Title: Finite - Temperature AdS/CFT Correspondence

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Abstract:

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Motivation

- QCD is weakly coupled at $T \gg \Lambda_{\mathrm{QCD}}$
- In practice, finite-temperature perturbation theory converges very slowly (expansion parameter g instead of α_s).
- There exist gauge theories where the strong coupling regime can be studied analytically using AdS/CFT correspondence
- Hope: learn more about QCD from models with analytic solutions

Zero-Temperature AdS/CFT

AdS/CFT correspondence

Maldacena; Gubser, Klebanov, Polyakov; Witten

between $N = 4$ supersymmetric Yang-Mills theory
and type IIB string theory on $AdS_5 \times S^5$

$$ds^2 = \frac{R^2}{z^2}(d\vec{x}^2 + dz^2) + R^2 d\Omega_5^2$$

Large 't Hooft limit in gauge theory $\Leftrightarrow$ small curvature limit in string theory

$$g^2 N_c = (R/l_s)^4$$

Correlation function are computable at large 't Hooft coupling, where string theory $\rightarrow$ supergravity.

The dictionary of gauge/gravity duality

gauge theory	gravity
operator $\hat{O}$	field ϕ
energy-momentum tensor $T_{\mu\nu}$	graviton $h_{\mu\nu}$
dimension of operator	mass of field
globar symmetry	gauge symmetry
conserved current	gauge field
anomaly	Chern-Simon term
...	...

$$\int e^{iS_{4\mathrm{D}}+\phi_0 O} = \int e^{iS_{5\mathrm{D}}}$$

where $S_{5\mathrm{D}}$ is computed with nontrivial boundary condition

$$\lim_{z\to 0} \phi(\vec{x}, z) = \phi_0(\vec{x})$$

Computing correlators

In the limit $N_c \to \infty$, $g^2 N_c \to \infty$, calculation of correlators reduces to solving classical e.o.m:

$$Z[J] = \int D\phi \, e^{iS[\phi]+i\int JO} = e^{iW[J]}$$

$$W[J] = S_{\text{cl}}[\varphi_{\text{cl}}] : \text{classical action}$$

φ_{cl} solves e.o.m., $\varphi_{\text{cl}}|_{z\to 0} \to J(x)$.

Example: correlator of R-charge currents

- R-charge current in 4D corresponds to gauge field in 5D
- Field equation for transverse components of gauge fields is

$$\partial_\mu(\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta}) = 0$$

- In the gauge $A_z = 0$, equation for transverse and longitudinal parts of A_μ decouple. The equation for the transverse part is

$$\partial_z \left(\frac{1}{z} \partial_z A_\perp(z,q) \right) - \frac{q^2}{z} A_\perp = 0 \Rightarrow A_\perp(z,q) = QzK_1(Qz)A_\perp(0,q)$$

Computing correlators (continued)

Two-point correlator = second derivative of classical action over boundary values of fields

$$\langle j^\mu j^\nu \rangle \sim \frac{\delta^2 S_{\text{Maxwell}}}{\delta A_\perp(0)^2} = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \frac{1}{z} \frac{\partial}{\partial z} [\underbrace{QzK_1(Qz)}_{1+Q^2z^2\ln(Qz)}]$$

$$= \#(g^{\mu\nu} q^2 - q^\mu q^\nu) \ln Q^2$$

This structure is the consequence of conformal symmetry, but the numerical coefficient can be checked with diagrammatic calculations (nonrenormalization theorem).

Similarities and differences

between $\mathcal{N} = 4$ SYM and QCD

Similarities:

- Are both gauge theories

Differences:

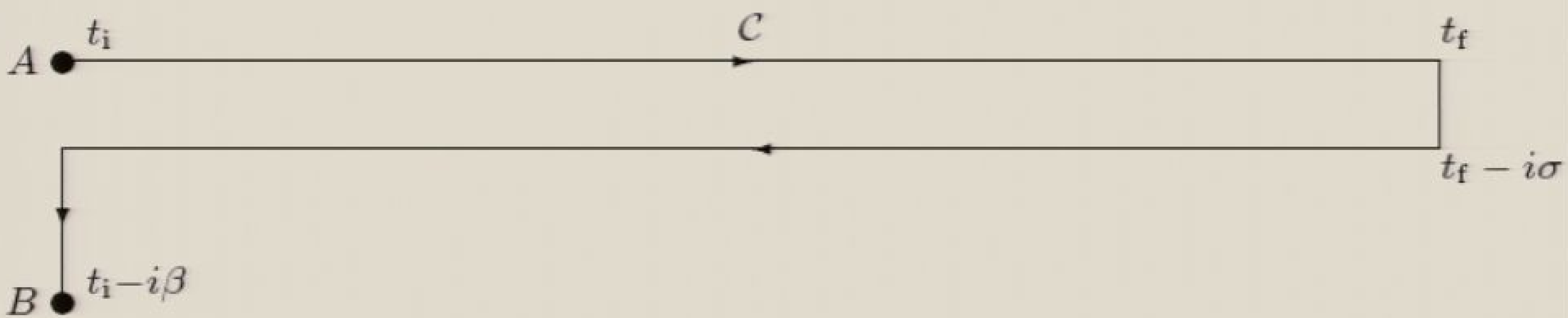
- Matter content: $\mathcal{N} = 4$ SYM contains adjoint fermions, scalars
- 't Hooft coupling $\lambda = g^2 N$ does not run in $\mathcal{N} = 4$ SYM, but runs in QCD (Λ_{QCD} is a parameter of QCD).
- The supergravity approximation of AdS/CFT works only in the strong coupling limit $\lambda \to \infty$, and large N.

Finite-temperature AdS/CFT correspondence

Field theory at finite temperature

Two formulations

- Euclidean formulation: periodic Euclidean time $\tau \sim \tau + \beta$.
  Suitable for lattice calculations
- Close-time-path (Schwinger-Keldysh) formulation



- One can turn on sources on the whole contour: J_1 on upper part, J_2 on lower part. Propagator is 2×2 matrix: G_{ab}, $a, b = 1, 2$

Different choices of σ

Choice of σ arbirary: changing σ rescales G_{12}, G_{21} by $e^{\#\omega}$, leaving G_{11}, G_{22} unchanged

Popular choices:

- $\sigma = 0$ (Keldysh): retarded Green's function has a simple form $G_R = G_{11} - G_{12} = G_{21} - G_{22}$
- $\sigma = \beta/2$ (Niemi-Semenoff): the propagator is symmetric, $G_{12} = G_{21}$

All 2-point functions can be expressed through the retarded one by fluctuation-dissipation theorem.

Black hole

Black 3-brane solution:

$$ds^2 = \frac{r^2}{R^2}[-f(r)dt^2 + d\vec{x}^2] + \frac{R^2}{r^2 f(r)}dr^2 + R^2 d\Omega_5^2, \qquad f(r) = 1 - \frac{r_0^4}{r^4}$$

- $r_0 = 0$, $f(r) = 1$: is $AdS_5 \times S^5$, $r = R^2/z$.
- $r_0 \neq 0$: corresponds to $\mathcal{N} = 4$ SYM at temperture

$$T = T_H = \frac{r_0}{\pi R^2}$$

Entropy density

Entropy $= A/4G$

A is the area of the event horizon
G is the 10D Newton constant.

$$A = \int dx\, dy\, dz\, \sqrt{g_{xx} g_{yy} g_{zz}} \times \underbrace{\pi^3 R^5}_{\text{area of } S^5} = V_{3D} \pi^3 r_0^3 R^2 = \pi^6 V_{3D} R^8 T^3$$

On the other hand, from AdS/CFT dictionary

$$R^4 = \frac{\sqrt{8\pi G}}{2\pi^{5/2}} N_c$$

Therefore

$$S = \frac{\pi^2}{2} N_c^2 T^3 V_{3D}$$

This formula has the same N^2 behavior as at zero 't Hooft coupling $g^2 N_c = 0$ but the numerical coefficient is 3/4 times smaller.

