Title: Fictional Forces in a SYM Thermal Bath

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Abstract:

Frictional forces in strongly coupled superYang-Mills plasmas

M. Edalati, J. Vázquez-Poritz (hep-th/0608118, 0612157)

P. Moomaw, R. Wijewardhana, J. Wittig (in progress)

I Light-like Wilson Loop in N=4 SYM

II AdS representation of SYM frictional forces

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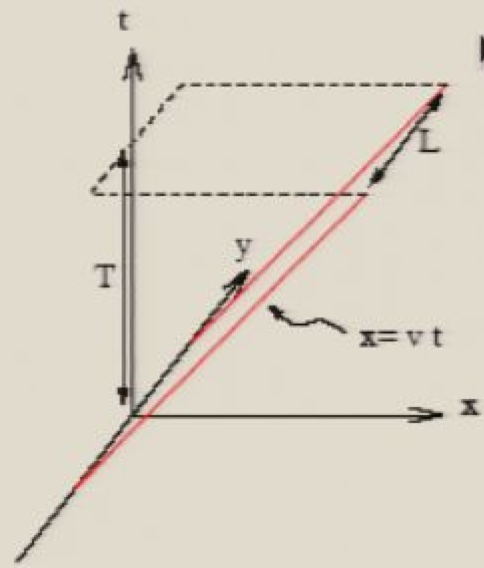
I Light-like Wilson Loop in N=4 SYM

Liu, Rajagopal, Wiedemann (hep-ph/0605178, 0607062) proposed that a certain lightlike Wilson loop W measures the jet quenching parameter $\hat{q}$ non-perturbatively:



$$\langle W \rangle \sim \exp\left\{-T(\cdots + \hat{q}L^2 + \cdots)\right\}$$

with $v \to 1^+$, $T \to \infty$, and $m^{-1} \ll L \ll \beta$, where m is the probe quark mass, and β the inverse temperature. Quark self-energies are to be subtracted.

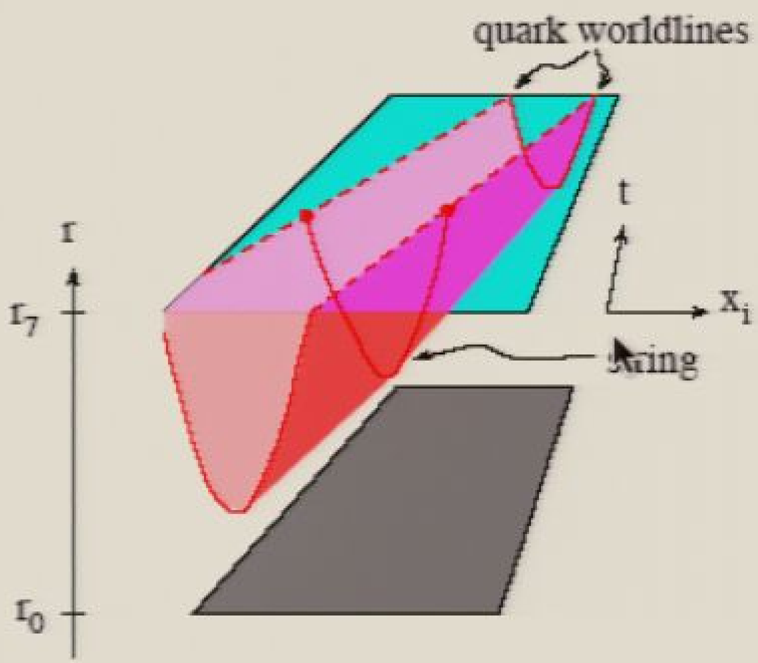
We consider this Wilson loop in the $N \gg g_{\rm ym}^2 N \gg 1$ $\mathcal{N}=4$ SU(N) SYM theory, dual to IIB strings on AdS$_5\times$S^5:

SYM theory			string theory
number of colors	N	$\Leftrightarrow$	$R^4/(4\pi\alpha'^2 g_s)$
't Hooft coupling	$\lambda = g_{\rm ym}^2 N$	$\Leftrightarrow$	R^4/α'^2
inverse temperature	β	$\Leftrightarrow$	$\pi R^2/r_0$
probe quark mass	m	$\Leftrightarrow$	$r_7/(2\pi\alpha')$
probe quark worldline		$\Leftrightarrow$	∂(string worldsheet)

Background geometry: AdS$_5$ black 3-brane

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\frac{r^4 - r_0^4}{r^2 R^2}\,dt^2 + \frac{r^2}{R^2}(dx^2 + dy^2 + dz^2) + \frac{r^2 R^2}{r^4 - r_0^4}\,dr^2.$$

with probe D7 brane down to $r = r_7$.



- The classical string dynamics governed by the Nambu-Goto action:

$$S = \frac{-1}{2\pi\alpha'}\int d^2\sigma\,\sqrt{-G}, \qquad G := \det\left[g_{\mu\nu}\frac{\partial x^\mu}{\partial\sigma^\alpha}\frac{\partial x^\nu}{\partial\sigma^\beta}\right].$$

- e^{iS} for its solutions with spacelike worldsheets give the semi-classical (saddlepoint) approximation contributions to the Wilson loop.

