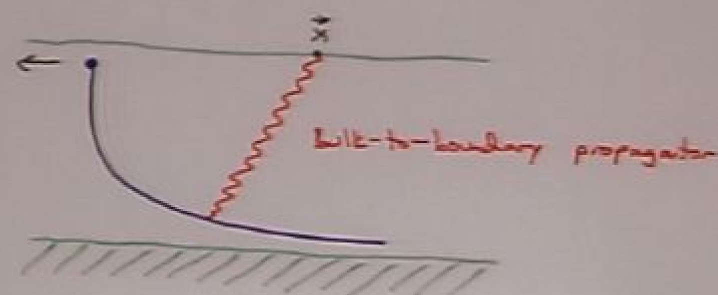


Do 4-d Fourier transform,
use gauge invariant linear combinations
of metric perturbations

$\Rightarrow$ 5 decoupled 2^{nd} order ODEs

helicity 0:

$$Z_q = q^2 h_{00} + 2\omega q_i h_{i0} + \omega^2 q_i q_j h_{ij} + q^2 (1 - f(u) - \omega^2) \tfrac{1}{2} (\delta_{ij} - \hat{q}_i \hat{q}_j) h_{ij}$$

$$Z_q''(u) + A_q(u) Z_q'(u) + B_q(u) Z_q(u) = S_q(u)$$

$$A_q = \cdots, \quad B_q = \cdots$$

$$S_q = (\cdots)\, \delta(\omega - v \cdot q)\, e^{-i\omega X_{string}(u)}$$

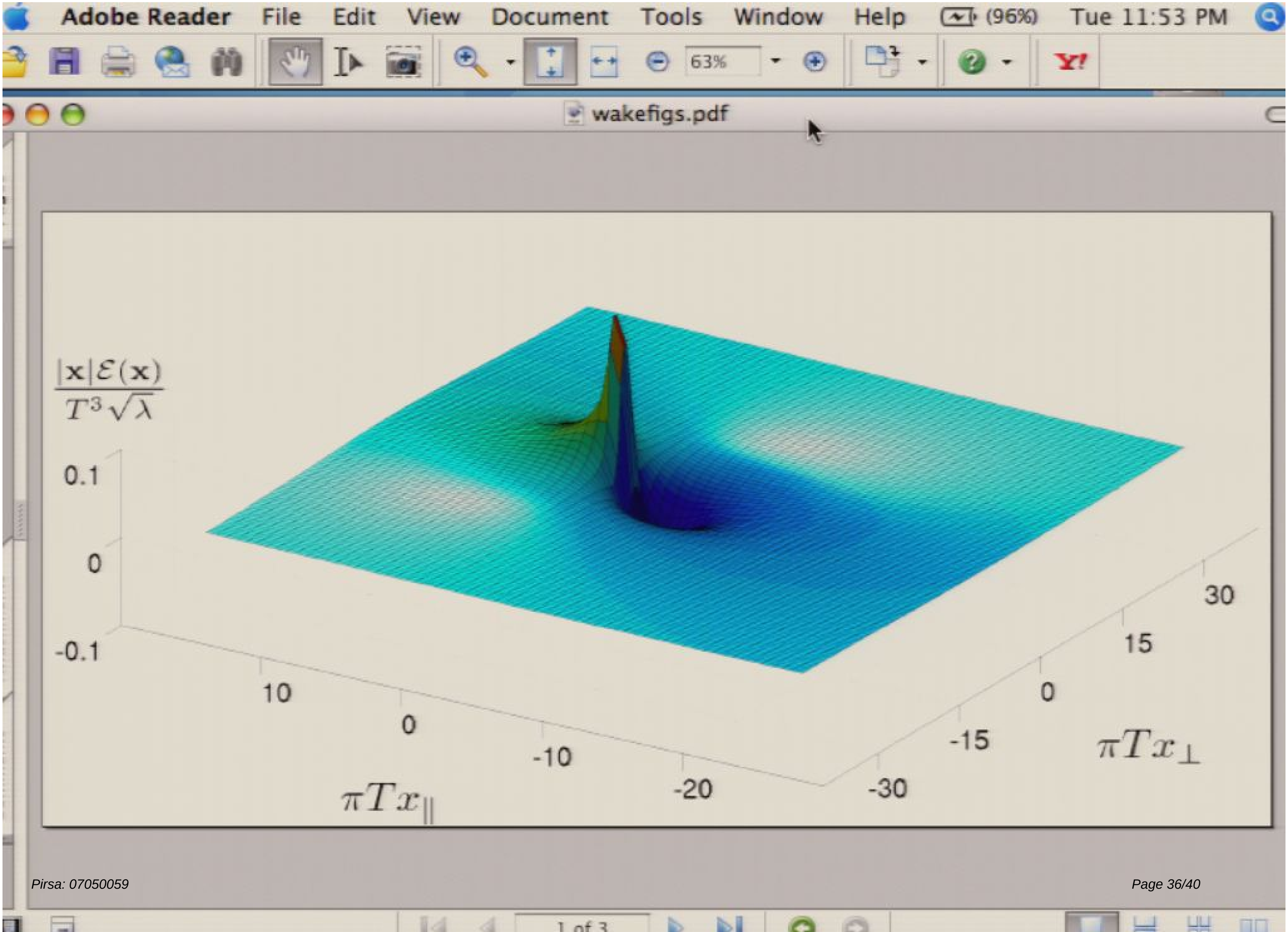
$$\langle T^{00}(x) \rangle = \lim_{u \to \infty} \int d^4 q \; e^{iq \cdot x} (\cdots)\, Z_q(u)$$