Thermodynamics

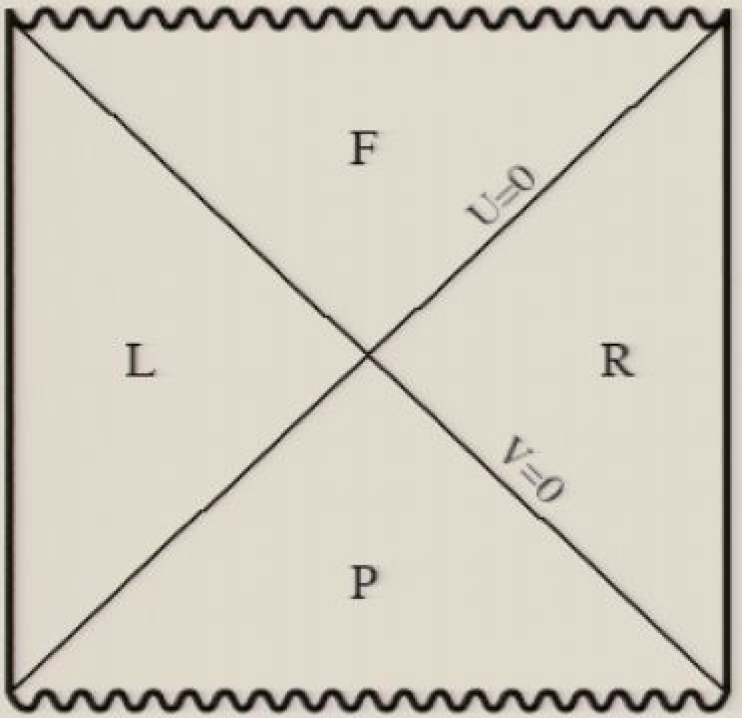
$$S = f(g^2 N_c)\frac{2\pi^2}{3} N_c^2 T^3 V_{3D}$$

where the function f interpolates between weak-coupling and strong-coupling values, which differ by a factor of 3/4:

$$f(\lambda) = \begin{cases} 1 - \frac{3}{2\pi^2}\lambda + \frac{\sqrt{2}+3}{\pi^3}\lambda^{3/2} + \cdots, & \lambda \ll 1 \\ \frac{3}{4} + \frac{45\zeta(3)}{32\lambda^{3/2}} + \cdots, & \lambda \gg 1 \end{cases} \quad (1)$$

Thermal correlators from AdS/CFT

Maldacena; Herzog and Son



- Penrose diagram of AdS black hole: two boundaries
- Identify boundary values at the 2 boundaries with the 2 sources on the 2 parts of the CTP contour
- L and R quadrants know about each other through the boundary condition at the horizon, which are conveniently formulated in Kruskal coordinates
  - incoming positive frequency modes
  - outgoing negative frequency modes,

Clearly L and R quadrants are symmetric: corresponds to $\sigma = \beta/2$ in the CTP contour.

For 2-point function: one can compute retarded Green's function by solving equation in one quadrant (R) with incoming-wave boundary condition at the horizon.

Hydrodynamics

Nice thing about real-time finite-temperature field theory: universal low-energy effective theory, hydrodynamics

- Valid in the hydrodynamic regime: at distances $\gg$ mean free path, time $\gg$ mean free time.
- At these length/time scales: local thermal equilibrium: T, μ vary slowly in space.
- Simplest example: relativistic plasma with no conserved charge

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} - \sigma^{\mu\nu}$$

$$\sigma^{\mu\nu} = P^{\mu\alpha} P^{\nu\beta} [\eta(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \tfrac{2}{3} g_{\alpha\beta} \nabla \cdot u) + \zeta g_{\alpha_\beta} \nabla \cdot u]$$

$(P^{\mu\alpha} = g^{\mu\alpha} + u^\mu u^\alpha)$

- All microscopic physics reduces to EOS and a small number of *kinetic coefficients* (shear viscosity η, bulk viscosity ζ).

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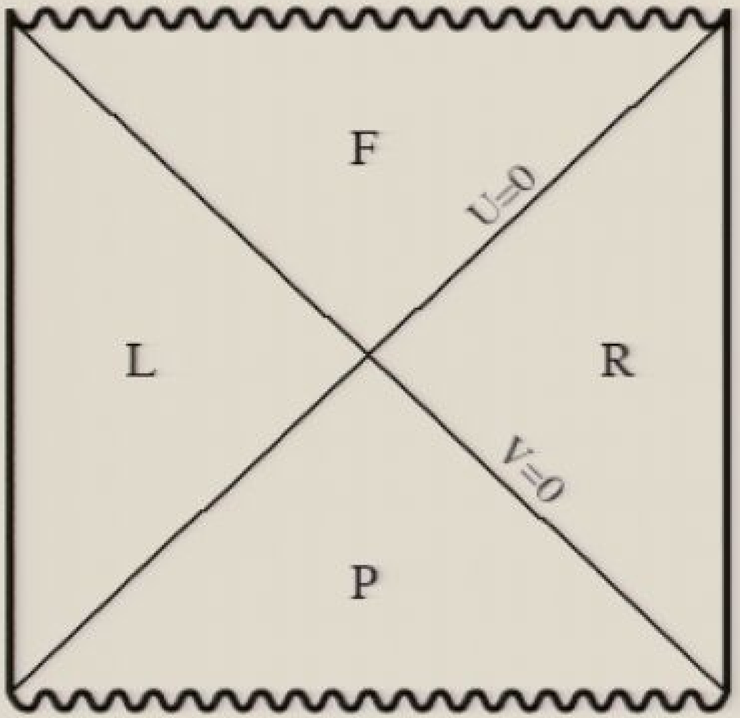

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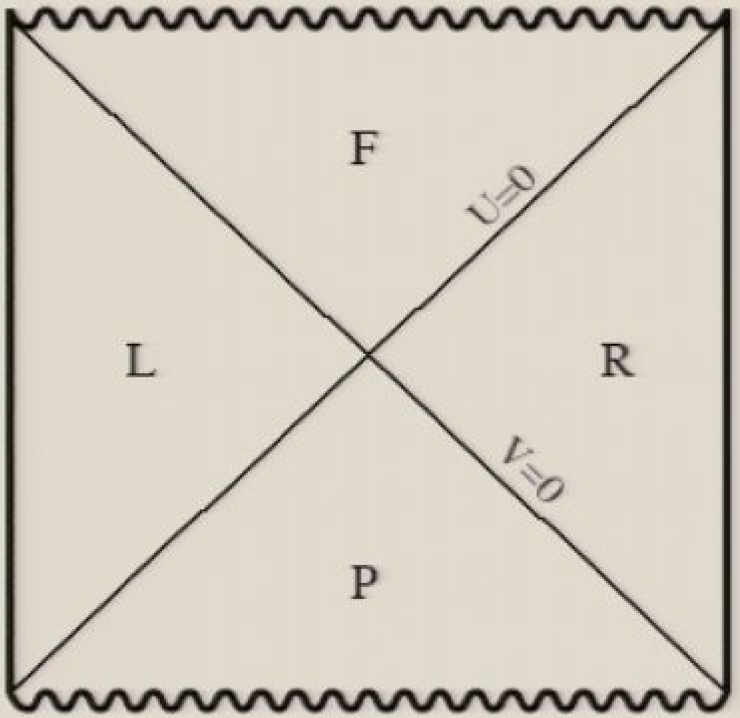

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Kubo's formulas for viscosities

Viscosities can be expressed in terms of Green's functions by Kubo's formula

- Hydrodynamics = effective theory describing response of a system to external long-distance perturbations.
- Example of such perturbation: gravitational waves

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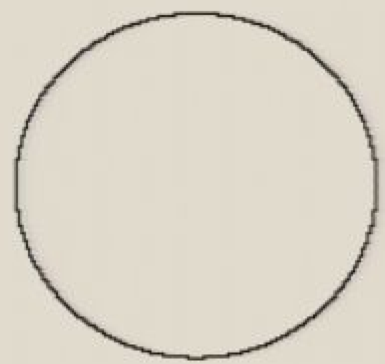

Derivation of Kubo's formula

Let us consider an external perturbation, uniformed in space but time-dependent:

$$g_{ij} = \delta_{ij} + h_{ij}(t), \quad h_{ii} = 0$$

Assuming $h_{ij} \ll 1$, let us discuss the linear response of the system to such a perturbation.
Tensor mode $\rightarrow T =$ const, $u^\mu = (1, 0, 0, 0)$.
The only nontrivial contribution: from Christoffel symbols

$$T^{ij} = \cdots - \eta(\nabla_i u_j + \nabla_j u_i)$$

but

$$\nabla_i u_j = \underbrace{\partial_i u_i}_{=0} - \Gamma^0_{ij} u_0 = \partial_t h_{ij}$$

So in the respsose to the gravitational perturbation is

$$T^{ij} = -P h_{ij} + \eta \partial_t h_{ij}$$

Derivation of Kubo's formula (continued)

However, linear response theory tells us that

$$\langle T^{\mu\nu}(x)\rangle = \frac{1}{2}\int dy\, \langle T^{\mu\nu}(x) T^{\lambda\rho}(y)\rangle_R h_{\lambda\rho}(y)$$

from which we find

$$\langle T^{xy} T^{xy}\rangle(\omega, \mathbf{0}) = -i\eta\omega + \text{real contact terms}$$

from which follows the Kubo formula:

$$\eta = -\lim_{\omega\to 0} \operatorname{Im} G_R^{xy,xy}(\omega, \mathbf{0})$$

AdS/CFT duality and hydrodynamics

We want to use AdS/CFT correspondence to explore the hydrodynamic regime of thermal gauge theory.