- Describe by a worldsheet embedding

$$t=\tau,\quad x=v\tau,\quad y=\sigma,\quad z=0,\quad r=r(\sigma),$$

with boundary conditions

$$0\le\tau\le T,\qquad -L/2\le\sigma\le L/2,\qquad r(\pm L/2)=r_7.$$

- Get equation of motion

$$r'^2=\frac{r^4-r_0^4}{a^2r_0^4R^4}\left[(v^2-1)r^4-(a^2-1)r_0^4\right],$$

where a^2 is a positive integration constant (its sign chosen so that the worldsheet is spacelike).

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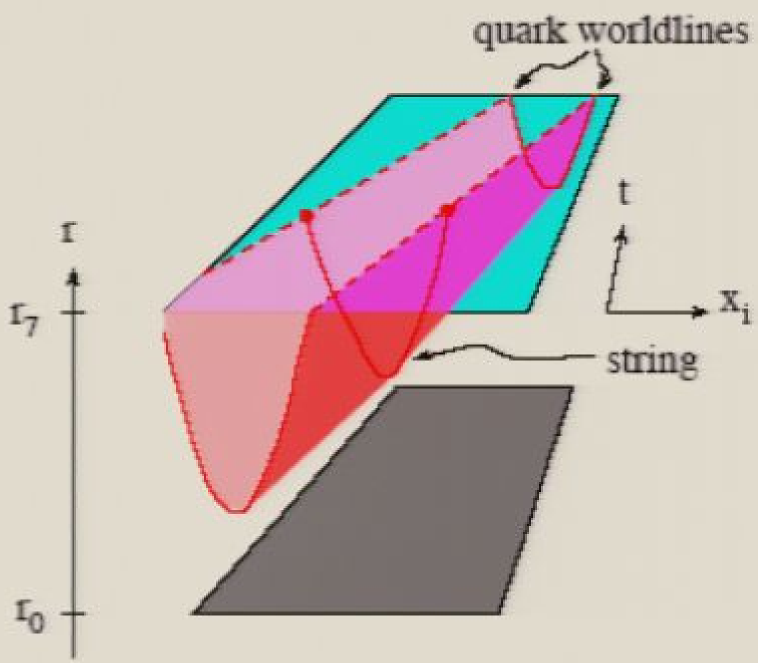

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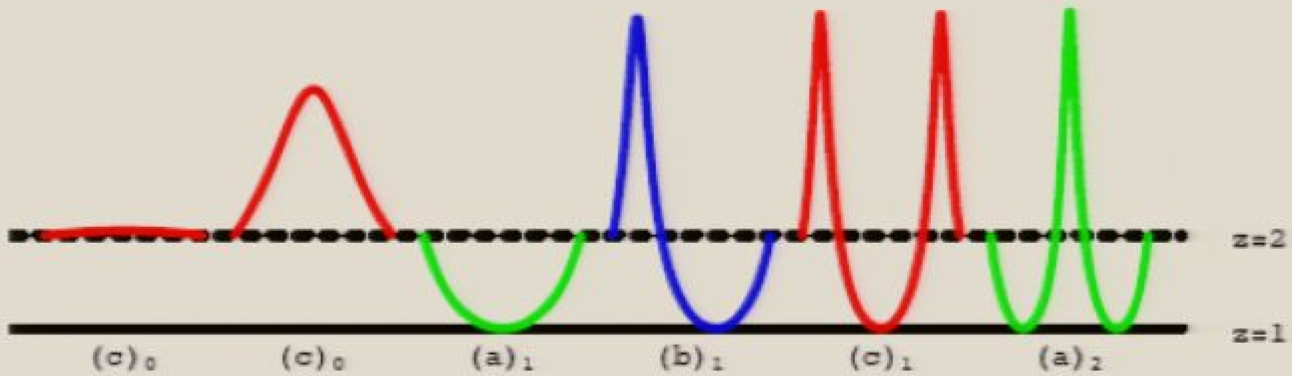
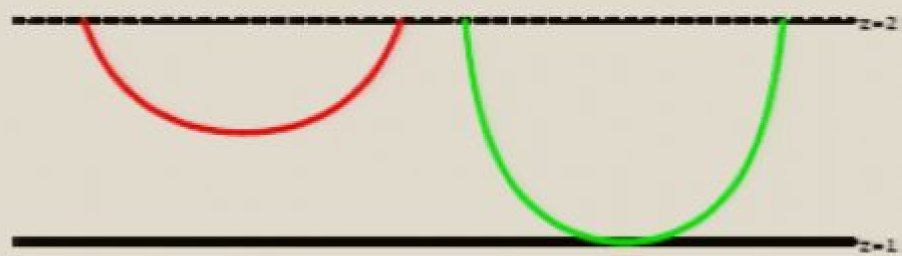
Find infinitely many spacelike string solutions with $v < 1$ (with same boundary condition):



Find two spacelike strings with $v > 1$:



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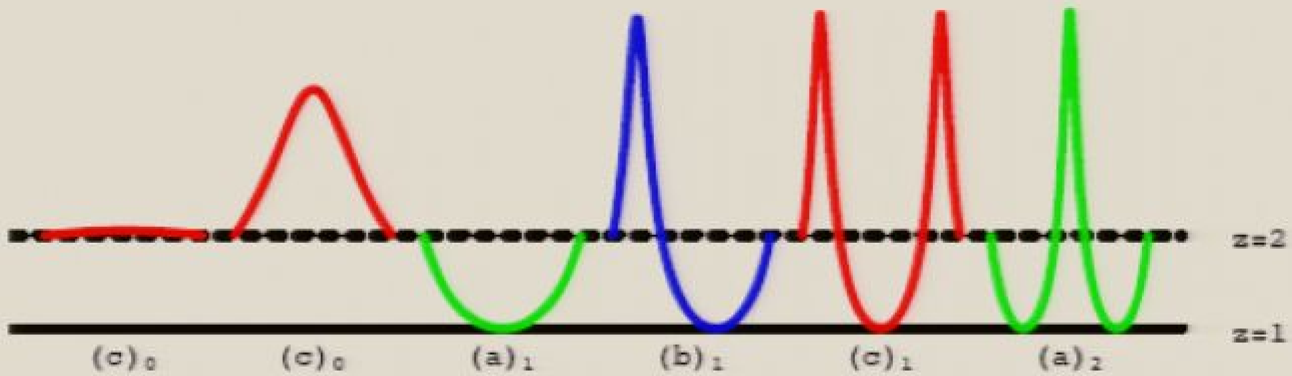

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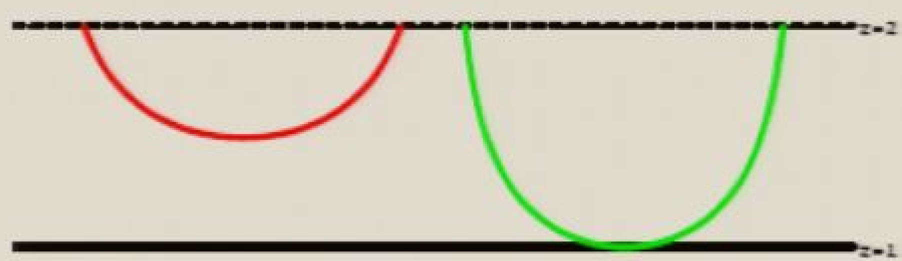

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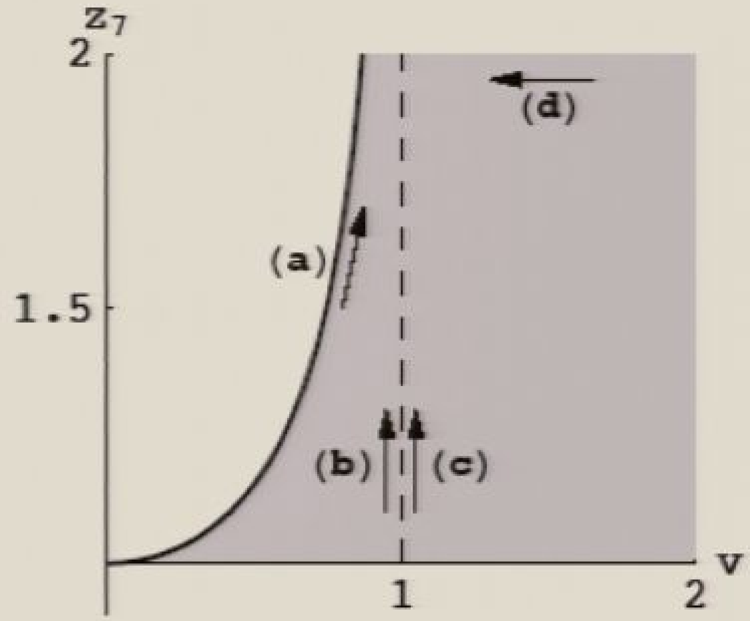

The answer is robust: independent of how the limit is taken

- The shortest (red) string gives $iS \sim -c_1 TL$ and has no L^2 piece.
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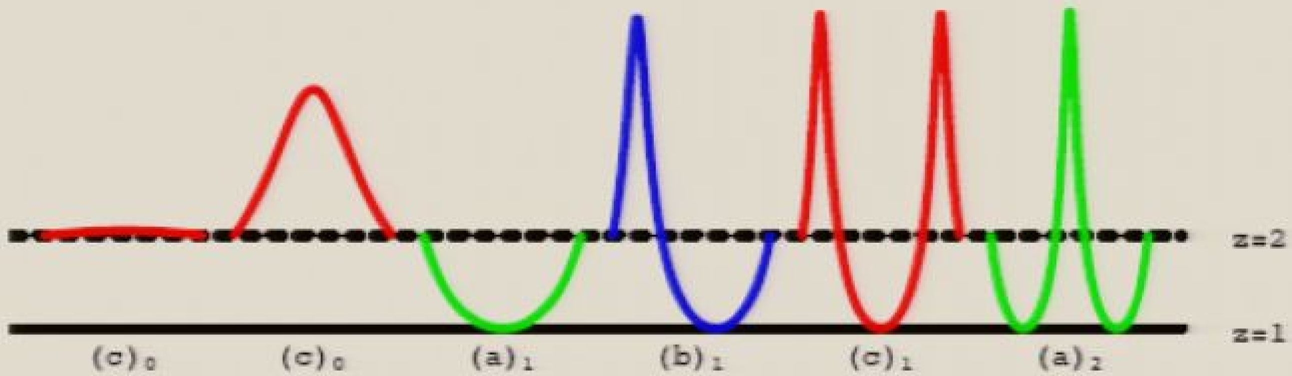