$$Z_q(u) = \int du' \, G_q(u, u')\, S_q(u')$$

Green's fcn. $G_q(u,u') = g^<(u_<)\, g^>(u_>) \,/\, w(u')$

homogeneous solutions

wronskian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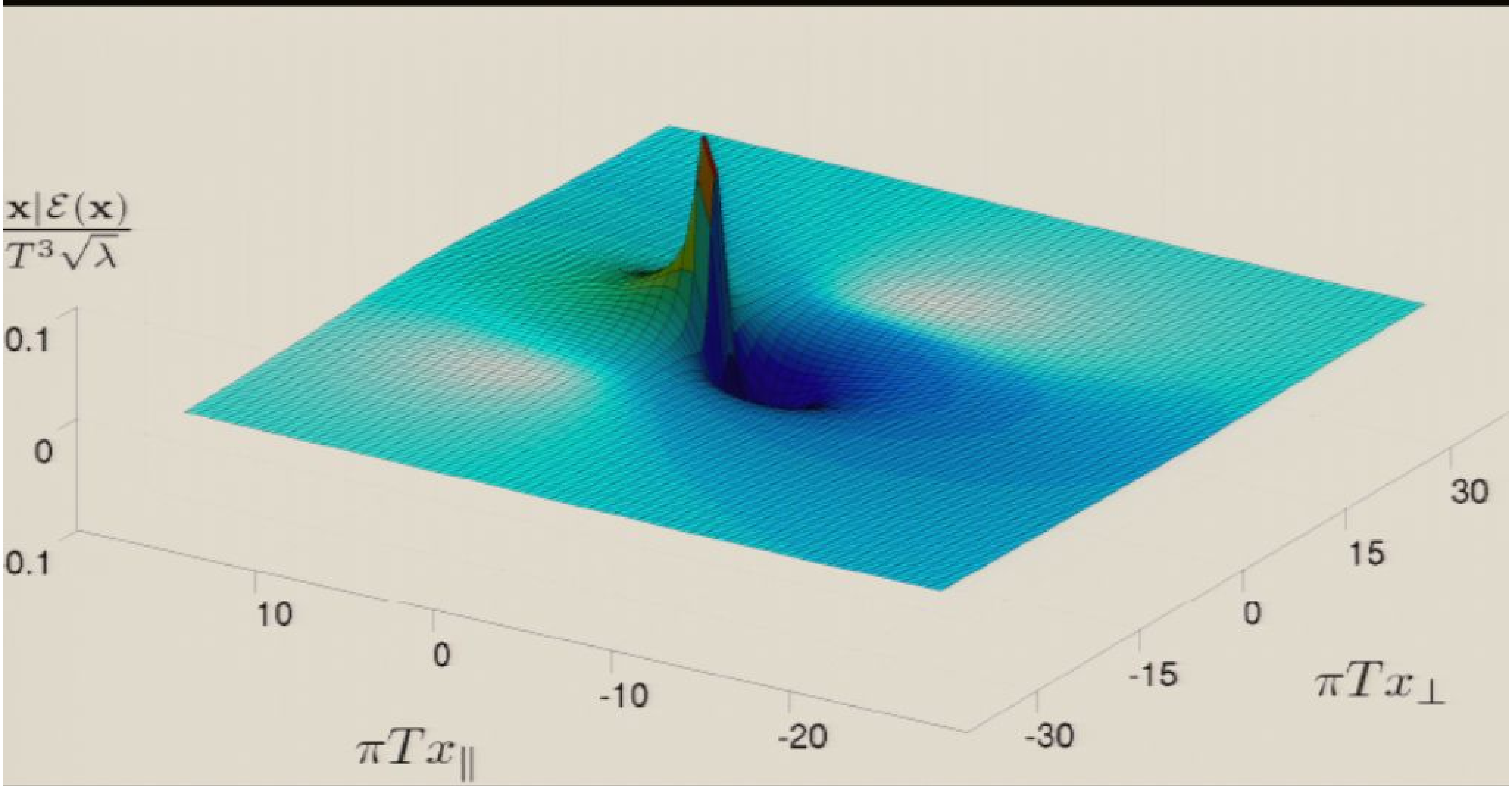
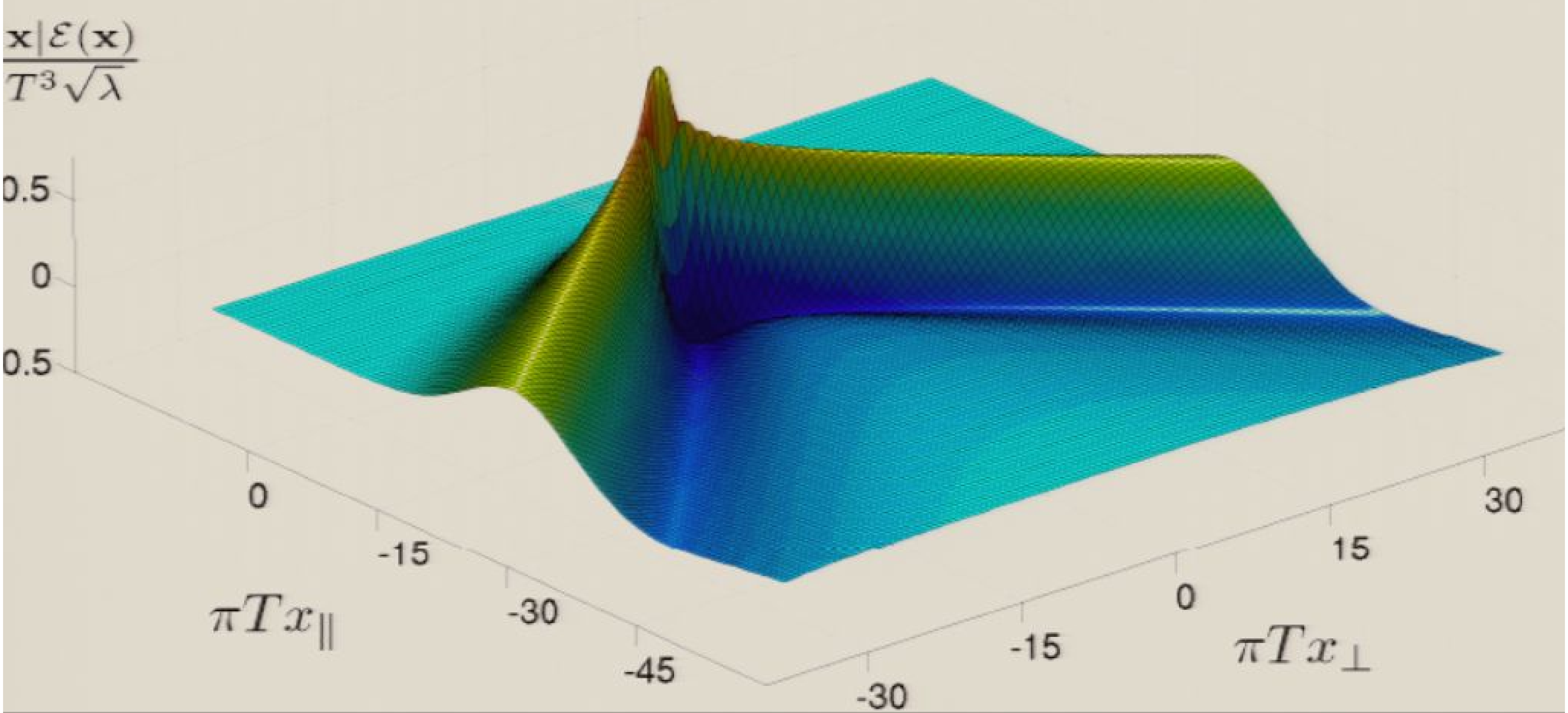
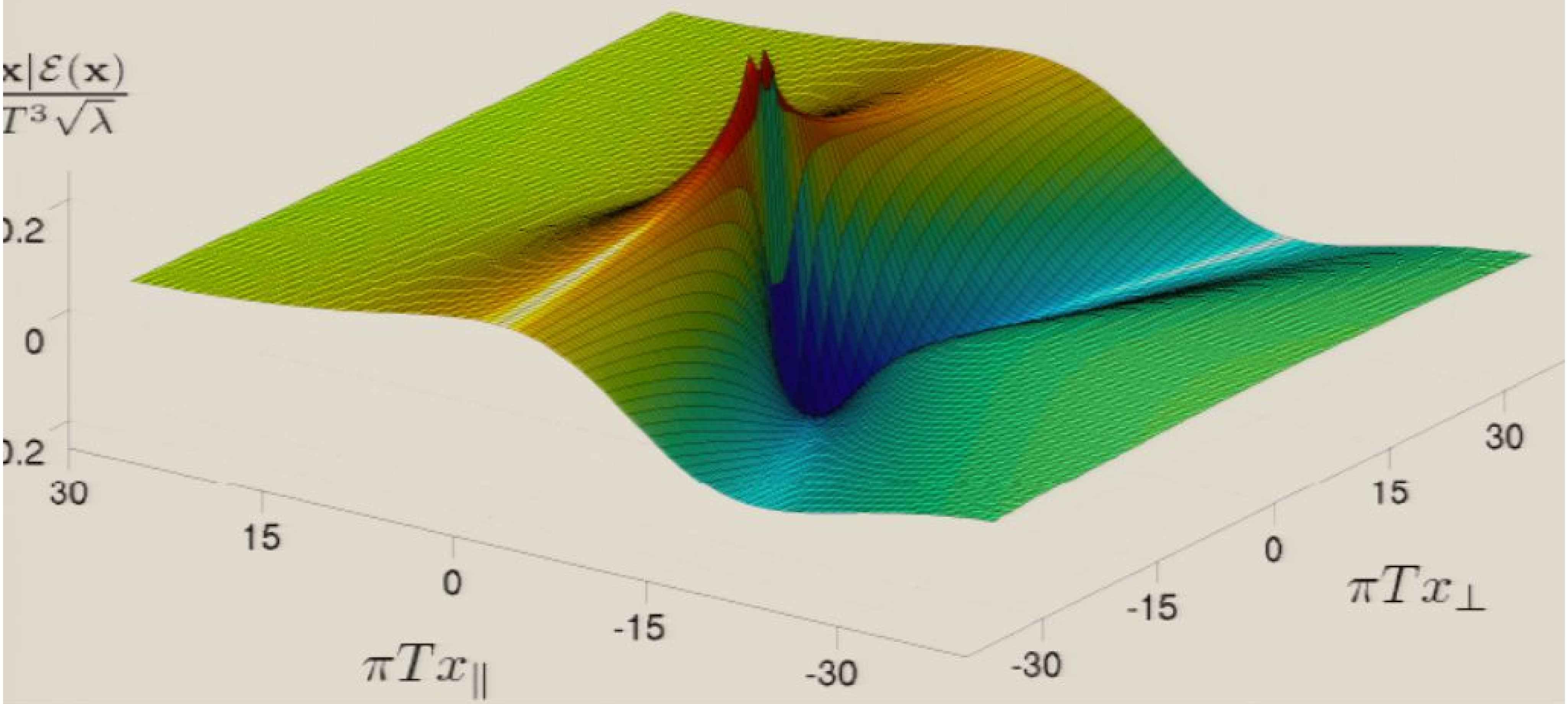
(96%)
Tue 11:53 PM
63%
wakefigs.pdf
$\frac{|\mathbf{x}|\mathcal{E}(\mathbf{x})}{T^3\sqrt{\lambda}}$