- Finite-T QFT ⇔ black hole with translationally invariant horizon

$$ds^2 = \frac{r^2}{R^2}(-f dt^2 + d\vec{x}^2) + \frac{R^2}{r^2}(\frac{dr^2}{f} + r^2 \partial\Omega_5^2)$$

horizon: $r = r_0$, $\vec{x}$ arbitrary.

- Local thermal equilibrium ⇔ parameters of metric (e.g., r_0) slowly vary with $\vec{x}$.

  remember that $T \sim r_0/R^2$.

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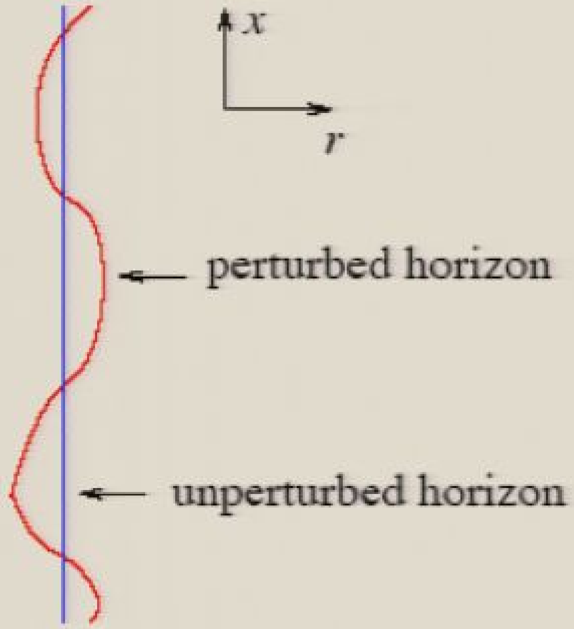
Dynamics of the horizon



$T \sim r_0 = r_0(\vec{x})$

Generalizing black hole thermodynamics M, Q,... to black brane hydrodynamics
$T = T_H(\vec{x})$, $\mu = \mu(\vec{x})$

Gravity counterpart of Kubo's formula

One can use the standard AdS/CFT prescription to compute the correlation function in Kubo's formula

Klebanov; Policastro, Son, Starinets: $\mathrm{Im}G^R$ is proportional to the absorption cross section by the black hole.

$$\sigma_{\mathrm{abs}} = -\frac{16\pi G}{\omega} \mathrm{Im}\, G^R(\omega)$$

That means viscosity = absorption cross section for low-energy gravitons

$$\eta = \frac{\sigma_{\mathrm{abs}}(0)}{16\pi G}$$

The absorption cross section can be found classically.

There is a theorem that the cross section at $\omega = 0$ is equal to the area of the horizon.

But the entropy is also proportional to the area of the horizon

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

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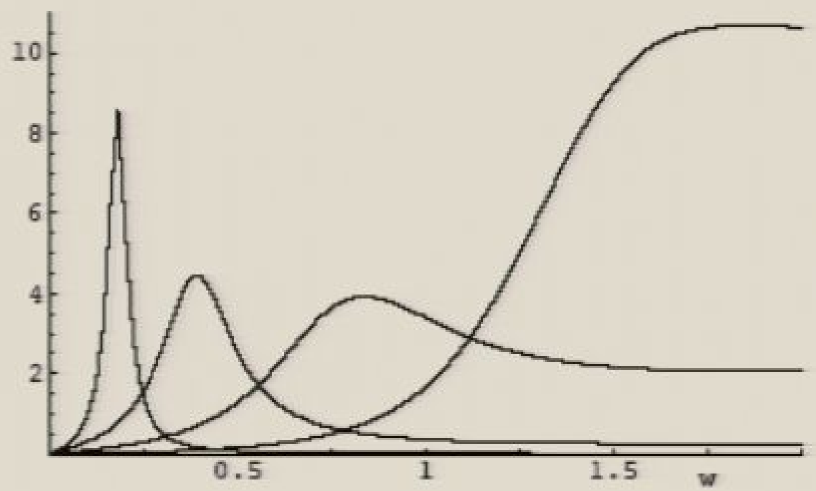
Remarks

- Hydrodynamic modes can be seen directly from correlators. E.g. spectral density of T_{00} correlator (Kovtun, Starinets)



Curves correspond to $q/2\pi T = 0.3, 0.6, 1.0, 1.5.$

- The proof of universality of η/s above is intuitive but has a limited range of applicability (metric can be extended to Minkowski flat)
- More general proofs are available (Buchel, Liu; Buchel)
- $\eta/s = 1/4\pi$ also when chemical potentials are turned on (Benincasa et al.)
- Correction $\sim 1/\lambda^{3/2}$ (Buchel, Liu, Starinets)

Viscosity/entropy ratio and uncertainty principle

Estimate of viscosity from kinetic theory

$$\eta \sim \rho v \ell, \qquad s \sim n = \frac{\rho}{m}$$

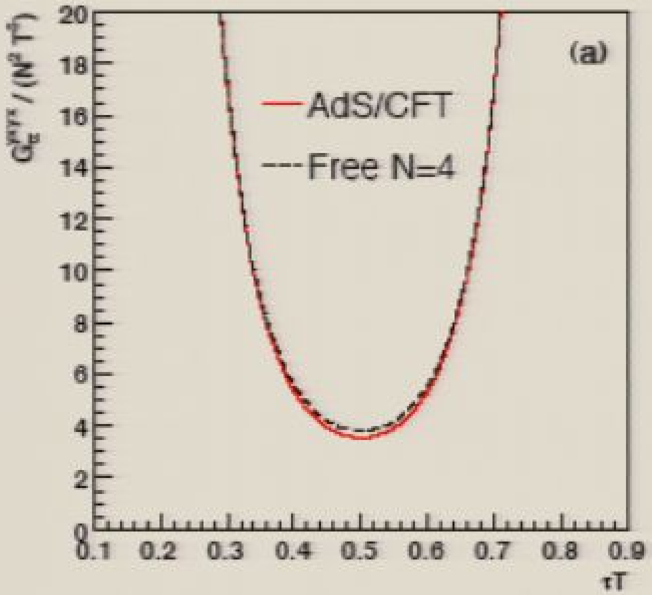
$$\frac{\eta}{s} \sim m v \ell \sim \hbar \frac{\text{mean free path}}{\text{de Broglie wavelength}}$$

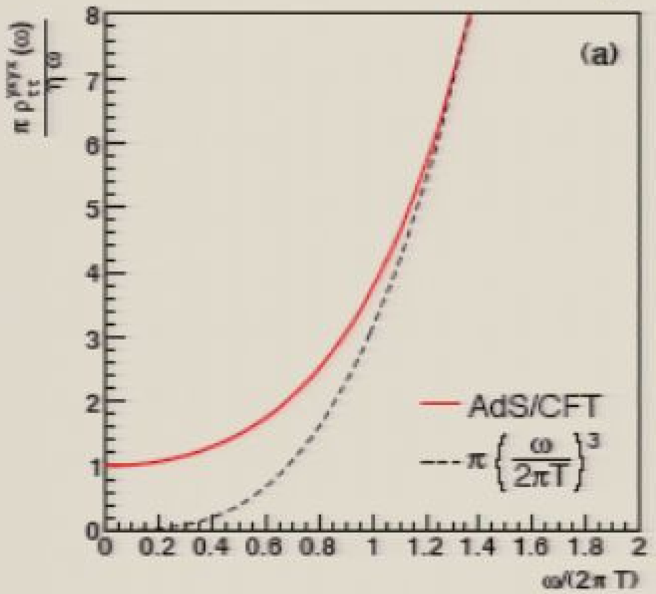
Quasiparticles: de Broglie wavelength $\lesssim$ mean free path

Therefore $\eta/s \gtrsim \hbar$

- Weakly interacting systems have $\eta/s \gg \hbar$.
- Theories with gravity duals have universal η/s, but we don't know how to derive the constancy of η/s without AdS/CFT.

Can one find η of QGP from lattice QCD?