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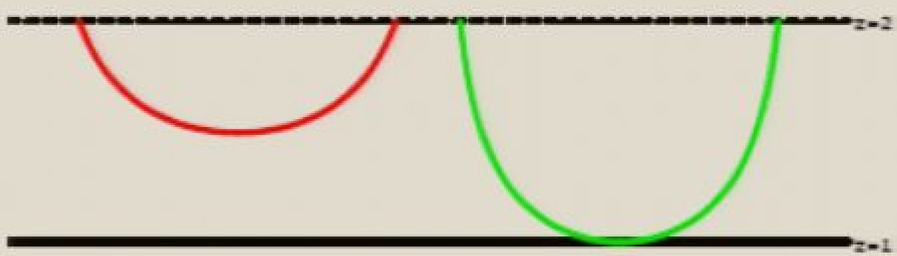

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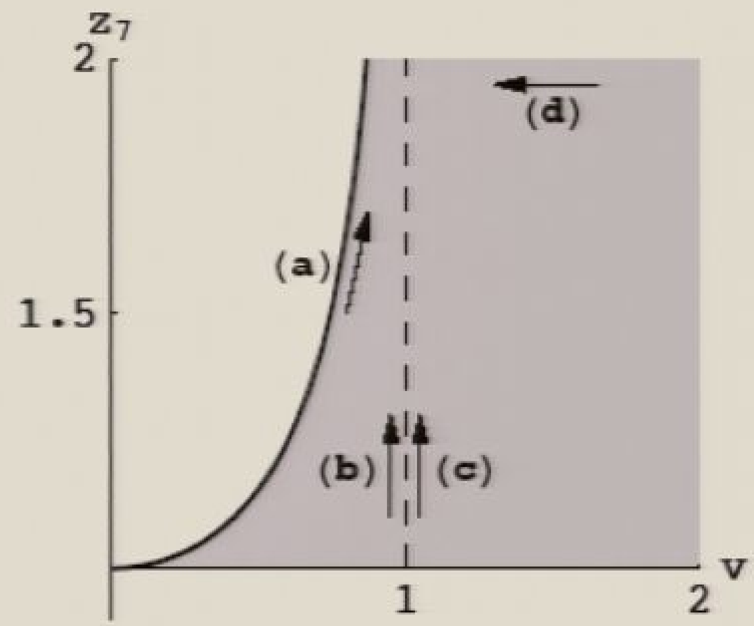

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Summing the saddlepoints:

$$\langle W \rangle \;=\; C_1 \exp\left\{-T\frac{\sqrt{\lambda}}{\beta}\left(-1.3 + \frac{\pi L}{2\beta}\right)\right\} + C_2 \exp\left\{-T\frac{\sqrt{\lambda}}{\beta}\left(0 + 0.94\frac{L^2}{\beta^2} + \mathcal{O}(L^4)\right)\right\}.$$

Only the first term survives the $T \to \infty$ limit for any L/β.

Comments:

• Infinite self-energy subtraction was made; finite pieces are terms in red.

Such subtractions are *ad hoc*: one does not have the freedom to make them in a gravitational theory. Better prescription: Drukker-Gross-Ooguri (hep-th/9904191)?

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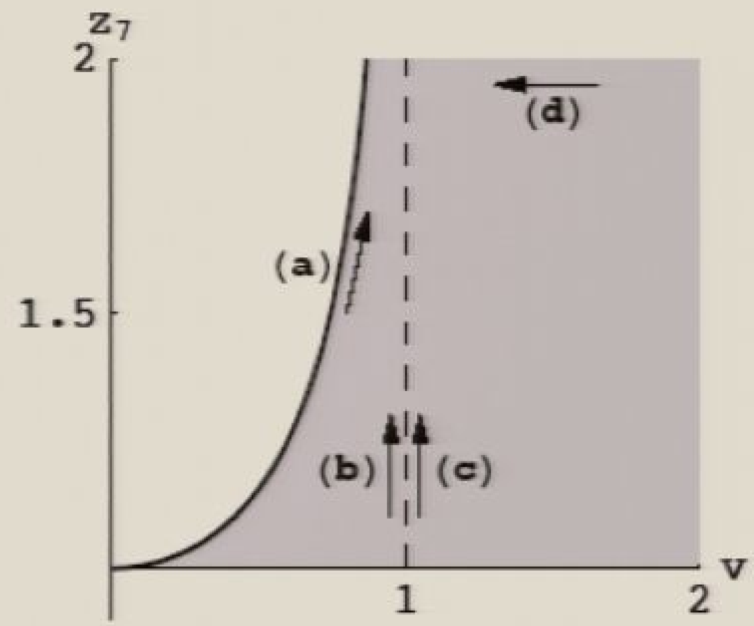

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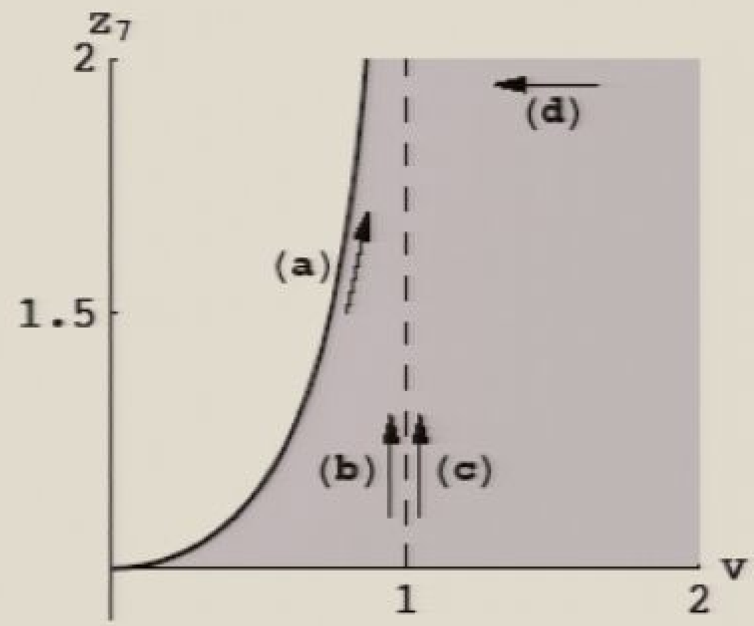