0.1
0
-0.1
10
0
-10
-20
$\pi T x_\parallel$
30
15
0
-15
-30
$\pi T x_\perp$
1 of 3



$\frac{\mathbf{x}|\mathcal{E}(\mathbf{x})}{T^3\sqrt{\lambda}}$
0.1
0
0.1
10
0
-10
-20
$\pi T x_{\parallel}$
-30
-15
0
15
30
$\pi T x_{\perp}$



$\frac{\mathbf{x}|\mathcal{E}(\mathbf{x})}{T^3\sqrt{\lambda}}$
0.5
0
0.5
0
-15
-30
-45
$\pi T x_\parallel$
-30
-15
0
15
30
$\pi T x_\perp$


0
30
15
0
-15
-30
$\pi T x_{\|}$
30
15
0
-15
-30
$\pi T x_{\perp}$

Open questions:

Non-uniform $\lambda \to \infty$, $\gamma \to \infty$ limits

$\frac{dE}{dx} \propto p$ valid for $p \ll \lambda^{?} T$?

Finish $\langle T^{\mu\nu}(x) \rangle_{\text{quark}}$

plasma heating or flow ?

$\langle T^{\mu\nu}(x) \rangle_{\text{light } q\bar{q}}$

$1/\lambda$ corrections

$1/N_c$ corrections

More QCD-like theories

$\mathcal{N} = 2^*$, K-S, ...

⋮