- Lattice QCD: Euclidean-time formulation
- Teaney: compare the Euclidean correlation function

$$\int d\vec{x}\,\langle T_{xy}(\tau,\vec{x})T_{xy}(0,\mathbf{0})\rangle$$

at $g^2N_c=\infty$ and $g^2N_c=0$

- Difference at 10% level
- at the same time the spectral densities of free and strongly coupled theories differ a lot at low ω
- Needs high precision lattice data
- Going from Euclidean correlator to spectral function requires prior knowledge of the latter.

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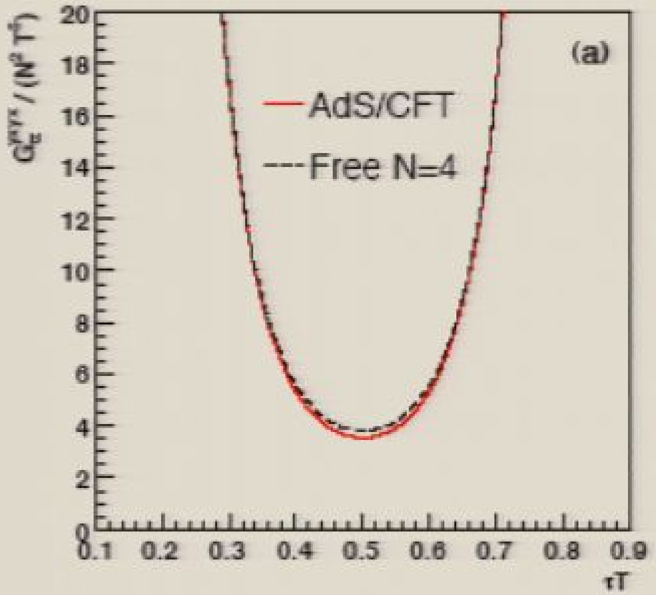

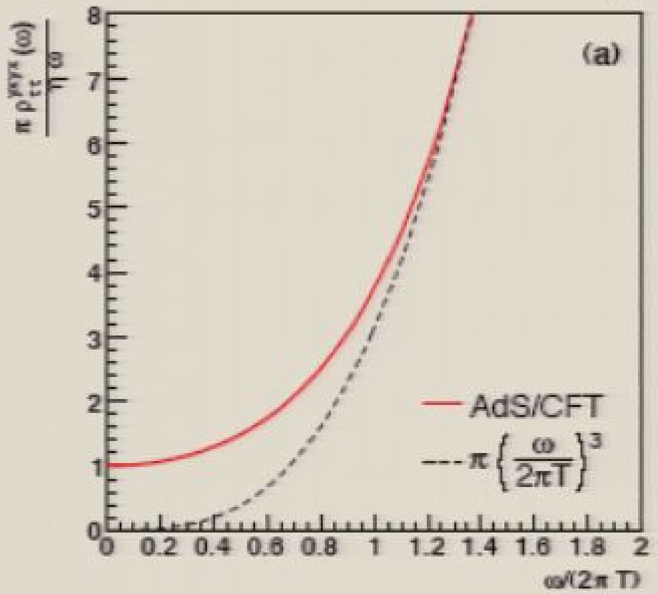

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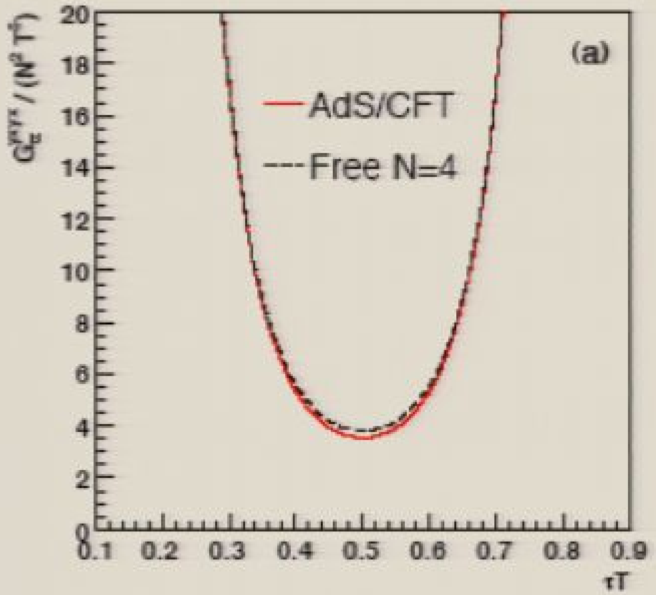

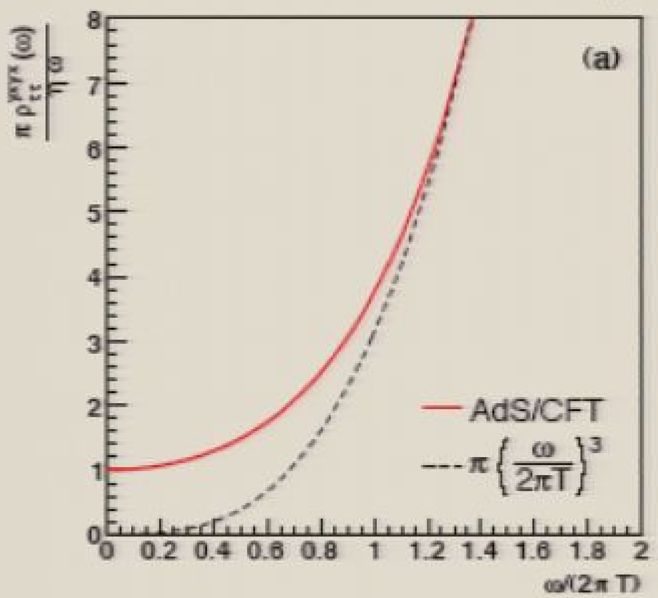

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Bulk viscosity

In conformal field theories bulk viscosity is zero: $\langle T^{\mu}_{\mu} \rangle = 0$ while

$$T_{\mu\nu} = \cdots + \zeta(g_{\mu\nu} + u_\mu u_\nu)\nabla u$$

However one can break conformal invariance in $\mathcal{N} = 4$ SYM (introducing masses to fermions and scalars).
Gravitational description are more complicated, but well defined $(\mathcal{N} = 2^*)$
Benincasa, Buchel, Starinets:

$$\zeta \sim -\#(v_s^2 - \frac{1}{3})$$

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Reverse information flow?

So far we have tried to extract information about gauge theories from gravity
Is there any consequence for gravity/string theory that can be obtained from gauge theory?

- Large η/s at weak coupling: absorption cross section $\gg$ horizon area if AdS radius $\gg$ string length?
  Who needs that?
- Nonanalytic next corrections to hydrodynamic correlators (Kovtun, Yaffe):

$$G^{xy,xy}(\omega, 0) = -i\eta\omega + \frac{\#}{N}\omega^{3/2}$$

  coefficient calculation calculable in the framework of hydrodynamics.
  Infrared dominated graviton loop in gravity description.
- Maybe something for rotating (in S^5) black hole, dual to finite R-charge chemical potential?

Finite chemical potential

- $\mathcal{N} = 4$ SYM theory has SO(6) global conserved charges.
- Dual description: black hole rotating in S^5, solution exactly known.
- It is know from gravity that the black hole is unstable for if $\mu/T >$ some critical value
- Can be understood from field theory perspective: existence of flat directions carrying R-charge