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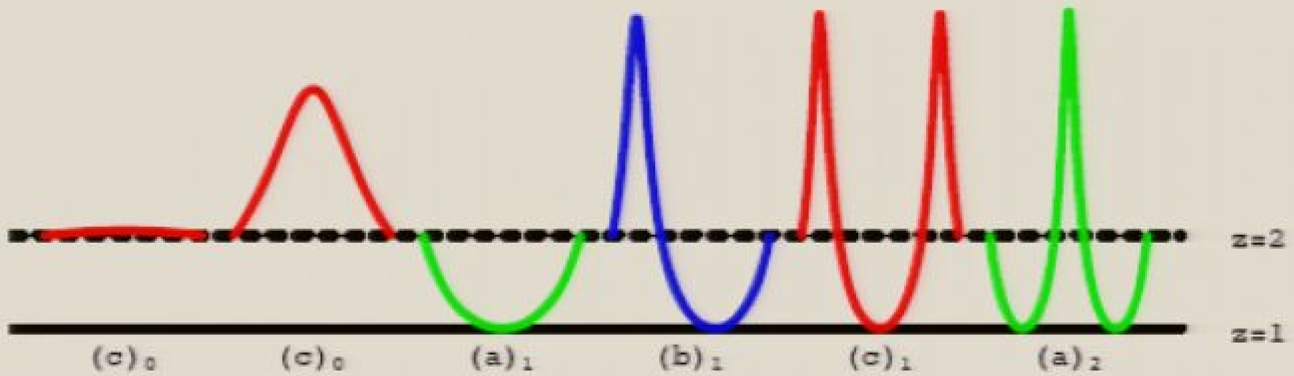

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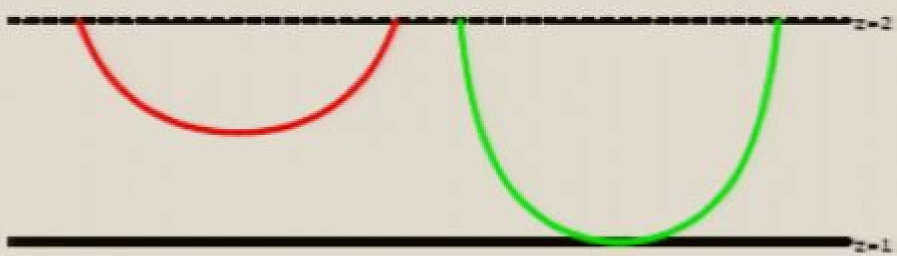

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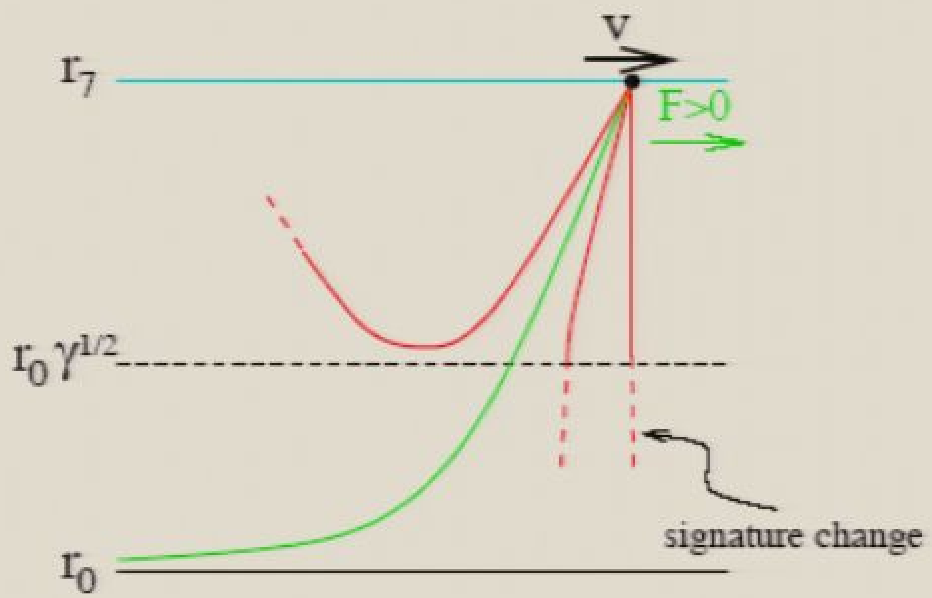
II Frictional forces in AdS

A. N=4 SYM plasma: Single quark friction parameter.

[Herzog et al hep-th/0605158, Gubser hep-th/0605182, Herzog hep-th/0605191]

Drag on a single colored probe pulled through the SYM thermal bath at constant velocity v:



Less drag and string changes signature below $r = r_0\sqrt{\gamma}$.
More drag and string rises back to D7 brane.
Energy flows down infinite string, “deposited in black hole”.