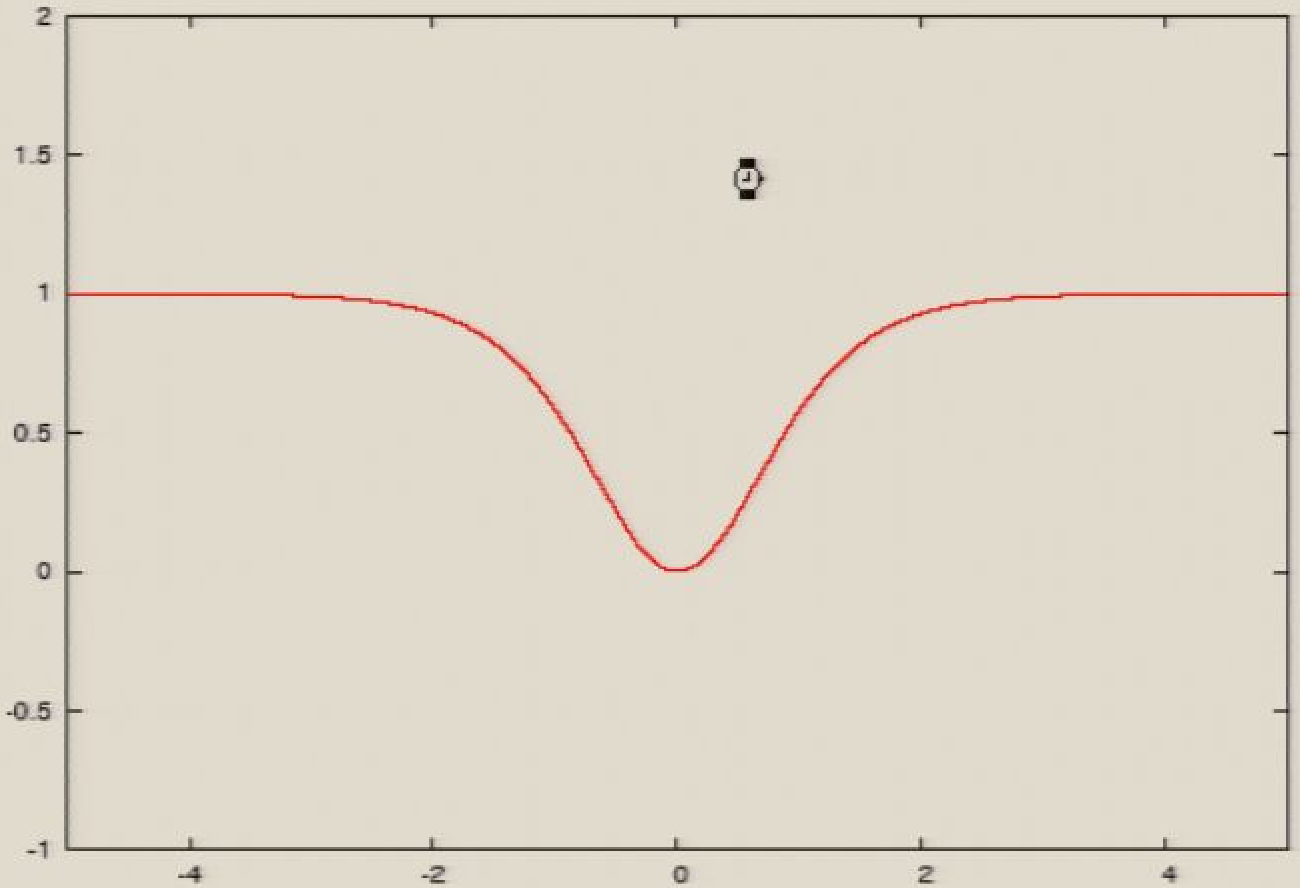
At finite μ, zero temperature:

$$L = |(\partial_0 - i\mu)\phi|^2 - V(|\phi|) \Rightarrow V_{\text{eff}}(|\phi|) = V(|\phi|) - \mu^2|\phi|^2$$

for flat directions $V(|\phi|) = 0$, V_{eff} unbounded from below
Finite temperature: flat directions are lifted, thermal mass for ϕ, needs finite μ for $\phi = 0$ to become a maximum.
But let us look more closely at the effective potential

Effective potential at finite T

$V_{\text{eff}}(|\phi|, T)$ goes to a constant at $|\phi| \to \infty$
Roughly speaking, at $|\phi| \to \infty$ some d.o.f. become infinitely massless and do not contribute to pressure ($P \downarrow$, free energy $\uparrow$) but *the number of such modes is finite*



Finite chemical potential

- $\mathcal{N}=4$ SYM theory has SO(6) global conserved charges.
- Dual description: black hole rotating in S^5, solution exactly known.
- It is know from gravity that the black hole is unstable for if $\mu/T >$ some critical value
- Can be understood from field theory perspective: existence of flat directions carrying R-charge

At finite μ, zero temperature:

$$L = |(\partial_0 - i\mu)\phi|^2 - V(|\phi|) \Rightarrow V_{\text{eff}}(|\phi|) = V(|\phi|) - \mu^2|\phi|^2$$

for flat directions $V(|\phi|) = 0$, V_{eff} unbounded from below
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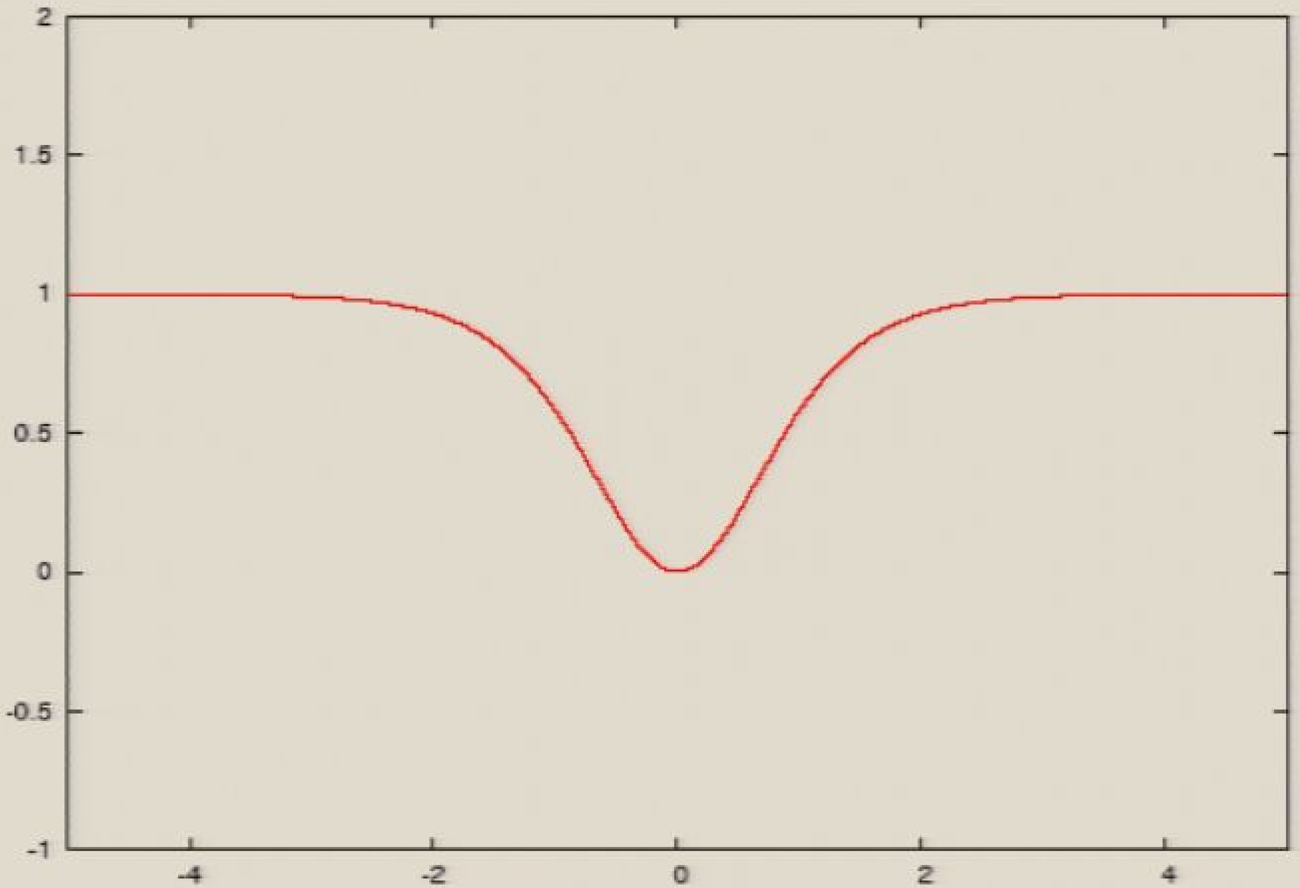