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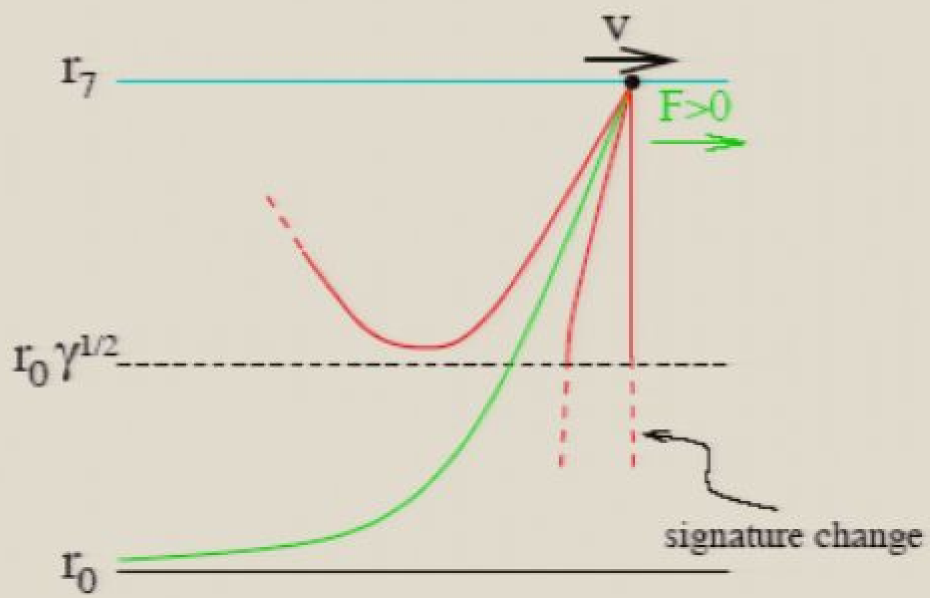

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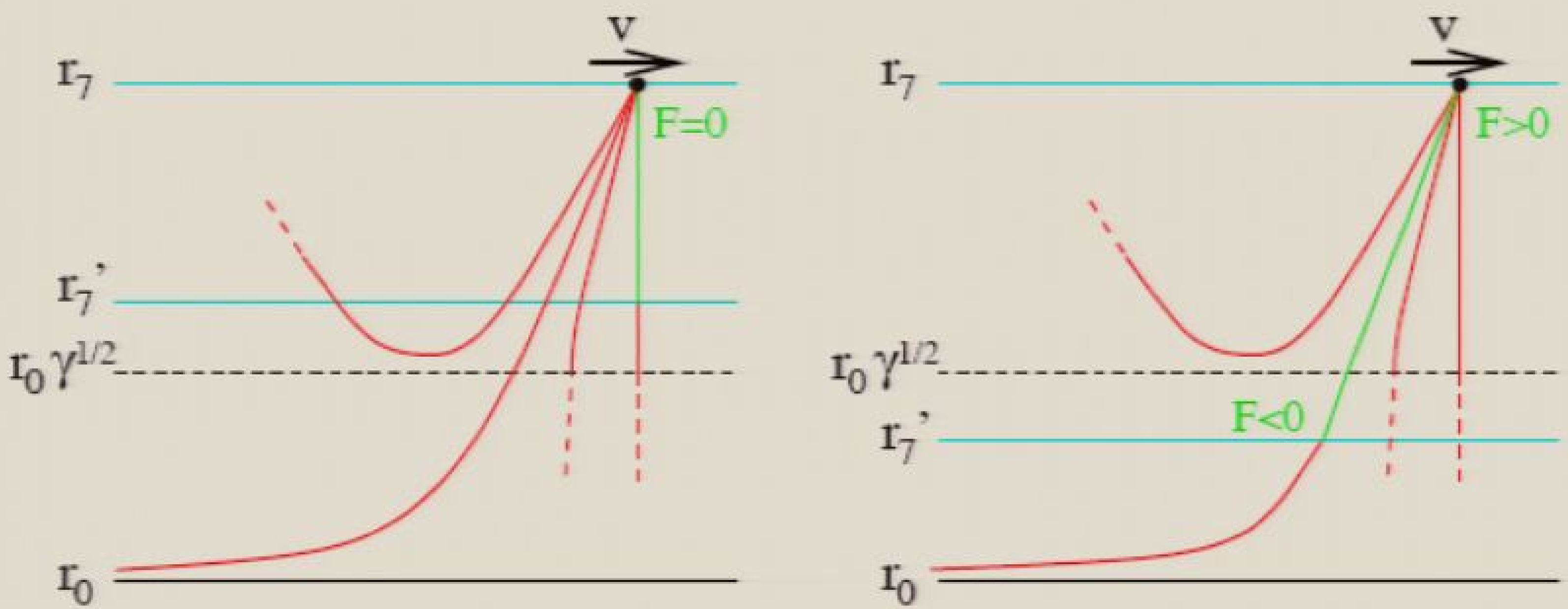
But if put a second (lighter quark) 7-brane at lower altitude r_7', and only pull on the heavy quark, then only upright solution:



Once $r_7' < r_0\sqrt{\gamma}$, jump to HK^{3}Y+G solution. But still not dragged: instead deposits energy on second brane?

Does some (small) effect destabilize the $F = 0$ solution when $r_7' > r_0\sqrt{\gamma}$, and select an $F \neq 0$ one?