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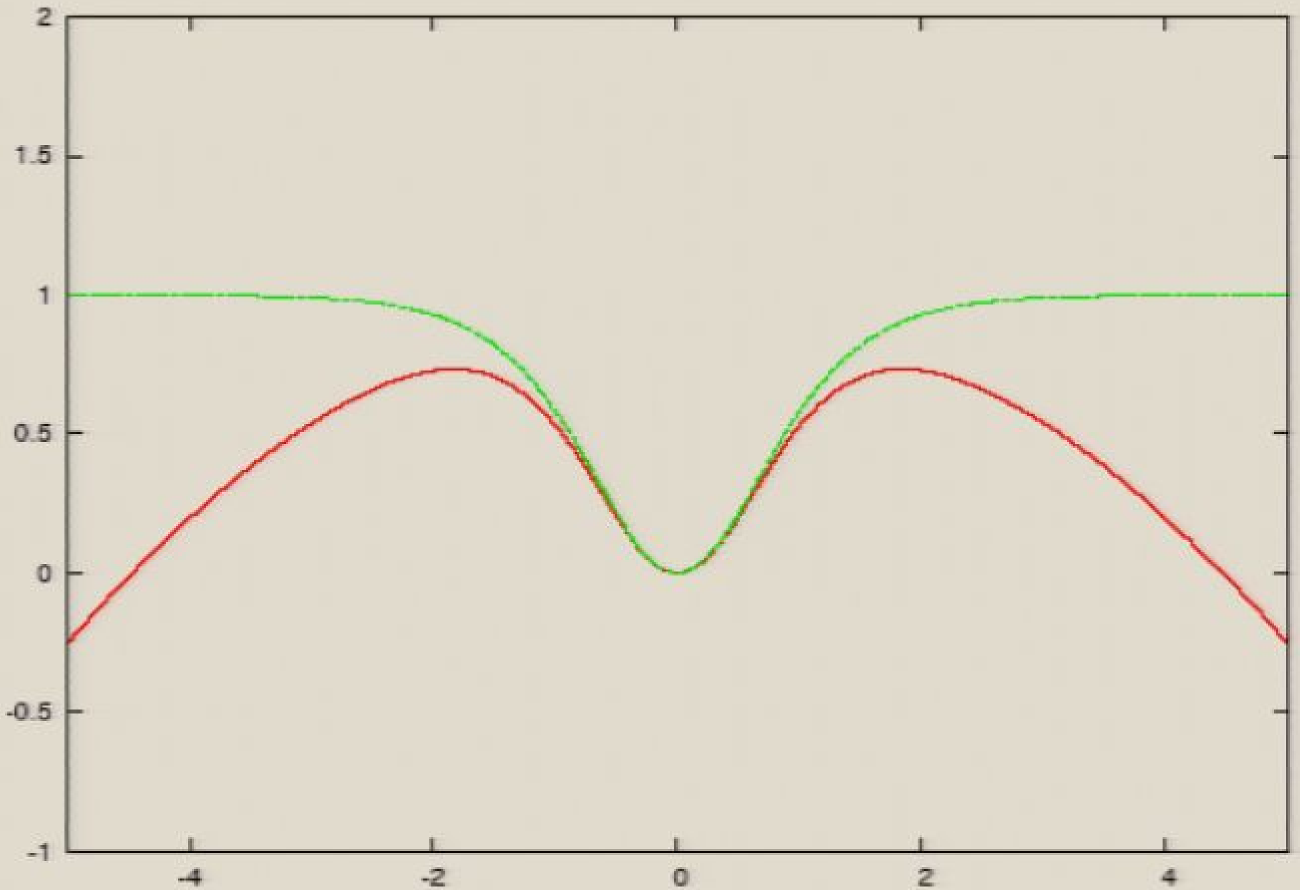

Any finite μ makes $\phi = 0$ globally unstable

Fate of rotating black holes

So we come to a conclusion that rotating black holes can only be metastables. How do they decay is not clear...

Wish list

- Cross-over or 2nd order deconfinement
  In confining theories with gravity duals, the deconfiment phase transition is first order (large N): black hole horizon appears suddenly, with nonzero area
- Description of non-equilibrium processes
  like heavy ion collisions
- Gravity dual of cold phases of quark matters (e.g., Fermi liquid)
- Gravity dual of nonrelativistic theory with no scale parameters: fermions with zero-range interaction fine-tuned to infinite scattering length

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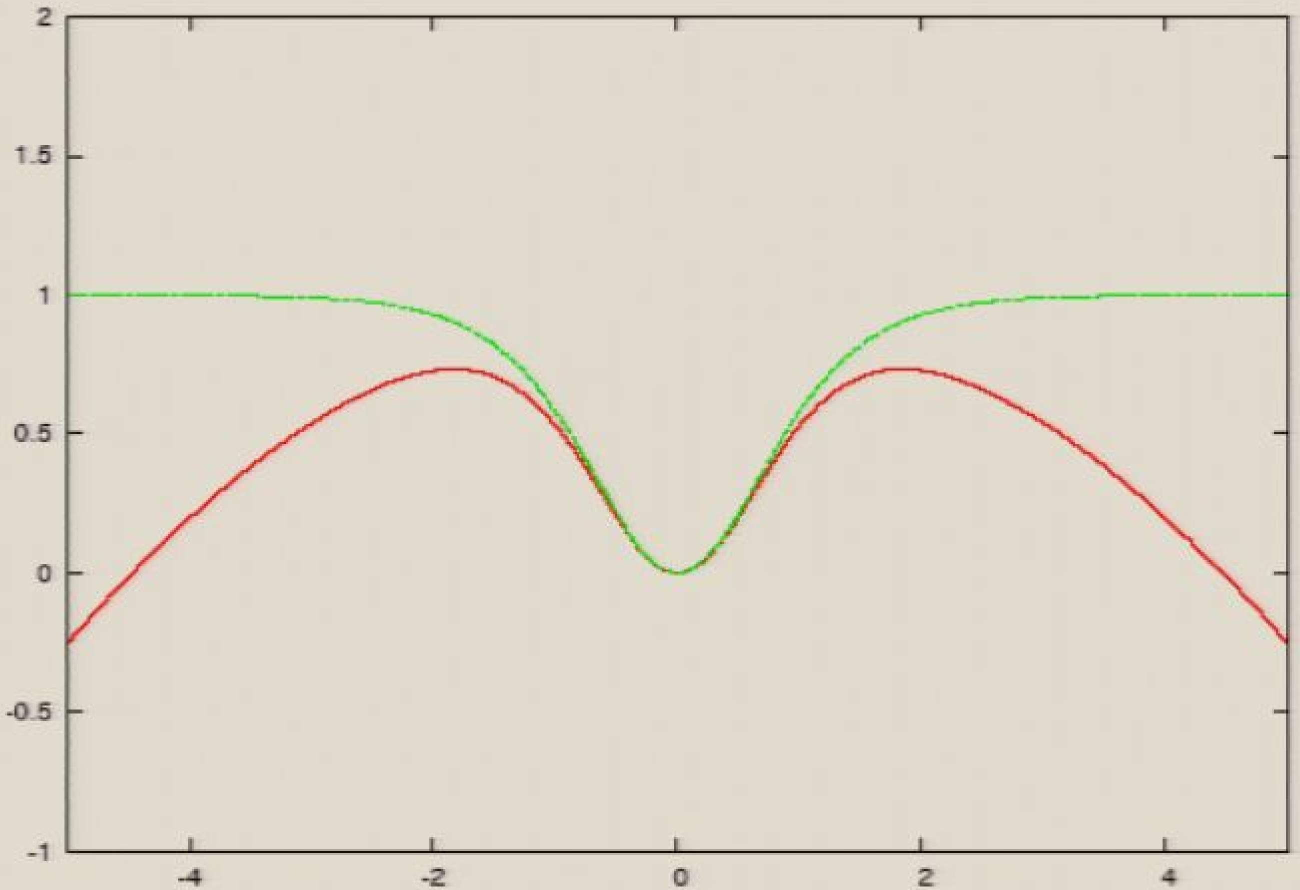