C. String friction in AdS space?

These examples seem to suggest that we need to include explicit drag forces on the gravity side. Some possible sources (all $1/N$ effects):

1) Drag in the black hole Hawking radiation: down by $1/N$...

2) Radiation reaction:
– gravitational *etc.* radiation of strings.
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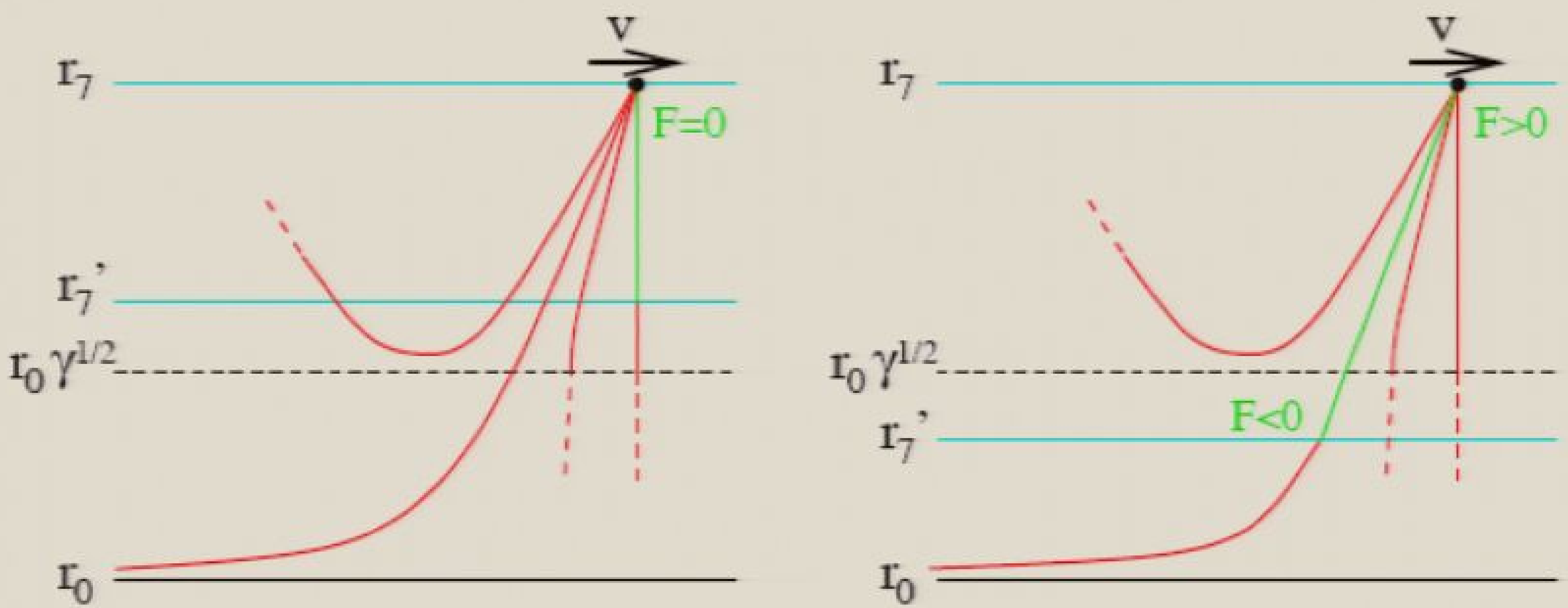

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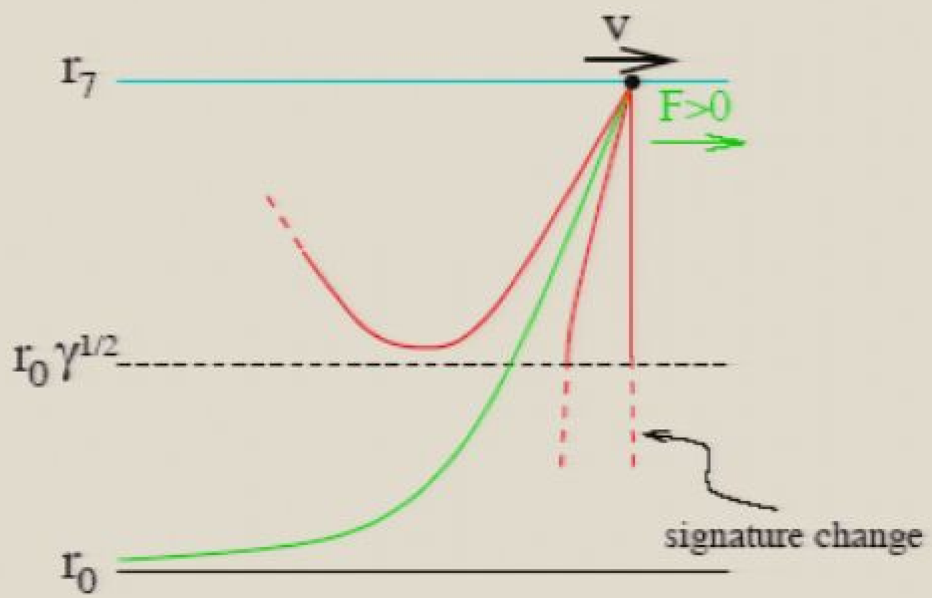

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B. Velocity-dependent interquark potential.