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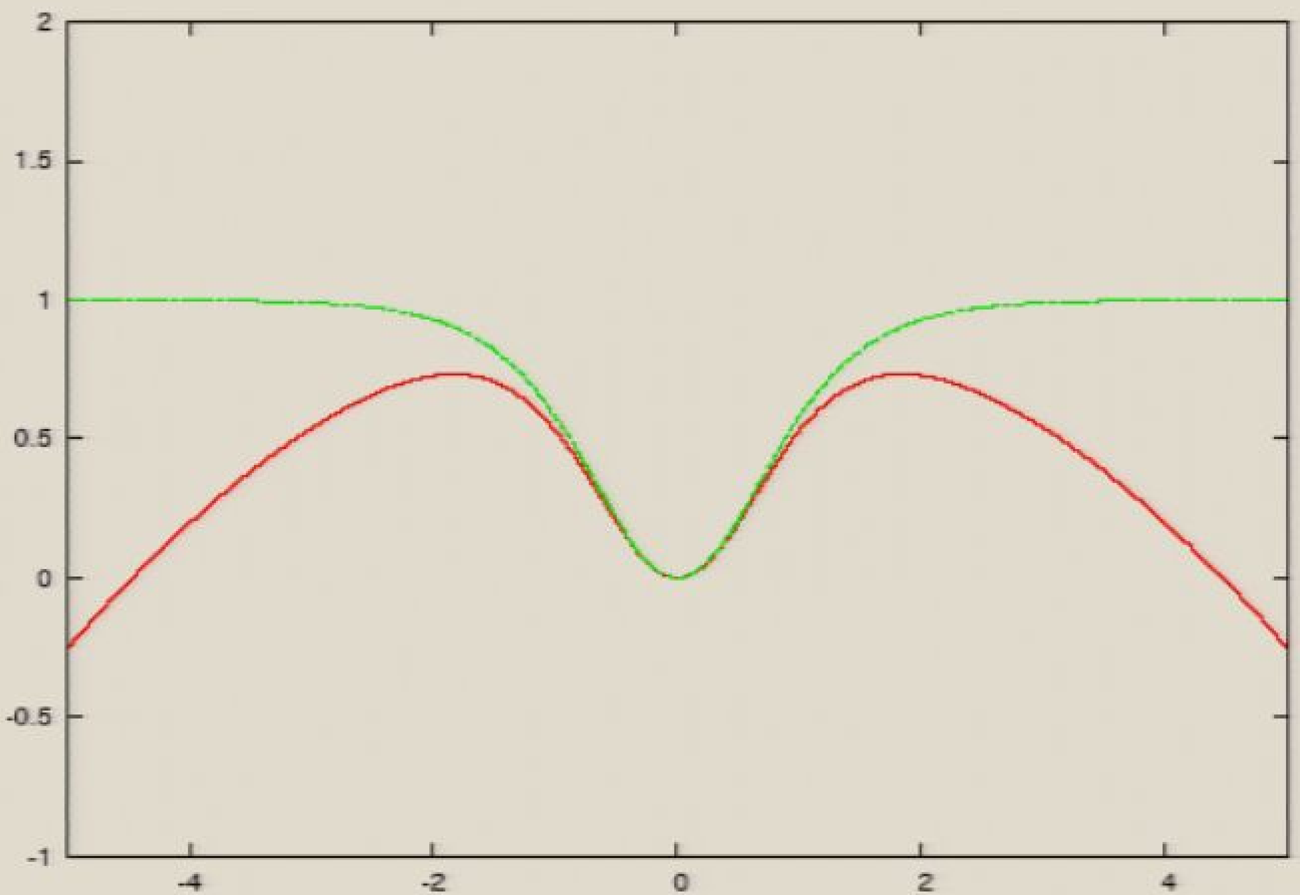

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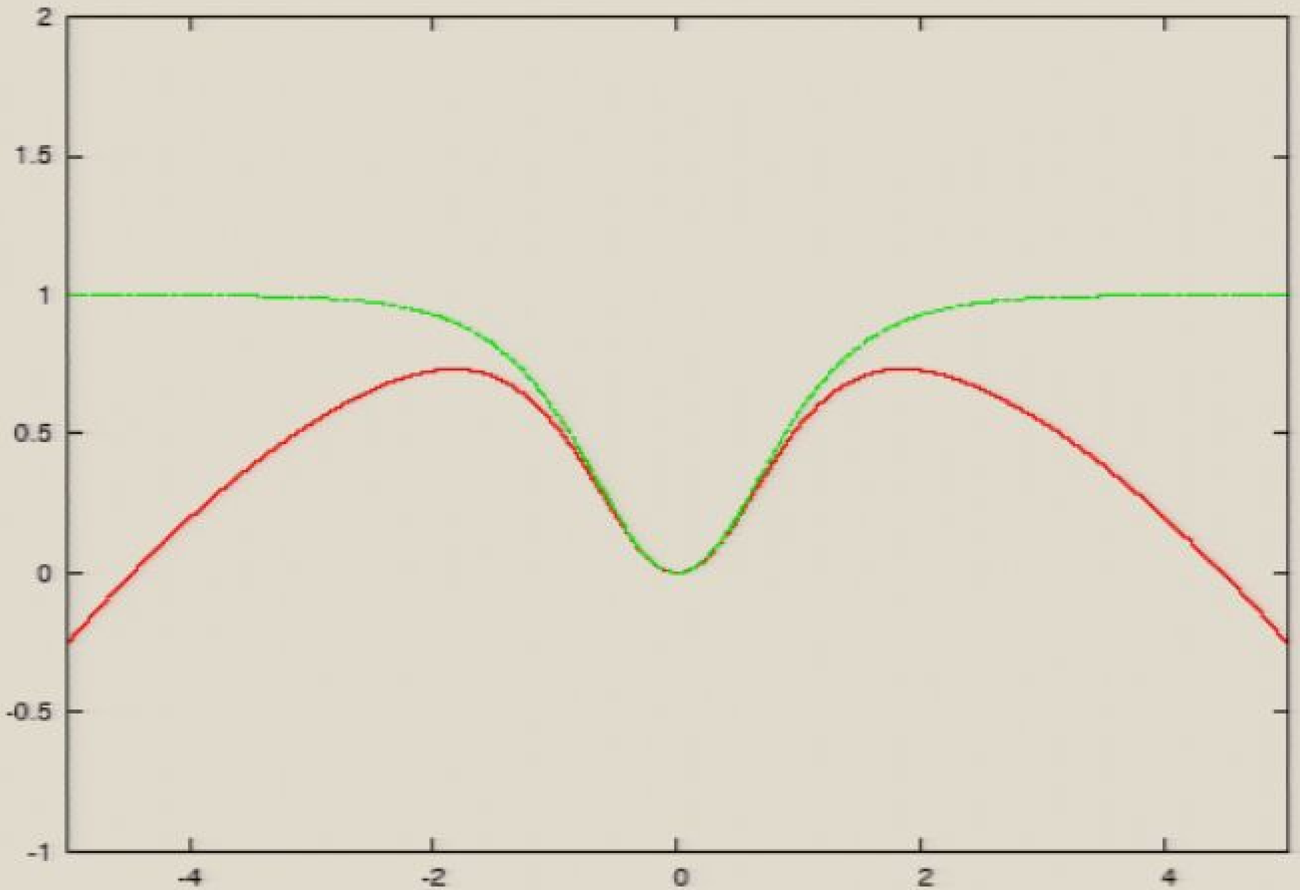

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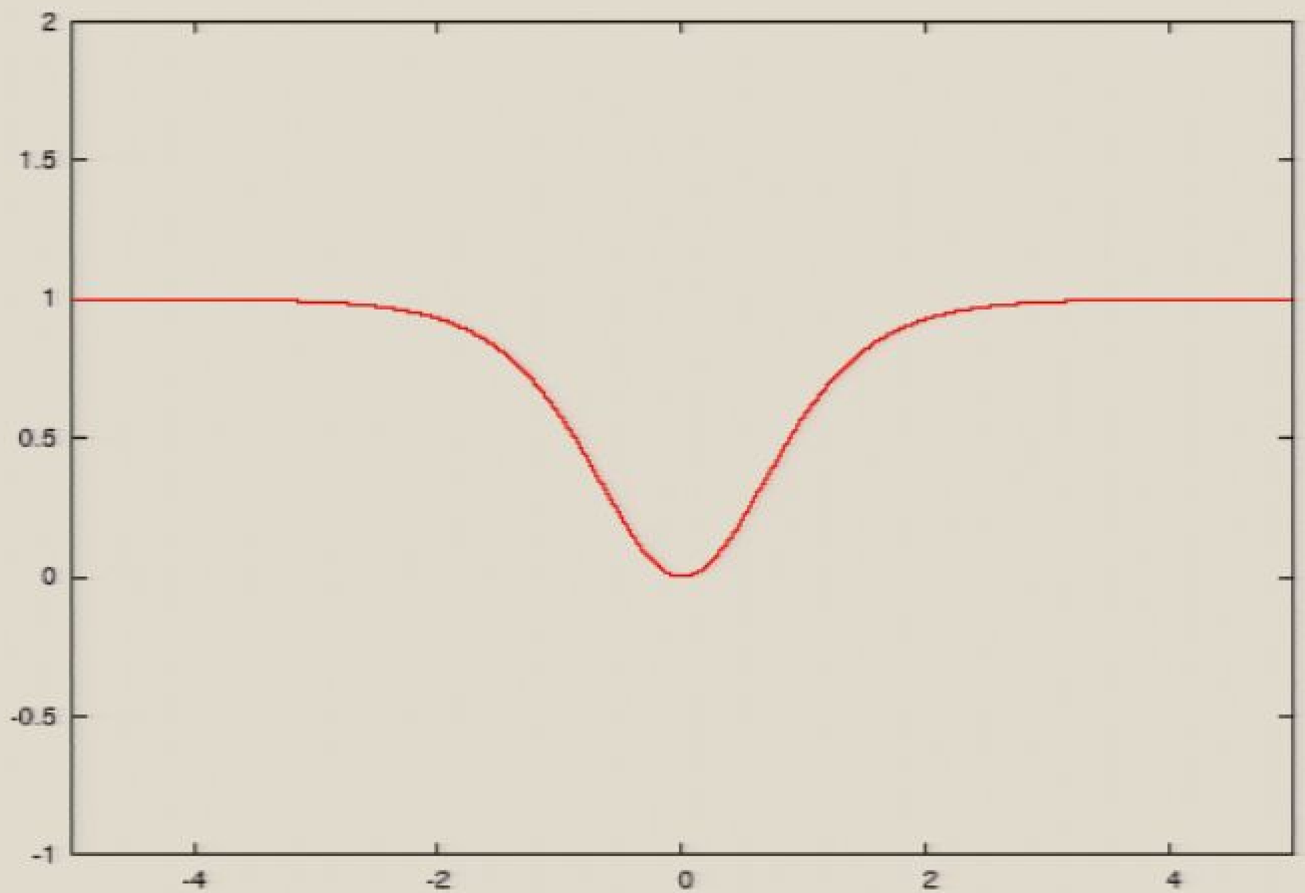

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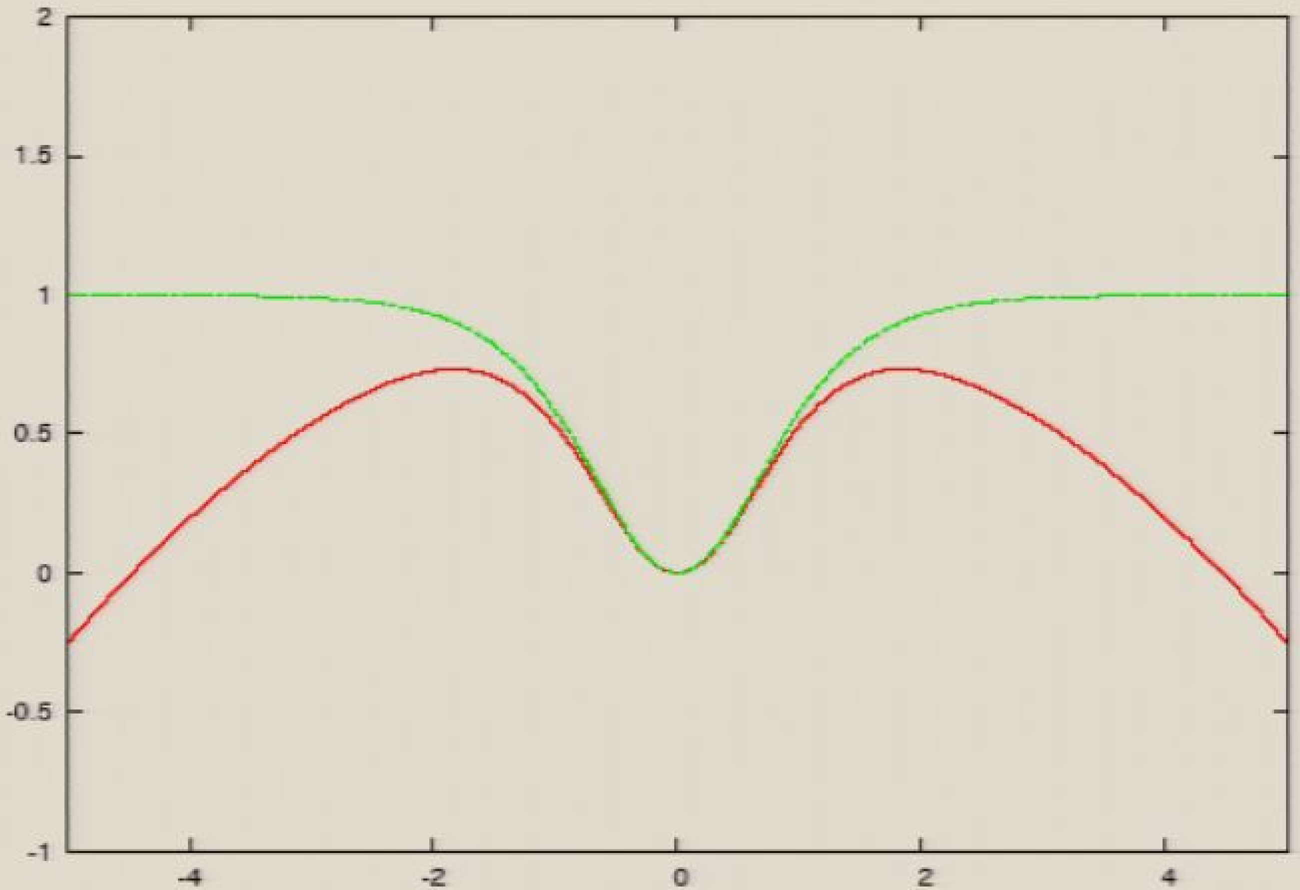

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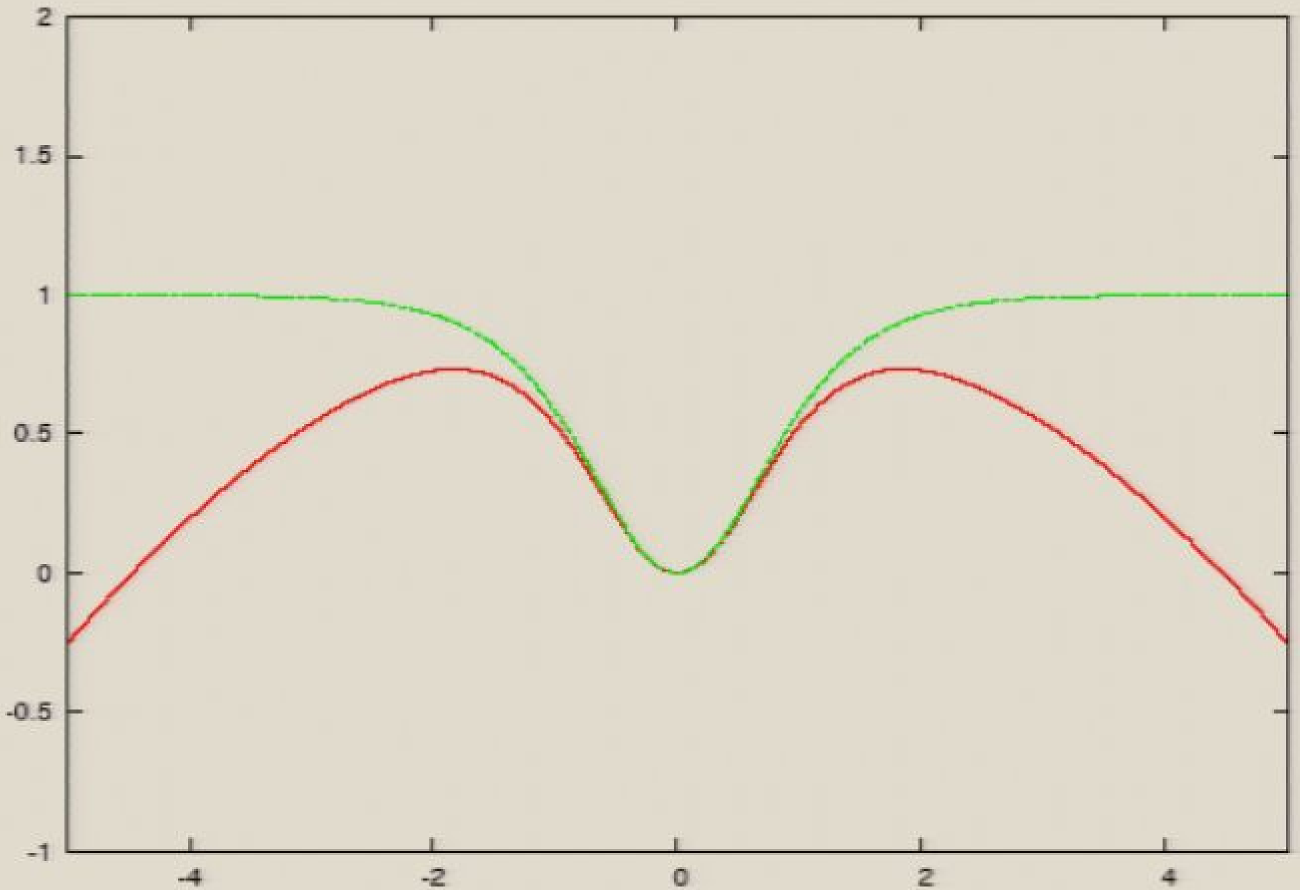

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