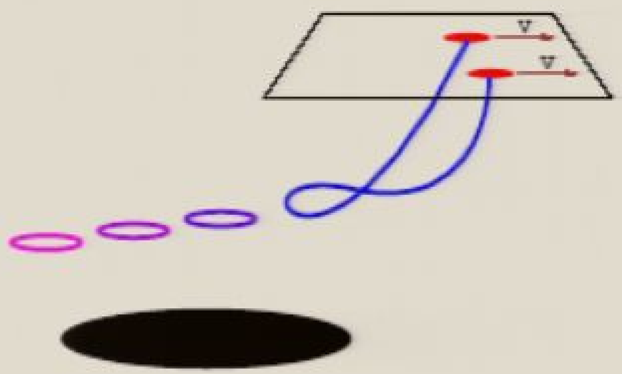
- No stationary dragged string solutions. M. Chernicoff *et. al.*, hep-th/0607089
  Interpretation: color-neutral mesons feel no drag.

- Quasi-static potential at small v; subtract KE. H. Liu *et. al.*, hep-ph/0607062; M. Chernicoff *et. al.*, hep-th/0607089; K. Peeters *et. al.*, hep-th/0606195; E. Cáceres *et. al.*, hep-th/0607233; S. Avramis *et. al.*, hep-th/0609079

- However one computes, the results indicate a non-trivial velocity dependence of the interquark potential in contrast to the lack of drag felt by these quark-antiquark configurations.

3) String looping:

perhaps favored solutions are not stationary, despite stationary boundary conditions:



E.g. development of turbulence in steady flows, resulting in spontaneous breaking of time-translation invariance. *Cf.* Friess et al hep-th/0609137

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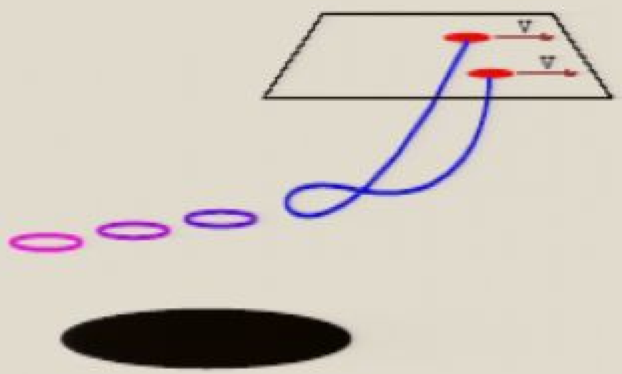

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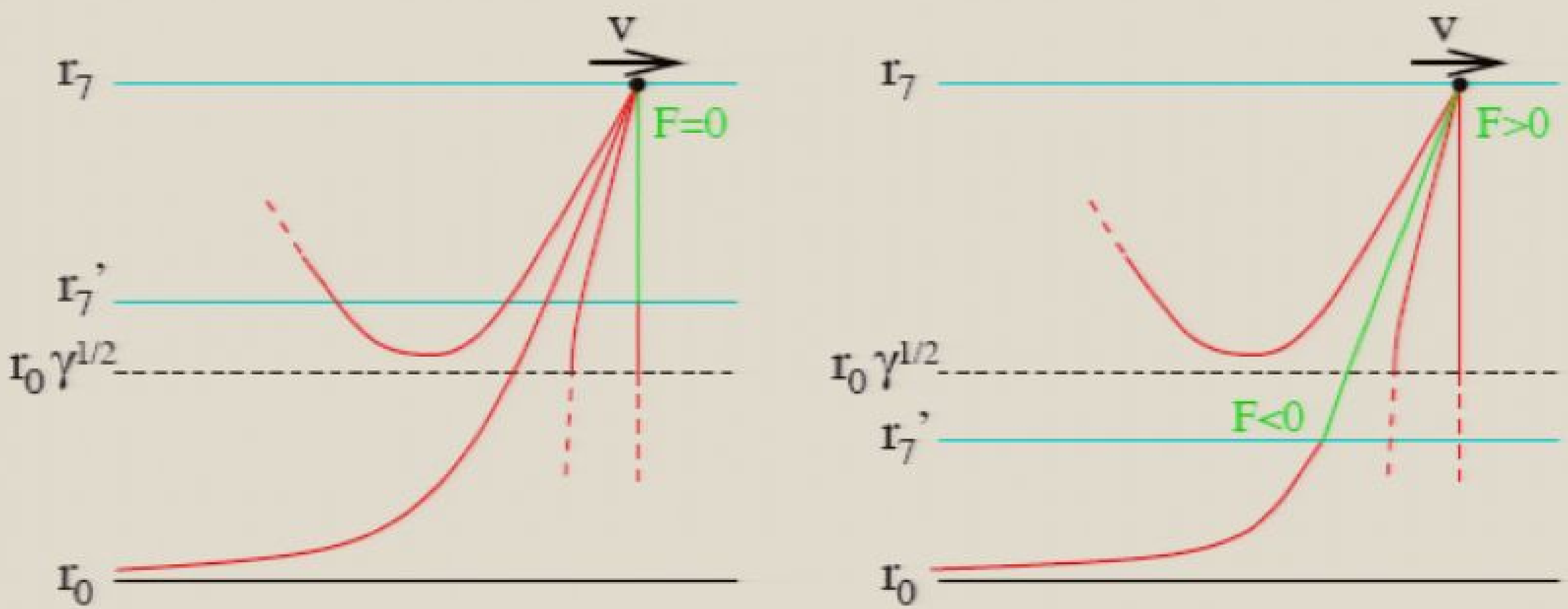

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